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NELSON J. GROOM

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National Aeronautics and Space Administration

Langley Research Center
Hampton, Virginia 23665

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FIXED-BASE AND TWO-BODY EQUATIONS OF MOTION FOR AN ANNULAR MOMENTUM CONTROL DEVICE (AMCD)

By Nelson J. Groom
Langley Research Center

SUMMARY

Fixed-base and two-body equations of motion for an Annular Momentum Control Device (AMCD) are presented. An AMCD consists of a spinning annular rim which is suspended by noncontacting magnetic bearings and powered by a noncontacting linear electromagnetic motor. The fixed-base equations are for a rigid AMCD rim suspended by magnetic bearings attached to a rigid fixed-base and the two-body equations are for a rigid AMCD rim suspended by magnetic bearings attached tc a rigid body spacecraft. The fixed-base equations are applicable to any potential ground based AMCD application such as energy storage.

INTRODUCTION

The Annular Momentum Control Device (AMCD) represents a new concept in momentum storage devices (ref. 1), and is proposed for the control and stabilization of spacecraft and other applications requiring momentum storage (ref. 2). Aside from the advantages of the AMCD as a momentum storage device, it appears that the concept may also have significant advantages as an energy storage device for Earth-based applications (refs. 3-5). The basic AMCD concept (shown schematically in fig. 1) consists of a spinning annular rim which is suspended by a minimum of three noncontacting magnetic bearing suspension stations and driven by a noncontacting linear electromagnetic drive motor. The magnetic suspension stations consist of electromagnetic actuators which produce axial and radial control forces on the rim. The electromagnetic actuators are controlled by servos which use rim position sensors to obtain the required relative displacement information. The purpose of this paper is to present a set of fixed-base and two-body equations of motion for an AMCD which can be utilized for the analysis of suspension system control concepts for Earth-based and spacecraft applications. The fixed-base equations are for a rigid AMCD rim suspended by magnetic bearings attached to a rigid fixed-base and the two-body equations are for a rigid AMCD rim suspended by magnetic bearings attached to a rigid body spacecraft.

LIST OF SYMBOLS

$\{D\}$	Initial displacement of the AMCD axis system from the inertially fixed 1_I, 2_I, 3_I axis system.
$\{E_S\}$	External disturbance torque acting on the spacecraft written in the S system.
$\{F_A\}$	Translational force on the AMCD rim written in the A system.
$\{F_{RA}\}$	Radial force on the AMCD rim produced by a given magnetic bearing station.
$\{F_{RS}\}$	Radial force on the spacecraft produced by a given magnetic bearing station.
$\{F_S\}$	Translational force on the spacecraft written in the S system.
$\{F_{XA}\}$	Axial force on the AMCD rim produced by a given magnetic bearing station.
$\{F_{XS}\}$	Axial force on the spacecraft produced by a given magnetic bearing station.
F_{R1}, F_{R2}	Radial forces on the AMCD rim resolved along the 1_A and 2_A axes, respectively.
$\{G_A\}$	Total moment acting about the center of mass of the AMCD rim written in the A system.
$\{G_S\}$	Reaction torque on the spacecraft caused by motion of the AMCD rim acting through the magnetic bearings written in the S system.
$\{g\}$	Relative displacement in inertial coordinates of the AMCD rim with respect to the spacecraft at a given bearing station.
$\{g_S\}$	$\{g\}$ transformed into spacecraft coordinates.
$\{H_A\}$	Absolute angular momentum of the AMCD rim (AMCD components)
$\{H_S\}$	Absolute angular momentum of the spacecraft (spacecraft components).
$\{H_{AO}\}$	$[I_A]\{\Omega_{AO}\}$ (eqn. 7)

LIST OF SYMBOLS (cont'd)

$[I_A]$	Moments of inertia of the AMCD rim about principal body axes.
$[I_S]$	Moments of inertia of the spacecraft about principal body axes.
$\{M_S\}$	Total moments acting on the spacecraft (spacecraft components).
m_A	Mass of the AMCD rim.
m_S	Mass of the spacecraft.
$\{r_A\}$	Vector, fixed in AMCD coordinates, which locates a point in the AMCD rim that coincides with the center point of a given bearing station.
$\{r_d\}$	Displacement of the center of mass of the AMCD rim in inertial coordinates.
r_m	Length of $\{r_A\}$ (radius of the AMCD rim).
$\{r_S\}$	Spacecraft coordinates of a given bearing station.
$\{r_{AI}\}$	$\{r_A\}$ in inertial coordinates.
$\{r_{CA}\}$	Position of the AMCD rim center of mass with respect to the origin of the inertial coordinate system.
$\{r_{CS}\}$	Position of the spacecraft center of mass with respect to the origin of the inertial coordinate system.
$\{r_{SI}\}$	$\{r_S\}$ in inertial coordinates.
$[T_A]$	Vector transformation matrix, AMCD components to inertial components.
$[T_S]$	Vector transformation matrix, spacecraft components to inertial components.
$\{T_{MA}\}$	Torques on the AMCD rim produced by a given bearing station written in the A system.
$\{T_{MS}\}$	Torques on the spacecraft produced by a given bearing station written in the S system.
$[U_A]$	Transformation matrix, AMCD body rates to AMCD Euler rates.

LIST OF SYMBOLS (cont'd)

$\{V_A\}$	AMCD rim translational rates written in the A system.
$\{V_S\}$	Spacecraft translational rates written in the S system.
$\{\dot{\theta}_A\}$	AMCD Euler rates.
$\{\theta_A\}$	AMCD Euler angles with respect to inertial axes developed with a 3, 2, 1 rotation sequence.
$\{\dot{\theta}_S\}$	Spacecraft Euler rates.
$\{\theta_S\}$	Spacecraft Euler angles with respect inertial axes developed with 3, 2, 1 rotation sequence.
ϕ_{SA}	Spin angle of axes 1_{AB}, 2_{AB} referenced to axes 1_A, 2_A, respectively.
$\{\bar{\Omega}_A\}$	Absolute angular velocity of the AMCD rim expressed in the A system.
$\{\Omega_A\}$	Angular velocity of the 1_A, 2_A, 3_A axis system written in the A system with respect to the I system.
$\{\Omega_{AO}\}$	Angular velocity of the 1_{AB}, 2_{AB}, 3_{AB} axis system with respect to the 1_A, 2_A, 3_A axis system, written in the AB system.
$\{\Omega_S\}$	Angular velocity of the 1_S, 2_S, 3_S axis system with respect to the I system written in the S system.
1, 2, 3,	Orthogonal axis system.

Notation:

$[\ ]$	Rectangular matrix
$[\ ]^{-1}$	Inverse of $[\ ]$
$[\ ]^T$	Transpose of $[\ ]$
$\{\ \}$	Column vector
$\lfloor\ \rfloor$	Row vector

LIST OF SYMBOLS (concl'd)

A single dot over a symbol denotes a first derivative with respect to time. Double dots denote a second derivative.

Subscripts:

1, 2, 3	**Orthogonal components along 1, 2, 3 axes, respectively.**
A	**AMCD coordinate system.**
AB	**AMCD body-fixed coordinate system.**
a, b, c	**Magnetic bearing station a, b, or c.**
I	**Inertially fixed coordinate system.**
S	**Spacecraft coordinate system.**

TWO-BODY EQUATIONS OF MOTION

AMCD - Spacecraft Equations

This section presents the AMCD-Spacecraft equations of motion in general terms. The coordinate systems used for the AMCD are shown in figure 2. A set of orthogonal 1_{AB}, 2_{AB}, and 3_{AB} body axes are fixed in the AMCD rim with the 3_{AB} axis being the rim spin axis. In order to simplify the equations, a set of nonspinning 1_A, 2_A, and 3_A axes are used to define the motion of the AMCD rim with respect to inertial space. The 3_A axis is coincident with the 3_{AB} axis and the remaining two axes lie in the plane defined by the 1_{AB} and 2_{AB} axes.

A set of orthogonal 1_S, 2_S, and 3_S body fixed axes defines the motion of the spacecraft with respect to inertial space. The initial alinement of the axis systems is shown in figure 3. The spacecraft axis system is initially alined with an orthogonal 1_I, 2_I, and 3_I system fixed in inertial space. The AMCD axis system is displaced from the I axis system by a distance D. The 1_A, 2_A, and 3_A axes are parallel to the 1_I, 2_I, and 3_I axes, respectively.

The assumptions used for deriving the equations of motion are: 1) The AMCD rim and spacecraft are rigid bodies, 2) the AMCD rim and spacecraft have negligible products of inertia, 3) the spin rate of the AMCD rim is constant with respect to the 1_A, 2_A, 3_A axis system, and 4) the spacecraft is nonspinning.

AMCD.- Referring to figure 2, the 3_{AB} axis is an axis of symmetry so that

$$I_{AB1} = I_{A1} = I_{AB2} = I_{A2} = I_A \tag{1}$$

The absolute (inertially referenced) angular velocity, $\{\bar{\Omega}_A\}$, of the AMCD rim can be written as

$$\{\bar{\Omega}_A\} = \{\Omega_A\} + \{\Omega_{AO}\} \tag{2}$$

where $\{\Omega_A\}$ is the absolute angular velocity of the 1_A, 2_A, 3_A system and $\{\Omega_{AO}\}$ is the angular velocity of the 1_{AB}, 2_{AB}, 3_{AB} system with respect to the 1_A, 2_A, 3_A system (for a general discussion of rigid body dynamics see ref. 6). The absolute angular momentum, $\{H_A\}$, of the AMCD rim then can be written as

$$\{H_A\} = [I_A]\{\bar{\Omega}_A\} \tag{3}$$

where

$$[I_A] = \begin{bmatrix} I_A & 0 & 0 \\ 0 & I_A & 0 \\ 0 & 0 & I_{A3} \end{bmatrix} \tag{4}$$

$\{\Omega_{AO}\}$ is given by (see figure 2)

$$\{\Omega_{AO}\}^T = \lfloor 0 \quad 0 \quad \dot{\phi}_{SA} \rfloor \tag{5}$$

From equations 2 and 3, $\{H_A\}$ can be written as

$$\{H_A\} = [I_A]\{\Omega_A\} + [I_A]\{\Omega_{AO}\} \tag{6}$$

Making the substitution

$$\{H_{AO}\} = [I_A]\{\Omega_{AO}\} \tag{7}$$

results in

$$\{H_A\} = [I_A]\{\Omega_A\} + \{H_{AO}\} \tag{8}$$

$\left\{H_{AO}\right\}$ is given by (from eqn. 4 and 5)

$$\left\{H_{AO}\right\}^T = \lfloor 0 \quad 0 \quad I_{A3}\,\dot{\phi}_{SA} \rfloor \tag{9}$$

and is constant since $\dot{\phi}_{SA}$ is assumed constant. The total moment, $\left\{G_A\right\}$, acting about the center of mass of the AMCD rim and referenced to the AMCD axis system is then

$$\left\{G_A\right\} = \left\{\dot{H}_A\right\} + \left[\Omega_A\right]\left\{H_A\right\} \tag{10}$$

where $\left[\Omega_A\right]$ is the skew symmetric cross product matrix defined by

$$\left[\Omega_A\right] = \begin{bmatrix} 0 & -\Omega_{A3} & \Omega_{A2} \\ \Omega_{A3} & 0 & -\Omega_{A1} \\ -\Omega_{A2} & \Omega_{A1} & 0 \end{bmatrix} \tag{11}$$

$\left\{G_A\right\}$ is the reaction torque on the AMCD rim caused by motion of the spacecraft acting through the magnetic bearings. Differentiating equation 8 results in

$$\left\{\dot{H}_A\right\} = \left[I_A\right]\left\{\dot{\Omega}_A\right\} + \left\{\dot{H}_{AO}\right\} \tag{12}$$

$\left\{\dot{H}_{AO}\right\}$ can be set to zero since the spin rate of the AMCD is assumed constant. Substituting equations 8 and 12 into 10 and rearranging terms results in

$$\left\{\dot{\Omega}_A\right\} = \left[I_A\right]^{-1}\left(\left\{G_A\right\} - \left[\Omega_A\right]\left(\left[I_A\right]\left\{\Omega_A\right\} + \left\{H_{AO}\right\}\right)\right) \tag{13}$$

The AMCD rates are obtained by integrating equation 13.

The AMCD Euler rates can be written

$$\left\{\dot{\theta}_A\right\} = \left[U_A\right]\left\{\Omega_A\right\} \tag{14}$$

where

$$\left[U_A\right] = \begin{bmatrix} 1 & \tan\theta_{A2}\sin\theta_{A1} & \tan\theta_{A2}\cos\theta_{A1} \\ 0 & \cos\theta_{A1} & -\sin\theta_{A1} \\ 0 & \sec\theta_{A2}\sin\theta_{A1} & \sec\theta_{A2}\cos\theta_{A1} \end{bmatrix} \qquad (15)$$

gives the transformation matrix from AMCD body rates to AMCD Euler rates for a 3, 2, 1 rotation sequence.

The translational force on the AMCD rim can be written in AMCD coordinates as

$$\left\{F_A\right\} = m_A\left(\left\{\dot{V}_A\right\} + \left[\Omega_A\right]\left\{V_A\right\}\right) \qquad (16)$$

where $\left\{F_A\right\}$ is the translational force of the magnetic bearings acting on the rim, m_A is the mass of the rim, $\left\{V_A\right\}$ is the rim velocity in AMCD coordinates, and $\left[\Omega_A\right]$ is the cross product matrix defined above. Rearranging 16 results in

$$\left\{\dot{V}_A\right\} = \frac{1}{m_A}\left\{F_A\right\} - \left[\Omega_A\right]\left\{V_A\right\} \qquad (17)$$

The rim translational rates are obtained by integrating equation 17. The rim translational rates in inertial coordinates, $\left\{\dot{r}_d\right\}$, become

$$\left\{\dot{r}_d\right\} = \left[T_A\right]\left\{V_A\right\} \qquad (18)$$

where

$$\left[T_A\right] = \begin{bmatrix} c\theta_{A3}c\theta_{A2} & (c\theta_{A3}s\theta_{A2}s\theta_{A1}-s\theta_{A3}c\theta_{A1}) & (c\theta_{A3}s\theta_{A2}c\theta_{A1}+s\theta_{A3}s\theta_{A1}) \\ s\theta_{A3}c\theta_{A2} & (s\theta_{A3}s\theta_{A2}s\theta_{A1}+c\theta_{A3}c\theta_{A1}) & (s\theta_{A3}s\theta_{A2}c\theta_{A1}-c\theta_{A3}s\theta_{A1}) \\ -s\theta_{A2} & c\theta_{A2}s\theta_{A1} & c\theta_{A2}c\theta_{A1} \end{bmatrix} \qquad (19)$$

gives the vector transformation from AMCD coordinates to inertial coordinates. Because of the size of the expression, sin has been shortened to s and cos shortened to c. The position of the rim center of mass with respect to the origin of the I coordinate system is, from figure 4,

$$\left\{r_{CA}\right\} = \left\{r_d\right\} + \left\{D\right\} \qquad (20)$$

where $\left\{D\right\}$ is given by

$$\left\{D\right\}^T = \lfloor d_1 \quad d_2 \quad d_3 \rfloor \qquad (21)$$

Spacecraft.- The momentum of the spacecraft in spacecraft coordinates is

$$\left\{H_S\right\} = \left[I_S\right]\left\{\Omega_S\right\} \tag{22}$$

The total moment, $\left\{M_S\right\}$, about the center of mass of the spacecraft referred to spacecraft coordinates is

$$\left\{M_S\right\} = \left[I_S\right]\left\{\dot{\Omega}_S\right\} + \left[\Omega_S\right]\left[I_S\right]\left\{\Omega_S\right\} \tag{23}$$

where $\left[\Omega_S\right]$ is the cross product matrix. $\left\{M_S\right\}$ can be written

$$\left\{M_S\right\} = \left\{G_S\right\} + \left\{E_S\right\} \tag{24}$$

where $\left\{G_S\right\}$ are the reaction torques on the spacecraft caused by motion of the AMCD rim acting through the magnetic bearings and $\left\{E_S\right\}$ are the external disturbance torques acting on the spacecraft. $\left\{G_S\right\}$ consists of torques due to the rotation of the AMCD rim plus torques due to the translational forces, produced by rim motion, which act through a point displaced from the center of mass of the spacecraft. These translational forces, $\left\{F_S\right\}$, cause a translation of the spacecraft center of mass and can be expressed in spacecraft coordinates as

$$\left\{F_S\right\} = m_S\left(\left\{\dot{V}_S\right\} + \left[\Omega_S\right]\left\{V_S\right\}\right) \tag{25}$$

where m_S is the mass of the spacecraft and $\left\{V_S\right\}$ is the spacecraft velocity. Rearranging 25 results in

$$\left\{\dot{V}_S\right\} = \frac{1}{m_S}\left\{F_S\right\} - \left[\Omega_S\right]\left\{V_S\right\} \tag{26}$$

The spacecraft translational rates are obtained by integrating equation 26. The spacecraft translational rates in inertial coordinates, $\left\{\dot{r}_{CS}\right\}$, become

$$\left\{\dot{r}_{CS}\right\} = \left[T_S\right]\left\{V_S\right\} \tag{27}$$

where $\left[T_S\right]$ is of the same form as $\left[T_A\right]$. The position of the spacecraft center of mass with respect to the I coordinate system is $\left\{r_{CS}\right\}$ (see fig. 4).

Magnetic Bearing Equations

The magnetic bearing configuration considered is one that consists of three bearing stations spaced equidistantly around the AMCD rim. The locations of the bearings are shown in figure 5. Each bearing station, which is fixed to the spacecraft, consists of an axial and radial bearing

segment and associated gap sensor for detecting the relative displacement of the AMCD rim.

In obtaining the equations for the bearing forces and torques, the following assumptions are made: 1) The AMCD and spacecraft are free to move with respect to each other with the only restraining forces between them being the forces generated by the magnetic bearings, 2) the axial bearings produce no force components in the radial direction, and 3) the radial bearings produce no force components in the axial direction.

Bearing Gaps.- The magnetic bearing forces are a function of the relative displacement between the AMCD and spacecraft. In order to calculate this displacement at a given bearing station, define a vector, $\{r_S\}$, which is fixed in spacecraft coordinates and which locates the center point of the bearing station under consideration. Next, define a vector, $\{r_A\}$, which is fixed in AMCD coordinates and which, when the coordinate systems are alined (see figure 5), locates a point in the rim that coincides with the center point of the bearing station under consideration. The vector $\{r_S\}$ in inertial coordinates becomes

$$\{r_{SI}\} = [T_S]\{r_S\} \tag{28}$$

and $\{r_A\}$ in inertial coordinates becomes

$$\{r_{AI}\} = [T_A]\{r_A\} \tag{29}$$

The displacement $\{g\}$, in inertial coordinates, of the AMCD relative to the spacecraft bearing station under consideration then becomes

$$\{g\} = \left(\{r_{CA}\} + \{r_{AI}\}\right) - \left(\{r_{CS}\} + \{r_{SI}\}\right) \tag{30}$$

Since the magnetic bearing segments and associated sensors are fixed to the spacecraft, the equivalent change in bearing gaps at a given bearing station then can be approximated by transforming $\{g\}$ into spacecraft coordinates and considering the axial and radial components at the center point of the station as illustrated in figure 6. In this figure the components of $\{g\}$ for bearing "a" are shown in spacecraft coordinates. The axial gap change is g_{Sa3} and the radial gap change is g_{Ra}. g_{Ra} is obtained by resolving g_{Sa1} and g_{Sa2} along the radial line labeled r_{Aa}.

Bearing torques.- The torque on the spacecraft due to the forces at a given bearing station will be

$$\{T_{mS}\} = [r_S]\{F_{XS}\} + [r_S]\{F_{RS}\} \tag{31}$$

where $[r_S]$ is the cross product matrix, $\{F_{XS}\}$ is the axial force on the spacecraft produced by the bearing, and $\{F_{RS}\}$ is the radial force on the spacecraft produced by the bearing. Since the forces on the rim produced by the radial bearing segments are assumed to act in the 1_A, 2_A plane, the torque on the AMCD rim produced by a given bearing station becomes

$$\{T_{mA}\} = [r_A] \{F_{XA}\} \tag{32}$$

where $[r_A]$ is the cross product matrix and $\{F_{XA}\}$ is the axial force on the rim produced by the bearing. The vectors $\{r_S\}$ and $\{r_A\}$ were defined previously.

Linearized Equations

In this section the equations of motion and magnetic bearing equations are expanded and linearized. In linearizing the equations, small angle assumptions are made and second order terms involving spacecraft and AMCD motion are neglected. Also, since the distance from the AMCD rim to a fixed point, i.e., a magnetic bearing location, is unaffected by rotation of the AMCD axis system about the 3_A axis, motion about this axis is set to zero.

<u>AMCD rotational equations</u>.-Expanding the term $[\Omega_A]\{H_A\}$ and neglecting products of rates results in

$$[\Omega_A]\{H_A\} = \begin{Bmatrix} \Omega_{A2} \; H_{AO_3} \\ -\Omega_{A1} \; H_{AO_3} \\ 0 \end{Bmatrix} \tag{33}$$

Using equation 33, the expansion of equation 13 becomes

$$\begin{Bmatrix} \dot{\Omega}_{A1} \\ \dot{\Omega}_{A2} \\ \dot{\Omega}_{A3} \end{Bmatrix} = \begin{Bmatrix} 1/I_A (G_{A1} - \Omega_{A2} \; H_{AO_3}) \\ 1/I_A (G_{A2} + \Omega_{A1} \; H_{AO_3}) \\ 0 \end{Bmatrix} \tag{34}$$

The AMCD body rates are obtained by integrating equation 34. By using small angle and rate assumptions, the Euler rates given by equation 14 reduce to

$$\begin{Bmatrix} \theta_{A1} \\ \theta_{A2} \\ \theta_{A3} \end{Bmatrix} = \begin{Bmatrix} \Omega_{A1} \\ \Omega_{A2} \\ 0 \end{Bmatrix} \tag{35}$$

__AMCD translational equations__.- The term $\left[\Omega_A\right]\left\{V_A\right\}$ consists of products of rates and will be neglected. Equation 17 then becomes

$$\begin{Bmatrix} \dot{V}_{A1} \\ \dot{V}_{A2} \\ \dot{V}_{A3} \end{Bmatrix} = 1/m_A \begin{Bmatrix} F_{A1} \\ F_{A2} \\ F_{A3} \end{Bmatrix} \qquad (36)$$

Equation 18, again using small angle and rate assumptions, can be written as

$$\begin{Bmatrix} \dot{r}_{d1} \\ \dot{r}_{d2} \\ \dot{r}_{d3} \end{Bmatrix} = \begin{Bmatrix} V_{A1} \\ V_{A2} \\ V_{A3} \end{Bmatrix} \qquad (37)$$

Finally, equation 20 becomes

$$\begin{Bmatrix} r_{CA1} \\ r_{CA2} \\ r_{CA3} \end{Bmatrix} = \begin{Bmatrix} r_{d1} + d_1 \\ r_{d2} + d_2 \\ r_{d3} + d_3 \end{Bmatrix} \qquad (38)$$

__Spacecraft rotational equations__.- The spacecraft rotational equations can be expanded in the same manner as the AMCD equations with equation 23 becoming

$$\begin{Bmatrix} \dot{\Omega}_{S1} \\ \dot{\Omega}_{S2} \\ \dot{\Omega}_{S3} \end{Bmatrix} = \begin{Bmatrix} 1/I_{S1}(G_{S1} + E_{S1}) \\ 1/I_{S2}(G_{S2} + E_{S2}) \\ 1/I_{S3}(G_{S3} + E_{S3}) \end{Bmatrix} \qquad (39)$$

The spacecraft Euler rates become

$$\begin{Bmatrix} \dot{\theta}_{S1} \\ \dot{\theta}_{S2} \\ \dot{\theta}_{S3} \end{Bmatrix} = \begin{Bmatrix} \Omega_{S1} \\ \Omega_{S2} \\ \Omega_{S3} \end{Bmatrix} \qquad (40)$$

Spacecraft translational equations.- The spacecraft translational equations can also be expanded in the same manner as the AMCD equations. Equation 26 becomes

$$\begin{Bmatrix} \dot{V}_{S1} \\ \dot{V}_{S2} \\ \dot{V}_{S3} \end{Bmatrix} = 1/m_S \begin{Bmatrix} F_{S1} \\ F_{S2} \\ F_{S3} \end{Bmatrix} \tag{41}$$

and equation 27 becomes

$$\begin{Bmatrix} \dot{r}_{CS1} \\ \dot{r}_{CS2} \\ \dot{r}_{CS3} \end{Bmatrix} = \begin{Bmatrix} V_{S1} \\ V_{S2} \\ V_{S3} \end{Bmatrix} \tag{42}$$

Bearing forces.- The vector $\{r_S\}$ for bearing station a (see fig. 5) can be written as

$$\{r_{Sa}\}^T = \lfloor (d_1 + r_m \cos 60^\circ)\ (d_2 + r_m \cos 30^\circ)\ d_3 \rfloor$$

or

$$\{r_{Sa}\}^T = \lfloor (d_1 + (\tfrac{1}{2})\ r_m)\ \ (d_2 + (\sqrt{3}/2)\ r_m)\ d_3 \rfloor \tag{43}$$

$\{r_S\}$ for bearing station b becomes

$$\{r_{Sb}\}^T = \lfloor (d_1 + (\tfrac{1}{2})\ r_m)\ \ (d_2 - (\sqrt{3}/2)\ r_m)\ d_3 \rfloor \tag{44}$$

$\{r_S\}$ for bearing station c becomes

$$\{r_{Sc}\}^T = \lfloor (d_1 - r_m)\ d_2\ \ d_3 \rfloor \tag{45}$$

$\{r_A\}$ for magnetic bearing stations a, b, and c, respectively, beomce

$$\{r_{Aa}\}^T = \lfloor (\tfrac{1}{2}) r_m \quad (\sqrt{3}/2) r_m \quad 0 \rfloor \tag{46}$$

$$\{r_{Ab}\}^T = \lfloor (\tfrac{1}{2}) r_m \quad -(\sqrt{3}/2) r_m \quad 0 \rfloor \tag{47}$$

$$\{r_{Ac}\}^T = \lfloor -r_m \quad 0 \quad 0 \rfloor \tag{48}$$

The vector $\{g\}$ in spacecraft coordinates becomes

$$\{g_S\} = [T_S]^T \{g\} \tag{49}$$

From equations 30, 28, and 29, then

$$\{g_S\} = \left([T_S]^T \{r_{CA}\} + [T_S]^T [T_A]\{r_A\}\right) - \left([T_S]^T \{r_{CS}\} + \{r_S\}\right) \tag{50}$$

using the same assumptions as used previously, $[T_S]^T [T_A]$ reduces to

$$[T_S]^T [T_A] = \begin{bmatrix} 1 & \theta_{S3} & (\theta_{A2}-\theta_{S2}) \\ -\theta_{S3} & 1 & (\theta_{S1}-\theta_{A1}) \\ (\theta_{S2}-\theta_{A2}) & (\theta_{A1}-\theta_{S1}) & 1 \end{bmatrix} \tag{51}$$

Equation 50 for magnetic bearing "a" then becomes

$$\{g_{Sa}\} = \left\{ \begin{array}{l} (r_{d1}-r_{CS1}) + \theta_{S3}(d_2 + (\sqrt{3}/2)r_m) - \theta_{S2}d_3 \\ (r_{d2}-r_{CS2}) - \theta_{S3}(d_1 + (\tfrac{1}{2})r_m) + \theta_{S1}d_3 \\ (r_{d3}-r_{CS3}) + \theta_{S2}d_1 - \theta_{S1}d_2 + (\theta_{S2}-\theta_{A2})(\tfrac{1}{2})r_m + \\ \qquad (\theta_{A1}-\theta_{S1})(\sqrt{3}/2)r_m \end{array} \right\} \tag{52}$$

Similarly, for bearings b and c, respectively

$$\{g_{Sb}\} = \left\{ \begin{array}{l} (r_{d1}-r_{CS1}) + \theta_{S3}(d_2-(\sqrt{3}/2)r_m) - \theta_{S2}d_3 \\ (r_{d2}-r_{CS2}) - \theta_{S3}(d_1+(\tfrac{1}{2})r_m) + \theta_{S1}d_3 \\ (r_{d3}-r_{CS3}) + \theta_{S2}d_1 - \theta_{S1}d_2 + (\theta_{S2}-\theta_{A2})(\tfrac{1}{2})r_m - \\ \qquad (\theta_{A1}-\theta_{S1})(\sqrt{3}/2)r_m \end{array} \right\} \tag{53}$$

$$\{g_{Sc}\} = \left\{ \begin{array}{l} (r_{d1}-r_{CS1}) + \theta_{S3}d_3 - \theta_{S2}d_3 \\ (r_{d2}-r_{CS2}) + \theta_{S3}(r_m-d_1) + \theta_{S1}d_3 \\ (r_{d3}-r_{CS3}) + \theta_{S2}d_1 - \theta_{S1}d_2 + (\theta_{A2}-\theta_{S2})r_m \end{array} \right\} \tag{54}$$

The radial gaps for bearings a, b, and c, respectively, become

$$\left(\begin{array}{lll} g_{Ra} & = & (\tfrac{1}{2})\ g_{Sa1} + (\sqrt{3}/2)\ g_{Sa2} \\ g_{Rb} & = & (\tfrac{1}{2})\ g_{Sb1} - (\sqrt{3}/2)\ g_{Sb2} \\ g_{Sc} & = & -\ g_{Sc1} \end{array}\right) \tag{55}$$

The torques on the spacecraft due to the axial and radial forces at the a, b, and c bearing stations, from equation 31, become

$$\left\{T_{mSa}\right\} = \left\{\begin{array}{l} (d_2 + (\sqrt{3}/2)r_m)F_{XSa} - (\sqrt{3}/2)d_3F_{RSa} \\ -(d_1 + r_m(\tfrac{1}{2}))F_{XSa} + (\tfrac{1}{2})d_3F_{RSa} \\ -(\tfrac{1}{2})d_2F_{RSa} + (\sqrt{3}/2)d_1F_{RSa} \end{array}\right\} \tag{56}$$

$$\left\{T_{mSb}\right\} = \left\{\begin{array}{l} (d_2 - (\sqrt{3}/2)r_m)F_{XSb} + (\sqrt{3}/2)d_3F_{RSb} \\ -(d_1 + (\tfrac{1}{2})r_m)F_{XSb} + (\tfrac{1}{2})d_3F_{RSb} \\ -(\tfrac{1}{2})d_2F_{RSb} - (\sqrt{3}/2)d_1F_{RSb} \end{array}\right\} \tag{57}$$

$$\left\{T_{mSc}\right\} = \left\{\begin{array}{l} d_2F_{XSc} \\ -(d_1-r_m)F_{XSc} - d_3F_{RSc} \\ d_2F_{RSc} \end{array}\right\} \tag{58}$$

Similarly, for torques on the AMCD, from equation 32,

$$\left\{T_{mAa}\right\} = \left\{\begin{array}{c} -(\sqrt{3}/2)r_mF_{XAa} \\ (\tfrac{1}{2})r_mF_{XAa} \\ 0 \end{array}\right\} \tag{59}$$

$$\left\{T_{mAb}\right\} = \left\{\begin{array}{c} (\sqrt{3}/2)r_mF_{XAb} \\ (\tfrac{1}{2})r_mF_{XAb} \\ 0 \end{array}\right\} \tag{60}$$

$$\left\{T_{mAc}\right\} = \left\{\begin{matrix} 0 \\ -r_m F_{XAc} \\ 0 \end{matrix}\right\} \tag{61}$$

The total torques on the spacecraft and AMCD due to bearing forces then become

$$\left\{G_S\right\} = \left\{T_{mSa}\right\} + \left\{T_{mSb}\right\} + \left\{T_{mSc}\right\} \tag{62}$$

$$\left\{G_A\right\} = \left\{T_{mAa}\right\} + \left\{T_{mAb}\right\} + \left\{T_{mAc}\right\} \tag{63}$$

FIXED-BASE EQUATIONS

The fixed-base equations of motion are obtained from the AMCD-spacecraft equations by making the spacecraft axis system a fixed reference system and setting the offset D to zero. The equations for the gaps become (equations 52-54).

$$\left\{g_{Sa}\right\} = \left\{\begin{matrix} r_{d1} \\ r_{d2} \\ r_{d3} + (\sqrt{3}/2)\ r_m\theta_{A1} - (\tfrac{1}{2})\ r_m\theta_{A2} \end{matrix}\right\} \tag{64}$$

$$\left\{g_{Sb}\right\} = \left\{\begin{matrix} r_{d1} \\ r_{d2} \\ r_{d3} - (\sqrt{3}/2)\ r_m\theta_{A1} - (\tfrac{1}{2})\ r_m\theta_{A2} \end{matrix}\right\} \tag{65}$$

$$\left\{g_{Sc}\right\} = \left\{\begin{matrix} r_{d1} \\ r_{d2} \\ r_{d3} + r_m\theta_{A2} \end{matrix}\right\} \tag{66}$$

The axial gaps, in terms of rim rotations and translation become

$$\left\{\begin{matrix} g_{Xa} \\ g_{Xb} \\ g_{Xc} \end{matrix}\right\} = \begin{bmatrix} (\sqrt{3}/2)\ r_m & -(\tfrac{1}{2})\ r_m & 1 \\ -(\sqrt{3}/2)\ r_m & -(\tfrac{1}{2})\ r_m & 1 \\ 0 & r_m & 1 \end{bmatrix} \left\{\begin{matrix} \theta_{A1} \\ \theta_{A2} \\ r_{d3} \end{matrix}\right\} \tag{67}$$

The radial gaps in terms of translations become (from equation 55)

$$\begin{Bmatrix} g_{Ra} \\ g_{Rb} \\ g_{Rc} \end{Bmatrix} = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ 1/2 & -\sqrt{3}/2 \\ -1 & 0 \end{bmatrix} \begin{Bmatrix} r_{d1} \\ r_{d2} \end{Bmatrix} \tag{68}$$

The torques on the rim due to the axial bearings become (from eqn. 63)

$$\begin{Bmatrix} G_{A1} \\ G_{A2} \\ G_{A3} \end{Bmatrix} = \begin{bmatrix} (\sqrt{3}/2)r_m & -(\sqrt{3}/2)r_m & 0 \\ -(1/2)r_m & -(1/2)r_m & r_m \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} F_{XAa} \\ F_{XAb} \\ F_{XAc} \end{Bmatrix} \tag{69}$$

and the radial forces on the rim resolved along the 1 and 2 axes become (see fig. 5)

$$\begin{Bmatrix} \bar{F}_{R1} \\ \bar{F}_{R2} \end{Bmatrix} = \begin{bmatrix} 1/2 & 1/2 & -1 \\ \sqrt{3}/2 & \sqrt{3}/2 & 0 \end{bmatrix} \begin{Bmatrix} F_{RAa} \\ F_{RAb} \\ F_{RAc} \end{Bmatrix} \tag{70}$$

From equations 34, 35, and 38 the rim rotational and axial translation dynamics become

$$\left(\begin{array}{l} I_A \ddot{\theta}_{A1} = G_{A1} - H_{A03} \dot{\theta}_{A2} \\ I_A \ddot{\theta}_{A2} = G_{A2} + H_{A03} \dot{\theta}_{A1} \\ m_A \ddot{r}_{CA3} = \bar{F}_A \end{array} \right) \tag{71}$$

where

$$\bar{F}_A = F_{XAa} + F_{XAb} + F_{XAc} \tag{72}$$

From equations 69 and 70

$$\begin{Bmatrix} G_{A1} \\ G_{A2} \\ \bar{F}_A \end{Bmatrix} = \begin{bmatrix} (\sqrt{3}/2)r_m & -(\sqrt{3}/2)r_m & 0 \\ -(1/2)r_m & -(1/2)r_m & r_m \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} F_{XAa} \\ F_{XAb} \\ F_{XAc} \end{Bmatrix} \tag{73}$$

Finally, from equations 36, 37, and 38 the radial translation dynamics become

$$m_A \begin{Bmatrix} \ddot{r}_{CA1} \\ \ddot{r}_{CA2} \end{Bmatrix} = \begin{Bmatrix} \bar{F}_{R1} \\ \bar{F}_{R2} \end{Bmatrix}$$

where $\bar{F}_{R1}$ and $\bar{F}_{R2}$ are defined by equation 70.

CONCLUDING REMARKS

This paper has developed fixed base and two-body equations of motion for an Annular Momentum Control Device (AMCD). The fixed-base equations are for a rigid AMCD rim suspended by magnetic bearings attached to a rigid base and the two-body equations are for a rigid AMCD rim suspended by magnetic bearings attached to a rigid body spacecraft. The fixed-base equations are applicable to the analysis of any potential ground based AMCD application such as energy storage.

REFERENCES

1. Anderson, Willard W. and Groom, Nelson J.: Annular Momentum Control Device. U.S. Letters Patent No. 3,915,416, October 28, 1975.

2. Anderson, Willard W. and Groom, Nelson J.: The Annular Momentum Control Device (AMCD) and Potential Applications. NASA TN D-7866, March 1975.

3. Schlieben, Ernest W.: Systems Aspects of Energy Wheels. Proceedings of the 1975 Flywheel Technology Symposium, November 10-12, 1975.

4. Aaland, Kristian and Lane, Joe E.: Ideas and Experiments in Magnetic Interfacing. Proceedings of the 1975 Flywheel Technology Symposium, November 10-12, 1975. ERDA 76-85, 1976, pp. 123-132.

5. Kirk, James A.; Studer, Phillip A.; and Evans, Harold E.: Mechanical Capacitor. NASA TN D-8185, March 1976.

6. Greenwood, Donald J.: Principles of Dynamics. Prentice-Hall, Inc., 1965.

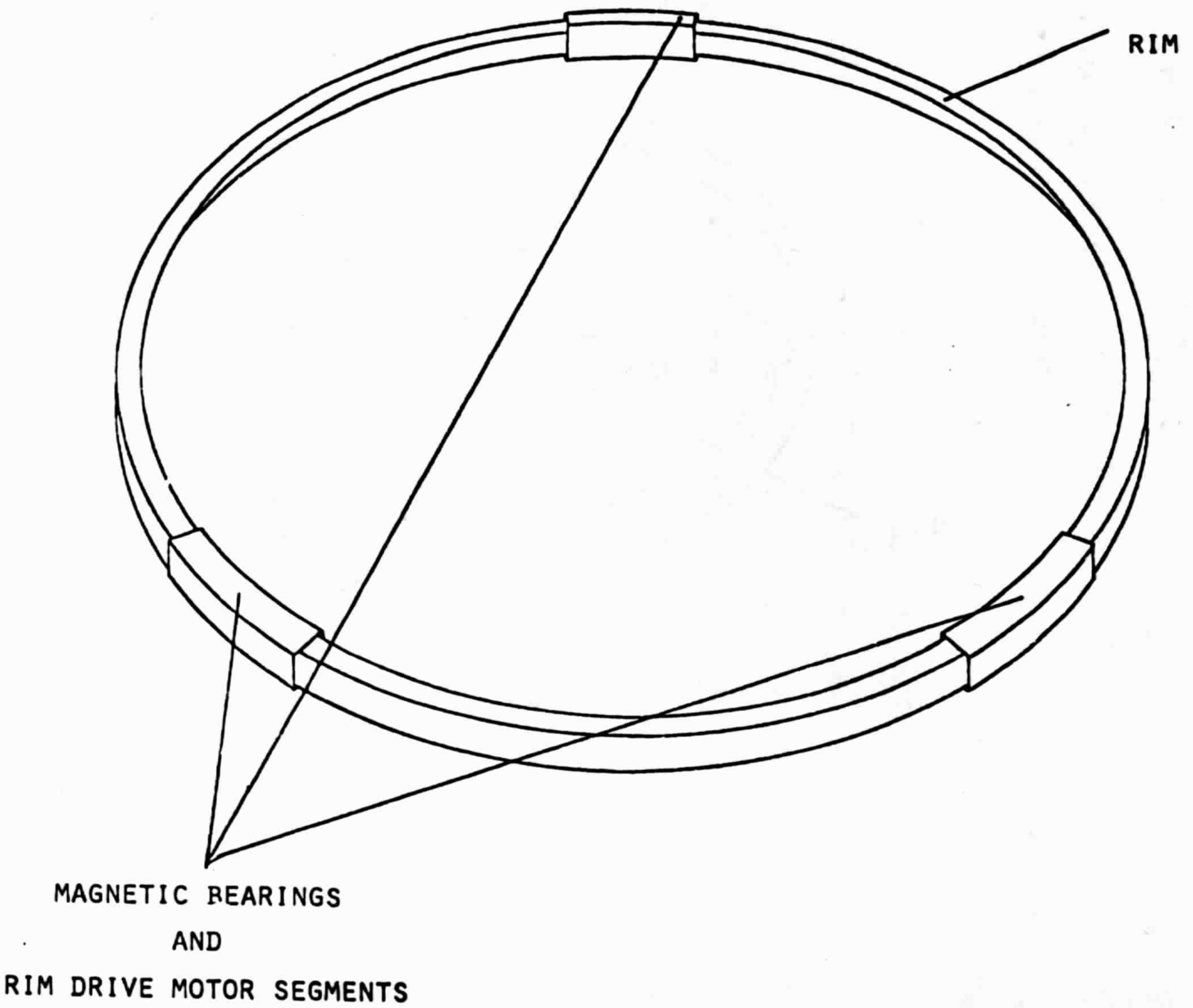

Figure 1.- Annular Momentum Control Device Concept.

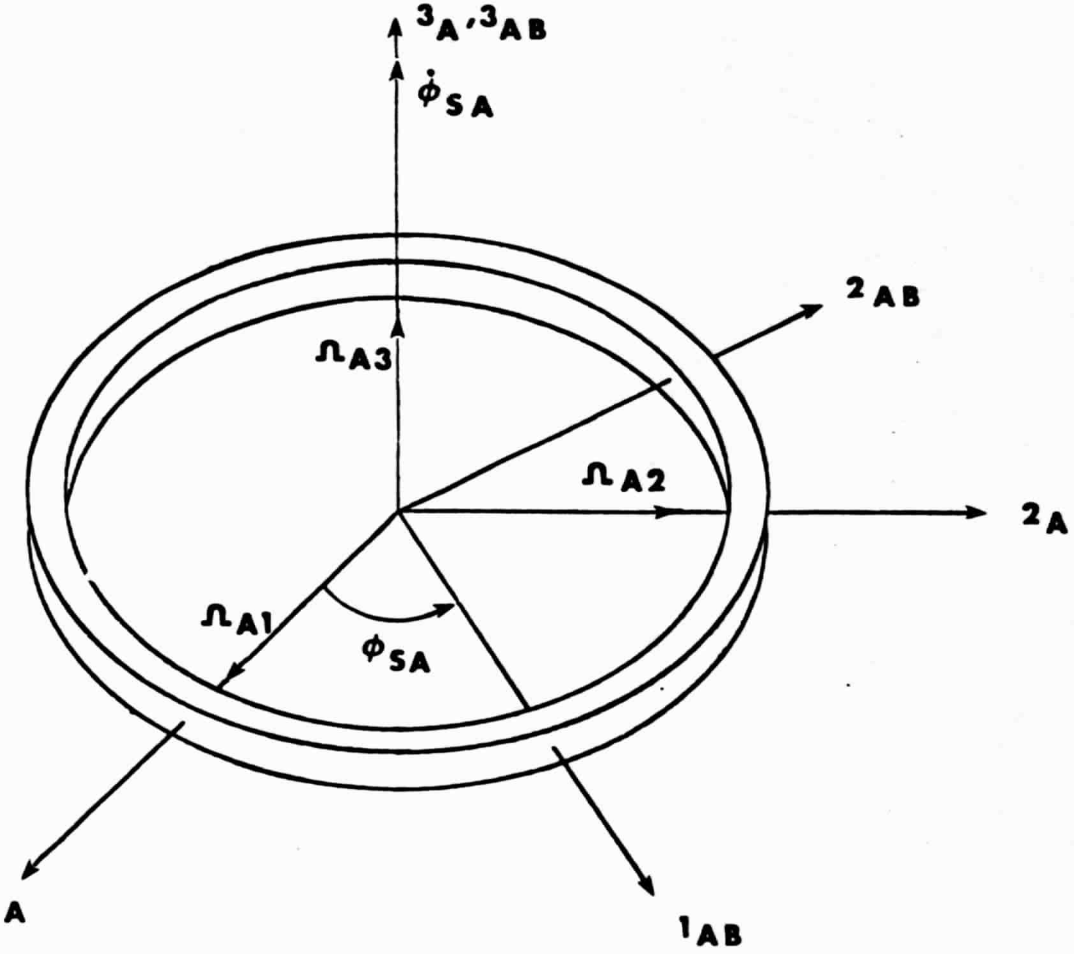

Figure 2.- AMCD coordinate systems.

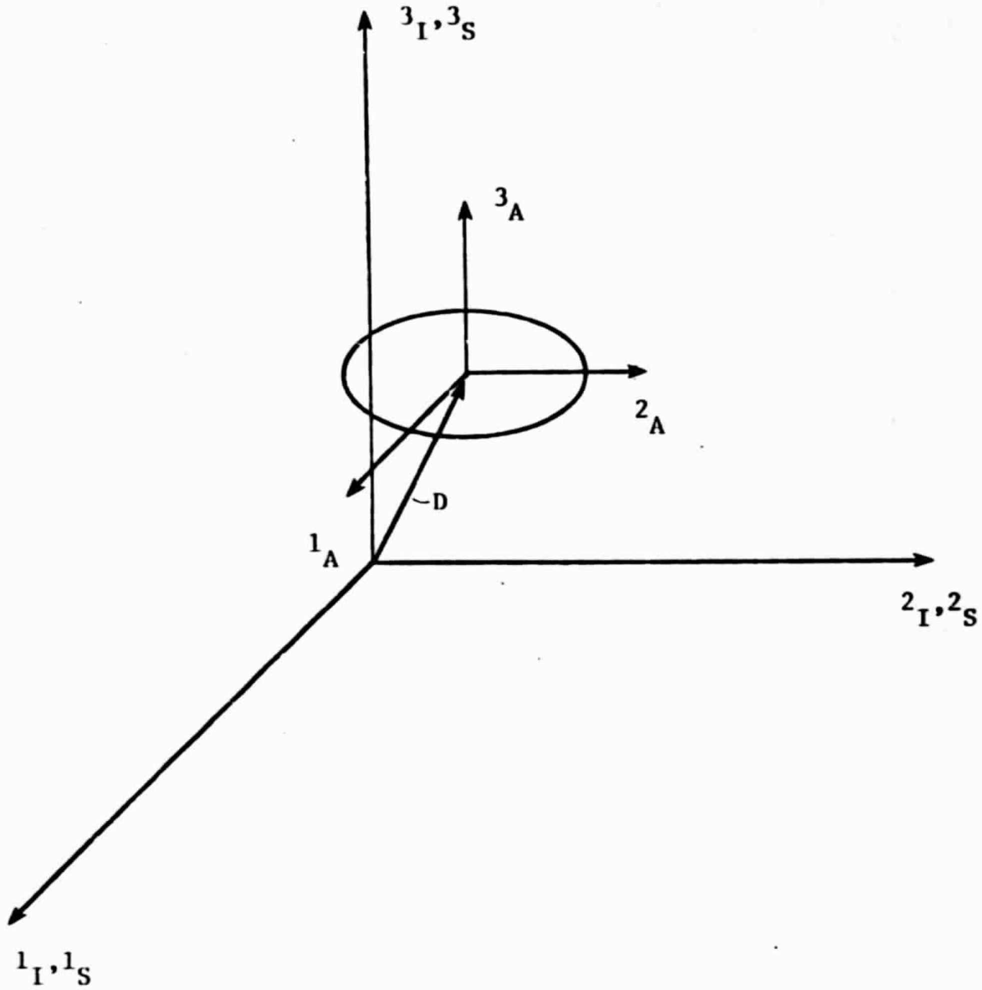

Figure 3.- Initial AMCD-spacecraft coordinate system alinement.

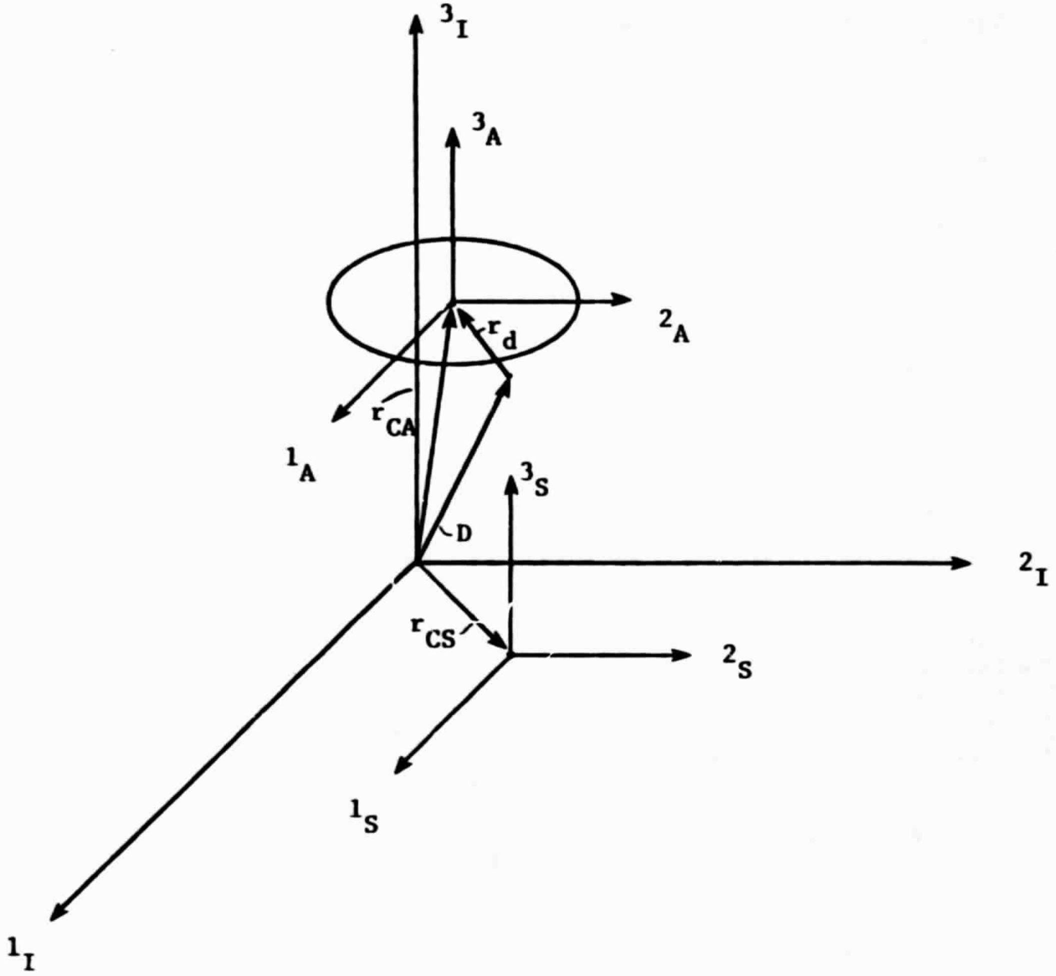

Figure 4.- Arbitrary displacement of AMCD and spacecraft coordinate system.

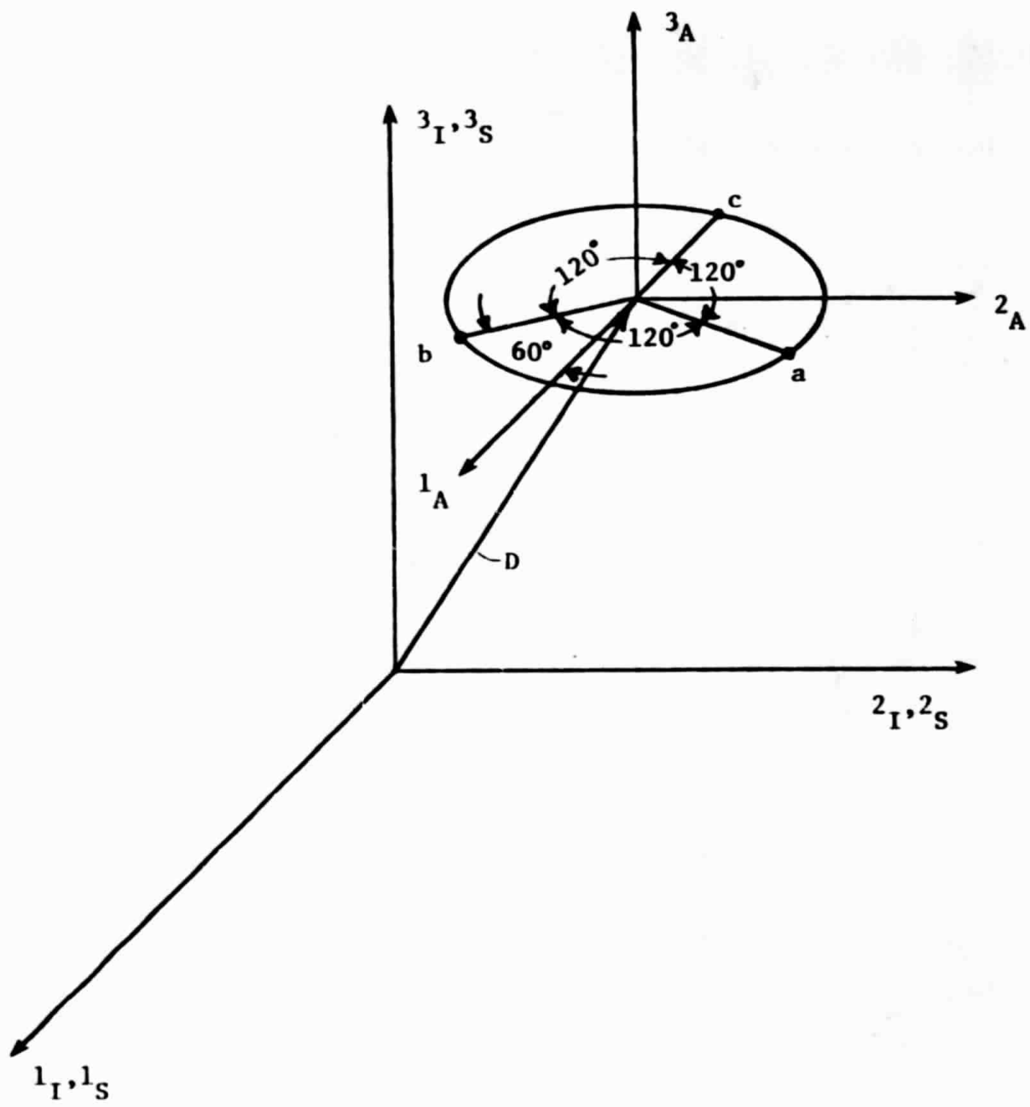

Figure 5.- Location of AMCD magnetic bearings.

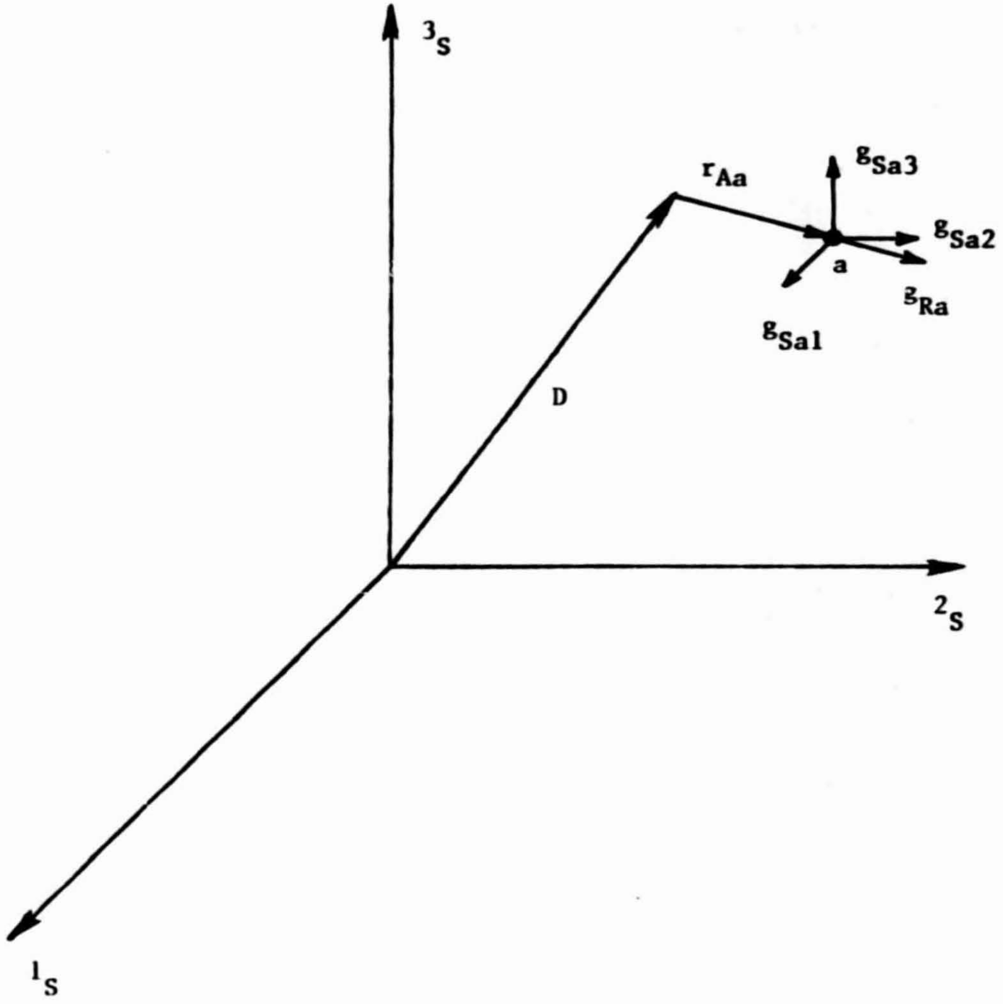

Figure 6.- Components of g for bearing a.

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16. Abstract

Fixed-base and two-body equations of motion for an Annular Momentum Control Device (AMCD) are presented. An AMCD consists of a spinning annular rim which is suspended by noncontacting magnetic bearings and powered by a noncontacting linear electromagnetic motor. The fixed-base equations are for a rigid AMCD rim suspended by magnetic bearings attached to a rigid fixed-base and the two-body equations are for a rigid AMCD rim suspended by magnetic bearings attached to a rigid body spacecraft. The fixed-base equations are applicable to any potential ground based AMCD application such as energy storage.

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