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MATHEMATICAL SIMULATION OF POWER CONDITIONING SYSTEMS. VOL. 1: FINAL REPORT   SIMULATION OF ELEMENTARY UNITS. REPORT ON SIMULATION METHODOLOGY

R. Prajous, J. Mazankine and J.C. Ippolito

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16. Abstract
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Methods and algorithms used for the simulation of elementary power conditioning units buck, boost and buck-boost, as well as shunt PWM are described. Definitions are given of similar converters and reduced parameters. The various parts of the simulation to be carried out are dealt with: local stability, corrective network, measurements of input-output impedance and global stability. A simulation example is given.

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R. Prajous, J. Mazankine and J.C. Ippolito
Centre National de la Recherche Scientifique, Toulouse (France)
Lab. d'Automatique et d'Analyse des Systèmes

Part 1: Buck, Boost, Buck-Boost

1. Introduction

The purpose of this first part is to describe from a
theoretical and conceptual standpoint the methods and algorithms
used to simulate the elementary units buck, boost and buck-boost.

To begin with we will discuss important general points:
- the definition of similar converters, a definition which
  determines the generality of the results obtained and the
  effectiveness of the simulation (there is a safeguard
  against simulating the same system several times);
- the definition of reduced parameters which characterize
  the converters and derive directly from the idea of
  similar converters;
- the transition from reduced parameters to machine magnitudes;
- the calculation of the corrector gain giving at the minimum
  the required "except everything" regulation of ±2%;
- the general philosophy of simulations and "optimizations"
  to be carried out.

Then we will discuss in the following order the different
parts of the simulations to be done:
- study of local stability;
- investigation of a good corrector network;
- measurement of the output impedance, input impedance
  and input-output impedance;
- study of overall stability.

Finally, we will end up by discussing the utilization of the
results obtained by the simulation and we will give an example.

2. Electric Systems and Similar Converters

Let Fig. 2.1 represent 2 electric circuits $\underline{C}_1$ and $\underline{C}_2$ on
which we impose the following restriction: $\underline{C}_1$ and $\underline{C}_2$ have the
same number of elements of type R, L or C. At any moment t,

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* Numbers in the margin indicate pagination in the foreign text.

such a circuit is thus completely determined if we know the m currents passing through it and the m voltages at the terminals of the m elements. We label these currents I_{1i} and I_{2i} and these voltages V_{1i} and V_{2i} for $\underline{C}_1$ and $\underline{C}_2$ respectively, with the subscript i varying from 1 to m.
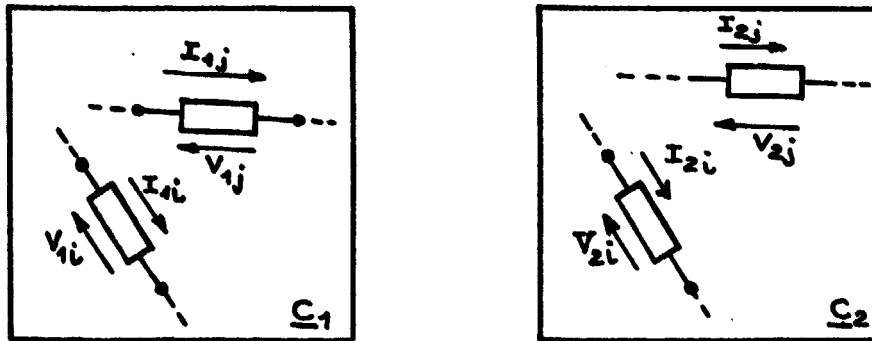
Figure 2.1

Definition:

Circuit $\underline{C}_2$ is similar to circuit $\underline{C}_1$ if and only if:

$$\exists k_I, k_V, k_T \quad \text{such that: } I_{2i}(k_T t) = k_I\, I_{1i}(t), \; \forall i \in [1,\ldots m]$$

$$V_{2i}(k_T t) = k_V\, V_{1i}(t), \; \forall i \in [1,\ldots m]$$

It is obvious that if we know the behavior of the circuit $\underline{C}_1$ during the time interval $[t_1,t_2]$ then we know the behavior of the circuit $\underline{C}_2$ during the time interval $[k_T t_1,\ k_T t_2]$.

Among other things:
- any non-periodic steady state of circuit $\underline{C}_1$ corresponds to a non-periodic steady state of circuit $\underline{C}_2$;
- any steady state of period T for circuit $\underline{C}_1$ corresponds /3 to a steady state of period k_T T for circuit $\underline{C}_2$;
- if such a state is stable for $\underline{C}_1$, it is also stable for $\underline{C}_2$.

It is clear that the above definition defines an equivalent class $\mathscr{C} = \{\underline{c}_1, \underline{c}_2, \dots\}$ which includes only circuits which are similar to one another.

Thus in order to analyse all the electric circuits, it is sufficient to analyse all the equivalence classes, i.e. in practice to analyse one cirucit per class.

With respect to the studies to be done within the framework of this contract, such an idea is very important because it guarantees that a minimum quantity of simulations will be done but that nevertheless all the desired information will be present.

It is very easy to see that two circuits $\underline{C}_1$ and $\underline{C}_2$ are only similar if:
- at two similar instants t and k_T T they have the same electric structure,
- any element R_2, L_2, C_2 of the circuit $\underline{C}_2$ is related to the corresponding element R_2, L_2, C_2 of circuit $\underline{C}_1$ by one of the following relationships:

$$R_2 = \frac{k_V}{k_I} R_1 \; , \; L_2 = \frac{k_V}{k_I} k_T L_1 \; , \; \dot{c}_2 = \frac{k_I}{k_V} k_T c_1 \; .$$

3. Reduced Parameters Characterizing the Converters

We have just shown the importance of the idea of a converter class.

It is now fitting to numerically characterize a given class with the help of parameters. These parameters are necessarily dimensionless.

Moreover, we have tried to give these parameters the greatest possible practical significance. Thus, electronic engineers should find them easy to use.

3.1 Schematic Structure of Buck, Boosts and Buck-Boost Converters

The converters we are considering may be represented as shown in Fig. 3.1.
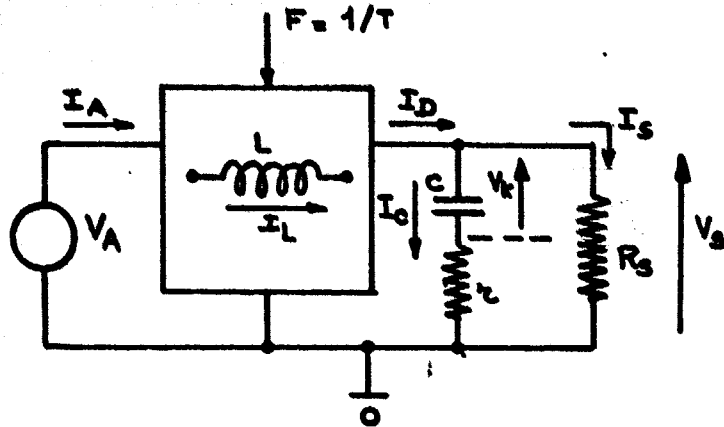
Fig. 3.1

The converter is powered by a voltage source V_A.

The output network always consists of the output capacitor C (which is given an internal resistance r) and the charge, represented by a resistor R_S, at the terminals of which appears the output voltage V_S.

Besides the above elements, the power stage includes the basic element consisting of the storage inductor L through which passes a current I_L.

<u>The three converters under consideration differ from one another only in the way in which V_A, V_S, L and the switches are connected to one another</u>.

The circuit shown in Fig. 3.1 suffices to define the main magnitudes V and I of the power stage as well as the operating frequency of the converter:

$$\boxed{F = \frac{1}{T}}$$ (3.1)

Note: transformer buck-boost. For this converter it is sufficient to express all of the magnitudes V, I of the power stage by basing them in the traditional way either on the primary or secondary winding of the transformer. Everything we say will then be equally valid in this case.

In addition, the parameters will be defined as follows:
- starting with the steady state of the system;
- idealizing this steady state at the output:
$V_B = C^{te}$, $I_B = C^{te}$ (the residual ripple of the output is ignored).

3.2 Parameter M, Ratio of Input and Output Voltages

We set up the following equation:

$$\boxed{M \triangleq \left| \frac{V_B}{V_A} \right|}$$ (3.2)

This parameter, the physical meaning of which is very clear, characterizes the operating point of the system.

Obviously we have the following:

- BUCK   : $0 < M < 1$
- BOOST   : $1 < M < +\infty$
- BUCK-BOOST : $0 < M < +\infty$

3.3 Parameter Δ, Current Ripple in the Inductor

The current I_L always has one of the two forms shown in Fig. 3.2.
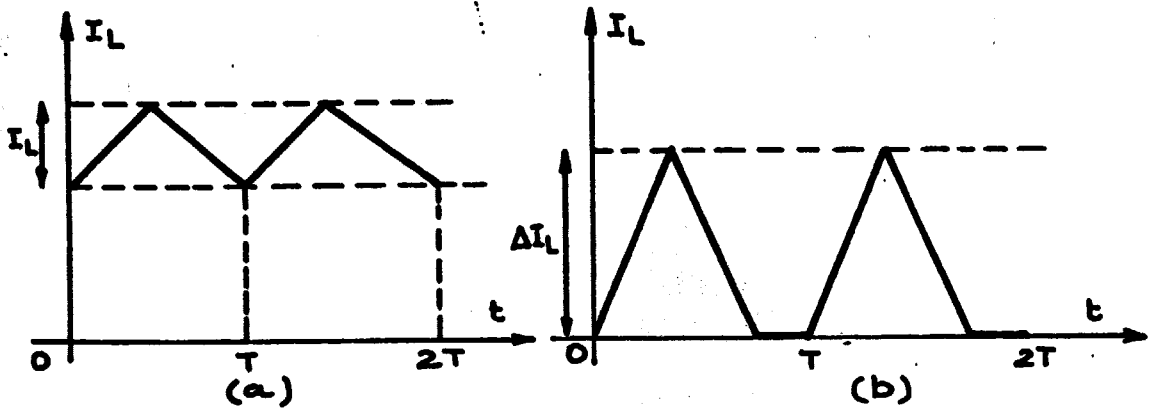
Fig. 3.2

If we assume (Fig. 3.2):

$$\Delta I_L \triangleq I_L \max - I_L \min \tag{3.3}$$

We then set up the following:

$$\boxed{\Delta \triangleq \frac{\Delta I_L}{I_s}} \tag{3.4}$$

This parameter characterizes above all the inductance value L: the greater L is, the smaller Δ is and vice versa. The value of Δ likewise enables us to know immediately if conduction is continuous or discontinuous.

We simply show that:

(3.5)

	Continuous Conduction	Discontinuous Conduction
BUCK	$\Delta < 2$	$\Delta > 2$
BOOST	$\Delta < 2 M$	$\Delta > 2 M$
BUCK-BOOST	$\Delta < 2 (M + 1)$	$\Delta > 2 (M + 1)$

3.4 Parameter ϕ, Value of the LC Filter

We set up the following equation:

$$\phi \triangleq \frac{1}{2\pi F \sqrt{LC}}$$

(3.6)

This parameter establishes the importance of the "filter" made up of the inductor L and the output capacitor C by comparing its imaginary tuning frequency with the operating frequency.

- The smaller ϕ is, the smaller the residual ripple (effect of r not included) because it varies as ϕ^2.
- The smaller ϕ is, the smaller the high frequency gain of the power stage, effect of r not included; it also varies as ϕ^2.
- Since the value of L is fixed by Δ, ϕ infact characterizes for a given Δ, the value of the output capacitor C.

3.5 Parameter Γ, Quality of the Output Capacitor

We set up the following:

$$\Gamma = 2\pi F C r$$

(3.7)

Γ is the ratio between the operating frequency and the frequency at which the output capacitor becomes purely resistive.

- If Γ is small, the quality of this capacitor is good.
- If Γ is large, the quality of this capacitor is bad.
- In addition, Γ enables us to get an idea of the effect of r on the dynamic behavior of the power stage, the threshold being situated very roughly at $\Gamma = 1$.
- The same is true for the effect of r on the residual ripple.

These last two points are summarized in Table (3.8).

Table (3.8)

Effect of r	$\Gamma < 1$	$\Gamma > 1$
On the dynamic behavior of the power stage	negligible	not neglibible
On the residual ripple	negligible	Ripple about Γ times greater than if r = 0

3.6 Other Reduced Parameters

In addition to the 4 above parameters, we can define other useful, but less important, parameters.

3.6.1 Parameter J, Value of the Output Current

We set up the following:

$$J \triangleq \frac{I_S}{I_{S\ max}} \qquad (3.9)$$

This parameter is defined as follows:
- for a given converter, i.e. V_A, V_S, L, C, r being constants,
- when its output current is varied, i.e. the charging resistance R_S.

The parameter J cannot be used to characterize a class of converters since, by definition, I_S cannot assume several values for the same system.

By contrast, J is useful in studying a given converter, as we shall see later on.

3.6.2 Parameter β, Saturation of the Magnetic Cores

For a magnetic circuit, in principle a transformer, we set up the following equation:

$$\beta \triangleq \frac{B_{saturation}}{B_{max}}$$

(3.10)

We will then always take β = 1.33.

3.6.3 Maximum current in the Switch

For a real system the current in the switch transistor for transistors is limited to a given value.

Without having to define a new symbol, we agree that:

$$\frac{I_L \text{ limitation}}{I_L \text{ max nominal}} = 1,33$$

(3.11)

3.6.4 Reduce Time Constants $\mathscr{C}$

It is obvious that any time constant, in order to be reduced, must be reduced to the period T.

We therefore set up:

$$\mathscr{C} \triangleq \frac{\tau}{T/2\pi}$$

(3.12)

From this it follows that F and the cut-off frequency corresponding to τ are in the ratio $\mathscr{C}$:

$$\frac{1}{2\pi\tau} = \frac{F}{\mathscr{C}}$$

(3.13)

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3.7 Remark 1: Modulator Gain

The definition of the similar circuits given in section 2 can of course be applied to a converter in its entirety.

Nevertheless, we will consider the basic circuit shown in Fig. 3.3
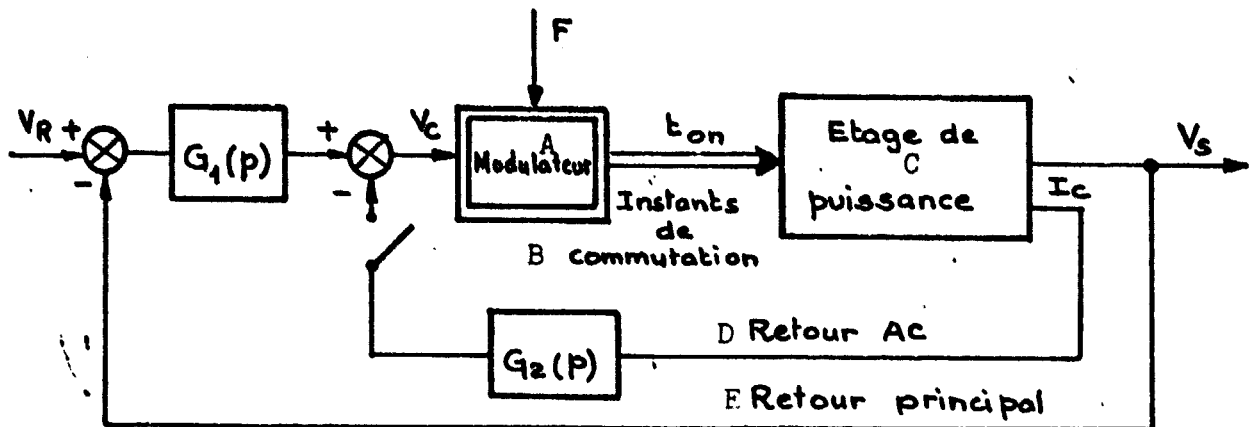
Fig. 3.3

Key: A) modulator   D) AC return
      B) switching instants   E) main return
      C) power stage

This circuit reveals three distince parts of the converter:
- the power stage,
- the modulator,
- the correcting networks $G_1(p)$ and $G_2(p)$.

The similarity rules should be strictly applied to the power stage.

By contrast, we can see that although the modulator also appears in the form of an electric circuit, it is not necessary that this circuit obey the similarity rule in its totality, but only at its output and input.

In other words, in comparing a converter 2 to a converter 1,
it suffices that:

$$G_{M2} = \frac{dt_{on2}}{dv_{c2}} = \frac{k_T}{k_V} \, G_{M1} \tag{3.14}$$

In particular, the characteristic equation of the coincidence
modulator:

$$v_c(t) - \mathcal{A}(t) = 0 \qquad \text{for} \qquad t = t_n \tag{3.15}$$

may be written as follows:

$$k \, v_c(t) - k \, \mathcal{A}(t) = 0 \qquad \text{for} \qquad t = t_n$$

From this it follows that the gain of the coincidence
modulator is unchanged if we multiply the amplitude of the
sawtooth by k and the control signal by the same factor k.

For the sampling-delay modulator with a gain of $G_M = T/A$,
we end up with the same conclusion.

As for the magnetic modulator, it was directly defined in
a reduced way in the "Implementation Report."

3.8 Remark 2: Correcting Networks

The correcting networks are defined by their transfer
function and not by a circuit.

For two similar converters, it thus suffices that the
reduced time constants are identical and that the gains:
- are the same for $G_1(p)$
- are in the ratio k_V/k_I for $G_2(p)$.

4. Transition from Reduced Parameters to True Parameters or to Machine Parameters

The problem which we pose in this section is to go from reduced parameters to finding a class of systems to the parameters of a particular converter of the class. Naturally this converter may be the one simulated and the equations which we are going to give are thus also those used for implementing the simulation.

To make this transition from reduced parameters to true parameters it is of course necessary to start with numerical values which characterize the converter in question within the class. To do this, it is clear that the values for the following must be known at the outset:

- a voltage,
- a current,
- a time.

It is equivalent and more convenient if we know the following:
- a voltage,
- a resistance,
- a time.

In fact, the following equations assume that V_S, R_S and the operating period T are known.

4.1 Equations Specific to the BUCK

In continuous conduction ($\Delta < 2$):

$$L = \frac{1 - M}{\Delta} \cdot R_S \cdot T$$

(4.1)

In discontinuous conduction ($\Delta > 2$):

$$L = \frac{2(1 - M)}{\Delta^2} \cdot R_S \cdot T$$

(4.2)

4.2 Equations Specific to the BOOST

In continuous conduction ($\Delta < 2\,M$):

$$L = \frac{M-1}{M^2} \cdot \frac{1}{\Delta} \cdot R_S \cdot T$$

(4.3)

In discontinuous conduction ($\Delta > 2\,M$):

$$L = \frac{M-1}{M} \cdot \frac{2}{\Delta^2} \cdot R_S \cdot T$$

(4.4)

4.3 Equations Specific to the BUCK-BOOST

In continuous conduction ($\Delta < 2\,(M+1)$):

$$L = \frac{1}{M+1} \cdot \frac{1}{\Delta} \cdot R_S \cdot T$$

(4.5)

In discontinuous conduction ($\Delta > 2\,(M+1)$):

$$L = \frac{2}{\Delta^2} \cdot R_S \cdot T$$

(4.6)

4.4 Equations Common to BUCK, BOOST, BUCK-BOOST

It follows directly from the definitions given in section 3 that:

$$V_A = \frac{V_S}{M}$$

(4.7)

$$C = \frac{1}{4\pi^2 \cdot \rho^2 \cdot F^2 L}$$

(4.8)

$$r = \frac{\Gamma}{2\pi \cdot C \cdot F}$$

(4.9)

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5. Gain of the Correcting Amplifier ensuring the ±2%
Regulation Required

A given converter is essentially subjected to two perturbations:

- the variations in input voltage V_A
- the variations in output current I_S.

The problem here is to calculate how these perturbations cause the output voltage V_S to vary <u>as a result of their simultaneous action</u> and, in particular, to calculate the <u>minimum gain</u> G_0 of the network $G_1(p)$ ensuring in these conditions the ±2% regulation required by the specifications (cf. the reports of the ESTEC-LAAS).

It is useless to complicated calculations to make this determination. It suffices to calculate the maximum variation in the control voltage V_c of the modulator ($V_{c\,max} - V_{c\,min}$) when V_A and I_S vary in the specified limits, then to write that:

$$\frac{V_{c\,max} - V_{c\,min}}{G_0} = 4 \cdot V_R \tag{5.1}$$

The problem thus becomes to determine $V_{c\,max}$ and $V_{c\,min}$ for each converter, and for these values it is clear that we have:

For the coincidence modulators and sampling-delay modulators

$$\left| \begin{array}{l} V_{c\,max} = t_{ON\,max} \dfrac{A}{T} \\[3mm] V_{c\,min} = t_{ON\,min} \dfrac{A}{T} \end{array} \right. \tag{5.2}$$

For the magnetic moculator (cf. the implementation report)

$$\left| \begin{array}{l} V_c = V_1 \left(1 - t_{ON} \dfrac{3}{2T}\right) \\[3mm] 0 < t_{ON} < \dfrac{2T}{3} \end{array} \right. \tag{5.3}$$

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All of the converters to be considered operate at J = 1 in
continuous conduction (cf. Report of the ESTEC-LAAS Meeting of
June 25, 1975, p. 5) then end up by changing into discontinuous
conduction when J decreases. We know that:

- in discontinuous conduction, t_{on} decreases when J decreases,
- in continuous conduction, t_{on} is independent of J.

It is then very clear that:

- $t_{ON_{max}}$ occurs in continuous conduction for a value of M to

  be determined,
- $t_{ON_{min}}$ occurs in discontinuous conduction for:

$$J = J_{min} = 0.05.$$

In these conditions very little error is committed by
assuming the following <u>in all cases</u>:

$$\boxed{t_{ON_{min}} = 0}$$

(5.4)

5.1 Buck

In continuous conduction we have:

$$\frac{t_{ON}}{T} = \frac{V_S}{V_A} = M$$

Hence $t_{ON_{max}} = T.M_{max} = 0.9\ T$ (cf. section 6.1)

and
$$\begin{cases} V_{o\ max} = 0.9\ T\ \frac{A}{T} = 0.9\ A \\ V_{o\ min} = 0 \end{cases}$$

From equation (5.1) we then obtain:

$$G_O = \frac{0.9 \; A}{4 \; \bullet \; V_R}$$

For the coincidence and
sampling-delay modulators

$$\boxed{G_{O \; BUCK} = 22.5 \; \frac{A}{V_R}} \qquad (5.5)$$

Thus in the case of the
simulation:

$$G_O = 22.5 \; \frac{0.1}{0.4}$$

$$\boxed{G_{O \; BUCK} = 5.6} \qquad (5.6)$$

For the magnetic modulator equations (5.3) gives us:

$$V_{c \; max} = V_1 \; (1 - t_{ON \; min} \; \frac{3}{2T}) = V_1$$

$$V_{c \; min} = V_1 \; (1 - t_{ON \; max} \; \frac{3}{2T})$$

We see that with this modulator we cannot exceed M = 0.666.

Under these conditions:

$$V_{c \; min} = 0$$

Hence:

$$G_O = \frac{V_1}{4 \bullet V_R}$$

$$\boxed{G_{O \; BUCK} = 25 \; \frac{V_1}{V_R}}$$

(5.7)

For the magnetic modulator

Thus, in the case of the
simulation:

$$G_O = 25 \; \frac{V_R}{V_R}, \; \text{since it is assumed } V_1 = V_R$$

$$\boxed{G_{O \; BUCK} = 25}$$

(5.8)

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5.2 Boost

In continuous conduction we have:

$$\frac{T_{ON}}{t_{OFF}} = \frac{V_S - V_A}{V_A} \quad \longrightarrow \quad \frac{T_{ON}}{T} = \frac{V_S - V_A}{V_S} = \frac{N-1}{N} , \quad N > 1$$

Hence $\quad t_{ON_{max}} = T \dfrac{N_{max}-1}{N_{max}} = T \dfrac{2-1}{2} = \dfrac{T}{2}$ (cf. section 6.1)

and $\quad \left| \begin{array}{l} V_{c\,max} = \dfrac{T}{2} \dfrac{A}{T} = \dfrac{A}{2} \\[2mm] V_{c\,min} = 0 \end{array} \right.$

Equation (5.1) gives us $\quad G_O = \dfrac{A/2}{4 \theta \, V_R}$

$$\boxed{G_{O\ BOOST} = 12,5 \ \frac{A}{V_R}} \tag{5.9}$$

Thus, in the case of the simulation:

$$G_O = 12,5 \ \frac{0,1}{0,4}$$

$$\boxed{G_{O\ BOOST} = 3,2} \tag{5.10}$$

For the coincidence and sampling-delay modulators

For the magnetic modulator it happens that:

$$\left| \begin{array}{l} V_{c\,max} = V_1 \\[2mm] V_{c\,min} = V_1 \left(1 - \dfrac{T}{2} \dfrac{3}{2T}\right) = \dfrac{V_1}{4} \end{array} \right.$$

Hence:
$$G_O = \frac{3\,V_1/4}{4\,t\,V_R}$$

(5.11)
$$\boxed{G_{O\ BOOST} = 19\,\frac{V_1}{V_R}}$$

(5.11)

For the magnetic modulator

Thus, for the simulation

$$\boxed{G_{O\ BOOST} = 19}$$

(5.12)

5.3 Buck-Boost

In continuous conduction, we have:

$$\frac{t_{ON}}{t_{OFF}} = \left|\frac{V_S}{V_A}\right| \longrightarrow \frac{t_{ON}}{T} = \frac{|V_S|}{|V_S| + |V_A|} = \frac{M}{M+1}$$

Hence $t_{ON\ max} = T\,\dfrac{M_{max}}{M_{max}+1}$

Since we consider that M = 1 (cf. the Report of the EXTEC-LAAS meeting of June, 25, 1975, p.5):

$$t_{ON\ max} = T\,\frac{1}{2} = \frac{T}{2}$$

In these conditions we obtain the same result as for the boost and it is therefore not necessary to re-do the calculations.

$$\boxed{G_{O\ BUCK\text{-}BOOST} = 12,5\,\frac{A}{V_R}}$$

(5.13)

For the simulation

For the coincidence and
sampling-delay modulators

(5.14)

$$\boxed{G_{O\ BUCK\text{-}BOOST} = 3,2}$$

$$G_{0\ \text{BUCK-BOOST}} = 19\ \frac{V_1}{V_R} \tag{5.15}$$

For the simulation

$$G_{0\ \text{BUCK-BOOST}} = 19 \tag{5.16}$$

For the magnetic moculator

6. Guidelines for the Simulations and "Optimizations" to be Carried Out /20

6.1 Variation Ranges of the Parameters

In agreement with the ESTEC (meeting of June 25, 1975), the following values of the reduced parameters will be considered;

ϕ	0,1	0,033	0,01

(6.1)

For the three types of converters

Γ	0	1	3	7

(6.2)

For the three types of converters

M BUCK	0,5	0,6	0,7	0,8	0,9

(6.3)

Δ BUCK	0,2	0,6	1,8	2,2	6	15

(6.4)

M BOOST	1,2	1,4	1,6	1,8	2,0

(6.5)

Δ BOOST	0,24	0,8	2,6	3,8	8	16

(6.6)

M BUCK-BOOST	1

(6.7)

Δ BUCK-BOOST	0,4	1,2	3,5	4,5	8,5	16

(6.8)

The extreme values of the given Δ's are compatible with the /21 extreme values of the given M's and with the converters:
- operation in continuous conduction from J = 1 to 0.1
- operation in discontinuous conduction from J = 0.75 to 0.05

(cf. ESTEC-LAAS Meeting of June 25, 1975).

For this compatibility we used the variation equations of Δ with M and J which we will give in the section which makes use of the results.

The variation ranges for the reduced parameters defined above are correct for each parameter taken separately. It may happen that some combinations are not realistic. In particular if Δ is small (large L) and Φ is large (C very very small), it is possible, when Γ ≠ 0, to end up with a very large value of r, larger than the charging resistance R_s! We have therefore eliminated any combination (Δ,Φ,Γ) resulting in a resistance r greater than $R_s/10$.

6.2 Quantity of Simulations to be Done

The preceding section shows that we have:
- 3 values of Φ
- 4 values of Γ
- 6 values of Δ
- 5 values of M for the buck and boost and 1 for the buck-boost.

We can thus study:
- 360 classes of the power stage for the buck
- 360 classes of the power stage for the boost
- 72 classes of the power stage for the buck-boost, i.e.

i.e. a total of 792 power stage classes.

In addition, we must consider the following:

- 3 types of modulators,
- 5 types of correcting networks.

Thus, in effect, we are considering 11,880 classes of converters.

For each class we are interested in the following:

- the local stability,
- the overall stability,
- the response at a step of output current,
- the output, input and input-output impedances.

Thus we end up with 71,280 elementary results to be obtained.

We give these figures to emphasize that:

- it would be absurd to want to obtain and formulate such a mass of results,
- at the limit, it would be nearly impossible to use this mass of results,
- the simulation is a powerful tool of analysis: all of the desired results may be obtained for one well determined system,
- the difficulty which exists, correlatively, in obtaining from them synthetic results.

The rest of this section is going to explain how we avoided this obstacle, i.e. how we reduced the quantity of elementary results in order to bring it to a realistic, though high, value while keeping a significant value for the results.

6.3 Simulations and "Optimizations" to be Carried Out

With respect to the different elementary results to be obtained, the following general remarks can be made:

a) The local stability:
- perhaps obtained using a mean calculation time;
- the search for a local stability limit only makes sense for a non "optimized" system, i.e. a converter whose amplifier-correcter is a simple flat gain. In effect, we suppose, for example, that we have determined that such a converter operates "best" with a correcting amplifier of the type $G \ 1+\tau_p/P$ and with $G = 5$ and $\tau = 0.1$. It is clear that at this moment it is meaningless to vary G in order to determine a local stability limit.

b) The overall stability: demands too much calculation time.

c) The response at a step of output current:
- demands rather little calculation time;
- is meaningless for a system functioning at its "best."

d) The determination of input, output and input-output impedances:
- demands enormous amount of calculation time;
- is meaningless for a system operating at its "best."

In addition, the reduced parameters Δ, Φ, Γ and M have different implications:

a) Γ and Φ are genuine <u>construction parameters</u>. Their value may be chosen intentionally by the designer: Δ enables him to decide in what current range the converter in question is going to operate in continuous or discontinuous conduction or to choose the inductance L; Φ then enables him to choose the size of the output capacitor.

b) Γ relates to an imperfection, that of the output capacitor. The designer <u>does not choose</u> a value of Γ. Rather he tries to use a component of which Γ is as small as possible or as a tolerable value.

c) M relates to an operating point. In a given application, M varies because V_A varies. Thus, M has a nominal value and may vary by a certain percentage more or less around this nominal value.

Taking into account all of the facts which we have just enumerated, we adopted the following procedure which may be broadly outlined as follows:
- exhaustive study of the local stability with coincidence modulator,
- simplified study of the local stability for the two other modulators,
- investigation of the better correction system for all of the doublets (Δ, Φ) with the conincidence modulator and study of the variations caused by the extreme values of M and by $\Gamma = 3$ or 7,
- readjustment of the correction system for the two other modulators and for one value of Φ and two values of Δ,
- plot of impedances and study the overall stability for only the optimized systems.

This procedure is shown in more detail in the synoptic diagram, Table 6.1, <u>for one unit</u> (buck or boost type). For the buck-boost unit the diagram has to be modified very slightly to allow for the fact that only one value of M is to be considered.

All the doublets
ϕ, Δ

All the doublets
M, Γ

1 central value
of M
$\Gamma = 0$

Local stability with
coincidence modulator

Local stability

Study of best
correction system
with coincidence
modulator

· Indicating
response

Impedances

Overall
stability

With sampling-delay
modulator

With magnetic
modulator

2 extreme values
of M
Γ = 3 or 7

ϕ = 0,033
$\Delta \rightarrow$ continuous conduction
$\Delta \rightarrow$ discontinuous conduction

1 central value of m
$\Gamma = 0$

Adjustment of the
correction system

With sampling-delay
modulator
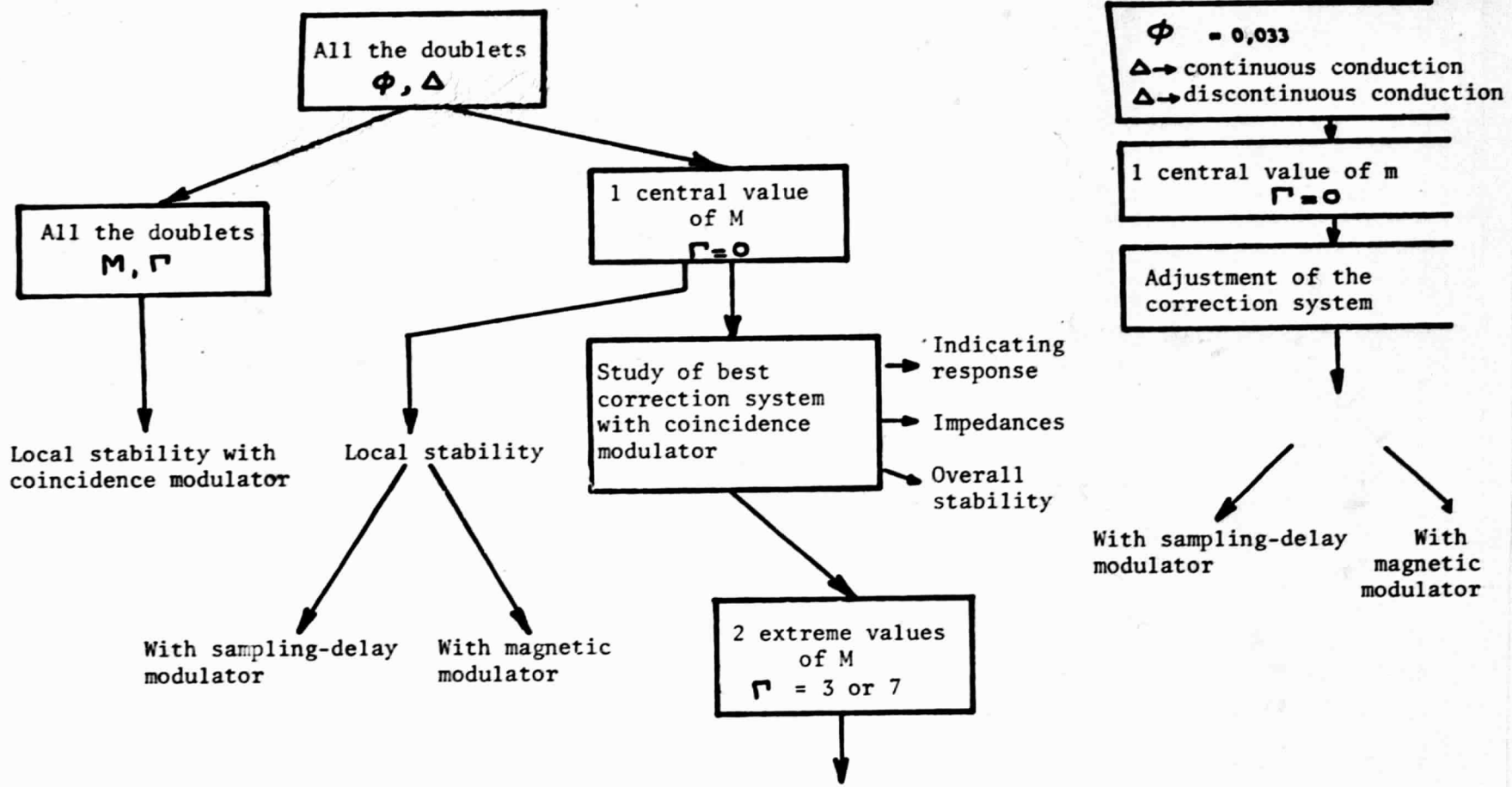
With
magnetic
modulator

Table 6.1. Diagram of the simulation procedure for one type of unit.

7. Study of Local Stability

When we consider a linear system subjected to constant inputs, this system possesses an equilibrium state $\underline{Z}^o$ which is such that (using variable of state notation):

$$\underline{z}(t_o) = \underline{z}^o \longrightarrow \underline{z}(t) = \underline{z}^o \ , \ t_o, \ t$$

If the system is described by the traditional equation:

$$\underline{\dot{z}} = A \, \underline{z} + B \, \underline{u}$$

(with, in our case $\underline{u} = \underline{u}^o = c^{\underline{z}}$)
it is easily shown that:

$$\underline{z}^o = - A^{-1} \, B \, \underline{u}^o$$

If we set:

$$\underline{\Delta z} = \underline{z} - \underline{z}^o$$

it is also easily shown that

$$\underline{\Delta z}(t) = \emptyset(t-t_o) \cdot \underline{\Delta z}(t_o), \quad \phi = \text{continuous state} \qquad (7.1)$$
$$\text{transition matrix}$$

The system is asymptotically stable $(\lim_{t \to \infty} \underline{z}(t) = 0)$. when $\phi(x)$ has all its eigenvalues less than 1 (absolute value) for $\forall x > 0$.

If we are dealing with a discrete linear system, Eq. (7.1), takes the form:

$$\underline{\Delta z}_n = \emptyset^n \, \underline{\Delta z}_o \ , \qquad \phi = \text{discrete state} \qquad (7.2)$$
$$\text{transition matrix}$$

We arrive at the asymptotic stability from the eigenvalues of ϕ in the same way as in the continuous case.

The linearity of the system implies that:
- $\underline{Z}^\circ$ is unique,
- the definition of the stability is valid whatever the value of $\underline{\Delta Z}_\circ$.

If we now consider a discrete non-linear system, this system may be described by the autonomous non-linear recurrence:

$$\underline{z}_{n+1} = \underline{f}(\underline{z}_n) \tag{7.3}$$

and the equilibrium point $\underline{Z}^\circ$ is defined by:

$$\underline{z}^\circ = \underline{f}(\underline{z}^\circ), \quad \text{this is first order cycle of the recurrence (7.3).} \tag{7.4}$$

Taking the definition:

$$\underline{\Delta z}_i \triangleq \underline{z}_i - \underline{z}^\circ \tag{7.5}$$

It is clear that:

$$\begin{cases} \text{If} \quad \underline{\Delta z}_n \rightarrow 0 \\ \underline{\Delta z}_{n+1} \rightarrow \emptyset . \underline{\Delta z}_n \\ \text{with } \emptyset = \dfrac{d\underline{f}}{d\underline{z}_n} \Bigg|_{\underline{z}_n = \underline{z}^\circ} \end{cases} \tag{7.6}$$

Eqs. (7.6) define the linear approximation of the system around $\underline{Z}^\circ$.

The discrete non-linear system (7.3) is said to be locally stable if its linear approximation around $\underline{Z}^\circ$ is asymptotically stable.

In contrast to linear systems:

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- $\underline{Z}^\circ$ is not necessarily unique,
- there may exist other equilibrium points than the first order cycles, cf. [2-3-4],
- $\underline{Z}_n$ is infinitely small.

The converters which we are interested in here are discrete non-linear systems [5] for which we know how to determine the local stability [5].

There exists a basic difficulty in analyzing the local /28 stability using a simulation of electric behavior. This difficulty is connected to the fact that the local stability is a purely mathematical idea in particular calling for a transition to the limit.

When a linear system is unstable and when it is "initialized" at a value $\underline{Z}_o \neq \underline{Z}^\circ$, the absolute value of $\underline{Z}_n$ increases indefinitely with n.

When we are dealing with a locally unstable non-linear system, the amplitude of $\underline{Z}_n$ remains limited. The system oscillates, either at a subharmonic frequency of F = 1/T (m order cycles, cf. [2]), or in a quasi-stochastic manner (cf. [3-4]). We eliminate the non-realistic case where other stable first order cycles would exist apart from $\underline{Z}^\circ$ (these cycles would correspond to non-oscillatory operating points different from the nominal point and which are eliminated by the electronic engineer for any real system by adequate safeguards). Thus, in all cases, the instability is characterized by the disappearance, in the steady state, of the strictly periodic operation of period T. By way of example, we can look at such oscillations in the similar graphs in Figs. 7.1 and 7.2 where the comparison saw-tooth $\sigma\!\!*(t)$ is given to serve as a time base.

In principle, the oscillation is present for various magnitudes existing in the system. In particular, it is impossible for the system to be unstable and for the successive t_{ON} to be equal to one another. As a result, the oscillation can be observed for the command voltage V_C.

In addition, it may also be noted that I_L is a more "sensitive" quantity (i.e. "whose variations are greater") than V_C since, in a first approximation, I_L varies as the integral of V_C. In fact, in the presence of a very low frequency oscillation, the relative value of I_L varies much more than that of V_C.

Since the information is discrete, it is necessary to have a value of I_L and a value of V_C per period. Since $\underline{Z}(t)$ at the end of a t_{ON} is a discrete vector of state [1], it is clear that we should take I_L and V_C at the end of the t_{ON} periods, which we will note $I_L(t_{ON})$ and $V_C(t_{ON})$.

Thus the method of analyzing the local stability is based on the following procedure:
a) The gain G of the error amplifier is set at the value G_o calculated in section 5.
b) The vector of state $\underline{Z}_o$ of the system is set (close) to the equilibrium point $\underline{Z}°$. Under the circumstances, the voltage at the terminals of the ideal part of the output capacitor is set as close as possible to the deduced value of $\underline{Z}°$ and I_L is set at 10% above the deduced value of $\underline{Z}°$. This 10% difference is arbitrary and simply results in the fact that systems which are mathematically stable but whose stability range around $\underline{Z}°$ is very small are considered to be locally unstable.
c) The system is allowed to develop long enough so that the transietn behavior is eliminated: we choose to take 110 T.
d) We take the average of $I_L(t_{ON})$ and $V_C(t_{ON})$ obtained for t going from 110 T to 150 T, i.e. I_L ave. and V_C ave.

Vs

I_L
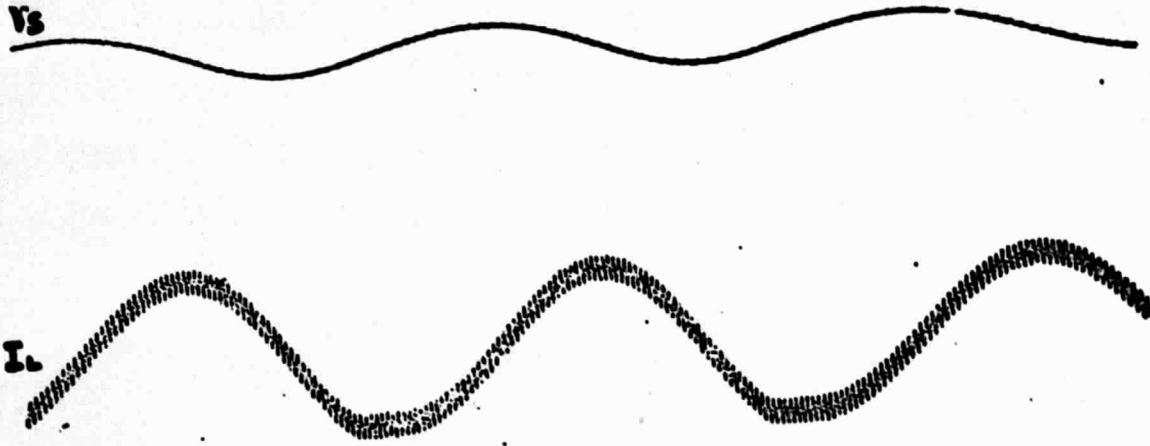
High amplitude oscillation at 0.014 F

BOOST. $\phi = 0.01$ $\Delta = 0.24$ $\Gamma = 0$ $M = 1.6$. $G = 0.56$

Figs. 7.1 and 7.2. Oscillation Examples

BUCK. $\phi = 0.033$ $\Delta = 0.6$ $\Gamma = 0$ $M = 0.5$. $G = 1.6$

$S(t)$

I_L

Vs
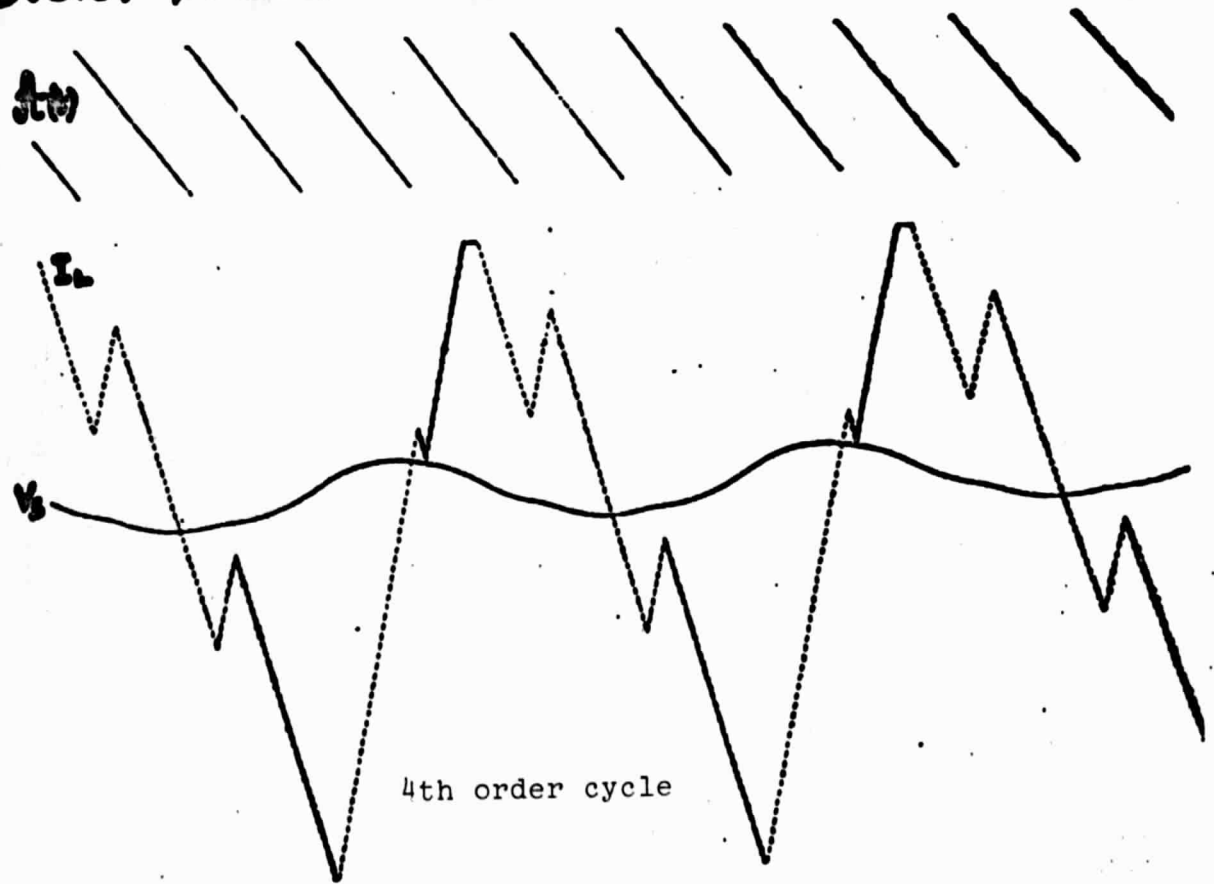
4th order cycle

e) We determine the maximum difference of $I_L(t_{ON})$ and
   $V_C(t_{ON})$ with respect to I_L ave. and V_C ave., i.e.
   ΔI_L max and ΔV_C max for t varying from 145 T to 150 T.

f) We decide that the system is locally unstable if:

$$\left| \frac{\Delta I_L}{I_L \text{ moy}} \right| > 5 \%$$

or if

$$\left| \Delta V_C \right| > 5 \% A = 0,005$$

g) If the system is stable, we increase the gain of G to
   1.5 G and we return to (b).

h) If it is unstable, we decrease the gain of G to 0.9 G
   until the system becomes stable again and we obtain the
   value of G denoted G_F, the limit gain of local stability.

Remark 1: Systems which are always stable.   /31

Some systems may remain stable even if $G \to \infty$ or may only
become instable for very high gains.

In this case, the following procedure is followed.

An increase in G causes an increase in the residual ripple
in V_C. We measure the gain G_M' where $V_C(t)$ reaches a value of
+A or a value of 0 and we stop the increase in gain at the
following value of G_M:

$$G_M = 5 \, G_M'$$

Although the coefficient 5 is arbitrary, G_M may be considered
in practice as an upper limit of the gain.

One might think of using only I_L for doing the convergence test. Some systems have a non-observable instability for $I_L(t_{ON})$. An example of this is given in the analog graph shown in Fig. 7.3.
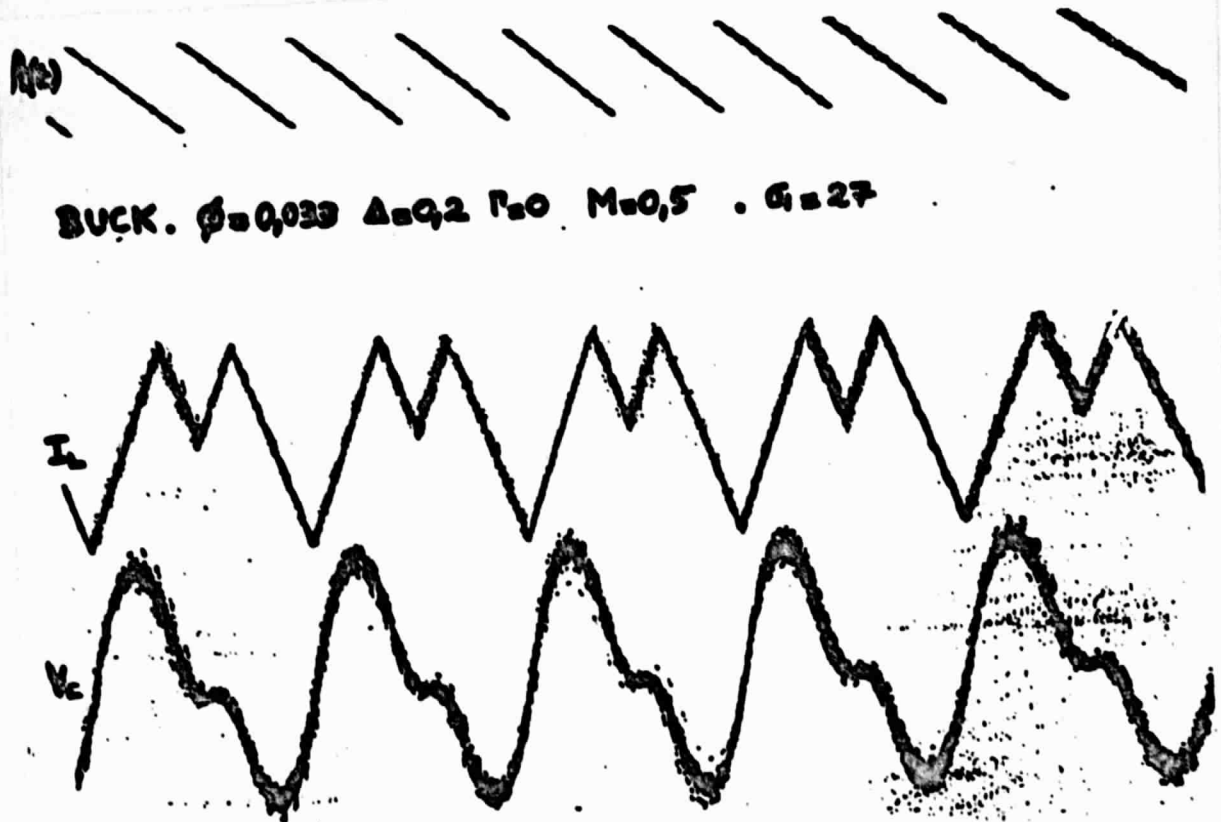
Fig. 7.3. Non-observable oscillation for I_L.

Remark 3: Limit systems of indistinct stability.

It is possible to describe in an approximate manner the behavior of the system in question by a continuous linear model to which we will again refer in the following section.

By means of this description it can be shown that some converters possess the following property: the phase shift caused by the power stage remains very close to $-\pi$ for a wide frequency

range.   From this it follows that <u>in a closed loop, for a wide range of gain, these systems are stable but have a damped oscillatory behavior with a very small damping coefficient.</u> An example of such a system is shown in Figs. 7.4 and 7.5. In this system the gain may be varied approximately in a ratio of 1 to 300 while maintaining this behavior (only the resonance frequency changes).

For a real converter or for the analog simulation the result is that the system reacts to noises or to parasitic perturbations while oscillating constantly with a high amplitude with respect to the perturbations.

We call these "indistinct stability limit" systems and, in agreement with the ESTEC (cf. Meeting of Dec. 12, 1975), we will study their local stability.
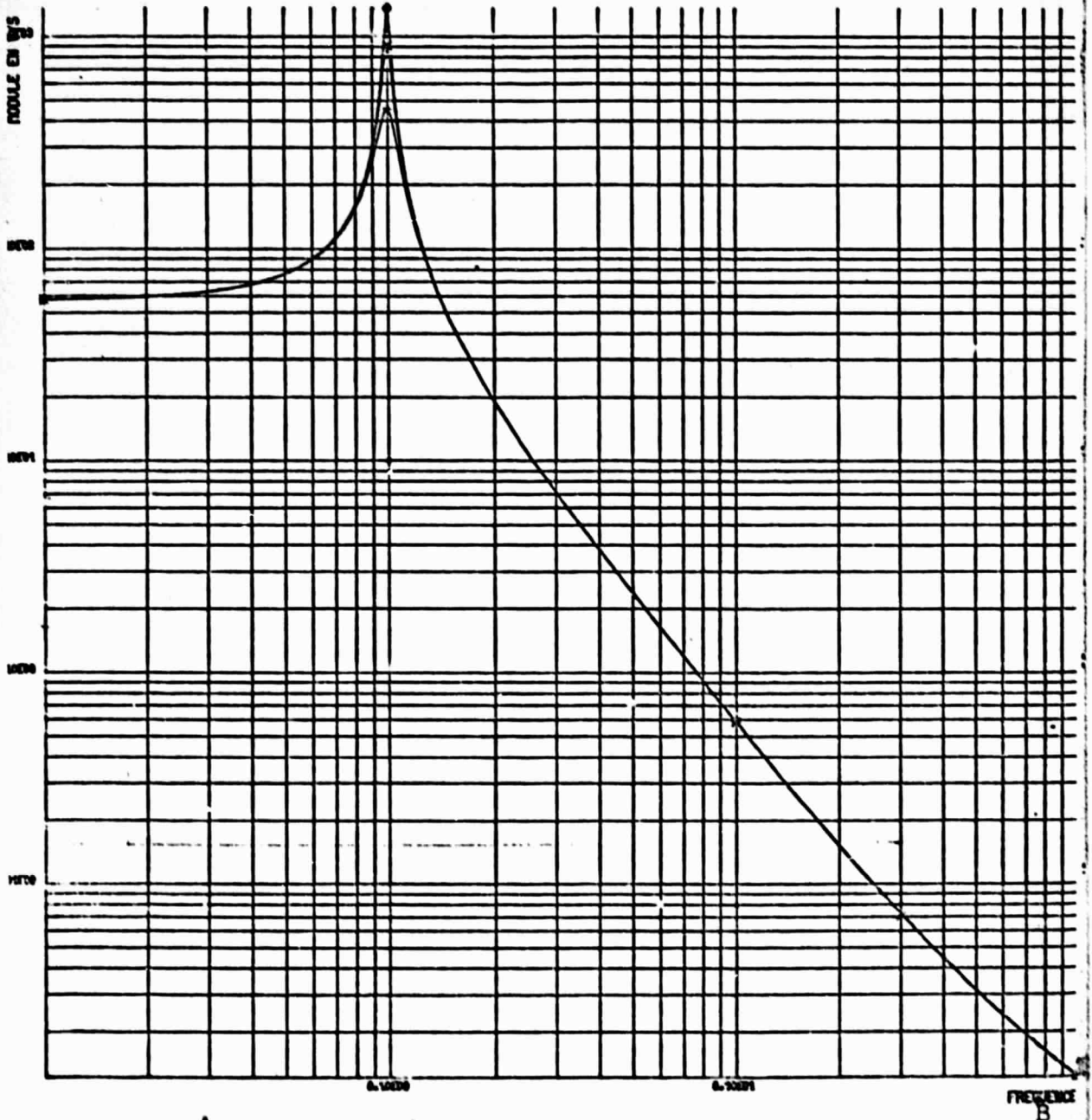
BUCK CONTINU (MODULE) DELTA=0.2 0.6 1.8
PHI=0.01 GAM=0. M=0.7

Fig. 7.4

Key: A) continuous   B) frequency

BUCK CONTINU (PHASE) DELTA=0.2 0.6 1.8
PHI=0.01 GAM=0. M=0.7

Fig. 7.5

Key: A) continuous   B) frequency

8. Study of a Correction Network Giving Good Performance

8.1 Correction Networks Considered

Using the symbols used in Fig. 3.3, the different correction networks which we will consider are the following, in order of increasing complexity:

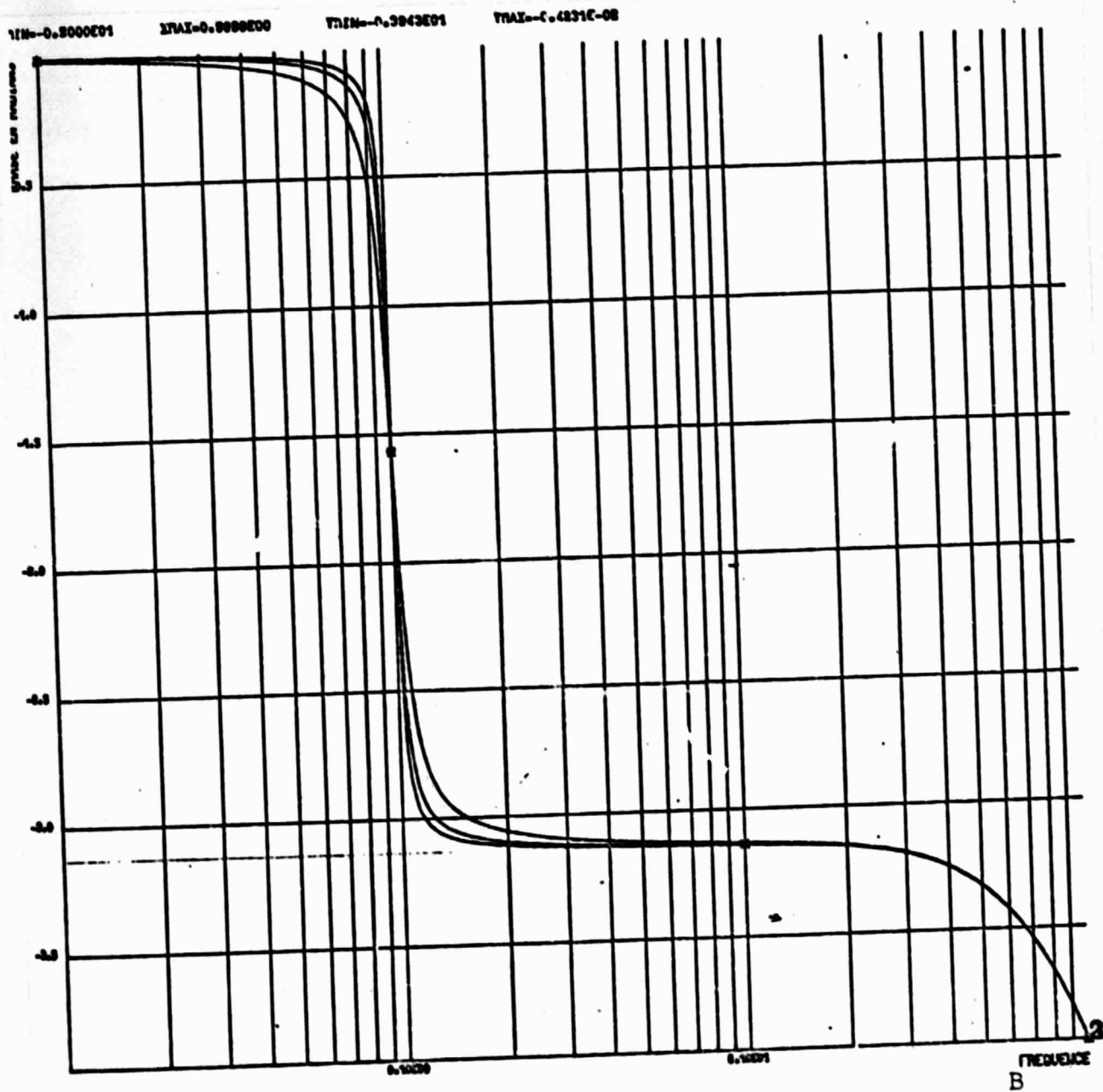$$G_1(p) = G \; ; \tag{8.1}$$

$$G_1(p) = G \frac{1 + a \, \tau p}{1 + \frac{\tau}{a} \, p}$$, advance-retard centered on the frequency $f = 1/2\pi\tau$.

We have chosen to take $a = \sqrt{5}$, this value constituting a good compromise between the obtained phase leading which equals 42° at the frequency f and the high-frequency gain which equals 5 G, thus:

$$G_1(p) = G \frac{1 + \sqrt{5} \, \tau p}{1 + \frac{\tau}{\sqrt{5}} \, p} \; ; \tag{8.2}$$

$$G_1(p) = G \frac{1 + \tau p}{\tau p}$$, integrator with a 0 at the frequency $f = 1/2\pi\tau$; $\tag{8.3}$

$$G_1(p) = G \frac{1 + \tau_1 p}{\tau_1 p} \; \frac{1 + \sqrt{5} \, \tau_2 p}{1 + \frac{\tau_2}{\sqrt{5}} \, p},$$ combination of the two preceding equations with $\tau_2 < \tau_1$ $\tag{8.4}$

$$\left| \begin{array}{l} G_1(p) = G \\[2mm] G_2(p) = G_{AC} \dfrac{1}{1 + \tau p} \end{array} \right.$$, current loop $\tag{8.5}$

Examination of this list of networks immediately leads to the following remarks:

- since networks (8.3) and (8.4) possess a pure integrator they obviously result in a static error of 0 and should

only be optimized with respect to their transient response;

- networks (8.1), (8.2) and (8.5) must necessarily be such that $G \geq G_o$ (minimum gain defined in section 5. We end up with this conclusion for the current loop (8-5) because since the mean value of the current I_C is 0, in continuous conductance, the current loop cannot improve the static regulation of the main loop.

8.2 Method of Studying the "Best" Correction Network

Among the factors which characterize the performance of a corrected system, some play the role of constraints in the sense that any system which does not observe a given limit is rejected as being unsatisfactory. These factors are the following:

- the local stability;
- the precision of regulation (cf. section 5); the
- the indicative response, response to a 50% step in output current (from I_S to $I_S/2$), which must enter into a determined window. This window is shown in Fig. 8.1 (cf. Meeting of June 25, 1975).
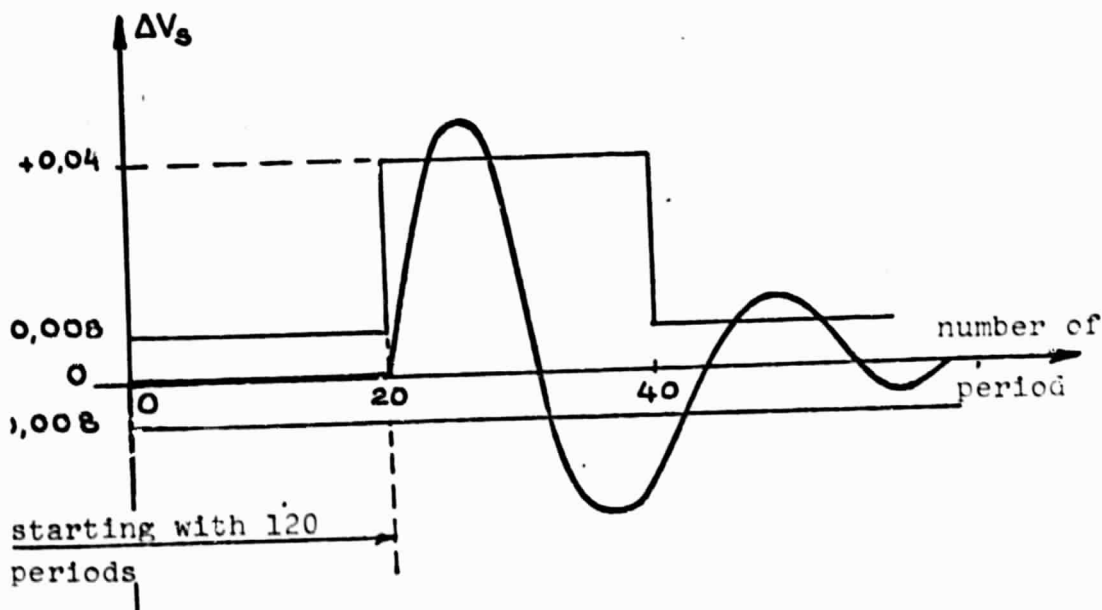

Fig. 8.1. Indicative response.

The preceding constraints thus have to be completely satisfied. By contrast:

- the output, input and input-output impedances appear rather like a property of a given system for which it is up to the designer, the person who uses the result, to decide whether or not it is satisfactory;
- the same applies for the overall stability, in certain cases, when the range of instability in the space of the initial conditions is such that the designer may decide that it cannot be reached or that adequate limitations are foreseeable.

Thus we have adopted a method which takes into account the above points and which is illustrated in the flow chart shown in Fig. 8.2.

The main point, with respect to this flow chart, is that all of the studies involved here are not done on the hybrid simulation system.

Since the simulation is only a method of analysis, such a task would be insurmountable.

As a result of studies in done in another connection in this area of the LAAS on the theoretical level, we have 6 approximate continuous linear models of the three converters buck, boost and buck-boost in continuous conduction and discontinuous conduction [6]. The models, although approximate, give a good indication of the behavior of the system in question by giving the gain and phase diagrams of the approximate transmittances:

$$\frac{\Delta v_s}{\Delta t_{ON}} (p) \quad \text{and} \quad \frac{\Delta I_c}{\Delta t_{ON}} (p)$$
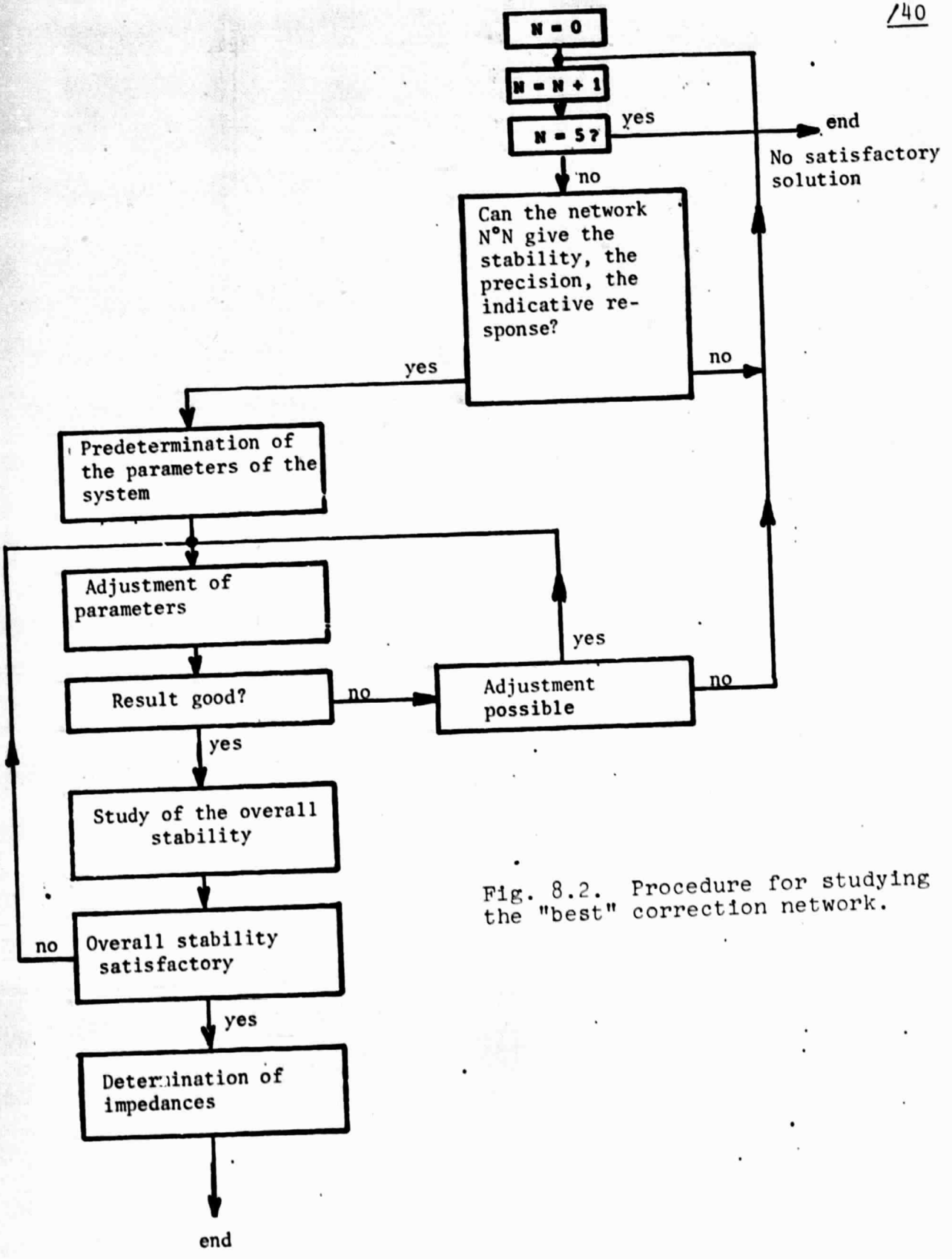
Fig. 8.2. Procedure for studying the "best" correction network.

from specific values of Λ, Φ, Γ and M and from the machine values $V_S = 0.4$; $F = 10$; $R_S = 1$ (see, for example, the diagrams in Figs. 7.4 and 7.5).

The following parts of the flow chart shown in Fig. 8.2 were done using these models:

- what is the simplest network which can be adapted?
- predetermination of the parameters of this network.

The simulation is used to:
- adjust the parameters of the network taking into account the exact behavior of the system;
- determine the exact impedances;
- give the overall stability, which the approximate model cannot give.

Thus it was possible to carry out this basic phase of the study of elementary units only by the joint use of a powerful but approximate theoretical method on the level of synthesis, and a method of exact analysis using simulation.

9. Determination of Output, Input and Input-Output Impedances

The notion of impedance is a notion purely related to continuous linear circuits.

The converters in question are discrete non-linear systems. This implies the following obvious consequences:
- the notion of impedance in the case of a converter is necessarily an approximate notion;
- the corresponding measurement can be made only at a small signal;
- the significance of the values obtained increases as the frequency in question decreases and their significance decreases as one approaches the frequency $F/2$ which represents a limit (SHANNON theorem).

9.1 Theoretical Method Used

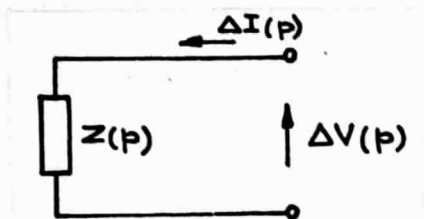Let us assume, therefore, Fig. 9.1, an impedance dipole $Z(p)$ represented symbolically as follows:

Fig. 9.1. Elementary dipole.

The problem which we have to solve is one of determining experimentally (in some way) the function $|Z(j\omega)|$ for a certain interval of variation of the angular frequency ω.

The difficulty basically stems from the fact that for the physical system (or for the analog simulation, which amounts to the same thing) we have temporary values $\Delta v(t)$ and $\Delta i(t)$.

The most obvious method consists in applying to the dipole a signal Δv (or Δi) of sinusoidal form $A\sin\omega t$ and measuring in a steady state the absolute value of the response Δi (or Δv) and setting up the ratio of the absolute values.

This extremely traditional method certainly leads to the result but in our case it has the following drawbacks:
 - since the measurement is made for a small signal, the noise present in the machine is not negligible and we cannot be content with a measurement of the peak value; the data obtained must be processes, for example a correlation process, and this treatment must be carried out for each value of ω, which extends the simulation time;
 - when ω is small, the measurement time required for one vlaue of ω is long, approximately $2 \times 2\pi/\omega$.

By way of example, it took us about 35 minutes to plot a curve $|z(j\omega)|$ using only 20 values of ω.

Therefore we used the method which we are now going to describe and which is based on the property that $Z(j\omega)$ is the FOURIER transformation of $z(t) = \mathcal{L}^{-1}[z(p)]$.

Therefore let:

$$z(t) \triangleq \mathcal{L}^{-1}\left[z(p)\right] \tag{9.1}$$

We break $Z(p)$ down into simple elements:

$$z(p) = \frac{A_1}{p-a_1} + \frac{A_2}{p-a_2} + \ldots \tag{9.2}$$

assuming that there are no multiple poles nor zero poles and that the number of zeros of $Z(p)$ is less than the number of poles.

We then have:

$$z(t) = A_1 e^{a_1 t} + A_2 e^{a_2 t} + \ldots \tag{9.3}$$

The FOURIER transformation $F(\omega)$ of $z(t)$ is:

$$F(\omega) \triangleq \int_0^\infty z(t) \, e^{-j\omega t} \, dt \tag{9.4}$$

This gives us:

$$F(\omega) = A_1 \int_0^\infty e^{(a_1 - j\omega)t} \, dt + A_2 \int_0^\infty e^{(a_2 - j\omega)t} \, dt + \ldots \tag{9.5}$$

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$$F(\omega) = -\frac{A_1}{a_1 - j\omega}\left[e^{(a_1 - j\omega)t}\right]_0^\infty + \frac{A_2}{a_2 - j\omega}\left[e^{(a_2 - j\omega)t}\right]_0^\infty + \dots \quad (9.6) \qquad \underline{/43}$$

Hence:

$$F(\omega) = \frac{A_1}{j\omega - a_1} + \frac{A_2}{j\omega - a_2} + \dots \quad (9.7)$$

Comparison of Eq. (9.2) and Eq. (9.7) clearly shows that:

$$F(\omega) = Z(j\omega) \quad (9.8)$$

The basic principle of the method is thus to apply a DIRAC delta function $\Delta i = I_0 \delta(t)$, to measure the output $\Delta v = I_0 z(t)$ and to perform the integral (9.4) off-line.

Theoretically, a single simulation giving $z(t)$ is sufficient to obtain the entire curve $|Z(j\omega)|$, which represents a great improvement with respect to the preceding method.

9.2 Adaptation of the Preceding Method to Our Case
9.2.1 Determination of the Delta Function Response

It is clear that it is not possible to directly apply a DIRAC delta function to the dipole in question.

One possibility is to apply an impulse of finite amplitude H for a sufficiently short time τ, Fig. 9.2, and compare this impulse with a DIRAC delta function of area $H\tau$. The amplitude H must be compatible with the possibilities of the circuits in question. Moreover, since τ is small, the energy supplied to the system is low and the corresponding response $H\tau\, z(t)$ is of low amplitude.
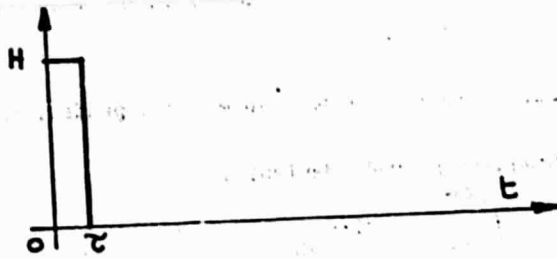
Fig. 9.2

Another method can be used.

The behavior of the circuit is determined by the following equation of state:

$$\begin{vmatrix} \dot{\underline{x}} = A\underline{x} + \underline{B}i \\ v = C\underline{x} \end{vmatrix} \tag{9.9}$$

If i is a DIRAC delta function $i = I_o \delta(t)$, we have:

$$\begin{vmatrix} \underline{x}(0^-) \triangleq 0 \\ \underline{x}(0^+) = \underline{B}I_o \end{vmatrix}$$

Thus:

$$\underline{x}(t) = e^{At} \underline{x}(0^+) = e^{At} \underline{B}I_o$$
$$\scriptstyle t>0$$

and

$$v(t) = I_o C e^{At} \underline{B} = I_o z(t)$$
$$\scriptstyle t>0 \tag{9.10}$$

The impulse response being sought $z(t)$ is thus only the response of the cirucit in a free state but conveniently set to the value $\underline{B}I_o$.

For a real circuit, this adequate initial setting of $\underline{B}I_o$ can generally be determined very simply by examining the circuit.

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We give to I_o the maximum value compatible for $\underline{B}I_o$ with the assumption of small signals.

9.2.2 Discrete Evaluation and Integration for a Finite Time

In the hybrid simulation we have infact only one value of v per period T of the converter, given the discrete nature of the behavior of these systems.

In addition, v is only measured for a finite time kT.

Eq. (9.4) becomes:

$$F'(\omega) = \int_0^{kT} z(t)\, e^{-j\omega t}\, dt \tag{9.11}$$

As a result, modified Eq. (9.6) gives:

$$F'(\omega) = \frac{A_1}{a_1 - j}\left[e^{(a_1 - j\omega)kT} - 1 \right] + \frac{A_2}{a_2 - j\omega}\left[e^{(a_2 - j\omega)kT} - 1 \right] + \ldots \tag{9.12}$$

$$F'(\omega) = F(\omega) + \underbrace{\frac{A_1}{a_1 - j\omega}\left[e^{(a_1 - j\omega)kT} + \frac{A_2}{a_2 - j\omega}\, e^{(a_2 - j\omega)kT} + \ldots \right]}_{\text{error term}} \tag{9.13}$$

In the discrete form we in fact calculate:

$$F''(\omega) = \sum_{n=1}^{k} z_n \cdot e^{-j\omega nT}\, T \tag{9.14}$$

By way of testing, we apply this method to a circuit of the second order wired on the analog part and such that:

- continuous impedance is 0.1
- resonance of 7 dB at frequency 1.

The result is shown in Fig. 9.3 and one can see that it is very satisfactory. The oscillatory appearance of the curve at the low frequencies is explained very well by the form of the error term of Eq. (9.13) which, at the given kT, is sinusoidal in terms of ω with an amplitude which increases when ω decreases. In practice, this error is not detrimental because it manifests itself in the zones where the behavior of the system is the best known.

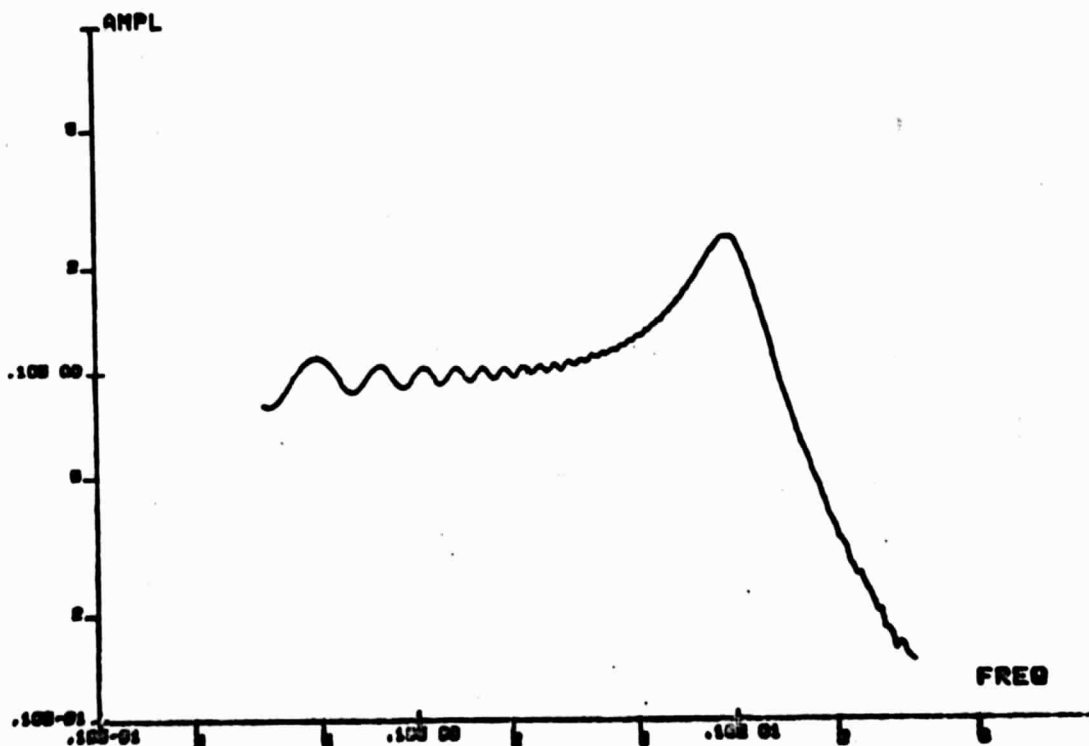Circuit of the Second Order

Fig. 9.3. Example of impedance obtained by FOURIER transformation.

In order to make a comparison with the first method, the obtaining of such a curve requires 90 seconds (essentially 90

seconds of processing on our CII-MITRA 15 computer) <u>for 200</u>
<u>values of ω</u> (thus for a very detailed curve).

Since the main portion of the time is the processing time,
in order to improve the results while decreasing the noise, we
in fact make <u>10 successive firings</u> in acquiring the $z(t)$ values
(which requires only 30 seconds) and we take the average of the
samples of the same order before actually processing the data.

9.3 Output Impedance

Let us consider the output network of the converters
(Fig. 9.4) to which a DIRAC impulse $I_o\delta(t)$ is applied.
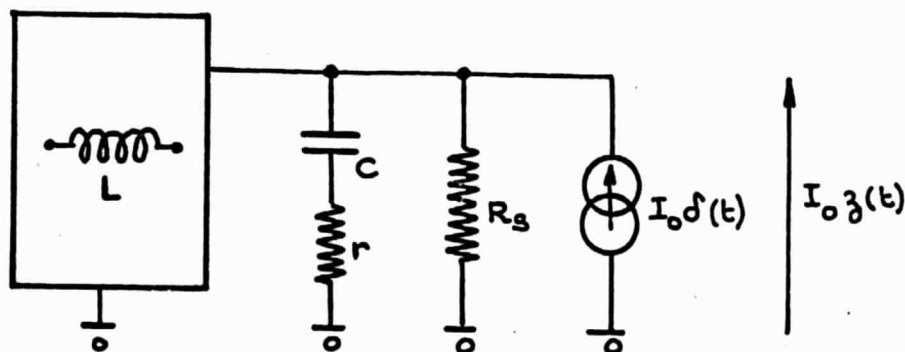
Fig. 9.4.   Application of a DIRAC impulse of current
to the output network.

Immediately we see that the effect of applying the impulse
at $t = 0^+$ is to create at the terminals of C a discontinuity of
the first kind of value

$$\frac{I_o}{C}$$

(9.15)

On the input side the network is either in the air or
connected to the inductor L. In both cases this connection
does not occur.

We therefore obtain $z_s(t)$ by the diagram shown in Fig. 9.5.
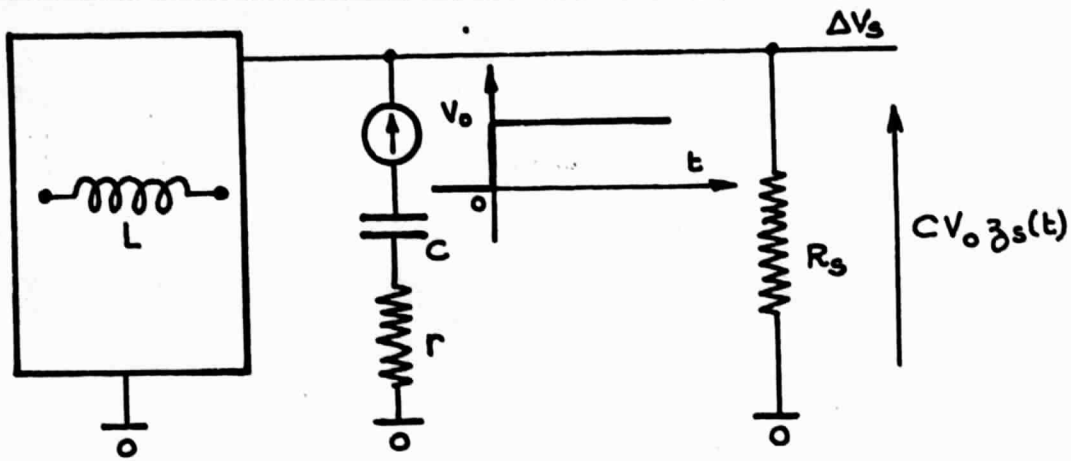
Fig. 9.5. Obtaining in practice the output impulse response.

The result is given in the reduced form:

$$\boxed{\dfrac{\left| z_s(j\omega) \right|}{R_s}} \tag{9.16}$$

In a first approximation, the output voltage V_s may be broken down into the sum of a continuous value and a residual ripple whose form does not vary at small signals. It is for this reason that it is sufficient to measure ΔV_s per period to obtain directly the variation in the mean value of V_s.

9.4 Input-Output Transmittance

By definition, this is the following quantity:

$$\boxed{T_{ES}(p) \triangleq \dfrac{\Delta I_A \text{moy}}{\Delta I_a \text{moy}} (p)} \tag{9.17}$$

This is a dimensionless quantity for which we must speak of transmittance and not impedance.

Since the measurement of this quantity requires the application of a DIRAC impulse on the output, T_{ES} is determined at the same time as Z_S. It suffices to measure ΔI_A and ΔV_S after applying the step V_o.

The determination of the mean ΔI_A is not as obvious as that of the mean ΔV_S. Fig. 9.6 shows what I_A can look like during the course of a steady state period and a transient state period.
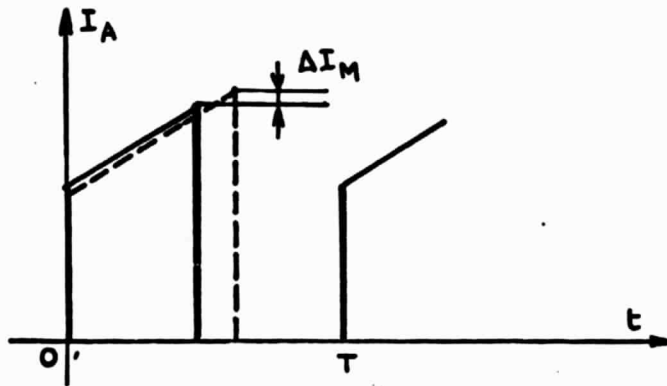
Fig. 9.6. Variations in I_A.

We see that it is not sufficient to know ΔI_M in order to know ΔI_A ave.

The following procedure is then used: an auxillary integrator reset to zero at the beginning of each period gives at the end of the period the quantity $\int_0^T I_A(t)\, dt = T\, I_A$ ave. and I_A ave. becomes directly acessible on the analog wiring no matter what the form of I_A, i.e. whatever the converter.

Remark: T_{ES} in continuous conduction.

Continuous T_{ES}, i.e. $T_{ES}(0)$ may be calculated in a very simple manner.

In the steady state we have:

$$V_A \, I_A \text{ moy} = V_S \, I_S \text{ ave.} \tag{9.18}$$

Thus: $\dfrac{I_A \text{ moy}}{I_S \text{ moy}} = \dfrac{V_S}{V_A} = M$ ave. (it is assumed that the gain is sufficient /50 such that $V_S = C \frac{D}{\quad}$).

We therefore have:

$$\boxed{T_{ES}(0) = M} \tag{9.19}$$

which gives us a very simple way of verifying the curve $|T_{ES}(j\omega)|$ obtained.

9.5 Input Impedance

Given the electric nature of the system connected to the input terminals, in order to measure the input impedance it is clear that it is necessary this time to apply a voltage ΔV_A and measure the variation ΔI_A ave resulting from this.

We thus determine an input admittance.

The correctness of the above reasoning is verified by the fact that the input admittance decreases with the frequency, which must necessarily be the case in order for the method discussed in section 9.1 to be applicable (number of zeros less than the number of poles).

Depending on the moment considered within the period, the source V_A is connected to the inductor L or it is not connected (Fig. 9.7).
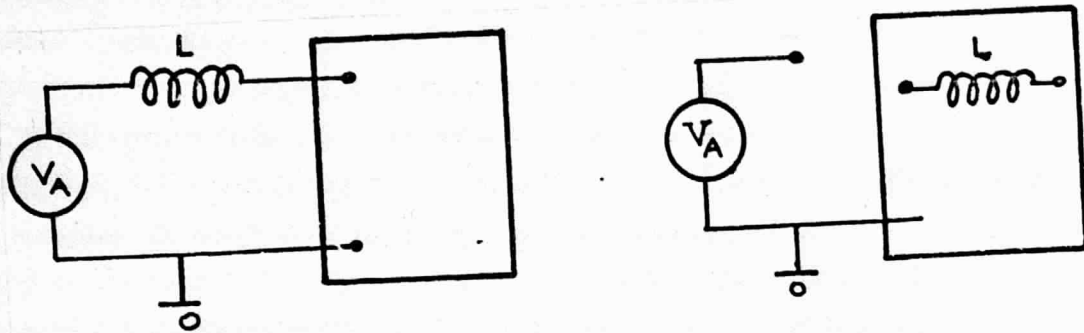
Fig. 9.7. Connections of V_A to the converter.

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We define:

$$K \triangleq \frac{\text{time in which L is connected to } V_A}{T} \qquad (9.20)$$

Suppose that we add to V_A a DIRAC impulse $V_o\delta(t)$. This procedure is purely imaginary: it does not have any meaning as such because of the discrete nature of the converter.

Nevertheless, we can deduce from it an <u>equivalent effect</u> on the system. If L is connected to B_A, applying the impulse causes a discontinuity of the first kind on I_L and there is no instantaneous effect on the other parts of the circuit. The equivalent discontinuity on I_L is equal to:

$$\boxed{I_o = \frac{V_o}{L} \, K} \qquad (9.21)$$

For all of the converters the impulse response at the input is obtained by means of the circuit shown in Fig. 9.8.

Remark 1: <u>Values of K</u>

The coefficient K defined in Eq. (9.20) equals:

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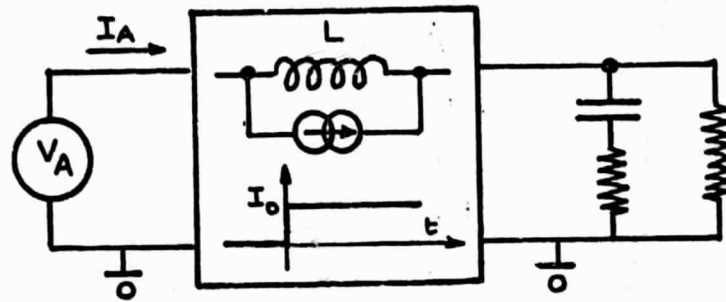
Fig. 9.8. Obtaining the input impulse response.

$K = t_{ON}/T$ for the buck and buck-boost in the two conduction modes

$K = 1$ for the boost in continuous conduction

$K = t_{ON} + t_{OFF}/T$ for the boost in discontinuous conduction.

The following values are easily calculated:

$$K = M \qquad \text{Buck in continuous conduction} \quad (9.22)$$

$$K = 1 \qquad \text{Boost in continuous conduction} \quad (9.23)$$

$$K = \frac{M}{M+1} \qquad \text{Buck-boost in continuous conduction} \quad (9.24)$$

$$K = \frac{2M}{\Delta} \qquad \text{in discontinuous conduction} \quad (9.25)$$

Remark 2: Presentation of the result

Although we determined an input admittance $Y_A(j\omega)$, we in fact plot the input impedance $z_A(j\omega) = \frac{1}{Y_A(j\omega)}$, presented in the reduced form:

$$\frac{|z_A(j\omega)|}{R_S} \qquad (9.26)$$

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Remark 3: Value of Z_A in continuous conduction.

We have: $V_A I_A$ ave. $= V_s I_s$ ave. $= c^{te}$

(We assume that the gain is sufficient so that V_s and I_s are nearly constant.)

Thus:

$$I_A \text{ moy.} dV_A + V_A \cdot d I_A \text{ moy} = 0$$

$$\frac{dV_A}{dI_A \text{ moy}} = - \frac{V_A}{I_A} = - \frac{V_s/M}{M I_s \text{ moy}} = - \frac{R_s}{M^2}$$

(9.27)

Key: moy = ave.

Thus we have:

$$\boxed{\frac{Z_A(0)}{R_s} = - \frac{1}{M^2}}$$

(9.28)

10. Study of the Overall Stability

For this section we will refer to references [1-2-3-4-7].

10.1 Definition

Let us assume a discrete non-linear system described in the space of state by the recurrence:

$$\underline{z}_{n+1} = \underline{f}(\underline{z}_n)$$

(10.1)

A cycle C of order m of the recurrence (10.1) is by definition a set of m vectors: $c = \{\underline{z}_{c1}, \underline{z}_{c2}, \ldots, \underline{z}_{cm}\}$ all consequent:

$$\left|\begin{array}{l} \underline{z}_{c\,k+1} = \underline{f}(\underline{z}_{c\,k}), \quad k \ [1, \ldots, m] \quad \text{and} \\ \underline{z}_{c1} = \underline{f}(\underline{z}_{cm}) \end{array}\right.$$

(10.2)

In other words:

$$\text{if } \underline{z}_n \in C, \ \underline{z}_{n+m} = \underline{z}_n \qquad (10.3)$$

C is an equilibrium set of the recurrence (10.1).

C is a stable cycle if the matrix S_c:

$$S_c \triangleq \frac{d\underline{z}_{n+m}}{d\underline{z}_n} \bigg|_{\underline{z}_n \in C} \qquad (10.4)$$

at all eigenvalues less than 1 in absolute terms.

There exists another type of equilibrium set called "stochastic behavior zone" which we only mention in the frame-work of this report. Let D be the closed space of the admissable initial conditions for the system described by Eq. (10.1).

By definition, the system described by Eq. (10.1) has /54 an overall stability if the only stable equilibrium set included in D is a cycle of the first order.

This cycle of the first order corresponds to the normal operation in a steady state of the system. Any other stable equilibrium set would correspond either to a subharmonic oscillation or to a pseudo-stochastic oscillation. We have already given examples of such oscillations in section 7.

Corollary: if the system (10.1) is on the whole stable within the range D, any state set to any value whatsoever of $Z_o \in D$ converges towards a stable cycle of the first order.

10.2 Method Used

Very elaborate methods exist for determining what

the equilibrium sets are for a given recurrence, stable or unstable, and for plotting the limits of the stability ranges of the stable sets (cf. references mentioned at the beginning).

These methods, which are especially designed for making a detailled analysis of the properties of a given recurrence, are not used in our case for the following reasons:
- they are essentially algorithms which can be used in numerical calculation,
- they require much calculation time,
- the precise plotting of the boundaries of the stability ranges is only of little use for the user of the system.

As a result, we used a method based on the following points:

a) In the case of the converters in question the recurrence (10.1) is at least of the second order, since the power stage itself is of the second order. Of the correction networks given in section 8.1, the most complex is network (8.4) which is of the second order. The recurrence corresponding to the system corrected in this way would thus be of the fourth order. Analysis of this system would present very great difficulties, both in terms of calculation time and result evaluation time.

As a result, in agreement with the ESTEC (cf. ESTEC-LAAS Meeting of Jan. 12, 1976), we decided to limit ourselves to systems of the third order. More specifically, we will study the systems of the fourth order in an approximate manner, as if they were third order systems. This approximation is perfectly justified. In effect, the pole $p = -\sqrt{5}/\tau_2$ present in networks (8.2) and (8.4) is situated beyond the equivalent cut-off frequency of the system.

In summary:
- the "optimized" converters provided with networks (8.1)
  and (8.2) will be analyzed as second order systems;
- those provided with networks (8.3), (8.4) and (8.5) will
  be analyzed as third order systems, the variable of state
  coming in addition to those of the power stage which is the
  output of the main integrator of the network.

b) The discrete vector of state in question is $\begin{bmatrix} V_K \\ I_L \end{bmatrix}$ at the
beginning of t_{ON} in the case of second order systems
and $\begin{bmatrix} V_K \\ I_L \\ Z \end{bmatrix}$ in the case of third order systems.

For each variable of state x we define a maximum value and
a minimum value x_{max} and x_{min}.

We consider that the range d of the admissable initial
conditions is the parallel piped limited by the different maxima
and minima.

D is discretely evaluated by giving to each variable of
state x the 4 values $(x_{min}), (x_{min} + \frac{x_{max} - x_{min}}{3}), (x_{max} - \frac{x_{max} - x_{min}}{3}), (x_{max})$.

The domain D is thus made discrete by 16 points in the case
of the second order and by 64 points in the case of the thrid
order.

c) Then we set the system to each of the values $\in$ D
and which we have just defined, and we observe if the system
converges or not towards the normal steady state (first order
cycle). To do this we use exactly the same algorithm as for the
local stability. The only slight difference is that the con-
vergence is not tested at 5% by at 10% or even more, because   /56
the quality of the analog wiring used for the overall stability

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is inferior to that of the wiring used for the local stability (in this regard, cf. the first part of volume 2).

10. 10.3 Presentation of Results

Fig. 10.1 shows an example of how the results are presented. We use <u>a perspective representation of the domain</u> <u>D which has been divided into discrete units</u>
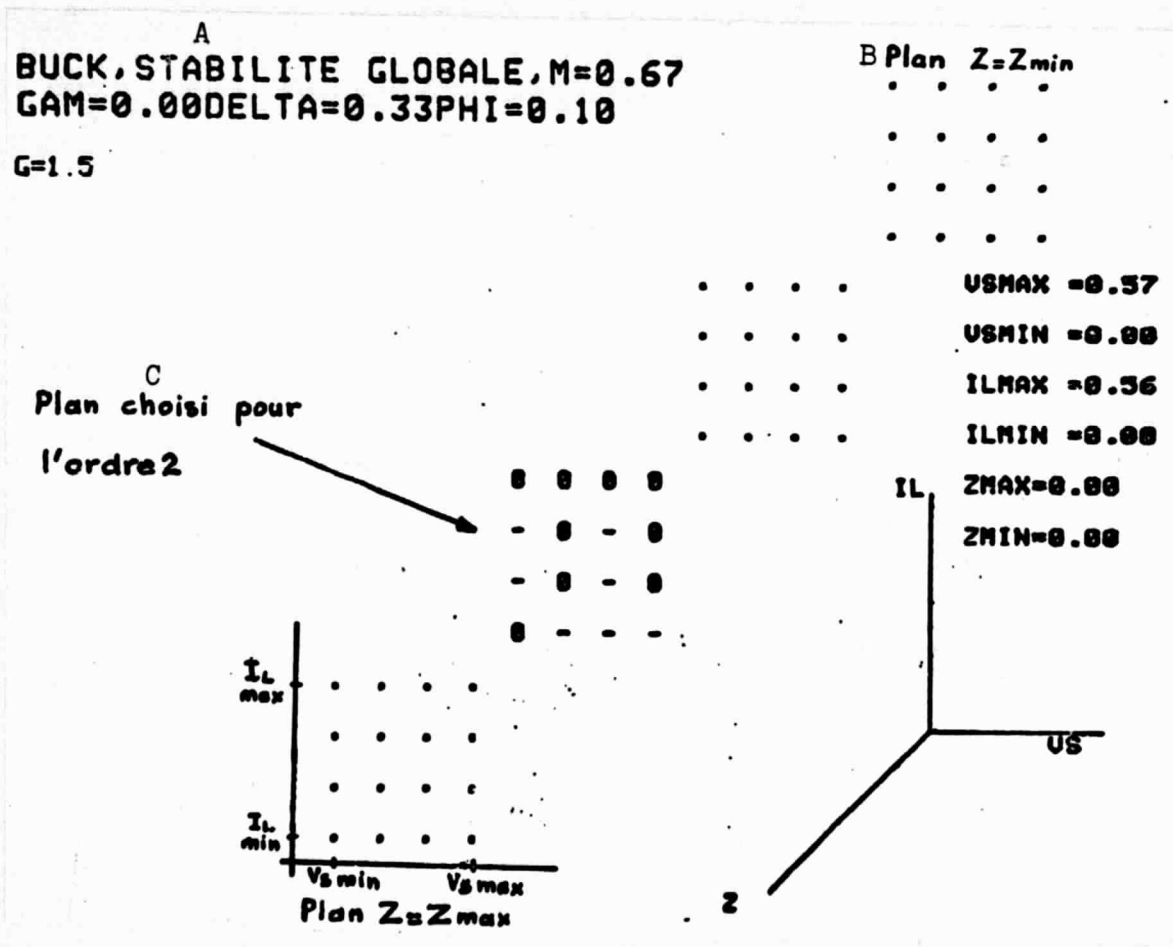

Fig. 10.1 Example of how the overall stability is represented.

Key: A) overall stability   c) plane chosen for the
       B) plane   second order

Each point of the domain D is represented by an alphanumeric character with a precise meaning:

- the dot "." means that this setting has not been analyzed;
- the minus sign "-" means that the corresponding setting converged towards the stable first order cycle;
- the zero "∅" means that the corresponding setting did not converge towards the first order cycle (instability).

The symbols "-" and "∅" were choses so as to be very easily distinguishable by sight.

Thus we have overall stability if and only if all of domain D contains only minus signs "-".

To be sure, strictly speaking we are not absolutely certain because the discrete evaluation of the domain D is not very detailed, but nevertheless there is very little chance that any instability present will not be detected.

For the second order systems the results are presented in the same way as for the third order systems, but only one of the four planes corresponding to the discrete evaluation of Z is used, the other three then being filled with points.

The coordinates of the parallel piped D are clearly indicated and for more clarity we have discussed V_s instead of V_K (since $V_s \simeq V_K$).

Remark: Systems close to the local instability.

- Some of these systems are locally stable and globally unstable. In this case, it is very common that the stability range of the first cycle has a very restricted surface area (or volume), but nevertheless its form is very complicated allowing it to attain very elongated zones of D of the stable first order cycle.

- In addition, such systems generally meet with a high degree of sensitivity $\frac{d\underline{z}_{n+1}}{d\underline{z}_n}$ and, <u>a fortiori</u>, a sensitivity $\frac{d\underline{z}_{n+k}}{d\underline{z}_n}$ which incerases as k increases.

In physical terms, this means that a small variation $\Delta\underline{Z}_n$ is going to result, for example, in a substantial variation in $\Delta\underline{Z}_{n+50}$.

- In the case of the analog simulation, these two facts combine along with the imperfections and noises to lead to the following phenomenon: <u>the same system, set several times in succession to the same value, each time follows a different development</u>.

It may happen, for example, that an initial condition taken out of the domain of stability of the first cycle, at the end of a certain number of periods may lead to a discrete vector of state penetrating into the domain of stability and remaining there.

As a result, <u>when we are faced with a domain D simultaneously containing "-'s" and "∅'s", the only valid information is that the system is locally stable and globally unstable. It is impossible to get any indication of the topography of the "-'s" and "∅'s" in D</u>, except for certain cases which will be pointed out when we discuss the actual results.

The example shown in Fig. 10.1 is precisely in this case. The example shown in Figs. 10.2 and 10.3 show the same system with a larger and smaller gain respectively of the error amplifier. Fig. 10.2 clearly shows that there are no longer initial conditions leading to stability and Fig. 10.3 shows that the system is globally stable.

A
BUCK,STABILITE GLOBALE,M=0.67
GAM=0.00DELTA=0.33PHI=0.10

G=5.

USMAX =0.57
USMIN =0.00
ILMAX =0.56
ILMIN =0.00
IL ZMAX=0.00
ZMIN=0.00

US

Z
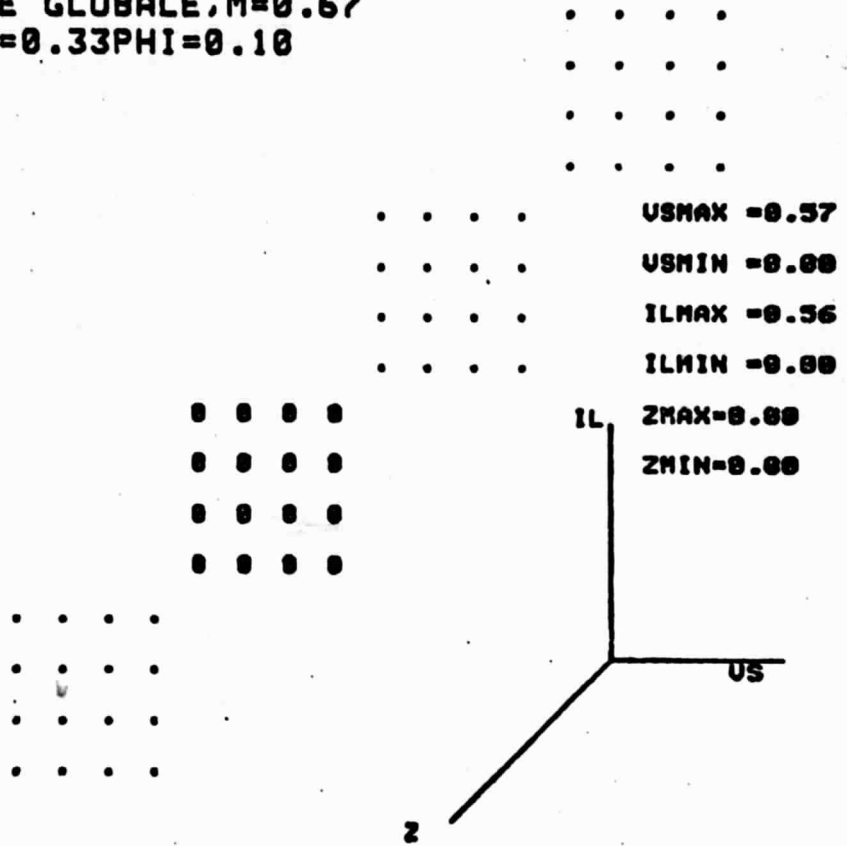
Figs. 10.2 and 10.3 (next page). The system shown in
Fig. 10.1 with different gains.

Key: A) overall stability

BUCK; STABILITE GLOBALE, M=0.67
GAM=0.00DELTA=0.33PHI=0.10

G=0.5

VSMAX =0.57
VSMIN =0.00
ILMAX =0.56
ILMIN =0.00
IL ZMAX=0.00
ZMIN=0.00

US

Z
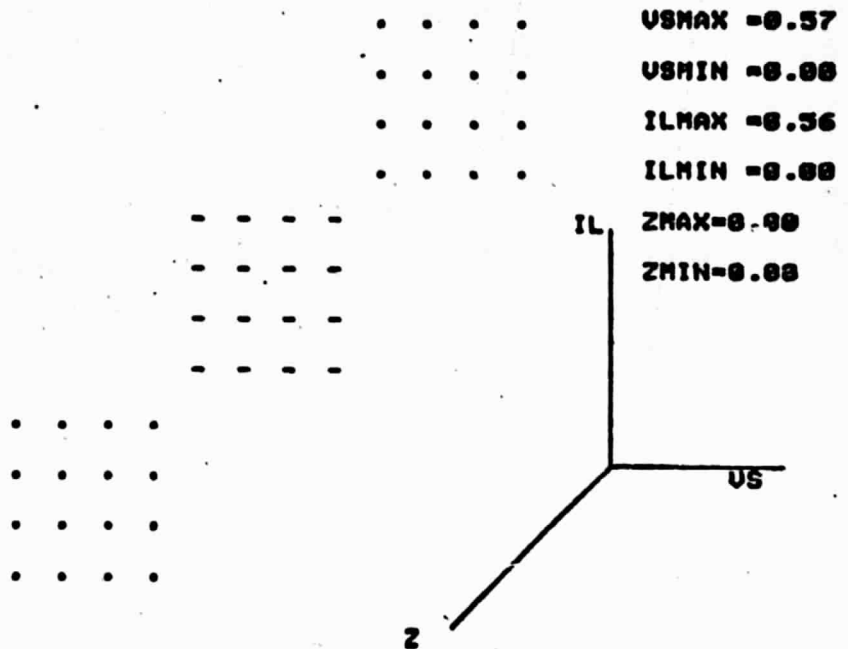
Fig. 10.3

11. Utilization of the Results

In this section we will use the following notation:
- subscript S for the variables relating to the simulation;
- subscript V (true) for the variables of the real
  converter in question.

11.1 Transition from the Numeric Values of the Curves and Hard-Copies to True Values

It is now assumed that we have curves or hard-copies
corresponding to the values of Δ, Φ, Γ and M of the real
converter which we intend to "optimize."

The problem here is to go from the numeric values of the
simulation to true numeric values.

11.1.1 Gain

The gain G of the error amplifier is involved in all of the results. When this amplifier does not have any correction function, i.e. when $G_1(p) = G$, there is no ambiguity. In the opposite case, the constant G has the meaning defined by Eqs. (8.1) - (8.5).

When the simulation and the real converter belong to the same class their loop gains are identical. This:

$$G_S \cdot G_{MS} \cdot G_{PS} = G_V \cdot G_{MV} \cdot G_{PV} \tag{11.1}$$

with G_M = modulator gain $\triangleq \dfrac{dt_{ON}}{dv_c}$

G_P = power stage gain $\triangleq \dfrac{dv_S}{dt_{ON}}$

For the coincidence modulators and sampling-delay modulators, G_M is of the form T/A. We therefore obtain:

$$G_S \frac{T_S}{A_S} \left(\frac{dv_S}{dt_{ON}} \right)_S = G_V \frac{T_V}{A_V} \left(\frac{dv_S}{dt_{ON}} \right)_V \tag{11.2}$$

The quantity $T\, dV_S/dt_{ON}$ is proportional to V_S. Thus:

$$G_S \frac{V_{SS}}{A_S} = G_V \frac{V_{SV}}{A_V} \tag{11.3}$$

Hence:

$$G_V = G_S \frac{A_V}{A_S} \frac{V_{SS}}{V_{SV}} = G_S \frac{A_V}{0,1} \frac{0,4}{V_{SV}} \tag{11.4}$$

Thus, in order to go from the gain read on the G_S curves to the ture gain, we have:

$$\boxed{G_V = 4 \cdot G_S \cdot \frac{A_V}{V_{SV}}}$$

for the coincidence and
sampling-delay   (11.5)
modulators

The operation of the magnetic modulator, such as it has been defined, leads to a constant quantity $\frac{dt_{on}}{dV_c} \frac{V_1}{T}$ which is equal to 2/3. G_M in the case of the magnetic moculator is thus proportional to $T/V_1 = T/V_S$ since it is assumed that $V_1 = V_S$.

Eq. (11.1) then becomes:

$$G_S \cdot \frac{T_S}{V_{SS}} \left(\frac{dV_S}{dt_{ON}} \right)_S = G_V \ \frac{T_V}{V_{SV}} \left(\frac{dV_S}{dt_{ON}} \right)_V \qquad (11.6)$$

As above, the quantity $T \ dV_S/dT_{ON}$ is proportional to V_S and we obtain:

$$G_S \cdot \frac{V_{SS}}{V_{SS}} = G_V \ \frac{V_{SV}}{V_{SV}}$$

Thus:

$$\boxed{G_V = G_S}$$

for the magnetic   (11.7)
modulator

The same line of reasoning is carried out for the gain of the current loop, but Eq. (11.4) becomes:

$$G_{ACV} = G_{ACS} \ \frac{A_V}{A_S} \ \frac{I_{CS}}{I_{CV}} = G_{ACS} \ \frac{A_V}{A_S} \cdot \frac{I_{SS}}{I_{SV}} = G_{ACS} \ \frac{A_V}{A_S} \ \frac{V_{SS}}{V_{SV}} \ \frac{R_{SV}}{R_{SS}} \qquad (11.8)$$

Since we have assumed $R_{SS} = 1$, we obtain:

$$\boxed{G_{ACV} = G_{ACS} \; \frac{G_V}{G_S} \; R_{SV}}$$

for the coincidence and
sampling-delay   (11.9)
modulators

For the magnetic modulator, Eq. (11.6) becomes:

$$G_{ACS} \cdot \frac{T_S}{V_{SS}} \left(\frac{dI_C}{dt_{ON}} \right)_S = G_{ACV} \; \frac{T_V}{V_{SV}} \left(\frac{dI_C}{dt_{ON}} \right)_V \qquad (11.10)$$

The quantity $T \; dI_C/dt_{ON}$ is proportional to V_S/R_S, hence:

$$G_{ACS} \; \frac{V_{SS}}{V_{SS}} \; \frac{1}{R_{SS}} = G_{ACV} \; \frac{V_{SV}}{V_{SV}} \; \frac{1}{R_{SV}} \qquad (11.11)$$

Thus:

$$\boxed{G_{ACV} = G_{ACS} \cdot R_{SV}}$$

for the magnetic   (11.12)
modulator

11.1.2 Time Constants, Poles, Zeros

This time we will consider in a correction
network a term of the form $(1 + \tau p)$ located either in the
numerator (the zero case) or in the denominator (the pole case).

For all of the elements it is clearly indicated "pole
(or zero) at F/x." Everything is produced to the operating
frequency F and, in accordance with the definition given in
section (3.6.4), the corresponding reduced time constant is:

$$\boxed{\mathscr{C} = x} \qquad (11.13)$$

As a result:

$$\tau_v = \frac{\mathscr{C} T_v}{2\pi} = \frac{x\, T_v}{2\pi}$$

(11.14)

11.1.3 Curves as a Function of Time

These are curves of response to a jump in output current.

They may be represented in two forms:

a) $\Delta V_S(t)$. We use the variable expressed in ordinate values (ΔV_S) by means of the following equation:

$$\Delta V_{SV} = \Delta V_{SS}\, \frac{V_{SV}}{0.4}$$

(11.15)

b) $\Delta I_L(t)$. We have

$$\Delta I_{LV} = \Delta I_{LS}\, \frac{I_{LV\,max}}{I_{LS\,max}}$$

(11.16)

(By I_{Lmax} is meant the value of I_L at the end of t_{ON}.)

As for the scale along the abscissa, it is directly divided into the number of samples, i.e. the number of periods.

11.1.4 Curves as a Function of Frequency

a) Output and Input Impedances

Since the ordinate of these curves is divided into values of $|Z/R_S|$, it is clear that

$$|Z_V| = \left|\frac{Z}{R_S}\right| R_{SV}$$

(11.17)

Direct utilization, the input-output trans-mittance being dimensionless.

As for the scale along the abscissa, we have

$$\boxed{f_V = f_S \; \bar{F}_V}$$

(11.18)

11.1.5 Overall Stability Diagrams

As above, we go from a quantity x_S to a quantity x_V by means of a proportionality relationship:

$$\text{Scale } v_S \; : \; v_{SV} = v_{SS} \; \frac{v_{SV}}{0,4}$$

(11.19)

$$\text{Scale } I_L \; : \; i_{LV} = i_{LS} \; \frac{1,33 I_{LV\,max}}{I_{LS\,max\,lu}}$$

(11.20)

$$\text{Scale } z \; : \; z_V = z_S \; \frac{z_{V\,max}}{z_{S\,max\,lu}}$$

(11.21)

[note: lu = read]

11.2 Change in Operating Point of a Given Converter

Let us assume a given converter, i.e. operating at a nearly constant voltage V_S as a result of closed loop operation and with the elements L, C and r with given values.

The operating point of this converter changes when V_A varies and/or when R_S (or I_S) varies. During such an operating point change, the converter changes class. Its behavior then

corresponds to curves other than those used previously.

During a change in operating point, ϕ and Γ are constant, but M and/or Δ vary.

Below we are going to give the equations allowing us to calculate these variations.

Consider a converter for which all of the values of the elements have been calculated for nominal values $\Delta_{nominal}$, $M_{nominal}$, $J_{nominal}$. If we use this converter with a different M (different V_A) and a different J (different I_S), the equations which follow give the value of Δ for the new operating point.

11.2.1 Buck (0 < M < 1)

1. Nominal values in continuous conduction

This is the case if $\boxed{\Delta_{nominal} < 2}$

$$\Delta = \Delta_{nominal} \cdot \frac{J_{nom}}{J} \cdot \frac{1-M}{1-M_{nominal}} \quad \text{if } \Delta < 2 \qquad (11.22)$$

The operating point in question is in continuous conduction.

$$\Delta = \sqrt{2\Delta_{nominal} \cdot \frac{J_{nom}}{J} \cdot \frac{1-M}{1-M_{nominal}}} \quad \text{if } \Delta > 2 \qquad (11.23)$$

The operating point in question is in discontinuous conduction.

2. Nominal values in discontinuous conduction.

This is the case if $\boxed{\Delta_{nominal} > 2}$

$$\Delta = \Delta_{nominal} \sqrt{\frac{J_{nom}}{J} \cdot \frac{1-M}{1-M_{nominal}}} \qquad \text{if } \Delta > 2 \qquad (11.24)$$

The operating point in question is in discontinuous conduction.

$$\Delta = \frac{\Delta^2_{nominal}}{2} \cdot \frac{J_{nom}}{J} \cdot \frac{1-M}{1-M_{nominal}} \qquad \text{if } \Delta < 2 \qquad (11.25)$$

The operating point in question is in continuous conduction.

11.2.2 Boost (M>1)

1. Nominal values in continuous conduction.

This is the case if: $\boxed{\Delta_{nominal} < 2\,M_{nominal}}$

$$\Delta = \Delta_{nominal} \cdot \frac{J_{nom}}{J} \cdot \left(\frac{M_{nominal}}{M}\right)^2 \cdot \frac{M-1}{M_{nominal}-1} \qquad \text{if } \Delta < 2M \qquad (11.26)$$

The operating point in question is in continuous conduction.

$$\Delta = \sqrt{2\Delta_{nominal} \cdot \frac{J_{nom}}{J} \cdot \frac{M^2_{nominal}}{M} \cdot \frac{M-1}{M_{nominal}-1}} \qquad \text{if } \Delta > 2M \qquad (11.27)$$

The corresponding operating point is in discontinuous conduction.

2. Nominal values in discontinuous conduction.

This is the case if $\boxed{\Delta_{nominal} > 2\,M_{nominal}}$

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$$\Delta = \Delta_{nominal} \sqrt{\frac{J_{nom}}{J} \frac{M_{nominal}}{M} \frac{M-1}{M_{nominal}-1}} \qquad \text{if } \Delta > 2M \qquad (11.28)$$

The operating point in question is in discontinuous conduction.

$$\Delta = \frac{\Delta_{nominal}^2}{2} \frac{J_{nom}}{J} \frac{M_{nominal}}{M^2} \frac{M-1}{M_{nominal}-1} \qquad \text{if } \Delta < 2M \qquad (11.29)$$

The operating point in question is in continuous conduction.

11.2.3 Buck-Boost (M>0)

1. Nominal values in continuous conduction.

This is the case if: $\boxed{\Delta_{nominal} < 2 (M_{nominal}+1)}$

$$\Delta = \Delta_{nominal} \frac{J_{nom}}{J} \frac{1+M_{nominal}}{1+M} \qquad \text{if } \Delta < 2(M+1) \qquad (11.30)$$

The operating point in question is in continuous conduction.

$$\Delta = \sqrt{\frac{\Delta_{nominal}^2}{2} \frac{J_{nom}}{J} (1 + M_{nominal})} \qquad \text{if } \Delta > 2(M+1) \qquad (11.31)$$

The operating point in question is in discontinuous conduction.

2. Nominal values in discontinuous conduction

This is the case if $\boxed{\Delta_{nominal} > 2 (M_{nominal}+1)}$

$$\Delta = \Delta_{nominal} \sqrt{\frac{J_{nom}}{J}} \qquad \text{if } \Delta > 2(M+1) \qquad (11.32)$$

The operating point in question is in discontinuous conduction.

$$\Delta = \frac{\Delta_{nominal}^2}{2} \; \frac{\mathcal{J}_{nom}}{\mathcal{J}} \; \frac{1}{1+M} \qquad \text{if} \quad \Delta < 2 \, (M+1) \qquad (11.33)$$

The operating point in question is in continuous conduction.

11.3 Change of Frequency for a Given Converter

The given converter is defined as in section 11.2. If this converter has been calculated for the nominal values Δ_{nom}, ϕ_{nom}, Γ_{nom} and F_{nom} it changes class if we change its operating frequency expressed as F.

We obtain:

$$\phi = \phi_{nom} \; \frac{F_{nom}}{F} \qquad (11.34)$$

$$\Gamma = \Gamma_{nom} \; \frac{F}{F_{nom}} \qquad (11.35)$$

$$\Delta = \Delta_{nom} \; \frac{F_{nom}}{F} \qquad \begin{array}{l} \text{if we go from continuous con-} \\ \text{duction to continuous conduction} \end{array} \qquad (11.36)$$

$$\Delta = \Delta_{nom} \; \sqrt{\frac{F_{nom}}{F}} \qquad \begin{array}{l} \text{if we go from discontinuous con-} \\ \text{duction to discontinuous conduction.} \end{array} \qquad (11.37)$$

If we change the mode of conduction as a result of the change in F, we use the following procedure:

$$\left.\begin{array}{l} F_{nom} \\ \\ \Delta_{nom} \end{array}\right\} \begin{array}{c} \text{Equation of} \\ \text{mode 1} \end{array} \longrightarrow \left.\begin{array}{l} F_{lim} \\ \\ \Delta_{lim} \end{array}\right\} \longrightarrow \left.\begin{array}{l} F \\ \\ \Delta \end{array}\right\} \text{mode 2}$$

<center>Transition between   Equation of mode 2 using
mode 1 and mode 2   the limit values instead
of the nomimal values</center>
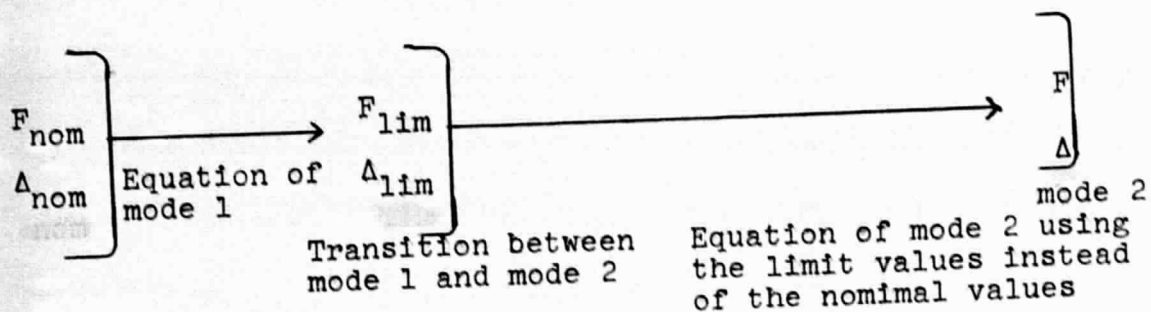
11.4 Method of Utilization

In this section we do not claim to give the only method for using the simulation results.

In effect, any designer familar with the results will be liable to use them in his own individual way.

Our intention here is only to provide the following: /69
- a certain number of remarks of a general character which may guide the user in his interpolations, increasing safety margins, etc;
- a utilization method which should be capable of being used in the greatest number of cases.

11.4.1 General Remarks on the Effects of Variations in Parameters

The data which follow are approximate and may turn out to be in part inexact for some systems. Nevertheless, in the majority of cases they are satisfacotry.

a) The converters in the discontinuous mode have their gain in high frequency which decreases 20 dB/decade.

The boosts in continuous conduction have their gain in high frequency which decreases 20 dB/decade.

The bucks and buck-boosts in continuous production have their gain in high frequency which decreases 40 dB/decade.

b) The high-frequency gain:
- varies as $1/M$
- varies as ϕ^2
- practically does not vary with Δ for the continuous mode
- varies as $1/\Delta$ for the discontinuous mode.

As a result, <u>if we want to keep the same pass band equivalent for a given system</u>, we must:

- use the gain $\quad G_{optim} \dfrac{M}{M_{optim}}$

- use the gain $\quad G_{optim} \left(\dfrac{\phi_{optim}}{\phi} \right)^2$

- use G_{optim} when Δ varies, in the continuous mode

- use $G_{optim} \Delta/\Delta_{opt}$ in the discontinuous mode

- in the case of the current loop, vary G_{AC} <u>and</u> G as indicated above.

c) If we want to <u>increase the safety margin</u> of a system by making its equivalent pass band go from f_{optim} to f_{optim}/x, we must:
- use the G_{optim}/x in the discontinuous mode and for the boost in the continuous mode,
- use the gain G_{optim}/x^2 for the bucks and buck-boosts in the continuous mode,
- use G_{optim}/x^2 <u>and</u> $G_{AC\ optim} / x$

d) The presence of $\Gamma \neq 0$:
- is unfavorable for the discontinuous mode and for the boost in continuous conduction,
- is favorable for the buck and buck-boosts in continuous conduction,

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– is negligible if $\Gamma < \dfrac{F}{\text{equivalent pass band}}$

11.4.2 Proposal for a Utilization Method

The <u>main point</u> which happens to complicate the optimization of the systems in question is the fact, described above, that <u>the properties of a given converter vary with the operating point</u>.

Therefore we should investigate what the worst operating conditions are with respect to the stability and correct the system under these conditions. Then, conversely, we determine what are the operating conditions leading to the lowest possible high-frequency gain of the output stage and we decide whether or not the resulting reduction in performance is acceptable.

Taking into account what we have said in section 11.4.1, the high-frequency gain of the power stage is maximum if:
- M is minimum <u>and</u>
- Δ is minimum.

The various equations relating the variations of Δ to those of J show that Δ decreases when J increases. <u>The worst case with respect to the stability is thus the one in which</u>:
- M is minimum <u>and</u>
- J = 1, i.e. the converter delivers its maximum current.

The opposite case thus corresponds to the following:
- M is maximum and
- J is minimum.

12. Utilization Examples

The following examples are examples relating to boost converters and the reader is thus requested to refer to part 1 of volume 3.

12.1 Example 1: Boost Operating Always in Discontinuous Conduction

Let us suppose a boost converter possessing the following characteristics:

$$V_S = 40 \text{ V} \quad I_S = 1 \text{ A}_{max} \quad \text{thus} \quad R_S \geq 40 \, \Omega.$$

V_A may vary from 30 V to 20 V

$$F = 20 \text{ kHz} \quad \text{thus} \quad T = 50.10^{-6} \qquad (12.1)$$

We want the converter always to operate in discontinuous conduction and we expect to use it from $I_S = 1$ A to $I_S = 0.05$ A. We can be content with a theoretical residual ripple on the order of 1% or even greater, such that C is small. We use a coincidence modulator with A = 5 V.

The parameter M may vary from 40/30 to 40/20, thus

$$\boxed{1,33 \leq M \leq 2}$$

$$(12.2)$$

Eq. (11.28) easily shows that, for a boost in discontinuous conduction, the quantity Δ/M decreases when M increases. Now we are in discontinuous conduction if $\Delta/M > 2$. Thus as M increases we come closer and closer to continuous conduction. As a result, we begin the proportioning at M maximum and at I_S maximum.

Let $M = 2$.

We should have $\Delta > 2M$, i.e. $\Delta > 4$. We take a margin of $\Delta = 5$. Thus we elect to operate with $\Delta = 5$ when $M = 2$.

If we use Eq. (11.28) to calculate the value of Δ for /73 $M = 1.33$ always at I_S max:

$$\Delta = 5 \sqrt{\frac{2}{1,33} \cdot \frac{0,33}{1}} = 3,5$$

we are going to proportion the converter for the nomimal values:

$$
\left|
\begin{array}{l}
M_{nom} = 1,33 \\[2mm]
\Delta_{nom} = 3,5 \\[2mm]
\emptyset = 0,1 \text{ (the relative residual ripple is} \\
\qquad \text{very roughly equal to } \phi^2) \\[2mm]
R_S \text{ nom} = 40\,\Omega
\end{array}
\right. \qquad (12.3)
$$

Eq. (4.4) gives us:

$$L = \frac{0,33}{1,33} \quad \frac{2}{3,52} \cdot 40.50.10^{-6}$$

$$L = 80 \ \mu H \qquad\qquad (12.4)$$

Eq. (4.8) gives us:

$$C = \frac{1}{4\pi^2.0,1^2.(2.10^4)^2.80.10^{-6}}$$

$$C = 80 \ \mu F \qquad\qquad (12.5)$$

The capacitor we have becomes purely resistive at 02 kHz.

Thus we have:

$$\Gamma = 1 \qquad\qquad (12.6)$$

The power stage is at its gain maximum for M = 1.33 and
Δ = 3.5.

It is for these values that we are now going to determine
the correction networks.

We begin by analyzing the local stability curves for which
we have the value Δ = 3.8, sufficiently close to that which we
are interested in. We find that for the values $\phi = 0.1$,
$\Gamma = 1$ and Δ = 3.8 it is impossible to obtain the local stability
for the gain G_0 leading to a regulation rate of ±2%. Thus a
correction network is necessary.

We now refer to the indicative response curves for the
values $\phi = 0.1$, Δ = 3.8, $\Gamma = 0$ and M = 1.6. On the curve in
question we read that the optimized correction network is a
network of the type $G \frac{1+\tau p}{\tau p}$ for which we have a simulation
value of G = 1.7 and a reduced value for τ of 25 (Eq. 3.13).
If we substitute the true value of G using Eq. (11.5):

$$G = 4.1,7 \; \frac{5}{40} = 0,85$$

Since we are using values of M and Δ which are slightly
different from those of the simulation, we correct this
value by using the suggestions given in section 11.4.1:

$$\boxed{G = 0,85 \; \frac{1,33}{1,6} \; \frac{3,5}{3,8} = 0,65}$$

We now calculate the value of the true time constant using
Eq. (11.14):

$$\tau = \frac{25.50.10^{-6}}{2} = 2.10^{-4}$$

We should therefore insert the following network:

$$G_1(p) = 0,65 \; \frac{1+2.10^{-4}p}{2.10^{-4}p}$$

(12.7)

We can now refer to the other diagrams concerning this
case and we not that:
- the overall stability is satisfactory, the reservations
  of a general character expressed on this subject in the
  report on the boost of course remain valid.

We now see what the minimum power stage gain conditions
are.

Since I_S has to vary from 1 A to 0.05 A, we have:

$$0,05 \leqslant J \leqslant 1$$

(12.8)

Eq. (11.28) gives us, for M = 2 and J = 0.05:

$$\Delta = 3,5 \sqrt{\frac{1}{0,05} \; \frac{1,33}{2} \; \frac{1}{0,33}} = 22$$

(12.9)

The power stage is at its gain minimum for M = 2 and
Δ = 22.

If the correction network was a simple flat gain G = 1.7
expressed as a simulation value, these new operating conditions
would not pose any stability problems, proof of which can be
obtained by looking at the local stability curve. The
equivalent path band would simply have moved back from 0.2 F
to:

$$0,2 \; F \; \frac{1,33}{2} \; \frac{3,5}{22} = 0,02 \; F = 400 \; Hz$$

But in the inserted network the zero which is involved there is at $F/25 = 0.04$ F. There is therefore a risk of instability since the network causes more phase shift at 0.02 F than at 0.04 F.

However, given the margin which exists for the indicative response diagram of the optimized system, we can move back the zero by a factor of three and place it at $F/75 = 0.0133$ F and thus use the network with greater certainty in a vacuum:

$$G_1(p) = 0,65 \frac{1+6.10^{-4}\rho}{6.10^{-4}p}$$
(12.10)

Remark: If, for example we were working with an instruction
$V_R = 5$ V and thus a dividing ridge of ratio $40/5 = 8$ on the output voltage, it is clear that it would be necessary to multiply the gain of the network by 8 in making it change to 0.65×8 5.

12.2 Example 2: Boost Functioning in Continuous Conduction from J = 1 to J = 0.2

Let us take a boost converter having the following characteristics:

$V_S = 40$ V $I_S = 1$ A max   thus $R_S \geqslant 40\,\Omega$

V_A may vary from 30 V to 20 V

$F = 20$ kHz   thus $T = 50.10^{-6}$

We want the converter to operate in continuous conduction from $I_S = 1$ A up to $I_S \leq 0.2$ A, but the converter will nevertheless be used from $I_S = 1$ A up to $I_S = 0.05$ A. There is no particular requirement with respect to the residual ripple.

We use a coincidence modulator with A = 5 V.

(12.11)

This time we will consider Eq. (11.26). The most unfavorable case for the transition into discontinuous conduction is when M = 1.33. Thus this is in the case when we should be at the boundary between the modes. Therefore we have:

$$M = 1,33 \longrightarrow \Delta_{lim} = 2 M = 2,66 \qquad \text{for} \qquad J = 0,2$$

Hence, for M = 1.33, we have $\Delta = 2.66 \; 0.2/1 = 0.53$ when $J = 1$.

We are going to use the following nominal values:

$$\left| \begin{array}{l} M_{nom} = 1,33 \\ \Delta_{nom} = 0,53 \\ R_{Snom} = 40 \end{array} \right. \qquad (12.12)$$

For the results relating to the boost we find that it is impossible to arrive at a satisfactory result with $\phi = 0.1$.
Therefore we assume:

$$\boxed{\phi = 0,03}$$

$$(12.13)$$

Eq. (4.3) gives us:

$$L = \frac{0,33}{1,33^2} \; \frac{1}{0,53} \cdot 40.50.10^{-6}$$

$$L = 0,7 \text{ mH} \qquad (12.14)$$

Eq. (4.8) gives us:

$$C = \frac{1}{4\pi^2 \cdot (0,03)^2 (2.10^4)^2 \; 0,7.10^{-3}}$$

$$C = 100 \; \mu F \qquad (12.15)$$

The condensor we are using becomes resistive at 10 kHz. Thus we have:

$$\Gamma = 2 \qquad (12.16)$$

The power stage is at its gain maximum for M = 1.33 and Δ = 0.53.

It is for these values that we are going to calculate the correction network.

Consulting all of the results relative to the boost, we then find that for φ = 0.03 no satisfactory solution exists for a Δ as low as 0.53.

Two approaches may then be adopted:
a) We can try to work with a larger Δ, in other words we allow the transition into discontinuous conduction to take place for a current clearly greater than 0.2 A. This does not seem to us to be a logical solution because it amounts to optimizing the converter in the continuous mode (which is the most delicate) while it will operate most often in discontinuous conduction.
b) We decrease φ. For example, we take: /78

$$\phi = 0.01 \qquad (12.17)$$

which amounts to using a capacitor:

$$C \simeq 1000 \ \mu F \qquad (12.18)$$

In effect, for the results of the boost we then find that the problem has a solution by using a current loop.

While observing the development of numeric values when
Δ varies from 0.2 to 0.8 then to 2.6, we see that this
development is roughly of the type:

$$\frac{\Delta_1}{\Delta_2} = C^{\underline{a}} \longrightarrow \frac{G_1}{G_2} = C^{\underline{a}}.$$

Therefore, as a result, we are going to interpolate from
the values $\Delta = 0.2$ and $\Delta = 0.8$.

For $\Delta = 0.2$, we read $G = 1.4$ and $G_{AC} = 1/17$.

For $\Delta = 0.8$, we read $G = 4.6$ and $G_{AC} = 1/30$.

Interpolation:
We have $G \simeq B\Delta^x$

thus $\quad \frac{4,6}{1,4} = \left(\frac{0,8}{0,2}\right)^x \longrightarrow x = 0,858$

$\quad \frac{G}{4,6} = \left(\frac{0,53}{0,8}\right)^{0,858} \longrightarrow G = 3,2$

likewise $\quad \frac{17}{30} = \left(\frac{0,8}{0,2}\right)^x \longrightarrow x = -0,41$

$\quad \frac{G_{AC}}{1/30} = \left(\frac{0,53}{0,8}\right)^{-0,41} \longrightarrow G_{AC} = 0,04$

Note: we may content ourselves with a simplified interpolation
in this case.

Now let us correct these values to bring them to the
good value of M:

$$\left.\begin{array}{l} G = 3,2 \; \frac{1,33}{1,6} = 2,66 \\[3mm] G_{AC} = 0,04 \; \frac{1,33}{1,6} = 0,033 \end{array}\right\} \quad \text{in simulation values}$$

We calculate the true values:

$$G = 4.2,66 \frac{5}{40} = 1,33$$

$$G_{AC} = 0,033 \frac{1,33}{2,66} \ 40 = 0,66 \quad \text{(Eq. 11.9)}$$

When at the pole of the current loop, $\tau = 5$ in reduced
value, thus:

$$\tau = \frac{5.50.10^{-6}}{2\pi} = 4.10^{-5} \quad \text{(Eq. 11.14)}$$

We thus insert the networks:

$$\boxed{\begin{array}{l} G_1(p) = 1,33 \\[2mm] G_2(p) = \dfrac{0,66}{1+4.10^{-5}p} \end{array}} \quad \text{(12.19)}$$

Finally, we go to the minimum gain conditions of the
power stage, i.e. M = 2 and J = 0.5.

Using Eq. (11.27) we obtain:

$$\Delta = \sqrt{2.0,53 \ \frac{1}{0,05} \ \frac{1,33^2}{2} \ \frac{1}{0,33}} = 7,5$$

The power stage is at its gain minimum when M = 2 and
when Δ = 7.5 (for J = 0.05). Therefore, as predicted, it is
in discontinuous conduction when it will operate correctly for
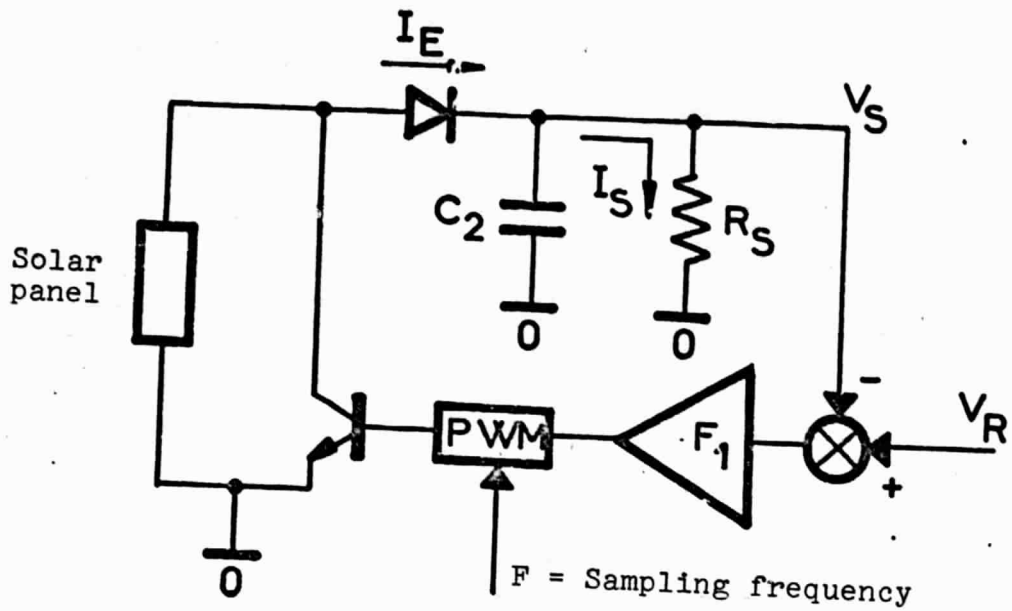the gains in question.

Part II.   PWM Shunt

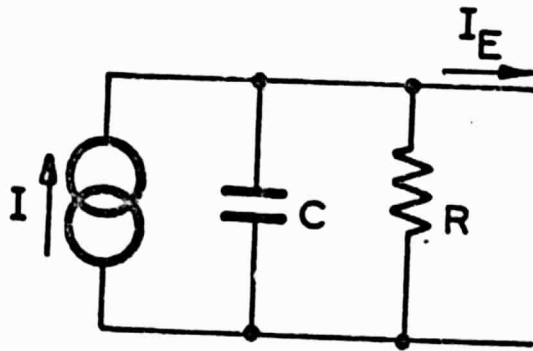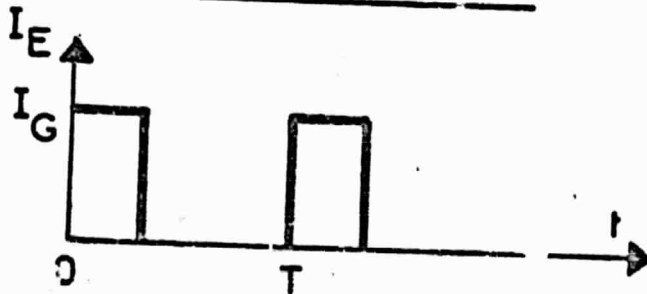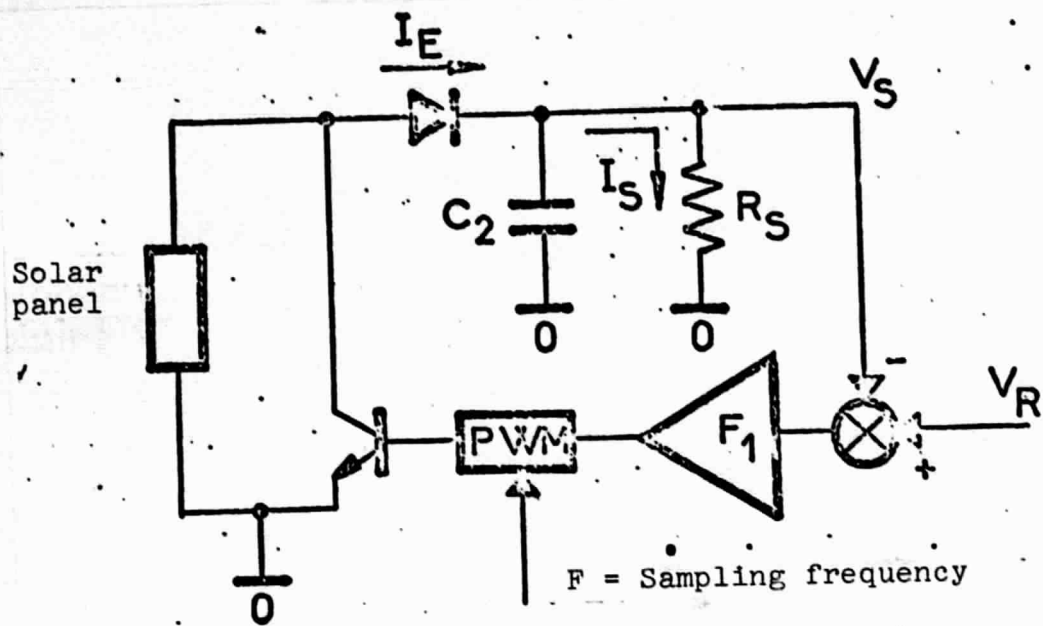___ Figure 1 ___

___ Figure 2 ___

Fig. 1.1.   Basic diagram
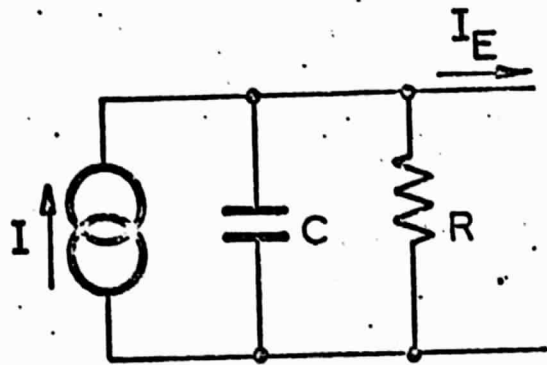
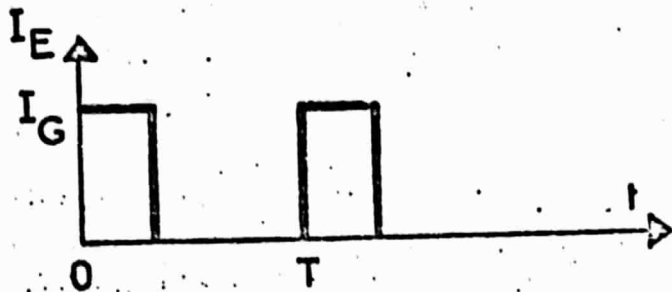Fig. 1.2.   Diagram of the solar panel.

Fig. 1.3.   Shape of the current panel.

As for the elementary units, these reduced parameters will characterize each class of PWM shunt converter.

These parameters are dimensionless and they have the greatest possible practical significance.

For defining these parameters we will make the following assumptions:
- the values of the reduced parameters are defined on the basis of the steady state of the system,
- this steady state is idealized at the level of the output = V_S = cte; I_S = cte.

These parameters were defined in accordance with the ESTEC during the meeting of Jan. 12, 1976. We are going to discuss their definition in detail.

I.1 Reduced Parameter Γ (GAM)

It is defined as: $\Gamma \triangleq 2\pi . R_c.C_2.F,$

F being the operating frequency of the PWM shunt.

As in the other types of elementary units, this parameter characterizes the quality of the output capacitor. If Γ is small, the capacitor is of good quality. If Γ is large, the capcitor is of poor quality.

I.2 Reduced Parameter ΓE (GAE)

By definition: $\Gamma E \triangleq 2\pi.R.C.F.$

This parameter defines the delay when the diode D of the shunt is put into conduction. The greater ΓE is, the longer the delay.

I.3 Reduced Parameter ΓS (GAS)

We define:

$$\Gamma_S \triangleq 2\pi.R_S.C_2.F.$$

This parameter determines the size of the output capacitor C2 of the PWM shunt by comparing the cut-off frequency of the output network with the operating frequency of the converter. At the same time, this parameter determines the size of the residual ripple at the output.

I.4 Parameter MJ, Ratio of Solar Panel Current to Output Current

We set up the following equation:

$$MJ \triangleq \frac{I_S}{IG}$$

Obviously we have:

$$0 < M < 1.$$

This parameter defines the current operating point of the system, i.e. the cyclic ratio of the transistor of the power stage.

I.5 Miscellaneous

In addition, we have defined the ratio ρ which exists between output resistor R_S and the resistor R_E in parallel on the current generator of the solar panel.

$$\rho \triangleq \frac{R}{R_S}$$

Everything which has already been said in the "Methodology Report" in Part I concerning the reduced time constants and the gains of the different modulators remains valid.

II. Transition from the Reduced Parameters to the Machine Parameters

In what follows we have assumed as known the following:
- the output voltage V_S,
- the charging resistance R_S,
- the operating period of the converter T.

These quantities V_S, R_S and T relate as well to a simulated system as to a real system. We deduce from them the quantities of the circuit to be considered:

$$R = \varrho \cdot R_S$$

$$C = \frac{\Gamma E.T}{2\pi. \varrho \cdot R_S}$$

$$C2 = \frac{\Gamma S.T}{2\pi \cdot R_S}$$

$$IG = \frac{V_S}{R_S \cdot MJ}$$

$$RC = \frac{R_S \cdot \Gamma}{\Gamma S}$$

III. Gain of the Correcting Amplifier Insuring the Required ±2% Regulation

In the same way as for the other elementary units, to find out the minimum gain which ensures the ±2% regulation, it suffices to calculate what the maximum variation is of the control voltage V_C of the modulator ($V_{cmax}-V_{cmin}$) when I_S and I_s vary within the specified limits, then to write the following equation:

$$\frac{V_{cmax} - V_{cmin}}{GO} = 4 \cdot V_R$$

For the coincidence modulator we have:

$$\begin{cases} v_{c_{max}} = t_{ON_{max}} \cdot \dfrac{A}{T} \\[2mm] v_{c_{min}} = t_{ON_{min}} \cdot \dfrac{A}{T} \end{cases}$$

For the magnetic modulators, we have:

$$\begin{cases} v_c = v_1 \left(1 - T_{ON} \cdot \dfrac{3}{2T}\right) \\[2mm] 0 < t_{ON} < \dfrac{2T}{3} \end{cases}$$

With the definition and the range of variation of MJ (cf. section 4) we easily have:

$$0.1.T \;<\; t_{ON}.\;<\; 0.9.T$$

Thus, finally we have:

$$\left.\begin{array}{l} v_{c_{max}} = 0.9 \; A \\[2mm] v_{c_{min}} = 0.1 \; A \end{array}\right\} \quad \text{coincidence modulator}$$

Therefore, for the shunt with the coincidence modulator we will have:

$$G_0 = \frac{0.9 \; A}{4 \cdot v_R} \;=\; 22.5 \; \frac{A}{v_R}$$

In the case of the simulation, we will have:

$$G_0 \; SHUNT = 5.6$$

For the magnetic modulator we see that we cannot go below MJ = 0.333.

$$v_{c_{max}} = v_1$$

$$v_{c_{min}} = 0$$

$$G_0 = 25 \; \frac{v_1}{v_R} = 25 \text{ because we take } v_1 = v_R$$

$$G_0 \; SHUNT = 25$$

IV. Guidelines for Simulations and Optimizations to be Carried Out

IV.1 Variation Range of the Parameters

In accordance with the ESTEC, we defined on Jan. 12, 1976 the following values for the reduced parameters:
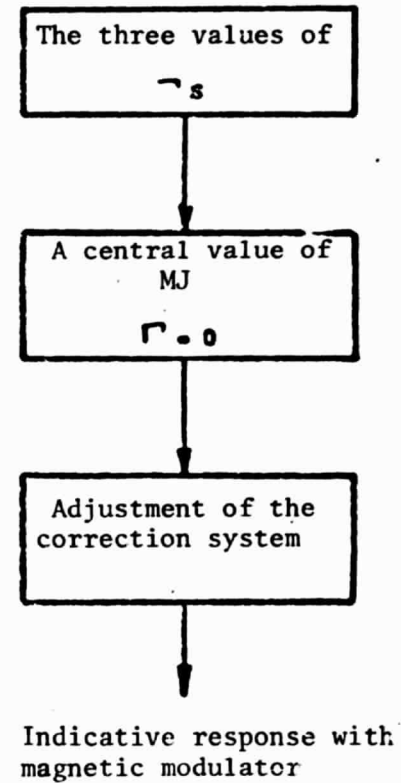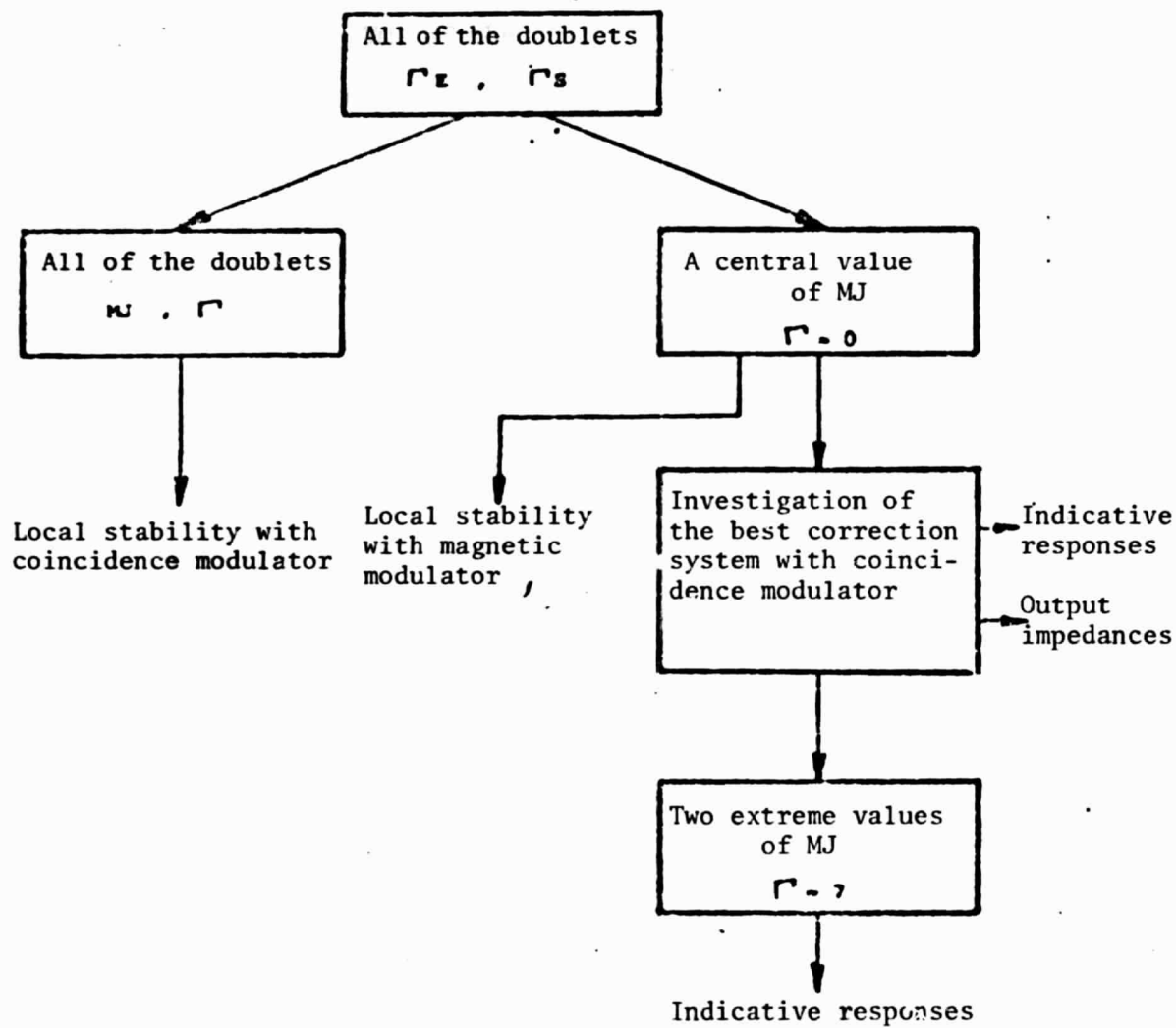
$$\Gamma \; : \quad 0. \qquad 3. \qquad 7.$$
$$\Gamma_E \; : \quad 0.3 \qquad 0.6 \qquad 1.5$$
$$\Gamma_S \; : \quad 100. \qquad 500. \qquad 1000.$$
$$MJ \; : \quad 0.1 \qquad 0.5 \qquad 0.9$$

From these we deduced the simulations and "optimizations" to be carried out which retain a significant value for the results.

IV.2 Simulations and "Optimizations" to be Carried Out

- Exhaustive study of the local stability with coincidence modulator,
- simplified study of the local stability with magnetic modulator,
- investigation of the best correction system for the doublets (ΓE, ΓS) with the coincidence modulator and study of the variations caused by the extreme values of MJ and by Γ,
- readjustment of the correction system for the magnetic modulator,
- plotting of output impedances only for optimized systems.

As for the elementary units studied previosuly, we constructed a flow diagram of the simulations to be carried out.

All of the doublets
Γ_E , Γ_S

All of the doublets
MJ , Γ

A central value
of MJ
$\Gamma = 0$

Local stability with
coincidence modulator

Local stability
with magnetic
modulator

Investigation of
the best correction
system with coinci-
dence modulator

Indicative
responses

Output
impedances

Two extreme values
of MJ
$\Gamma = 7$

Indicative responses

The three values of
$\neg_S$

A central value of
MJ
$\Gamma = 0$

Adjustment of the
correction system

Indicative response with
magnetic modulator

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