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by

Athanasios Dermanis

Prepared for the

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The Ohio State University
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PREFACE

OSURF Project 3820-A1 is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, The Ohio State University, and under the technical direction of Mr. James P. Murphy, Special Programs, Office of Applications, Code ES, NASA Headquarters, Washington, D.C. The contract is administered by the Office of University Affairs, NASA, Washington, D.C. 20546.

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1. Introduction

Very Long Baseline Interferometry (VLBI) is one of the new techniques which will probably dominate geodesy and geophysics in the near future. Its main advantage lies in the fact that it brings the accuracy of direction measurements to a level previously possible only for range measurements. This closes the gap between powerful range determination techniques such as laser ranging and the much less accurate determination of directions through photographic tracking of artificial earth satellites.

The technique is geometric in the sense that the observations are independent of the gravity field of the earth. However, the "orbits" of the observed extragalactic radio sources with respect to an earth-fixed system are dominated and perturbed by the rotation of the earth with respect to inertial frame. This allows the determination of polar motion, precession-nutation and length-of-the-day variations, and the technique becomes also "dynamic" in this respect.

The capability of determining the geometry of a network of stations within a short time interval and with a centimeter level accuracy also allows the study of the variation of network geometry with time caused by earth tides and other periodic or secular station drifts.

The obvious importance of VLBI for both geodetic and geophysical applications is only limited by the effect of atmospheric refraction and unwanted noise associated with instrumentation [Shapiro and Knight, 1970, Section 3]. At present the development of the technique is at the stage of establishing its capabilities by direct comparison with classical surveys in relatively short baselines where geophysical effects are negligible, and of making the system more transportable.

In the present work the effect of nonwhite instrumentation noise, atmospheric refraction and earth tides are ignored, and the emphasis is on the possibility of recovering earth rotation and network geometry (baseline) parameters. It is assumed that nonwhite instrumentation noise is either eliminated or separately determined through calibration except for a linear effect which cannot be distinguished from clock errors included in the adjustment parameters. Errors in the determination of atmospheric refraction are assumed to be either absent or uncorrelated in which case their effect is incorporated in the variance of the considered instrumentation white noise. The effect of earth tides on the variation of station coordinates is assumed to be calculated from separate information and subtracted from the actual observations.

The numerical simulated experiments performed here are set up in an environment where station coordinates vary with respect to inertial space according to a simulated earth rotation model similar to the actual but unknown rotation of the earth.

In Chapter 2 the basic technique of VLBI and its mathematical model are presented. The parametrization of earth rotation chosen is described in Chapter 3, and the resulting model is linearized in Chapter 4. Cartesian station coordinates are not estimable quantities in the analysis of VLBI observations. It is possible to use a model where only estimable quantities are considered (see, for example, [Arnold, 1974]). However, the choice of Cartesian coordinates leads to a simpler model, and the problem of coordinate system definition is resolved in Chapter 5 with the use of inner constraints. A simple analysis of the geometry of the observations in Chapter 6 leads to some useful hints on achieving maximum sensitivity of the observations with respect to the parameters considered. The basic philosophy for the simulation of data and their analysis through standard least squares adjustment techniques is presented in Chapter 7.

The main objective of the present work is the exploration of the capabilities of VLBI for the recovery of earth rotation and baseline parameters. For this purpose, a number of characteristic network designs based on present and candidate station locations is chosen in Chapter 8. The results of the simulations for each design are presented in Chapter 9 together with a summary of the conclusions.

2. VLBI—Basic Technique and Mathematical Model

Very Long Baseline Interferometry (VLBI) is a technique for observing time intervals and/or their derivatives at two widely separated antennas between the two instants of reception of the same wavefront emitted from extragalactic radio sources. The fundamental difference with respect to classical radio (short baseline) interferometry is in the fact that the antennas are not directly connected by cables for the comparison of the received signals. Sufficiently stable atomic frequency standards at each site make it possible to record the signals as functions of time. The recorded signals are then crosscorrelated afterwards in a computer. The fact that maximum correlation occurs when the two recordings are shifted in time by an amount equal to the wave travel time between the two instances of reception, makes it possible in principle to determine this time interval as well as its time derivative. Extragalactic radio sources are sufficiently far away for the wavefront to be considered as a plane and thus from the known velocity of the wave (velocity of light) the geometric distance corresponding to the observed time delay can be calculated.

The crosscorrelation procedure is described in detail in a series of articles by Thomas (1972a, 1972b and 1973), and we shall be concerned here only with the geometry of the observations. In Fig. 1, XYZ is a geocentric Cartesian reference frame fixed with respect to the radio sources (assumed to be an inertial frame). A certain wavefront arrives at station i at epoch t_1, when the position vector of the station is $X_i(t_1)$, and at the station j at a slightly different epoch t_2 when the position vector of this station is $X_j(t_2)$. The true time delay $t_2 - t_1$ is the travel time corresponding to the projection D_{ij} of the retarded baseline $X_j(t_2) - X_i(t_1)$ on the direction of the radio source. If e is the unit vector in the radio source direction and c the velocity of light,

$$D_{ij} = c\,(t_1 - t_2) = [X_j(t_2) - X_i(t_1)] \cdot e \qquad (2\text{-}1)$$

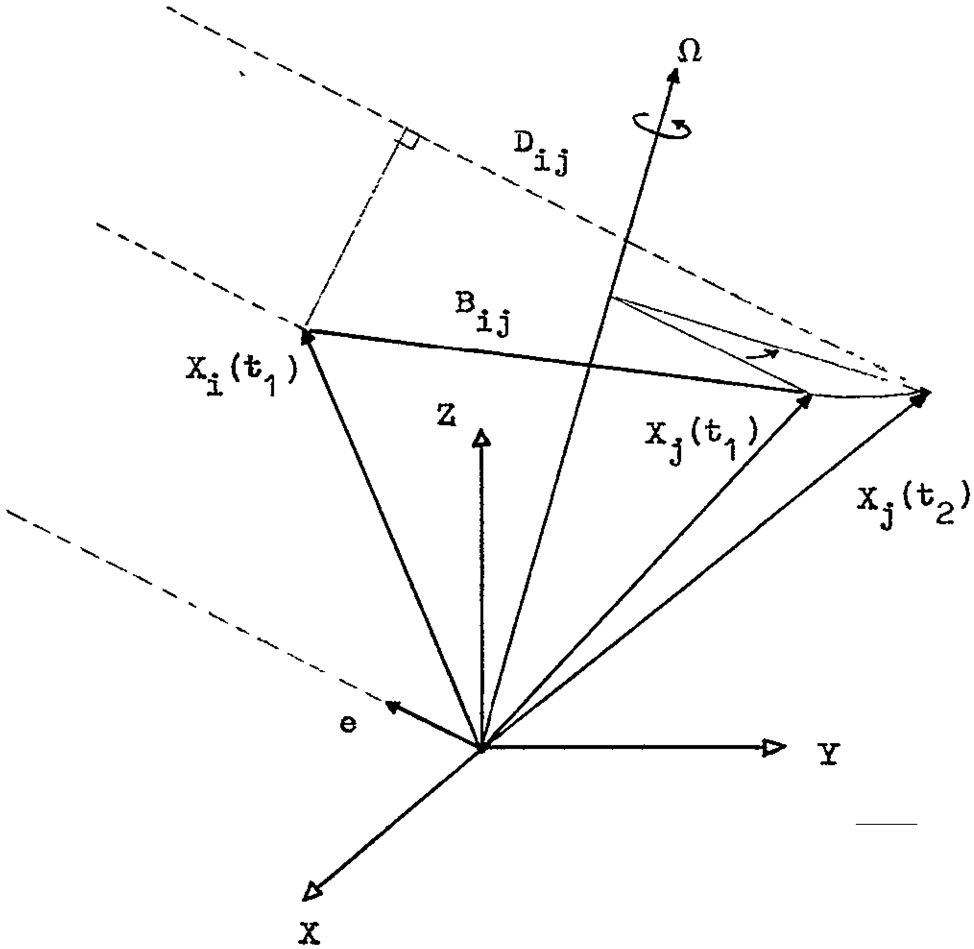

Fig. 2. 1 The geometry of VLBI observations.

Since $(t_2 - t_1)$ is a very short time interval, the time variation of the position vector $X_j(t)$ may be assumed to be solely due to the earth's rotation. Therefore

$$X_j(t_2) = X_j(t_1) + \int_{t_1}^{t_2} \Omega(t) \times X_j(t) \, dt \tag{2-2}$$

where $\Omega(t)$ is the rotational velocity vector of the earth. $\Omega(t)$ is practically constant over the short time interval $(t_1 - t_2)$, and thus, by setting $\Omega(t) = \Omega(t_1)$

$$X_j(t_2) = X_j(t_1) + (t_2 - t_1) [\Omega(t_1) \times X_j(t_1)] \tag{2-3}$$

Setting $B_{ij}(t) = X_j(t) - X_i(t)$ and $V_j(t) = \Omega(t) \times X_j(t)$, one obtains

$$D_{ij} = B_{ij}(t_1) \cdot e - \frac{1}{c} D_{ij} \; V_j(t_1) \cdot e \tag{2-4}$$

If approximate values of $\Omega(t_1)$, $X_j(t_1)$ and e are known with sufficient accuracy such that the corresponding uncertainty in the retardation effect

$$\delta D_{ij} = \frac{1}{c} D_{ij} V_j \cdot e \tag{2-5}$$

is negligible, one obtains, by setting $d_{ij} = D_{ij} + \delta D_{ij}$ and switching to matrix (column vector) notation, for an observation from the ij baseline to the radio source p at epoch t_k, the much simpler model

$$d_{ijpk} = B^T_{ij} (t_k) \, e_p \tag{2-6}$$

Assume that d_{ijpk} has been corrected for effects giving rise to discrepancies between modeled and true distance, e.g., atmospheric errors, relativistic effects, aberration, except for clock errors. Assuming that the two local oscillators at stations i and j differ only by a constant offset C_{oij} (at some epoch t_o) and a linear drift C_{ij}, the model becomes

$$d_{ijpk} = B^T_{ij}(t_k) \, e_p + c[C_{oij} + C_{ij}(t_k - t_o)] \tag{2-7}$$

Since the inner product of two vectors is invariant with respect to the choice of reference frame, the model may be rewritten

$$d_{ijpk} = b^T_{ij} \, e^*_p(t_k) + c[C_{oij} + C_{ij} (t_k - t_o)] \tag{2-8}$$

with

$$b_{ij}^{T} = [x_j - x_i, \quad y_j - y_i, \quad z_j - z_i] \tag{2-9}$$

where b_{ij} and e_p^* are the baseline and the radio source direction unit vector, respectively, with respect to an earth-fixed reference frame x, y, z. One also has

$$e_p^* (t_k) = M(t_k)\, e_p \tag{2-10}$$

where M(t) is the earth rotation matrix of transformation from the inertial to the earth-fixed frame. The model with respect to the earth-fixed reference frame becomes

$$d_{ijpk} = b_{ij}^{T} M(t_k)\, e_p + c[C_{oij} + C_{ij}(t_k - t_o)] \tag{2-11}$$

Further development of the model depends on the particular parametrization chosen for the rotation of the earth.

3. Earth Rotation Parametrization

The most straightforward way of parametrizing earth rotation is that of expressing the inertial to earth-fixed system transformation matrix M(t) in terms of three rotation angles at every epoch t. The most logical choice seems to be that of the Eulerian angles φ, θ, ψ, defined as

$$M(t) = R_3[\varphi(t)]\; R_1[\theta(t)]\; R_3[\psi(t)] \tag{3-1}$$

where R_1, R_2, R_3 denote conventional rotation matrices about the x, y, z axes, respectively. The rotating earth can be viewed as a dynamical system whose state evolves in time according to a second-order differential equation of the form

$$\frac{d^2 E}{dt^2} = f(E, t) \tag{3-2}$$

where $E = [\varphi, \theta, \psi]^T$. If such equations of motion could be exactly known and solved, their solution

$$E(t) = F(E(t_o), \dot{E}(t_o), t) \qquad (\dot{E} = dE/dt) \tag{3-3}$$

would provide a parametrization of the transformation matrix

$$M(t) = M(\varphi_o, \theta_o, \psi_o, \dot{\varphi}_o, \dot{\theta}_o, \dot{\psi}_o, t_o, t) \tag{3-4}$$

in terms of the six initial values (integration constants):

$$\varphi_o = \varphi(t_o) \qquad \theta_o = \theta(t_o) \qquad \psi_o = \psi(t_o)$$

$$\dot{\varphi}_o = \frac{d\varphi}{dt}\bigg|_{t_o} \qquad \dot{\theta}_o = \frac{d\theta}{dt}\bigg|_{t_o} \qquad \dot{\psi}_o = \frac{d\psi}{dt}\bigg|_{t_o}$$

However, such an approach is not practical because of uncertainties surrounding the current knowledge of the earth's rotation. The alternative is the representation of the functions $\varphi(t)$, $\theta(t)$, $\psi(t)$ in terms of a finite number of parameters to be estimated from the observations. Such representations as polynomials, trigonometric series, etc. can sufficiently approximate a wide class of functions, provided that appropriate number of terms is included The efficiency of the approximation can be judged only when the function to be approximated is known over some time interval, while in our case an unknown function is to be approximated. The unknown approximation error (representation error, modeling error) may well be significant compared to the observational errors, thus affecting the validity of the final parameter error estimates based upon real data analysis.

Among various finite parameter representations, the simplest perhaps is that of representing a continuous bounded function over an interval by a step function, which has a constant value over each subinterval, into which the original time interval has been partitioned.

To formally represent a step function, we introduce the characteristic or indicator function χ_A of a set A, defined as follows:

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} \tag{3-5}$$

Let [a, b] be the original interval and

$$a = t_0 < t_1 < t_2 < \ldots < t_{n-1} < t_n = b$$

If χ_i denotes the characteristic function of the set $[t_{i-1}, t_i]$ $(i = 1, 2, \ldots, n-1)$ and χ_n that of $[t_{n-1}, t_n]$, a step function with respect to the above partition of $[a,b]$ can be written in the form

$$y(t) = \sum_{i=1}^{n} \alpha_i \chi_i(t) \tag{3-6}$$

A continuous bounded function f(t) can be approximated arbitrarily well by a step function y(t) with an appropriate choice and increase in the number of subintervals. In the case of this investigation, the functions to be approximated are the Eulerian angles $\varphi(t)$, $\theta(t)$, $\psi(t)$, and the effectiveness of step function approximation may be increased if the known parts of φ, θ, ψ are filtered out. Let $\varphi^o(t)$, $\theta^o(t)$, $\psi^o(t)$ be the "reference" Eulerian angles as computable from classical astronomy. Then the earth rotation transformation matrix may be represented as

$$M(t) = R_3[\varphi^o + \delta\varphi] \; R_1[\theta^o + \delta\theta] \; R_3[\psi^o + \delta\psi] \tag{3-7}$$

where $\delta\varphi(t)$, $\delta\theta(t)$ and $\delta\psi(t)$ are step functions. An alternative parametrization may be of the form

$$M(t) = M^o(t) \; \delta M(t) = R_3(\varphi^o) \, R_1(\theta^o) \, R_3(\psi^o) \, R_3(\delta\varphi) \, R_1(\delta\theta) \, R_3(\delta\psi) \tag{3-8}$$

Traditionally, earth rotation representations are separated into three parts with the help of the instantaneous rotation vector $\omega(t)$ related to the Eulerian angles through Euler's geometric equations

$$\begin{aligned} \omega_x &= \sin\theta \sin\varphi \; \dot{\psi} + \cos\varphi \; \dot{\theta} \\ \omega_y &= \sin\theta \cos\varphi \; \dot{\psi} + \sin\varphi \; \dot{\theta} \\ \omega_z &= \cos\theta \; \dot{\psi} + \quad + \dot{\varphi} \end{aligned} \tag{3-9}$$

where ω_x, ω_y, ω_z are the rotation vector components with respect to the earth-fixed system. The variation in the direction of the vector ω with respect to the inertial system constitutes precession and nutation, while polar motion is the corresponding variation with respect to the earth-fixed system. The third part of earth rotation is the variation in the length of the vector ω, i.e., the variation of the rotational velocity (length of the day) $\Omega = \|\omega\|$.

To obtain a parametrization of earth rotation where precession-nutation, polar motion and rotational velocity are separated, we introduce an intermediate moving geocentric Cartesian reference frame x', y', z'. The z'-axis direction coincides at any epoch with the direction of the instantaneous rotation axis, while the x'-axis direction is arbitrary. The transformation from the X, Y, Z (inertial) to the x', y', z' frame may be expressed in terms of three rotations:

$$R_3(\Theta_2)\ R_2(\Xi)\ R_1(H)$$

and the transformation from x', y', z' to the x, y, z (earth-fixed) frame can be similarly expressed as

$$R_1(-\eta)\ R_2(-\xi)\ R_3(\Theta_1)$$

Setting $\Theta = \Theta_1 + \Theta_2$, the total transformation from the inertial to the earth-fixed frame becomes

$$M = R_1(-\eta)\, R_2(-\xi)\, R_3(\Theta)\, R_2(\Xi)\, R_1(H) \tag{3-10}$$

where η, ξ, Θ, Ξ, H are all functions of time. (Note that the Θ_1 and Θ_2 components of Θ cannot be separated, and this justifies the arbitrariness of the x'-axis direction.) In order to introduce the instantaneous rotational velocity Ω in the model, noting that the $R_3(\Theta)$ rotation is about the instantaneous rotation axis (diurnal rotation), one may set

$$\Theta(t) = \Theta_0 + \int_{t_0}^{t} \Omega(\tau)\, d\tau \qquad \Theta_0 = \Theta(t_0) \tag{3-11}$$

The three angles Ξ, H, Θ_0 represent the traditional precession-nutation, and the angles η and ξ the polar motion. The angle $(\Theta - \Theta_0)$ is similar to GAST (Greenwich Apparent Sidereal Time). The functions η, ξ, Ω, Ξ, H can now be approximated by step functions. To improve the approximation, we shall include in the transformation M the traditional precession-nutation based on current knowledge, as follows:

$$M = R_1(-\eta)\ R_2(-\xi)\ R_3(\Theta)\ R_2(\Xi)\ R_1(H)\ R^0 \tag{3-12}$$

where the known part of precession and nutation may be computed from [Mueller, 1969, Ch. 4].

$$R^{o} = R_1(-\epsilon - \Delta\epsilon)\ R_3(-\Delta\psi)\ R_1(\epsilon)\ R_3(-z)\ R_2(\theta)\ R_3(-\zeta_o) \qquad (3\ 13)$$

In the above expression, z, θ, ζ_o are the Newcomb components of precession, and $\Delta\psi$ and $\Delta\epsilon$ the nutations in longitude and obliquity by Woolard. The transformation R^o defines the traditional "true" frame of reference, an intermediate, noninertial, moving reference frame X', Y', Z' of known orientation, such that the Z'-axis is at any epoch near the instantaneous rotation axis. The angles Ξ, H, Θ_o are thus small corrections to the reference precession-nutation parameters used in (3-13), and as such they may be considered as the transformation parameters between the "traditional (Newcomb/Woolard) true" frame and the "actual true" frame.

There are two alternatives in the step function representation of the angle Θ. One may represent Ω as a step function with constant value Ω_i over the i^{th} time interval (step), in which case

$$\Theta(t) = \Theta_o + \sum_{k=1}^{i-1} \Omega_k (t_k - t_{k-1}) + \Omega_i (t - t_{i-1}) \qquad t_{i-1} < t \leq t_i \qquad (3\text{-}14)$$

A second choice is to approximate Θ over each step by a function of the form

$$\Theta_i(t) = \Theta_{oi} + \Omega_i (t - t_{i-1}) \qquad t_{i-1} \leq t < t_i \qquad (3\text{-}15)$$

This is a weaker representation from the adjustment point of view because of the larger number of parameters (Θ_{oi}, $i = 1, 2, \ldots, N$ vs. Θ_o), but it leads to estimates of variances of the estimates of the step function constants η_i, ξ_i, Θ_{oi}, Ω_i, Ξ_i, H_i, which are uniform (equal) over all steps (t_{i-1}, t_i). On the contrary, the first representation leads to variances which are smaller for steps in the middle and larger for steps at the ends of the original time span of the available observations.

Over each step the model for the VLBI observations becomes

$$\begin{aligned} d_{ijpk} = {} & b_{ij}^{T}\ R_1(-\eta)\ R_2(-\xi)\ R_3[\Theta_o + \Omega(t_k - t_o)]\ R_2(\Xi)\ R_1(H)\ R^o e_p + \\ & + c\,[C_{oij} + C_{ij}(t_k - t_o)] \end{aligned} \qquad (3\text{-}16)$$

where η, ξ, Θ_o, Ω, Ξ, H are constants over the considered step (the subscripts identifying the steps have been dropped for simplicity), t_k is the epoch of observation, and t_o is the beginning of the step containing t_k.

4. Linearization of Observation Equations

For the time span of available observations considered in this study (maximum of one month), the effect of precession-nutation on the experimental design (relative radio sources - stations configuration) is negligible. One can therefore drop the known transformation matrix R^0 from the model without any significant change on the simulation results. This simplifies somewhat the equations following, but it must be understood that appropriate modifications must be made when analyzing real data by including the effect of R^0.

The angles ξ, η, Ξ, H are very small since both the z and Z axes (actually the noninertial Z'-axis on account of ignoring R^0) are or can be chosen to be near the rotation axis. The usual approximations, $\cos\alpha = 1$, $\sin\alpha = \alpha$, $\alpha^2 = 0$, for a small angle α may be used, leading to

$$R_1(-\eta)\ R_2(-\xi) = \begin{pmatrix} 1 & 0 & \xi \\ 0 & 1 & -\eta \\ -\xi & \eta & 1 \end{pmatrix}$$

and

$$R_2(\Xi)\ R_1(H) = \begin{pmatrix} 1 & 0 & -\Xi \\ 0 & 1 & H \\ \Xi & -H & 1 \end{pmatrix}$$

The model now becomes

$$d_{ijpk} = (r_j - r_i)^T\ S^T(\xi,\eta)\ R_3(\Theta_0 + \Omega\ \tau_k)\ S(\Xi,\ H)\ e_p\ + \\ +\ c\,[C_{0ij} + C_{ij}\ \tau_k] \qquad (4\text{-}1)$$

where $\tau_k = t_k - t_0$; $e_p = [\cos\delta_p \cos\alpha_p,\ \cos\delta_p \sin\alpha_p,\ \sin\delta_p]^T$, α_p and δ_p are the right-ascension and declination of the p^{th} radio source with respect to the X, Y, Z inertial frame; $r_i = [x_i,\ y_i,\ z_i]^T$, x_i, y_i, z_i are the coordinates of the i^{th} station with respect to the x, y, z earth-fixed frame; and

$$S(a,\ b) = \begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & b \\ a & -b & 1 \end{pmatrix} \qquad (4\text{-}2)$$

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The available observations d_{ijpk} form a vector of observations L_b, corresponding to a vector of observables $L_a = F(X_a)$, where X_a is a vector of unknown parameters. The unknowns are the station coordinates x_i, y_i, z_i, $i = 1, 2, \ldots, N_s$, the earth rotation parameters ξ, η, Θ_0, Ω, Ξ, H (one set of six per each step), radio source coordinates α_p, δ_p, $p = 1, 2, \ldots, N_{rs}$, and clock synchronization parameters C_{oij}, C_{ij}, $i,j = 1, 2, \ldots, N_s$.

The d^a_{ijpk} components of the vector L_a are linear with respect to most of the parameters with the exception of Θ_0, Ω, α_p, and δ_p. A completely linearized model can be obtained by means of approximate values X^0 of the parameters X_a. Expansion in Taylor series and neglect of second- and higher-order terms leads to

$$L_a = F(X^0) + \left.\frac{\partial F(X_a)}{\partial X_a}\right|_{X^0} (X_a - X^0) \qquad (4\text{-}3)$$

The actual observations L_b differ from the observables L_a because of the presence of unknown observations errors V, according to $L_a = L_b + V$. Setting $A = (\partial F/\partial X_a)|_{X^0}$, $L^0 = F(X^0)$, $X = X_a - X^0$, and $L = L^0 - L_b$, the linearized observation equations are obtained:

$$V = AX + L \qquad (4\text{-}4)$$

A and L are known, while X and V are unknown. The vector V is assumed to be an outcome of a random vector with $E[V] = 0$ and known covariance matrix $E[V V^T] = S$.

Application of standard least squares techniques leads to a solution X, minimizing the quadratic form V^TPV where $P = S^{-1}$, which satisfies the "normal equations,"

$$(A^TPA)X = -A^TPL \qquad \text{or} \qquad NX = U \qquad (4\text{-}5)$$

To compute the elements of the desing matrix A, the partial derivatives of d_{ijpk} with respect to the parameters are required. Introducing the notation, $\psi_k = \Theta_0 + \Omega\,\tau_k$, $\chi_{kp} = \psi_k - \alpha_p$, $x_{ij} = x_j - x_i$, $y_{ij} = y_j - y_i$, $z_{ij} = z_j - z_i$, and dropping subscripts where confusion is not likely to arise, the partial derivatives are as follows:

$$\frac{\partial d}{\partial x_j} = -\frac{\partial d}{\partial x_i} = \cos\delta\cos\chi + \sin\delta\,(-\Xi\cos\psi + H\sin\psi + \xi)$$

$$\frac{\partial d}{\partial y_j} = -\frac{\partial d}{\partial y_i} = -\cos\delta\sin\chi + \sin\delta\,(\Xi\sin\psi + H\cos\psi - \eta)$$

$$\frac{\partial d}{\partial z_j} = -\frac{\partial d}{\partial z_i} = \sin\delta - \cos\delta\,(\xi\cos\chi + \eta\sin\chi - \Xi\cos\alpha + H\sin\alpha)$$

$$\begin{aligned}\frac{\partial d}{\partial \xi} = {} & x_{ij}\,[\sin\delta + \cos\delta\,(\Xi\cos\alpha - H\sin\alpha - \xi\cos\chi)] + \\ & + y_{ij}\,[\eta\cos\delta\cos\chi] + \\ & + z_{ij}\,[-\cos\delta\cos\chi - \sin\delta\,(-\Xi\cos\psi + H\sin\psi + \xi)]\end{aligned}$$

$$\begin{aligned}\frac{\partial d}{\partial \eta} = {} & y_{ij}\,[-\sin\delta + \cos\delta\,(\xi\cos\chi + \eta\sin\chi - \Xi\cos\alpha + H\sin\alpha)] + \\ & + z_{ij}\,[-\cos\delta\sin\chi + \sin\delta\,(\Xi\sin\psi + H\cos\psi - \eta)]\end{aligned}$$

$$\begin{aligned}\frac{\partial d}{\partial \Theta_o} = {} & x_{ij}\,[-\cos\delta\sin\chi + \sin\delta\,(\Xi\sin\psi + H\cos\psi)] + \\ & + y_{ij}\,[-\cos\delta\cos\chi + \sin\delta\,(\Xi\cos\psi - H\sin\psi)] + \\ & + z_{ij}\,[\cos\delta\,(\xi\sin\chi - \eta\cos\chi)]\end{aligned}$$

$$\frac{\partial d}{\partial \Omega} = \tau_k\frac{\partial d}{\partial \Theta_o}, \qquad \frac{\partial d}{\partial C_{oij}} = c\,, \qquad \frac{\partial d}{\partial C_{ij}} = c\,\tau_k$$

$$\begin{aligned}\frac{\partial d}{\partial \Xi} = {} & x_{ij}\left\{-\cos\psi\cos\delta + \cos\delta\,[\xi\cos\alpha + \cos\psi\,(-\Xi\cos\alpha + H\sin\alpha)]\right\} + \\ & + y_{ij}\left\{\sin\psi\sin\delta + \cos\delta\,[-\eta\cos\alpha + \sin\psi\,(\Xi\cos\alpha - H\sin\alpha)]\right\} + \\ & + z_{ij}\,[\cos\delta\cos\alpha + \sin\delta\,(\xi\cos\psi + \eta\sin\psi - \Xi)]\end{aligned}$$

$$\begin{aligned}\frac{\partial d}{\partial H} = {} & x_{ij}\,[\sin\delta\sin\psi + \cos\delta\sin\alpha\,(\Xi\cos\psi - H\sin\psi - \xi)] + \\ & + y_{ij}\,[\sin\delta\cos\psi + \cos\delta\sin\alpha\,(-\Xi\sin\psi - H\cos\psi + \eta)] + \\ & + z_{ij}\,[-\cos\delta\sin\alpha + \sin\delta\,(-\xi\sin\psi + \eta\cos\psi - H)]\end{aligned}$$

$$\begin{aligned}\frac{\partial d}{\partial \alpha} = {} & x_{ij}\,[\cos\delta\sin\chi] + y_{ij}\,[\cos\delta\cos\chi] + \\ & + z_{ij}\cos\delta\,(-\xi\sin\chi + \eta\cos\chi - \Xi\sin\alpha - H\cos\alpha)\end{aligned}$$

$$\frac{\partial d}{\partial \delta} = x_{ij} [- \sin\delta \cos\chi + \cos\delta (- \Xi \cos\psi + H \sin\psi + \xi)] + $$
$$+ y_{ij} [\sin\delta \sin\chi + \cos\delta (\Xi \sin\psi + H \cos\psi - \eta)] +$$
$$+ z_{ij} [\cos\delta + \sin\delta (\xi \cos\chi + \eta \sin\chi - \Xi \cos\alpha + H \sin\alpha)] \tag{4-6}$$

A second type of possible observations is the time derivative of d_{ijpk}

$$f_{ijkp} = \frac{\partial d_{ijpk}}{\partial t} = \Omega (r_j - r_i)^T S^T(\xi,\eta) P_3 R_3(\Theta_o + \Omega \tau_k) S(\Xi,H) e_p + c C_{ij} \tag{4-7}$$

where

$$P_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4-8}$$

The related partial derivatives of f_{ijkp} with respect to the parameters are as follows:

$$\frac{1}{\Omega} \frac{\partial f}{\partial \xi} = x_{ij} [\xi \cos\delta \sin\chi] - y_{ij} [\eta \cos\delta \sin\chi]$$
$$+ z_{ij} [\cos\delta \sin\chi - \sin\delta (\Xi \sin\psi + H \cos\psi)]$$

$$\frac{1}{\Omega} \frac{\partial f}{\partial \eta} = y_{ij} [\cos\delta (- \xi \sin\chi + \eta \cos\chi)]$$
$$+ z_{ij} [- \cos\delta \cos\chi + \sin\delta (\Xi \cos\psi - H \sin\psi)]$$

$$\frac{1}{\Omega} \frac{\partial f}{\partial \Xi} = x_{ij} [\sin\delta \sin\psi - \cos\delta \sin\psi (- \Xi \cos\alpha + H \sin\alpha)]$$
$$+ y_{ij} [\sin\delta \cos\psi - \cos\delta \cos\psi (- \Xi \cos\alpha + H \sin\alpha)]$$
$$+ z_{ij} [\sin\delta (- \xi \sin\psi + \eta \cos\psi)]$$

$$\frac{1}{\Omega} \frac{\partial f}{\partial H} = x_{ij} [\cos\psi \sin\delta - \cos\delta \sin\alpha (\Xi \sin\psi + H \cos\psi)]$$
$$+ y_{ij} [- \sin\psi \sin\delta + \cos\delta \sin\alpha (- \Xi \cos\psi + H \sin\psi)]$$
$$+ z_{ij} [\sin\delta (- \xi \cos\psi - \eta \sin\psi)]$$

$$\frac{1}{\Omega}\frac{\partial f}{\partial \Theta_o} = x_{ij}\,[-\cos\delta\cos\chi + \sin\delta\,(\Xi\cos\psi - H\sin\psi)\,]$$
$$+\, y_{ij}\,[\cos\delta\sin\chi - \sin\delta\,(-\,\Xi\sin\psi - H\cos\psi)\,]$$
$$+\, z_{ij}\,[\cos\delta\,(\xi\cos\chi + \eta\sin\chi)\,]$$

$$\frac{1}{\Omega}\frac{\partial f}{\partial x_j} = -\frac{1}{\Omega}\frac{\partial f}{\partial x_i} = -\cos\delta\sin\chi + \sin\delta\,(\Xi\sin\psi + H\cos\psi)$$

$$\frac{1}{\Omega}\frac{\partial f}{\partial y_j} = -\frac{1}{\Omega}\frac{\partial f}{\partial y_i} = -\cos\delta\cos\chi + \sin\delta\,(\Xi\cos\psi - H\sin\psi)$$

$$\frac{1}{\Omega}\frac{\partial f}{\partial z_j} = -\frac{1}{\Omega}\frac{\partial f}{\partial z_i} = \cos\delta\,(\xi\sin\chi - \eta\cos\chi)$$

$$\frac{1}{\Omega}\frac{\partial f}{\partial \alpha} = x_{ij}\,[\cos\delta\cos\chi\,] + y_{ij}\,[-\cos\delta\sin\chi\,]$$
$$+\, z_{ij}\,[\cos\delta\,(-\,\xi\cos\chi - \eta\sin\chi)\,]$$

$$\frac{1}{\Omega}\frac{\partial f}{\partial \delta} = x_{ij}\,[\sin\delta\sin\chi + \cos\delta\,(\Xi\sin\psi + H\cos\psi)\,]$$
$$+\, y_{ij}\,[\sin\delta\cos\chi + \cos\delta\,(\Xi\cos\psi - H\sin\psi)\,]$$
$$+\, z_{ij}\,[\sin\delta\,(-\,\xi\sin\chi + \eta\cos\chi)\,]$$

$$\frac{\partial f}{\partial \Omega} = \tau_k\,\frac{\partial f}{\partial \Theta_o}, \qquad \frac{\partial f}{\partial C_{oij}} = 0, \qquad \frac{\partial f}{\partial C_{ij}} = c \tag{4-9}$$

5. Coordinate System Definition and Inner Constraints

The combination of all observations leads to the linearized observation equations

$$ {}_nV_1 = {}_nA_u X_1 + {}_nL_1 \tag{5-1} $$

Minimization of $V^T PV$ leads to a solution satisfying the normal equations [Uotila, 1967]

$$ N X = U \tag{5-2} $$

where

$$ N = A^T P A \quad \text{and} \quad U = -A^T P L. $$

Because of the lack of coordinate system definition, N is singular with rank (N) = $r < u$, where u is the number of parameters and $s = u - r$ is the rank deficiency of N.

Originally $s = 9$, corresponding to the six degrees of freedom in the definition of the earth-fixed coordinate system (three for origin position and three for orientation) plus the three degrees of freedom in the inertial system definition (orientation only)

Since a radio source catalogue of a certain accuracy is assumed to be available, radio source coordinates are treated as observations (weighted unknowns) rather than as unknown parameters. The inertial frame is therefore defined through the catalogued radio source coordinates and the remaining rank deficiency of N becomes $s = 6$. A unique solution to the normal equations may be obtained if in addition a set of minimal linear constraints are imposed on the parameters, of the form

$$ C^T X = 0 \tag{5-3} $$

where C is a $u \times s$ matrix and rank (C) = s.

Among the various possible solutions to the normal equations, the unique one given by $X = N^+ U$, where N^+ is the pseudoinverse of N, has the following properties [Blaha, 1971]: $X^T X = \text{min.}$ and trace $N^+ = \text{min.}$ In view of the interpretation of N^+ as the variance-covariance matrix of the parameters, the second property makes the solution optimal in the sense that smaller variances of the parameters provide a better representation of parameter related estimable quantities in terms of the nonestimable parameters. To avoid the use of pseudoinverse computation algorithm, one may resort to a particular set of minimal constraints called inner constraints [Blaha, 1971],

$$E^T X = 0 \tag{5-4}$$

which leads to the same solution for X as the one obtained with the use of the pseudoinverse N^+. It can be shown that such a set of inner constraints can be obtained with the help of a $u \times s$ matrix E (rank (E) = s) satisfying

$$N E = 0 \tag{5-5}$$

An algebraic type of proof can be found in [Pope, 1971] which settles the truth of the matter but throws little light on the interrelation between inner constraints and pseudoinverse. A different type of proof is given in Appendix A of this work, based on the geometry of the operator represented by the matrix N.

In view of the fact that $AE = 0 \Rightarrow NE = A^T PAE = 0$, a set of inner constraints has been analytically constructed using six independent solutions to the set of equations

$$A^j y = 0 \qquad j = 1, 2, \ldots, u \tag{5-6}$$

where A^j denotes the j^{th} row of A. We assume that the approximate values of η, ξ, Ξ, H are zero in the computation of partials of observations with respect to the unknowns, thus obtaining simpler analytical expressions for the elements of A. Letting the order of the unknowns in X to be the following

$$X^T = [d\eta, d\xi, d\Theta_o, d\Omega, dx_1, dy_1, dz_1, \ldots, dx_N, dy_N, dz_N]$$

one may set

$$y^T = [w_1, w_2, w_3, w_4, \alpha_1, \beta_1, \gamma_1, \ldots, \alpha_i, \beta_i, \gamma_i, \ldots, \alpha_N, \beta_N, \gamma_N]$$

For a row of A corresponding to a d_{ijkp} observation, one has

$$w_1 \frac{\partial d}{\partial \eta} + w_2 \frac{\partial d}{\partial \xi} + w_3 \frac{\partial d}{\partial \Theta_o} + w_4 \frac{\partial d}{\partial \Omega} + \alpha_i \frac{\partial d}{\partial x_i} + \beta_i \frac{\partial d}{\partial y_i} + \gamma_i \frac{\partial d}{\partial z_i} +$$

$$+ \alpha_j \frac{\partial d}{\partial x_j} + \beta_j \frac{\partial d}{\partial y_j} + \gamma_j \frac{\partial d}{\partial z_j} = 0 \tag{5-7}$$

With the analytical expressions of the partials and after a considerable algebraic effort, one arrives at six independent solutions y_i, $i = 1, 2, \ldots, 6$, and the inner constraint matrix E is formulated by setting

$$E = [y_1\ y_2\ \ldots\ y_6]$$

The final result is

$$\begin{array}{ccccccccccc} d\eta & d\eta & d\Theta_0 & d\Omega & dx_1 & dy_1 & dz_1 & \ldots & dx_N & dy_N & dz_N \end{array}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \ldots & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & -z_1^o & y_1^o & & 0 & -z_N^o & y_N^o \\ 0 & 1 & 0 & 0 & z_1^o & 0 & -x_1^o & & z_N^o & 0 & -x_N^o \\ 0 & 0 & -1 & 0 & -y_1^o & x_1^o & 0 & & -y_N^o & x_N^o & 0 \end{bmatrix} \tag{5-8}$$

where x_i^o, y_i^o, z_i^o are the approximate values of station coordinates used in the computation of the design matrix A More explicitly there are two sets of constraints. The first one

$$\sum_i \begin{pmatrix} dx_i \\ dy_i \\ dz_i \end{pmatrix} = 0 \tag{5-9}$$

"defines" the origin of the earth-fixed system, while the second one

$$\begin{pmatrix} -d\eta \\ -d\xi \\ d\Theta_0 \end{pmatrix} = \sum_i \begin{pmatrix} 0 & -z_i^o & y_i^o \\ z_i^o & 0 & -x_i^o \\ -y_i^o & x_i^o & 0 \end{pmatrix} \begin{pmatrix} dx_i \\ dy_i \\ dz_i \end{pmatrix} . \tag{5-10}$$

"defines" its orientation.

In the discussion above, the problem of epoch and time scale definition for the set of station clocks has been ignored The parameters C_{0ij} and C_{ij} refer only to relative offsets and drifts between the clocks at stations i and j.

Usually epoch and time scale are provided by the readings of one of the clocks (master clock), and they are transformed to other station clocks through the parameters C_{omj}, C_{mj} where the master clock is at the m^{th} station.

Another way to provide epoch and time scale is the use of inner constraints defining a fictitious master clock. We may shift to a new set of parameters by setting

$$C_{oij} = C_{opj} - C_{opi} \qquad C_{ij} = C_{pj} - C_{pi} \tag{5-11}$$

The new parameters are offsets C_{opi} and drifts C_{pi} with respect to the fictitious master clock, and the additional inner constraints are

$$\sum_i dC_{opi} = 0 \quad \text{and} \quad \sum_i dC_{pi} = 0 \tag{5-12}$$

6. Remarks Related to Design of Experiment Concepts

Since the usual objective of a geodetic experiment is the estimation of certain parameters, an experimental design may be considered optimal when the errors in parameter estimates are minimized in some certain sense. Within the linear least squares model, the a posteriori variance-covariance matrix of parameter estimates is given in general by

$$S_X = \sigma_0^2 \, (A^T PA)^+ = \sigma_0^2 \, Q^+ \tag{6-1}$$

where Q^+ denotes the pseudoinverse of a square matrix Q, P is the weight matrix and σ_0^2 the a posteriori estimate of the variance of unit weight. The "errors" in parameter estimates therefore depend on the observational accuracy and on the design matrix A. The design matrix $A = A(X^o)$ itself depends on the approximate values of parameters X^o. The determination of optimal X^o values minimizing some risk function (e g., trace S_X) is usually referred to as the "configuration problem," or the "first-order optimal design" [Grafarend, 1974] On the assumption that the risk function chosen is insensitive to small changes of X^o, the approximate values of parameters may be identified with the true ones. In a similar way the small angles η, ξ, Ξ, H in our model may be considered to be zero. Of the remaining parameters Θ_0 depends on the choice of reference frames, Ω is approximated by the fixed average rotational velocity of the earth, while in the case of transcontinental baselines the choice of radio source coordinates α_p, δ_p is strongly restricted by observability conditions. Therefore, the first-order optimal design is practically limited to the determination of the optimal configuration of the network of observing stations.

The order of magnitude of elements in a row of the design matrix A determines the sensitivity of the corresponding observation with respect to each of the parameters. Next a short geometric interpretation of these partials is given to provide possible "hints" towards the determination of the optimal design.

The partial derivatives with respect to η, ξ, Θ_0, Ω, Ξ, H (earth rotation parameters) and α_p, δ_p are proportional to the baseline length. Indeed, they all contain linear terms in $x_{ij} = r_{ij}\, x^o_{ij}$, $y_{ij} = r_{ij}\, y^o_{ij}$ and $z_{ij} = r_{ij}\, z^o_{ij}$, where r_{ij} is the length of the ij baseline and x^o_{ij}, y^o_{ij}, z^o_{ij} are the components of the unit vector r^o_{ij} in the direction of the baseline. It is therefore possible to define "sensitivity per unit of baseline length" by means of $\frac{1}{r_{ij}} \frac{\partial d}{\partial \beta}$ where β stands for any of the earth rotation or radio source coordinates parameters. It can be shown that

$$\frac{1}{r_{ij}} \frac{\partial d}{\partial \beta} = (r^o_{ij})^T S_\beta \tag{6-2}$$

i.e., that the sensitivity per unit of baseline length is the projection of some vector S_β (called the parameter β sensitivity vector) on the direction of the baseline. It can be easily verified that the parameter sensitivity vectors are given by.

$$\begin{aligned}
S_\eta &= -R_1(\pi/2)\; P_{yz}\; R_3(\chi)\; e_6 \\
S_\xi &= -R_2(\pi/2)\; P_{xz}\; R_3(\chi)\; e_6 \\
S_{\Theta_0} &= \tau_k^{-1} S_\Omega = P_{xy}\; R_3(\chi + \pi/2)\; e_6 \\
S_\Xi &= R_3(\psi)\; R_2(\pi/2)\; P_{xz}\; R_3(-\alpha)\; e_6 \\
S_H &= R_3(\psi)\; R_1(\pi/2)\; P_{yz}\; R_3(-\alpha)\; e_6 \\
S_\alpha &= -R_3(\chi + \pi/2)\; P_{xy}\; e_6 \\
S_\delta &= R_3(\chi)\; R_2(\pi/2)\; e_6
\end{aligned} \tag{6-3}$$

where

$$e_\delta = [\cos\delta,\ 0,\ \sin\delta]^T, \qquad e_p = R_3(-\alpha)\, e_{\delta_p} \text{ and}$$

$$P_{xy} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad P_{yz} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad P_{xz} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

are projection matrices on the xy, yz and xz planes, respectively. The position of the sensitivity vectors varies with time, and their loci, all contained within the unit sphere, are depicted in Fig. 6.1. Maximum sensitivity occurs when the baseline is parallel to the corresponding parameter sensitivity vector, while an observation is insensitive to a parameter when the baseline and parameter sensitivity vector are perpendicular. With this in mind and with the help of Fig. 6.1, Table 6.1 is constructed, summarizing the sensitivity to various parameters for extreme cases of baseline and radio source directions.

Table 6.1

Sensitivity of Baseline - Radio Source Extreme Configurations with Respect to Various Parameters

Parameters	Baseline Parallel to Equator ($z_{ij} = 0$)		Baseline Parallel to Rotation Axis ($x_{ij} = y_{ij} = 0$)	
	Equatorial Radio Sources ($\delta = 0°$)	Polar Radio Sources ($\delta = 90°$)	Equatorial Radio Sources ($\delta = 0°$)	Polar Radio Sources ($\delta = 90°$)
ξ, η	no	yes	yes	no
Ξ, H	no	yes	yes	no
Θ_0, Ω, α	yes	no	no	no
δ	no	yes	yes	no

Note that observations in the fourth column are impossible due to observability conditions (a baseline parallel to the rotation axis must be located near the equator and observe equatorial radio sources only).

The sensitivity with respect to Ω as well as to C_{ij} is proportional to the time span τ_k of the available observations while that to C_{ij} and C_{oij} is independent of the baseline - radio source configuration.

Sensitivity with respect to station coordinates is independent of baseline length. If e_x, e_y, e_z denote unit vectors in the directions of the x, y, z axes, respectively, the relevant partial derivatives under the same approximations are

$$\frac{\partial d}{\partial x_j} = e_x^T S_c, \qquad \frac{\partial d}{\partial y_j} = e_y^T S_c, \qquad \frac{\partial d}{\partial z_j} = e_z^T S_c$$

where $S_c = R_3(-\chi)\, e_\delta$. The sensitivity of observations with respect to station coordinates is determined by the projection of the coordinate sensitivity vector S_c on the coordinate axes. The locus of S_c is depicted in Fig. 6.1. It can readily be seen that observations to equatorial radio sources are insensitive to the z station coordinates, while observations to polar radio sources are insensitive to x and y station coordinates.

All the above remarks are of a rather general nature and simply provide "hints" towards the design of an optimal station configuration for the VLBI experiment, which also depends on which of the parameters in particular is to be determined.

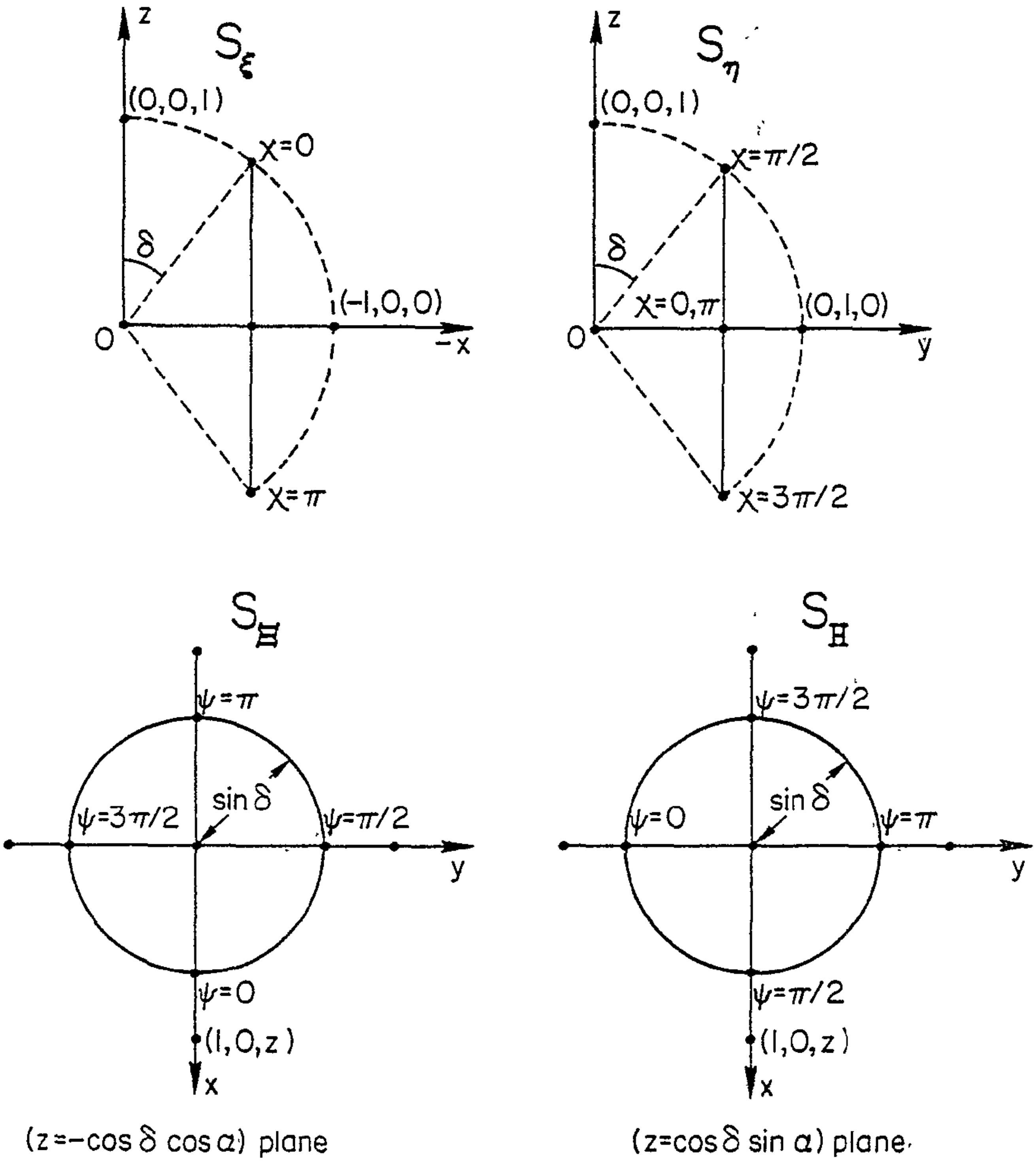
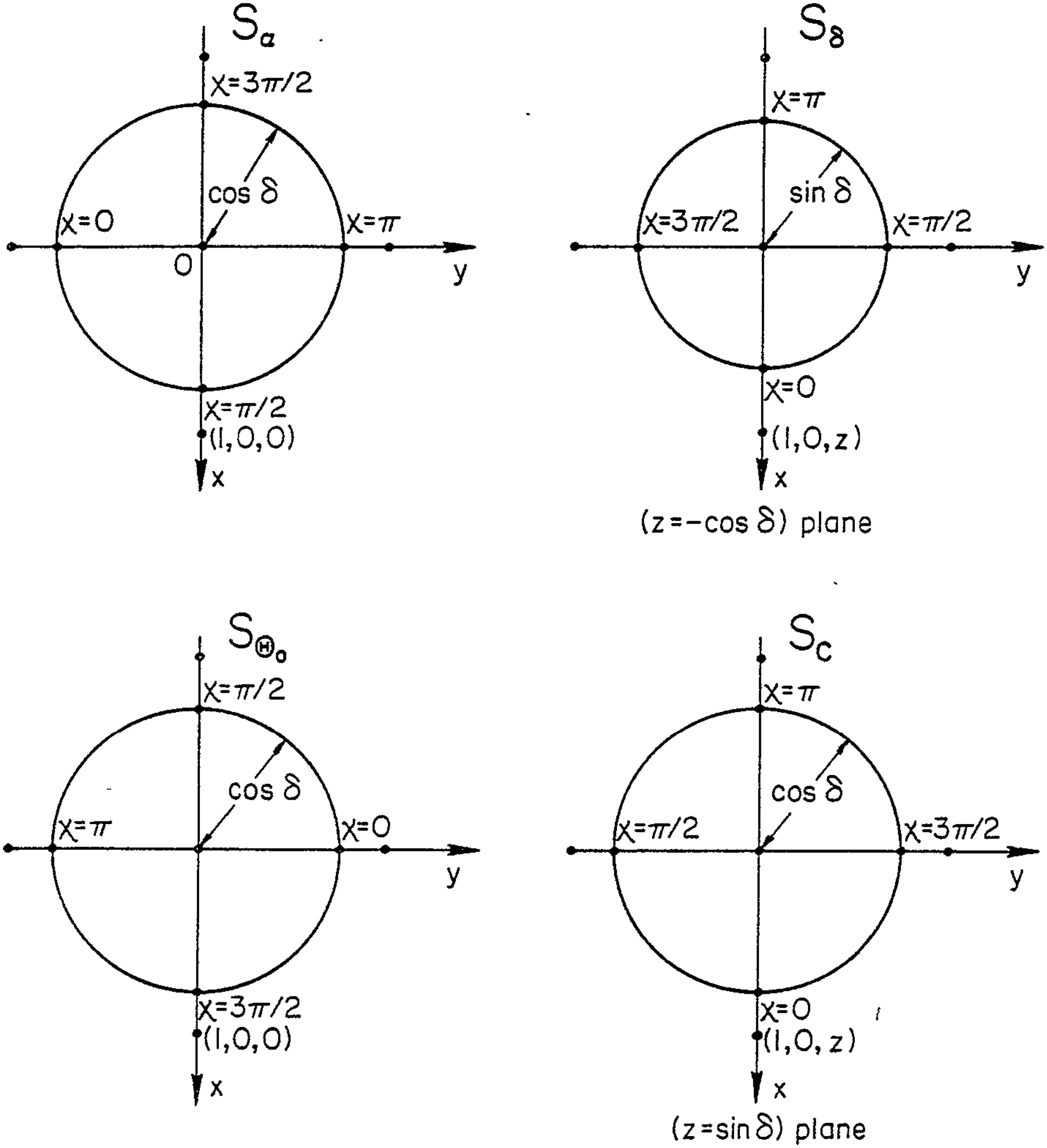


Fig. 6.1 Geometric loci of parameter sensitivity vectors.



S_α
$\chi=3\pi/2$
$\cos\delta$
$\chi=0$
$\chi=\pi$
O
y
$\chi=\pi/2$
(1,0,0)
x
S_δ
$\chi=\pi$
$\sin\delta$
$\chi=3\pi/2$
$\chi=\pi/2$
y
$\chi=0$
(1,0,z)
x
$(z=-\cos\delta)$ plane
S_{Θ_0}
$\chi=\pi/2$
$\cos\delta$
$\chi=\pi$
$\chi=0$
y
$\chi=3\pi/2$
(1,0,0)
x
S_C
$\chi=\pi$
$\cos\delta$
$\chi=\pi/2$
$\chi=3\pi/2$
y
$\chi=0$
(1,0,z)
x
$(z=\sin\delta)$ plane

Fig. 6.1 (cont'd)

7. Simulation and Adjustment Philosophy

The objective of this simulation is the determination of standard deviations of parameter estimates relevant to VLBI observations for a number of characteristic network designs. In addition to this design (relative configuration of station - radio sources and arrangement of observations in time), the a posteriori standard deviations of parameters also depend on a number of other factors. These include the accuracy of observations, their total time span, and the modeling of earth rotation and relative station motions in the adjustment. Since step function models are used here, the time interval over which earth rotation parameters are considered to be constant ("earth step") and the time interval over which station coordinates are considered constants ("station step") become also variables. It has also been assumed that a radio source coordinate catalogue is available, and its accuracy is one additional variable affecting final results. Summarizing, the problem is to examine the variation of standard deviations of the following parameters:

i. station coordinates
ii. earth rotation parameters

with respect to the following independent variables:

a. observational noise
b. time span of observations
c. earth step
d. station step
e. accuracy of radio source catalogue
f. station - radio source configuration, arrangement of observations in time.

With respect to the last item above, three different variables appear in the same group. This is justified in view of the fact that once a transcontinental network design has been decided upon, observability conditions impose strong limitations on both the choice of radio sources to be observed and the arrangement of observations in time.

The fact that observations are not instantaneous, but averages over some time interval (integration time of 7-10 minutes), imposes also limitations on the maximum density of observations in time.

The choice of earth and station step in real data analysis depends on the frequency content of the corresponding time functions. In view of the aliasing effect [Koopmans, 1974], the smaller the step size, the higher the spectral resolution of the corresponding functions. On the other hand, larger step size leads to a smaller number of parameters to be estimated and a "stronger" adjustment with more degrees of freedom. The choice of optimal step size in the analysis of real data constitutes a very interesting problem worthy of a separate study.

In the present simulation study, step size has been considered as a variable, while the corresponding functions generated in the simulation of observations are continuous rather than step functions. However, precautions have been taken so that the total variation $V_{f,I}$ of the function f(t) in question over each step time interval I is insignificant compared to observational noise. In fact,

$$V_{f,I_i} = \max_{t \in I_i} [f(t)] - \min_{t \in I_i} [f(t)] \leq \sigma_b/N \qquad i = 1, 2, \ldots, n$$

where σ_b is the standard deviation of the observations, and N is a large-integer (specifically here N = 10).

In view of a 7-10-minute integration time, and allowing for antenna motion, observations (time delays and their time derivatives) are uniformly spaced at 15-minute intervals over the total time span considered. Their corresponding standard deviations σ_b and σ_b' are taken to be of a constant ratio $\sigma_b/\sigma_b' = 1/2$, where σ_b is in nsec and σ_b' in nsec/hr. This value is typical, but the ratio may be somewhat different depending on antenna and observed signal characteristics. The choice of a fixed ratio reduces the number of considered parameters by one with a corresponding significant reduction in computational effort.

Once a certain observational pattern has been decided upon, a simulation program (VLBI SIMULATOR) produces perfect observations using the simplified model (Eq. 4-1). This program requires as input station and radio source true coordinates and the pattern of the observations (baseline - radio source - epoch of observation). The station coordinates are taken to be constants, while the values of earth rotation parameters at the epoch of observation are provided by a subroutine (SUBROUTINE EARTH). Θ_0 is fixed to a realistic value for the beginning of the observations t_0. The effect of traditional precession-nutation is absent from the simulation as well as from the adjustment model for computational simplicity. The inertial frame thus is taken to be the true equatorial system of the epoch t_0 so that its Z-axis is near the rotational axis. For the same reason, the earth-fixed frame is taken to be the CIO/BIH zero-meridian terrestrial system.

The perfect observations are processed through a program which adds "Observational errors" drawn from a Gaussian population with zero mean and desired variance.

For the adjustment of the observations, two separate programs are used. The first one (PROG01) processes observations of total time span equal to both earth and station step (one day). An option is available for processing any desired number of different radio source catalogue accuracies in the same run.

The second program (PROG02) processes observations of time span equal to the station step only, which in turn is taken to be an integer multiple of the earth step. It is, however, possible to process observations using unequal earth steps within the total span of available observations. Within each earth step the linearized observations equations are of the form

$$V_i = \dot{A}_i\dot{X} + \ddot{A}_i\ddot{X}_i + L_i$$

where $\dot{X}$ refers to parameters common to all earth steps (station and radio source coordinates) and $\ddot{X}_i$ refers to parameters particular to each earth step (earth rotation parameters).

The total set of such individual observation equations is combined in a unified solution through a "second-order partitioned linear regression" scheme, extended to incorporate inner constraints. The detailed algorithm is presented in Appendix B. The related programs and subroutines are in Appendix C.

8. Arrangement of Experiments

To obtain more realistic results, station locations considered here are not completely arbitrary, but limited to present and prospective locations of VLBI stations. Such candidate station locations are presented in Table 8.1. In the same table the stations actually used in this study are indicated by asterisks. Two criteria have been considered in the selection. First, stations close to plate boundaries, i.e., in geophysically active areas, have been excluded to avoid possible comparatively large station drifts deviating from those considered in our simulations. Secondly, from groups of stations close to each other, only one station has been included on the assumption that final results would be similar for a nearby station. It is recognized that these criteria probably exclude the most active stations currently in operation, but this should not influence the general conclusions sought.

An experiment consists of a group of stations simultaneously participating in the observations. In Table 8.3 the station groups for the experiments considered in this study are presented. After a certain station group has been selected, the arrangement of observations is one observations every 15 minutes, which interval and choice of the radio sources are limited by observability conditions. The observability condition naturally imposed is that the observed radio source must be above the horizon at both ends of the observing baseline. To avoid large atmospheric refraction errors, the zenith distance of the radio source is limited, in general, to be less than $z_{max} = 45°$, with some exceptional cases where $z_{max} = 60°$.

Such observability conditions limit the observable part of the sky to within the intersection of two cones with the local station verticals as axes and z_{max} as the vertex angle. For transcontinental baselines, the region of observability is small and the choice is between observing a certain quasar or a neighboring one without significant effect on the experimental design.

In setting out a certain sequence of observations, the choice between possible alternatives has been guided by the hints obtained in Chapter 6, with emphasis on sensitivity with respect to earth motion and station coordinate parameters.

Table 8.1

Present and Prospective VLBI Station Locations and Their Approximate Coordinates

No.	Location	Longitude	Latitude
	Kashima, Japan	137°	35°
1	Canberra, Australia *	149	-35
2	Kauai, Hawaii *	200	22
3	Fairbanks, Alaska *	212	65
	Goldstone, California	243	35
	Algonquin, Canada	277	51
	Greenbank, Virginia	282	38
4	Haystack, Massachusetts *	288	41
	Santiago, Chile	289	-34
	Arecibo, Puerto Rico	294	18
5	Sao Paulo, Brazil *	313	-24
6	Madrid, Spain *	356	40
	Bonn, Germany	7	51
7	Onsala, Sweden *	17	59
8	Johannesburg, So. Africa *	28	-26
	Crimea, USSR	34	45

* indicates stations considered in this study

Table 8.2

Fictitious Stations Used in Experiments 6 and 9

No.	Location	Longitude	Latitude
9	equatorial	0°	0°
10	"	60	0
11	"	120	0
12	"	180	0
13	"	240	0
14	"	300	0
15	meridian	0	0
16	"	0	60
17	"	180	60
18	"	180	0
19	"	180	-60
20	"	0	-60

Table 8.3

Experiments and Participating Baselines
(Station Nos. Refer to Numbering in Table 8.1 or 8.2)

Experiment No.	Participating Baselines
1	2-3, 2-4, 3-4
2	3-4, 3-7, 4-7
3	4-5, 4-6, 5-6
4	5-6, 5-8, 6-8
5	1-2, 2-3, 2-4, 3-4, 3-7, 4-5, 4-6, 4-7, 5-6, 5-8, 6-7, 6-8
6	9-10, 10-11, 11-12, 13-14, 14-9
7	2-4, 4-5
8	2-4, 4-6
9	15-16, 16-17, 17-18, 18-19, 19-20, 20-15
10	3-4, 4-7

An effort has also been made to observe radio sources of a variety of declinations to avoid the critical configuration appearing when, for example, all radio sources are of the same declination. From a larger set of available radio sources, only a rather uniformly distributed set has been considered. The criterion for inclusion has been the use of the radio source in previous experiments so that its appropriateness for VLBI observations is assured.

Since a radio source catalogue of a certain accuracy is assumed to be available, no effort has been made to optimize radio source coordinate recovery. As a result, this study is of no "astrometric" value, and no significant improvement of radio source coordinate accuracy can be found in the results of the adjustment performed.

One sidereal day can be taken as the "unit" for an observational pattern since a sequence of observations performed over one day can be repeated over the next days. For observations of a total time span of a number of days, the same daily pattern is repeated in the simulation. Various experimental designs are compared to each other on the basis of one day of observations where earth rotation and baseline parameters are considered as constants over the whole day.

The choice of experiments performed has been primarily directed towards the possibility of recovering earth rotation parameters from a minimum number of observing stations. The minimum number is the three stations necessary for the definition of an earth-fixed system through minimal or inner constraints. Experiments 1, 2, 3 and 4 are cases of such minimal "triangle networks" where all the sides of the triangle are observing baselines.

A weaker desing (Experiments 7, 8, 10) is a three station configuration where only two of the triangle sides are observing baselines. In particular, Experiment 10 is a counterpart of Experiment 2, such that the effect of removing one baseline (other design parameters remaining the same) can be studied. The opposite to such minimal three-station designs is provided in Experiment 5 where all eight stations participate in the observations.

In addition to these "realistic" station locations, two "fictitious" designs are also considered, their common characteristic being the closure of a network of stations on a great circle around the earth. Experiment 6 is such a network of uniformly spaced stations along the equator, while Experiment 9 is of a similar design along a meridian. The coordinates of stations involved in these experiments are listed in Table 8.2.

Fig. 9.1 depicts the geographic locations of the networks in the above-mentioned experiments.

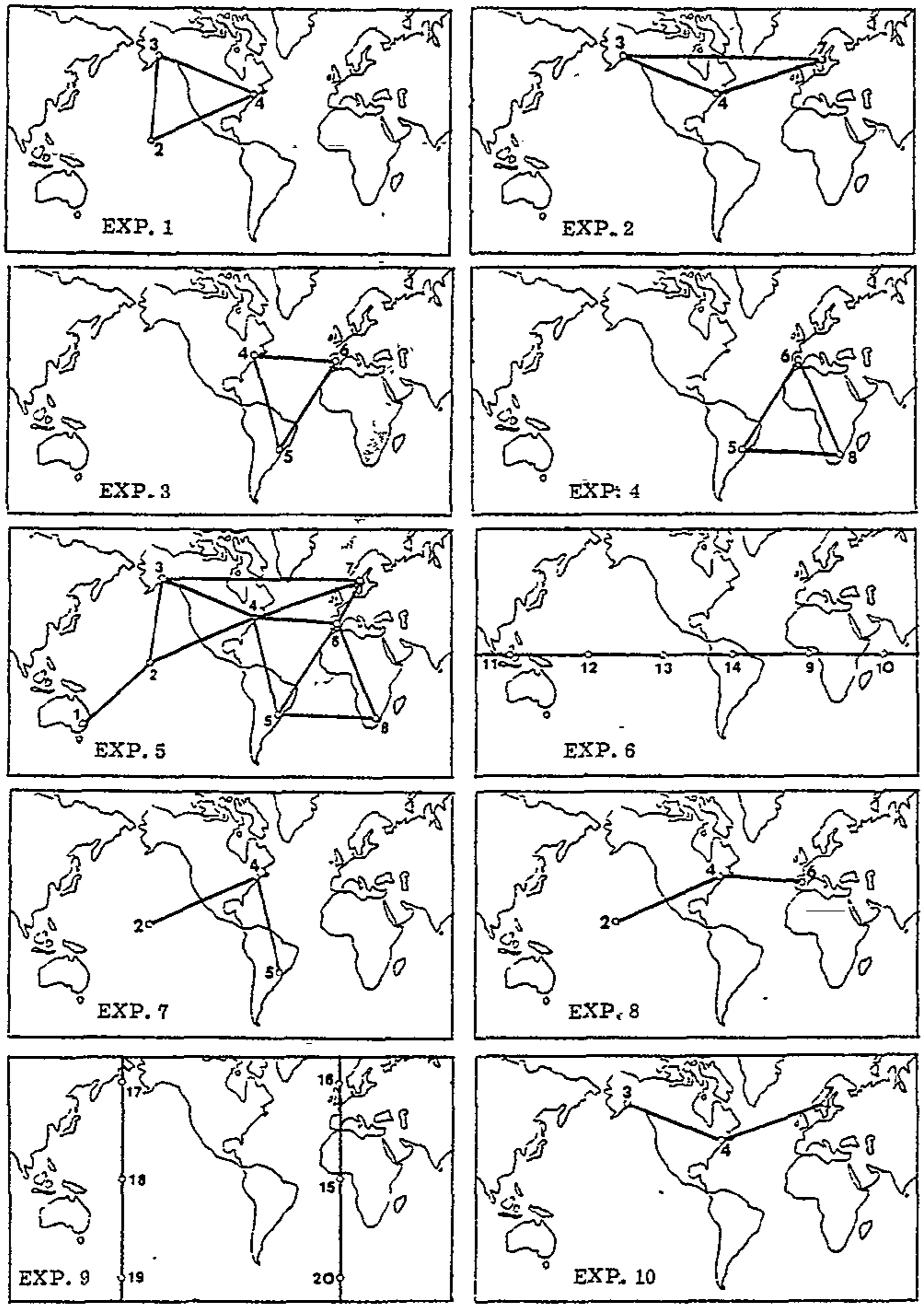


Fig. 9.1 Geographic locations of stations and baselines in Experiments 1 through 10.

9. Results and Conclusions

The results of the simulations are presented in three parts. The first part (Figs. 9.2 - 9.3) deals with the comparison of the ten designs described in the previous chapter, on the basis of one day of observations. Fig. 9.2 depicts the recovery (standard deviations) of earth rotation parameters (ξ, η, Ξ, Θ_o, Ω), while Fig. 9-3 shows the recovery of network geometry in terms of baseline lengths and angles (estimable quantities).

Standard deviations σ for each parameter are given as a function of the standard deviation σ_Q of radio source coordinates and for various values of observational precision σ_b. Values used in the simulation are σ_b, σ_Q = 1, 5, 10, 20, 50 cm. To achieve uniformity, σ_Q is expressed as arc length on a reference sphere with a radius of 6371 km (1" $\approx$ 30 m).

The second part (Figs. 9.4 - 9 5) is a study of the improvement of the standard deviations of the parameters with the total time interval T of available observations. Station coordinates are assumed to be constant over T while earth rotation parameters are constant only over the subintervals of duration Δt (earth step). The ratio $\rho = \sigma/\sigma_b$ is given for each parameter as a function of T for various values of Δt. In the simulations, the following values have been used: T = 1, 2, 3, 4, 5, 10, 30 days; Δt = 6, 12, 24 hours. Experiment 2 has been chosen for this study. Radio source coordinate standard deviations are assumed to be $\sigma_Q = \sigma_b$ for all cases. However, the variation of the results with σ_Q is insignificant for T $\geq$ 5 days.

The third part (Figs. 9.6 and 9.7) is a study of the variation of $\rho = \sigma/\sigma_b$ with Δt (earth step) based on one day of observations (T = 1 day) for some of the other designs (Experiments 1, 3, 4, 6, 9, 10) is shown.

Since the results of the simulations are fully presented here, one may draw his own conclusions. However, the following general remarks are made in an attempt to summarize the results:

(1) Earth Rotation Parameters from One-Day Observations

For one day of observations and the recovery of earth rotation parameters, the following are noted:

(a) Three-baseline "triangle" designs (Experiments 1, 2, 3, 4 in Fig. 9.2) are capable of recovering polar motion (ξ, η) with a precision of the same order of magnitude as that of the observations, even for low radio source accuracies (σ_Q = 50 cm).

For two-baseline designs (Experiments 7, 8, 10 in Fig. 9.2), polar motion recovery is of the same quality as for the "triangle" station configurations. However, the removal of one baseline in Experiment 10 results in an increase of polar motion standard deviations by a factor of 1.5 compared to Experiment 2. The more complex design of Experiment 5 offers no dramatic improvement in polar motion recovery. An essential improvement can be seen in the equatorial design of Experiment 6 and even more in Experiment 9 which is the "best," but admittedly not very practical design for polar motion recovery.

(b) The standard deviations of the (precession-nutation) parameters Ξ and H are practically the same for all two- or three-baseline designs (Experiments 1, 2, 3, 4, 7, 8, 10). In particular, the removal of one baseline in Experiment 10 with respect to Experiment 2 has no effect on the recovery of Ξ and H. The best recovery of Ξ, H can be seen in Experiments 5 and 9. Although the station configuration in Experiment 6 is of the same shape as in Experiment 9, the change in their position (equatorial vs. meridian) has a great effect on Ξ, H standard deviations. Experiment 6 has the weakest recovery, while Experiment 9 provides the strongest one.

(c) The standard deviation of Θ_0 varies with the station configurations. It is highest for Experiments 4, 8, 7; medium for Experiments 10, 3, 5; and smallest for Experiments 1, 2, 9, and especially 6 (best).

(d) Recovery of Ω varies strongly even for designs of the same type. It is generally weak for two-baseline designs (Experiments 7, 8, 10) and somewhat better for three-baseline designs with the exception of the extremely weak recovery in Experiment 4. Many-station designs (Experiments 5, 6, 9) improve the recovery especially in the equatorial configuration of Experiment 6.

(e) Ω and Θ_0 are the only parameters favored by the artificial equatorial design of Experiment 6 vs. the meridian one of Experiment 9. Recovery of the two components ξ and η of polar motion is not equal but depends on the position of the stations in each experiment with the exception of Experiment 6. On the contrary, both the Ξ and H parameters of precession-nutation are recovered with practically the same precision for every design.

The recovery of earth rotation parameters varies with the accuracy (σ_Q) of radio source coordinates. However, this dependence is much stronger for Ξ, H; milder for the rest of the parameters; and even disappearing for Ω, ξ, η in the strongest designs of Experiments 6 and 9.

(f) The results of the idealized designs of Experiments 6 and 9 can be used to verify some of the results of Chapter 6. Recovery of ξ, η, Ξ, H is better for Experiment 9 than 6, while recovery of Θ_o, Ω is better for Experiment 6. This is explained by the fact that Experiment 9 involves baselines parallel to the equator (16-17, 19-20) observing polar radio sources, a design sensitive to ξ, η, Ξ, H. Experiment 6 involves equatorial baselines observing equatorial radio sources, the design most sensitive to Θ_o, Ω (see Table 6.1).

(g) The recovery of baselines varies with station configuration. However, larger number of baselines does not in general improve the individual recovery as in the case of earth rotation parameters. For example, recovery in the three-baseline design of Experiment 2 is much better than in any of the many-station designs (Experiments 5, 6, 9). The relative recovery of angles in the same experiment is inversely proportional to the recovery of the opposite baselines. Dependence on radio source accuracy varies and disappears in the case of the equatorial configuration of Experiment 6, especially in comparison to the meridian configuration of Experiment 9.

(2) <u>Earth Rotation and Baseline Parameters As a Function of the Length of Observations and Earth Step</u>

For the variation of the recovery of earth rotation and baseline parameters with respect to the total time interval of observations T, it is noted that the standard deviations σ of the parameters are directly proportional to the standard deviations σ_b of the observations. (Note that $\sigma_Q = \sigma_b$.) For this reason only the ratio $\rho = \sigma/\sigma_b$ is depicted in Figs. 9.4 and 9.5.

The dependence of ρ on the earth step Δt is very strong for earth rotation parameters, especially for Ω (see Fig. 9.4), while not significant for baseline parameters (see Fig. 9.5). In general, the recovery improves as Δt increases since the corresponding number of unknown parameters in the adjustment decreases. For earth rotation parameters (Fig. 9.4), standard deviation decreases strongly as T increases from 1 to 5 days. An additional increase of T from 5 to 10 days offers a small improvement, while no significant additional improvement is gained from the extension of T from 10 to 30 days. The same is true for baseline parameters (lengths and angles, see Fig. 9.5) with the exception of an additional small improvement with the increase of T from 10 to 30 days.

(3) Earth Rotation and Baseline Parameters As a Function of Earth Step

As mentioned, the recovery of earth rotation and network geometry parameters improves with larger earth step Δt (Figs. 9.6 and 9.7). The only exceptions are with respect to the ξ, η parameters in Experiments 1, 2, 4, 10 and with respect to Θ_0 in Experiment 4 where $\Delta t = 24$ hours results in worse recovery than $\Delta t = 12$ hours or even $\Delta t = 6$ hours in some cases. This "inversion" is correlated to the weak recovery of polar motion in Experiments 1, 2, 4, 10 (see Fig. 9.2) and to the weak recovery of Θ_0 in Experiment 4. On the contrary, for Experiments 6 and 9, where ξ, η and Θ_0 recovery is better, variation of ρ with respect to Δt is "normal." Such variation is negligible for Θ_0 in Experiment 6 and also for network geometry parameters in Experiments 6 and 9. Naturally an increase in the length of the earth step is a disadvantage when short periodic variations in the earth rotation parameters are sought.

Fig. 9.2 Recovery of earth rotation parameters, Experiments 1 - 10.

Total time span of observations: $T = 1$ day

Earth step: $\Delta t = 1$ day

σ = a posteriori standard deviation of earth rotation parameters

σ_Q = a priori standard deviation of radio source coordinates

σ_b = standard deviation of observational errors

Note: Angular quantities when expressed in length units are arc lengths on a sphere of radius $R = 6\ 371\ 000$ m ($1'' \approx 30$ m).

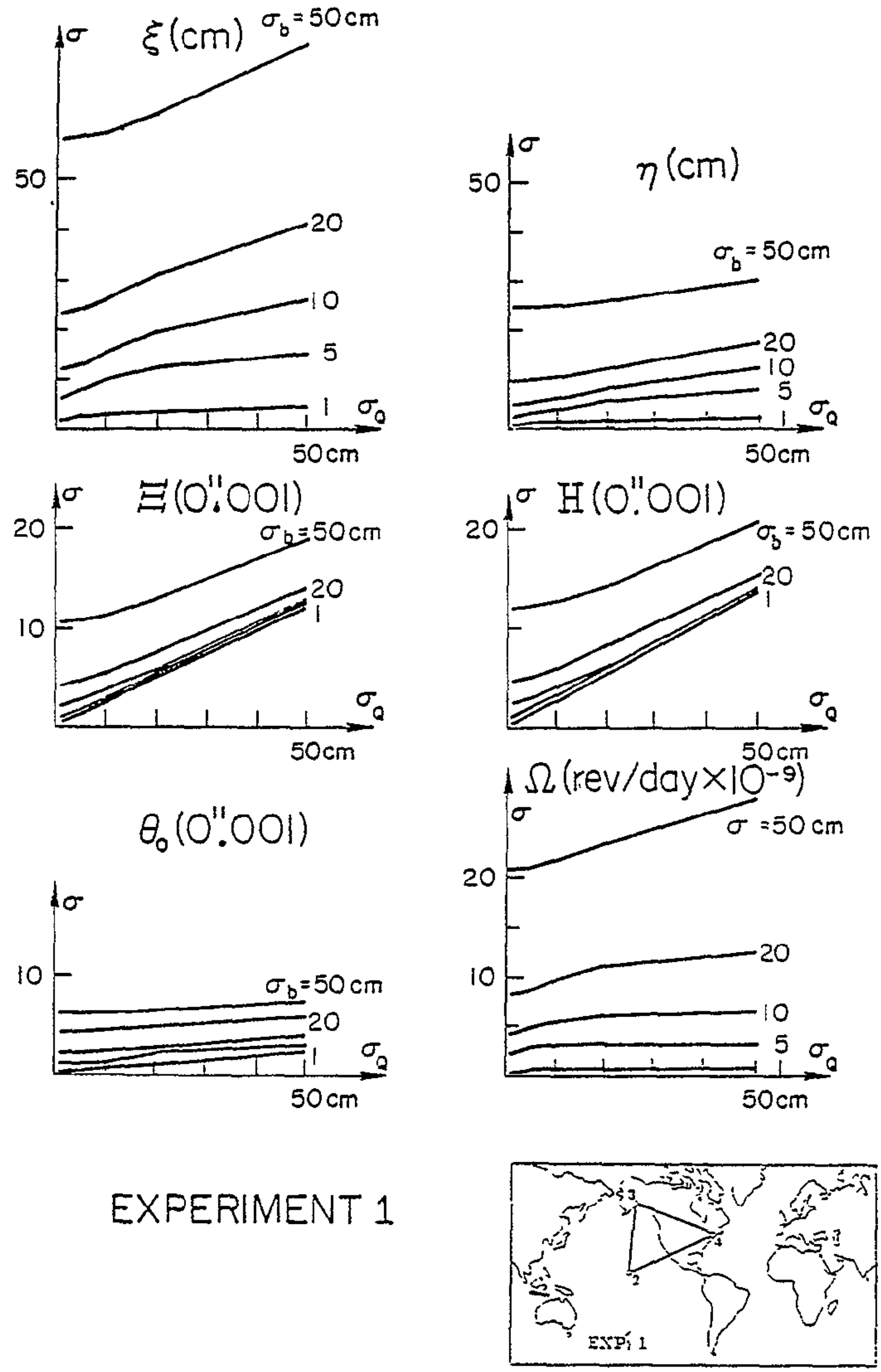


Fig. 9.2 Recovery of earth rotation parameters.

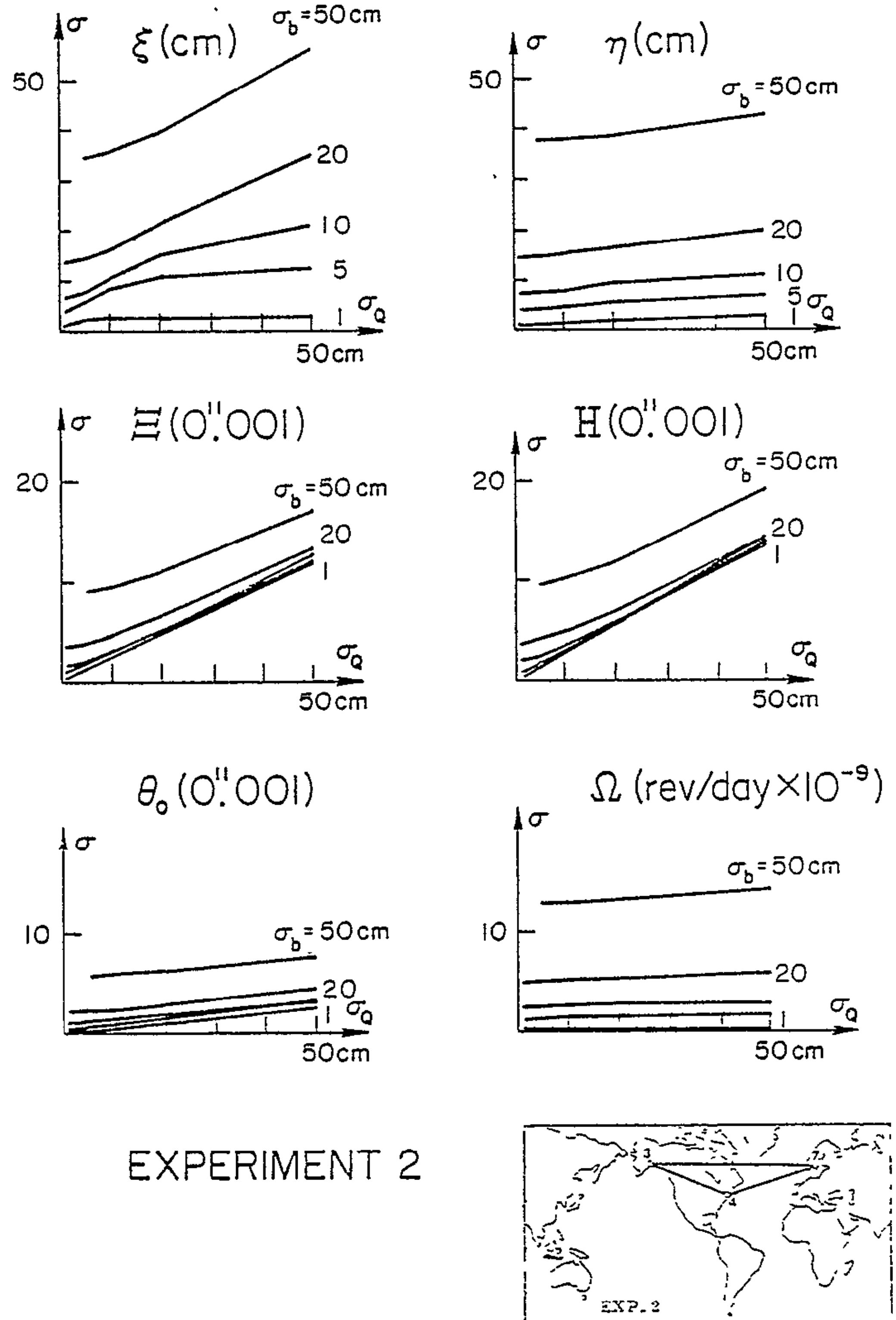
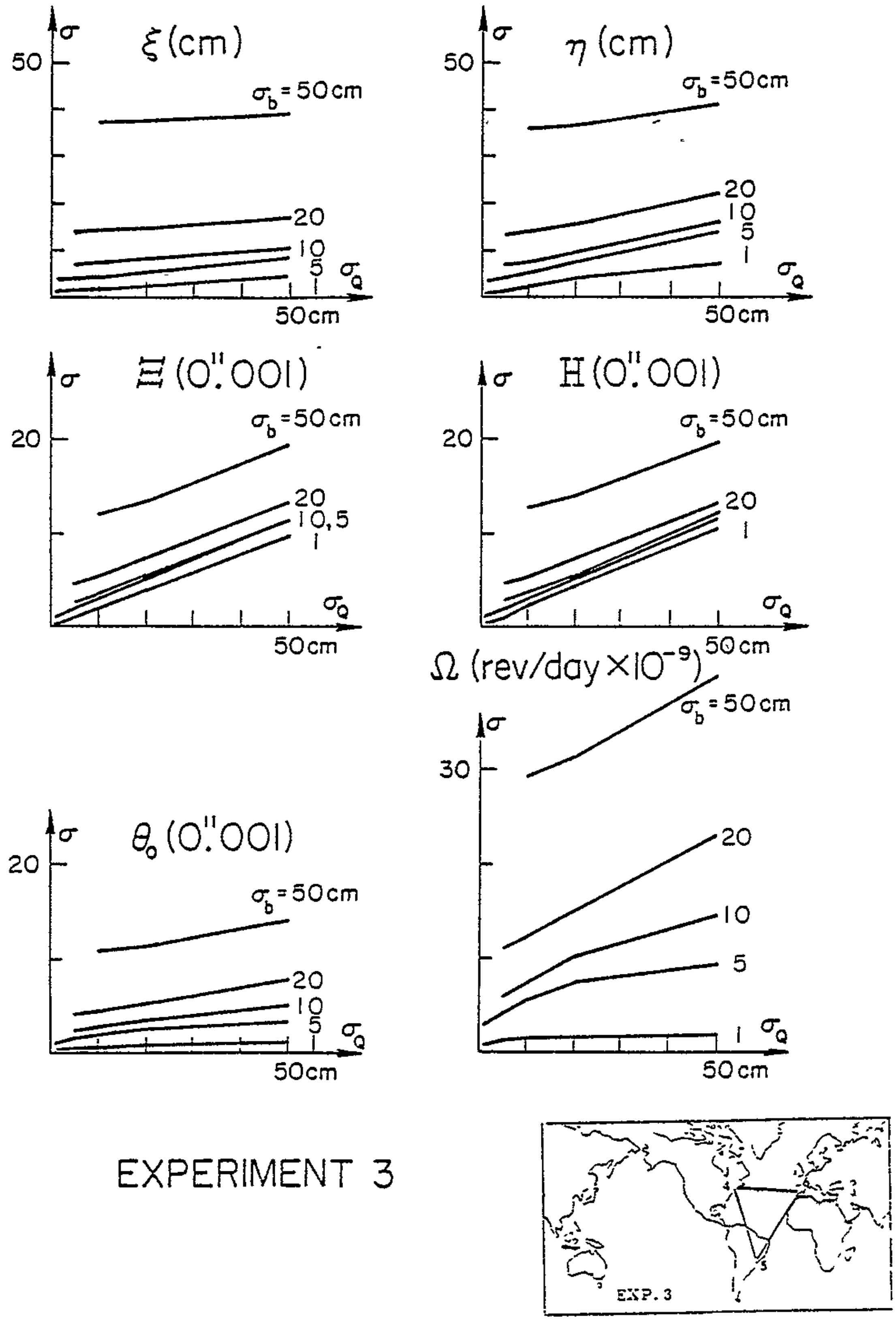
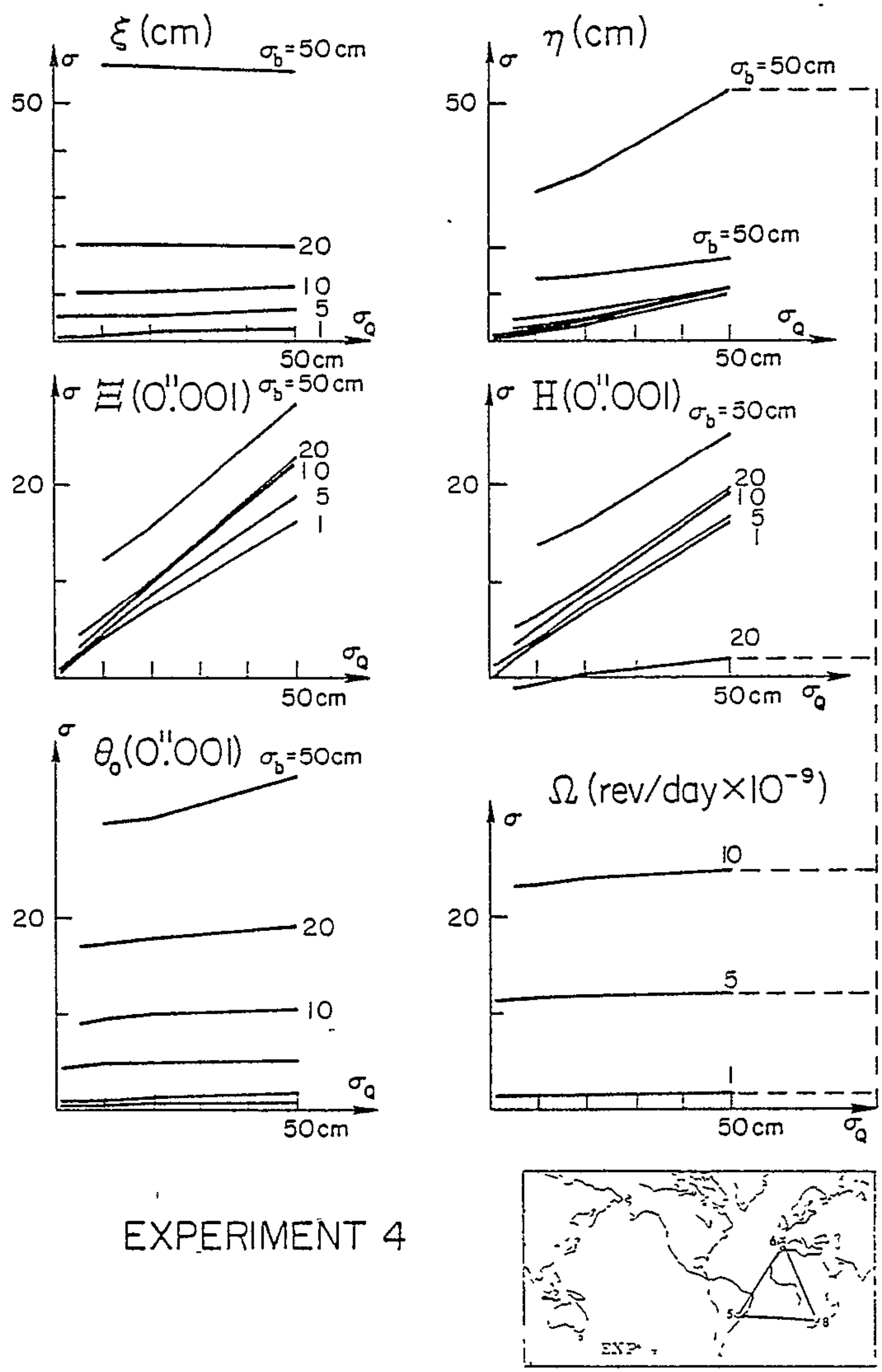

ξ (cm)
σ
50
σ_b=50cm
20
10
5
1
σ_Q
50cm
η (cm)
σ
50
σ_b=50cm
20
10
5
1
σ_Q
50cm
Ξ (0.″001)
σ
20
σ_b=50 cm
20
1
σ_Q
50cm
H (0.″001)
σ
20
σ_b=50cm
20
1
σ_Q
50cm
θ_0 (0.″001)
σ
10
σ_b=50cm
20
1
σ_Q
50cm
Ω (rev/day × 10^{-9})
σ
10
σ_b=50cm
20
1
σ_Q
50cm
EXPERIMENT 2
EXP. 2

Fig. 9.2 (cont'd)



σ
ξ (cm)
50
σ_b = 50cm
20
10
5
1
σ_Q
50cm
η (cm)
Ξ (0".001)
H (0".001)
θ_o (0".001)
10,5
Ω (rev/day $\times 10^{-9}$)
30
EXPERIMENT 3
EXP. 3

Fig. 9.2 (cont'd)



ξ (cm)
σ
σb = 50 cm
50
20
10
5
1
σQ
50cm
η (cm)
σb = 50cm
Ξ (0".001)
H (0".001)
θ0(0".001)
Ω (rev/day×10⁻⁹)
EXPERIMENT 4

Fig. 9.2 (cont'd)

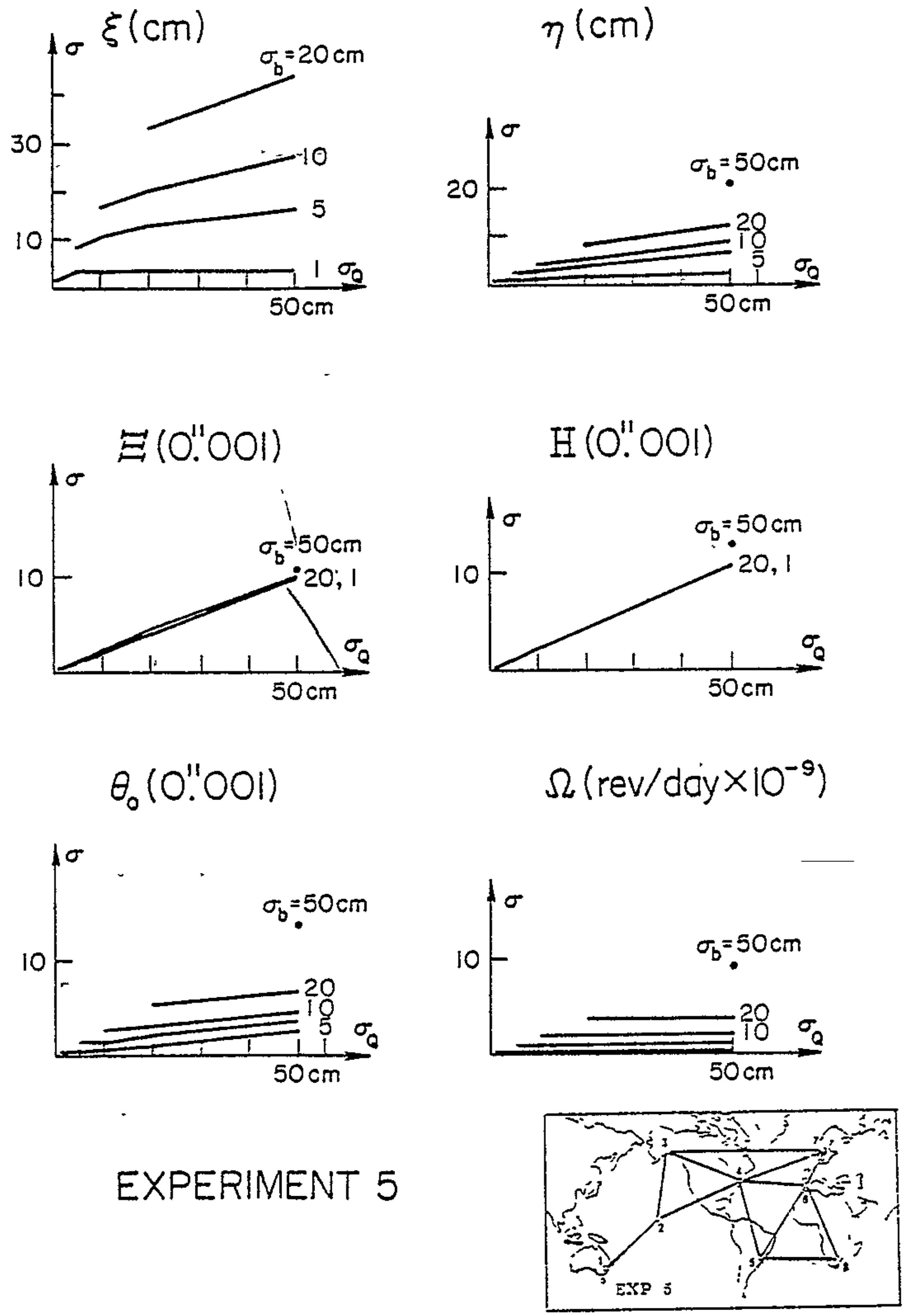

ξ (cm)
η (cm)
Ξ (0.″001)
H (0.″001)
θ₀ (0.″001)
Ω (rev/day × 10⁻⁹)
σ
σ_Q
σ_b = 20 cm
σ_b = 50 cm
50 cm
EXPERIMENT 5
EXP 5

Fig. 9.2 (cont'd)

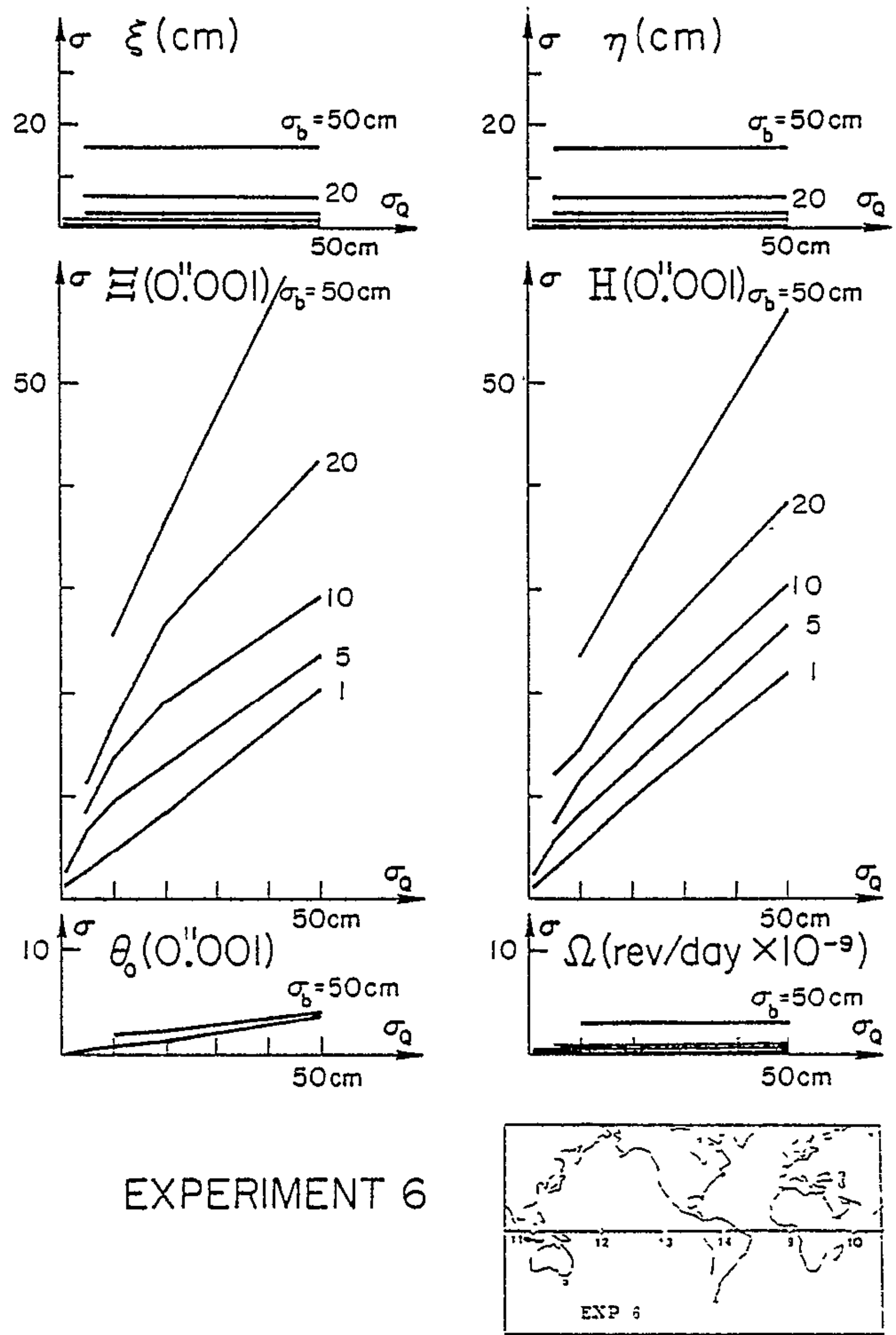

σ ξ(cm)
20
σb=50cm
20
σQ
50cm
σ η(cm)
20
σb=50cm
20
σQ
50cm
σ Ξ(0."001)
σb=50cm
50
20
10
5
1
σQ
50cm
σ Η(0."001)
σb=50cm
50
20
10
5
1
σQ
50cm
10
σ θ0(0."001)
σb=50cm
σQ
50cm
10
σ Ω(rev/day ×10⁻⁹)
σb=50cm
σQ
50cm
EXPERIMENT 6
EXP 6

Fig. 9.2 (cont'd)

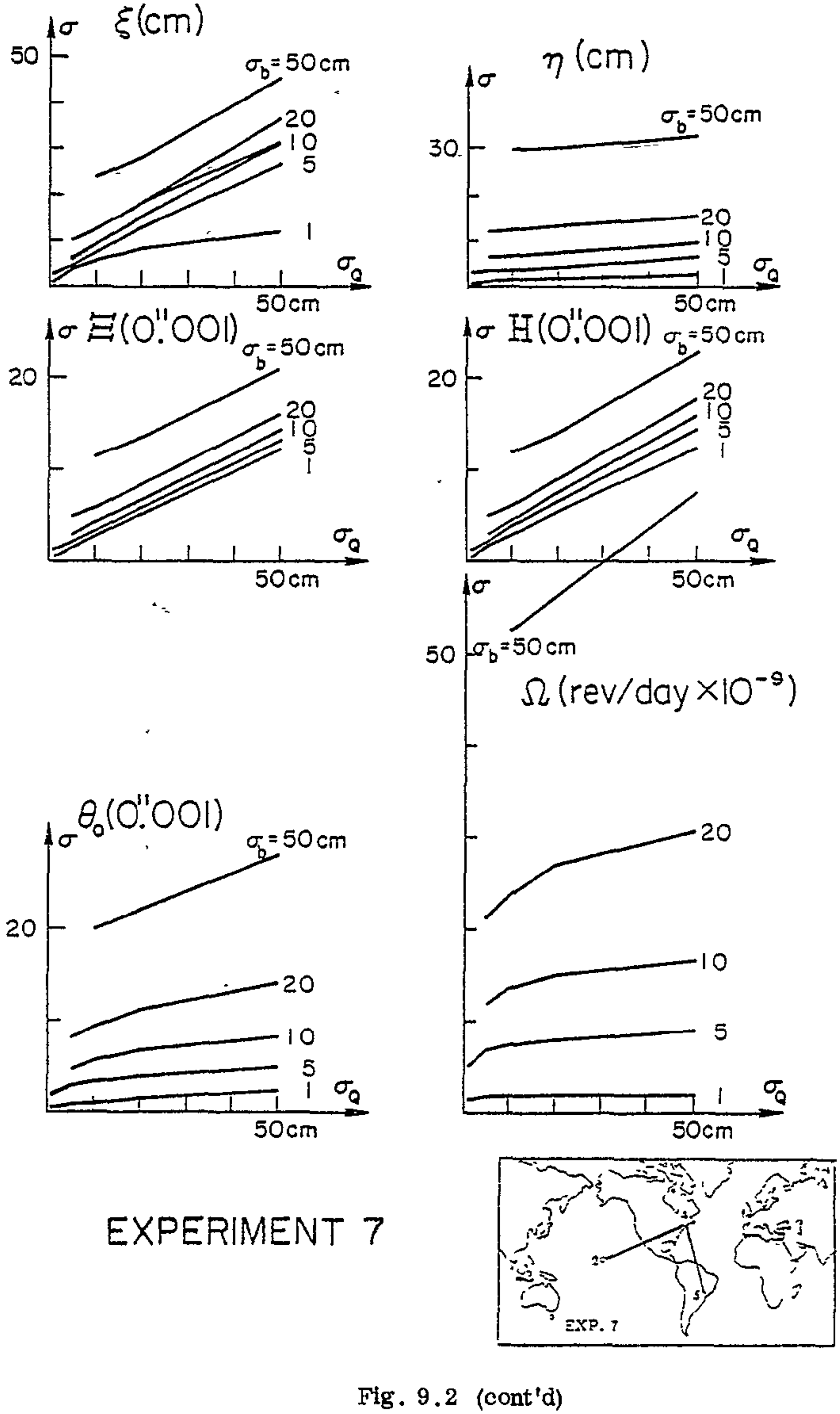

σ ξ(cm)
50
σb=50cm
20
10
5
1
σQ
50cm
σ η(cm)
30
σb=50cm
20
10
5
1
σQ
50cm
σ Ξ(0."001)
σb=50cm
20
20
10
5
1
σQ
50cm
σ H(0."001)
σb=50cm
20
20
10
5
1
σQ
50cm
σ
50
σb=50cm
Ω (rev/day ×10⁻⁹)
20
10
5
1
σQ
50cm
σ θo(0."001)
σb=50cm
20
20
10
5
1
σQ
50cm
EXPERIMENT 7
EXP. 7

Fig. 9.2 (cont'd)

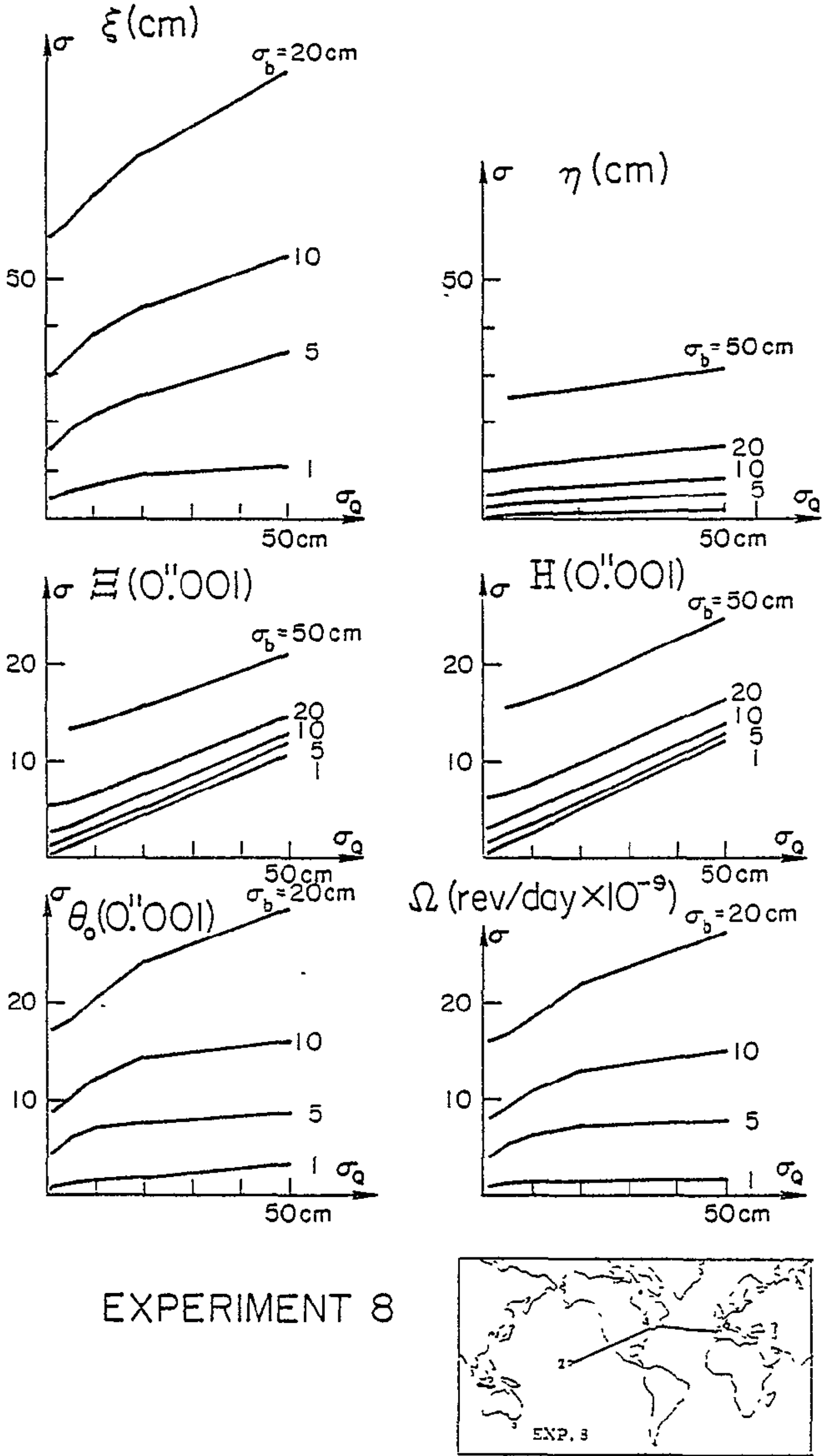

σ ξ (cm)
σb = 20 cm
10
5
1
50
σQ
50 cm
σ η (cm)
50
σb = 50 cm
20
10
5
1
σQ
50 cm
σ Ξ (0".001)
σb = 50 cm
20
10
20
10
5
1
σQ
50 cm
σ H (0".001)
σb = 50 cm
20
10
20
10
5
1
σQ
50 cm
σ θo (0".001)
σb = 20 cm
20
10
10
5
1
σQ
50 cm
Ω (rev/day × 10⁻⁹)
σb = 20 cm
σ
20
10
10
5
1
σQ
50 cm
EXPERIMENT 8
EXP. 8

Fig. 9.2 (cont'd)

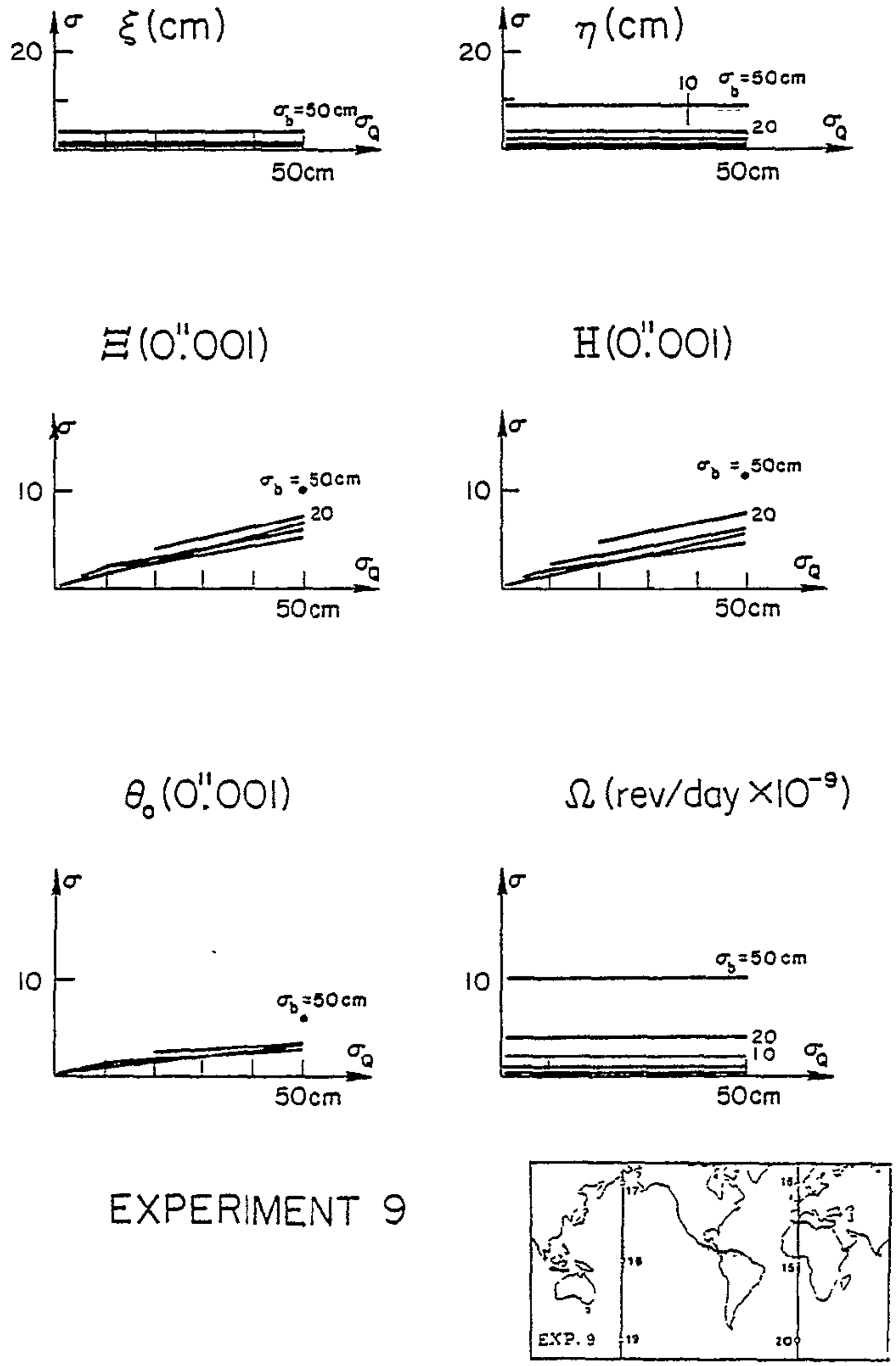

σ ξ (cm)
20
σb=50cm
σQ
50cm
σ η (cm)
20
10
σb=50cm
20
σQ
50cm
Ξ (0".001)
σ
10
σb = 50cm
20
σQ
50cm
H (0".001)
σ
10
σb = 50cm
20
σQ
50cm
θ0 (0".001)
σ
10
σb=50cm
σQ
50cm
Ω (rev/day ×10^-9)
σ
10
σb=50 cm
20
10
σQ
50cm
EXPERIMENT 9
EXP. 9

Fig. 9.2 (cont'd)

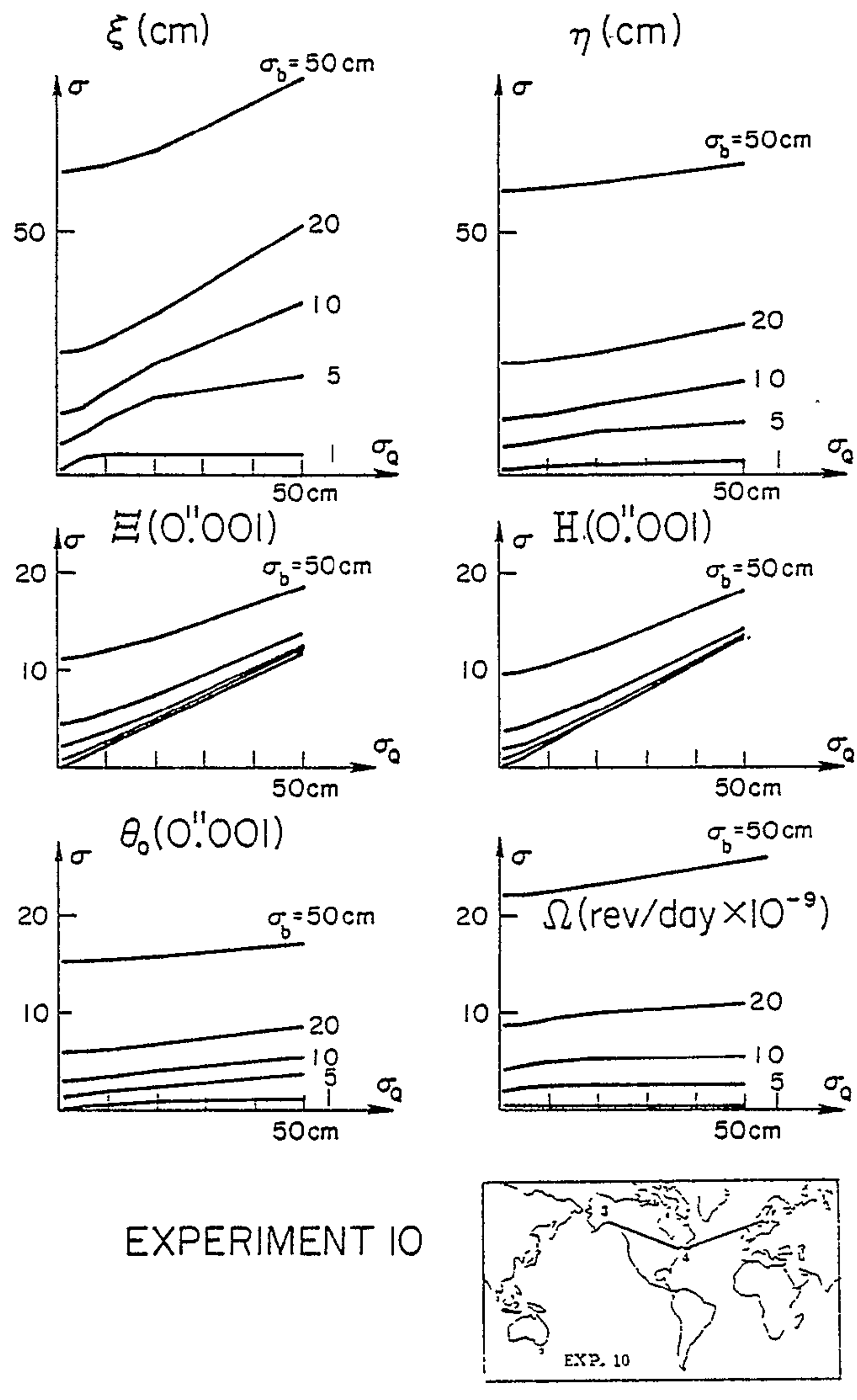

ξ (cm)
σ
σb = 50 cm
50
20
10
5
1
σQ
50cm
η (cm)
σ
σb = 50cm
50
20
10
5
1
σQ
50cm
Ξ (0."001)
σ
20
10
σb = 50cm
σQ
50cm
H (0."001)
σ
20
10
σb = 50cm
σQ
50cm
θ0 (0."001)
σ
20
10
σb = 50cm
20
10
5
1
σQ
50cm
σ
σb = 50cm
20
10
Ω (rev/day × 10^-9)
20
10
5
σQ
50cm
EXPERIMENT 10
EXP. 10

Fig. 9.2 (cont'd)

Fig. 9.3 Recovery of baseline lengths and angles, Experiments 1 - 10

Total time span of observations: T = 1 day

Earth step: Δt = 1 day

For explanation of other symbols, see Fig. 9.2, p. 40.

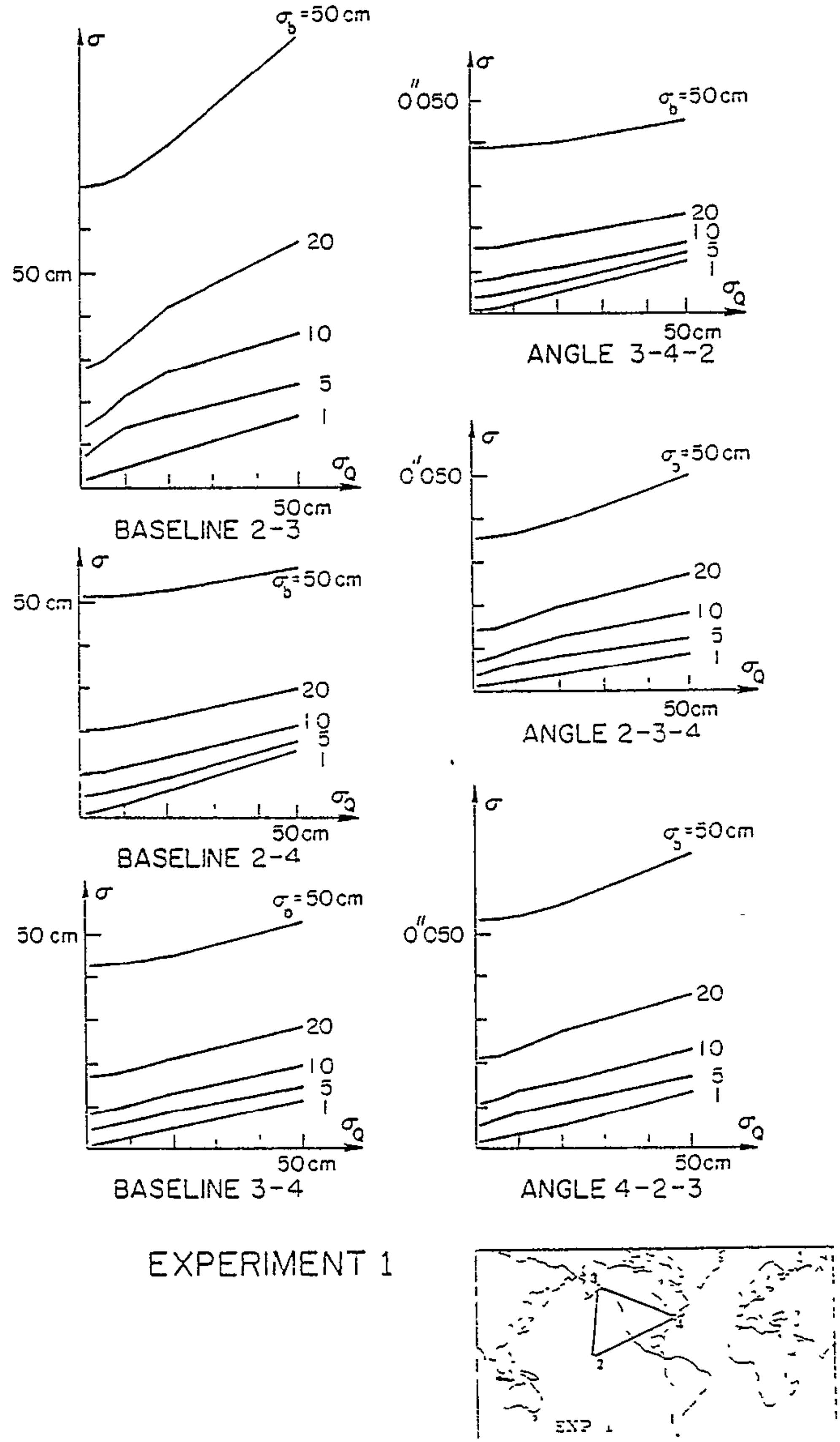


Fig. 9.3 Recovery of baseline lengths and angles.

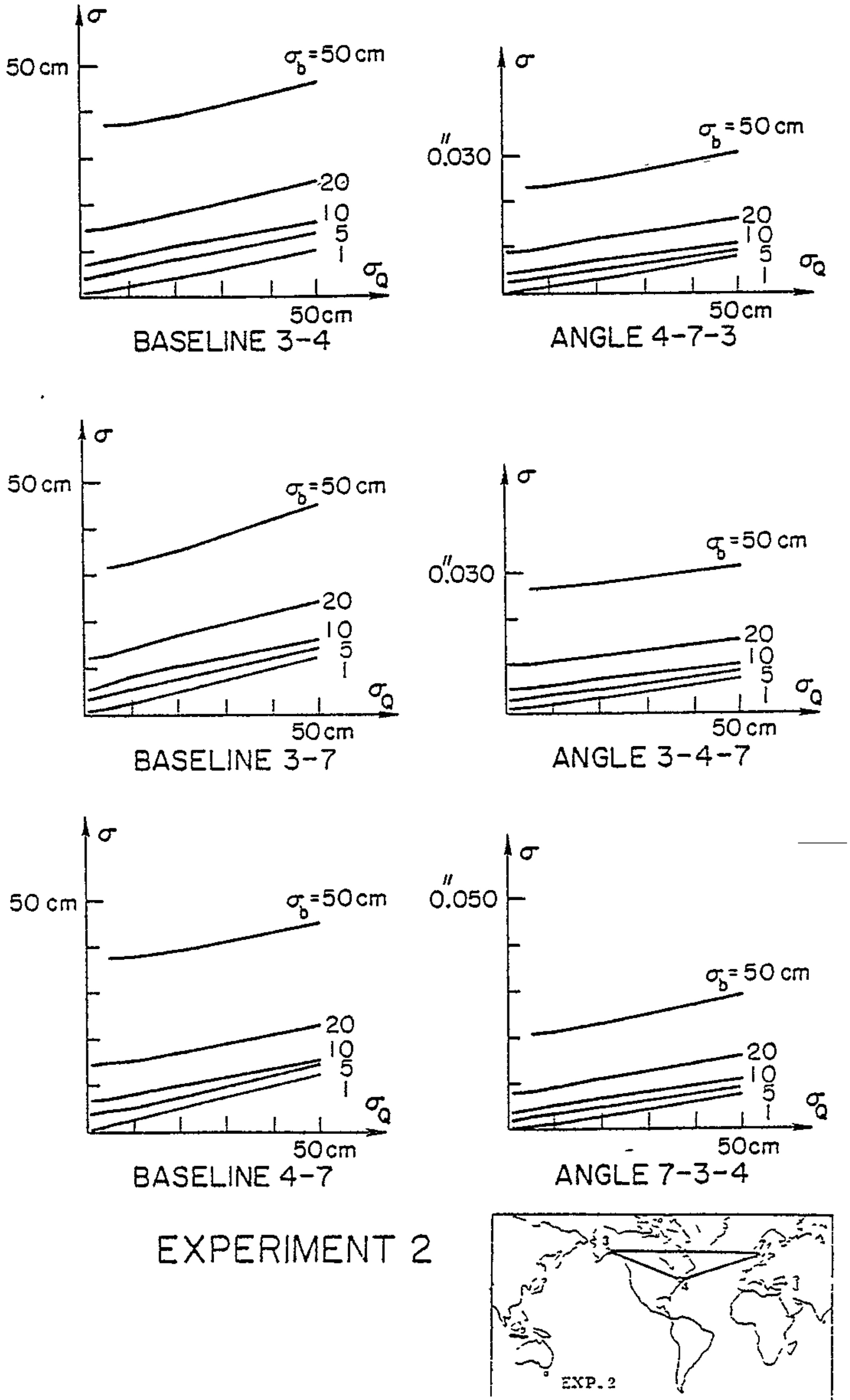
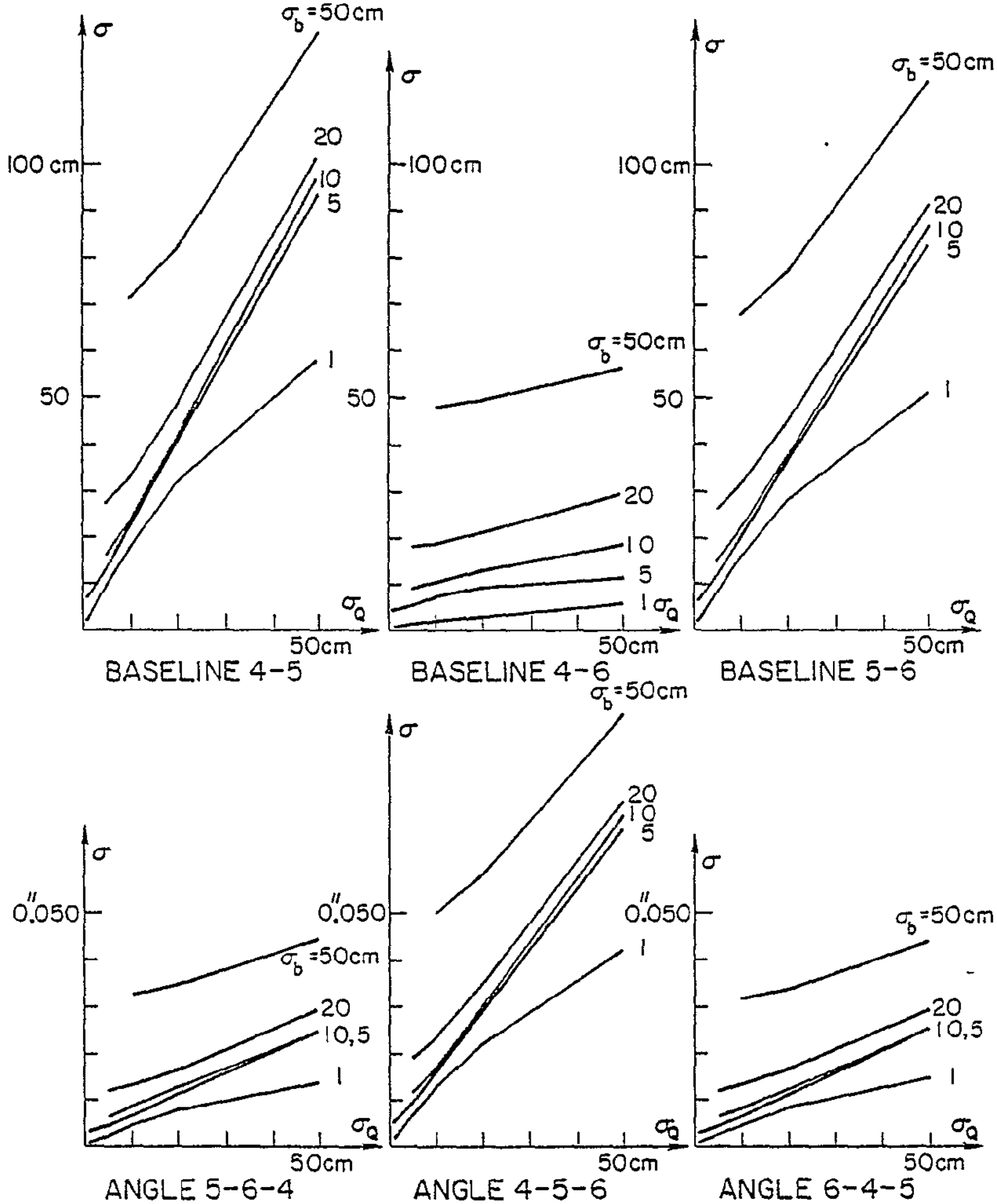

σ
50 cm
σ_b = 50 cm
20
10
5
1
σ_Q
50cm
BASELINE 3-4
σ
0.″030
σ_b = 50 cm
20
10
5
1
σ_Q
50cm
ANGLE 4-7-3
σ
50 cm
σ_b = 50 cm
20
10
5
1
σ_Q
50 cm
BASELINE 3-7
σ
0.″030
σ_b = 50 cm
20
10
5
1
σ_Q
50 cm
ANGLE 3-4-7
σ
50 cm
σ_b = 50 cm
20
10
5
1
σ_Q
50cm
BASELINE 4-7
σ
0.″050
σ_b = 50 cm
20
10
5
1
σ_Q
50cm
ANGLE 7-3-4
EXPERIMENT 2
3
4
7
EXP. 2

Fig. 9.3 (cont'd)



EXPERIMENT 3

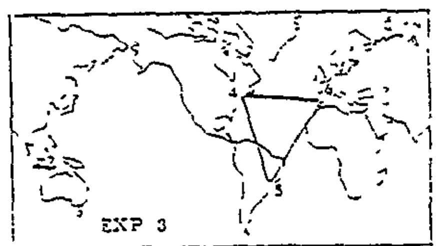
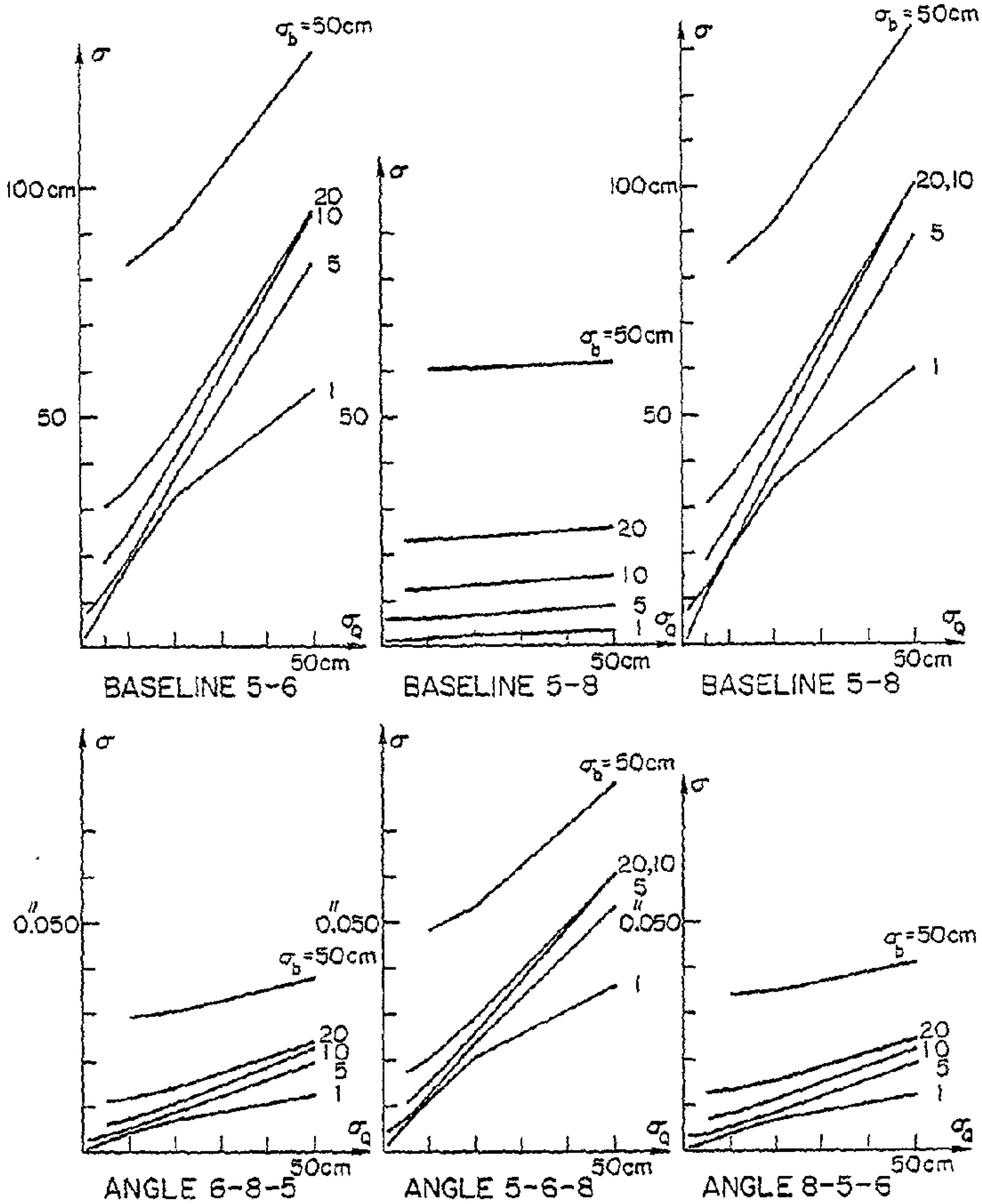


Fig. 9.3 (cont'd)



EXPERIMENT 4

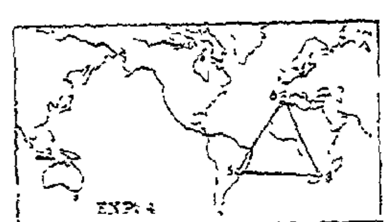
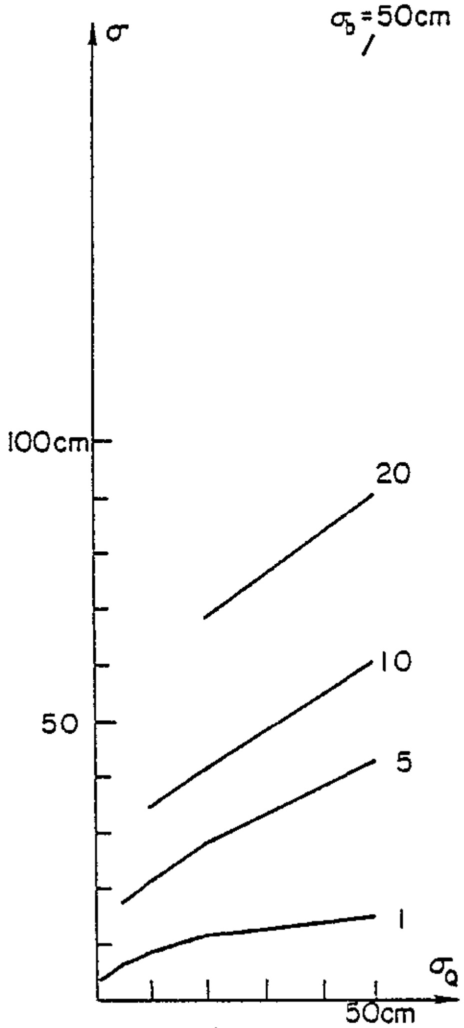
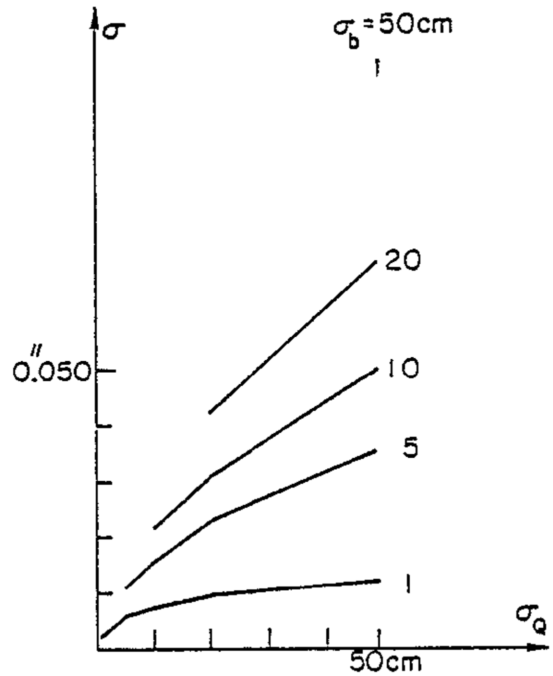


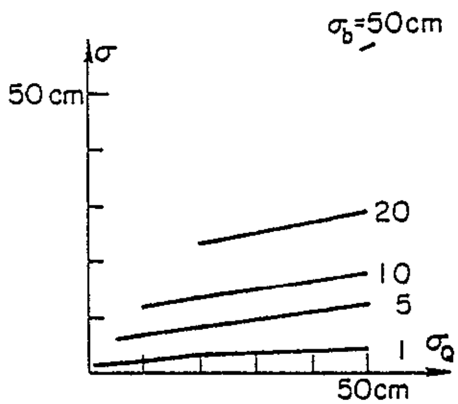
Fig. 9.3 (cont'd)



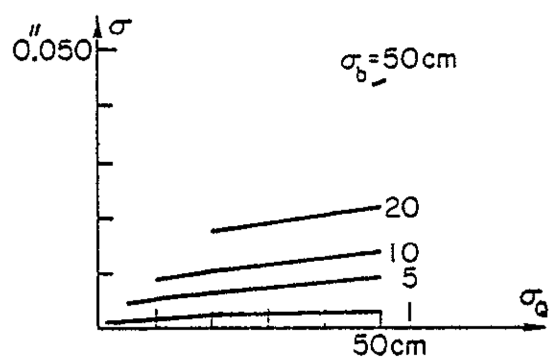
BASELINE 1-2 (WORST)



ANGLE 6-8-4 (WORST)



BASELINE 4-7 (BEST)



ANGLE 3-4-7 (BEST)

EXPERIMENT 5

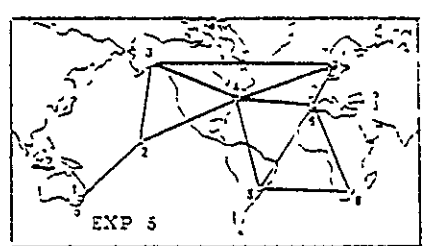
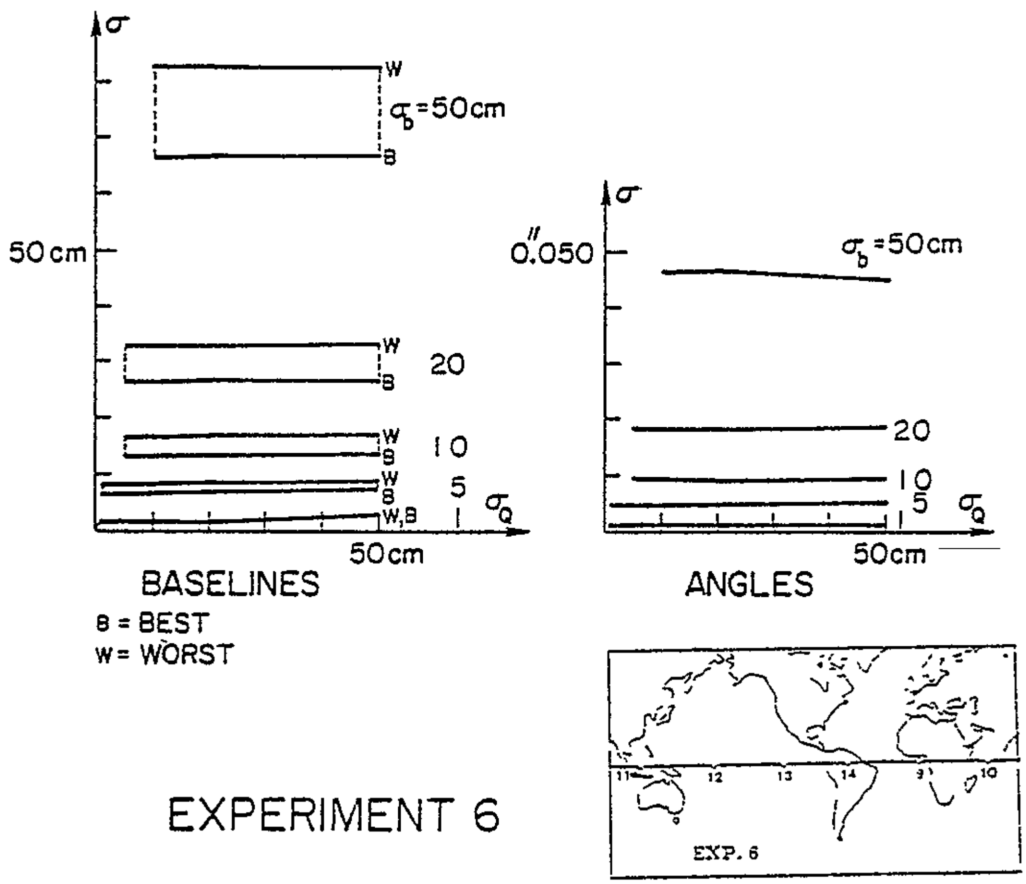
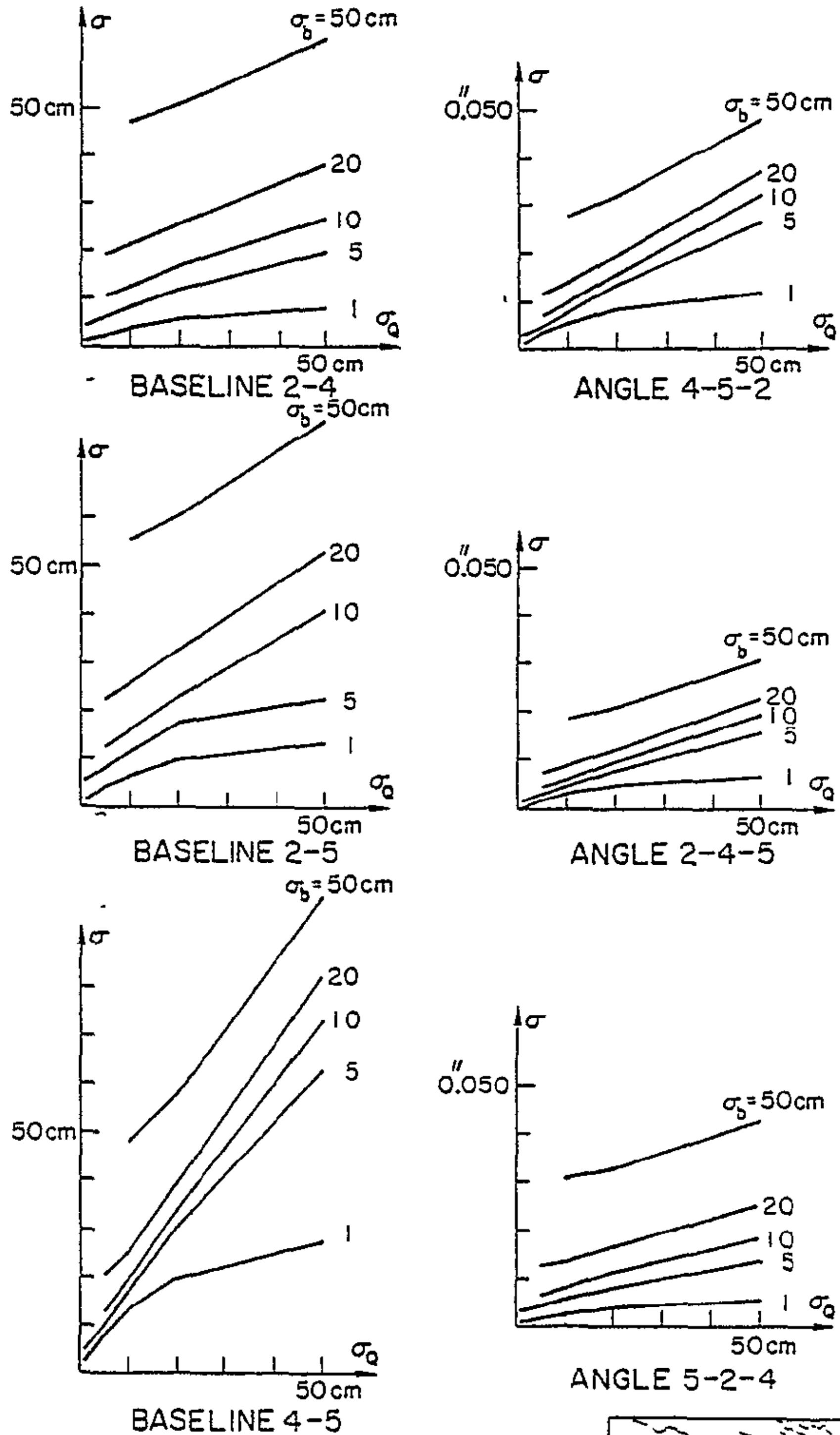


Fig. 9.3 (cont'd)



σ
W
σ_b=50cm
B
50 cm
W
20
B
W
10
B
W
5
B
W,B
1
σ_Q
50cm
BASELINES
B = BEST
W = WORST
σ
0.″050
σ_b=50cm
20
10
5
1
σ_Q
50cm
ANGLES
11
12
13
14
9
10
EXP. 6
EXPERIMENT 6

Fig. 9.3 (cont'd)



EXPERIMENT 7

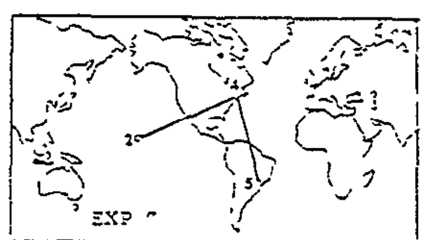
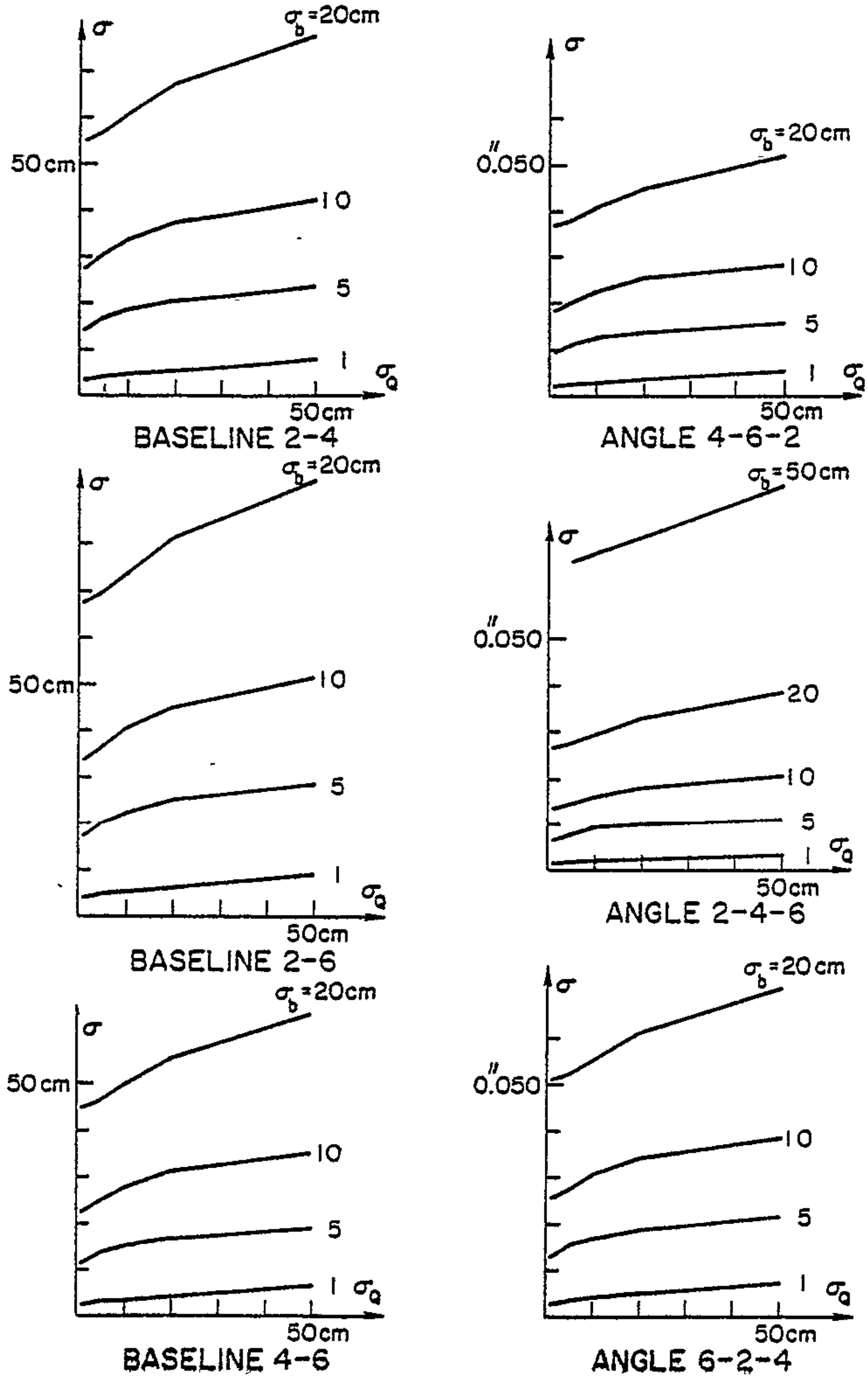


Fig. 9.3 (cont'd)



EXPERIMENT 8

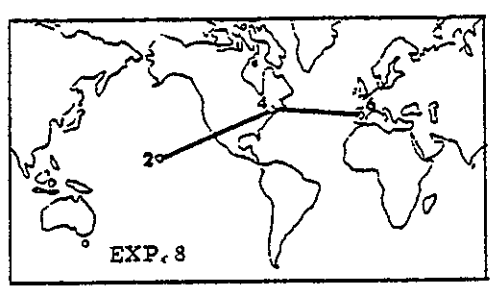
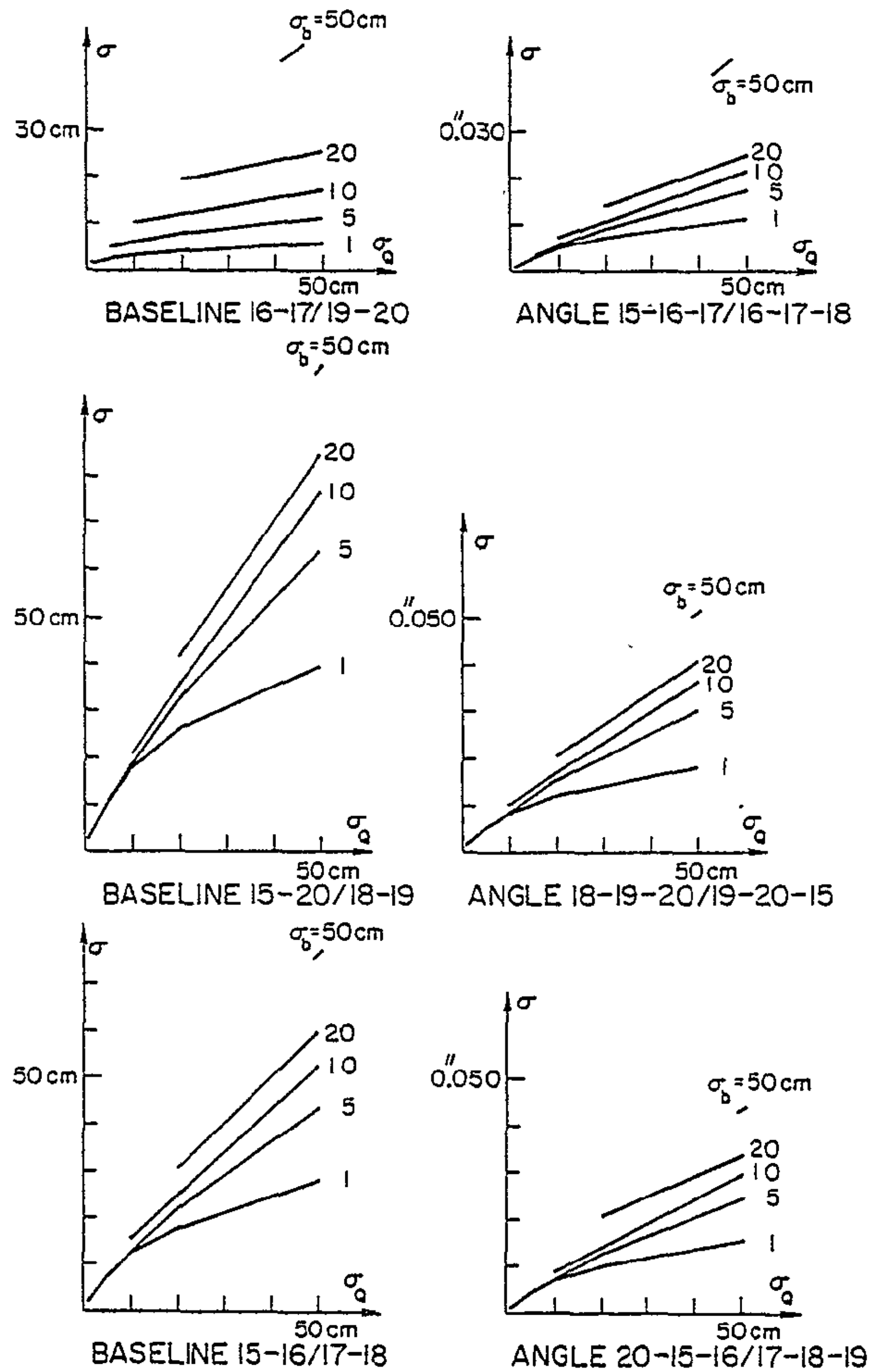


Fig. 9.3 (cont'd)



EXPERIMENT 9

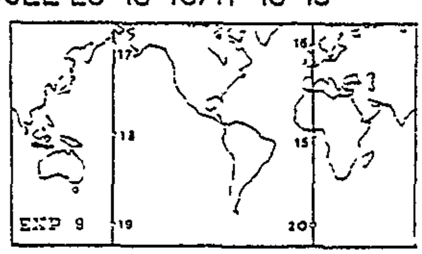


Fig. 9.3 (cont'd)

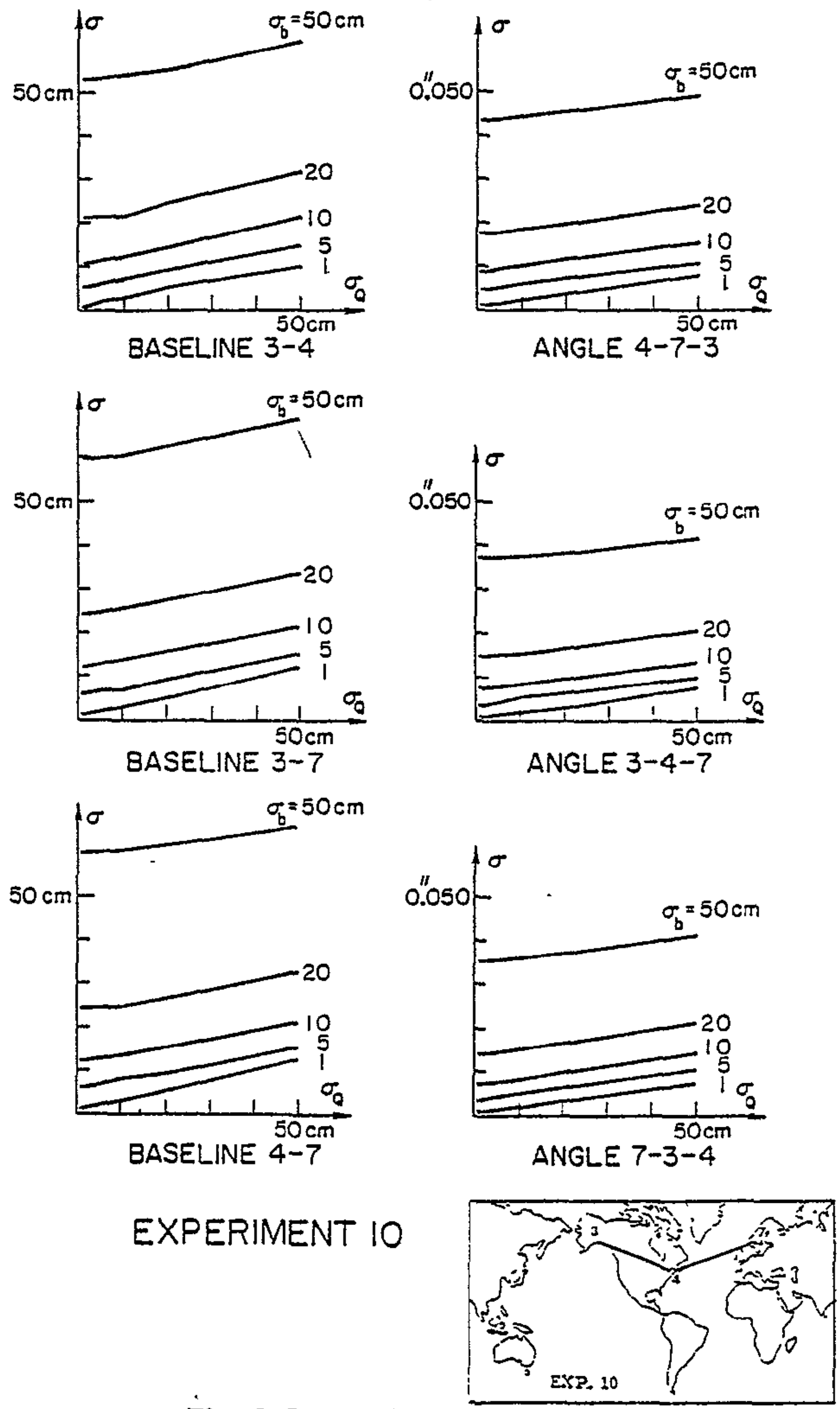

σ
σb = 50cm
50cm
20
10
5
1
σQ
50cm
BASELINE 3-4
σ
0.″050
σb = 50cm
20
10
5
1
σQ
50 cm
ANGLE 4-7-3
σ
σb = 50cm
50cm
20
10
5
1
σQ
50cm
BASELINE 3-7
σ
0.″050
σb = 50cm
20
10
5
1
σQ
50cm
ANGLE 3-4-7
σ
σb = 50cm
50 cm
20
10
5
1
σQ
50cm
BASELINE 4-7
σ
0.″050
σb = 50cm
20
10
5
1
σQ
50cm
ANGLE 7-3-4
EXPERIMENT 10
EXP. 10

Fig. 9.3 (cont'd)

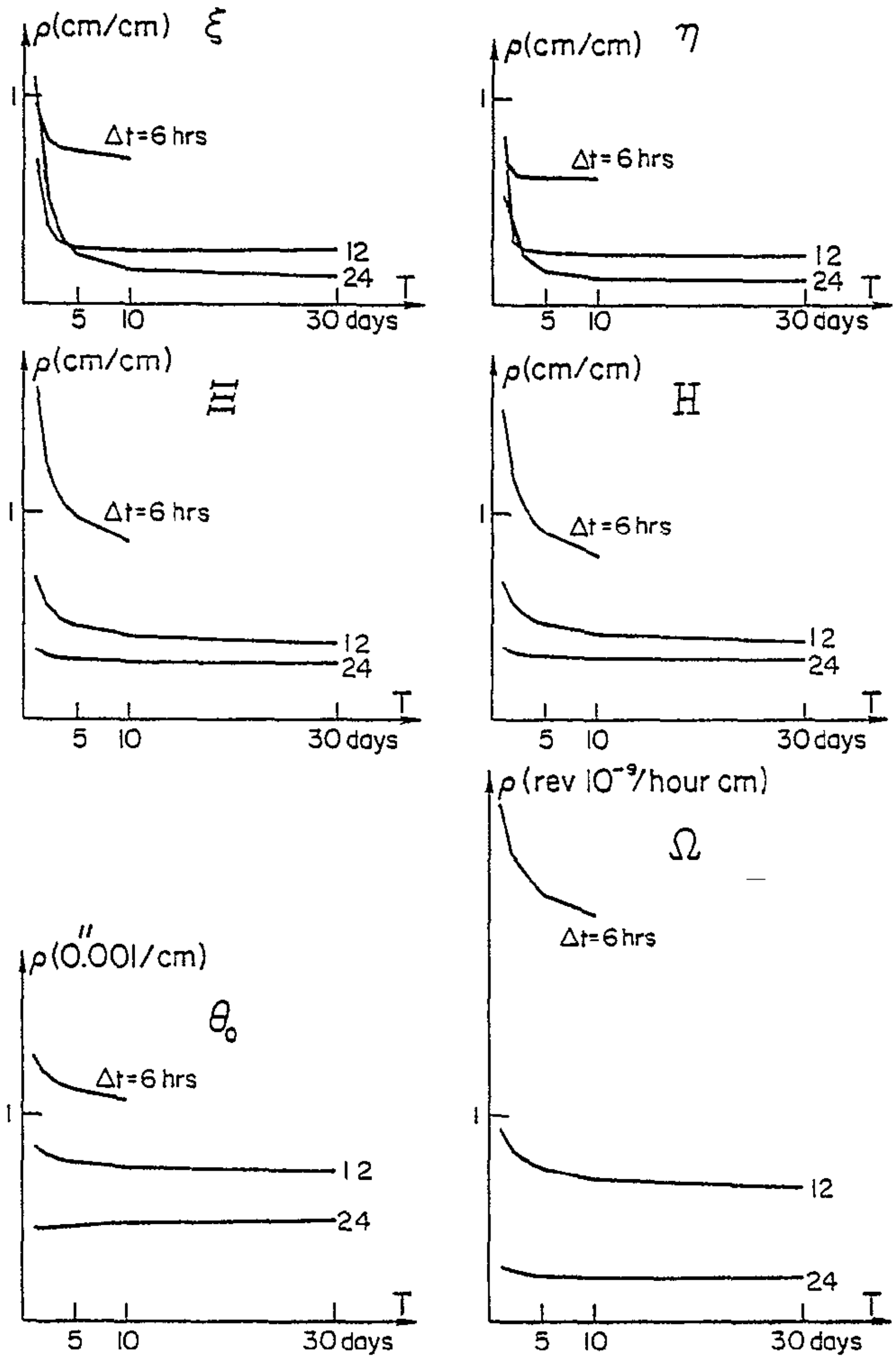


Fig. 9.4 Variation of earth rotation parameter recovery with stepsize and the total time interval of observations (Experiment 2).

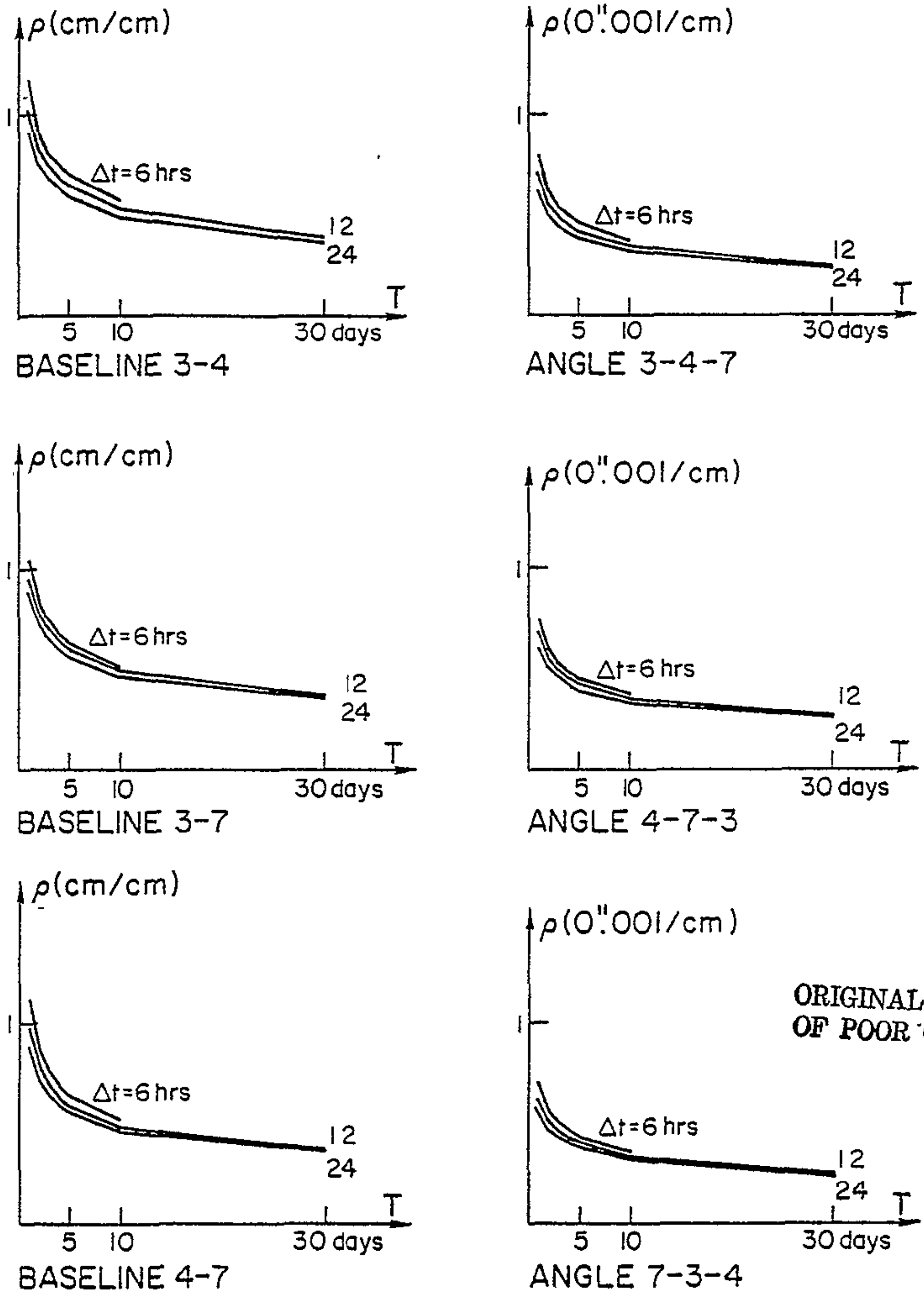


Fig. 9.5 Variation of baseline length and angle recovery with stepsize and the total time interval of observations (Experiment 2).

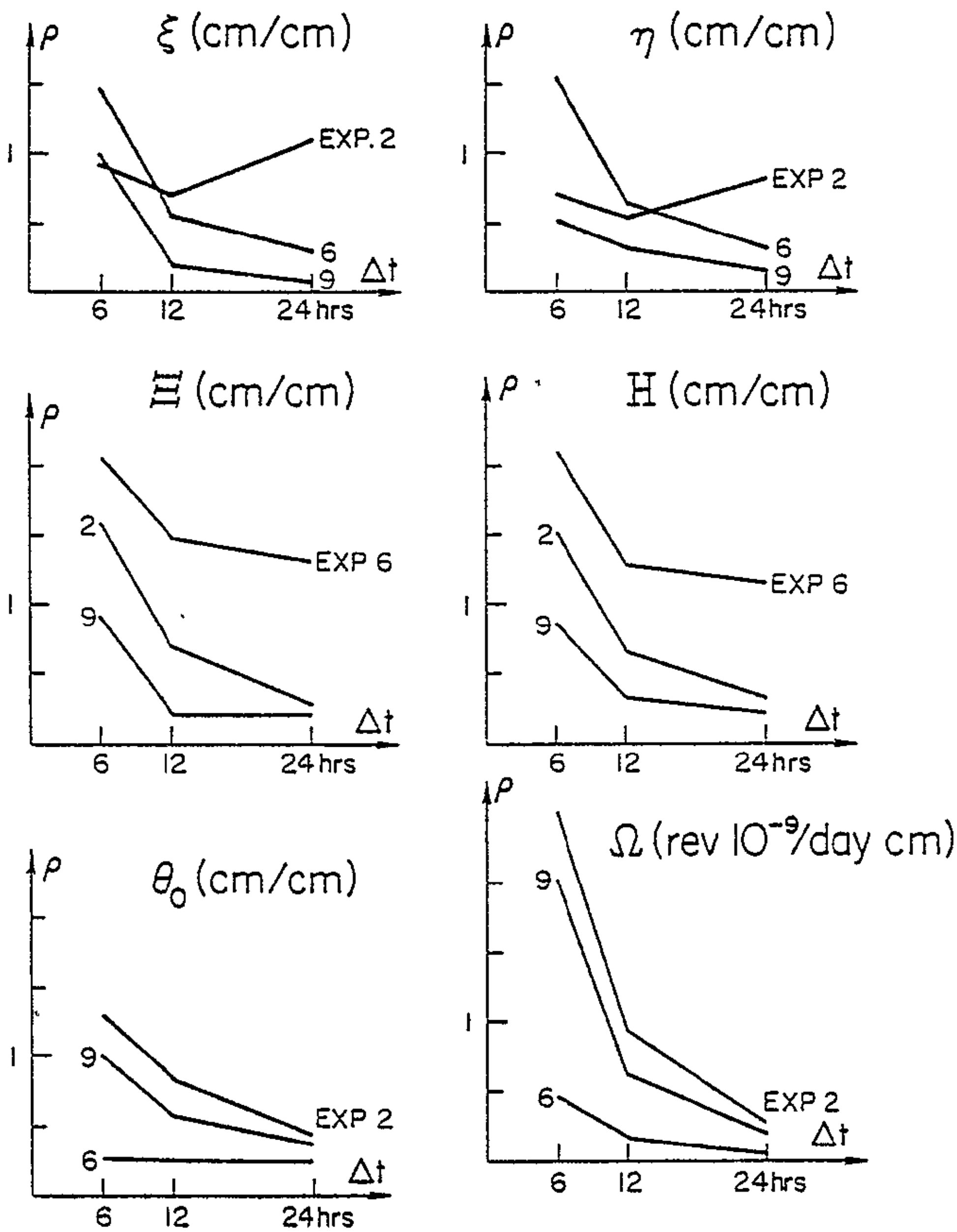


Fig. 9.6 Variation of earth rotation parameter recovery with earth step.

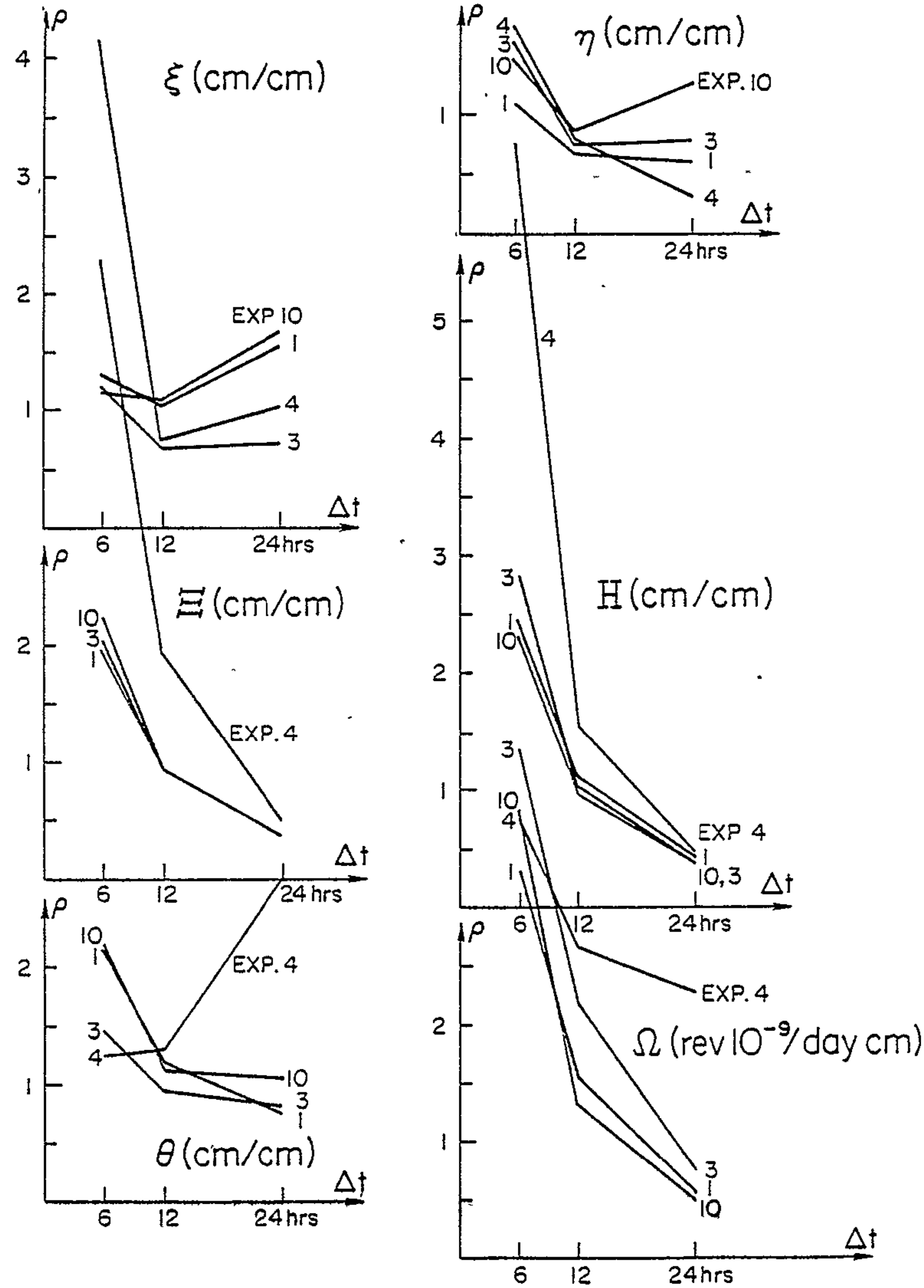

ξ (cm/cm)
EXP 10
4
3
1
Δt
6
12
24hrs
Ξ (cm/cm)
10
3
1
EXP. 4
θ (cm/cm)
EXP. 4
η (cm/cm)
EXP. 10
H (cm/cm)
EXP 4
10,3
Ω (rev 10⁻⁹/day cm)
EXP. 4
ρ

Fig. 9.6 (cont'd)

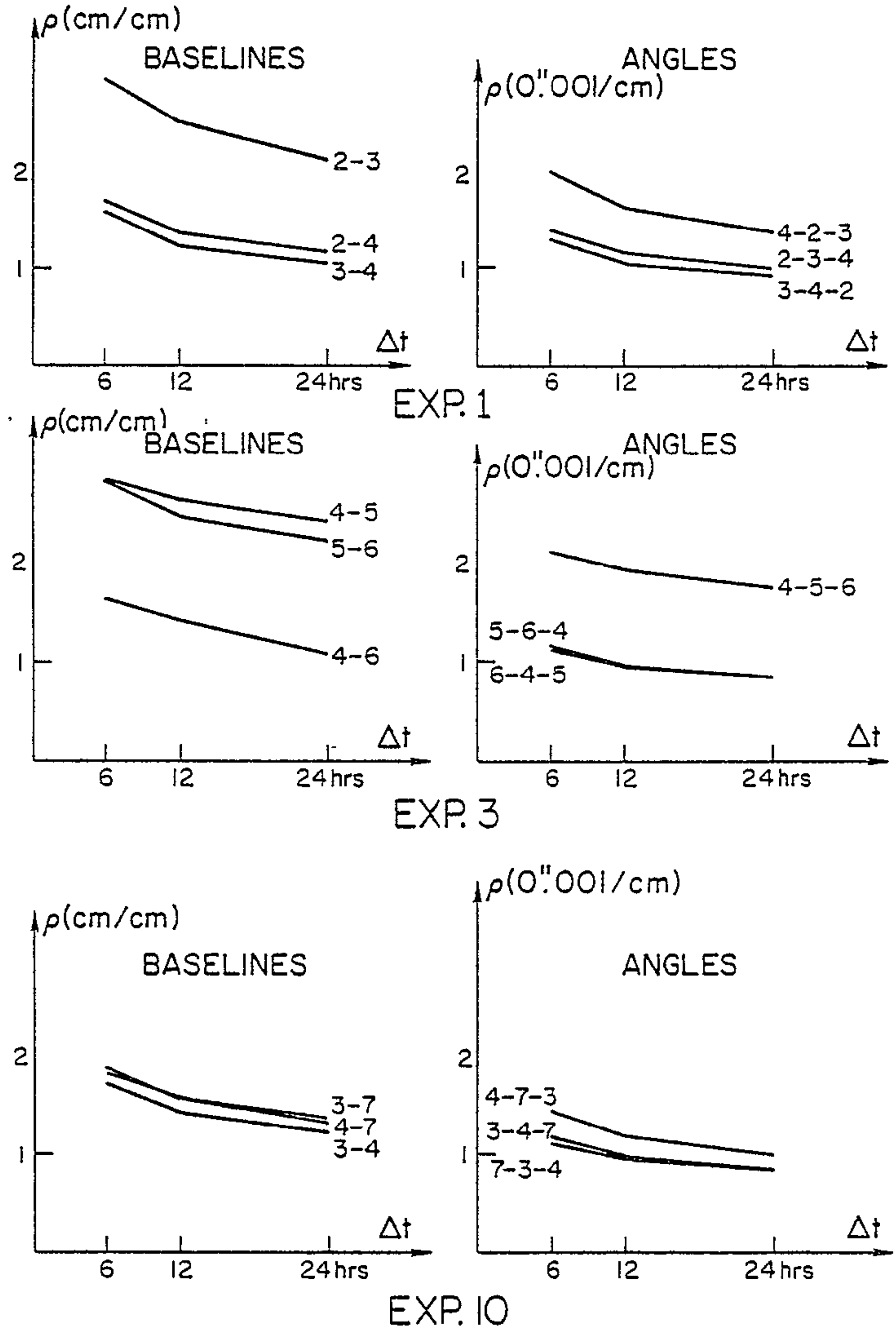
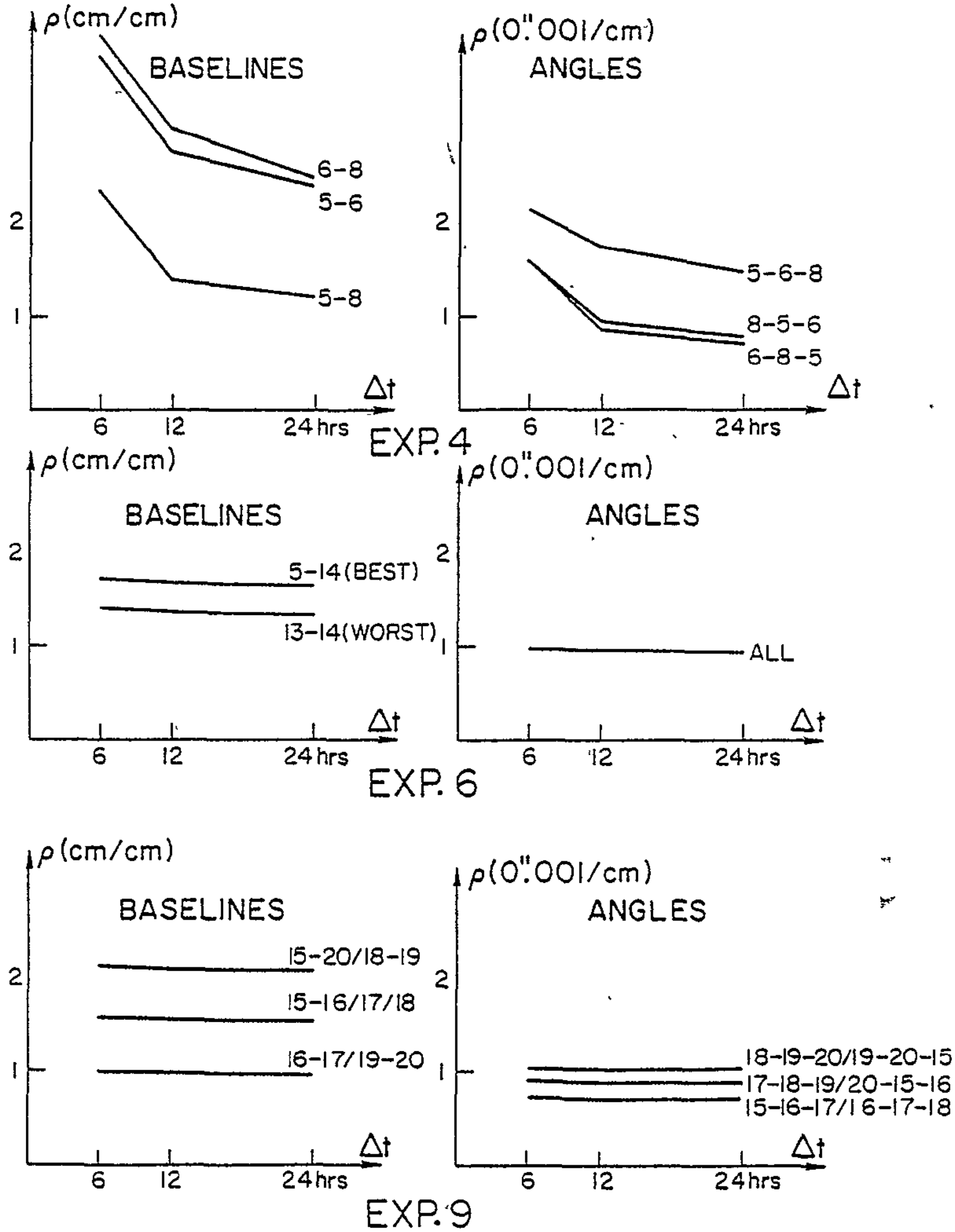


Fig. 9.7 Variation of baseline length and angle recovery with earth step.



ρ(cm/cm)
BASELINES
6-8
5-6
5-8
2
1
Δt
6
12
24 hrs
ρ(0."001/cm)
ANGLES
5-6-8
8-5-6
6-8-5
2
1
Δt
6
12
24 hrs
EXP. 4
ρ(cm/cm)
BASELINES
5-14 (BEST)
13-14 (WORST)
2
1
Δt
6
12
24 hrs
ρ(0."001/cm)
ANGLES
ALL
2
1
Δt
6
12
24 hrs
EXP. 6
ρ(cm/cm)
BASELINES
15-20/18-19
15-16/17/18
16-17/19-20
2
1
Δt
6
12
24 hrs
ρ(0."001/cm)
ANGLES
18-19-20/19-20-15
17-18-19/20-15-16
15-16-17/16-17-18
2
1
Δt
6
12
24 hrs
EXP. 9

Fig. 9.7 (cont'd)

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APPENDIX A

Inner Constraints and Their Relation to the "Geometry" of the Pseudoinverse of an Operator

Suppose that a system of linear equations

$$\underset{u \times u}{N} \quad \underset{u \times 1}{x} = \underset{u \times 1}{y}$$

is given (N, y known; x unknown) which is consistent (i.e., it has at least one solution), and furthermore it has more than one solution. The objective is to show that there exists a set of linear constraints

$$E^T x = 0$$

called inner constraints, such that the solution to the simultaneous system of equations $N x = y$ and $E^T x = 0$ is unique and identical to the solution $x_0 = N^+ y$, where N^+ is the pseudoinverse of the matrix N.

The matrix N is a representation of an operator $N: X \rightarrow Y$, where X and Y are both u-dimensional inner product spaces with elements $u \times 1$ vectors and the usual Euclidean inner product

$$< f, g > = g^T f \qquad f, g \in X \quad \text{or} \quad f, g \in Y$$

Two subspaces can be introduced with respect to the operator N, the range $R(N) = R$ of N defined as

$$R = [y; \; y \in Y \; \text{ and } \; y = Nx \; \text{ for some } \; x \in X], \qquad R \subset Y$$

and the kernel or null space $K(N) = K \subset X$ of N,

$$K = [x; \; x \in X \; \text{ and } Nx = 0]$$

The consistency of the linear equations $Nx = y$ refers to the fact that $y \in R$, while the non-uniqueness of solution to the fact that $K \cap \{0\}^c$ is non-empty, i.e., K has other elements in addition to 0. If x' is a solution to $Nx = y$, then $x = x' + \bar{x}$ is also a solution where $\bar{x}$ is any element of K. We can therefore identify the solution space $S_y \subset X$, defined as

$$S_y = [x;\ x \in X \quad \text{and} \quad Nx = y,\ y \in R \text{ fixed}]$$

with a linear variety

$$S_y = [x;\ x = x' + \bar{x},\ \bar{x} \in K, \quad x' \text{ fixed with } Nx' = y]$$

If $K^{\perp}$ is the orthogonal complement of K with respect to X, the projection $x_0 = \mathcal{P}_{K^{\perp}}(x)$ of any element $x \in S_y$ on $K^{\perp}$ is unique, and furthermore [Dermanis, 1977, §3.5]

$$\| x_0 \| = \min_{x \in S_y} \| x \|$$

We can now define the pseudoinverse N^+ of the operator N as an operator N^+: $R \rightarrow X$ where

$$N^+ y = x_0 = \mathcal{P}_{K^{\perp}}(x) \qquad \text{for any } y \in R \text{ and } x \in S_y$$

N^+ is therefore the ordinary inverse of the restriction of the operator N to $K^{\perp}$ (see [Desoer and Whalen, 1963, p. 444] for a more rigorous definition). The domain of definition of N^+ can be extended to the whole space Y (N^+: $Y \rightarrow X$) by setting

$$N^+(y) = N^+(y')$$

where $y' = \mathcal{P}_R(y)$ is the projection of $y \in Y$ on R.

The range of N^+ is $K^{\perp}$, while its kernel (null) space is $R^{\perp}$. For $y \in R$ we have $x_0 = N^+ y$ and x_0 can be alternatively defined as the unique element in the intersection $S_y \cap K^{\perp}$. This means that x_0 is uniquely defined by its two properties:

$$x_0 \in S_y \qquad \text{and} \qquad x_0 \in K^{\perp}$$

The first property is expressed by the original equations $Nx = y$, while a similar expression must be established for the second property. Let $r = \text{rank}(N) < u$ and $s = u - r$ be the rank deficiency of N. The dimension of the linear subspace $K \subset X$ is then s, and suppose that $\{e_i\}$, $i = 1, 2, \ldots, s$ is a basis in K. Since $x_0 \in K^{\perp}$, we have $x_0 \perp e_i$ for $i = 1, 2, \ldots, s$, i.e.,

$$\langle x_0, e_i \rangle = e_i^T x_0 = 0 \qquad \text{for } i = 1, 2, \ldots, s$$

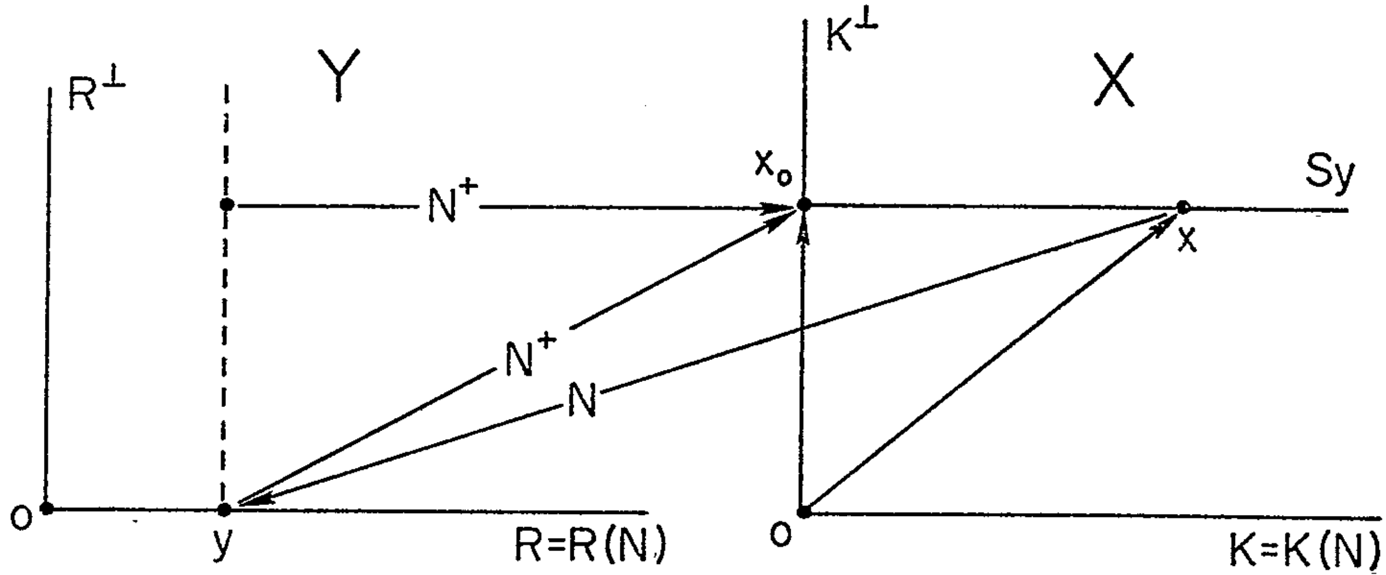

Fig. A.1 The geometry of the pseudoinverse operator.

Setting

$$\underset{u \times s}{E} = [e_1 \ e_2 \ \dots \ e_s]$$

the above equations can be written in the compact matrix form

$$E^T x_0 = 0$$

Therefore, x_0 is uniquely determined as the vector satisfying both equations

$$Nx = y \quad \text{and} \quad E^T x = 0$$

where the columns of E constitute a basis in K, the kernel (null space) of N. The constraints $E^T x = 0$ are referred to as the "inner" constraints.

The problem has now been reduced to that of finding a basis in K, i.e., of finding a set of s linearly independent u × 1 vectors $\{e_i\}$ satisfying

$$N e_i = 0 \qquad \text{for } i = 1, 2, \dots, s$$

The equations above can be written again in matrix form

$$N E = 0 \quad \text{where} \quad E = [e_1 \ e_2 \ \dots \ e_s]$$

We therefore only need to find a u × s matrix E with rank (E) = s = u - rank (N) (to secure linear independence of its columns e_i) satisfying NE = 0. Summarizing, we have the following theorem

> Given the system of linear equations
>
> $$\underset{u \times u \times 1}{N x} = \underset{u \times 1}{y} \qquad \text{(N, y known; x unknown)}$$
>
> where rank (N) = r < u, s = u - r, and a u × s matrix E with rank (E) = s satisfying N E = 0, the vector x satisfying simultaneously N x = y and $E^T x = 0$ is unique and identical to $x = N^+ y$, where N^+ is the pseudoinverse of N.

References

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APPENDIX B

Algorithm for First-Order Partitioned Linear Regression Including Inner Constraints

The problem in question is the construction of an algorithm for the least squares adjustment of a group of linearized observation equations of the form

$$V_i = \ddot{A}_i \ddot{X}_i + \dot{A}_i \dot{X} + L_i \qquad i = 1, 2, \ldots, m \qquad \text{(A 1)}$$

with $E\{V_i\} = 0$ and $E\{V_i V_i^T\} = P_i$. $\dot{X}$ refers to parameters common to all sets of observations, while $\ddot{X}_i$ refers to those appearing only in the i^{th} set of observations. Each set of observation equations gives rise to a contribution to the normal equations of the form

$$\begin{bmatrix} \ddot{N}_i & \overline{N}_i \\ \overline{N}_i^T & \dot{N}_i \end{bmatrix} \begin{bmatrix} \ddot{X}_i \\ \dot{X} \end{bmatrix} + \begin{bmatrix} \ddot{U}_i \\ \dot{U}_i \end{bmatrix} = 0 \qquad \text{(A 2)}$$

where

$$\ddot{N}_i = \ddot{A}_i^T P_i \ddot{A}_i \qquad \ddot{U}_i = \ddot{A}_i^T P_i L_i$$

$$\dot{N}_i = \dot{A}_i^T P_i \dot{A}_i \qquad \dot{U}_i = \dot{A}_i^T P_i L_i$$

and

$$\overline{N}_i = \ddot{A}_i^T P_i A_i$$

The combination of such individual sets of normal equations leads to the final set of normal equations incorporating all observations

$$\begin{bmatrix} \ddot{N} & \overline{N} \\ \overline{N}^T & \dot{N} \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \dot{X} \end{bmatrix} + \begin{bmatrix} \ddot{U} \\ \dot{U} \end{bmatrix} = 0 \qquad \text{(A 3)}$$

where

$$\ddot{N} = \begin{bmatrix} \ddot{N}_1 & 0 & & 0 \\ 0 & \dot{N}_2 & & \\ & & \ddots & \\ & 0 & & \ddot{N}_m \end{bmatrix}, \quad \overline{N} = \begin{bmatrix} \overline{N}_1 \\ \overline{N}_2 \\ \vdots \\ \overline{N}_m \end{bmatrix}, \quad \ddot{U} = \begin{bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \\ \vdots \\ \ddot{U}_m \end{bmatrix}, \quad \ddot{X} = \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \vdots \\ \ddot{X}_m \end{bmatrix}$$

$$\dot{N} = \sum_i \dot{N}_i , \qquad \dot{U} = \sum_i \dot{U}_i$$

In case $N = \begin{bmatrix} \ddot{N} & \overline{N} \\ \overline{N}^T & \dot{N} \end{bmatrix}$ is nonsingular (det $N \neq 0$), the normal equations above can be solved through a First-Order Partitioned Linear Regression scheme as explained in [Brown and Trotter, 1969; and Uotila, 1973]. We give here an extension of this scheme for the case when N is singular and a unique solution is ascertained only after the introduction of a set of inner constraints

$$\ddot{E}^T \ddot{X} + \dot{E}^T \dot{X} = 0 \tag{A 4}$$

with

$$\ddot{E}^T = [\ddot{E}_1^T \ \ddot{E}_2^T \ \ldots \ \ddot{E}_m^T] = [\ddot{E}_0^T \ \ddot{E}_0^T \ \ldots \ \ddot{E}_0^T]$$

The normal equations can now be augmented as follows [Uotila, 1967]:

$$\begin{bmatrix} \ddot{N} & \overline{N} & \ddot{E} \\ \overline{N}^T & \dot{N} & \dot{E} \\ \ddot{E}^T & \dot{E}^T & 0 \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \dot{X} \\ K_L \end{bmatrix} + \begin{bmatrix} \ddot{U} \\ \dot{U} \\ 0 \end{bmatrix} = 0 \tag{A 5}$$

The new augmented coefficient matrix is nonsingular and inversion with the introduction of some new notation leads to

$$\begin{bmatrix} \ddot{N} & \overline{N} & \ddot{E} \\ \overline{N}^T & \dot{N} & \dot{E} \\ \ddot{E}^T & \dot{E}^T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \ddot{M} & \overline{M} \\ \overline{M}^T & \dot{M} \end{bmatrix}^{-1} = \begin{bmatrix} \ddot{Q} & \overline{Q} \\ \overline{Q}^T & \dot{Q} \end{bmatrix} = \begin{bmatrix} \ddot{G} & \overline{G} & \ddot{F} \\ \overline{G}^T & \dot{G} & \dot{F} \\ \ddot{F}^T & \dot{F}^T & F \end{bmatrix} \tag{A 6}$$

With the help of this inverse, the solution of the normal equations becomes

$$\begin{aligned} -\ddot{X} &= \ddot{G}\,\ddot{U} + \overline{G}\,\dot{U} \\ -\dot{X} &= \overline{G}^T\ddot{U} + \dot{G}\,\dot{U} \end{aligned} \tag{A 7}$$

If $\overline{G}_i$, $\ddot{G}_{ij}$ denote submatrices of $\overline{G}$ and $\ddot{G}$, respectively, with dimensions equal to those of $\overline{N}_i$ and $\ddot{N}_{ij}$, respectively, we have

$$\begin{aligned} -\dot{X} &= \sum_j \overline{G}_j^T \ddot{U}_j + \dot{G}\,U \\ -\ddot{X}_i &= \sum_j \ddot{G}_{ij}\,\ddot{U}_j + \overline{G}_i\,\dot{U} \end{aligned} \tag{A 8}$$

The inversion of Eq. (A 6) leads to

$$\dot{Q} = [\ddot{M} - \overline{M}^T \ddot{M}^{-1} \overline{M}]^{-1}$$

$$\overline{Q} = - \ddot{M}^{-1} \overline{M} \dot{Q} \tag{A 9}$$

$$\ddot{Q} = \ddot{M}^{-1} + \ddot{M}^{-1} \overline{M} \dot{Q} \overline{M}^T \ddot{M}^{-1}$$

Replacing $\ddot{M}$, $\dot{M}$, $\overline{M}$ from (A 6) in terms of submatrices of $\ddot{N}$, $\dot{N}$, $\overline{N}$, and introducing the following notation

$$H_i = \ddot{N}_i^{-1} \overline{N}_i \qquad R_i = N_i^T H_i$$

$$S_i = N_i - R_i \qquad S = \sum_i S_i$$

$$H = \sum_i H_i \qquad K = \sum_i \ddot{N}_i^{-1}$$

$$\overline{L}_i = H_i^T \ddot{U}_i \qquad L = \sum_i \overline{L}_i$$

$$T_i = \ddot{N}_i^{-1} \ddot{U}_i \qquad T = \sum_i T_i$$

we finally obtain after some algebraic manipulations

$$\dot{Q}^{-1} = \begin{bmatrix} S & (\dot{E} - H^T \ddot{E}_0) \\ (\dot{E}^T - \ddot{E}_0^T H) & (-\ddot{E}_0^T K \ddot{E}_0) \end{bmatrix} \tag{A 10}$$

$$\dot{X} = \dot{G} L + \dot{F} \ddot{E}_0^T T - \dot{G} \dot{U} \tag{A 11}$$

$$\ddot{X}_i = - T_i - \ddot{N}_i^{-1} [\ddot{E}_0 \dot{F}^T (-\dot{U} + L) + (\ddot{E}_0 F \ddot{E}_0^T) T] - H_i \dot{X} \tag{A 12}$$

$$\ddot{G}_{ij} = \delta_{ij} \ddot{N}_i^{-1} + H_i \dot{G} H_j^T + H_i (F \ddot{E}_0^T) \ddot{N}_j^{-1} +$$

$$+ \ddot{N}_i^{-1} (\ddot{E}_0 \dot{F}^T) H_j^T + \ddot{N}_i^{-1} (\ddot{E}_0 F \ddot{E}_0^T) \ddot{N}_j^{-1} \tag{A 13}$$

The algorithm can now be summarized in the following steps:

1. Compute $\ddot{A}_i, \dot{A}_i, L_i; \dot{E}, \ddot{E}_0$

2. Compute $\ddot{N}_i, \dot{N}_i, \overline{N}_i, \ddot{U}_i, \dot{U}_i$

3. Compute $H_i, R_i, S_i, T_i, \overline{L}_i$

4. Compute H, S, $\dot{N}$, $\dot{U}$, K, L, T

5. Obtain $\dot{G}, \dot{F}, F$ by inversion

$$\begin{bmatrix} S & E - H^T \ddot{E}_0 \\ (\dot{E} - H^T \ddot{E}_0)^T & -\ddot{E}_0^T K \ddot{E}_0 \end{bmatrix}^{-1} = \begin{bmatrix} \dot{G} & \dot{F} \\ \dot{F}^T & F \end{bmatrix}$$

6. Compute $\dot{X}$ from (A 11), $\ddot{X}_i$ from (A 12), and $\ddot{G}_{ij}$ from (A 13)

References

Brown, D.C. and J.E. Trotter. 1969. "SAGA, A Computer Program for Short Arc Geodetic Adjustment of Satellite Observations." DBA Systems Inc., Melbourne, Florida.

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APPENDIX C

Computer Programs

The simulation of observations and their adjustment are carried out with the help of three programs, VLBI SIMULATOR, PROGØ1, and PROGØ2 as explained in Chapter 7. These programs are presented here in detail together with their supporting subroutines.

1. Name of Program: VLBI SIMULATOR

   Function: Simulates distance and distance rate VLBI observations corresponding to time delays and their derivatives, according to a previously decided observational pattern.

   Input: Number of participating stations and radio sources, radio source and station coordinates, initial and final epochs of observations, time interval between successive observations, identifying numbers of stations and radio source participating in each observation.

   Output: Identification numbers for participating stations and radio source, observed (perfect) distance and distance rate, epoch of observation.

Subroutines required:

GRESID: Calculates Greenwich sidereal time for a given epoch
JULIA: Converts Universal Time to Julian date
EARTH: Provides simulated earth rotation parameters for a given epoch
REDPI: Converts an angle in radians to the $(-\pi, \pi)$ interval.

The following is a listing of VLBI SIMULATOR program and supporting subroutines.

```
      C******* CONVERT RA AND DEC TO RADIANS
            RAR=RA(I)*PI/180.D0
            DECR=DEC(I)*PI/180.D0
            CD=DCOS(DECR)
            SD=DSIN(DECR)
            CA=DCOS(RAR)
            SA=DSIN(RAR)
            E1(I)=CD*CA
            E2(I)=CD*SA
            E3(I)=SD
      5     CONTINUE
      C
      C
      C******* READ STATION COORDINATES
      C******* PHI(LATITUDE) AND LONG ARE IN DEGREES, R IS IN METERS
            DO 6 I=1,IN
            READ(5,701) IDUM,IL1,IL2,LONG(I),PHI(I),R(I)
      701   FORMAT(1X,I2,1X,A4,A2,3F15.1)
            WRITE(6,701) I,IL1,IL2,LONG(I),PHI(I),R(I)
      101   FORMAT(3F15.5)
      C******* CONVERT PHI AND LONG TO RADIANS
            PHIR=PHI(I)*PI/180.D0
            LONGR=LONG(I)*PI/180.D0
            CF=DCOS(PHIR)
            SF=DSIN(PHIR)
            CL=DCOS(LONGR)
            SL=DSIN(LONGR)
            X(I)=CF*CL
            Y(I)=CF*SL
            Z(I)=SF
      6     CONTINUE
      C
      C
            OMG=PI/12.D0
      C
      C
      C******* READ INITIAL EPOCH IN UT
            READ(5,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
            WRITE(6,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
      201   FORMAT(1X,I4,4I5,F15.1)
      C
      C
      C******* FIND GREENWITCH SIDERIAL TIME AT EPOCH TO
            CALL GRESID(IYEAR,IMO,IDAY,IHOUR,IMIN,SEC,THO)
      C
      C
      C******* READ FINAL EPOCH TF AND TIME INTERVAL INHOURS
            READ(5,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
            WRITE(6,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
            READ(5,101) DT
            WRITE(6,101) DT
      C
      C
            IF(JYEAR.NE.IYEAR) GO TO 88
            IF(JMO.NE.IMO) GO TO 88
      C******* NOTICE THAT IF TO, TF ARE CLOSE BUT IN DIFFERENT MONTHS SECOND
      C******* DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33
      79    TF=DFLOAT(JDAY-IDAY)*24.D0+DFLOAT(JHOUR-IHOUR)+DFLOAT(JMIN-IMIN)
```

```
     ?/60.D0+(SECJ-SEC)/3600.D0
C
C
      T0=0.D0
      T=0.D0
33    IF(T.GT.TF) GO TO 40
C
      DO 8 I=1,IM
C******* TRANSFORM QUASAR UNIT VECTOR TO EARTH FIXED SYSTEM AT EPOCH T
      ANGLE=OMG*(T-T0)+THO
      CA=DCOS(ANGLE)
      SA=DSIN(ANGLE)
      Q1=CA*E1(I)+SA*E2(I)
      Q2=-SA*E1(I)+CA*E2(I)
      Q3=E3(I)
C
      DO 8 J=1,IN
      ZNT=DARCOS(X(J)*Q1+Y(J)*Q2+Z(J)*Q3)
      ARGU=X(J)*Q1+Y(J)*Q2+Z(J)*Q3
      IF(ARGU.LT.-1.D0) ARGU=-1.D0
      IF(ARGU.GT.1.D0) ARGU=1.D0
      ZNT=DARCOS(ARGU)
      ZNT=DABS(ZNT)*180.D0/PI
      ZNTMAX=60.D0
      IF(ZNT.GT.ZNTMAX) ZNT=10.D0**10
      INDEX(I,J)=ZNT
8     CONTINUE
C
      WRITE(6,207) T


207   FORMAT('1'//10X,'EPOCH = ',F15.5//20X,'1',2X,'2',2X,'3',2X,

     ?'4',2X,'5',2X,'6',2X,'7',2X,'8',2X,'9',1X,'10',1X,'11',1X,'12',

     ?1X,'13',1X,'14',1X,'15',1X,'16',1X,'17',1X,'18',1X,'19',1X,'20',

     ?1X,'21',1X,'22',1X,'23',1X,'24',1X,'25',1X,'26',1X,'27',1X,'28',
     ?1X,'29',1X,'30'//)
C
      DO 9 I=1,IM
      WRITE(6,105) I,RA(I),DEC(I),(INDEX(I,J),J=1,IN)
105   FORMAT(1X,I2,2F7.0,1X,30(1X,I2))
9     CONTINUE
C
C
      INM=IN-1
      DO 555 I=1,IM
      DO 555 J=1,INM
      IF(INDEX(I,J).GT.1000) GO TO 555
      JP=J+1
      DO 555 K=JP,IN
```

```
      IF(INDEX(I,K).GT.1000) GO TO 555
      WRITE(7,705) J,K,I,T
  705 FORMAT(3I5,F15.2)
  555 CONTINUE
C
C
   99 T=T+DT
      GO TO 33
   88 WRITE(6,500)
  500 FORMAT('1'///////10X,'WRONG INITIAL AND FINAL DATE'/
     210X,'PROGRAM INTERRUPTED THROUGH STATEMENT 88')
   40 STOP
      END
```

```
C******* RA AND DEC ARE IN DEGREES
      DO 5 I=1,IM
      READ(5,321) IDM,IL1,IL2,IL3,RA(I),DEC(I)
      WRITE(6,321)  I,IL1,IL2,IL3,RA(I),DEC(I)
321   FORMAT(1X,I3,7X,3A4,F17.7,F20.9)
C******* CONVERT RA AND DEC TO RADIANS
      RAR=RA(I)*PI/180.D0
      DECR=DEC(I)*PI/180.D0
      CD=DCOS(DECR)
      SD=DSIN(DECR)
      CA=DCOS(RAR)
      SA=DSIN(RAR)
      E1(I)=CD*CA
      E2(I)=CD*SA
      E3(I)=SD
5     CONTINUE
C
C
C******* READ STATION COORDINATES
C******* PHI(LATITUDE) AND LONG ARE IN DEGREES, R IS IN METERS
      DO 6 I=1,IN
      READ(5,701) IDM,IL1,IL2,LONG(I),PHI(I),R(I)
      WRITE(6,701)  I,IL1,IL2,LONG(I),PHI(I),R(I)
701   FORMAT(1X,I2,1X,A4,A2,3F15.1)
C******* CONVERT PHI AND LONG TO RADIANS
      PHIR=PHI(I)*PI/180.D0
      LONGR=LONG(I)*PI/180.D0
      CF=DCOS(PHIR)
      SF=DSIN(PHIR)
      CL=DCOS(LONGR)
      SL=DSIN(LONGR)
      X(I)=CF*CL
      Y(I)=CF*SL
      Z(I)=SF
6     CONTINUE
C
C
C******* READ CORRECTION TO GREENWITCH SIDERIAL TIME DDTH IN ARC SEC
      READ(5,101) DDTH
      WRITE(6,101) DDTH
101   FORMAT(3F15.5)
C******* CONVERT TO RADIANS
      DDTH=DDTH*PI/(180.D0*3600.D0)
C
C
C******* READ INITIAL EPOCH IN UT
      READ(5,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
      WRITE(6,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
201   FORMAT(5I5,F15.5)
C
C
C******* FIND GREENWITCH SIDERIAL TIME AT EPOCH TO
      CALL GRESID(IYEAR,IMO,IDAY,IHOUR,IMIN,SEC,THO)
      COPY(1)=THO
      THO=THO*(180.D0/PI)
      WRITE(6,136) THO
136   FORMAT(///10X,'APPR. THO= ',F25.9//)
      WRITE(7,101) THO
```

```
      C*****************************************************************************
      C*********                                                         **********
      C*********          VLBI NETWORK, OBSERVATION SIMULATOR            **********
      C*********                                                         **********
      C*****************************************************************************
      C*********                                                         **********
      C*********          CONTINUOUS VERSION :   EARTH MOTION PARAMETERS **********
      C*********                    PROVIDED BY SUBROUTINE EARTH         **********
      C*********                                                         **********
      C*****************************************************************************
      C*********                                                         **********
      C*********          THE MODEL USED IS:                             **********
      C*********                                                         **********
      C*********                                 T                       **********
      C*********          D(I,J,P,K)=(XJ-XI) *S(XP,HP)*                  **********
      C*********                                       T                 **********
      C*********                    *R3(OMG*TK+THO)*S  (XX,HH)*QP        **********
      C*********                                                         **********
      C*********                   (  1  0  A )                          **********
      C*********          S(A,B)=  (  0  1 -B )                          **********
      C*********                   ( -A  B  1 )                          **********
      C*********                                                         **********
      C*********            T                                            **********
      C*********          QP   = (COS(DECP)*COS(RAP),                    **********
      C*********                  COS(DECP)*SIN(RAP), SIN(DECP)  )       **********
      C*********                                                         **********
      C*********          XJ,XI      : POSITION VECTORS OF J AND I       **********
      C*********                       STATIONS IN EARTH FIXED SYSTEM    **********
      C*********          DECP,RAP   : DECLINATION AND RIGHT ASCENSION   **********
      C*********                       OF P QUASAR IN QUASQR FIXED SYSTEM **********
      C*********          QP         : UNIT VECTOR OF P QUASAR IN        **********
      C*********                       QUASAR FIXED SYSTEM               **********
      C*********          OMG        : ROTATIONAL VELOCITY OF EARTH      **********
      C*********          XP, HP     : POLAR MOTION SMALL ANGLES         **********
      C*********          XX, HH     : PRECESSION NUTATION SMALL ANGLES  **********
      C*********          D(I,J,P,K): OBSERVED DISTANCE INVOLVING IJ     **********
      C*********                       BASELINE AND P QUASAR AT TIME TK  **********
      C*********                       AFTER INITIAL REFERENCE EPOCH TO  **********
      C*********          THO        : GREENWITCH SIDERIAL TIME AT EPOCH TO **********
      C*********          THETA      : UT1 ANGLE                         **********
      C*********                                                         **********
      C*****************************************************************************
      C*********                                                         **********
0001        IMPLICIT REAL*8(A-H,L-Z)
0002        DIMENSION X(4),Y(4),Z(4),E1(18),E2(18),E3(18),RA(18),DEC(18)
           2,PHI(4),LONG(4),R(4),COPY(3)
      C        X(IN),Y(IN),Z(IN),E1(IM),E2(IM),E3(IM),INDEX(IM,IN),RA(IM),DEC(IM),
      C        PHI(IN),LONG(IN),R(IN)
      C        IN=NO OF STATIONS, IM=NO OF QUASARS, IN.LT.35
      C
0003        PI=4.DO*DATAN(1.DO)
0004        REART=6371000.DO
      C
0005        READ(5,100) IN,IM
0006        WRITE(6,100) IN,IM
0007  100   FORMAT(2I5)
      C
      C******* READ QUASAR COORDINATES
```

```
      THO=COPY(1)+DDTH
C
C
C******* READ FINAL EPOCH TF AND TIME INTERVAL INHOURS
      READ(5,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
      WRITE(6,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
      READ(5,101) DT
      WRITE(6,101) DT
C
C
      IF(JYEAR.NE.IYEAR) GO TO 88
      IF(JMO.NE.IMO) GO TO 88
C******* NOTICE THAT IF TO, TF ARE CLOSE BUT IN DIFFERENT MONTHS SECOND
C******* DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33
 79   TF=DFLOAT(JDAY-IDAY)*24.D0+DFLOAT(JHOUR-IHOUR)+DFLOAT(JMIN-IMIN)
     ?/60.D0+(SECJ-SEC)/3600.D0
C
C
      TO=0.D0
      T=0.D0
 33   IF(T.GT.TF) GO TO 40
C
 555  READ(5,107) I,J,K
      IF(I.EQ.0) GO TO 99
C******* I=0 IS A CODE TO INDICATE END OF OBSERVATIONS FOR THIS EPOCH
C
      CALL EARTH(T,XP,HP,OMG,THETA,XX,HH)
C
C     MODIFY EARTH PARAMETERS AND WRITE / PUNCH
      XPD=XP*REART
      HPD=HP*REART
      XXD=XX*REART
      HHD=HH*REART
      OMGD=OMG*(12.D0/PI)
      THET=THETA
      THET=THET*(180.D0/PI)
      ITH1=THET
      TH2=(THET-DFLOAT(ITH1))*60.D0
      ITH2=TH2
      TH3=(TH2-DFLOAT(ITH2))*60.D0
      WRITE(6,801) T,XPD,HPD,XXD,HHD,ITH1,ITH2,TH3,OMGD
 801  FORMAT(3X,F7.2,4F10.4,1X,2I4,1X,F8.4,F12.9)
C
      COPY(1)=   E1(K)          -XX*E3(K)
      COPY(2)=          +   E2(K)+HH*E3(K)
      COPY(3)=XX*E1(K)-HH*E2(K)+   E3(K)
C
      ANGLE=THETA+THO
      CA=DCOS(ANGLE)
      SA=DSIN(ANGLE)
C
      Q1= CA*COPY(1)+SA*COPY(2)
      Q2=-SA*COPY(1)+CA*COPY(2)
      Q3=COPY(3)
C
      QF1=-SA*COPY(1)+CA*COPY(2)
      QF2=-CA*COPY(1)-SA*COPY(2)
      QF3=-XP*QF1+HP*QF2
```

```
                C
 0088                 COPY(1)=     Q1       +XP*Q3
 0089                 COPY(2)=           Q2-HP*Q3
 0090                 COPY(3)=-XP*Q1+HP*Q2+    Q3
                C
 0091                 DS=(X(J)*R(J)-X(I)*R(I))*COPY(1)+(Y(J)*R(J)-Y(I)*R(I))*COPY(2)
                     2 +(Z(J)*R(J)-Z(I)*R(I))*COPY(3)
                C
 0092                 FRNG=(X(J)*R(J)-X(I)*R(I))*QF1+(Y(J)*R(J)-Y(I)*R(I))*QF2
                     2+(Z(J)*R(J)-Z(I)*R(I))*QF3
                C     FRNG= DERIVATIVE OF DELAY DISTANCE DS (METERS/HOUR)
 0093                 FRNG=FRNG*OMG
                C
 0094                 WRITE(6,107) I,J,K,DS,FRNG,T
 0095                 WRITE(7,107) I,J,K,DS,FRNG,T
 0096           107   FORMAT(3I5,3F15.5)
 0097                 GO TO 555
                C
 0098           99    T=T+DT
 0099                 GO TO 33
 0100           88    WRITE(6,500)
 0101           500   FORMAT('1'///////10X,'WRONG INITIAL AND FINAL DATE'/
                     210X,'PROGRAM INTERRUPTED THROUGH STATEMENT 88')
 0102           40    STOP
 0103                 END
```

```
      SUBROUTINE GRESID(IYEAR,IMO,IDAY,IHOUR,IMIN,SEC,THO)
C*********                                                           **********
C******************************************************************************
C*********                                                           **********
C*********            CALCULATES THE GREENWITCH SIDERIAL TIME        **********
C*********            THO, FOR EPOCH WITH DATE IYEAR IMO IDAY        **********
C*********            IHOUR IMIN SEC  UT.                            **********
C*********                                                           **********
C******************************************************************************
C*********                                                           **********
      IMPLICIT REAL*8(A-H,L-Z)
      PI=4.D0*DATAN(1.D0)
      I1=0
      I2=0
      A=0.D0
      CALL JULIA(IYEAR,IMO,IDAY,I1,I2,A,MJD)
      T=(MJD-2415020.D0)/36525.D0
      T2=T*T
      THO=99.6909833D0+36000.7689D0*T+0.00038708D0*T2
      THO=THO*PI/180.D0
      CALL REDPI(THO)
      DT=DFLOAT(IHOUR)*60.D0+DFLOAT(IMIN)+SEC/60.D0
      DTHDT=4.3752695D0/(10.D0**3)
      THO=THO+DT*DTHDT
      CALL REDPI(THO)
      RETURN
      END
```

```
0001        SUBROUTINE JULIA(IYEAR,IM,IDAY,IHHH,IMMM,S,MJD)
      C**************************************************************************
      C*********                                                       **********
      C*********                                                       **********
      C*********         SUBROUTINE TO CONVERT UNIVERSAL TIME TO       **********
      C*********                       JULIAN DATE                     **********
      C*********                                                       **********
      C*********                                                       **********
      C**************************************************************************
0002        IMPLICIT REAL*8 (A-H,L-Z)
0003        DIMENSION IMONTH(12)
0004        DATA IMONTH /0,31,59,90,120,151,181,212,243,273,304,334/
0005        H=DFLOAT(IHHH)
0006        M=DFLOAT(IMMM)
0007        ICOD=0
0008        IDIS=(IYEAR-1897)/4
0009        IF(IYEAR.GT.1900) IDIS=IDIS-1
0010        ICH=4*(IYEAR/4)
0011        IF(IYEAR.EQ.ICH.AND.IM.GT.2)  ICOD=1
0012        IF(IYEAR.EQ.1900) ICOD=0
0013        MJD=2415020+(IYEAR-1900)*365.D0+IDIS+IMONTH(IM)+ICOD-0.5D0+IDAY
           X+H/24.D0+M/1440.D0+S/86400.D0
0014        RETURN
0015        END
```

```
0001            SUBROUTINE EARTH(T,XP,HP,OMG,THETA,XX,HH)
          C
          C         T     : TIME INTERVAL AFTER INITIAL EPOCH  (HOURS)
          C         XP,HP: POLAR MOTION COORDINATES  (RADIANS)
          C         THETA: UT1 ANGLE IN RADIANS
          C         OMG  : ROTATIONAL VELOCITY OF EARTH  (RADIANS PER HOUR)
          C         XX,HH: PRECESSION-NUTATION SMALL ANGLES  (RADIANS)
0002            IMPLICIT REAL*8(A-H,L-Z)
0003            DIMENSION P(10),A(10),B(10),PS(10),F(10)
          C
0004            DATA P/8760.D0,4380.D0,2190.D0,720.D0,360.D0,
               2168.D0,24.D0,12.D0,1.D0,0.5D0/,
               3A/1.16D0,0.63D0,0.28D0,0.12D0,0.06D0,
               40.03D0,0.002D0,0.001D0,0.000D0,0.000D0/,
               5B/0.95D0,0.70D0,0.22D0,0.13D0,0.05D0,
               60.01D0,0.004D0,0.001D0,0.000D0,0.000D0/,
               7PS/3.15D0,5.28D0,1.32D0,0.15D0,0.0D0,0.99D0,2.0D0,
               86.14D0,2.75D0,4.13D0/,
               9F/1.13D0,2.85D0,6.17D0,4.35D0,0.05D0,5.18D0,3.75D0,1.24D0,
               70.35D0,1.67D0/
          C
0005            PI=4.D0*DATAN(1.D0)
0006            PI2=2.D0*PI
0007            REART=6371000.D0
0008            M10=6.D0
0009            M20=0.D0
0010            PC=429.D0*24.D0
0011            C=DCOS(PI2*T/PC)
0012            S=DSIN(PI2*T/PC)
0013            M1=C*M10-S*M20
0014            M2=S*M10+C*M20
          C
0015            DO 5 I=1,10
0016            ARG=(PI2*T)/P(I)
0017            M1=M1+A(I)*DCOS(ARG+PS(I))
0018            M2=M2+B(I)*DCOS(ARG+ F(I))
0019      5     CONTINUE
          C
0020            XP=-M2/REART
0021            HP=-M1/REART
0022            XX=0.06D0+0.01D0*DCOS(2.34D0+T/24.D0)-0.003*DCOS(0.1D0+T/12.D0)
0023            HH=0.04D0+0.02D0*DCOS(3.11D0+T/24.D0)-0.005D0*DCOS(0.07D0+T/12.D0)
0024            XX=XX/REART
0025            HH=HH/REART
0026            TD=285.D0+T/24.D0
0027            ARG1=PI2*TD/365.25D0-0.499D0
0028            ARG2=2.D0*PI2*TD/365.25D0-0.862D0
0029            THETA=0.00175D0*(T/24.D0)+25.1D0*DSIN(ARG1)-9.2D0*DSIN(ARG2)
0030            FACTOR=PI2/86400000.D0
0031            THETA=PI2*(T/24.D0)+THETA*FACTOR
0032            OMG=0.00175D0/24.D0+(25.1D0*DCOS(ARG1)-18.4D0*DCOS(ARG2))*
               2*(PI2/8766.D0)
0033            OMG=PI2/24.D0+FACTOR*OMG
0034            RETURN
0035            END
```

```
0001          SUBROUTINE REDPI(A)
        C*********************************************************************************
        C*********                                                              **********
        C*********              REDUCES AN ANGLE A IN RADIANS TO THE            **********
        C*********              ( -PI, +PI ) INTERVAL                           **********
        C*********                                                              **********
        C*********************************************************************************
        C*********                                                              **********
0002          IMPLICIT REAL*8(A-H,L-Z)
0003          PI=4.D0*DATAN(1.D0)
0004          PI2=PI*2.D0
0005          COPY=A/PI2
0006          IA=COPY
0007          IF(A.GE.0.D0) A=A-DFLOAT(IA)*PI2
0008          IF(A.LT.0.D0) A=A+DFLOAT(IA)*PI2
0009    5     IF(A.GT.-PI.AND.A.LE.PI) GO TO 8
0010          IF(A.LT.0.D0) A=A+PI2
0011          IF(A.GT.0.D0) A=A-PI2
0012          GO TO 5
0013    8     RETURN
0014          END
```

2. Name of Program: PROGØ1

Function:

Adjustment of a set of observations with total time-span equal to both earth and station step. Earth rotation parameters and station coordinates are treated as constants over the total time span of observations. A number of weighting options for radio source coordinates are available.

Input:

Number of weighting options for radio source coordinates, standard deviations of observations and radio source coordinates, approximate values of parameters, identification numbers of participating stations and radio source, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

REDPI.

(Following is a listing of PROGØ1.)

```
      C*******************************************************************************
      C*********                                                          **********
      C*********                    PROGO1                                **********
      C*********                                                          **********
      C*********          VLBI  ADJUSTMENT  PROGRAM                       **********
      C*********                                                          **********
      C*********          THIS VERSION PERFORMS AN ADJUSTMENT OF          **********
      C*********          ONE DAY OF OBSERVATIONS TREATING ALL            **********
      C*********          PARAMETERS AS CONSTANTS OVER THE ONE DAY        **********
      C*********          INTERVAL.                                       **********
      C*********          AN OPTION IS AVAILABLE FOR APPLYING A           **********
      C*********          NUMBER OF DIFFERENT WEIGHTS ON RADIO            **********
      C*********          SOURCE COORDINATES                              **********
      C*********                                                          **********
      C*********          OBSERVATIONS: DELAY DISTANCES AND               **********
      C*********                        THEIR DERIVATIVES                 **********
      C*********                                                          **********
      C*********          UNKNOWNS: EARTH MOTION PARAMETERS               **********
      C*********                    STATION & QUASAR COORDINATES          **********
      C*********                    CLOCK OFFSETS AND DRIFTS              **********
      C*********                                                          **********
      C*********          SYSTEMS DEFINITION:                             **********
      C*********          QUASAR FIXED: WEIGHTED QUASAR COORDINATES       **********
      C*********          EARTH FIXED: BY MEANS OF INNER CONSTRAINTS      **********
      C*********                                                          **********
      C*********          SUBROUTINES REQUIRED :   REDPI                  **********
      C*********                                                          **********
      C*********                                                          **********
      C*******************************************************************************
      C*********                                                          **********
      C*********                                                          **********
      C
      C          A     ROW OF DESIGN MATRIX CONTAINING PARTIALS OF DELAY
      C                OBSERVATIONS WITH RESPECT TO PARAMETERS
      C          B     ROW OF DESIGN MATRIX CONTAINING PARTIALS OF DELAY
      C                DERIVATIVE OBSERVATIONS WITH RESPECT TO PARAMETERS
      C                CC         COEFFICIENT MATRIX OF INNER CONSTRAINTS
      C                N          COEFFICIENT MATRIX OF NORMAL EQUATIONS
      C                U          CONSTANT VECTOR OF NORMAL EQUATIONS
      C                XX         VECTOR OF UNKNOWNS (CORRECTIONS TO PARAMETERS)
      C          THE ORDER OF THE PARAMETERS IS:
      C                XP, HP     POLAR MOTION ANGLES KSI AND ETA
      C                XN, HN     PRECESSION-NUTATION ANGLES (CAPITAL) KSI & ETA
      C                TH         THETA ZERO ANGLE (GAST AT INITIAL EPOCH)
      C                OMG        ANGULAR VELOCITY OF EARTH ROTATION
      C                X(I),Y(I),Z(I)
      C                           COORDINATES OF STATION I   (I=1,2,...,IN)
      C                RA(J), D(J)
      C                           RIGHT ASCENSION AND DECLINATION OF
      C                           RADIO SOURCE  J   (J=1,2,...,IM)
      C                CLOCK OFFSET AND DRIFT AT STATION I  (I=1,2,...,IN)
      C
0001        IMPLICIT REAL*8(A-H,L-Z)
0002        DIMENSION A(37),N(45,45),X(3),Y(3),Z(3),RA(8),D(8),U(37)
          2,XX(37),L1(45),L2(45),CC(8,37),DO(3),DD(3),B(37),NKEEP(45,45)
          3,SDQUA(10),PRATIO(10),F(13)
      C     DIMENSION A(KK),N(K8,K8),X(N),Y(N),Z(N),RA(M),D(N),U(KK),XX(KK)
      C    2,L1(K8),L2(K8),                         CC(8,KK),DO(N),DD(N)
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
              C     3,B(KK),NKEEP(K8,K8),SDQUA(10),PRATIO(10)
              C      WHERE: N=NO OF STATIONS, M=NO OF QUASARS, K=6+3*N+2*M, KK=K+2*N, K8=KK+8
              C
              C          READ EXPERIMENT IDENTIFICATION NO
 0003                READ(5,100) IEXP
 0004                WRITE(6,977) IEXP
 0005         977    FORMAT('1'////10X,'EXPERIMENT NO =',I5/10X
                    2,'*******************',/////)
              C
              C          READ NUMBER OF WEIGHTING OPTIONS
 0006                READ(5,100) IOPT
 0007                WRITE(6,978) IOPT
 0008         978    FORMAT(10X,'REQUESTED NO OF WEIGHTING OPTIONS =',I5/////)
              C
              C          READ A PRIORI STANDARD DEVIATIONS:
              C          SDBD : OF DELAY DISTANCE IN CM
              C          SDBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
              C          SDQUA: OF QUASAR COORDINATES IN CM
              C
 0009                READ(5,101) SDBD,SDBF
 0010         101    FORMAT(5F15.5)
              C
 0011                DO 181 I=1,IOPT
 0012                READ(5,101) SDQUA(I)
 0013                PRATIO(I)=SDBD**2/SDQUA(I)**2
 0014         181    CONTINUE
              C
              C
 0015                R=6371000.D0
 0016                CLIGHT=3.D0*(10.D0**8)
 0017                CLIGHT=CLIGHT/R
 0018                PI=4.D0*DATAN(1.D0)
 0019                R100=R*100.D0
 0020                RHRDM9=(12.D0/PI)*(10.D0**9)
 0021                RASM3=(180.D0/PI)*3600000.D0
              C
 0022                PF=(SDBD/R100)/(SDBF/R)
 0023                PF=PF**2
              C          WEIGHTS :
              C          PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2
              C
              C         READ IN= NO OF STATIONS & IM= NO OF QUASARS
 0024                READ(5,100) IN, IM
 0025         100    FORMAT(5I5)
 0026                WRITE(6,920) IN,IM
 0027         920    FORMAT(5X,'NO OF STATIONS =',I5/5X,'NO OF QUASARS  =',I5)
 0028                K=6+3*IN+2*IM
 0029                KK=K+2*IN
 0030                K8=KK+8
              C
              C          INITIALIZE POLAR MOTION (XP,HP) AND PRECESSION-NUTATION (XN,HN)
              C          PARAMETERS (APPROXIMATE VALUES)  TO ZERO
 0031                XP=0.D0
 0032                HP=0.D0
 0033                XN=0.D0
 0034                HN=0.D0
              C
 0035                DO 51 I=1,IN
```

```
0036              DO(I)=0.D0
0037              DD(I)=0.D0
0038        51    CONTINUE
            C
            C*************************************************************************
            C          READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, OMG,
            C          X(I), Y(I), Z(I), (I=1,IN),  RA(J), D(J), (J=1,IM)
            C          ANGLES ARE IN DEGREES, DISTANCES IN METERS AND OMG IN RADIANS PER HOUR
            C*************************************************************************
            C
0039              READ(5,101) TH
0040              WRITE(6,860) TH
0041        860   FORMAT(//5X,'THETA ZERO = ',F20.9/)
0042              TH=TH*PI/180.D0
0043              OMG=PI/12.D0
            C
0044              WRITE(6,450)
0045        450   FORMAT(////,10X,'STATION & RADIO SOURCES APPROXIMATE COORDINATES'
                 ?,//3X,'STATIONS',10X,'LONG'
                 2,12X,'LAT',11X,'R',/)
            C
0046              DO 5 I=1,IN
0047              READ(5,701) IDM,IL1,IL2,ALONG,APHI,AR
0048              WRITE(6,701)  I,IL1,IL2,ALONG,APHI,AR
0049        701   FORMAT(1X,I2,1X,A4,A2,3F15.1)
0050              APHI=APHI*PI/180.D0
0051              ALONG=ALONG*PI/180.D0
            C          CONVERT TO CARTESIAN
0052              X(I)=AR*DCOS(APHI)*DCOS(ALONG)
0053              Y(I)=AR*DCOS(APHI)*DSIN(ALONG)
0054              Z(I)=AR*DSIN(APHI)
            C          CONVERT TO EARTH RADII UNITS
0055              X(I)=X(I)/R
0056              Y(I)=Y(I)/R
0057              Z(I)=Z(I)/R
0058        5     CONTINUE
            C
            C
0059              WRITE(6,451)
0060        451   FORMAT(//11X,'RADIO SOURCE',10X,'RA',17X,'DEC',//)
            C
0061              DO 6 J=1,IM
0062              READ(5,321) IDM,IL1,IL2,IL3,RA(J),D(J)
0063              WRITE(6,321) J,IL1,IL2,IL3,RA(J),D(J)
0064        321   FORMAT(1X,I3,7X,3A4,F17.7,F20.9)
            C          CONVERT TO RADIAN UNITS
0065              RA(J)=RA(J)*PI/180.D0
0066              D(J) = D(J)*PI/180.D0
0067        6     CONTINUE
            C
            C*************************************************************************
            C          INITIATE N, U, XX, CC  TO ZERO
            C*************************************************************************
            C
0068              DO 7 I=1,KK
0069              XX(I)=0.D0
0070              U(I)=0.D0
0071              DO 7 J=1,8
```


ORIGINAL PAGE IS OF POOR QUALITY

```
0072              CC(J,I)=0.D0
0073        7     CONTINUE
            C
0074              DO 16 I=1,K8
0075              DO 16 J=1,K8
0076              N(I,J)=0.D0
0077              NKEEP(I,J)=0.D0
0078        16    CONTINUE
            C
            C         FORMULATE CONSTRAINT MATRIX CC
            C
0079              CC(5,1)=1.D0
0080              CC(4,2)=1.D0
0081              CC(6,5)=-1.D0
            C
0082              DO 3 I=1,IN
0083              IB1=4+3*I
0084              IB2=IB1+1
0085              IB3=IB2+1
0086              CC(1,IB1)=1.D0
0087              CC(2,IB2)=1.D0
0088              CC(3,IB3)=1.D0
0089              CC(4,IB2)=-Z(I)
0090              CC(4,IB3)=Y(I)
0091              CC(5,IB1)=Z(I)
0092              CC(5,IB3)=-X(I)
0093              CC(6,IB1)=-Y(I)
0094              CC(6,IB2)=X(I)
0095              IC1=K+I
0096              IC2=IC1+IN
0097              CC(7,IC1)=1.D0
0098              CC(8,IC2)=1.D0
0099        3     CONTINUE
            C
            C*******************************************************************************
            C
            C        READ OBSERVATIONS AND IDENTIFIERS:
            C        DS= DISTANCE IN METERS OF IJ BASELINE OBSERVATION TO QUASAR IP
            C            AT TIME TK (IN HOURS) AFTER SOME INITIAL EPOCH TO
            C
            C*******************************************************************************
            C
            C
0100              ICOUNT=0
            C
0101        33    READ(5,104) I,J,IP,DS,FRNG,TK,ICHEK
0102        104   FORMAT(3I5,3F15.5,2I5)
            C
            C        ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1
0103              IF(ICHEK.EQ.1) GO TO 66
0104              ICOUNT=ICOUNT+1
0105              WRITE(8,100) I,J,IP
            C
0106        96    DS=DS/R
0107              FRNG=FRNG/R
0108              DX=X(J)-X(I)
0109              DY=Y(J)-Y(I)
0110              DZ=Z(J)-Z(I)
```

C-2


```
0111              CA=DCOS(RA(IP))
0112              SA=DSIN(RA(IP))
0113              CD=DCOS(D(IP))
0114              SD=DSIN(D(IP))
0115              Y1=OMG*TK+TH
0116              Y2=Y1-RA(IP)
0117              CK=DCOS(Y1)
0118              SK=DSIN(Y1)
0119              CKP=DCOS(Y2)
0120              SKP=DSIN(Y2)
            C
            C        FORMULATE  PARTIALS
            C
0121              F1=-CD*DZ*CKP+DX*SD
0122              F1=F1+(-DX*XP+DY*HP)*CD*CKP-DZ*XP*SD
0123              F1=F1+DX*CD*(XN*CA-HN*SA)+DZ*SD*(XN*CK-HN*SK)
0124              F2=-CD*DZ*SKP-DY*SD
0125              F2=F2+DY*CD*(XP*CKP+HP*SKP)-DZ*HP*SD
0126              F2=F2+DY*CD*(-XN*CA+HN*SA)+DZ*SD*(XN*SK+HN*CK)
0127              F3=DX*(CD*CK*(-XN*CA+HN*SA)-SD*CK+XP*CD*CA)
                 2    +DY*(-CD*SK*(-XN*CA+HN*SA)+SD*SK-HP*CD*CA)
                 3    +DZ*(SD*(XP*CK+HP*SK-XN)+CD*CA)
0128              F4=DX*(CD*SA*(XN*CK-HN*SK-XP)+SK*SD)
                 2    +DY*(CD*SA*(-XN*SK-HN*CK+HP)+CK*SD)
                 3    +DZ*(SD*(-XP*SK+HP*CK-HN)-CD*SA)
0129              F5=-(DX*SKP+DY*CKP)*CD
0130              F5=F5+DZ*CD*(XP*SKP-HP*CKP)
0131              F5=F5+DX*SD*(XN*SK+HN*CK)+DY*SD*(XN*CK-HN*SK)
0132              F6= TK*F5
0133              F7=CD*CKP
0134              F7=F7+XP*SD
0135              F7=F7+SD*(-XN*CK+HN*SK)
0136              F8=-CD*SKP
0137              F8=F8-HP*SD
0138              F8=F8+SD*(XN*SK+HN*CK)
0139              F9=SD
0140              F9=F9+CD*(-XP*CKP-HP*SKP)
0141              F9=F9+CD*(XN*CA-HN*SA)
0142              F10=CD*(DX*SKP+DY*CKP)
0143              F10=F10+DZ*CD*(-XP*SKP+HP*CKP)
0144              F10=F10+DZ*CD*(-XN*SA-HN*CA)
0145              F11=(-DX*CKP+DY*SKP)*SD+DZ*CD
0146              F11=F11+CD*(DX*XP-DY*HP)+DZ*SD*(XP*CKP+HP*SKP)
0147              F11=F11+DX*CD*(-XN*CK+HN*SK)+DY*CD*(XN*SK+HN*CK)
                 2         -DZ*SD*(XN*CA-HN*SA)
0148              F12=1.D0
0149              F13=TK/24.D0
            C
0150              G1=DZ*CD*SKP
0151              G1=G1+DX*XP*CD*SKP-DY*HP*CD*SKP
0152              G1=G1-DZ*SD*(XN*SK+HN*CK)
0153              G1=OMG*C1
0154              G2=-DZ*CD*CKP
0155              G2=G2+DY*CD*(-XP*SKP+HP*CKP)
0156              G2=G2+DZ*SD*(XN*CK-HN*SK)
0157              G2=OMG*G2
0158              G3=(DX*SK+DY*CK)*SD
0159              G3=G3-(DX*SK+DY*CK)*CD*(-XN*CA+HN*SA)
```

```
      G3=G3+DZ*SD*(-XP*SK+HP*CK)
      G3=OMG*G3
      G4=DX*CK*SD-DY*SK*SD
      G4=G4+DZ*SD*(-XP*CK-HP*SK)
      G4=G4+DX*CD*SA*(-XN*SK-HN*CK)+DY*CD*SA*(-XN*CK+HN*SK)
      G4=OMG*G4
      G5=-DX*CD*CKP+DY*CD*SKP
      G5=G5+DZ*CD*(XP*CKP+HP*SKP)
      G5=G5+DX*SD*(XN*CK-HN*SK)-DY*SD*(XN*SK+HN*CK)
      G5=OMG*G5
      G6=TK*G5
      G7=-CD*SKP
      G7=G7+SD*(XN*SK+HN*CK)
      G7=OMG*G7
      G8=-CD*CKP
      G8=G8+SD*(XN*CK-HN*SK)
      G8=OMG*G8
      G9=0.D0
      G9=G9+CD*(XP*SKP-HP*CKP)
      G9=OMG*G9
      G10=DX*CD*CKP-DY*CD*SKP
      G10=G10-DZ*CD*(XP*CKP+HP*SKP)
      G10=OMG*G10
      G11=DX*SD*SKP+DY*SD*CKP
      G11=G11+DZ*SD*(-XP*SKP+HP*CKP)
      G11=G11+DX*CD*(XN*SK+HN*CK)+DY*CD*(XN*CK-HN*SK)
      G11=OMG*G11
      G12=0.D0
      G13=1.D0/24.D0
C
C        A, B ARE ROWS OF THE DESIGN MATRIX CORRESPONDING TO DELAY
C        AND DELAY DERIVATIVE OBSERVATIONS RESPECTIVELY
C
C        INITIATE A AND B TO ZERO
C
      DO 8 II=1,KK
      A(II)=0.D0
      B(II)=0.D0
    8 CONTINUE
C
C       SET PARTIALS IN APPROPRIATE LOCATIONS OF A AND B ROWS
C
      A(1)=F1
      A(2)=F2
      A(3)=F3
      A(4)=F4
      A(5)=F5
      A(6)=F6
C
      B(1)=G1
      B(2)=G2
      B(3)=G3
      B(4)=G4
      B(5)=G5
      B(6)=G6
C
      I1=3*I+4
      I2=I1+1
```

```
            I3=I1+2
            A(I1)=-F7
            A(I2)=-F8
            A(I3)=-F9
      C
            B(I1)=-G7
            B(I2)=-G8
            B(I3)=-G9
            IC1=K+I
            IC2=IC1+IN
            A(IC1)=-F12
            A(IC2)=-F13
      C
            B(IC1)=-G12
            B(IC2)=-G13
      C
            J1=3*J+4
            J2=J1+1
            J3=J1+2
            A(J1)= F7
            A(J2)= F8
            A(J3)= F9
      C
            B(J1)=G7
            B(J2)=G8
            B(J3)=G9
      C
            JC1=K+J
            JC2=JC1+IN
            A(JC1)=F12
            A(JC2)=F13
      C
            B(JC1)=G12
            B(JC2)=G13
      C
            IP1=3*IN+2*IP+5
            IP2=IP1+1
            A(IP1)=F10
            A(IP2)=F11
      C
            B(IP1)=G10
            B(IP2)=G11
      C
      C     FORMULATIOM OF L VECTOR ENTRY
      C
      50    DSO=DX*F7+DY*F8+DZ*F9
            DSO=DSO+DO(J)-DO(I)+(DD(J)-DD(I))*(TK/24.DO)
            L=DSO-DS
      C
      C
      C*****************************************************************************
      C
      C        STORE PARTIALS / NON-ZERO ELEMENTS OF A & B ROWS ON DISK 8 FOR
      C        LATER USE IN COMPUTATION OF RESIDUALS AND VTPV
      C        ADD CONTRIBUTIONS OF CURRENT OBSERVATIONS TO NORMAL EQUATIONS
      C
      C*****************************************************************************
      C
```

```
 0244                WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L
 0245          351   FORMAT(10X,F40.15)
               C
 0246                DO 12 II=1,KK
 0247                U(II)=U(II)-A(II)*L
 0248                DO 12 JJ=1,KK
 0249                NKEEP(II,JJ)=NKEEP(II,JJ)+A(II)*A(JJ)
 0250          12    CONTINUE
               C
               C
 0251                FRNGO=DX*G7+DY*G8+DZ*G9
 0252                FRNGO=FRNGO+(DD(J)-DD(I))/24.DO
 0253                L=FRNGO-FRNG
               C
 0254                WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L
               C
 0255                DO 65 II=1,KK
 0256                U(II)=U(II)-B(II)*L*PF
 0257                DO 65 JJ=1,KK
 0258                NKEEP(II,JJ)=NKEEP(II,JJ)+B(II)*B(JJ)*PF
 0259          65    CONTINUE
               C
 0260          350   FORMAT(80X,2F19.9)
               C
 0261                GO TO 33
               C
               C*************************************************************************
               C
               C          SOLVE NORMAL EQUATIONS, COMPUTE PARAMETER CORRECTIONS
               C
               C*************************************************************************
               C
               C
 0262          66    DO 1 I=1,8
 0263                DO 1 J=1,KK
 0264                IK=I+KK
 0265                NKEEP(IK,J)=CC(I,J)
 0266                NKEEP(J,IK)=NKEEP(IK,J)
 0267          1     CONTINUE
               C
 0268                DO 182 IW=1,IOPT
               C
 0269                DO 183 I=1,K8
 0270                DO 183 J=1,K8
 0271                N(I,J)=NKEEP(I,J)
 0272          183   CONTINUE
               C
 0273                IN1=7+IN*3
 0274                DO 2 I=IN1,K
 0275                N(I,I)=N(I,I)+PRATIO(IW)
 0276          2     CONTINUE
               C
 0277                CALL DMINV(N,K8,DET,L1,L2)
               C
 0278                DO 187 I=1,KK
 0279                XX(I)=0.D0
 0280          187   CONTINUE
               C
```

```
            DO 14 I=1,KK
            DO 14 J=1,KK
            XX(I)=XX(I)+N(I,J)*U(J)
      14    CONTINUE
      C
      C     WRITE     XX
      C
            DO 9 I=1,5
            CALL REDPI(XX(I))
      9     CONTINUE
      C
            ISTART=7+3*IN
      C
            DO 26 I=ISTART,K
            CALL REDPI(XX(I))
      26    CONTINUE
      C
      C
      C*****************************************************************************
      C
      C          READ A AND B (ROWS OF DESIGN MATRIX) FROM DISK 8,
      C          FORMULATE RESIDUALS AND COMPUTE VARIANCE COVARIANCE MATRIX
      C
      C*****************************************************************************
      C
      85    REWIND 8
            VTPV=0.0D0
      C
            VABS1=0.D0
            VABS2=0.D0
      C
            DO 22 II=1,ICOUNT
            READ(8,100) I,J,IP
      C
            DO 27 IDUM=1,2
            READ(8,351) (F(JJ),JJ=1,13),L
            V=0.D0
      C
            DO 23 JJ=1,6
            V=V+F(JJ)*XX(JJ)
      23    CONTINUE
      C
            I1=3*I+4
            I2=I1+1
            I3=I1+2
            J1=3*J+4
            J2=J1+1
            J3=J1+2
            V=V+F(7)*(XX(J1)-XX(I1))+F(8)*(XX(J2)-XX(I2))
           2   +F(9)*(XX(J3)-XX(I3))
      C
            IP1=3*IN+2*IP+5
            IP2=IP1+1
            V=V+F(10)*XX(IP1)+F(11)*XX(IP2)
      C
            IC1=K+I
            IC2=IC1+IN
            JC1=K+J
```

```
0317              JC2=JC1+IN
0318              V=V+F(12)*(XX(JC1)-XX(IC1))+F(13)*(XX(JC2)-XX(IC2))
            C
0319              V=V+L
            C
0320              IF(IDUM.EQ.2) GO TO 28
0321              VABS1=VABS1+DABS(V)
0322              VTPV=VTPV+V**2
0323              GO TO 27
            C
0324        28    VABS2=VABS2+DABS(V)
0325              VTPV=VTPV+PF*V**2
0326        27    CONTINUE
            C
0327        22    CONTINUE
            C
0328              VABS1=VABS1/ICOUNT
0329              VABS2=VABS2/ICOUNT
0330              IIN=3*IN+6
0331              IIN1=IIN+1
            C
            C         ALTERNATIVE FOR SO COMPUTATION
            C     SO=VTPV+DFLOAT(IM*2)*(SDBD/R100)**2
            C     SO=SO/DFLOAT(ICOUNT*2-KK+2*IM)
            C
0332              DO 172 I=IIN1,K
0333              VTPV=VTPV+PRATIO(IW)*XX(I)**2
0334        172   CONTINUE
0335              SO=VTPV/DFLOAT(2*ICOUNT-KK+2*IM)
            C
0336              DO 24 I=1,KK
0337              DO 24 J=1,KK
0338              N(I,J)=N(I,J)*SO
0339        24    CONTINUE
            C
            C
0340              K1=K+1
0341              KIN=K+IN
0342              KIN1=KIN+1
            C
0343              DO 41 J=1,KK
0344              N(1,J)=R100*N(1,J)
0345              N(2,J)=R100*N(2,J)
0346              N(3,J)=R100*N(3,J)
0347              N(4,J)=R100*N(4,J)
0348              N(5,J)=RASM3*N(5,J)
0349              N(6,J)=RHRDM9*N(6,J)
            C
0350              DO 42 I=7,IIN
0351              N(I,J)=N(I,J)*R100
0352        42    CONTINUE
            C
0353              DO 43 I=IIN1,K
0354              N(I,J)=N(I,J)*RASM3
0355        43    CONTINUE
0356              DO 47 I=K1,KIN
0357              N(I,J)=(N(I,J)/CLIGHT)*1000.D0
0358        47    CONTINUE
```

```
                C
 0359                 DO 48 I=KIN1,KK
 0360                 N(I,J)=(N(I,J)/CLIGHT)*1000.D0
 0361           48    CONTINUE
                C
 0362           41    CONTINUE
                C
 0363                 DO 44 I=1,KK
 0364                 N(I,1)=R100*N(I,1)
 0365                 N(I,2)=R100*N(I,2)
 0366                 N(I,3)=R100*N(I,3)
 0367                 N(I,4)=R100*N(I,4)
 0368                 N(I,5)=RASM3*N(I,5)
 0369                 N(I,6)=RHRDM9*N(I,6)
                C
 0370                 DO 45 J=7,IIN
 0371                 N(I,J)=N(I,J)*R100
 0372           45    CONTINUE
                C
 0373                 DO 46 J=IIN1,K
 0374                 N(I,J)=N(I,J)*RASM3
 0375           46    CONTINUE
                C
 0376                 DO 49 J=K1,KIN
 0377                 N(I,J)=(N(I,J)/CLIGHT)*1000.D0
 0378           49    CONTINUE
                C
 0379                 DO 52 J=KIN1,KK
 0380                 N(I,J)=(N(I,J)/CLIGHT)*1000.D0
 0381           52    CONTINUE
                C
 0382           44    CONTINUE
                C
 0383           227   FORMAT(10X,3F20.4)
 0384           229   FORMAT(10X,2F22.6)
 0385           222   FORMAT(5X,F40.9)
                C
 0386                 DO 86 I=1,KK
 0387                 N(I,I)=DSQRT(N(I,I))
 0388           86    CONTINUE
                C
 0389                 ICNT2=ICOUNT*2
 0390                 IDF=ICNT2-KK+2*IM
 0391                 WRITE(6,696) ICNT2,IDF
 0392           696   FORMAT(/////5X,'NO OF OBSERVATIONS =',I5//5X
                     2,'DEGREES OF FREEDOM =',I5/)
                C
 0393                 WRITE(6,850) IEXP,SDBD,SDBF,SDQUA(IW)
 0394           850   FORMAT('1'///15X,'EXPERIMENT NO =',I5///10X
                     1,'A PRIORI STANDARD DEVIATIONS',//5X
                     2,'DELAY DISTANCES (CM)',10X,F15.5/5X
                     3,'DISTANCE DERIVATIVES (M/HOUR)',1X,F15.5/5X
                     4,'QUASAR COORDINATES (CM)',7X,F15.5/)
                C
 0395                 WRITE(6,356) (N(I,I),I=1,6)
 0396           356   FORMAT(///20X,'STANDARD DEVIATIONS'/20X
                     1,'*******************'//5X
                     2,'POLAR MOTION =',7X,F14.5,3X,'CM',/26X,F14.5,3X,'CM'/5X
```

```
                    3,'PRECESSION-NUTATION =',F14.5,3X,'CM',/26X,F14.5,3X,'CM',/5X
                    4,'THETA ZERO =',9X,F14.5,3X,'ARCSEC/1000'/5X
                    5,'OMG =',16X,F14.5,3X,'REV PER DAY/10**9'//10X
                    6,'STATION COORDINATES (X,Y,Z) IN CM'/)
0397                 WRITE(6,357) (N(I,I),I=7,IIN)
0398         357     FORMAT(10X,3F20.5)
0399                 WRITE(6,358)
0400         358     FORMAT(/10X,'QUASAR COORDINATES (RA,DEC) IN ARCSEC/1000'/)
0401                 WRITE(6,359) (N(I,I),I=IIN1,K)
0402         359     FORMAT(10X,2F20.5)
             C
0403                 WRITE(6,361)
0404         361     FORMAT(//10X,'STATION CLOCK OFFSETS IN MSEC'/)
0405                 WRITE(6,362) (N(I,I),I=K1,KIN)
0406         362     FORMAT(2X,F20.9)
0407                 WRITE(6,363)
0408         363     FORMAT(//10X,'STATION CLOCK DRIFTS IN MSEC PER DAY'/)
0409                 WRITE(6,362) (N(I,I),I=KIN1,KK)
             C
             C
0410         162     CONTINUE
             C
0411         86      STOP
0412                 END
```

3. Name of Program: PROGØ2

Function:

Adjustment of observations with a time span equal to the station step. Station coordinates are considered constant over the whole set of observations, while earth rotation parameters are treated as constant but different over a prearranged set of subintervals.

Input:

Standard deviations of observations and radio source coordinates, approximate values of parameters, station and radio source identification numbers, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

MADD, MATPRO, TRAPRO, PROTRA for matrix operations and REDPI.

(Following is a listing of PROGØ2 and supporting subroutines, except for REDPI already included in VLBI SIMULATOR.)

```
      C*************************************************************************
      C*********                                                      **********
      C*********                    PROG02                            **********
      C*********                                                      **********
      C*********          VLBI  ADJUSTMENT  PROGRAM                   **********
      C*********                                                      **********
      C*********          THIS VERSION PERFORMS THE ADJUSTMENT OF     **********
      C*********          A SET OF OBSERVATIONS TREATING STATION      **********
      C*********          COORDINATES AS CONSTANTS. EARTH ROTATION    **********
      C*********          PARAMETERS ARE CONSTANT BUT DIFFERENT       **********
      C*********          OVER SUBINTERVALS OF THE ORIGINAL TOTAL     **********
      C*********          TIME INTERVAL OF THE AVAILABLE OBSERVATIONS.**********
      C*********                                                      **********
      C*********          OBSERVATIONS: DELAY DISTANCES AND           **********
      C*********                        THEIR DERIVATIVES             **********
      C*********                                                      **********
      C*********          UNKNOWNS: EARTH MOTION PARAMETERS           **********
      C*********                    STATION & QUASAR COORDINATES      **********
      C*********                    CLOCK OFFSETS AND DRIFTS          **********
      C*********                                                      **********
      C*********          SYSTEMS DEFINITION:                         **********
      C*********          QUASAR FIXED: WEIGHTED QUASAR COORDINATES   **********
      C*********          EARTH FIXED: BY MEANS OF INNER CONSTRAINTS  **********
      C*********                                                      **********
      C*********                                                      **********
      C*********          SUBROUTINES REQUIRED :                      **********
      C*********                                   REDPI              **********
      C*********                                   MADD               **********
      C*********                                   MATPRO             **********
      C*********                                   TRAPRO             **********
      C*********                                   PROTRA             **********
      C*********                                                      **********
      C*************************************************************************
      C*********                                                      **********
0001         IMPLICIT REAL*8(A-H,L-Z)
0002         DIMENSION DO(3),DD(3),X(3),Y(3),Z(3),RA(9),D(9),ED(33,8),U(39)
            2,NKEEP(39,39),A(39),B(39),NB(6,33),UDI(33),HI(6,33),RI(33,33)
            3,QB(41,41),RLI(33),H(6,33),RL(33),UD(33),HTEDO(33,8),L1K(41)
            4,L2K(41),GB(33,33),FB(33,8),GBL(33),FET(33),GU(33),FBEOT(33,6)
            5,GBHT(33,6),FETN(33,6),XBI(6,20),STD(153),XB(33)
            6,EDO(6,8),UDDI(6),NDD(6,6),L16(6),L26(6),TI(6),RK(6,6),RT(6)
            7,RKEDO(6,8),ETKE(8,8),FF(8,8),ETOT(8),FEDT(8),FBTL(8),FBTU(8)
            8,SOL(6),EDF(6,8),EFET(6,6),GDD(6,6),HGHT(6,6),HFEN(6,6),EFEN(6,6)
            9,NEFEN(6,6),HIXB(6),NSOL(6),F(13)
      C      DIMENSION DO(IN),DD(IN),X(IN),Y(IN),Z(IN),RA(IM,D(IM),ED(K,8)
      C     2,U(KK),NKEEP(KK,KK),A(KK),B(KK),NB(6,K)UDI(K),HI(6,K),RI(K,K)
      C     3,QB(K8,K8),RLI(K),H(6,K),RL(K),UD(K),HTEDO(K,8),L1K(K),L2K(K)
      C     4,GB(K,K),FB(K,8),GBL(K),FET(K),GU(K),FBEO (K,6),GBHT(K,6)
      C     5,FETN(K,6),XBI(6,IDCNT),STD(6*IDCNT+K),XB(K),
      C     6,EDO(6,8),UDDI(6),NDD(6,6),L16(6),L26(6),TI(6),RK(6,6),RT(6)
      C     7,RKEDO(6,8),ETKE(8,8),FF(8,8),ETOT(9),FEDT(8),FBTL(8),FBTU(8)
      C     8,SOL(6),EDF(6,8),EFET(6,6),GDD(6,6),HGHT(6,6),HFEN(6,6),EFEN(6,6)
      C     9,NEFEN(6,6)HIXB(6),NSOL(6),F(13)
      C          WHERE:  K=5*IN+2*IM,KK=K+6,K8=K+8,IN=NO OF STATIONS,IM=NO OF QUASARS
0003         EQUIVALENCE (HTEDO,FB),(UDI,L1K,GBL),(RLI,L2K,FET),(RI,GB)
            2,(RKEDO,EDF,GBHT),(EDO,FETN),(RT,HIXB),(L16,TI),(L26,NSOL)
            3,(RK,NEFEN),(HGHT,HFEN,EFEN),(ETKE,FF),(ETOT,FBTL,FBTU)
      C
```

```
              C          READ EXPERIMENT IDENTIFICATION NO
 0004                 READ(5,100) IEXP
 0005                 WRITE(6,977) IEXP
 0006         977     FORMAT('1'////10X,'EXPERIMENT NO =',I5/10X
                    2,'***********************',/////)
              C
              C
              C          READ A PRIORI STANDARD DEVIATIONS:
              C          SDBD : OF DELAY DISTANCE IN CM
              C          SDBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
              C          SDQUA: OF QUASAR COORDINATES IN CM
              C
 0007                 READ(5,101) SDBD,SDBF
 0008         101     FORMAT(5F15.5)
              C
 0009                 WRITE(6,240) SDBD,SDBF
 0010         240     FORMAT(10X,'OBSERVATIONAL NOISE'/15X,'DISTANCES
                    2,F10.5/15X,'DISTANCE RATES ',F10.5//)
              C
 0011                 READ(5,101) SDQUA
 0012                 PRATIO=SDBD**2/SDQUA**2
              C
              C          READ EARTH STEP INTERVAL IN HOURS
              C
 0013                 READ(5,101) ERSTEP
 0014                 WRITE(6,243) ERSTEP
 0015         243     FORMAT(//10X,'EARTH STEP INTERVAL (HRS) =',F10.2/)
              C
 0016                 R=6371000.D0
 0017                 CLIGHT=3.D0*(10.D0**8)
 0018                 CLIGHT=CLIGHT/R
 0019                 PI=4.D0*DATAN(1.D0)
 0020                 R100=R*100.D0
 0021                 RHRDM9=(12.D0/PI)*(10.D0**9)
 0022                 RASM3=(180.D0/PI)*3600000.D0
              C
 0023                 PF=(SDBD/R100)/(SDBF/R)
 0024                 PF=PF**2
              C           PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2
              C
              C         READ IN= NO OF STATIONS & IM= NO OF QUASARS
              C*************************************************************************
 0025                 READ(5,100) IN, IM
 0026         100     FORMAT(10I5)
 0027                 WRITE(6,920) IN,IM
 0028         920     FORMAT(5X,'NO OF STATIONS =',I5/5X,'NO OF QUASARS  =',I5)
 0029                 K=5*IN+2*IM
 0030                 KK=K+6
 0031                 K8=K+8
              C
              C
 0032                 DO 51 I=1,IN
 0033                 DO(I)=0.D0
 0034                 DD(I)=0.D0
 0035         51      CONTINUE
              C
              C         READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, OMG,
              C*************************************************************************
```

```
              C        X(I), Y(I), Z(I), (I=1,IN),  RA(J), D(J), (J=1,IM)
              C        ANGLES ARE IN DEGREES, DISTANCES IN METERS AND OMG IN RADIANS PER HOUR
 0036                 READ(5,101) TH
 0037                 WRITE(6,860) TH
 0038         860     FORMAT(//5X,'THETA ZERO = ',F20.9/)
 0039                 THET=TH*PI/180.D0
 0040                 OMG=PI/12.D0
              C
 0041                 WRITE(6,450)
 0042         450     FORMAT(////,10X,'STATION & RADIO SOURCES APPROXIMATE COORDINATES'
                     ?,//3X,'STATIONS',10X,'LONG'
                     2,12X,'LAT',11X,'R',/)
              C
 0043                 DO 5 I=1,IN
 0044                 READ(5,701) IDM,IL1,IL2,ALONG,APHI,AR
 0045                 WRITE(6,701)  I,IL1,IL2,ALONG,APHI,AR
 0046         701     FORMAT(1X,I2,1X,A4,A2,3F15.1)
 0047                 APHI=APHI*PI/180.D0
 0048                 ALONG=ALONG*PI/180.D0
              C          CONVERT TO CARTESIAN
 0049                 X(I)=AR*DCOS(APHI)*DCOS(ALONG)
 0050                 Y(I)=AR*DCOS(APHI)*DSIN(ALONG)
 0051                 Z(I)=AR*DSIN(APHI)
              C        CONVERT TO EARTH RADII UNITS
 0052                 X(I)=X(I)/R
 0053                 Y(I)=Y(I)/R
 0054                 Z(I)=Z(I)/R
 0055         5       CONTINUE
              C
              C
 0056                 WRITE(6,451)
 0057         451     FORMAT(//11X,'RADIO SOURCE',10X,'RA',17X,'DEC',//)
              C
 0058                 DO 6 J=1,IM
 0059                 READ(5,321) IDM,IL1,IL2,IL3,RA(J),D(J)
 0060                 WRITE(6,321) J,IL1,IL2,IL3,RA(J),D(J)
 0061         321     FORMAT(1X,I3,7X,3A4,F17.7,F20.9)
              C        CONVERT TO RADIAN UNITS
 0062                 RA(J)=RA(J)*PI/180.D0
 0063                 D(J) = D(J)*PI/180.D0
 0064         6       CONTINUE
              C
              C
              C          FORMULATE CONSTRAINT MATRICES ED, EDO
 0065                 DO 11 J=1,S
 0066                 DO 9 I=1,6
 0067                 EDO(I,J)=0.D0
 0068         9       CONTINUE
 0069                 DO 11 I=1,K
 0070                 ED(I,J)=0.D0
 0071         11      CONTINUE
              C
 0072                 EDO(1,5)=1.D0
 0073                 EDO(2,4)=1.D0
 0074                 EDO(5,6)=-1.D0
              C
 0075                 DO 3 I=1,IN
 0076                 IB1=3*I-2
```

```
0077            IB2=IB1+1
0078            IB3=IB2+1
0079            ED(IB1,1)=1.D0
0080            ED(IB2,2)=1.D0
0081            ED(IB3,3)=1.D0
0082            ED(IB2,4)=-Z(I)
0083            ED(IB3,4)=Y(I)
0084            ED(IB1,5)=Z(I)
0085            ED(IB3,5)=-X(I)
0086            ED(IB1,6)=-Y(I)
0087            ED(IB2,6)=X(I)
0088            IC1=3*IN+2*IM+I
0089            IC2=IC1+IN
0090            ED(IC1,7)=1.D0
0091            ED(IC2,8)=1.D0
0092      3     CONTINUE
0093            IDCNT=0
0094            ICOUNT=0
          C
          C          INITIATE H, RK, RL, RT, UD, QB  TO ZERO
          C*************************************************************************
          C
0095            DO 71 I=1,K
0096            RL(I)=0.D0
0097            UD(I)=0.D0
0098            DO 71 J=1,6
0099            H(J,I)=0.D0
0100      71    CONTINUE
          C
0101            DO 72 I=1,6
0102            RT(I)=0.D0
0103            DO 72 J=1,6
0104            RK(I,J)=0.D0
0105      72    CONTINUE
0106            DO 73 I=1,K8
0107            DO 73 J=1,K8
0108            QB(I,J)=0.D0
0109      73    CONTINUE
          C
          C
          C          INITIATE NKEEP,U TO ZERO
          C*************************************************************************
0110      55    DO 16 I=1,KK
0111            U(I)=0.D0
0112            DO 16 J=1,KK
0113            NKEEP(I,J)=0.D0
0114      16    CONTINUE
          C
0115            TH=THET+(EKSTEP/12.D0)*PI*DFLOAT(IDCNT)
          C
          C*************************************************************************
          C
          C          READ OBSERVATIONS AND IDENTIFIERS:
          C          DS= DISTANCE IN METERS OF IJ BASELINE OBSERVATION TO QUASAR IP
          C              AT TIME TK (IN HOURS) AFTER SOME INITIAL EPOCH TO
          C*************************************************************************
          C
          C
```

```
              C
 0116         33     READ(5,104) I,J,IP,DS,FRNG,TK,ICHEK
 0117         104    FORMAT(3I5,3F15.5,2I5)
              C
              C         ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1
 0118                IF(ICHEK.EQ.1) GO TO 66
 0119                IF(ICHEK.EQ.2) GO TO 66
 0120                ICOUNT=ICOUNT+1
 0121                TK=TK-FRSTEP*DFLOAT(IDCNT)
              C
 0122                DS=DS/R
 0123                FRNG=FRNG/R
 0124                DX=X(J)-X(I)
 0125                DY=Y(J)-Y(I)
 0126                DZ=Z(J)-Z(I)
 0127                CA=DCOS(RA(IP))
 0128                SA=DSIN(RA(IP))
 0129                CD=DCOS(D(IP))
 0130                SD=DSIN(D(IP))
 0131                Y1=OMG+TK+TH
 0132                Y2=Y1-RA(IP)
 0133                CK=DCOS(Y1)
 0134                SK=DSIN(Y1)
 0135                CKP=DCOS(Y2)
 0136                SKP=DSIN(Y2)
              C
              C               FORMULATE PARTIALS
              C
 0137                F1=-CD*DZ*CKP+DX*SD
 0138                F2=-CD*DZ*SKP-DY*SD
 0139                F3=-DX*SD*CK+DY*SD*SK+DZ*CD*CA
 0140                F4=DX*SK*SD+DY*CK*SD-DZ*CD*SA
 0141                F5=-(DX*SKP+DY*CKP)*CD
 0142                F6= TK*F5
 0143                F7=CD*CKP
 0144                F8=-CD*SKP
 0145                F9=SD
 0146                F10=CD*(DX*SKP+DY*CKP)
 0147                F11=(-DX*CKP+DY*SKP)*SD+DZ*CD
 0148                F12=1.D0
 0149                F13=TK/24.D0
              C
 0150                G1=DZ*CD*SKP
 0151                G1=OMG*G1
 0152                G2=-DZ*CD*CKP
 0153                G2=OMG*G2
 0154                G3=(DX*SK+DY*CK)*SD
 0155                G3=OMG*G3
 0156                G4=DX*CK*SD-DY*SK*SD
 0157                G4=OMG*G4
 0158                G5=-DX*CD*CKP+DY*CD*SKP
 0159                G5=OMG*G5
 0160                G6=TK*G5
 0161                G7=-CD*SKP
 0162                G7=OMG*G7
 0163                G8=-CD*CKP
 0164                G8=OMG*G8
 0165                G9=0.D0
```

```
0166          G10=DX*CD*CKP-DY*CD*SKP
0167          G10=OMG*G10
0168          G11=DX*SD*SKP+DY*SD*CKP
0169          G11=OMG*G11
0170          G12=0.D0
0171          G13=1.D0/24.D0
        C
        C          INITIATE A AND B TO ZERO
        C*************************************************************************
        C
0172          DO 8 II=1,KK
0173          A(II)=0.D0
0174          B(II)=0.D0
0175    8     CONTINUE
        C
        C         SET PARTIALS IN APPROPRIATE LOCATIONS OF A ROW
        C*************************************************************************
        C
0176          A(1)=F1
0177          A(2)=F2
0178          A(3)=F3
0179          A(4)=F4
0180          A(5)=F5
0181          A(6)=F6
        C
0182          B(1)=G1
0183          B(2)=G2
0184          B(3)=G3
0185          B(4)=G4
0186          B(5)=G5
0187          B(6)=G6
        C
0188          I1=3*I+4
0189          I2=I1+1
0190          I3=I1+2
0191          A(I1)=-F7
0192          A(I2)=-F8
0193          A(I3)=-F9
        C
0194          B(I1)=-G7
0195          B(I2)=-G8
0196          B(I3)=-G9
        C
0197          IC1=6+3*IN+2*IM+I
0198          IC2=IC1+IN
0199          A(IC1)=-F12
0200          A(IC2)=-F13
        C
0201          B(IC1)=-G12
0202          B(IC2)=-G13
        C
0203          J1=3*J+4
0204          J2=J1+1
0205          J3=J1+2
0206          A(J1)= F7
0207          A(J2)= F8
0208          A(J3)= F9
        C
```

```
0209             B(J1)=G7
0210             B(J2)=G8
0211             B(J3)=G9
           C
0212             JC1=6+3*IN+2*IM+J
0213             JC2=JC1+IN
0214             A(JC1)=F12
0215             A(JC2)=F13
           C
0216             B(JC1)=G12
0217             B(JC2)=G13
           C
0218             IP1=3*IN+2*IP+5
0219             IP2=IP1+1
0220             A(IP1)=F10
0221             A(IP2)=F11
           C
0222             B(IP1)=G10
0223             B(IP2)=G11
           C
           C     FORMULATIOM OF L VECTOR ENTRY
           C
0224       50    DSO=DX*F7+DY*F8+DZ*F9
0225             DSO=DSO+DO(J)-DO(I)+(DD(J)-DD(I))*(TK/24.DO)
0226             L=DSO-DS
           C     LWI=L*R100
           C
           C
           C*********************************************************************
           C        STORF A, B, L AND OBSERVATION IDENTIFIERS ON DISK 8 FOR LATER
           C         USE IN COMPUTATION OF RESIDUALS AN COVARIANCES
           C*********************************************************************
           C
0227             WRITE(8,100) ICHEK,I,J,IP
0228             WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L
0229       351   FORMAT(10X,F40.15)
           C
0230             DO 12 II=1,KK
0231             U(II)=U(II)+A(II)*L
0232             DO 12 JJ=1,KK
0233             NKEEP(II,JJ)=NKEEP(II,JJ)+A(II)*A(JJ)
0234       12    CONTINUE
           C
           C
0235             FRNGO=DX*G7+DY*G8+DZ*G9
0236             FRNGO=FRNGO+(DD(J)-DD(I))/24.DO                          ???????
0237             L=FRNGO-FRNG
           C     LW2=L*R
           C     WRITE(6,350) LW1,LW2
           C
0238             WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L
0239             DO 65 II=1,KK
0240             U(II)=U(II)+B(II)*L*PF
0241             DO 65 JJ=1,KK
0242             NKEEP(II,JJ)=NKEEP(II,JJ)+B(II)*B(JJ)*PF
0243       65    CONTINUE
           C
0244       350   FORMAT(80X,2F19.9)
```

```
                C
0245                  GO TO 33
                C
                C
                C*****************************************************************************
                C
                C
0246            66    WRITE(8,100) ICHEK
                C
0247                  DO 20 I=1,6
0248                  UDDI(I)=U(I)
0249                  DO 20 J=1,6
0250                  NDD(I,J)=NKEEP(I,J)
0251            20    CONTINUE
0252                  DO 21 I=1,6
0253                  DO 21 J=1,K
0254                  J6=J+6
0255                  NB(I,J)=NKEEP(I,J6)
0256            21    CONTINUE
                C
0257                  DO 22 I=1,K
0258                  I6=I+6
0259                  UDI(I)=U(I6)
0260            22    CONTINUE
                C
0261                  CALL DMINV(NDD,6,DFT,L16,L26)
0262                  CALL MATPRO(NDD,NB,HI,6,6,K)
0263                  CALL TRAPRO(NB,HI,RI,K,6,K)
0264                  DO 23 I=1,K
0265                  DO 23 J=1,K
0266                  I6=I+6
0267                  J6=J+6
0268                  QB(I,J)=QB(I,J)+NKEEP(I6,J6)-RI(I,J)
0269            23    CONTINUE
                C
0270                  CALL MATPRO(NDD,UDDI,TI,6,6,1)
0271                  CALL TRAPRO(NB,TI,RLI,K,6,1)
0272                  CALL MADD(H,HI,6,K)
0273                  CALL MADD(RK,NDD,6,6)
0274                  CALL MADD(RL,RLI,K,1)
0275                  CALL MADD(RT,TI,6,1)
0276                  CALL MADD(UD,UDI,K,1)
                C
                C*****************************************************************************
                C                STORE UDDI,NDD,HI,TI  ON DISK 9
                C*****************************************************************************
                C
0277                  DO 25 I=1,6
0278                  DO 25 J=1,6
0279                  WRITE(9,351) NDD(I,J)
0280            25    CONTINUE
                C
0281                  DO 26 I=1,6
0282                  DO 26 J=1,K
0283                  WRITE(9,351) HI(I,J)
0284            26    CONTINUE
                C
0285                  DO 27 I=1,6
```

```
                WRITE(9,351) TI(I)
         27     CONTINUE
                IDCNT=IDCNT+1
                IF(ICHEK.EQ.2) GO TO 67
                GO TO 55
         C
         C
         C*****************************************************************************
         C
         C
         67     CALL TRAPRO(H,EDO,HTEDO,K,6,8)
         C
                DO 28 I=1,K
                DO 28 J=1,8
                JK=J+K
                QB(I,JK)=ED(I,J)-HTEDO(I,J)
                QB(JK,I)=QB(I,JK)
         28     CONTINUE
         C
                CALL MATPRO(RK,EDO,RKEDO,6,6,8)
                CALL TRAPRO(EDO,RKEDO,ETKE,8,6,8)
         C
                DO 29 I=1,8
                IK=I+K
                DO 29 J=1,8
                JK=J+K
                QB(IK,JK)=-ETKE(I,J)
         29     CONTINUE
         C
                IPF=3*IN+1
                IPL=3*IN+2*IM
                DO 34 I=IPF,IPL
                QB(I,I)=QB(I,I)+PRATIO
         34     CONTINUE
         C
                CALL DMINV(QB,K8,DET,L1K,L2K)
         C
                DO 30 I=1,K
                DO 30 J=1,K
                GB(I,J)=QB(I,J)
         30     CONTINUE
         C
                DO 31 I=1,K
                DO 31 J=1,8
                JK=J+K
                FB(I,J)=QB(I,JK)
         31     CONTINUE
         C
                DO 32 I=1,8
                IK=I+K
                DO 32 J=1,8
                JK=J+K
                FF(I,J)=QB(IK,JK)
         32     CONTINUE
         C
                CALL MATPRO(GB,RL,GBL,K,K,1)
                CALL TRAPRO(EDO,RT,ETOT,8,6,1)
                CALL MATPRO(FB,ETOT,FET,K,8,1)
```

```
            CALL MATPRO(GB,UD,GU,K,K,1)
      C
            DO 41 I=1,K
            XB(I)=GBL(1)+FET(I)-GU(I)
      41    CONTINUE
      C
            CALL MATPRO(FF,ETOT,FEDT,8,8,1)
            CALL TRAPRO(FB,RL,FBTL,8,K,1)
            CALL MADD(FEDT,FBTL,8,1)
            CALL TRAPRO(FB,UD,FBTU,8,K,1)
      C
            DO 44 I=1,8
            FEDT(I)=FEDT(I)-FBTU(I)
      44    CONTINUE
      C
            CALL MATPRO(EDO,FEDT,SOL,6,8,1)
            CALL PROTRA(FB,EDO,FBEOT,K,8,6)
            CALL MATPRO(EDO,FF,EDF,6,8,8)
            CALL PROTRA(EDF,EDO,EFET,6,8,6)
      C
            REWIND 9
      C
            DO 60 IDAY=1,IDCNT
      C
      C
            DO 35 I=1,6
            DO 35 J=1,6
            READ(9,351) NDD(I,J)
      35    CONTINUE
      C
            DO 36 I=1,6
            DO 36 J=1,K
            READ(9,351) HI(I,J)
      36    CONTINUE
      C
            DO 37 I=1,6
            READ(9,351) TI(I)
      37    CONTINUE
      C
            DO 38 I=1,6
            DO 38 J=1,6
            GDD(I,J)=NDD(I,J)
      38    CONTINUE
      C
            CALL PROTRA(GB,HI,GBHT,K,K,6)
            CALL MATPRO(HI,GBHT,HGHT,6,K,6)
            CALL MADD(GDD,HGHT,6,6)
            CALL MATPRO(FBEOT,NDD,FETN,K,6,6)
            CALL MATPRO(HI,FETN,HFEN,6,K,6)
      C
            DO 39 I=1,6
            DO 39 J=1,6
            GDD(I,J)=GDD(I,J)+HFEN(I,J)+HFEN(J,I)
      39    CONTINUE
            CALL MATPRO(EFET,NDD,EFEN,6,6,6)
            CALL MATPRO(NDD,EFEN,NEFEN,6,6,6)
            CALL MADD(GDD,NEFEN,6,6)
      C
```

```
C
C
C
      DO 48 I=1,6
      IPF=6*IDAY-6+I
      STD(IPF)=GDD(I,I)
48    CONTINUE
C
C
      CALL MATPRO(NDD,SOL,NSOL,6,6,1)
      CALL MATPRO(HI,XB,HIXB,6,K,1)
C
      DO 40 I=1,6
      XBI(I,IDAY)=-TI(I)-NSOL(I)-HIXB(I)
40    CONTINUE
60    CONTINUE
C
C
C***********************************************************************
C
C
      DO 49 I=1,K
      IPL=6*IPCNT+I
      STD(IPL)=GB(I,I)
49    CONTINUE
C
      REWIND 8
      IDAY=1
      VTPV=0.D0
C
77    READ(8,100) ICHEK,I,J,IP
      IF(ICHEK.EQ.1) GO TO 88
      IF(ICHEK.EQ.2) GO TO 89
      IS1=3*I-2
      IS2=IS1+1
      IS3=IS2+1
      JS1=3*J-2
      JS2=JS1+1
      JS3=JS2+1
      IP1=3*IN+2*IP-1
      IP2=IP1+1
      ID1=3*IN+2*IM+I
      JD1=3*IN+2*IM+J
      ID2=ID1+IN
      JD2=JD1+IN
      DO 45 JJ=1,2
      READ(8,351) (F(II),II=1,13),L
      V=L
      DO 42 II=1,6
      V=V+F(II)*XBI(II,IDAY)
42    CONTINUF
C
      V=V+F(7)*(XB(JS1)-XB(IS1))+F(8)*(XB(JS2)-XB(IS2))
     2   +F(9)*(XB(JS3)-XB(IS3))
      V=V+F(10)*XB(IP1)+F(11)*XB(IP2)
      V=V+F(12)*(XB(JD1)-XB(ID1))+F(13)*(XB(JD2)-XB(ID2))
      IF(JJ.EQ.2) GO TO 43
C     VW=V*R100
```

```
      C     WRITE(6,453) VW
            VTPV =VTPV+V**2
            GO TO 45
      43    VTPV=VTPV+PF*V**2
      C     VW=V*R
      C     WRITE(6,454) VW
      C454  FORMAT(50X,F30.15)
      C453  FORMAT(20X,F30.15)
      45    CONTINUE
            GO TO 77
      88    IDAY=IDAY+1
            GO TO 77
      89    IPF=3*IN+1
            IPL=3*IN+2*IM
            DO 46    I=IPF,IPL
            VTPV=VTPV+PRATIO*XB(I)**2
      46    CONTINUE
      C
            IS2=2*ICOUNT
            JS2=5*IN+6*IDCNT
            IS3=IS2-JS2
            IDC=(24.D0/ERSTEP+0.1D0)
            IDC=IDCNT/IDC
            WRITE(6,242) IDC,IS2,JS2,IS3
      242   FORMAT(///10X,'DAYS OF OBSERVATION=',I5/10X,'NO OF OBSERVATIONS ='
           2,I5/10X,'NO OF UNKNOWNS      =',I5/10X,'DEGREES OF FREEDOM =',I5//)
            S0=VTPV/DFLOAT(IS3)
      C
            IPL=K+6*IDCNT
            DO 47 I=1,IPL
            IF(STD(I).GT.0.D0) GO TO 999
            WRITE(6,998) I,STD(I)
      998   FORMAT(/////'***** ERROR *****    I=',I5/10X'STD=',F90.30/////)
      999   STD(I)=DSQRT(STD(I)*S0)
      47    CONTINUE
      C
      C*********      WRITE STANDARD DEVIATIOS
            WRITE(6,888)
      888   FORMAT('1'//10X,'STANDARD DEVIATIONS'/10X,'*******************'//
           2,3X,'DAY',10X,'POLAR MOTION',13X,'PRECESSION - NUTATION',7X
           3,'THETA ZERO',7X,'OMEGA'/12X,'KSI (CM)',7X,'ETA (CM)',7X
           4,'KSI (CM)',7X,'ETA (CM)',9X,'(CM)',6X,'(REV/DAY)*10**9'//)
      C
            DO 85 IDAY=1,IDCNT
            IPL=6*IDAY
            IPF=IPL-5
            DO 86 II=1,5
            JJ=IPF+II-1
            CALL REDPI(STD(JJ))
            STD(JJ)=STD(JJ)*R100
      86    CONTINUE
            STD(IPL)=STD(IPL)*RHRDM9
            WRITE(6,889) IDAY,(STD(J),J=IPF,IPL)
      889   FORMAT(/1X,I4,6F15.5)
      85    CONTINUE
            WRITE(6,221)
      221   FORMAT('1'///10X,'STATION  COORDINATES',//,17X,'X',9X,'Y',9X
           2,'Z'/15X,'(CM)',6X,'(CM)',6X,'(CM)'//)
```

```
0459              IPF=6*IDCNT+1
0460              IPL=6*IDCNT+3*IN+2*IM
0461              II=IPL-2*IM+1
0462              DO 91 I=II,IPL
0463              CALL REDPI(STD(I))
0464       91     CONTINUE
           C
0465              DO 75 I=IPF,IPL
0466              STD(I)=STD(I)*R100
0467       75     CONTINUE
           C
0468              IPL=IPL-2*IM
0469              WRITE(6,222) (STD(I),I=IPF,IPL)
0470       222    FORMAT(10X,3F10.2)
           C
0471              WRITE(6,223)
0472       223    FORMAT(/////10X,'QUASAR COORDINATES',//16X,'RA',8X,'DEC',/15X
                 2,'(CM)',6X,'(CM)'//)
0473              IPF=IPL+1
0474              IPL=IPL+2*IM
0475              WRITE(6,224) (STD(I),I=IPF,IPL)
0476       224    FORMAT(10X,2F10.2)
           C
0477              WRITE(6,225)
0478       225    FORMAT(/////10X,'STATION CLOCK OFFSETS AND DRIFTS'/13X
                 2,'(MSEC)',6X,'(MSEC/DAY)'//)
           C
0479              DO 76 I=1,IN
0480              IC1=IPL+I
0481              IC2=IC1+IN
0482              STD(IC1)=(STD(IC1)/CLIGHT)*1000.D0
0483              STD(IC2)=(STD(IC2)/CLIGHT)*1000.D0
0484              WRITE(6,226) STD(IC1),STD(IC2)
0485       226    FORMAT(4X,2F16.12)
0486       76     CONTINUE
           C
0487              STOP
0488              DEBUG SUBCHK
0489              END
```

```
 0001                  SUBROUTINE MADD(A,B,L,M)
                 C
                 C
                 C           A   =  A   +  B
                 C          L*M    L*M    L*M
                 C
                 C
 0002                  IMPLICIT REAL*8(A-H,O-Z)
 0003                  DIMENSION A(L,M),B(L,M)
 0004                  DO 5 I=1,L
 0005                  DO 5 J=1,M
 0006                  A(I,J)=A(I,J)+B(I,J)
 0007            5     CONTINUE
 0008                  RETURN
 0009                  END
```

```
 0001                 SUBROUTINE MATPRO(A,B,C,L,M,N)
                C
                C
                C          C   =   A * B
                C         L*N    L*M M*N
                C
                C
 0002                 IMPLICIT REAL*8(A-H,O-Z)
 0003                 DIMENSION A(L,M),B(M,N),C(L,N)
 0004                 DO 5 I=1,L
 0005                 DO 5 J=1,N
 0006                 C(I,J)=0.D0
 0007                 DO 5 K=1,M
 0008                 C(I,J)=C(I,J)+A(I,K)*B(K,J)
 0009           5     CONTINUE
 0010                 RETURN
 0011                 END
```

```
      SUBROUTINE TRAPRO(A,B,C,L,M,N)
C
C
C                T
C        C  =  A * B
C      L*N    L*M M*N
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(M,L),B(M,N),C(L,N)
      DO 5 I=1,L
      DO 5 J=1,N
      C(I,J)=0.D0
      DO 5 K=1,M
      C(I,J)=C(I,J)+A(K,I)*B(K,J)
    5 CONTINUE
      RETURN
      END
```

```
 0001                SUBROUTINE PROTRA(A,B,C,L,M,N)
           C
           C
           C                             T
           C            C   =  A * B
           C          L*N    L*M  M*N
           C
           C
 0002                IMPLICIT REAL*8(A-H,O-Z)
 0003                DIMENSION A(L,M),B(N,M),C(L,N)
 0004                DO 5 I=1,L
 0005                DO 5 J=1,N
 0006                C(I,J)=0.D0
 0007                DO 5 K=1,M
 0008                C(I,J)=C(I,J)+A(I,K)*B(J,K)
 0009      5         CONTINUE
 0010                RETURN
 0011                END
```