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Ground Distance Covered During Airborne Horizontal Deceleration of an Airplane

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SUMMARY

An analysis is presented of the distance an airplane floats with respect to the ground during deceleration at constant altitude, taking into account the effects of a constant wind. By use of suitable nondimensionalizing parameters suggested previously in the literature, data applicable to all airplanes are presented by means of a single family of curves. The application of these curves in a typical example is included.

INTRODUCTION

A study of the phase of flight in which an airplane decelerates at constant altitude has several practical applications. If an airplane completes the landing flare with excessive speed, the pilot may elect to hold the airplane off the ground until a suitable touchdown speed or attitude is reached. General-aviation airplanes operating at large airports frequently approach at speeds higher than normal to maintain their spacing in the traffic pattern. Measurements have shown that, even in routine operations, many pilots land with excessive speed (ref. 1). The flight path of an airplane after an aborted take-off is another situation which may involve deceleration at constant altitude. Some situations exist involving an excessive tendency to float during a landing. For example, landing with a tail wind increases the distance covered. The effect of encountering a head wind at low altitude is an increased initial airspeed which may also lengthen the floating distance. Sailplanes, because of their low drag, may float excessively in landing, particularly if a deceleration device such as a drag parachute fails to work. In all these applications, the presence of wind has an important effect on the ground distance covered.

A rational analysis of the landing of an airplane is given in reference 2. This analysis includes the phases of glide, flare, deceleration at constant altitude (called the float phase), and the ground run. Constant deceleration (corresponding to constant lift-drag ratio) was assumed during the float phase, an approximation that was adequate for the purpose under consideration. Approximate methods for the calculation of landing distance sometimes neglect the float phase entirely (ref. 3). Some consideration of the role of the float phase in determining certification standards for the landing distance of transport airplanes is given in reference 4 but again the assumption of constant lift-drag ratio was made. In cases in which the flare is completed with considerable excess speed, however, a more accurate method for calculating the distance covered during the float phase would be desirable.

A solution of the equations governing deceleration in level flight is presented in reference 5. The analysis of reference 5 shows that by use of suitable nondimensionalizing parameters, the solutions for air distance and airspeed may be expressed by means of a pair of curves applicable to all types of airplanes. The present report extends these results by including the effects of a constant wind and solving for the ground distance covered during deceleration at constant altitude. By use of the same nondimensionalizing parameters as those suggested in reference 5, the results for a series of values of wind velocity are given by a single family of curves applicable to all types of airplanes.

SYMBOLS

C constant of integration

C_D drag coefficient, $\dfrac{D}{\frac{\rho}{2} U^2 S}$

$C_{D,o}$ profile drag coefficient

C_L lift coefficient, $\dfrac{L}{\frac{\rho}{2} U^2 S}$

$C_{L,max}$ maximum lift coefficient

D drag

g acceleration due to gravity ($1g = 9.807$ m/sec^2)

K variation of C_D with $C_L{}^2$

L lift

l_p aerodynamic penetration, $\dfrac{2m}{\rho S C_{D,o}}$

m mass

S wing area

s float distance with respect to ground

s' nondimensional float distance with respect to ground, $\dfrac{s}{l_p}$

U true airspeed

U_g ground speed

U_r reference velocity, $\sqrt{g l_p}\,(K C_{D,o})^{1/4}$

U_s true stall speed

U_w wind speed

U'	nondimensional airspeed, $\frac{U}{U_r}$
U_g'	nondimensional ground speed, $\frac{U_g}{U_r}$
U_s'	nondimensional stall speed, $\frac{U_s}{U_r}$
U_w'	nondimensional wind speed, $\frac{U_w}{U_r}$
W	weight
x	$= U'^2$
γ	flight-path angle
ρ	air density

A dot over a symbol denotes differentiation with respect to time.

ANALYSIS

The vertical and longitudinal forces acting on an airplane in gliding flight with wings level are shown in figure 1. The longitudinal equations in flight-path axes are

$$\dot{U} + g \sin \gamma + (C_{D,o} + KC_L{}^2)\frac{\frac{\rho}{2} U^2 S}{m} = 0 \tag{1}$$

$$U\dot{\gamma} + g \cos \gamma - C_L \frac{\frac{\rho}{2} U^2 S}{m} = 0 \tag{2}$$

In level flight, $\gamma = \dot{\gamma} = 0$. Equation (2) then becomes

$$C_L = \frac{mg}{\frac{\rho}{2} U^2 S} \tag{3}$$

Substituting this value in equation (1) gives

$$\dot{U} + C_{D,o} \frac{\frac{\rho}{2} U^2 S}{m} + K \frac{mg^2}{\frac{\rho}{2} U^2 S} = 0 \tag{4}$$

If the velocity over the ground is defined as $U_g = U + U_w$, where U_w is a steady wind directed along the flight path, the differential equation governing the distance an airplane will float over the ground at constant altitude is

$$\dot{U}_g = U_g \frac{dU_g}{ds} = -\frac{C_{D,o}\rho S}{2m}(U_g - U_w)^2 - \frac{2Kmg^2}{\rho S(U_g - U_w)^2}$$

or

$$\frac{dU_g}{ds} = -\frac{C_{D,o}\rho S}{2m}\,\frac{(U_g - U_w)^2}{U_g} - \frac{2Kmg^2}{\rho S U_g (U_g - U_w)^2} \qquad (5)$$

As shown in reference 5, the solution of this equation may be simplified by expressing distance in terms of a reference length called the aerodynamic penetration which is defined by the following equation:

$$l_p = \frac{2m}{\rho S C_{D,o}}$$

and by expressing velocity in terms of a reference velocity given by

$$U_r = \sqrt{\frac{2mg}{\rho S}}\left(\frac{K}{C_{D,o}}\right)^{1/4} = \sqrt{g l_p}(KC_{D,o})^{1/4}$$

This reference velocity is the speed at which $C_{D,o} = KC_L^2$ and is therefore the speed for the maximum ratio of lift to drag. Let $s' = s/l_p$, $U_g' = U_g/U_r$, and $U_w' = U_w/U_r$. Equation (5) then becomes

$$\frac{dU_g'}{ds'} = -\frac{(U_g' - U_w')^2}{U_g'} - \frac{1}{U_g'(U_g' - U_w')^2} \qquad (6)$$

Separating the variables yields

$$-ds' = \frac{dU_g'}{\dfrac{(U_g' - U_w')^2}{U_g'} + \dfrac{1}{U_g'(U_g' - U_w')^2}} \qquad (7)$$

If equation (7) is integrated,

$$-s' = \int \frac{U_g'(U_g' - U_w')^2\, dU_g'}{(U_g' - U_w')^4 + 1} \qquad (8)$$

Let $U' = U_g' - U_w'$, or $U_g' = U' + U_w'$, and $dU_g' = dU'$. Then equation (8) becomes

$$-s' = \int \frac{(U' + U_w')U'^2}{U'^4 + 1}\, dU' = \int \frac{U'^3 + U_w'U'^2}{U'^4 + 1}\, dU' \quad . \qquad (9)$$

Considering the terms separately gives

$$-s' = \int \frac{U'^3\,dU'}{U'^4 + 1} + U_w' \int \frac{U'^2\,dU'}{U'^4 + 1} \tag{10}$$

Let $x = U'^2$ and $dU' = dx/2U'$. Equation (10) can then be written

$$-s' = \frac{1}{2} \int \frac{x\,dx}{x^2 + 1} + \frac{U_w'}{2} \int \frac{\sqrt{x}\,dx}{x^2 + 1} \tag{11}$$

These integrals may be found in a table of integrals such as that given in reference 6. The solution is

$$-s' + C = \frac{1}{4} \ln(x^2 + 1) - \frac{U_w'}{2\sqrt{2}}\left[\frac{1}{2} \ln\left(\frac{x + \sqrt{2x} + 1}{x - \sqrt{2x} + 1}\right) - \tan^{-1} \frac{\sqrt{2x}}{1 - x}\right] \tag{12}$$

The numerical solution of equation (12) may lead to difficulties if $x = 1$, because of the infinity occurring in the argument of the $\tan^{-1}$ term. The term itself, however, approaches $\pi/2$ under these conditions. The correct quadrant for the $\tan^{-1}$ term must be used. If the value of the term $\sqrt{2x}/(1 - x)$ is positive $(x < 1)$, the angle should be between 0 and $\pi/2$, whereas if the value is negative $(x > 1)$, the angle should be between $\pi/2$ and π.

RESULTS

The curves of figure 2 show in nondimensional form the float distance s' required to decelerate as a function of airspeed U', for various values of tail wind or head wind. The results, based on equation (12), are presented in terms of the nondimensional parameters

$$s' = \frac{s}{l_p} \qquad U' = \frac{U}{\sqrt{g l_p}(KC_{D,o})^{1/4}} \qquad U_w' = \frac{U_w}{\sqrt{g l_p}(KC_{D,o})^{1/4}}$$

where l_p is the aerodynamic penetration defined as $2m/\rho S C_{D,o}$. As was mentioned previously, these nondimensionalizing parameters are suggested in reference 5.

The values of s' are plotted as positive quantities in figure 2. The negative sign of s' shown in equation (12) is ignored. This negative sign in the formula results from the fact that the distance is increasing in a positive direction as the airplane decelerates. If the point $s' = 0$ corresponds to the point where the ground speed is zero, the airplane is, strictly speaking, in the region of negative values of s' during the period of deceleration.

The constant of integration C in equation (12) may be used to place the zero of the scale of s' at some arbitrary location. In the data presented, the value of C is determined such that, for tail winds, the value of $s' = 0$ when $U' = 0$, and for head winds, the value of $s' = 0$ when $U_g' = 0$ or $U' = -U_w'$.

In practice, the float distance required is the distance covered between some initial and final values of airspeed. This distance may be obtained as the difference between the values of s' at these values of airspeed. The maximum float distance is usually obtained if the final airspeed is the stall speed. The nondimensional stall speed is

$$U_s' = \frac{1}{\sqrt{C_{L,max}}}\left(\frac{C_{D,o}}{K}\right)^{1/4}$$

A vertical line may be drawn in figure 2 showing the airspeed at touchdown to facilitate reading the values of float distance for various values of wind speed. In the unlikely event that a head wind exists with velocity greater than the touchdown speed, the forward progress of the airplane stops when $U' = -U_w'$. In this case, the maximum forward penetration of the airplane may be obtained from the value of s' at the initial airspeed, inasmuch as the final value is plotted as zero when $U' = -U_w'$.

DISCUSSION

The results of this analysis are based on the assumption that the drag is given by an expression of the form $C_D = C_{D,o} + KC_L^2$. If the airplane is floating at low altitude, the values of $C_{D,o}$ and K may be influenced by ground effect. Values appropriate to the altitude under consideration should therefore be used. The primary effect of proximity to the ground is a reduction in the induced drag as expressed by the value of K. At high values of lift coefficient, the ground effect may also reduce the effective dynamic pressure. If the airspeed is considered to be the speed with respect to the distant air mass, as is required for calculation of the distance covered, the effect of this reduction in dynamic pressure may be approximated by appropriate modifications in $C_{D,o}$ and K. The effect of an idling propeller or jet engine may introduce drag or thrust terms that differ in form from the assumed expression $C_D = C_{D,o} + KC_L^2$. A more detailed analysis would be required to account for these effects.

The value of float distance for a particular case may be read from figure 2 and converted back to dimensional form. If more accuracy is required, the value may be calculated from equation (12). Alternatively, the entire plot presented in figure 2 may be converted back to dimensional form for application to a particular airplane. An example is shown in figure 3, which presents the float distances for a general-aviation airplane with the characteristics shown in the figure. The drag characteristics assumed correspond to a lift-drag ratio of 6.98 at the stall ($C_L = 1.2$). This value of lift-drag ratio is representative of the condition of flaps and landing gear down.

In reference 5, the float distance and time are each given as functions of airspeed by expressions similar to the first and second terms, respectively, of equation (12). Both time and distance are involved in the present analysis because, with a head wind, the distance the airplane moves with respect to the air must be corrected for the movement of the air with respect to the ground during the time of flight. In the derivation presented herein, however, the ground distance is determined as a function of airspeed directly, without the intermediate step of determining time of flight.

CONCLUDING REMARKS

An analysis is presented of the distance an airplane floats with respect to the ground during deceleration at constant altitude, taking into account the effects of a constant wind. By use of suitable nondimensionalizing parameters, data applicable to all airplanes are presented by means of a single family of curves. These results provide a rational approach for estimating the contribution of this floating regime of flight to the distance needed for landings or aborted take-offs.

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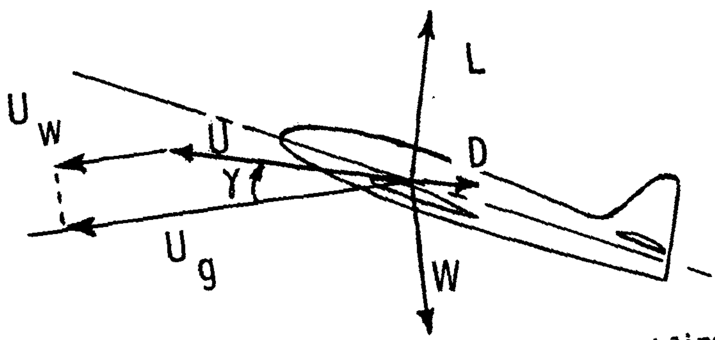

Figure 1.- Forces acting on an airplane in gliding flight.

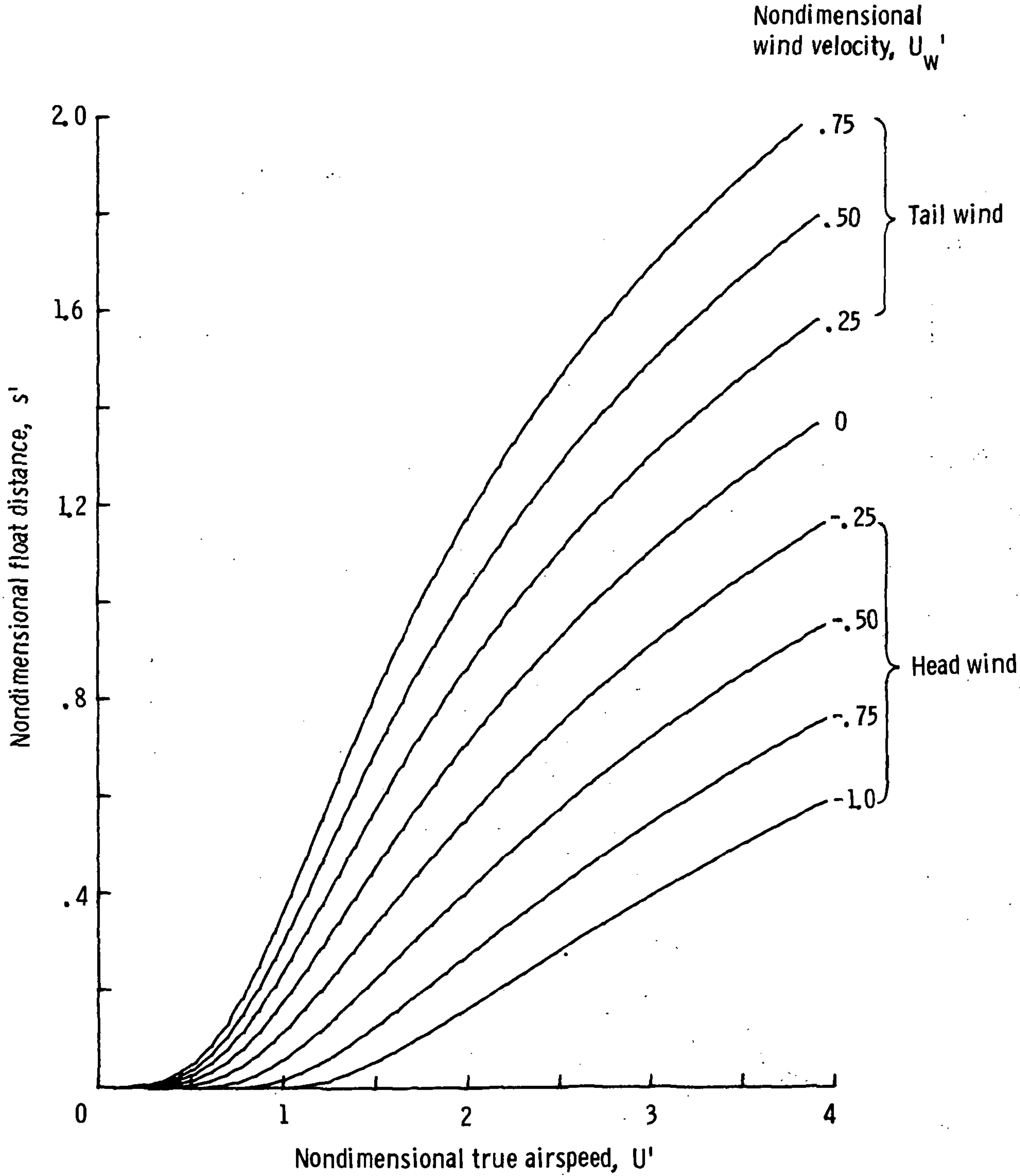

Figure 2.- Nondimensional float distance required to decelerate as a function of initial airspeed for various values of wind. $s' = \frac{s}{l_p}$; $U' = \frac{U}{\sqrt{g l_p}(KC_{D,o})^{1/4}}$; $U_w' = \frac{U_w}{\sqrt{g l_p}(KC_{D,o})^{1/4}}$; $l_p = \frac{2m}{\rho S C_{D,o}}$.

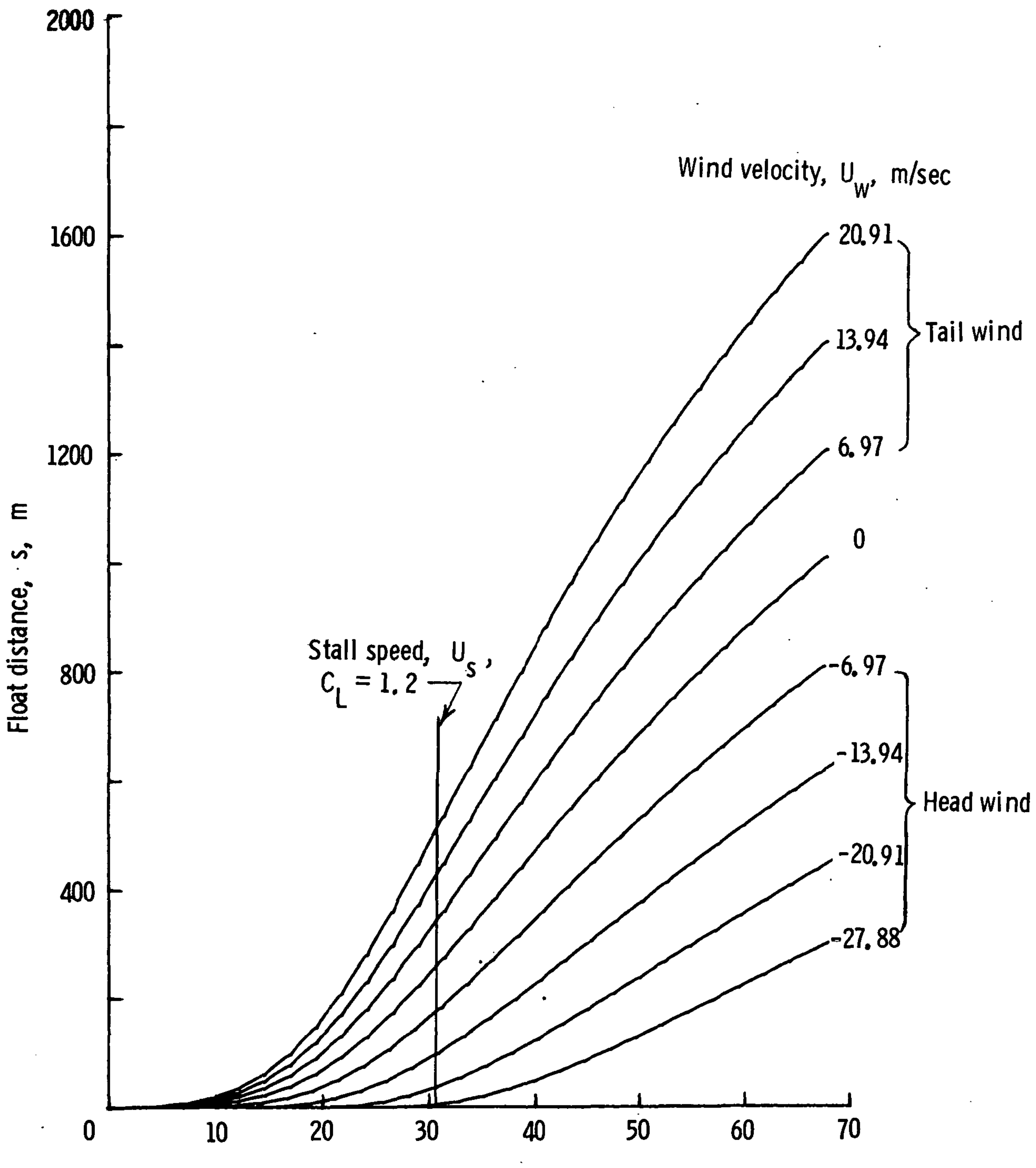

Figure 3.- Float distance for a typical general-aviation airplane as a function of initial airspeed. $C_D = 0.1 + 0.05C_L^2$; $m/S = 68.72\ kg/m^2$; $\rho = 1.226\ kg/m^3$; $l_p = 1121$ m.

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16. Abstract

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