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INVESTIGATION OF THE PHENOMENA OF THE BIG COMET OF 1858

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16. Abstract

Various aspects of the large comet of 1858 - including the luminosity of the core and the shape, intensity and position of the taile with respect to the sun and stars - are described and then compared with the large comet of 1744 described by Heinsius and Halley's comet of 1835. The purpose of these observations is to try to gain a clearer understanding of the nature of the polar force from the sun acting on the comet. This force is said to differ from the usual force of gravity.

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C.F. Pape

The appearance of such a bright and, in its development, such an instructive comet like the big one discovered by Donati must necessarily direct our attention to those items whose observation and careful investigation is highly suited to expanding our knowledge on the nature of these heavenly bodies and the forces which result in their development. During the time of its visibility our comet displayed phenomena which were so surprisingly similar to those of earlier comets -- I mention here only the large comet of 1744 described by Heinsius and Halley's Comet of 1835 -- that the mere comparison of these phenomena offers rich material for furthere considerations. Without doubt thesesphenomena will be carefully observed by the owners of large telescopes which are now so numerous and hopefully we will be indebted to the reports of their observations for a considerable increase in our knowledge. It is out of the question here to attempt to compare these observations of other astronomers with those made here using relatively moderate means. Nevertheless, I report them in what follows because the course of the phenomena was so obvious that it could certainly be perceived in its general features even when observed with weaker telescopes. /313*

So far I have only spoken of the appearance of the comet in a telescope. Nearly as much attention is merited by what appears to the naked eye, namely the tail of the comet. During the time of greatest brightness I carefully plotted the tail in celestial charts, comparing it with nearby stars, and in what follows I present the investigation of the position of the tail based on observations of its boundaries. However the results produced by this investigation pertaining to the forces which

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* Numbers in the margin indicate pagination in the foreign text.

determined the shape of the tail I considered to be only provisional, but I hope by reporting them at least those observers who have carefully observed the tail will be moved to publish their observations in detail.

I am adding a number of illustrations of the comet to this article. The first group shows the tail of the comet in its development over time. The shapes are copied from my drawings which are traced directly from the outlines of the comet recorded in Argelander's Atlas. The configuration of the stars is as faithful a copy as is possible of the comet's surroundings based on Argelander's Charts. The second group of illustrations gives the appearance of the comet with different (144-216 ff.) magnifications of a 4-foot Fraunhofer. From my drawings I have selected those which in their sequence most clearly show the development of the emanating light.

I will now present the observations made in chronological order.

1

During August and in the first days of September I did not see the comet, partly because nearby buildings concealed it from the observatory, but especially because other tasks detained me. On Sept. 13 I saw it for the first time, however only at low magnifications of a 3 1/2-foot Fraunhofer. The core-like condensed shape did not seem to me to display anything striking. It was surrounded on all sides by a light coma which, on the side opposite the sun, merged into the tail. The tail was almost straight and about 4° long in the comet viewfinder. The leading side, however, was slightly curved with the convexity of the curve pointing in the direction in which the comet was moving. This side was somewhat lighter and more sharply delineated than the side opposite it.

On Sept. 15 with the same telescope I saw the appearance of the comet similar to that of Sept. 13. Using greater magnifications I noticed a small core in the center of the thickest portion of the coma.

On Sept. 17 I observed the comet together with Mr. Paschen in Schwerin at different magnifications using a 4 1/2-foot Fraunhoffer. It seemed to me (looking through the inverting telescope) as if a projection in the form of a small emanation was visible on the core somewhat to the left of the line to the crest of the coma. However the arrangement of the telescope was not sufficinetly secure to be able to calmly observe this phenomenon. The direction ....[1] .... heavier, perhaps because it could not be so easily /315 distinguished from the parabolic zone which becomes lighter. By contrast, the other emanation on the left side was distinct and could be observed far into the tail. At twilight the left branch of the tail was considerably brighter than the right (leading) branch.

After darkness had set in at 7:10 I saw the comet with the greatest magnifications of the 4-foot Fraunhofer and found the shape of the emanation sector somewhat different than I had seen it just a little while before in bright twilight. The sector had a periphery of about 240° and its shape was more parabolic. It was brighter on the right side than on the left where the otherwise distince border was completely indeterminate and crooked. The emanation noticed earlier on this side was still distinctly visible and, as it seemed to me, had become wider than just a little while before, perphas to the fact that night had fallen in the meantime. The zone surrounding the sector had a brightly emerging edge on the side facing the sun which I had

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[1] Translators note, page of text at this point appears to be missing.

not noticed on previous evenings. Both branches of the tail which still stood out sharply from the haze surrounding them on Oct. 1 and 2 were completely washed out today. The dark zone between them was poorly delineated and lighter than before. The appearance remained so until the comet set (cf. Fig. 9).

The baseline measurements at 6:22 gave 190° 70 for the approximate angle of the sector and 31.20 for the tail.

On the following evening of Oct. 5 as the comet began to brighten up it showed a quite different appearance. The core of the comet was surrounded by a double sector, so to speak a double halo, so that only a space of about 100-120° remain free on the side pointing towards the tail. The interior emanation sector was by far brighter than the exterior sector, nearly as bright as the core, but it was washed out on its right leading side. This was how the comet appeared at 60-70 times magnification. The use of stronger magnifications soon showed that the boundary of the exterior sector was very washed out and at a magnification of 216 times it could be distinguished only with difficulty from the two bordering branches of the tail into which the luminous material forming it obviously flowed. I estimated the radius of the inside sector to be 25", and that of the outside sector 40-45". The two branches of the tail were definitely no longer sharply delineated, the dark zone between the two was brighter than before and the inclination of the two branches towards each other had become distinctly stronger since Oct. 2. The extent of the haze on the side towards the sun might have been /316
about 3-4'. At 6:25 I obtained the following apex angles: 230°35 for the sector and 36.65 for the tail. Prof. Peters estimates the direction of the sector at the same time to be 246°.

October 6. At 6:45 the comet became visible between clouds. The outside sector observed the day before at low magnifications had disappeared. The diameter of the inside sector had increased, while its periphery was not more than 180°. The form appeared to be parabolic and indeed the core did not lie in the focal point of the figure but it was much closer to the right border which was very pale. I estimated the radius of the sector to be 30" and that of the parabolic surrounding it about 40-45". The outer envelope of mist around the comet could be observed at a distance of over 4' from the core. Its form was not at all regular but rather was bent out in the direction of the forward side. Directly on the core could be seen a very small but bright internal emanation. However the telescope used was too weak to show this appearance distinctly.(cf. Fig. 10).

At 6:50 the apex angle measurements gave 236°40 for the direction of the sector and 42.40 for the direction of the tail.

On Oct. 7 the comet appeared only for a few minutes between clouds. Only in glimpses I saw that the right side of the sector was very washed out and that it at least took some effort to distinguish its borders from the light background on which it lay. At 6:37 I obtained the following measurements: 226°25 for the apex angle of the sector and 44.25 for the tail.

On the following evening, Oct. 8, the comet looked much like it did on Oct. 6. At 7:00 the sector had a periphery of 180° with a radius of about 30". From time to time I saw a fine dark stripe on the left side. The object was obviously too fine for the telescope used. I mention this observation only because it has been confirmed by the observations of other astronomer using larger telescopes. The appearance of the comet was very washed out and the dark central zone was poorly delineated and considerably wider than before. The brightness of the branch of the tail on the lower left border of the sector was striking. At 6:25 the measurements of the apex angle gave 229°25 for the sector and 52.60 for the tail.

So far I have included little on the appearance of the tail because it remained nearly the same. This evening, however, an observation was made which was subsequently confirmed and about which I must speak here in somewhat more detail. The forward /317
convex edge of the tail had a regular curve to it up to Oct. 4. On Oct. 5 a small irregularity in the curve could not fail to be noticed and thereafter it increased each day with the outcome that the line bordering the core of the comet had first proceeded in a nearly straight line to a distance of about 8° but then curved off considerably to the right and then described a curve somewhat parallel to the previous border. Above this unevenness appeared today a few fine stripes emerging like columns from the border of the tail. The direction of the stripes made an angle of about 10° with the front edge of the tail.

On the following evening, Oct. 9, the observations of the previous evening were confirmed in general. The brightness of the sector had somewhat decreased, its radius had again increased and indeed not less than 33" while the apex radius of the zone surrounding it remained nearly the same (45"). The left side of the sector and the forward branch of the tail were considerably brighter than the opposite side and the overall appearance was very washed out (cf. Fig. 11).

At 6:22 I obtained the following apex angles: 244°15 for the sector and 59.15 for the tail.

The tail afforded a peculiar aspect. At a distance of about 24° from the core a bright luminous column emerged from the forward convex side about 30' to the left of sCoronae. I could observe this several degrees beyond the tail. Its light was brighter than that of the adjacent portions of the tail so that it could be observed deep down into the tail. It was surrounded on both sides by similar but less intense and shorter columns. These penetrated the border of the tail on the left side and gave it an irregular appearance, while on the right side they always merged with the light background of the tail onto which they projected. Thus the tail was divided into two parts, a lower bright and narrow section and an upper section which was very diffuse and spread out. This evening on the whole provided the most spectacular appearance of the tail whose extreme border, which could still be perceived with effort, was, based on calculation, 50° of a very large circle away from the core. I have tried to reproduce the appearance for this day by means of a drawing (Fig.4).

On the following evening of Oct. 10 the same appearance of showed up. Just as on the preceding evening the tail was divided in two. However the projecting columns on the left side could be observed for a considerably longer distance than yesterday. The brightness of the upper parts, however, had diminished. The length of the tail was still 40° and the greatest extent of the /318 width, which on this evening reached its maximum, was not less than 10°. Compared with what was seen on preceding days observation of the comet through the telescope showed no essential change. The appearance was more washed out than before. The radius of the emanation sector at 7:10 was about 35" and its preiphery was not more than 180°. The angle which the extentions of the branches of the tail formed with one another was greater than on the previous days so that since the end of September a steady increase in this angle had taken place.

Since Oct. 6 I had seen a small, internal very bright emanation adjacent to the core which reappeared each evening. Its brightness could hardly be distinguished from that of the core and it was so small in size that I could not observe anything certain concerning its shape. It seemed to me only as if it was stronger on the left side than on the right. This evening I thought it looked larger than before and I therefore suspected that this emanation was the begining of a new sector in the process of being formed. However I cannot deduce anything certain on this point from my observations. At 6:20 the apex angle measurements gave 250°95 for the sector and 63.57 for the tail.

On Oct. 11 it was overcast and on Oct. 12 it cleared up at 6:00. The appearance of the comet had changed considerably. The emanation sector included an arc of only 150° and the radius in th this case was only 15". Its border at the apex was washed out and towards the left side a bright emanation merged into the tail. The entire left side of the coma and of the core next to the tail was much brighter than the right side. The extent of the mist on the side towards the sun might have been about 2-3'. The parabolic zone, which previously had been set off so sharply against this enveloping haze, had lost its sharp outline. The entire appearance was rather quite nebulous, which is perhaps explained by the low position of the comet. At 6:20 I obtained the following apex angle measurements: 239°67 for the sector and 74.0 for the tail.

The appearance of the tail was entirely different from what had been seen before. At a distance of about 6° from the core the border of the leading side bent strongly to the right, then almost 20° away it formed a slightly convex curve and on the upper end, where the tail goes to a point under σHerculis, it curves still somewhat further to the right. The right side was border by a doubly bent curve, but which was obviously still surrounded by a very small amount of mist whose borders were /319
indeterminate. Fig. 5 shows an illustration of this appearance. The faintness of the light of the tail was striking in comparison with the bright luminosity seen between Oct. 4 and 9.

The last evening on which I saw the comet was that of Oct. 16. At 7:40 only the core and the emanation was visible in the bright twilight. The figure of the emanation was irregular, the border at the apex was very washed out and the angle enclosed by the edges on the core was about 100°, the apex radius perhaps 15-20". As night fell the low position of the comet prevented more exact observation. I beleived I just still perceived that the darkzone between the branches of the tail had practically disappeared. At 5:40 four settings gave an apex angle of 300°15 for the emanation.

I saw very little of the tail, it view being obscured by the mists of the horizon and the bright moonlight.

2

In the above description I did not mention anything about the brightness of the comet in order to avoid unnecessary repititions. I will now compile the few facts which I find in my notes pertaining to this. From September 20 to 22 in bright moonlight the comet appeared to me to have the same brightness as a second order star. It might still somewhat exceed the brightness of the nearby stars of Ursa Major. On Sept.28 when the moon was not present its brightness equaled that of a star of the first magnitude, however did not equal the brightness of Sagittarius. On Oct. 1 it was certainly brighter than this star and the brightest object in the sky. The brightness of the comet steadily kept increasing until about Oct. 6 or 8. On Oct. 9 it had already become less but it still exceeded that of Sagittarius. On Oct. 10 its brightness equaled that of Sagittarius, however on Oct. 12 when it was in a lower position it was considerably weaker. Since the comet was found for a rather long period of time at about the same altitude as Sagittarius this data, even without correction for differences in transparency of the air, will suffice in order to determine the brightness of its appearance for the naked eye.

This bright luminosity of the comet suggested to me and indeed also to other astronomers that it would be possible to see it by day. However all attempts which I made to this end in the final days of September in order to see it in the 5-foot telescope of the local Meridian line were in vain. A few days later I plainly convinced myself of the impossibility of seeing /320 during the day. At sunset on the particularly clear evening of Oct. 4 I directed the 3 1/2-foot telescope of the small local equatorial on the location of the comet in order to perceive the comet when it first became visible in the twilight. Only at 5:51, i.e. 20 minutes after sunset, did I see the core and emanation for certain. Thus it is not surprising that it remains invisible during bright daylight in a telescope with a 4-inch opening. That it can also not be seen in telescopes of the greatest optical power during bright daylight is shown by the data reported by Mädler in No. 1167 of the <u>Astronomisch Nachrichten</u>.

The faintness of the core of the comet, compared for example with the bright luminosity of the big comet of 1853, was very striking. I do not beleive that the light of the core was brighter than that of a star of say the third or fourth magnitude. In bright twilight I have seen stars of the fifth order in the telescope nearly as early as the comet. In this respect the comet differs considerably from the comet of 1744 which it otherwise so much resembles. In his description of this comet, which is worth reading, Heinsius reports that its brightness equals that of Venus and that farsighted persons can still see it in the twilight with the naked eye a few minutes prior to sunrise. Cassini, who by the way states that the greatest length of the tail of this comet is 34°, even mentions that it can be perceived in bright daylight with the naked eye. Compared with this appearance our comet must be considered as one of moderate brightness.

3

The most interesting feature shown by the Donati Comet was indisputably the bright, semi-circular emanation of the core on the side towards the sun and its continual development. If this phenomenon on its own attracted our attention to a high degree, it had to be attracted to an even greater degree if one compared the comet seen here with the description of earlier comets. Everyone who had an opportunity to compare the 8 excellent drawings by Heinsius of the comet of 1744 with the appearance of our comet must notice the extraordinary similarity of the two. On particular days one could have, with small changes, passed off one of the Heinsius drawings for an illustration of our comet. Even the development which appears in the shapesd of the emanations in the Heinsius drawings is exactly similar to that of the Donati Comet. The third comet which is related to these two in terms of its [test illegible] is Halley's Comet [text illegible] ... so masterfully presented and explained in his classical treatement /321 (Astronomische Nachrichten, Vol. 13, p. 185). As everyone knows, Halley's Comet exhibited changes in the direction of its emanation whose observations were represented by Bessel by means of an oscilation of a constant period and whose explanation led him to the assumption of a polar force acting on the comet from the sun. Because of the similarity of the appearance Bessel assumes similar oscilation in the case of the comet of 1744 and regards the development of its emanation as a new proof of a presence of a polar force. For the same reason our comet would suggest the same conclusions.

In fact, careful observation of the direction of the emanation has established the fact that this direction was always subject to fluctuations. Yet with a diffuse, partially washed out and irregular shape of the emanation sector the difficulty of determining a definite base line was so great that it is not surprising if the measurements of various observers give constant or even varying differences. The result of this uncertainty is only that the observations will not permit us to recognize the presence or non-presence of an oscillation period. There can be no doubt about the existence of a change in the direction.

First of all I here give the list of our observations of the apex angle of the emanation with the direction of the comet to the sun along with the differences in the two directions. The apex angle is with reference to the axis of the shape of the emanation sector.

A m. Zt. Altona		B Beob. Pos.W. p'	C Richt zur ☉ p^0	$p^0 - p'$
Sept.20	8^h 0^m	148° 0′	178° 39′	+30° 39′
21	8 0	134 0	179 44	+45 44
22	7 10	180 0	180 57	+ 0 57
28	6 40	195 0	192 50	− 2 10
29	6 45	162 0	195 49	+33 49
30	7 5	192 0	199 8	+ 7 8
Octb. 1	8 27	218 15	203 4	−15 11
2	7 0	216 15	206 54	− 9 21
4	6 22	199 42	216 22	+16 40
5	6 25	233 21	221 48	−11 33
5	6 25	246 0	221 48	−24 12 D (Schätzung)
6	6 50	236 24	227 46	− 8 38
7	6 37	226 15	233 48	+ 7 33
8	6 25	229 15	239 55	+10 40
9	6 22	244 9	245 59	+ 1 50
10	6 20	253 57	251 46	− 2 11
12	6 20	239 40	261 53	+22 13
16	5 40	300 9::	274 28	−25 41

Key: A) Altona time
B) Apex angle
C) Direction to the sun
D) Estimated

/322 Assuming that the fluctuations in the emanation in the plane of the orbit had occured I tried to present the observations using a periodic formula. In general, a period of 4-5 days seemed to fit the observations, yet I was able to concur only with a portion of them while the ones remaining exhibited large deviations from the formula. I refrain from reporting these investigations here, especially since the obserrvations can be of only secondary value beside those made with large measuring instruments. The measurments published by Mädler in No. 1167 of the Astronomische Nachrichten in fact deviate very considerably from ours, and indeed in the constant sense so that no other explanation of this difference is possible other than that given the smallness of our instruments only the brighter portions of the sector obviously stand out, whereas others, which in larger instruments would be recognized as belonging to this sector, merge more with the surroundings. In general the deviation is such that Mädler states that the direction of the sector is further to the right in an inverting telescope. On this side the sector was constantly weaker and more washed out up to about Oct. 6 than on the opposite side so that the deviation seemed explainable to me for this reason alone.

The excellent agreement of the Dorpat Measurements caused me also to investigate them with respect to the observed fluctuations and here I give the result in more detail, since this series of measurments is certainly more important than my own. The observationstime is expressed in terms of the Berlin Meridian and is freed of aberations. I have averaged the various measurements made on a single evening.

Apex Angle of the Emanation Observed in Dorpat

A Berl. Zeit	B Beob. Pos. W. p'	C Richt. zur $\odot p^\circ$	$p^\circ - p'$
Sept. 17,2556	184° 11′	176° 13′	— 7° 58′
19,2802	156 57	177 44	+20 47
20,2581	145 42	178 38	+32 56
21,2902	146 15	179 43	+33 28
22,3493	145 10	181 2	+35 42
24,3185	158 51	184 4	+25 13
25,2381	167 30	185 45	+18 15
26,2536	175 7	187 50	+12 43
27,2309	170 31	190 6	+19 35
28,3089	166 54	192 56	+26 2
29,2487	167 41	195 43	+28 2
30,2147	167 38	198 50	+31 12
Octb. 6,3011	213 19	227 50	+14 31
7,2303	206 29	223 29	+27 0
8,1977	227 13	239 27	+12 14
Octb. 9,2046	238° 3′	245° 36′	+ 7° 33′
12,2195	236 11	261 41	+25 30
13,1927	237 8	266 52	+28 44
14,1922	237 17	269 81	+32 14

Key: A) Berlin time
B) Apex angle
C) Direction with respect to the sun

The observations of Sept. 19-30 show a very regular movement but the measurements of Oct. 6-9 and also the later measurements deviate considerably from this regualrity. Yet these relationships emerge more distinctly if one reduces this data to the plane in which the possible fluctuations in the emanation possibly took place, namely to the plane of the comet orbit. I will give these reductions below.

4

In his treatment of Halley's Comet Bessel (Astronomische Nachrichten Vol. 13 p.193) develops in detail the formulas by means of which one can compare the observed apex angles with any desired assumptions about the plane in which the oscillations took place, For the sake of relevance I will here only quote those formulas which are used in the following calculations.

Starting with the core of the comet consider a spherical triangle described on the celestrial sphere whose corners in turn are: the cometocentric location of the earth, the pole of the axis of rotation of the oscillations and the pole of the axis of the emanation which in the case of our comet is thus represented by the axis of the sector. Let S stand for the side of this triangle which is formed by the angle between the location of te the earth and the pole of the axis of rotation. Let u—P' stand for the angle at the pole of the axis of rotation and P—p the angle at the point of the earth. Then u is the apex angle of the emanation and P' is the apex angle of the point of the earth at the point of the rotation axis. P is the apex angle of the rotation axis at the geocentric point of the comet, and finally p is the observed apex angle of the emanation. If we designate the geocnetric AR and the declination of the comet by α and σ respectively and the same coordinates for the pole of the axis of rotation by A and D, then S. P. and P'. are determined by the following formulas:

$$\left.\begin{aligned} \cos S &= -\sin\delta \sin D - \cos\delta \cos D \cos(A-\alpha) \\ \sin S \cos P &= \cos\delta \sin D - \sin\delta \cos D \cos(A-\alpha) \\ \sin S \sin P &= \cos D \sin(A-\alpha) \\ \sin S \cos P' &= -\sin\delta \cos D + \cos\delta \sin D \cos(A-\alpha) \\ \sin S \sin P' &= -\cos\delta \sin(A-\alpha) \end{aligned}\right\} \ldots (1)$$

For logrithmatic calculations I have found the following transformation easier to work with:

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$$\left.\begin{aligned} &\operatorname{tang} P = \frac{\sin G \cdot \operatorname{tg}(A-\alpha)}{\cos(G+\delta)}, \quad \operatorname{tang} P' = \frac{\sin G' \operatorname{tg}(A-\alpha)}{\cos(G'+D)} \\ &\operatorname{cotg} S = -\cos P \cdot \operatorname{tg}(G+\delta) = \cos P' \cdot \operatorname{tg}(G'+D). \\ &\quad \text{Hier ist} \\ &\operatorname{tg} G = \operatorname{cotg} D \cos(A-\alpha) \quad \operatorname{tg} G' = \operatorname{cotg} \delta \cos(A-\alpha). \end{aligned}\right\} \ldots (2)$$

In the case under consideration it is assumed that all fluctuations have occured in the plane of the orbit and that the axis of rotation is perpendicular to this plane. From this it follows, taking as a basis the eliptical elements of Bruhns, that

A = 76°56'   D = −4°10'.

The relationship between angles u and p then follows from the following equations

$$\left.\begin{aligned} n \cos(p-P) &= -\cos(u-P') \cos S \\ n \sin(p-P) &= -\sin(u-P') \end{aligned}\right\} \ldots\ldots\ldots\ldots (3)$$

where n stands for the perspective shortening of the axis of the emanation. If we first substitute the observed apex angle p' for p and then substitute the apex angle of the sun (earlier designated with p°), then from the difference of the two above values of u we get the angle in the plane of the orbit between the emanation and the direction with respect to the sun.

In the following ephemerides I give a list of the quantities derived from these formulas which are used in the following study as also with the calculations pertaining to the tail. It should be noted only that u° is that angle which one obtains if one substitutes the apex angle of the sun for p in equations (3).

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A m.B.Z.	p	p'	s	u^0	A m.B.Z.	p	p'	s	u^0
Sept. 17,3	270° 10′	126° 15′	82° 45′	187° 46′	Oct. 1,3	283 12	120 54	59 2	229 1
18,3	270 54	126 14	81 43	190 16	2,3	283 47	119 33	56 10	232 8
19,3	271 41	126 13	80 38	192 51	3,3	284 6	117 54	53 5	235 18
20,3	272 31	126 10	79 31	195 33	4,3	284 4	115 53	49 51	238 30
21,3	273 24	126 5	78 18	198 21	5,3	283 33	113 25	46 27	241 35
22,3	274 21	125 56	76 58	201 15	6,3	282° 24′	110° 25′	42° 55′	244° 36′
23,3	275 19	125 43	75 32	204 13	7,3	280 28	106 46	39 19	247 32
24,3	276 20	125 28	73 59	207 14	8,3	277 34	102 17	35 47	250 23
25,3	277 21	125 10	72 19	210 18	9,3	273 30	96 45	32 26	253 10
26,3	278 24	124 49	70 30	213 24	10,3	267 58	90 0	29 26	255 55
27,3	279 27	124 24	68 34	216 30	11,3	260 46	81 50	26 49	258 38
28,3	280 30	123 48	66 29	219 38	12,3	252 10	72 21	24 46	261 20
29,3	281 31	122 58	64 11	222 46	13,3	242 29	61 54	23 31	264 0
30,3	282 26	122 0	61 43	225 54	14,3	232 7	51 9	23 1	266 38

Key: A) Meridian Berlin Time

If by using the quantities taken from this table one reduces the measurements of Mädler to the plane of the comet orbit then one gets the following data to which I immediately add the pertinent directions with respect to the sun along with the differences between the two directions.

	A P.W. in d. Bahnebene u'	B P.W. der Sonne u^0	$u^0 - u'$
Sept. 17	245° 42′	187° 39′	−58° 3′
19	145 39	192 48	+47 9
20	139 47	195 26	+55 39
21	141 3	198 19	+57 16
22	141 25	201 24	+59 53
24	153 31	207 18	+53 47
25	165 13	210 7	+44 54
26	178 19	213 16	+34 57
27	174 10	216 16	+40 6
28	166 12	219 40	+53 28
29	168 56	222 37	+53 41
30	167 43	225 40	+57 57
Octb. 6	228 10	244 30	+16 20
7	227 17	247 20	+20 3
8	238 13	250 53	+12 40
9	245 59	252 55	+6 56
12	237 54	261 7	+23 13
13	237 9	263 43	+26 34
14	236 1	266 22	+30 21

Key: A) Apex angle in the plane of the orbit
B) Apex angle of the sun

Attentive considerations of the differences between u° and u' shows that it is impossible to represent the course of these figures by means of a continuous function. The same thing will occur if one compares the observations with another assumption as to the location of the oscillation plane or as to the type of oscillations. If one ignores the first very deviant observation then it is indeed possible to give a good representation of the measurements from Sept. 19 to 26 by means of a simple sine function and only the later measurements deviate considerably. The measurements from Oct. 6 to 9 show large leaps and can in no way be integrated either with the preceding or subsequent measurements. A close look at the variation in these numbers gave me the impression as if the around the beginning of October a sudden disturbance had changed the former direction. I remember here the variable dark gaps which were /326 observed in the sector from Oct. 3 to 8 by all observers furnished with powerful telescopes. It is possible that at the time of the perihelion the emanation of different types of matter, which perhaps separated more and more as the comet came closer and closer to the sun, caused disturbances in the usual shape of the sector which made it impossible for us to distinguish a regular change in direction over time.

Thus with respect to what was being sought the study of the two series of measurements produced negative results. However it would be very risky to infer from this that no oscillations were present anywhere. Bessel, in his discussion (Astronomische Nachrichten Vol. 13, p. 200) draws attention to the fact that the oscilations in the emanation are not to be explained for explained by the emanation from different points of the core but that they are due only to oscillations of the core itself. This explanation which is valid for Halley's Comet is also applicable to our comet, which in many respects is so similar to Halley's Comet. With Halley's comet the shape of the emanation which is in the form of a long, narrow beam makes it easy to recognize directly the osscillations of the core from the oscillations in the shape of the emanation. With our comet the opposite occured. The emanation extended over a curve of more than 180°. Its uneven brightness and partly irregular shape made it almost impossible to determine a definite direction for the shape of the comet. Nevertheless, the fact that we observed changes in direction whose existence no one who observed the comet for a few evenings will refute makes it likely that the direction of the core was subject to changes which manifested themselves, only more concealed, by means of the emanation. However this also suggests to me the possibility of an oscillation of the core and at the same time the effect of a polar force on our comet.

Moreover, the existence of a polar force from the sun acting on the comet, i.e. a force which is considerably different from the usual force of gravity, clearly showed up as soon as the development of the emanation was observed. Around mid-September the first trace of the emanation appeared in the form of a beam of light on the side of the core next to the sun. The appearance was entirely similar to that which was observed by Bessel in the case of Halley's comet on Oct. 2 and by Heinsius in the case of the comet of 1744 on Jan. 25. As the comet approached the sun the emanation spread out over an ever larger periphery of the core until at the time of the perihelion and still somewhat later it extended over two quadrants. Later the /327 emanation angle diminished and on Oct. 16 it was considerably smaller than at the beginning of the month.

Along with this development in the shape of the emanation the shape of the particles rising towards the sun began to curve away from their initial direction and separate into two different branches which flow over into the tail with increasing intensity from one day to the next. These two phenomena are identical with those of the comet of 1744 and those conclusions which Bessel deduced from the phenomena of this comet on which to base the assumption of a polar force can be fully applied in the case of our comet.

I want to mention here at once yet another striking similarity which occured in the case of the shape of the emanation of the two comets and which merits further investigation. At the time of of the first development of the emanation the laterr was washed out in the direction of the apex of the coma. The appearance was that of an emanation coming directly out of the core and into the coma and then, varying in direction, into the tail. This was how the emanation still looked on Sept. 22. On Sept. 28, on the other hand, the sharply delineated sector had formed which other astronomers had already seen a few days earlier and which I certainly observed, however not with the same degree of sharp delineation, up to Oct. 12. Comparing the values of the radius of this sector on different days snow shows that from Oct. 1 to about Oct. 10 the steady increase in this radius had occured (which is not explained solely by the constant approach of the comet to the earth), so that on Oct. 9 or 10 the radius was about twice as long as on Oct. 1. These two pehnomena are now found to be quite similar to those of the comet of 1744. If one compares the shapes which Heinsius gives for Jan. 31 with those of Feb. 8, 9 and 16, then one will find the same change in the emanation and the same sharp delineation of the sector in the final shapes. Even the increase in the radius immediately emerges from his drawings. If one also compares with this the shape which Bessel drew of Halley's Comet on Oct. 22, then it seems as if the three comets had this sharply delineated shape in common and the explanation of this is found in one and the same cause. Bessed did not express his opinion on this point. Hopefully, however, the observations of this phenomenon in the case of our comet, which are still yet to be fully reported, will enable the true cause of this phenomenon to be discovered. However the special consideration of this matter, given the nature of the forces (perhaps /328 electrical) in question here, belongs more to physics than to astronomy.

5

The core of the comet on the side towards the tail was bordered by a very dark zone having a narrow width and which could be observed far up into the tail and whose axis described a straight line in its initial direction. The zone formed the inner border of the two branches of the tail which lay almost symmetrically on both sides of the same. They apparently represented the axis of the tail and measurement of the apex angle of their axis at the same time gave the apex angle of the initial direction of the tail. In No. 1167 Mädler gave a series of measurements of this direction. To these I will add those measurements made by me as well as a number of Figures which I owe to the kindness of Dr. Winnecke. Dr. Winnecke made the measurements from Sept. 2 to Oct. 14. However I omit here the measurements made from Sept. 2 to 17, while their study is of little value due to the unfavorable position of the earth to the comet orbit with respect to the purpose being pursued here. In the list below the observation times are converted to the Berlin Meridian and freed of aberations. Also included are the apex angles of the radius vector extending over the core along with the differences between this direction and the observed direction of the tail.

	A Beob.P.-W. p'	B Verl.d.R.V. p^0	p^0-p'	C Beob.-Ort.
Sept. 17,280	354°42′	356°14′	+1°32′	Dorpat (*Mädler*)
17,289	355 42	356 14	+0 32	Pulkowa (*Winnecke*)
18,294	356 18	356 57	+0 39	Pulkowa
19,253	353 2	357 42	+4 40	Dorpat
19,268	356 40	357 43	+1 3	Pulkowa
21,309	359 32	359 45	+0 17	Dorpat
23,349	1 44	2 30	+0 46	〃
24,319	0 38	4 4	+3 26	〃
24,407	4 33	4 15	−0 19	Pulkowa
25,244	4 2	5 45	+1 43	Dorpat
25,290	4 28	5 51	+1 23	Pulkowa
26,245	5 31	7 49	+2 18	Dorpat
27,358	8 40	10 25	+1 45	Pulkowa
28,283	10 17	12 52	+2 35	Altona
29,249	14 28	15 43	+1 15	Pulkowa
29,290	11 40	15 50	+4 10	Dorpat
30,227	18 34	18,53	+0 19	〃
Sept. 30,245	17°27′	18°56′	+1°29′	Pulkowa
30,301	16 0	19 8	+3 8	Altona

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	p'	p^0	p^0-p'	C Beob.-Ort
Oct. 1,356	18 45	23 4	+4 19	〃
2,298	20 15	26 54	+6 39	〃
4,272	31 12	36 22	+5 10	〃
5,253	38 18	41 43	+3 25	Pulkowa
5,274	36 39	41 49	+5 10	Altona
6,291	42 24	47 46	+5 22	〃
6,303	38 54	47 51	+8 57	Dorpat
7,234	49 29	53 30	+4 1	〃
7,235	48 7	53 30	+5 23	Pulkowa
7,282	44 15	53 48	+9 33	Altona
8,225	54 57	59 37	+4 40	Pulkowa
8,274	52 36	59 55	+7 19	Altona
9,207	56 8	65 36	+9 28	Dorpat
9,243	59 10	65 49	+6 39	Pulkowa
9,272	59 9	65 59	+6 50	Altona
10,271	63 45	71 46	+8 1	〃
12,230	74 15	81 43	+7 28	Dorpat
12,270	74 0	81 53	+7 53	Altona
13,222	82 8	85 59	+3 51	Pulkowa
13,232	75 40	86 1	+10 21	Dorpat
14,216	77 15	89 36	+12 21	〃

Key: A) Observed apex angle
B) Extension of the radius vector
C) Place of observation

The certainty of the measured apex angles varied considerably during the course of the appearance. Prior to Sept. 22 the dark axis of the tail was indeed narrow but washed out, whereas after Sept. 28 it appeared strikingly distinct with a sharply defined border up until Oct. 6 or 8. After this time the zone increased in width and became more and more washed out so that in the final days in which the comet was still veisible I found it extremely difficult to pick out its axis with certainty. A glance at the differences between p'' and p' shows that the variation in the measurements over time is quite in keeping with the circumstances. In the first 10 days there are considerable differences followed by a good agreement in the figures towards the middle period of the appearance, while rather large deviations again appear towards the end.

The observations seem to indicate a constant increase in the angle between the axis of the tail and the extension of the radius vector. However this behavior in the last series of figures is only a result of the changed position of the earth with respect to the comet orbit. It disappears as soon as one converts the measurements to the plane of the comet orbit according to the formulas given in § 4. in making this conversion I simply took the average of the different measurements made on a single evening.

The figures in the column under heading n indicate the perspective shortening in which the axis of the tail appeared to us. Consideration of these figures shows that from Sept. 27 to 30 the apparent shape of the comet exactly coincided with a section made through the axis. /330

	A Red.P.-W.u'	B Pos.d.R.V.u°	n	u°—u'	C Beob.
Sept. 17,285	1°44'	7°44'	0,827	+ 6° 0'	2
18,294	7 3	10 15	0,876	3 12	1
19,261	4 9	12 45	0,852	8 36	2
21,309	16 51	18 23	0,948	1 32	1
23,249	21 31	24 22	0,972	2 51	1
24,363	22 0	27 26	0,975	5 26	2
25,267	25 8	30 12	0,986	5 4	2
26,245	26 13	33 14	0,990	7 1	1
27,358	32 1	36 41	0,999	4 40	1
28,283	33 19	39 44	0,999	6 25	1
29,270	36 38	42 40	0,999	6 2	2
30,258	42 23	45 46	0,987	3 23	3
Oct. 1,356	41 30	49 11	0,987	7 41	1
2,298	41 4	52 8	0,986	11 4	1
4,272	51 33	58 27	0,944	6 54	1
5,264	55 27	61 29	0,897	6 2	2
6,297	56 41	64 36	0,915	7 55	2
7,250	61 6	67 24	0,895	6 18	3
8,249	64 35	70 15	0,886	6 40	2
9,241	66 0	73 0	0,887	7 0	3
10,271	68 40	75 50	0,889	7 10	1
12,250	74 14	81 12	0,912	6 58	2
13,227	77 6	83 48	0,922	6 42	2
14,216	74 34	86 25	0,932	+11 51	1

Key: A) Converted apex angle
B) Position of the radius vector
C) Number of observations

The marked change in the apex angles in the previous table has disappeared here. In order to present these figures more clearly I have divided them into the following six groups.

Mean Values

				A
Sept. 17—21	u°—u'	=	+5°59'	6 Beob.
23—28		=	+5 15	8
29—Oct. 4		=	+6 0	8
Oct. 5—7		=	+6 41	7
8—10		=	+6 55	6
12—14		=	+7 38	5

Key: A) Number of observations

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If from all of these figures we take a mean, then with respect to the number of observations we obtain +6°18' for the difference $u^o - u'$. Thus the initial direction of the tail in the plane of the orbit was inclined backwards by this angle towards the extension of the radius vector. This result seems very interesting to me, as a comparison of it with the above mean values shows that in the course of the phenomenon observed here, i.e. since the formation of the emanation, the initial direction of the tail formed a constant or at least very nearly constant angle with the direction to the sun in the plane of the orbit. This seems to imply a steady small increase, while one can justly attribute /331 the deviations present among the individual mean values to the uncertainty of the observations.

To my knowledge no one has been able to show with any other comet this constancy in the direction of the tail and its relationship to the comet orbit. In recent times one has paid more attention to this direction and from now on it will be of interest to determine each time the position of the same in the plane of the orbit. Later on in considering the remaining conditions of the tail I will return to the constancy of this angle in the case of our comet.

6

To the brilliant appearance of the large comet of 1811 we owe an hypothesis concerning the formation of the comet tail which was proposed and backed up by a number of reasons by Olbers.[1] Olbers assumes that the core of the comet expells particles in the direction of the sun and that a repelling force of the sun acts on these same particles which now as a result of this double effect describe paths which become visible to us in the tails of comets. By way of analogy this phenomenon further leads Olbers to compare the supposed repulsive forces with electrical forces. Olbers' hypothesis most simply explains the observed pehnomena. It is also the only one which up to now has been pursued theoretically.

Brandes was the first to base theoretical considerations on this hypothesis.[2] First of all he determined that curve in which

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[1] Monatl Correspond. Vol. 25 p. 1.
[2] *Ibid*, Vol. 26, p. 533.

the forces of the comet and the sun maintain themselves in equilibrium, with special consideration of the particular shape of the comet of 1811. But then, assuming that the sun and comet are suspended in space, he looks for the path in which a freely moving particle is carried off in the tail. Brandes later published a few calculations on the shape of the tail of this comet.[3] I do not know, however, whether he or any other astronomer who observed the appearance of the large comet of 1811 carried out further studies in the next 20 years on this subject.

The return of Halley's Comet in 1835 led Bessel to consider /332 these phenomena and at the same time to pursue and further develop Olbers' hypothesis. To Bessel we owe a complete theory of all the phenomena which he observed in connection with Halley's Comet. Under the same conditions which occured in the case of this comet one can apply the theory to all comet phenomena. Bessel assumes that the effect of the repulsive force of the sun is inversely proportional in different points of the orbit to the square of its distance away and that the tail particles, after they have left the shpere of influence of the comet with a given velocity and in a given direction, are to be considered as freely moving points which describe the curve of the tail as a result of the constant effect of the force of the sun. While he only considers their movement outside the sphere of influence of the comet this of course precludes the investigation of the same in the immediate vicinity of the core. However Bessel probably did this since at least with Halley's Comet this sphere of influence is a very small quantity. The second condition excludes the assumption of a force by means of which the particles could have a repulsive effect on one another. Should such an effect be present immeidately after their emanation from the core then to be sure after rising into the tail it would in-

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1. Zeitschrift für Astronomie [Journal for Astronomy] by Lindenau and Bohnenberger, Vol. 1.

creasingly fade away. It also seems impossible to determine the strength of this force by observations. Bessel ignored it entirely in his theory.

A third assumption in the application of Bessel's theory is that the movement of the tail particles through the medium in which they are speeding are not subject to any considerable resistance. The acceleration of the rotations of Encke's Comet can be explained by a resistance of the ether which the comet is subject to only in the parts of its path very close to the sun. At considerably larger distances away, i.e. at a distance in which the tail particles of our comet were located, this effect will be very much smaller if it is present at all. If we now compare the great brightness of the tail of Donati's Comet with the weak luminosity of Encke's Comet, which permits at least an approximate inference as to the density of these two substances, we will then arrive at the conclusion that the influence of a resistance caused by the ether will not be so great so as to significantly modify the results obtained without taking this resistance into account. In addition, it would also indeed be impossible at the present to calculate the influence of a resisting agent on the movement of the tail particles.

Bessel sets up the coordinates of a tail particle according /333
to increasing powers of the time which has elasped since the emergence of the particle from the sphere of influence of the comet. If we let ξ stand for the coordinate parallel to the radius vector of the comet in the plane of the orbit and η stand for the perpendicular to the radius vector in the same plane, then these coordinates are determined by the following equations:

$$\left.\begin{aligned} \xi &= a + b\tau' + c\frac{\tau'^2}{2} + d\frac{\tau'^3}{6} \\ \eta &= a' + b'\tau' + c'\frac{\tau'^2}{2} + d'\frac{\tau'^3}{6} \end{aligned}\right\} \cdots\cdots\cdots\cdots (4)$$

ξ is assumed to be positive in the direction from the core of the comet towards the tail and η is assumed to be positive in the direction opposite to that in which the comet is moving. The quantities a, b, c, d, a', b', c', and d' depend on the components of motion of the comet and on the constants determining the relative motion of the particles. If we assume that the particle at time $t-\tau$, where t stands for the time of the observation, has left the spehre of influence of the comet, then τ is the time elasped since leaving the sphere of influence. The coordinates ξ and η are not set up directly according to the powers of this quantity but rather according to the powers of the quantity τ' which is related to τ by the following equation:

$$\boxed{\tau = \tau' - \frac{2}{3}\frac{c \sin v}{r\sqrt{p}} \cdot \tau'\tau' \quad \ldots\ldots\ldots\ldots\ldots (5)}$$

The terms c, v, r and p here have their usual meanings. Bessel /334
used this transformation because by introducing it enabled him to derive the various constants of the motion of the particle separately from one another. If τ' is eliminated from the above equations (4), then the equation gives the curve in which particles are found at time t which prior to this time left the sphere of influence of the comet with the same velocity at the same point.

Before I perform this elimination I must still mention a few things about the quantities on which the coefficients in the above equations (4) depend. Bessel uses μ to designate the force with which the sun acts on the particles. He uses g to designate the velocity with which they leave the sphere of influence of the comet. G stands for the angle of direction of this movement with respect to the radius vector measured from the radius vector towards the direction from which the comet is moving. He uses f to designate the radius of the sphere of influence and F to designate the angle of its direction with respect to that of the radius vector. If we replace the values of the coefficients in equations (4) which can be expressed by these quantities and by those which determine the orbital motion of the comet, then we obtain the following equation:

$$\left.\begin{aligned}\xi = &-f\cos F-\left(g\cos G+f\sin F\frac{\sqrt{p}}{rr}\right)\tau' \\ &+\left\{\frac{(1-\mu)}{rr}-g\sin G\frac{2\sqrt{p}}{rr}+g\cos G\frac{4e\sin v}{3r\sqrt{p}}-f\cos F\left(\frac{2\mu}{r^3}+\frac{p}{r^4}\right)+f\sin F\frac{10e\sin v}{3r^3}\right\}\frac{\tau'\tau'}{2} \\ &-\left\{g\cos G\left(\frac{4\mu}{r^3}+\frac{3p}{r^4}\right)-g\sin G\frac{14e\sin v}{r^3}\right\}\frac{\tau'^3}{6} \\ \eta = &\ f\sin F+\left(g\sin G-f\cos F\frac{\sqrt{p}}{rr}\right)\tau'-\left\{g\cos G\frac{2\sqrt{p}}{rr}+g\sin G\frac{4e\sin v}{3r\sqrt{p}}+f\sin F\left(\frac{\mu}{r^3}-\frac{p}{r^4}\right)-f\cos F\frac{10e\sin v}{3r^3}\right\}\frac{\tau'\tau'}{2} \\ &+\left\{(1-\mu)\frac{2\sqrt{p}}{r^4}+g\sin G\left(\frac{\mu}{r^3}+\frac{3p}{r^4}\right)+g\cos G\frac{14e\sin v}{r^3}\right\}\frac{\tau'^3}{6}\end{aligned}\right\}\ldots(6)$$

Bessel now assumes that f and g are small quantities in comparison with (1—μ) whose products and squares can be ignored. On the basis of this assumption he eleimates τ' from the first equations (4) and substitutes the following approximate value:

$$\tau' = \frac{R-b}{c}-\frac{dRR}{6c^3}\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

where

$$R = \sqrt{\{2c(\xi-a)+bb\}}\ldots\ldots\ldots\ldots(8)$$

If this value of τ' is substituted in the second equation of equations (4) then, ignoring the quantities involving squares and products of f and g, we obtaint the following equation:

$$\eta = a'+\frac{b'}{c}R+\frac{cc'-bd'}{2c^3}R^2+\frac{d'}{6c^3}R^3\ \ldots\ldots(9)$$

If their values are substituted for the coefficients and if we also ignore quantities on the order of the radius of the sphere of influence f, then we obtain:

$$\eta = g \sin G\, r r \frac{R}{(1-\mu)} - g \sin G \frac{2r^3 e \sin v}{3\sqrt{p}} \cdot \frac{R^2}{(1-\mu)^2} + \frac{r r \sqrt{p}}{3} \frac{R^3}{(1-\mu)^3} \quad \text{(10)}$$

For very large values of ξ we can now regard the second term bb in equation (8) as being very small in comparison with the first term and thus we obtain the following approximate value: R = $\sqrt{2c\xi}$. /335

If we substitute this value we obtain:

$$\eta = g \sin G \frac{r}{\sqrt{(1-\mu)}} \sqrt{(2\xi)} - g \sin G \frac{2 r e \sin v}{3\sqrt{p}} \cdot \frac{\sqrt{(2\xi)}}{(1-\mu)} + \frac{\sqrt{p}}{3r} \frac{(2\xi)^{\frac{3}{2}}}{\sqrt{(1-\mu)}} \quad \text{(11)}$$

Leaving out the factor $\frac{1}{1-\mu}$ Bessel mistakingly sets the second term of this expression equal to

$$-g \sin G \frac{2 r e \sin v}{3\sqrt{p}} \sqrt{(2\xi)}$$

This error was also passed onto the following equation. However in using this equation to determine the repulsive force of the sun from observing the tail of Halley's Comet this error did not have any effect while this force was derived solely from the last term of the equation. If we divide equation (11) by ξ then we obtain ~~the~~ tangent of the angle which the line from the core to a point on the curve of the tail in the plane of the path forms with the extension of the radius vector. If we let this angle be designated by ϕ then we obtain the following equation:

$$\tan \varphi = g \sin G \left\{ \frac{r \cdot \sqrt{2}}{\sqrt{(1-\mu)}\sqrt{\xi}} - \frac{4 r e \sin v}{3\sqrt{p}(1-\mu)} \right\} + \frac{2\sqrt{2p}}{3r} \frac{\sqrt{\xi}}{\sqrt{(1-\mu)}} \quad \text{(12)}$$

7

The observation of two points lying in the delineation curve of the tail gives two equations if one derives the values of ϕ and ξ and substitutes them in equation (12). By combining the two unknowns the quantities (1—μ) and g sin G can be derived at the same time. In this manner I calculated from my observations of the curve of the tail the values of (1—μ) and g sin G for each day. However before reporting the results of this investigation here I must mention a few reductions which have to done to the observations.

The above equations refer only to a curve located in the plane of the comet path, whereas our observations refer to that bordering curve which lies in the section of the tail prependicular to our visual line. The assumption that the observed curve lies in the plane of the comet path would at least not inconsiderably distort the results. Hence it is necessary first of all to derive from the observed curve that in which the tail is cut by the plane of the orbit, or in other words one must reduce the observed points to the plane of the orbit. Consider a straight line drawn from the core of the comet to that point (C) in which the line from the earth to the observed point in the tail meets the orbital plane and a second straight line perpendicular to the plane of the orbit (which would thus coincide with the former axis of rotation).

Furthermore consider a spherical triangle described from the core of the comet throught the point at which these two lines meet the celestial spehre and through the point of the earth. In this triangle S has considered in § 4, is one side and p—P and P'—u are the angles formed with this side. The /336
third angle opposite side S I designate with t and the side opposite angle P—u by T. The third side is equal to 90". Thus in this triangle one has the following:

$$\left.\begin{aligned} \cos T &= \sin S \cos(u-P') \\ \sin T \cos(p-P) &= -\cos S \cos(u-P') \\ \sin T \sin(p-P) &= - \sin(u-P') \\ \sin t &= \sin S \sin(p-P) \end{aligned}\right\} \ldots\ldots (13)$$

I now let s stand for the arc of the largest circle between the observed point in the tail and the core of the comet and let p' stand for the apex angle of the direction of the same on the comet core. Substituting p' in equations (13) gives the corresponding angle u'. I let Φ' stand for the difference (180 (180° + u°) − u'. Now at the point observed at which the visual line is tangent to the curve of the tail consider a line h drawn perpendicular to the first perpendicular line which may meet the plane of the comet path at point D. The plane in which the straight line h is drawn is thus determined since it is perpendicular to the line from the core to point D. The line h can almost be assumed to be known, since it can be derived from the width of the tail in the vicinity of the observed point. For the curve in which the tail is cut by the above mentioned plane I have substituted a circle whose radius is line h.

Now consider a spherical triangle lying through the points at which the baselines from the core towards the observed point in the tail and towards points C and D meet the celestial sphere. In this triangle one angle equals 90° − t and another equals 90°. I let m stand for the side opposite the first angle and n for the side opposite the right angle. The third side is designated by ω. Then, if l stands for half the apparent width of the tail in the vicinity of the observed point, we obtain the following approximate values: /3

$$\left.\begin{aligned} \sin m' &= \frac{\sin T}{\sin s}\, tg\, l \\ \sin \sigma' &= tg\, m' \cdot tg\, t \end{aligned}\right\} \cdots\cdots\cdots\cdots (14)$$

and thus:

$$\left.\begin{aligned} \sin m &= \sin m' \cdot \frac{\sin (T+\sigma')}{\sin T} \\ \sin n &= \sin m \cdot sec\, t \\ \sin \sigma &= tg\, m \cdot tg\, t \end{aligned}\right\} \cdots\cdots\cdots (15)$$

Now if we let Δ stand for the distance of the observed point from the core and ρ for the distance of the latter from the earth, then we obtain:

$$\left.\begin{aligned}\varphi &= \varphi' + n - m,\\ \Delta &= \frac{\rho \sin s}{\sin(T+s+\sigma)}.\end{aligned}\right\}\ldots\ldots\ldots\ldots(16)$$

Equations (14) through (16) contian an approximate reduction to the orbital plain, but in the present case it is sufficiently accurate.

8

The following table contains the positions observed by me from Sept. 28 to Oct. 28 in the leading and trailing edge of the tail with respect to the beginning of 1858. After Oct. 8 the appearance and delineation of the tail had so changed the I will analyze the continuation of these observations separately below.

Observed Points in the Leading Edge of the Tail

A M. B. Z.	α	δ	α☄	δ☄
Sept. 28,308	193° 21′	+39° 5′	192° 17′	+32° 24′
28,309	190 13	+46 13		
Octb. 1,350	202 28	+37 55	200 17	+28 19
1,350	199 36	+50 26		
2,301	205 42	+39 16	202 57	+26 39
2,301	202 46	+49 44		
4,347	215 2	+36 10	209 3	+22 11
4,347	213 55	+46 38		
5,333	219 26	+35 55	212 6	+19 39
5,333	217 55	+50 25		
6,326	223 21	+28 55	215 12	+16 48
6,326	224 29	+48 26		
8,319	232 10	+25 26	221 27	+10 30
8,319	236 54	+42 51		

Key: A) Meridian Berlin Time

Observed Points in the Trailing Edge of the Tail

	α	δ
Octb. 1,350	198°30′	+41°20′
2,301	201 42	+39 31
4,347	206 51	+39 55
5,333	211 55	+35 55
6,326	216 11	+31 55

First of all let us examine the leading curve of the tail. The table below gives besides l the quantities p' and s derived directly from the observations and the final values of ϕ and ξ.

	l	p'	s	φ	ξ
Sept.28	0°30′	7° 5′	6°44′	20°29′	0,0916
28	1 0	354 2	13 54	44 55	0,2143
Octb. 1	0 31	10 36	9 26	26 15	0,1047
1	1 23	358 55	22 5	47 3	0,2635
2	0 36	9 38	12 50	32 7	0,1326
2	1 20	359 42	23 5	48 55	0,2458
4	0 43	19 5	14 55	26 37	0.1418
4	1 20	6 28	24 46	43 49	0.2234
5	0 32	20 7	11 41	23 46	0,1083
5	1 38	7 11	30 40	47 54	0.2597
6	0 42	30 17	14 15	21 21	0,1302
6	1 48	12 4	32 38	45 51	0.2633
8	0 56	32 49	18 3	28 31	0,1487
8	1 56	18 44	34 43	45 13	0,2504

Here I must add something about the way in which I derived the figures contained in column l. In the course of this examination I became convinced that the dimensions of the width of the tail varies quite considerably in different directions, thus for example in the plane of the orbit and in the plane perpendicular to this one. The diameter in the plane of the orbit was quite considerably larger than in any other direction. Had I assumed for l the apparent radius in each case, then the reduction to the orbital plane would have been considerably incorrect. However on Sept. 28 the direction to the comet was such that the apparent radius obtained on that day will have closely corresponded to the curve of the leading branch of the tail in the plane perpendicular to the axis for the duration of the phenomenon treated here. The remaining values of l are derived by calculation from the observations of Sept. 28, taking into consideration the distance of the observed point from the core of the comet and from the earth. By the way, I have only suggested this method here because I consider all of the following analysis to be only provisional.

If we set the quantities

$$g \sin G = \beta, \quad \sqrt{\frac{1}{(1-\mu)}} = \alpha,$$

then by applying equation (12) to my observations we obtain the /339
following equations:

$$\begin{array}{lrcl}
\text{Sept.28} & 9.57235 & = & \beta(0.43290\,\alpha + 8{,}82365\,\alpha^2) + 9.72335\,\alpha \\
28 & 9{,}99874 & = & \beta(0{,}24832\,\alpha + 8{,}82365\,\alpha^2) + 9.90793\,\alpha \\
\text{Oct. } 1 & 9.69298 & = & \beta(0.30347\,\alpha - 8{,}72997\,\alpha^2) + 9{,}75278\,\alpha \\
1 & 0{,}03110 & = & \beta(0.20302\,\alpha - 8{,}72997\,\alpha^2) + 9.95323\,\alpha \\
2 & 9.79776 & = & \beta(0{,}35336\,\alpha - 8{,}96024\,\alpha^2) + 9.80289\,\alpha \\
2 & 0{,}05956 & = & \beta(0{,}21935\,\alpha - 8{,}96024\,\alpha^2) + 9.93690\,\alpha \\
4 & 9{,}69995 & = & \beta(0{,}34328\,\alpha - 9.23552\,\alpha^2) + 9.81297\,\alpha \\
4 & 9.98206 & = & \beta(0{,}24456\,\alpha - 9{,}23552\,\alpha^2) + 9{,}91169\,\alpha \\
5 & 9{,}64381 & = & \beta(0{,}40471\,\alpha - 9.32125\,\alpha^2) + 9.75154\,\alpha \\
5 & 0.04404 & = & \beta(0{,}21482\,\alpha - 9{,}32125\,\alpha^2) + 9{,}94143\,\alpha \\
6 & 9.59205 & = & \beta(0.36841\,\alpha - 9{,}39434\,\alpha^2) + 9{,}78784\,\alpha \\
6 & 0.01289 & = & \beta(0.21547\,\alpha - 9{,}39434\,\alpha^2) + 0{,}94078\,\alpha \\
8 & 9.73507 & = & \beta(0.34830\,\alpha - 9{,}51009\,\alpha^2) + 9{,}80795\,\alpha \\
8 & 0.00328 & = & \beta(0{,}23521\,\alpha - 9{,}51009\,\alpha^2) + 9{,}92104\,\alpha
\end{array}$$

All the figures here are logarithms. From these equations we easily obtain the following:

$$\begin{array}{lrcl}
\text{Sept.28} & \alpha & = & 0.21098 - 8{,}69727\,\beta\alpha^2 \\
\text{Oct. } 1 & \alpha & = & 0{,}14941 + 8{,}56447\,\beta\alpha^2 \\
2 & \alpha & = & 0{,}23615 + 8{,}78417\,\beta\alpha^2 \\
4 & \alpha & = & 0{,}27413 + 9{,}06743\,\beta\alpha^2 \\
5 & \alpha & = & 0{,}20301 + 9{,}16344\,\beta\alpha^2 \\
6 & \alpha & = & 0{,}25355 + 9{,}22230\,\beta\alpha^2 \\
8 & \alpha & = & 0{,}24045 + 9{,}34089\,\beta\alpha^2
\end{array}$$

again the figures are logarithms. Even the first term on the right side of these equations gives a sufficient approxiamte value for α. When this term is substituted in the first equations the proper values of α and β can very quickly be found. In this way I obtained the following values:

Sept. 28	$\alpha = 1{,}640$	$\beta = -0{,}106$
Oct. 1	$\alpha = 1{,}405$	$\beta = -0{,}057$
2	$\alpha = 1{,}700$	$\beta = -0{,}127$
4	$\alpha = 1{,}805$	$\beta = -0{,}197$
5	$\alpha = 1{,}569$	$\beta = -0{,}126$
6	$\alpha = 1{,}626$	$\beta = -0{,}136$
8	$\alpha = 1{,}633$	$\beta = -0{,}183$

The unexpected agreement of these results surprised me. The agreement is such that one can regard the mean of these values as corresponding to the complex of the above observations. These mean values are as follows: α = 1.626, β = -0.140.

The examination of the trailing edge of the tail is of little value because in comparison with the leading edge the markedly lesser brightness and distinctness of the border to be sure did not permit accurate determination of the borders of the tail here. Therefore only for purposes of comparison I selected one observations point in this edge for each day and substituted the value of β in the equation for this point. In this way I obtained the value of valid for this limit curve. Next I give the values derived directly from the observations.

/340

	l	p'	s	φ	ξ
Oct. 1	0°50′	354° 5′	13° 6′	49°46′	0,1243
2	0 42	355 31	12 54	51 8	0,1096
4	1 2	354 29	17 50	57 17	0,1231
5	1 4	329 28	16 16	56 39	0,1007
6	1 0	3 12	16 8	55 20	0,0897

If we substitute these values of ϕ and ξ in equation (12) § 6, then we obtain:

$$\begin{array}{lll}
\text{Oct. } 1 & 0{,}07260 = \beta(0{,}36775\,\alpha - 8{,}72997\,\alpha^2) + 9{,}78830\,\alpha \\
2 & 0.09370 = \beta(0{,}39476\,\alpha - 8{,}96024\,\alpha^2) + 9{,}76149\,\alpha \\
4 & 0{,}19209 = \beta(0{,}37393\,\alpha - 9{,}23352\,\alpha^2) + 9{,}78232\,\alpha \\
5 & 0{,}18169 = \beta(0{,}42058\,\alpha - 9{,}32125\,\alpha^2) + 9{,}73567\,\alpha \\
6 & 0{,}16016 = \beta(0{,}44921\,\alpha - 9{,}39434\,\alpha^2) + 9{,}70704\,\alpha
\end{array}$$

Again the figures stand for logarithms. If we substitute the the values of α in these equations which correspond to the individual days and are derived from the previous analysis, then we obtain the following:

Oct. 1 $\beta = +0{,}101$
2 $\beta = +0{,}065$
4 $\beta = +0{,}126$
5 $\beta = +0{,}185$
6 $\beta = +0{,}158$

On average β = +0.126. The agreement of this value found with that for the leading edge shows that the velocity at which the particles emanate towards both sides of the radius vector was the same. The difference in sign is in keeping with the nature of the matter, while the angle of emanation G is assumed negative towards the leading edge and positive towards the trailing edge.

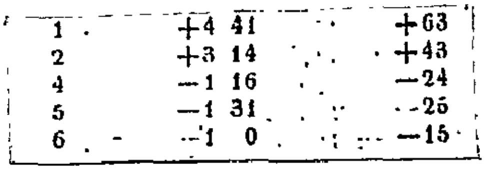
Now in addition, by substituting the two constants α = 1.625 and β = ∓ 0.140 derived above (where the upper sign applies to the leading edge of the tail and the lower sign to the trailing edge) in equation 12, § 6, I have attempted to derive the values for the curve of the tail using the equation for the duration of the entire phenomenon. I then compared the observed values with these calculated values. In this way I obtained the following differences (R–B).

Leading Tail Edge

	A Unterschiede im P.-W.	im gr. Kreise B
Sept.28	−3°17′	−24′
28	−1 51	−27
Oct. 1	−3 57	−39
1	+0 41	+15
2	−1 41	−24
2	1 23	−31
4	+3 22	+50
4	+0 34	+15
5	−0 49	−10
5	+0°19′	+10′
6	+6 2	+86
6	+2 3	+62
8	+3 11	+58
8	±1 43	+58

Key: A) Differences in the apex angle
B) In the large circle

Trailing Tail Edge



1	+4 41	+63
2	+3 14	+43
4	−1 16	−24
5	−1 31	−25
6	−1 0	−15

If one considers how uncertain the observations are of such a washed out appearance as that of the margin of the tail of a comet then in general one should not find the magnitude of the above deviations surprising. The margins of the tail were plotted by me on the basis of an estimate of their position made with the naked eye with respect to the surrounding stars in Argelander's Atlas, and the observation points were simply estimated from this curve. I believe that assuming the uncertainty of the final positions to be about 1/4 of a degree is too small rather than too large. If we compare with this the differences in the arc of the largest circle for the leading edge of Sept. 28 through Oct. 5 and for the trailing edge of Oct. 2 through 6, then these can be explained solely by the uncertainty of the positions. Actually I should not have included the observation of the trailing edge for Oct. 1, since I plotted this margin quite casually. The deviation present here can merely be attributed to this fact. Even the large deviations of Oct. 6 and 8 can be sufficiently explained. On Oct. 6 the sky was only partially clear and the tail of the comet was periodically covered by clouds so that I saw it moderately clearly only for a few moments. On this day I plotted several limit curves, which, however differ among themselves by more than 1°, and I took the mean to obtain the final positions. Thus we see that the figures for this day are subject to considerable uncertainty. The observations for Oct. 8 are uncertain for another reason. I have already mentioned in § 1 in the description of the appearances that on this day I first saw the leading edge washed out and somewhat jagged. The uncertainty with respect to the margins of the comet caused by this combined with the very low luminosity of the tail in its upper parts is sufficient to explain the large deviations for this day.

The general result of the above comparison seems very remarkable to me. Substituting two constants which determine the limit curve of the tail in the equation of this curve produces a curve so similar to this one for the entire duration of the appearance considered here that the remaining differences can be blamed for the most part only on the uncertainty of the observations and the influence of adverse circumstances. Furthermore it turns out that the forces which determine the limit curves can be deduced with considerable certainty from the observations of these limit curves, and finally that these forces were totally or nearly constant during the duration of the appearance. /342

9

The analysis in the preceding section is based on the assumption that the quantities ignored in deriving equation (12) § 6, do not have any effect, at least no considerable effect, on the result. In the case of comet this assumption is not totally correct. From the above found values of $\alpha = 1.625$ we find $\mu = 0.621$ and $(1-\mu) = 0.379$. If with these we compare the quantities β or $g \sin G = -0.140$ then we see that the products a and squares of these quantities, in comparison with $(1-\mu)$, can not be totally ignored, although in general the effect of ignoring them will not be considerable. However it does seem necessary to derive such a value of $tg\phi$ from the strict equations for ξ and η which is accurate up to quantities on the order of the square of $g \sin G$. This makes the calculation considerably more complicated and for this reason I have for the time being neglected repeating these calculations with my observations, while I hope to be able to considerably reinforce the positions given here with the data of other observers.

In what follows I will now give the results which I derived from my observations of the tail from Oct. 9 through 12. Already in the description of the tail in § 1 I mentioned that on Oct. 8, but especially after Oct. 9, the shape of the tail changed considerably. On Oct. 9 the central parts of the leading edge, which until then had been sharply delineated, was prenetrated by several bright columns which spread out to a rather large extent over the lower part of the tail and because of their brightness clearly stood out from the background on which they lay. The same was the case, only to a more marked degree, on Oct. 10. On Oct. 12 the leading edge was less curved back and the trailing edge was so the speak double as the drawing for this day shows more clearly than a description. Accordingly it seems as if beginning with Oct. 9, a second tail /343 emerged from the main tail whose particles followed a less curved path than those of the main tail. The main tail could be observed very plainly over a large area on Oct. 9 and 10, but it was washed out and each day became shorter.

The calculations confirmed this suspicion in so far as they show that these tail particles separating off in the form of columns require one of the earlier various repulsive forces of the sun to determine the curve which they describe. First of all I give there the table of observations:

Observed Points in the Tail

Am. B. Zt.		α	δ	α☄	δ☄
Oct.9,288	1	244° 8'	+52°57'	224°27'	+7°11'
	2	240 41	+29 27		
	3	238 26	+27 17		
	4	230 12	+18 37		
10,319	1	243 56	+24 27	227 36	+3 32
	2	234 54	+15 53		
	3	240 12	+17 27		
12,308	1	250 45	+13 30	233 26	—3 30
	2	245 58	+ 5 47		
	3	243 3	+10 0		

Key: A) Meridian Berlin time

In my examination of these figures I have assumed that the observed points lay in the plane of the orbit. Since with the exception of the first observation they refer only to the particles separating from the main tail -- but because of their irregular configuration they do not allow the earlier reduction to be used, -- it was thus not possible, even approximately, to determine the position with respect to the plane of the orbit. Moreover, on Oct. 9 through 12 the section of the plane of the tail perpendicular to the visual line coincided so closely with the orbital plane that absolutely no significant error was introduced by assuming that the two coincided.

The following table contains the quantities s, p', ϕ and ξ derived from the observations.

		s	p'	φ	ξ
Oct. 9	1	48°30′	15°43′	52°10′	0.3283
	2	27 0	32 26	33 17	0.2081
	3	24 5	31 46	34 1	0.1843
	4	12 43	25 32	40 54	0.0899
10	1	26 11	35 28	34 48	0.1964
	2	14 17	29 42	40 48	0.1005
	3	18 36	40 43	19 34	0.1642
12	1	24 10	44 58	34 39	0.1834
	2	16 43	63 30	26 41	0.1330
	3	16 32	35 14	43 48	0.1117

Substituting these values of ϕ and ξ and the values of r and v corresponding to the observation times in equation (12), § 6, we obtain the following equations: /344

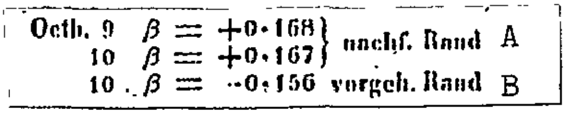
$$
\begin{array}{lll}
\text{Oct. 9} & 1 & 0{,}10980 = \beta\,(0{,}18135\,\alpha \quad 9{,}55623\,\alpha^2) + 9{,}97390\,\alpha \\
 & 2 & 9{.}81721 = \beta\,(0{,}28035\,\alpha - 9{,}55623\,\alpha^2) + 9{,}87490\,\alpha \\
 & 3 & 9{.}82926 = \beta\,(0{,}30672\,\alpha - 9{,}55623\,\alpha^2) + 9{,}81853\,\alpha \\
 & 4 & 9{,}93763 = \beta\,(0{,}46268\,\alpha - 9{,}55623\,\alpha^2) + 9{,}69257\,\alpha \\
10 & 1 & 9{.}84200 = \beta\,(0{,}29850\,\alpha - 9{,}59921\,\alpha^2) + 9{,}85775\,\alpha \\
 & 2 & 9{,}93610 = \beta\,(0{,}44395\,\alpha - 9{,}59921\,\alpha^2) + 9{,}71230\,\alpha \\
 & 3 & 9{.}55075 = \beta\,(0{,}33746\,\alpha - 9{,}59921\,\alpha^2) + 9{,}81879\,\alpha \\
12 & 1 & 9{.}83957 = \beta\,(0{,}32550\,\alpha - 9{,}67137\,\alpha^2) + 9{,}83075\,\alpha \\
 & 2 & 9{.}70121 = \beta\,(0{,}39541\,\alpha - 9{,}67137\,\alpha^2) + 9{,}76094\,\alpha \\
 & 3 & 9{.}98433 = \beta\,(0{,}43326\,\alpha - 9{,}67137\,\alpha^2) + 9{,}72309\,\alpha
\end{array}
$$

The first equation, Oct. 9, refers to a point in the leading edge of the main tail. In spite of the fact that the outline of of the tail was blured I consider this observation to be rather certain, Using the mean value of $\alpha = 1.625$ found earlier in § 8 this observation suffices to allow us to find the second quantity β. For β I obtained a value of -0.159, thus very nearly agreeing with the earlier mean value of -0.140. This proves that the leading curve of the main tail maintained the same survature as in the preceding days.

Equations 2 and 3, Oct 9, pertain to two points which lay very close together in the apparent axis of the part of the tail branching off from the main tail. If in the two equations we compare the behavior of the two quantities ϕ and ξ with one another we see that the particles moving in the direction of this axis cannot possibly be subjected to the same force of the sun as t those which were moving in the periphery or in the axis of the main tail. Assuming that both points lay close together in the axis just mentioned, then for both these points β is equal to 0. Thus for the first point this gives us a value for α of 0.876 and for the second point 0.957, or a mean value of α of 0.916. The first equation for Oct. 10 also refers to a point in the same axis. If we set β equal to 0 it gives us a value for α of 0.964. From the determinations of both days I have taken a mean and obtained a value for α of 0.940. Now since $\alpha = \sqrt{1/1-\mu}$, then $(1-\mu) = 1.131$ and $\mu = -0.131$, thus totally different from the values found above.

Now after at least an approximate value of α has been found we can also derive β from observations of point not on the axis. This is done by substituting α in equations 2 and 3 for Oct. 10 and equation 4 for Oct. 9. The last equation applies to a point in the trailing edge, likewise equation 2, Oct. 10. The third /31
equation for Oct. 10, on the other hand, applies to a point in the

leading edge of the part of the tail under special consideration here. I obtained the following values of β :



Octb. 9 $\beta = +0{\cdot}168$ } nachf. Rand A
10 $\beta = +0{\cdot}167$ }
10 $\beta = -0{\cdot}156$ vorgeh. Rand B

Key: A) Trailing edge
B) Leading edge

It now remains to derive the values of α and β from the equations of Oct. 12. The first two equations apply to two points lying in the same leading curve. Solving these equations gives us the following:

$\alpha = 1.342$ $\beta = 0.105$

The third equation applies to a point in the trailing edge. By using the just obtained value of α this equation gives a value for β of +0.077.

The following seems to me to emerge from this analysis. Little by little the core of the comet expelled different types of particles which were subjected to a quite different effect of the sun. After emerging from the sphere of influence of the comet these particles at first moved upward together in the main tail. At a greater distance from the core, where the difference in directions of the particles caused by various forces appeared more striking, the more strongly pushed particles separated from the others in the direction in which the comet was moving. This had to give rise to precisely that appearance which we observed on Oct. 8, 9 and 10. This also could explain the sudden bending of the leading curve of the tail in the vicinity of the core.

On Oct. 12 the appearance had changed in that the main tail appeared extremely shortened. The curvature of its leading

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edge merged in the lower parts with the less curved secondary tail whose trailing edge, by contrast, stood out plainly. The value for of 1.342 derived from the leading curve can therefore not be matched either with that found for the main tail or with that found for Oct. 9 and 10. The trailing edge would produce a considerably smaller value of α, such as $\alpha = 1$, which is very close to the value found on Oct. 9 and 10.

10

Dr. Winnecke in Pulkowa and Prof Listing and Mr. Auwers in Göttingen also saw a straight, narrow and very weak secondary tail which escaped my notice and that of many other observers completely. According to the descriptions given by Prof. Listing and Mr. Auwers in No. 1167 of the Astronomische Nachrichten this tail lay nearly in the extension of the radius vector. Thus /346
the particles forming it had to be subjected to an extremely strong repulsive force of the sun. The data contained in No. 1167 is sufficient in order to derive form the figures the position of the tail for a few days and from this position to derive the quantities α and β which are valid for the tail. From those figures I have taken the following determinations for the end-point of the tail and to them I add the data for ϕ and ξ :

A M.B.Z.	α 1868,0	δ	φ	ξ
Oct. 1,350	210°30′	+62° 0′	14°29′	0·2947
4,322	228 9	+48 57	16 9	0·3117
10,278	257 11	+24 58	19 34	0·3014

Key: A) Meridian Berlin time

The eqauations applying to these points are:

$$\begin{array}{lll}
\text{Oct. } 1 & 9{,}41214 = \beta\,(0{,}17882\,\alpha - 8{,}72997\,\alpha^2) + 9{,}97743\,\alpha \\
4 & 9{,}46177 = \beta\,(0{,}17212\,\alpha - 9{,}23152\,\alpha^2) + 9{,}98412\,\alpha \\
10 & 9{,}55075 = \beta\,(0{,}20523\,\alpha - 9{,}59675\,\alpha^2) + 9{,}95102\,\alpha
\end{array}$$

If we assume that the above points can be valid for the endpoints of the axis of this tail, then in these equations $\beta = 0$, and we obtain for α the following values:

Oct. 1	$\alpha = 0{\cdot}272$
4	$\alpha = 0{\cdot}299$
10	$\alpha = 0{\cdot}398$

For Oct. 10 I have before me a drawing of the tail communicated by Mr. Auwers in which the width of the tail at its end is about 1°. With this information one can derive the limits of g sin G from the equation for Oct. 10. If we used this for the basis of the calculation, then for the limits of β or g sin G ± 0.067.

The above value of α corresponds to an extremely large repulsive force of the sun. If I assume the data for Oct. 10 to be the most certain -- since I was able myself to take the observation on which it is based from the drawing given by Mr. Auwers -- then with $\alpha = 0.398$, I obtained $(1-\mu) = 6.317$ and $\mu = -5.317$.

If we compare this value of μ with the previous one derived for the main tail, then we are obliged to assumed that there is an extraordinary amount of variation in the particles expelled from the core. On the other hand, if we do not want to allow the assumption that the sun has repelled these particles with very differing degrees of intensity, then we are obliged to accept the second assumption that the specific weight of the particles differed very considerably and therefore moved upwards with unequal velocity in the ether gravitating towards the sun. In this case the usual attraction of the sun would explain the /347
phenomena. In both cases, however, one comes to the conslusion that the comet has expelled particles having very different properties.

If we compare the values of β derived for the main tail with one another, it seems as if in the course of appearance the component of the initial velocity perpendicular to the radius vector has changed somewhat. In gerneral, the values derived in the first days are smaller than the later values. Perhaps this indicates that as the periphery of the emanation spreads out, thus as the angle G increases, the component g sin G has also increased. This would argue against Bessel's assumption according to which the product g sin G becomes a constant. However there is yet another way to convince ourselves of the inadequacy of this assumption in the case of our comet.

Between the force with which the sun acts on the particles expelled from the comet and between the distance of the haze on side of the core towards the sun there exists a certain ratio. If s stands for the observed distance of the haze on the side towards the sun, then the following equation is valid for the particles which emanate in the direction of the radius vector, thus for which G and F = 0:

$$s = f + \frac{rrgg}{2(1-\mu)}$$ [1]

If (1—μ) is known and s is given by means of observation, then from this equation we can thus derive the velocity at which the particles emanate in the direction of the sun if we ignore f or the radius of the sphere of influence g.

I assume that the largest distance at which one could still perceive haze on the side of the nucleus towards the sun on Oct. 5 was about 4'. Thus s = 0.000679. If for (1—μ) I substitute its value of 0.379 derived from the mean value of α, then, with log r = 9.77158, g = 0.038. If we compare this value of g with

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[1] Bessel's discussion in Astronomische Nachrichten, Vol. 13, p. 217.

the earlier mean value of g sin G, 0.140, then we must assume that the comet expelled the particles with greater velocity in greater angles with the radius vector. This is in harmony with the result derived earlier of the constant increase in g sin G. However this assumption is admissable only as long as the above equation is absolutely correct. As soon as the particles emanating towards the sun met with resistance for instance in the haze of the coma or as soon as the sphere of influence of the coma was /348 not ver small, then the value found for g becomes incorrect along with the assumption based on this value.

If we now assume that for the value – 0.140 of g sin G the angle G was very close to a right angle -- which isnin good agreement with the observed shape of the emanation which extended from about + 90° to – 90° -- then g = 0.140. This g refers to the unit of time (1/k or 58.13244 days). If the day is chosen as the unit of time then g = 0.002409. This value corresponds to a velocity of about 0.58 statute miles per second. The particles located in the extreme leading edge of the tail exited with this velocity from the sphere of influence of the comet.

The assumption that these particles depart from the comet towards the radius vector under an angle of – 90° gives us an opportunity to make an approximate determination of the time which they required to reach any given observed point in the tail. For this determination I chose the point whose AR and Declination I observed on Oct. 5 to be 217°55' and + 50°25'. For this point $\xi = 0.2597$. Assuming that G = – 90° and g = 0.140, we obtain from the following equation (Bessel's discussion, Astronomische Nachrichten, Vol. 13, p. 223)

$$\tau' = \frac{r\sqrt{(2\xi)}}{\sqrt{(1-\mu)}}\left\{1-g\frac{\sqrt{p}}{1-\mu}\right\} + \frac{\xi}{(1-\mu)^2}\,g\,\frac{14}{3}\,c\,r\sin v$$

τ' = 0.787 and hence τ = 0.601 = 34.9 days. In order to climb from the sphere of influence of the coma up to the observed point in the tail the particles thus required almost 35 days, or they left the sphere of influence of the comet the beginning of September. However this result, as well as the preceding one, is changed considerably if the assumption about G was not entirely correct.

I now also performed the same analysis for the particles forming the weak secondary tail. I assume that on Oct. 10 the most extreme distance of the still visible haze from the core of the nucleus on the side towards the sun was about 4', thus giving $(1-\mu) = 6.317$ and $g == 0.143$. If we compare this value of the initial velocity of the particles in the secondary tail with the limit value found above of $g \sin G = \pm 0.067$, then it seems as if these particles were expelled chiefly in the direction towards the sun. At least for G we obtain only the limit values of ±28°.

From the value of g just derived above we now obtain the following values of $\tau\tau'$ and τ for the particles which were located in the axis and at the extreme end of the tail on Oct. 10:

$$\tau' = 0.1910, \quad \tau = 0.1723,$$ /349

or expressed in days, $\tau = 10.0$ days. Thus these particles left the sphere of influence of the comet on Sept. 30.

According to information imparted by correspondents Dr. Winnecke observed this weak secondary tail as early as mid-September. The particles located at that time at the extreme end thus left the comet about the beginning of September. The fact that this moment coincides so closely with the one derived above for the main tail and at the same time with the first visible appearance of the tail (about Aug. 30) seems to indicate more than a mere accidental coincidence. By pursuing this matter we may come up with further information.

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In In § 5 I have shown that the initial direction of the axis of the tail formed a constant angle with the extension of the radius vector in the plane of the orbit between Sept. 17 and Oct. 14. Averaging all the measurements I found this angle to be 6°18'. Equation (12), § 6, shows that for greater distances from the core the inclination of the axis of the tail is a function only of the constant $(1-\mu)$. For small distances from the core this is valid only with greater or lesser approximation. However since we are dealing here only with a provisional comparison, it will suffice if I compare the inclination found for the initial direction with the following equation:

$$\tan \varphi = \frac{2 \cdot \sqrt{2p}}{3r} \cdot \frac{\sqrt{\xi}}{\sqrt{1-\mu}}$$

The measurements made by Dr. Winnecke are valid for a point in /350
the axis at a distance of 13' from the core; my measurements are valid for a point perhaps somewhat closer. Assuming the distance to be about 12'; this gives us for Oct. 5, with ϕ = 6°18', ξ and thus $\sqrt{1/1-\mu} \approx \alpha = 1.33$. This is only a rough approximate value which is close enough to the mean value of 1.625 found above for α if one considers that for such small values of ξ such as occur here the ignored squared values of g still have considerable influence. However from this it follows that the strong rearward bending of the initial direction of the tail requires a considerably large value of $\sqrt{1/1-\mu}$, hence a small value of $(1-\mu)$.

The constancy of the angle of the direction of the axis and the radius vector shows, however, that the value of μ in the course of the appearance has remained the same, which is in agreement with the result of § 8. Besides the equation shows that this constancy occurs only in the vicinity of the perihelion and that with increasing r, ignoring other still possible influences, ϕ steadily increases.

In my description of the phenomena (§ 1) I frequently had to mention that I saw the leading edge of the tail much brighter and more sharply delineated than the trailing edge. The comet looked just the same in the telescope in that the forward branch of the tail was always wider and brighter up to about Oct. 4 than the left branch, so that apparently a larger amount of particles emanating from the comet were pushed in the direction where η is negative. The equation which Bessel derived for the coordinate η, namely:

$$\eta = f \sin F + \left\{ g \sin G - f \cos F \frac{\sqrt{p}}{rr} \right\} \tau - \left\{ g \cos G \frac{2\sqrt{p}}{rr} + f \sin F \left(\frac{\mu}{r^3} - \frac{p}{r^4} \right) - f \cos F \frac{2 e \sin v}{r^3} \right\} \frac{\tau^2}{2}$$
$$+ \left\{ \frac{(1-\mu)}{r^4} 2\sqrt{p} + g \sin G \left(\frac{\mu p}{r^3} + \frac{3p}{r^4} \right) + g \cos G \frac{6 e \sin v}{r^3} \right\} \frac{\tau^3}{6},$$

is modified as follows for the case where G and F are set equal to 0, thus in which we are considering particles emanating in the direction of the radius vector:

$$\eta = -f \frac{\sqrt{p}}{rr} \tau - \left\{ g \frac{\sqrt{p}}{rr} - f \frac{e \sin v}{r^3} \right\} \tau^2 + \left(\frac{1-\mu}{r^4} \sqrt{p} + g \frac{3 e \sin v}{r^3} \right) \frac{\tau^3}{3}.$$

Prior to passing through the perihelion, hence when v is still negative, all the terms are negative with the exception of that one containing (1-μ). If the value of (1-μ) is not very large -- which was certianly not the case with our comet -- then all the particles go over to the direction where η is negative. After the perihelion the third and final term become positive. However as soon as the value of g is very small in comparison with (1-μ), then for small values of τ, which are the only ones under consideration here, the first and second terms outweigh the sum of the remaining terms for quite a while after the perihelion. This proved to be the case with our comet. For the first time on Oct. 4 I saw for certain in the telescope that the trailing branch of the tail was considerably brighter than the leading edge. Thus from this day onward the value of η was positive. Moreover, it follows from this, as from the earlier analysis, that the value of g must have been considerably greater.

It should be noted hhere that the flowing across of the particles towards the leading side of the tail also explains the peculiar appearance of the very bright leading edge of the tail and the very dim trailing edge. Since this motion of the particles prior to the perihelion, i.e. in September, was especially strong and resulted in a large buildup of particles on the leading side, then the phenomenon necessarily had to appear which we observed with our comet. /351

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Looking at the dimensions of the tail revealed a quite striking increase in the width of the tail. From Sept. 28 up to Oct. 10 the width increased from 2° up to 10° and more. If the tail had been a cone-shaped shell with a circular base, then taking into account its approach to the earth and the position of its axis, its width on Oct. 10 would have had to be about twice as large as on Sept. 28. The fact that such a large increase in the width was not real scarcely needs to be mentioned, since this would have been expressed by means of an extraordinary increase in the size of g sin G. In fact the increase was only apparent. The reason for this lies in the fact that the tail was considerably more extensive in the plane of the orbit than in any other plane. If we assume that the extent of the tail in the plane of the orbit was about four times as greateas in the plane perpendicular to this and that a section perpendicular to the axis of the tail had approximately the shape of an elispe, then the figures for the width of the tail for the different days, taking perspective shortening into consideration, can be quite well reconciled with one another.

The observation that the angle at the core of the comet between the directions of the two branches of the tail steadily increases from Sept. 28 to Oct. 10 is also simply explained merely by this assumption. On Oct. 10 we saw the shape of the comet with a small amount of foreshortening. However due to the position towards the earth the angle between the two branches of the tail seemed somewhat larger than it actually was in the plane of the orbit. By contrast, in the preceding days, but especially towards the end of September, we saw the parts lying in the plane of the orbit considerably foreshortened. To me this observation seems to indicate that the comet expelled particles mainly in the plane of the orbit, and this to some extent substantiates the assumption on the basis of which the analysis pertaining to the oscillations in emanation and to the shape of the tail was applied especially to this plane.

Comparison of the phenomena of our comet with those of earlier comets leads to a few more observations which I don't intend to suppress altogether. In his often mentioned article Bessel explained the peculiar appearance of the comet of 1807 by assuming different kinds of particles which were repelled by various degrees by the sun, initially moving in a common tail which, at a greater distance from the core, split into two tails, one straight and one curved.

This same phenomenon was seen by a few observers in the case of our comet. Along with the bright, strongly curved tail they also saw a preceding secondary tail which was straight and dim.

Above I have shown that this phenomenon calls for the same explanation which Bessel gave for the comet of 1807.

The large comet of 1811 also exhibited a small secondary tail. It was plainly observed by Olbers on Oct. 9, 1811.[1]

With the same comet Olbers saw a phenomenon which was repeated in the case of our comet: the peculiar outflowing of column-shaped parts of the tail from the edges of the main tail. In his article on the comet of 1811 Olbers says the following about this phenomenon:[2] "The outer edge of the ring was already less sharply delineated from the last half of September onward than when the comet first appeared, but surrounded with a light haze which in November curved parabolically downward from the sun on the left side (its forward side with respect to its true motion) in strips 25-30' long. Thus little by little very different kinds of matter must have separated off from the comet on which both the sun and the comet itself exerted differing degrees of repulsive force." So much for Olbers. With small modifications his description and explanation fits our comet. Something similar must have appeared with the large comet of 1744. The shapes which Heinsius drew of the tail of this comet show a bulge in the trailing edge which can hardly be called a secondary tail as Cassini has termed this phenomenon. As far as one can tell from the crude drawings it was similar to the bend which appeared with our comet on Oct. 9 and 10, only it is further separated from the edge of the main tail. Little or nothing reliable concerning earlier appearances of comets can be added to what has been cited here, however it is probable that many of the characteristic /353 shapes of older comets can be explained by the same cause just like the appearances just described.

Continued investigation of the tails of comets, combined with

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[1] Monatl. Correspond. Vol. 25, p. 13.

[2] Ibid, p. 21.

the equally instructive observation of emanations, the study of which is perhaps suitable foremaking a large contribution to discovering the special characteristic of the forces at work here, promises further information in the future on the still so puzzling nature of these celestial bodies. Even the older appearances provide, if not rich, nevertheless ample material which for a long time has been waiting to be processed.

Altona, Dec. 1858