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CR 151716

APR 21 1978

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Ref: 642-6773  
Contract NAS 9-15200  
Job Order 73-743-21  
N78-23646

(NASA-CR-151716) SOIL MOISTURE MODELING  
REVIEW (Lockheed Electronics Co.) 32 p  
HC A03/MF A01      CSCL 08M

Unclas  
G3/46 16648

TECHNICAL MEMORANDUM  
SOIL MOISTURE MODELING REVIEW

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April 1978

LEC-11857

One advantage that the physical models have compared to the budget models is the ability to handle redistribution in the soil column in a precise manner, hence, to account for loss of water from drainage or gain from capillary rise. A disadvantage is the added difficulty of programming and coding the numerical solution techniques.

This literature review and limited evaluation indicate that the physical models have the potential to provide a more accurate and precise simulation of the profile changes of the soil moisture. Thus, it is recommended that further information regarding these models incorporating evapotranspiration be obtained and analyzed. In particular, documentation of the computer programs and a listing of the programs should be obtained, if available.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle SOIL MOISTURE MODELING REVIEW		5. Report Date April 1978	6. Performing Organization Code
7. Author(s) W. W. Hildreth		8. Performing Organization Report No. LEC-11857	
9. Performing Organization Name and Address Lockheed Electronics Company, Inc. Systems and Services Division Houston, Texas 77058		10. Work Unit No. 63-1557-3743	11. Contract or Grant No. NAS 9-15200
12. Sponsoring Agency Name and Address Earth Observations Division Space and Life Sciences Directorate - NASA/JSC Houston, Texas 77058		13. Type of Report and Period Covered Technical Memorandum	
15. Supplementary Notes		14. Sponsoring Agency Code	
16. Abstract  <p>In previous investigations, a number of soil moisture budget-type models that are based mainly on empirical relationships were documented. A determination of the state-of-the-art in soil moisture transport modeling based on physical or physiological principles was made and is presented in this report. In addition, some new models are presented.</p> <p>It was found that soil moisture models based on physical principles have been under development for more than 10 years. However, these models involve nonlinear equations that require numerical methods for their solution. Although these models have been shown to represent infiltration and redistribution of soil moisture quite well, evapotranspiration has not been as adequately incorporated into the models.</p>			
17. Key Words (Suggested by Author(s)) Budget models Evapotranspiration Modeling Physical models Soil moisture		18. Distribution Statement	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages	22. Price*

\*For sale by the National Technical Information Service, Springfield, Virginia 22161



## 1. INTRODUCTION

In two previous investigations [Hildreth (refs. 1, 2)], a number of soil moisture models that have been used in agricultural activities, such as irrigation scheduling, crop yield modeling, and precipitation runoff, were documented. These models are budget-type models in the sense that they keep track of the gains and losses of water in the soil layers through empirical relationships that incorporate only limited physical or physiological principles. Hence, these models may not have the best structure in which to incorporate remotely sensed data.

Presented in this report are the results of a literature search to determine what is the state-of-the-art in soil moisture transport modeling and to locate other existing soil moisture budgeting models, particularly those based on physical and physiological laws and principles.

## 2. TECHNICAL DISCUSSION — SOIL MOISTURE MODELS

The main objectives of soil moisture modeling for agricultural purposes are to keep track of the moisture distribution in the soil and of the transpiration by plants in order to determine the best estimate of crop yield. The soil moisture or water at any one point is affected by a number of factors which, acting in unison, tend to change the moisture content.

It is, perhaps, more illustrative to look first at the factors affecting a soil column that extends from the ground through the water table down to bedrock. One classification of this subsurface water system is shown in figure 1. For any location or time, the thickness of these zones may be quite different. In this study, the zone of aeration and, in particular, the soil water or root zone were the primary interests.

The factors which can account for the change in the amount of root zone moisture in a specific column during any given time interval are related by the following equation.

$$SM_t - SM_{t-1} = \Delta SM = P - RO + L + E - T + C - Q \quad (1)$$

where

$SM_t$  = soil moisture amount at one time

$SM_{t-1}$  = soil moisture amount at an earlier time

$\Delta SM$  = the change in soil moisture for the column layer

P = precipitation

RO = surface runoff

L = net subsurface lateral movement

E = evaporation or condensation

T = transpiration

C = capillary rise from lower levels

Q = percolation from one level to a lower level

The amount of water available for the column at the soil-atmosphere interface is the precipitation (P) minus the runoff (RO). This amount may be augmented by subsurface lateral gain or loss (L) or from condensation from the atmosphere (E) or from the capillary rise (C) from below. The loss of moisture from the root zone layer can be from evaporation (-E), from transpiration (T), or from lower boundary drainage (Q).

Moisture changes in lower layers below the surface layer are represented by

$$\Delta SM = L + C + Q \quad (2)$$

where now C and Q are the net changes in the layer caused by capillary rise and drainage, respectively.

The relative importance of each of the terms in the above equations depends on the soil, topography, depth of ground water table, agricultural practice, crop, and climate. In agricultural situations where nearly level fields are dominant, runoff and subsurface lateral change can generally be neglected. However, runoff can be important in heavy thunderstorm cases or in rainy periods of long duration.

There are two basic approaches presently in use to evaluate the above equations: (1) the accounting or budget approach and (2) the physical (dynamical) approach.

The budget approach employs empirical relations to estimate the terms on the right-hand side of equations (1) and (2) for each time period to determine the soil moisture change and the new soil moisture amount. The physical approach employs theoretical laws and principles, supplemented by empirical relations, to represent the physical processes.

The budget models have been discussed in some detail by Hildreth (ref. 2), a part of which is presented in modified and updated form in appendix A. The physical approach is given in the following sections.

## 2.1 THE PHYSICAL APPROACH

The equations that are generally used to model moisture movement through the soil are both theoretical and experimental. The first experimental equation represents Darcy's law stating that the saturated flow of water through a column of soil is directly proportional to the head difference and inversely proportional to the length of the column. Later it was determined that this law can be applied to unsaturated flow with low seepage velocities.

Thus, for three-dimensional flow in a homogeneous isotropic media, Darcy's law is represented by

$$\vec{V} = -K(\theta)(\nabla\phi) \quad (3)$$

where

$\vec{V}$  = the seepage velocity

$\phi$  = total potential (cm)

$\theta$  = moisture amount ( $\text{cm}^3/\text{cm}^3$ )

$K(\theta)$  = hydraulic conductivity (cm/sec) which is a function of the moisture amount

The theoretical equation needed is the equation of continuity which expresses the law that mass is continuous and is neither created or destroyed. Thus, the equation of continuity can be written

$$\frac{\partial(\rho\theta)}{\partial t} = -\nabla \cdot \rho\vec{V} = -\left[ \frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} \right] \quad (4)$$

where  $\rho$  is the density of the water. If density changes can be neglected, this becomes

$$\frac{\partial\theta}{\partial t} = -\nabla \cdot \vec{V} \quad (5)$$

The other experimental equation used relates the hydraulic conductivity  $K$  to the moisture content  $\theta$ .

$$K = K(\theta) \quad (6)$$

Combining Darcy's law and equation of continuity gives

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [K(\theta) \nabla \phi] \quad (7)$$

If it is assumed that the absorption potential, chemical potential, osmotic-pressure potential, and thermal potential may be neglected, then the total potential can be written:

$$\phi = \psi + Z \quad (8)$$

where

$\psi$  = moisture potential

$Z$  = gravitational potential  $\psi_g$  on an energy/unit weight basis

Equation (7) now becomes

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [K(\theta) \nabla (\psi + Z)] = \nabla \cdot [K(\theta) \nabla \psi] + \frac{\partial K(\theta)}{\partial z} \quad (9)$$

For homogeneous soil, let  $D(\theta) \equiv K(\theta)(d\psi/d\theta)$ , where  $D$  is the soil moisture diffusivity. Then

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [D(\theta) \nabla \theta] + \frac{dk}{d\theta} \frac{\partial \theta}{\partial z} \quad (10)$$

Equation (10) is sometimes known as the nonlinear Fokker-Planck equation.

If the last term in equation (10) is equal to zero (i.e., horizontal flow), then

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [D(\theta) \nabla \theta] \quad (11)$$

The above equation is known as the diffusion equation. For vertical flow only, equation (10) reduces to

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ D(\theta) \frac{\partial \theta}{\partial z} \right] + \frac{dk}{d\theta} \frac{\partial \theta}{\partial z} \quad (12)$$

These equations for moisture change are highly nonlinear, and exact solutions can be obtained only under highly restrictive conditions [e.g., Philip (ref. 4) and Parlange (ref. 5)]. In general, these equations are solved by numerical methods. Techniques and examples have been presented by Remson et al. (ref. 6). This is an excellent book by which to obtain a background on the necessary considerations for using the available numerical methods. A number of examples from the literature are referenced for each technique described. These references use some version of the Fokker-Planck equation, along with the appropriate boundary conditions, to form a numerical computer model for a solution of some soil moisture flow problem. Remson et al. (ref. 6) list about 30 papers that have numerical solutions.

The earliest and by far the most frequently referenced problem considered aspects of infiltration of water into the soil. Some of these models referenced by Remson et al. as well as several later models have been compared and evaluated by Haverkamp et al. (ref. 7). In this paper, six models, based on the soil moisture equation, each employing different ways of discretization of the equation were tested. The models were compared in terms of execution time, accuracy, and programming considerations. All models provided excellent agreement with measured soil water content and with the quasi-analytical solutions of Philip (ref. 4). The two explicit models, the  $\theta$ -based CSMP model and the  $h$ -based (i.e., soil moisture pressure) explicit model, required some 5 to 10 times more computer time than the implicit models but were easier to program. The authors stated that the results of the test indicate that numerical solutions of the soil moisture models can yield very accurate results at moderate costs in terms of computer time.

Numerical models dealing with other aspects of soil moisture movements in addition to infiltration, such as evaporation, drainage, and consequent redistribution, are more limited. A test of a typical soil water model capable of representing the changes occurring at several levels from infiltration, evaporation, and redistribution has been presented by Beese et al. (ref. 8). Their model is solved numerically in an explicit way using the IBM S/360 CSMP computer program. The test used data from a 218-day experiment

on a fallow loess soil to compare calculated infiltration and redistribution of soil moisture to measured values under natural field conditions. The test showed that the calculated values for all depths deviated less than 15-percent from the measured ones. The authors concluded that the numerical computer models of the moisture flow equation can be useful in calculating values to supplement field measurements.

Another similar numerical model programmed in CSMP language has been described in considerable detail by Hillel et al. (ref. 9). It was used later by Hillel and Van Bavel (ref. 10) to simulate moisture characteristics as affected by evaporation and drainage in a fallow soil.

## 2.2 CROP-SOIL-WATER MODELS

Numerical models representing moisture dynamics affected by transpiring plants are more limited than redistribution models under fallow conditions. For this problem, the basic soil moisture equation needs to be modified. This is done by the addition of a sink term which represents the water uptake by the roots.

Two main approaches have been taken to represent transpiring plants: (1) the microapproach, which considers the radial flow to a single root or group of roots, and (2) the macroapproach, which considers the integrated effect of the entire root system. In the macroapproach, the water uptake is represented by a volumetric sink term which is added to the continuity equation [Feddes et al. (ref. 11)]. The resulting equation for one-dimensional flow considering  $Z$  positive downward is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ D(\theta) \frac{\partial \theta}{\partial z} \right] - \frac{\partial K(\theta)}{\partial z} - S(\theta) \quad (13)$$

In order to solve this equation, the sink term  $S(\theta)$  has to be defined. Several different  $S(\theta)$  functions have been used. Feddes et al. (ref. 12), by means of a literature survey, pointed out that most current functions for  $S(\theta)$  are assumed to be directly proportional to the difference in pressure head between the soil and the root interior in the following manner.

$$S(\theta) = \frac{K(\theta)(h_s - h_r)}{b} \quad (14)$$

where  $h_s$  and  $h_r$  represent the pressure heads of the soil and the root, respectively, and where  $b$  can be considered an empirical root effectiveness function. Part of the difference among investigations is how this  $b$ -function is evaluated. Feddes et al. (ref. 12) showed that this function is proportional to root mass. Although they indicated that both varied exponentially with depth, they pointed out that the function will vary with soil type and rooting system and requires careful and expensive experimentation to determine the nature of the function. For this reason, Feddes et al. (ref. 11) evaluated an approach in which the water uptake by the roots was a function of the water content of the soil. Calculations made by a numerical model with an implicit finite difference formulation were compared to field data taken under red cabbage in the Netherlands. Although the calculated profiles indicated less moisture than the measured in the root zone towards the end of the growing season, the authors felt that the cumulative calculated evapotranspiration agreed well with the measured. An earlier model [Feddes et al. (ref. 12)] based on an estimated root zone distribution gave similar results for cumulative evapotranspiration except that the calculated profile had more water in the root zone toward the end of the period than was measured.

Another model using a more complicated  $S(\theta)$  term but a similar numerical formulation has been developed and tested by Nimah and Hanks (refs. 13, 14) but does not appear to give as good results as Feddes et al. (ref. 11).

Neuman et al. (ref. 15) have developed a two-dimensional model to simulate water uptake by roots which is solved by an approach similar to the Galerkin finite element (GFE) method. According to the authors, this method has several advantages over the conventional finite difference techniques. It can easily take care of nonuniform flow regions having irregular boundaries and arbitrary degrees of local anisotropy. Tests of this model by Feddes et al. (ref. 16) showed fair agreement with measurements.



The use of two integral methods to solve the problem of water flow in a soil-plant system has been presented by Neuman et al. (ref. 17) for several examples. The two methods were: (1) the GFE method and (2) the integrated finite difference (IFD) method. According to the authors, the basic difference between these two methods lies only in the manner in which the governing equations are discretized in space, not in the way the resulting differential equations are integrated in time. The IFD method appears better for isotropic soils, whereas the GFE method is better for anisotropic soils. In general, the GFE method is more difficult to program than the IFD method.

### 3. CONCLUSIONS AND RECOMMENDATIONS

This survey has indicated that there are basically only two different types of soil moisture models: (1) the water budget models and (2) the physical models. The water budget approach has been used in crop yield modeling and watershed hydrological modeling. The physical models which incorporate nonlinear moisture flow equations have been under development for over 15 years. More than 40 papers have been located or referenced that deal with some aspect of these equations.

Because these equations are nonlinear, they generally have to be solved by numerical techniques using a computer. Only under highly restrictive conditions can analytical solutions be obtained. However, these solutions can be important as a means of checking the computer programs.

Tests of some of the computer-programmed models using analytical solutions and field measurements have been made. These tests have indicated that the computer programs for the numerical solutions of the models representing infiltration can provide excellent simulations of actual conditions. A test of a model that includes evaporation, drainage, and redistribution also provided favorable simulation in that all points are within 15 percent of the measured value.

Although a large number of computer simulation models exist for infiltration and redistribution, there are only a few that include evaporation or evapotranspiration. These evapotranspiration models generally simulate transpiration by a root uptake function. These models appear to simulate cumulative seasonal evaporation and transpiration fairly well but have problems simulating conditions for shorter time scales. These models appear to have problems in at least two areas: (1) relating actual evaporation and water uptake by roots to the atmospheric demand and (2) relating the change of root distribution with time to the soil characteristics and moisture amount. However, these problems are similar to those encountered in the water budget approach.

APPENDIX A  
SOIL MOISTURE MODELING BY BUDGET TECHNIQUES

## APPENDIX A

### SOIL MOISTURE MODELING BY BUDGET TECHNIQUES

A number of the soil moisture models using budget techniques are presented in table A-1. The most detailed model with respect to the soil moisture profile is that of Stoff (ref. 18). The most detailed of the moisture budget-evapotranspiration models is that of Kanemasu (refs. 19, 20), which uses a modified Ritchie (ref. 21) evapotranspiration model. A typical budget model is the versatile soil moisture budget (VSMB) model.

#### THE VERSATILE SOIL MOISTURE BUDGET

The VSMB model developed by Baier and Robertson (ref. 22) is a fairly detailed soil moisture model and is the one used by Feyerherm (ref. 23) in his wheat yield model and by others in modified form. In this model, plant evapotranspiration is determined by the following equation.

$$AET_i = \sum_{j=1}^n \left\{ k_j \frac{SM_j(i-1)}{SP_j} z_j^{PET_i} \exp[-w(PET_i - \overline{PET})] \right\} \quad (A-1)$$

where

$AET_i$  = actual evapotranspiration for day  $i$  ending at the morning observation of day  $i + 1$

$\sum_{j=1}^n$  = summation carried out from zone  $j = 1$  to zone  $j = n$

$k_j$  = coefficient accounting for soil and plant characteristics in the  $j$ th zone

$SM_j(i-1)$  = available soil moisture in the  $j$ th zone at the end of day  $(i - 1)$ ; that is, at the morning observation of day  $i$

TABLE A-1.— SOIL MOISTURE MODELS

(a) Stuff (ref. 18); Shaw (ref. 24)

[Ten layers; corn]

Soil moisture equations

$$\Delta SM = P - AET - Q + C$$

$$AET = F_i \cdot E_p$$

where

$\Delta SM$  = daily change in available soil moisture

$P$  = daily precipitation amount

$RO$  = runoff

$AET$  = actual evapotranspiration

$Q$  = downward movement (percolation) or loss into drainage tiles

$C$  = upward capillary flow

$F_i$  = functions depending on soil, phenological day, and atmospheric conditions

$E_p$  = pan evaporation

TABLE A-1.- Continued.

(b) Baier and Robertson (ref. 22)  
[Six layers; spring wheat]

Soil moisture equations

$\Delta SM = P - RO - AET - Q$

$AET_i$  = (function of plant, soil, and soil moisture characteristics)<sub>PET</sub>

See text for actual equations and definitions.

where:

$\Delta SM$  = daily change in available soil moisture

$P$  = daily precipitation amount

$RO$  = runoff

$AET$  = actual evapotranspiration

$Q$  = downward movement to lower layer

PET equations

$PET = bLE$  [Baier (ref. 25)]

$LE = a_0 + a_1 T_X + a_2 (T_X - T_N) + a_3 Q_0 + a_4 Q_S + a_5 (e_s - e_a) + a_6 w_d$

[Baier and Robertson (ref. 26)]

where

$a_0 \dots a_6$  = empirical coefficients available for eight different combinations of variables

$T_X$  = maximum temperature, °F

$T_N$  = minimum temperature, °F

$Q_0$  = total daily solar radiation falling on a horizontal surface at the top of the atmosphere (cal cm<sup>-2</sup> day<sup>-1</sup>)

$Q_S$  = total daily solar and sky radiation falling on a horizontal surface at ground level (cal cm<sup>-2</sup> day<sup>-1</sup>); determined empirically from sunshine duration and day length

$e_s - e_a$  = vapor pressure deficit

$w_d$  = total daily wind run (miles)

TABLE A-1. - Continued.

(c) Kanemasu (refs. 19, 20)

[Five layers; wheat, corn, soybeans, and sorghum]

Soil moisture equations

$$\Delta SM = P - R_o - AET - Q$$

Similar to Baier and Robertson (ref. 22)

$$AET = E_s + T + A$$

where

$E_s$  = evaporation from soil surface

=  $F_1$ (location, soil moisture, crop, leaf area index (LAI), and growth stage)PET

$T$  = transpiration

$$= \sum_{j=1}^5 T_{j,PET}$$

$T_j$  =  $F_2$ (location, soil moisture, crop, LAI, growth stage, and growing degree days) ( $K_j$ )

$K_j$  = root distribution in the  $j$ th layer

$$A = C_s T_{max}$$

$T_{max}$  = daily maximum temperature

$C_s$  = a constant that depends on crop type and  $T_{max}$

Pet equation

$$PET(\text{mm/day}) = \frac{\alpha \delta R_n}{58.3}$$

where

$$R_n = A_n R_s + b_n$$

$a_n, b_n$  = empirical constants that depend on the crop type, LAI, and growth stage

$R_s$  = solar radiation

$\alpha$  = a constant that depends on location and crop

$\delta$  = a function of temperature

TABLE A-1.- Continued.

(d) EarthSat (ref. 27)

[Three layers; spring wheat]

Soil moisture equations

Simplified Baier and Robertson (ref. 22)

$$\Delta SM = P - AET - RO - Q$$

AET similar to Baier and Robertson (ref. 22)

PET equations

$$PET(\text{mm}/\text{time}) = \left[ \frac{\Delta R_n + 0.64f(w_d)(e_s - e_a)}{\Delta + 0.64} \right] \quad [\text{Penman (ref. 28)}]$$

where

$\Delta$  = slope of saturation vapor pressure versus temperature curve (mbar/°K)

$R_n$  = net total surface solar radiation (cal cm<sup>-2</sup> time interval<sup>-1</sup>)

$f(w_d)$  = 0.35(0.5 +  $w_d/100$ ) where  $w_d$  is wind movement (miles/time interval)

$e_s - e_a$  = vapor pressure deficit

(e) Feyerherm (ref. 23)

[Six layers; winter wheat]

Soil moisture equations

Modified Baier and Robertson (ref. 22; also see ref. 29)

$$\Delta SM = P - AET - RO - Q$$

AET similar to Baier (ref. 29)

PET equations

PET(inch/day) = 0.0037LE [Baier (ref. 25)]

For LE see Baier and Robertson (ref. 26)



TABLE A-1. - Continued.

(f) Rickman (ref. 30)

[One layer; winter wheat]

Soil moisture equations

$$\Delta SM = P - AET$$

$$AET = CPET$$

$$C = K_p + K_s \leq 1$$

where

$K_p$  = plant cover coefficient

$K_s$  = soil coefficient

PET equations

$$PET(\text{mm}/24 \text{ hr}) = 0.0171 \left[ \frac{\Delta(R_N - G) + \gamma(15.36)(1.0 + 0.01w_d)(e_s - e_a)}{\Delta + \gamma} \right]$$

[Jensen (ref. 31)]

where

$G$  = daily soil heat flux ( $\text{cal cm}^{-2}$ )

$\gamma$  = psychrometric constant ( $\text{mbar } ^\circ\text{C}^{-1}$ )

TABLE A-1.- Continued.

(g) Arkin (ref. 32)

[One layer; sorghum]

Soil moisture equations

$$\Delta SM = P - AET$$

$$AET = ET_p \cdot C + E_s$$

where

$$ET_p = 0.53(LAI)^{0.5} PET_0 ; LAI \leq 3$$

$$ET_p = PET_0 - E_s ; LAI > 3$$

$E_s$  =  $F_1$  (surface moisture,  $PET_{so}$ , and time)

$C$  =  $F_2$  (field capacity and available moisture)

PET equations

$LAI \geq 0.5$ :

$$PET_0 = 1.28 \left( \frac{\Delta}{\Delta + \gamma} \right) \left| \frac{(1 - \epsilon) R_s + R_l}{583} \right|$$

[Ritchie (ref. 21)]

where

$$\epsilon = 0.3367 - 0.1867 \exp(-0.6LAI)$$

= albedo for soil/canopy surface

$$R_l = F_3(R_s, T, R_0)$$

$$R_0 = 520 + 193 \sin[0.0172(i - 80)] ;$$

$i$  = a Julian date

$$PET_{so} = \left( \frac{\Delta}{\Delta + \gamma} \right) \left| \frac{(1 - \epsilon) R_s + R_l}{583} \right| \exp(-0.398LAI)$$

$LAI < 0.5$ :

$$PET_{so} = PET_0$$

TABLE A-1.- Continued.

(h) Cihlar and Ulaby (ref. 33)

[ Two basic layers; bare soil  
[microwave sensing of moisture]

<u>Soil moisture equations</u>	<u>PET equations</u>
$\Delta S = P + I - AE - D$	Arbitrary
Upper layer - 0 to 30 cm	
$AE = PET \sum_{j=1}^n k_j C_j$	
Lower layer - >30 cm (Relationship not defined)	
where	
$\Delta S$ = change in soil moisture	
$P$ = precipitation amount	
$I$ = irrigation	
$AE$ = evapotranspiration	
$D$ = drainage	
$PET$ = potential evapotranspiration	
$k_j$ = depth coefficient for the $j$ th layer	
$C_j$ = coefficient accounting for soil characteristics in $j$ th layer	

TABLE A-1.- Concluded.

(i) Palmer (ref. 34)  
[Two layers; drought]

Soil moisture equations

Upper layer - 1-inch moisture

Depth variable - depleted first

Lower layer - 5 to 10 inches

$L_s = S'_s$  or PET - P (whichever is smaller)

$L_\mu = (PET - P - L_s) \frac{S'_\mu}{AWC} ; L_\mu \leq S'_\mu$

where

$L_s$  = moisture loss from surface layer

$S'_s$  = available moisture in surface layer at start of month

PET = potential monthly evapotranspiration

P = monthly precipitation

$L_\mu$  = loss from underlying layers

$S'_\mu$  = available moisture stored in underlying layers

AWC = combined available water capacity

PET equations

PET = PET' · d · b [Thornthwaite (ref. 35)]

where

d = adjustment factor for day length

b = adjustment factor for the number of days in a month

PET' =  $1.6(10T_p/I)^a ; \left(\frac{cm}{30 \text{ days}}\right)$  unadjusted

where

$T_p$  = monthly mean temperature, °C

I = annual heat index

$$= \sum_{i=1}^{12} i ; i = (T/5) 1.514$$

a =  $6.75 \times 10^{-7} I^3 - 7.71 \times 10^{-5} I^2 + 0.01792 I + 0.49239$

- $SP_j$  = capacity for available water in the  $j$ th zone  
 $PET_i$  = potential evapotranspiration for day  $i$   
 $w$  = adjustment function or factor accounting for effects of varying PET rates on the AET/PET ratio  
 $\overline{PET}$  = long-term average daily PET for month or season  
 $Z_j$  = adjustment factor for different types of soil dryness curves

In equation (A-1),  $n$  is the number of zones or layers considered. The layers can be of fixed thickness or of variable thickness. Baier and Robertson (ref. 22) and Feyerherm (ref. 23) use zones that contain a certain percentage of the total water. Their standard zones have a variable thickness such that the zones contain 5.0, 7.5, 12.5, and 25 percent of the total available moisture of the plant in the soil profile. According to Baier and Robertson (ref. 22), the adoption of such standard zones makes it possible to use one set of crop coefficients for a given crop in all soil types. Feyerherm (ref. 23) uses 10 inches as a measure for the total amount of available water in the soil profile.

In order to use the AET equation for sequential calculations, a technique is needed to keep track of (i.e., budget) the soil moisture changes in the various layers. These changes can be determined by the following set of soil moisture change equations:

$$SM(i,1) = SM(i-1,1) + P(i) - RO(i) - AET(i,1)K(1) - Q(i,1)$$

Surface layer,  $j = 1$  (A-2)

$$SM(i,j) = SM(i-1,j) + Q(i,j-1) - Q(i,j) - AET(i,j)K_j$$

Surface  $j > 1$  (A-3)

where

$$Q(i,1) = P(i) - RO(i) - [SP(1) - SM(i-1,1)] \quad ; \quad j = 1$$
$$Q(i,1) > 0 \quad (A-4)$$

$$\left. \begin{aligned} Q(i,1) &= 0 \text{ if } Q(i,1) \leq 0 \\ Q(i,j) &= Q(i,j-1) - [SP(j) - SM(i-1,j)] \quad ; \quad j > 1 \\ Q(i,j) &= 0 \text{ if } Q(i,j) \leq 0 \end{aligned} \right\} \quad (A-5)$$

where

$Q(i,j)$  = percolation to lower layers on day  $i$

$P(i)$  = precipitation (rain) on day  $i$

$RO(i)$  = rainfall runoff on day  $i$

The above AET equation and soil moisture change equations have been programmed for computer calculations. The soil moisture budget (SMB) program is written in Fortran IV for the Univac Exec II. This SMB program uses meteorological, soil moisture, and plant rooting characteristics to calculate daily evapotranspiration and soil moisture amounts.

The initial data needed are the plant available water capacity in each layer ( $SP_j$ ), the actual available water in each layer ( $SM_j$ ), the plant root distribution factors ( $K_j$ ), the soil dryness adjustment factors ( $Z_j$ ), and the atmospheric demand adjustment factor ( $w$ ). In addition, a PET function is needed.

Meteorological data are needed on a daily basis. The amount of daily data required depends mainly on the PET function used. As a minimum, daily precipitation, maximum temperature, minimum temperature, and solar radiation outside the atmosphere are needed. If wind, humidity, and global surface solar radiation data are available, more complex PET equations can be used. (See table A-1.)

Consideration of the initial and daily data involved in the SMB calculations suggests that there can be a great deal of uncertainty in the results. In particular, it is difficult to determine the soil drying characteristics  $Z_j$  and the atmospheric demand factor  $w$ . In order to better interpret the results of experiments using the SMB program, a sensitivity and accuracy analysis of the AET equation has been performed (ref. 2). This analysis indicated that the soil moisture capacity (SP), the available soil moisture (SM) amount, the root distribution coefficient (K), and the soil dryness adjustment coefficient (Z) each gave a 10-percent error in output for a 10-percent error in input. PET and  $\overline{\text{PET}}$  each gave approximately a 5-percent error in output for a 10-percent error in input. The atmospheric demand coefficient ( $w$ ), on the other hand, gave only a 0.5-percent change in output for a 10-percent input error. In a simulation experiment, realistic values for SP, K, Z, and  $w$  provided a calculated value of the total water loss by evapotranspiration that was within 2 percent of the measured water loss. Further experiments with the same data set have indicated that the variation in calculated water loss is not as sensitive to uncertainties in the individual parameter variables as the sensitivity analysis indicated.

APPENDIX B  
REFERENCES



## APPENDIX B

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