(NASA-CR-135414-App) HELIOCENTRIC INTERPLANETARY LOW THRUST TRAJECTORY OPTIMIZATION PROGRAM, SUPPLEMENT 1, PART 2 Final Report (Business and Technological Systems, Inc.) 103 p HC A06/MF A01 CSCL 22A G3/12

N78-25106

Unclas
21601

BUSINESS AND TECHNOLOGICAL SYSTEMS, INC.

BTS

BTS-TR-78-54

NASA CR-135414

HILTOP SUPPLEMENT

HELIOCENTRIC INTERPLANETARY LOW THRUST TRAJECTORY OPTIMIZATION PROGRAM

SUPPLEMENT #1

F.I. Mann

J.L. Horsewood

FINAL REPORT

Part 2 of 2

Contract NAS 3-20950

February 1978

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Lewis Research Center
Cleveland, Ohio 44135

by

BUSINESS AND TECHNOLOGICAL SYSTEMS, INC.
10210 Greenbelt Road, Suite 440
Seabrook, Maryland 20801

SUMMARY

This report is the first supplement to the currently existing primary HILTOP program document [1] and describes the improvements made to the HILTOP electric propulsion trajectory optimization computer program since the publication of the primary document.

A new, more realistic propulsion system model has been implemented in the program, in which various thrust subsystem efficiencies and specific impulse are modeled as variable functions of power available to the propulsion system. The number of operating thrusters are staged, and the beam voltage is selected from a set of five (or less) constant voltages, based upon the application of variational calculus. The constant beam voltages may be optimized individually or collectively.

The new propulsion system logic is activated by a single program input key in such a manner as to preserve the old HILTOP logic.

The report contains the new analysis describing these features, a complete description of program input quantities, and sample cases of computer output illustrating the new program capabilities.

TABLE OF CONTENTS

PRECEDING PAGE BLANK NOT FILMED

NOMENCLATURE

Generally, upper-case symbols denote vectors and lower-case symbols denote scalars. Lower-case symbols with bars denote unit vectors. The abbreviations EPS for electric propulsion system and BVP for boundary value problem are used.

a	EPS instantaneous thrust acceleration, expressions (22) and (35); semi-major axis
a_c	Semi-major axis of primary-target capture orbit
a_i	Solar power law coefficients
$\bar{a}_1$, $\bar{a}_2$	Arbitrary unit vectors used in (132) and (139) of [1]
b	A coefficient in the efficiency law of the old spacecraft model; auxiliary quantity defined by (29)
b_1, b_2, b_3	Launch vehicle coefficients
C	Vector constant of optimal rocket problem, expression (63) of [1]
C^o	Radians-to-degrees conversion factor
C_1	Zeroth-order term in expansion for thrust reduction factor, expression (12)
C_2	Coefficient in first-order term in expansion for thrust reduction factor, expression (12)

PRECEDING PAGE BLANK NOT FILMED

C_3	Constant in expression (13) or (14) for η_u
C_4	Constant in expression (13) or (14) for η_u
C_5	Ratio of accelerator current to beam current, as used in (15) and (16)
C_6	Zeroth-order term in expansion for screen supply efficiency, expression (17)
C_7	Coefficient in first-order term in expansion for screen supply efficiency, expression (17)
c	EPS jet exhaust speed, constant in the "old" spacecraft model; defined by (20) or (36) or (56) in the improved model; abbreviation for cosine function
c_{acc}	Acceleration conversion factor, expression (35)
c_r	Retro stage jet exhaust speed
c_{vel}	Speed conversion factor, expression (36)
c_1	Auxiliary quantity given by expression (74) of [1]
c_1, c_2, c_3	Coefficients in quadratic expression for Δv_i, expression (78) of [1]
DIU	Digital Interface Unit
d	A coefficient in the efficiency law; an auxiliary quantity in the coast-phase solution; solar flux density
E	Eccentric anomaly (a scalar)

e	A coefficient in the efficiency law; the base of the natural logarithms; eccentricity; subscript denoting Earth
$\bar{e}_h$	Spacecraft unit angular momentum vector
$\bar{e}_r$	Spacecraft unit radius vector
$\bar{e}_t$	EPS unit thrust vector
$\bar{e}_v$	Spacecraft unit velocity vector
e_x	Retro stage characteristic speed exponential factor given by expression (76) of [1]
$\bar{e}_\lambda$	Unit primer vector
F	Auxiliary scalar function defined by (215) of [1]
F_t	See αF_t
F_v	Auxiliary quantity defined by (30)
f	EPS instantaneous thrust magnitude; f-function of the f and g series; subscript denoting a desired value; true anomaly; auxiliary variable defined by equation (147) of [1]
f_r	Retro stage thrust magnitude
f_t	Total thrust, expression (18)
f_x	Auxiliary quantity given by expression (77) of [1]
G	Auxiliary quantity defined by (28)
G_i	Auxiliary scalar functions in the coast-phase solution, equation (45) of [1]

g — EPS reference thrust acceleration (old spacecraft model); g-function of the f and g series; BVP point-constraint geometric mean of the weighting factors; parameter local to expression (47)

g' — Parameter local to expression (47)

g_r — Reference gravity acceleration constant, used in (19)

g_x — Auxiliary quantity given by expression (97) of [1]

H — Spacecraft angular momentum vector

h — Magnitude of spacecraft angular momentum vector

$\bar{h}$ — Spacecraft unit angular momentum vector

h_I — Component of h_v containing thrust acceleration, expression (41)

h_v — Variational hamiltonian, given by (54)

h_x, h_y, h_z — Cartesian components of spacecraft angular momentum vector

h_σ — Thrust-switching step-function (old spacecraft model only)

I_B — Beam current per thruster

I_{BP} — Phantom beam current (per thruster), equal to the beam current a thruster would have if there were no limitations on I_B, given by (26)

I_{BX} — Restricted phantom beam current, expression (55)

I_{max} — Maximum beam current (per thruster)

I_{min} Minimum beam current (per thruster)

I_{SP} EPS specific impulse, expression (19)

I_{SP_o} Reference value of I_{SP}, defined in discussion following expression (23)

i Subscript pertaining to an intermediate target; inclination to ecliptic; general subscript or running index; inclination of parking orbit about Earth

$\bar{i}$ Unit vector along x-axis

i_{max} Parking orbit inclination associated with range safety limit

J Index-set of the BVP dependent variables

$\bar{j}$ Unit vector along y-axis

j_a Jettison indicator for solar arrays (or other power source) prior to retro maneuver, used in (5)

j_p Unspecified-reference-power indicator (old spacecraft model only)

j_{ps} EPS propulsion system jettison indicator (retro maneuver) in old spacecraft model

j_r Retro stage existence indicator

j_t EPS tankage jettison indicator (retro maneuver)

j_{th} EPS thrust subsystem jettison indicator (retro maneuver), used in (5)

k Fundamental constant associated with Mercury propellent, used in (18) and (19); in [1], arbitrary positive constant associated with performance index (replaced by λ_π in this document); temporary variable ultimately equated to inverse of the characteristic degradation time

$\overline{k}$ Unit vector along z-axis

k_c Auxiliary quantity given by expression (75) of [1]

k_{drop} Intermediate-target drop-mass factor defined by expression (6) of [1]

k_{rt} Retro stage tankage mass factor defined by expression (11) of [1]

k_s EPS structure mass factor defined by expression (8) of [1]

k_{samp} Intermediate-target sample-mass factor defined by expression (6) of [1]

k_t EPS tankage mass factor defined by expression (7) of [1]

L Launch site latitude (scalar)

M Mean anomaly (scalar)

M_0, M_1, M_2, M_3, M_4, M_5 Coefficients used in computing nuclear and total magnitudes of a celestial body (scalars)

M_N Nuclear magnitude (scalar)

M_T Total magnitude (scalar)

m Spacecraft total mass variable

$\overline{m}$ Auxiliary unit vector given by expression (53)

$\dot{m}$ — Mass flow rate, expression (21)

m_a — Solar array (or other power source) mass, expression (2)

m_b — Constant mass component of m_a, expression (2)

m_{drop} — Intermediate-target drop-mass given by expression (6) of [1]

$\dot{m}_\ell$ — Thruster neutral propellant loss, used in (13) and (14)

$\dot{m}_n$ — Neutralizer propellant flow rate, used in (13) and (14)

m_{net} — Net spacecraft mass

m_o — Initial spacecraft mass (payload of launch vehicle) given by expression (2) of [1]

m_p — EPS propellant mass

m_{ps} — Electric propulsion system mass given by expression (4) of [1]

m_r — Retro stage mass, expression (4)

m_{rp} — Retro stage propellant mass given by expression (5), or (9) of [1]

m_{rs} — Retro stage structure mass defined by expression (11) of [1]

m_{rst} — Retro stage structure and tankage mass given by expression (11) of [1]

m_s — EPS structure mass

m_{samp} — Intermediate-target sample-mass given by expression (6) of [1]

m_t — EPS tankage mass

m_{th}	EPS thruster subsystem mass, expression (3)
Δm_p	Propellant mass increment due to primary-target spiral maneuver
n	Exponent in step-size law, expression (39) of [1]; subscript denoting time at the primary target; number of BVP dependent variables
$\overline{n}$	Unit vector normal to the solar arrays
n_{max}	Maximum number of operating thrusters
n_{min}	Minimum number of operating thrusters (>0)
$\overline{n}_p$	Unit vector directed along a planet's north pole
n_t	Number of operating thrusters (variable)
Δn_t	Increment in n_t , when staging thrusters
o	Subscript denoting launch time; subscript denoting the beginning of a computation step
P	A celestial body's position vector; BVP partial derivative matrix
PDS	Power Distribution System
PPU	Power Processing Unit
p	EPS instantaneous power in old spacecraft model; subscript denoting a perturbed, or neighboring, parameter; auxiliary variable in equations (79) of [1]
Δp	Ratio of housekeeping to reference power, p_h/p_{ref} , old model only
p_a	Total instantaneous power developed by arrays (or other power source), expression (6)

$P_{a_{max}}$ Maximum power output of power source that can be utilized by the thruster subsystem and other spacecraft modules, expression (25)

P_{ao} Reference power of solar arrays or other power source, used in (2) and (6)

P_b Beam power, expressions (10) and (23)

P_{conv} Housekeeping power in improved spacecraft model, expression (9)

P_d Power output of power distribution system, expression (7)

P_{diu} Digital interface unit power requirement, used in (9)

P_h Housekeeping power in old spacecraft model

$P_{\ell v}$ Power processor low voltage power requirement, used in (9)

P_{max} Maximum power input to an individual thruster, expression (25a)

P_{mm} Mission module power requirement, used in (9)

P_{ref} EPS reference power (old model only)

P_t Power input each PPU, expressions (8) and (27)

P_{ts} Thrust subsystem power requirement, used in (9)

P_{to} Reference power of each thruster, expressions (3) and (24)

P_1, P_2 Auxiliary quantities in coast-phase solution, expressions (54) and (55) of [1]

ORIGINAL PAGE IS
OF POOR QUALITY

q	Auxiliary variable in equations (79) of [1]; solar array radiation damage factor
R	Spacecraft position vector
r	Magnitude of R
r_a	Primary-target capture-orbit apocenter distance
r_c	Earth-to-spacecraft communication distance
$\overline{r}_n$	Unit vector along line of ascending node
r_p	Primary-target capture-orbit pericenter distance; primary-target swingby passage-distance
$\overline{r}_p$	Swingby passage-distance unit vector
r_{peak}	Value of r for which γ-curve is at a maximum
s	Abbreviation for sine function; auxiliary variable used in equations (79) of [1]; degradation time
$\overline{s}$	Unit vector directed toward Canopus
t	Time
t_b	Retro maneuver burn time given by expression (12) of [1]
t_{ratio}	Minimum throttling ratio, expression (25b)
Δt	Time-increment due to primary-target spiral maneuver
u	Generalized universal anomaly during thrust phases

Δu	Generalized universal anomaly increment, equivalent to the computation step-size during numerical integration
V_G	Neutralizer to beam coupling potential, used in (15) and (16)
V_I	Beam voltage (I^{th} value selected from set of up to five constant values)
$V_{I_{max}}$	Largest beam voltage
$V_{I_{min}}$	Smallest beam voltage
ΔV_I	Discharge voltage, expressions (15) and (16)
V_∞	Hyperbolic excess velocity (or encounter velocity)
$V_{\infty A}$	Swingby planet arrival hyperbolic excess velocity
$V_{\infty D}$	Swingby planet departure hyperbolic excess velocity
v	Magnitude of spacecraft velocity
v_c	Characteristic speed of a rocket maneuver
v_e	Escape speed from launch parking orbit
v_g	Minimum velocity impulse required for non-coplanar injection from a circular orbit to a given excess velocity
v_o	Speed of a spacecraft in a circular orbit
v_p	Planetocentric speed at primary-target swingby closest-approach point; auxiliary speed given by equation (72) of [1]

v_∞	Hyperbolic excess speed (or encounter speed)
Δv	Retro stage impulsive velocity increment magnitude; characteristic velocity associated with primary-target spiral maneuver; incremental speed required at powered swingby
$\Delta v'$	Retro stage total velocity increment magnitude
Δv_0	Minimum incremental velocity (magnitude) for coplanar boost out of circular orbit
Δv_g	Velocity penalty due to noncoplanar boost out of circular orbit
Δv_i	Velocity penalty due to launch azimuth
w	Auxiliary variable in equations (79) of [1]
x	First Cartesian component of position; a general variable; a general state variable; auxiliary variable in equations (79) of [1]
y	Second Cartesian component of position; auxiliary variable in equations (79) of [1]
z	Third Cartesian component of position
α	EPS specific mass; geocentric right ascension of launch excess velocity
αF_t	Thrust reduction factor due to double ions and beam divergence, given by (12)
$\left.\begin{matrix}\alpha_A \\ \alpha_D\end{matrix}\right\}$	Auxiliary parameters defined by equations (211) and (212) of [1]

α_a	Specific mass of the solar arrays, used in (2)
α_c	Communication angle (Sun-Earth-spacecraft)
α_t	Specific mass of the power conditioning and thruster subsystem, used in (3)
α_1, α_2	Arbitrary, independent angles defining orientation of excess velocity in (132) and (139) of [1]
β	Independent variable of coast-phase solution, also generalized to be the independent variable on the entire trajectory
β_o	Value of β at the beginning of a computation step
$\Delta\beta$	Computation step size (increment of trajectory independent variable)
γ	Normalized power function
γ'	$\partial\gamma/\partial r$
γ^*	$\partial\gamma/\partial d$, where d is the solar flux density
δ	Launch hyperbolic-excess-velocity asymptote declination; BVP dependent-variable tolerance
δ_A, δ_D	Bend angles of hyperbolic arrival and departure trajectories, expression (213) of [1]
δ_T	Total bend angle given by expression (214) of [1]
δ_{ij}	Kronecker delta function

ϵ	Auxiliary quantity in the coast-phase solution; obliquity of the Earth's equator to the ecliptic
ε_I	Discharge losses, used in (15) and (16)
η	EPS efficiency (old model)
η'	$d\eta/dc$
η_{conv}	DC-DC converter efficiency, used in (9)
η_D	Discharge supply efficiency, used in (16)
η_{pd}	Efficiency of the power distribution system, used in (7)
η_{PPU}	Power processor efficiency, given by (16)
η_{PTH}	Efficiency given by expression (15)
η_S	Screen supply efficiency, given by (17)
η_{TH}	Thruster efficiency, given by (11)
η_u	Propellant utilization efficiency, given by (13) and (14)
η_u'	$\partial\eta_u/\partial I_B$, given by (46) and (47)
θ	In-plane thrust angle
θ_i	Travel angle increment
θ_t	Travel angle
Λ	Primer vector (adjoint to spacecraft velocity)

λ	Magnitude of Λ ; a general adjoint variable; the iterator inhibitor
λ_c	Adjoint variable associated with jet exhaust speed
λ_g	Adjoint variable associated with reference thrust acceleration
λ_s	Adjoint variable associated with degradation time
λ_{V_I}	Adjoint variable associated with beam voltage V_I
λ_x	Thrust cone angle Lagrange multiplier
λ_ν	Adjoint variable associated with mass ratio
λ_π	Adjoint variable associated with performance index (symbol k in [1])
λ_τ	Adjoint variable associated with propulsion time
λ_ϕ	Adjoint variable associated with thrust cone angle
μ	Gravitational constant of the sun; a general gravitational constant
μ_t	Gravitational constant of the primary target
ν	Mass ratio
$\Delta\nu$	Mass ratio increment at an intermediate target
π	Performance index; ratio of circle circumference to diameter
π_x	Partial derivative of π with respect to arbitrary variable x

ρ	Auxiliary variable used in equations (79) of [1]
σ	Thrust switch function, given by (42)
σ^*	Special form of thrust switch function, given by equation (186) of [1]
σ_r	Portion of total thrust switch function, given by (193) of [1]
$\Delta\sigma$	Propulsion-corner-proximity tolerance-interval
τ	EPS propulsion time
τ_d	Characteristic degradation time
Φ	Transformation matrix for rotating from ecliptic to equatorial coordinate system
ϕ	Thrust cone angle (between thrust and radius)
χ	Angle between normal to solar arrays and the spacecraft-sunline
ψ	Out-of-plane thrust angle
Ω	Longitude of ascending node of an orbit
ω	Angular position from the ascending node of an orbit to the spacecraft; argument of perifocus of an orbit

1.0 INTRODUCTION

This document is the first supplement to the currently existing primary HILTOP program document (published in December 1974; see reference [1]) and describes the modifications and improvements made to the HILTOP electric propulsion trajectory optimization computer program up through February 1978.

A new, more realistic propulsion system model involving the actual ion beam current and voltage has been implemented in the program. The power processor efficiency, ion thruster efficiency, and thruster specific impulse are modeled as variable functions of the (solar array, nuclear, or other) power available to the propulsion system. The number of operating thrusters are staged, and the beam voltage is selected from a set of five (or less) constant voltages, based upon the application of variational calculus. The minimum and maximum number of operating thrusters, the minimum throttling ratio, and the maximum input power to an individual thruster are specified as input data. The constant beam voltages may be optimized individually or collectively.

The new propulsion system logic is activated by a single program input key (NAMELIST input "NEW"); program modifications have been designed to retain the "old" HILTOP program within the framework of the new logic, so that old input data files (with no modifications required) will run the new program version and produce identical results as before.

The capability of simulating solar array degradation with the new spacecraft model is not included in this program version; also not included is the capability of simulating the new spacecraft model under the Launch Vehicle Independent (LVI) mode. The simulation of array degradation and the LVI mode remain available with the old spacecraft model.

The execution step requirements of the new program version are a little less than 390K bytes of Main Core Storage. This compares to 350K for the old version.

The report contains the new analysis describing these features, a complete description of program input quantities, and sample cases of computer output illustrating the new program capabilities.

2.0 FORMULATION

2.1 Spacecraft and Trajectory Models

The following discussion is oriented toward the programming logic aspects of the new HILTOP computer program version. Equations and analysis which have not been affected by the implementation of the new spacecraft model are not repeated here and may be found in reference [1]. The new spacecraft model was obtained from the Lewis Research Center [2] .

2.1.1 Spacecraft Mass Components

In the new spacecraft model, the spacecraft is composed of an electric propulsion system and associated tankage and propellant masses, a structure mass component, a retro propulsion component (for maneuvers about a primary target), a set of instrument package masses to be dropped at intermediate targets and a net spacecraft mass as follows:

$$m_o = m_a + m_{th} + m_p + m_t + m_s + m_r + \sum_{i=1}^{n-1} m_{drop\ i} + m_{net}, \qquad (1)$$

where m_o is the initial spacecraft mass; m_a, m_{th}, m_p, m_t are the solar array (or other power source), thruster subsystem, propellant and tankage masses, respectively; m_s is the structure component; m_r is the retro propulsion mass; $m_{drop\ i}$ is the instrument package mass left at the i^{th} target; and m_{net} is net spacecraft mass (payload). In the analysis to follow, the subscript o denotes the launch body and n the primary (final) target. The net spacecraft mass consists of the scientific instruments, communications, navigation, and other engineering hardware, shielding, and any other mass components required to carry out the mission of interest. Equation (1) is identical to that of Reference [1] (the old spacecraft model) except that the quantity m_{ps} of the old model has been replaced by $m_a + m_{th}$.

The solar array and thruster subsystem masses are given by,

$$m_a = \alpha_a p_{ao} + m_b \tag{2}$$

$$m_{th} = \alpha_t n_{max} p_{to} \tag{3}$$

where p_{ao} is the reference power of the solar array or other power source (see Electric Propulsion System), α_a is the specific mass of the arrays, n_{max} is the maximum number of operating thrusters, p_{to} is the reference power of each thruster, α_t is the specific mass of the thruster and power conditioning subsystem, and m_b is a constant mass.

The propellant, tankage, and structure masses m_p, m_t, and m_s, respectively, are computed the same as in the old model. The retro propulsion mass m_r is composed of the two components (as before),

$$m_r = m_{rp} + m_{rst} \tag{4}$$

in which the retro structure and tankage mass component is computed identically as before and the retro propellant requirment, m_{rp} , is now given by

$$m_{rp} = j_r\left(m_o \nu_n - j_a m_a - j_{th} m_{th} - j_t m_t\right) e_x \quad , \tag{5}$$

j_a, j_{th} and j_t are input jettison indicators set equal to one if the solar array, thruster subsystem and tankage mass components are to be jettisoned prior to the retro maneuver and equal to zero otherwise, and j_r is an indicator (1 or 0) for the presence or absence respectively, of the retro stage; and e_x is the retro burn exponential factor discussed in reference [1]. j_a is a new indicator, and j_{th} replaces j_{ps} of the old model.

2.1.2 Electric Propulsion System

The electric propulsion system is comprised of the solar array and the thruster subsystem*. The solar array is characterized by a reference power, p_{ao}, defined as the power developed at 1 AU from the sun assuming the arrays are oriented normal to the sun. The instantaneous power developed by the arrays p_a, takes into account the effects of temperature, the distance from the sun and the orientation of the arrays with respect to the sun line. The instantaneous power developed is expressed

$$p_a = \gamma p_{ao} \tag{6}$$

where γ is a power function which accounts for temperature, distance and orientation effects and is computed identically as in reference [1], having the same options.

The thruster subsystem consists of a power distribution system which accepts the power delivered by the array and distributes it to a number of power processing units (PPU's) each of which is dedicated to a separate thruster. Each PPU processes the power input to the unit to deliver to the associated thruster the appropriate voltage and current parameters for efficient operation. The power output of the power distribution system, p_d, is modelled

$$p_d = \eta_{pd} p_a \tag{7}$$

where η_{pd} is the efficiency of the power distribution system, a specified constant. This power is distributed evenly among the current number n_t of the operating PPU/thruster modules. That is, the power input to each PPU is

$$p_t = \frac{p_d - p_{conv}}{n_t} \tag{8}$$

p_{conv} is the power requirement input to the system DC-DC converter to provide power to the digital interface units, the power processor low voltage section, the thrust system, and the mission module. p_{conv}, which is loosely denoted "housekeeping power" is given by

*A power flow schematic is displayed in Appendix A.

$$p_{conv} = \frac{(p_{diu}+p_{\ell v})n_t + p_{mm} + p_{ts}}{\eta_{conv}} \tag{9}$$

in which five program-input constants are

p_{diu} = digital interface unit power requirement,

$p_{\ell v}$ = power processor low voltage input power requirement,

p_{mm} = mission module power requirement,

p_{ts} = thrust subsystem power requirement,

η_{conv} = DC-DC converter efficiency.

The numerator of equation (9) represents the power output of the DC-DC converter. The power output of each PPU/thruster module is the beam power p_b which is related to the input power as follows

$$p_b = \eta_{TH}\eta_{PPU}p_t \tag{10}$$

where η_{TH} and η_{PPU} are the efficiencies of the thruster and power processing units, respectively. These efficiencies are dependent upon the operating conditions, expressed in terms of the beam current I_B and the beam voltage V_I , of the thruster. The beam voltage is selected at each instant in time from one of up to five input discrete values; the selection is made by the program as part of the problem solution as discussed in the section, Optimality Conditions. The beam current is throttled as necessary to make use of the input power p_t , but is subject to the constraint of a maximum operating value, I_{max} , which is specified by input. The thruster efficiency is modelled empirically

$$\eta_{TH} = (\alpha F_t)^2 \eta_u \eta_{PTH} \tag{11}$$

where αF_t is a thrust reduction factor due to double ions and beam divergence

$$\alpha F_t = C_1 + C_2 I_B \tag{12}$$

where C_1 and C_2 are input constants; η_u is the propellant utilization efficiency which is a function of the beam current

$$\eta_u = \frac{I_B}{\dot{m}_\ell + \dot{m}_n + I_B \left[1 - C_3\left(\frac{I_B + (I_B - 1)^2}{C_4 + (I_B - 1)^2}\right)\right]} \quad \text{for } I_B \geq 1 \text{ amp} \tag{13}$$

$$\eta_u = \frac{I_B}{\dot{m}_\ell + \dot{m}_n + I_B\,(1 - C_3 I_B / C_4)} \quad \text{for } I_B < 1 \text{ amp} \tag{14}$$

where $\dot{m}_\ell$ is the thruster neutral propellant loss in equivalent amps, $\dot{m}_n$ is the neutralizer propellant flow rate in equivalent amps, and $\dot{m}_\ell$, $\dot{m}_n$, C_3 and C_4 are input constants; and η_{PTH} is a function of the beam voltage

$$\eta_{PTH} = \frac{V_I}{(1 + C_5)(V_I + V_G - \Delta V_I) + \varepsilon_I + \Delta V_I} \tag{15}$$

where ε_I is the discharge losses in eV/ion, ΔV_I is the discharge voltage in volts, V_G is the neutralizer to beam coupling potential in volts, and C_5 is the ratio of accelerator current to beam current. ε_I, ΔV_I, V_G and C_5 are input constants. The power processor efficiency is of the form

$$\eta_{PPU} = \frac{(1+C_5)(V_I+V_G-\Delta V_I) + \varepsilon_I + \Delta V_I}{(1+C_5)(V_I+V_G-\Delta V_I)/\eta_S + (\varepsilon_I+\Delta V_I)/\eta_D} \quad (16)$$

where η_D is the discharge supply efficiency, an input constant, and η_S is the screen supply efficiency which is assumed to be a linear function of beam current; i.e.,

$$\eta_S = C_6 + C_7 I_B \quad (17)$$

For trajectory computation purposes, it is convenient to express the thruster subsystem performance in terms of the actual thrust output f_t by the n_t operating thrusters

$$f_t = k n_t (\alpha F_t) I_B \sqrt{V_I} \quad \text{(newtons)} \quad (18)$$

(where $k = 2.0391 \times 10^{-3}$) and the specific impulse I_{SP}

$$I_{SP} = \frac{2(\alpha F_t)\eta_u \sqrt{V_I}}{k g_r} \quad \text{(sec)} \quad (19)$$

where $g_r = 9.80665\ \text{m/sec}^2$. Other performance parameters of traditional importance include the jet exhaust speed c, the mass flow rate $\dot{m}$, the thrust acceleration a, and the beam power p_b. These parameters are evaluated as follows:

$$c = g_r I_{SP} \quad (20)$$

$$\dot{m} = -f_t/c \quad (21)$$

$$a = f_t/m \quad (22)$$

$$p_b = \tfrac{1}{2} f_t c \quad (23)$$

where m is the instantaneous mass.

ORIGINAL PAGE IS
OF POOR QUALITY

For identification and documentation purposes, it is useful to identify reference values of the thruster specific impulse and power. These reference values correspond to the maximum permissible value of beam current, I_{max}, an input parameter, and the largest of the discrete values of beam voltage, V_{Imax}. The reference value of specific impulse, denoted I_{SPo}, is evaluated with Equation (19) using I_{max} and V_{Imax} in the Equations (11) - (17) and (19). The unit thruster reference power p_{to} is defined to be the power allocable to each thruster that is input to the power distribution system assuming each thruster is operating at a beam current of I_{max} and a beam voltage of V_{Imax}. That is,

$$p_{to} = \frac{(1+C_5)(V_{Imax}+V_G-\Delta V_I) + \varepsilon_I + \Delta V_I}{\eta_{pd}\ \eta_{PPU}\Big|_{I_{max},V_{Imax}}}\ I_{max}$$

$$= \frac{\eta_D(1+C_5)(V_{Imax}+V_G-\Delta V_I) + (C_6+C_7 I_{max})(\varepsilon_I+\Delta V_I)}{\eta_{pd}\ \eta_D(C_6+C_7 I_{max})}\ I_{max} \tag{24}$$

Thus, if n_{max} denotes the maximum number of operating thrusters permitted, then the maximum power output of the arrays that can be utilized by the thruster subsystem and other spacecraft modules is

$$p_{a_{max}} = n_{max} p_{to} + \frac{p_{conv(max)}}{\eta_{pd}} \tag{25}$$

in which $p_{conv(max)}$ is p_{conv} evaluated with $n_t = n_{max}$.

The maximum power input to an individual thruster is computed,

$$p_{max} = 10^{-3} \times I_{max}\left[(1+C_5)(V_{I_{max}}+V_G-\Delta V_I) + (\varepsilon_I+\Delta V_I)\right] \tag{25a}$$

(in kw) when I_{max} is specified as a program input, and this formula is inverted to compute I_{max} when p_{max} is specified as a program input. The minimum throttling ratio is defined in terms of the beam current,

$$t_{ratio} = I_{min}/I_{max} \qquad (25b)$$

when I_{min} is specified as a program input, and this formula is inverted to compute I_{min} when t_{ratio} is specified as a program input.

Notice that the solar array tilt angle does not appear in this formulation; it is essentially replaced by I_B. When $I_B = I_{max}$, which corresponds to "operating below the power curve", then this model is not concerned with how any potentially-available excess power is avoided. Methods of avoidance include

(1) tilting the (solar) arrays
(2) shielding the arrays
(3) dumping excess power via radiators
(4) shunting the excess power for other use

The solar array tilt angle would become a factor in the model if array degradation were considered; however, including array degradation in the model would introduce considerable complexity to the equations and algorithms required for the optimal solution and is deemed beyond the scope of the current implementation.

When the spacecraft is "operating on the power curve" as specified by γ (i.e., using all available input solar power), the beam current for each operating thruster is given by

$$I_B = \frac{-b + \sqrt{b^2 + 4C_6C_7Gp_t}}{2C_7G} \qquad (26)$$

in which p_t , using equations (6), (7) and (8), is given by

$$p_t = \frac{\eta_{pd} p_{ao} \gamma}{\eta_t} - \frac{p_{conv}}{\eta_t} \quad (27)$$

and

$$G = (\varepsilon_I + \Delta V_I)/\eta_D \quad (28)$$

$$b = F_V - C_7 p_t + C_6 G \quad (29)$$

$$F_V = (1+C_5)(V_I+V_G-\Delta V_I) \quad (30)$$

In this spacecraft model, the solar array (or other power source) may not be perfectly matched to the thrust subsystem capability under reference conditions; instead, the power, p_{ao} , may be a specified constant, which may be less than, equal to, or greater than the thrust subsystem maximum power requirement $p_{a_{max}}$. In the case of solar electric propulsion, the analyst pre-selects the desired power curve γ via program inputs a_i (power curve coefficients) and γ_{max} . In fact, the same power curve options are available as in the simpler spacecraft model, and these options are discussed on pp. 11-13 of [1]. Then, at any given point in time, the value of γ will allow the computation of a hypothetical value for the beam current using equation (26), and if this value lies in the acceptable range $I_{min} \leq I_B \leq I_{max}$, it is used in the actual calculations; if the value is greater than I_{max} , then I_{max} is used, and if less than I_{min} (which is associated with the minimum throttling ratio), then special action is required involving an iteration to isolate the point along the trajectory at which $I_B = I_{min}$, at which point the control state of the thruster subsystem is optimally switched (as discussed in the section, Optimality Conditions). When either $I_B = I_{max}$ or the spacecraft is

ORIGINAL PAGE IS
OF POOR QUALITY

operating on a level portion of the power curve $\gamma = \gamma_{max}$, this model is not concerned with how any potentially-available excess power is avoided (as mentioned above). In most applications, the spacecraft will be operating on the non-constant portion of the power curve, representing the optimal situation in which the arrays are normal to the sun line and gathering as much power as possible. If the program user wishes to have the solar array output power matched to the remainder of the spacecraft (under the reference conditions of having the solar arrays at 1 AU from the sun and oriented normal to the sun line), a program input key is provided which causes the setting $p_{ao} = p_{a_{max}}$ internally.

2.1.3 Differential Equations

The differential equations of motion applicable with the new spacecraft model are (Consult Nomenclature for definitions of previously-defined symbols):

$$\ddot{\bar{R}} = a\bar{e}_t - \frac{\mu}{r^3} R \quad , \tag{31}$$

$$\dot{\nu} = - \frac{a\nu}{c} \quad , \tag{32}$$

$$\dot{V}_I = 0 \ , \quad I = 1,2,3,\ldots(\max 5) \quad , \tag{33}$$

$$\dot{\phi} = 0 \quad , \tag{34}$$

in which the thrust acceleration is given by

$$a = \frac{k(C_1+C_2I_B)\sqrt{V_I}\ n_tI_B}{c_{acc}m_o\,\nu} \quad , \tag{35}$$

and the jet exhaust speed is given by

$$c = \frac{2(C_1+C_2I_B)n_u\sqrt{V_I}}{c_{vel}k} \quad (36)$$

where $c_{acc} = 5.9301282604\times10^{-3}$ m/sec^2 and $c_{vel} = 29784.916613$ m/sec render the quantities a and c expressed in program internal units. It is therefore emphasized that the symbols a and c from this point of the discussion onward pertain to thrust acceleration and jet exhaust speed expressed in (normalized) program internal units, in contrast to the a and c of equations (22) and (20), respectively, which are expressed in MKS units. Relations (35) and (36) are valid regardless of the algorithm by which the beam current I_B is generated. The controls are n_t, V_I, $\overline{e}_t$, and I_B, where n_t is the number of operating thrusters, V_I is the beam voltage, $\overline{e}_t$ is a unit vector defining the direction of thrust, and I_B is the beam current per thruster.

The analysis pertaining to the thrust cone angle ϕ (the angle between the radius and thrust vectors) remains essentially unchanged compared to that of the old HILTOP model, and is included here for the sake of completeness. The propulsion time τ of the old model is not included in the new model since a spacecraft having thrusters which are staged according to available power does not have a "propulsion time" which can be simply implemented in a variational calculus approach.

The differential equations which govern the behavior of the adjoint variables are given by

$$\ddot{\Lambda} = -\frac{\mu}{r^3}\Lambda + \frac{3\mu}{r^5}(\Lambda\cdot R)R + \frac{\partial h_I}{\partial R} + \lambda_x(\overline{e}_t - \frac{R}{r}\cos\phi) , \quad (37)$$

$$\dot{\lambda}_\nu = \frac{a}{\nu}(\Lambda\cdot\overline{e}_t) , \quad (38)$$

$$\dot{\lambda}_{V_I} = -a\left\{\frac{(\Lambda\cdot\bar{e}_t)}{2V_I} + \left[(\Lambda\cdot\bar{e}_t)\left(\frac{C_2}{C_1+C_2I_B}\right) + \left((\Lambda\cdot\bar{e}_t)-\sigma\right)\frac{n_u'}{n_u} + \sigma\right]\frac{\partial I_B}{\partial V_I}\right\} ,$$

$$I = 1,2,3,\ldots(\max 5) , \tag{39}$$

$$\dot{\lambda}_\phi = \lambda_\chi R \cdot (\bar{m}\times\bar{e}_t) \tag{40}$$

The term h_I in equation (37) is the component of the variational hamiltonian containing the thrust acceleration:

$$h_I = a\sigma , \tag{41}$$

where the coefficient σ of the thrust acceleration is called the thrust switch function and is given by

$$\sigma = \Lambda\cdot\bar{e}_t - \frac{\nu\lambda_\nu}{c} . \tag{42}$$

The partial derivative $\partial h_I/\partial R$ is a somewhat lengthy expression, determined as follows; it is first written as the product of partial derivatives

$$\frac{\partial h_I}{\partial R} = \frac{\partial h_I}{\partial I_B}\frac{\partial I_B}{\partial R} . \tag{43}$$

When $I_B \equiv 0$ (during coast), $\dot{\lambda}_\nu \equiv 0$ and also $\partial h_I/\partial R \equiv 0$ because $\partial I_B/\partial R \equiv 0$.

Therefore $I_B > 0$ in what follows. The following partial derivatives are determined in a straightforward manner and the results are given:

$$\frac{\partial h_I}{\partial I_B} = h_I \left[\frac{1}{\sigma} \frac{\partial \sigma}{\partial I_B} + \frac{C_2}{C_1 + C_2 I_B} + \frac{1}{I_B} \right] \tag{44}$$

in which the "singularity" $\frac{1}{\sigma}$ is removed by using $h_I/\sigma = a$. Then

$$\frac{\partial \sigma}{\partial I_B} = (\Lambda \cdot \bar{e}_t - \sigma) \left[\frac{C_2}{C_1 + C_2 I_B} + \frac{\eta_u'}{\eta_u} \right] \tag{45}$$

and

$$\eta_u' = \frac{\partial \eta_u}{\partial I_B} = \frac{\eta_u}{I_B} \left[1 - \eta_u (1 - 2C_3 I_B / C_4) \right] \quad \text{when } I_B \leq 1 \tag{46}$$

and

$$\eta_u' = \frac{\partial \eta_u}{\partial I_B} = \frac{\eta_u}{I_B} \left[1 - \eta_u \left\{ 1 - C_3 \left(\frac{I_B + g}{C_4 + g} \right) \left[1 + I_B \left(\frac{1 + g'}{I_B + g} - \frac{g'}{C_4 + g} \right) \right] \right\} \right] \tag{47}$$

when $I_B > 1$; $g = (I_B - 1)^2$ and $g' = \frac{\partial g}{\partial I_B} = 2(I_B - 1)$.

The partial derivative $\partial h_I / \partial I_B$ is then determined. It remains to determine $\partial I_B / \partial R$. When $I_B \equiv 0$ or $I_B \equiv I_{max}$, $\partial I_B / \partial R \equiv 0$. It therefore remains to determine $\partial I_B / \partial R$ when the spacecraft is operating on the power-constraint curve (using all available input solar power). For this case, I_B is computed from equation (26), and, after some manipulation,

$$\frac{\partial I_B}{\partial R} = \frac{1}{2G} \frac{\partial p_t}{\partial R} \left[1 + \frac{2C_6G-b}{2C_7GI_B+b} \right] \tag{48}$$

in which

$$\frac{\partial p_t}{\partial R} = \frac{\eta_{pd} P_{ao}}{\eta_t} \frac{\partial \gamma}{\partial R} \tag{49}$$

and

$$\frac{\partial \gamma}{\partial R} = - \frac{2}{r^3} \bar{e}_r \frac{\partial \gamma}{\partial d} \tag{50}$$

In equation (50), $\bar{e}_r$ is the radius unit vector from the sun to the spacecraft and $\partial\gamma/\partial d$ is generated the same as in the old model.

In equation (39), $\partial I_B/\partial V_I$ is given by

$$\frac{\partial I_B}{\partial V_I} = - (1+C_5) \left[\frac{I_B}{2C_7GI_B+b} \right] \tag{51}$$

when the spacecraft is operating "on the power curve", and $\partial I_B/\partial V_I \equiv 0$ when $I_B \equiv I_{max}$ or $I_B \equiv 0$.

The expression for λ_x in equation (37) is identical to that of the old model when written in terms of the thrust acceleration,

$$\lambda_x = - a \frac{\Lambda \cdot (\bar{m} x \bar{e}_t)}{R \cdot (\bar{m} x \bar{e}_t)} \tag{52}$$

if the thrust cone angle is held constant and λ_x is identically zero if the cone angle is unconstrained. The unit vector $\bar{m}$ is defined the same as in the old model,

$$\bar{m} = \frac{R \times \Lambda}{|R \times \Lambda|} \quad . \tag{53}$$

From equation (39), it is seen that five (or less) additional differential equations (for $\dot{\lambda}_{V_I}$) are integrated along each trajectory, but only when at least one beam voltage transversality condition requires satisfaction, as requested by user program input. Actually, each $\dot{\lambda}_{V_I}$ can be non-zero only when the corresponding beam voltage V_I is being used to drive the spacecraft, and is identically-zero otherwise; therefore, at any given time along a trajectory, only one $\dot{\lambda}_{V_I}$ need be integrated. (Transversality is discussed in a later section.)

2.1.4 Optimality Conditions

The control variables available for optimization along the trajectory consist of the number of operating thrusters n_t, the beam voltage V_I (selectable from a set of from one to five constant values), the unit vector $\bar{e}_t$ defining the direction of thrust, and the beam current per thruster I_B . The control variable h_σ of the old model is replaced by I_B , such that $I_B \equiv 0$ defines a coast phase. As will be explained below, $I_B \equiv 0$ is used to conceptually define when the spacecraft is coasting rather than $n_t \equiv 0$ or $V_I \equiv 0$, since the optimal values of $n_t > 0$ and $V_I \neq 0$ must be maintained even during coast. (Of course, the voltage can be turned off on board the spacecraft during coast.)

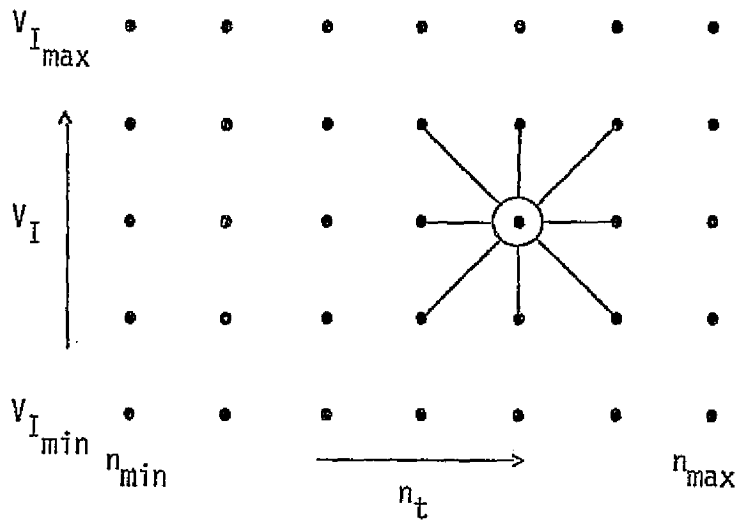
The application of the Maximum Principle of optimal control theory leads to the result that the proper choice of the control variables is that which maximizes the variational hamiltonian, h_V , at each point along the path. The variational hamiltonian for the problem forumlated here may be written

$$h_v = a\sigma - \frac{\mu}{r^3}(\Lambda\cdot R) - \dot{\Lambda}\cdot\dot{R} \tag{54}$$

where a is the thrust acceleration given by equation (35) and σ is the thrust switch function given by (42); σ gets its name "thrust switch function" because its sign determines, as usual, if the spacecraft is thrusting or coasting. Specifically, the variational hamiltonian is maximized with respect to the beam current I_B by setting I_B to its lowest permissible value (zero) when $\sigma<0$ and its largest permissible value when $\sigma>0$, for given values of n_t and V_I . The method of selecting and maintaining the optimal values of n_t and V_I is discussed below. For now, assume that the optimal values of n_t and V_I are known. Then, when $\sigma>0$, I_B is chosen as large as possible. This is accomplished by first computing the beam current according to (26), and using this value for I_B unless it is greater than I_{max}, in which case $I_B \equiv I_{max}$ is chosen. When $\sigma<0$, $I_B \equiv 0$ is the optimal choice. In this manner, the optimal choice of I_B is determined. There is also a value of the beam current, I_{min} , determined by the minimum throttling ratio, which does not enter into the discussion here for choosing the optimal beam current, but which plays a fundamental role in the algorithm for choosing optimal values for n_t and V_I . This will enter into the discussion below.

The variational hamiltonian h_v is maximized with respect to $\overline{e}_t$ identically the same whether employing the old or new spacecraft model; the optimal choice for $\overline{e}_t$ is discussed on p. 16 of [1].

It remains to determine the choices for n_t and V_I which maximize h_v at each point along the path. In general, n_t and V_I might be considered to generate a 2-dimensional control sub-space in which is found a finite, bounded, discrete rectangular set of permissible points:



Then, at the initial point in time, the variational hamiltonian is computed for each point of the entire grid (for each pair of values (V_I, n_t)), and such that I_B is chosen as a function of V_I and n_t according to the method discussed above*. Cases for which $I_B < I_{min}$ are discarded from the competition. When the grid mapping is complete, the optimal values of n_t and V_I are known, by simply saving the values associated with the largest h_V as the grid mapping progresses. It is assumed in the present implementation that all cases are not discarded from the competition for maximum h_V due to $I_B < I_{min}$, i.e., there is sufficient power available to thrust at the start of the mission for a sensibly designed spacecraft. The optimal values of n_t and V_I thus determined might be represented by the circled point in the grid above. If the thrust switch function σ associated with these optimal values is positive, the spacecraft thrusts (using the maximum value of I_B allowed); otherwise, $\sigma<0$ and the spacecraft coasts.

---

*This is depicted in Appendix B.

A critical distinction regarding the beam current I_B is made at this point. The notion of a "phantom" beam current I_{BP} is introduced and is defined as the value of beam current (per thruster) which the spacecraft would have if there were no physical restrictions (I_{min}, I_{max}) on the beam current. I_{BP} is therefore computed using expression (26). Also introduced is the restricted phantom beam current, I_{BX}, which is equal to I_{BP} unless $I_{BP} > I_{max}$, in which case $I_{BX} = I_{max}$. Therefore, both I_{BP} and I_{BX} may be less than I_{min}, but only I_{BP} may be greater than I_{max}. The actual beam current, which is used to drive the spacecraft when $\sigma>0$, cannot violate the (throttling ratio) bounds $I_{min} \leq I_B \leq I_{max}$ except for $I_B = 0$. Both I_{BP} and I_{BX} are always greater than zero.

It is the restricted phantom beam current I_{BX} which is always employed in the computation of the jet exhaust speed, c, in equation (36), even when $I_B = 0$. The jet exhaust speed (or equivalently, specific impulse) is therefore always a positive quantity, even when the spacecraft is not thrusting. The specific impulse is therefore conceived as a latent physical property which characterizes a thrust subsystem's potential capability, even when the spacecraft is not operating. In turn, the jet exhaust speed c, always computed using I_{BX}, is used in equation (42) for the thrust switch function σ. Then the sign of σ is used, in turn, to decide whether the spacecraft is thrusting or coasting, in which σ is computed using the optimal values of V_I and n_t. Therefore, using functional notation,

$$I_{BX} = \min \left\{ I_{BP}(V_I, n_t),\ I_{max} \right\} \tag{55}$$

Then, in terms of V_I and n_t, the jet exhaust speed is given by (again using functional notation)

$$c = \left(\frac{2}{c_{vel} k}\right)\left(C_1 + C_2 I_{BX}(V_I, n_t)\right) \eta_u\left(I_{BX}(V_I, n_t)\right) \sqrt{V_I} \tag{56}$$

Once optimal values for V_I and n_t are initialized, it remains to determine how to maintain optimal values along the path. A brute force approach would be to keep testing the entire grid of permissible points in the (V_I, n_t) control subspace along the path, and switch to a new value of (V_I, n_t) when the new controls produced a larger variational hamiltonian; the point of switchover would be isolated by iteration. However, for control grids having 30 or 40 points, this approach would be computationally very costly. An assumption is therefore made which is denoted "the neighboring solution assumption," for the purpose of building a computationally efficient optimal control algorithm. In this assumption, the only grid points in the control subspace which are considered as candidates for optimal control are those "directly" neighboring the presently existing optimal control point; the neighboring points are effectively defined by the connecting lines (to the circled optimal point) in the grid depicted above. In a two-dimensional control grid there are therefore (a maximum of) eight alternate control strategies to consider, and, obviously, fewer when the optimal control point is on a grid border or corner.

The tactic employed in the computer program for maintaining optimal values of V_I and n_t therefore consists of the above scheme of comparing candidate hamiltonian values, in which each point (in time) of optimal switchover of the controls V_I and n_t (i.e., the point where the difference between hamiltonian values associated with the optimal grid point and candidate grid point vanishes) is strongly isolated by iteration.

A few other considerations must be taken into account in the algorithm for maintaining optimal values of V_I and n_t. Roots of the two functions $I_{BP} - I_{max}$ and $I_{BP} - I_{min}$ must be strongly isolated (by iteration), corresponding respectively to the maximum throttling ratio (unity) and minimum throttling ratio thresholds. It is useful to introduce at this point the notion of an "imaginary spacecraft" associated with each point in the (V_I, n_t) control subspace neighboring the optimal point, such that the imaginary spacecraft has the corresponding neighboring values of beam voltage V_I and number of operating thrusters n_t. Then minimum throttling ratio thresholds must also be isolated for the imaginary spacecrafts associated with (some of the) points neighboring the optimal point in the (V_I, n_t) control subspace, because switches in this subspace are not allowed which would result in $I_B < I_{min}$. Switches in the (V_I, n_t) control subspace will occur regardless of the sign of the thrust switch function σ .

When the minimum throttling ratio threshold is attained and $\dot{I}_{BP}$ is negative, a switch must occur in the (V_I, n_t) subspace or else the spacecraft must commence coasting unless it was already coasting (σ<0). Or, if the throttling ratio of any comparative imaginary spacecraft attains the minimum threshold and $\dot{I}_{BP}$ for that imaginary spacecraft is positive, then a switch in the (V_I, n_t) control subspace may occur to the point associated with that imaginary spacecraft. It may also be that, for solar electric propulsion, the spacecraft will recede so far from the sun that all points of the (V_I, n_t) control subspace grid will "be" below the minimum throttling ratio threshold, so that the spacecraft will have no alternative but to coast; and it may also be that the spacecraft will commence thrusting again as it approaches the sun and some pair of values (V_I, n_t) have an associated acceptable throttling ratio with σ>0. Whenever one of the above situations (described in this paragraph) occurs, the primer derivative must be jumped so that the variational hamiltonian remains constant. The jump condition is

$$\dot{\Lambda}^{+} = \dot{\Lambda}^{-} + \left(\frac{h_I^{+} - h_I^{-}}{R \cdot \dot{R}}\right) R \tag{57}$$

in which $h_I = a\sigma$ is that portion of the variational hamiltonian associated with the engine parameters.

It is possible that the neighboring solution assumption (discussed above) will in fact be violated at some point in time along a particular trajectory, for a particular spacecraft configuration and a particular mission. This situation will occur extremely infrequently. However, when it does happen, the optimal controls (V_I, n_t) must be re-determined. It is a simple matter to detect when the neighboring solution assumption has been violated; whenever a switch occurs in the (V_I, n_t) control subspace to a point neighboring the prior point, the values of all variational hamiltonians associated with the points neighboring the new optimal (?) point are compared to the hamiltonian value associated with the new point. If the new point has the largest hamiltonian, it is (considered to be) the optimal point (V_I, n_t), and the neighboring solution assumption is not violated; otherwise, the assumption is violated, and one of the new neighboring points in the (V_I, n_t) control subspace is more optimal (the one having the largest hamiltonian), and an additional switch is immediately made to the new point, with a jump in $\dot{\Lambda}$ according to (57) to maintain hamiltonian constancy. Once the switch to the new point has occurred, the entire test of neighboring points for the optimality of the present values of (V_I, n_t) is conducted once again. The test of the neighboring (optimal) solution assumption is indeed carried out every time a switch occurs in the (V_I, n_t) control subspace, and the testing of neighboring points is repeated (as described above) until the optimal values of (V_I, n_t) are obtained.

Whenever the neighboring solution assumption is violated (which, again, happens extremely infrequently), a double-switch or multiple-switch occurs in the (V_I, n_t) control subspace, with a concurrent jump in $\dot{\Lambda}$; this represents a slightly sub-optimal control strategy, compared to the (computationally inefficient) globally optimum strategy of directly testing for the optimality of all points in the (V_I, n_t) subspace. The globally optimum strategy would therefore switch directly to the (not-necessarily-neighboring) new optimal point at a time along the trajectory very slightly less than that found by the neighboring solution strategy, and with no jump in $\dot{\Lambda}$ required. Nevertheless, the mechanization adopted for the general solution to the optimal rocket flight problem, as posed in this report, employs the neighboring solution assumption (or restriction) because:

- It is computationally efficient
- It is extremely rarely violated
- All violations are felt to have negligible impact on spacecraft masses and other performance parameters (compared to the globally optimum solution)

2.2 Boundary and Transversality Conditions

The basic boundary conditions and transversality conditions are described in [1]. This section discusses only those conditions which must be modified to accomodate the more sophisticated spacecraft model, and also some entirely new transversality conditions. Therefore, any boundary or transversality conditions appearing in [1] but not mentioned here remain unchanged.

The discussion in this section pertains solely to the new spacecraft model, and therefore certain quantities which are indigenous to the old model will be found entirely absent in the new analysis. Specifically, these quantities are the (constant) jet exhaust speed c of the old model, the reference thrust acceleration g, and the propulsion time τ. Therefore, all equations in [1] pertaining to these quantities are absent in the analysis describing the new spacecraft model, and any boundary or transversality conditions associated with these quantities are not applicable with the new model.

The general equation for the transversality conditions is written

$$\lambda_\pi d\pi + \sum_{i=1}^{n} \left[\Lambda \cdot d\dot{R} - \dot{\Lambda} \cdot dR + \lambda_\nu d\nu + \lambda_\phi d\phi + \sum_{I=1}^{n_{V\max}} \lambda_{V_I} dV_I - h_V dt \right]_{t_{i-1}}^{t_i} = 0 \qquad (58)$$

in which the performance index π is equal to the negative of the net spacecraft mass, $\pi = - m_{net}$, and λ_π is the arbitrary positive constant which renders the general transversality condition linear and homogeneous in the adjoint variables (λ_π replaces the symbol k in [1]). The convenient choice is made whereby each λ_{V_I} is forced to be continuous at each intermediate target, which means that only $\lambda_{V_I}(t_n)$ need appear in the derived transversality expressions rather than the cumbersome expression

$$\lambda_{V_I}(t_n) - \sum_{i=1}^{n-1}\left(\lambda^+_{V_I}(t_i) - \lambda^-_{V_I}(t_i)\right) - \lambda_{V_I}(t_o), \quad I = 1,2,3,\ldots(\max 5) \tag{59}$$

This is because $\lambda_{V_I}(t_n)$ alone, with $\lambda_{V_I}(t_o) = 0$ and $\lambda^+_{V_I}(t_i) = \lambda^-_{V_I}(t_i)$ for each i, has the same value as the cumbersome expression cited above if $\lambda_{V_I}(t_o)$ were not zero and $\lambda_{V_I}(t_i)$ were not continuous, and this is due to the absence of λ_{V_I} in the differential equations.

The expression for π may be written

$$\pi = j_r m_{rs} + m_o\{k_s + k_t - (1+k_t)\nu_n + j_r(1+k_{rt})e_x\left[(1+j_t k_t)\nu_n - j_t k_t \left(1 + \sum_{i=1}^{n-1}(k_{samp\ i} - k_{drop\ i})\right)\right] + (1+k_t)\sum_{i=1}^{n-1} k_{samp\ i} - k_t \sum_{i=1}^{n-1} k_{drop\ i}\}$$

$$+ (\alpha_a p_{ao} + m_b)\left[1 - j_a j_r(1+k_{rt})e_x\right] + (\alpha_t n_{max} p_{to})\left[1 - j_{tn} j_r(1+k_{rt})e_x\right], \tag{60}$$

where symbol definitions may be found in Nomenclature. π may be written functionally in its most general form,

$$\pi = \pi(v_{\infty o}, v_{\infty n}, \nu_n, V_{I_{max}}, \delta, i) \tag{61}$$

Using the notation $\pi_x = \partial\pi/\partial x$, the general variation of π may be written

$$d\pi = \pi_{v_{\infty o}} dv_{\infty o} + \pi_{v_{\infty n}} dv_{\infty n} + \pi_{\nu_n} d\nu_n + \pi_{V_{I_{max}}} dV_{I_{max}} + \pi_\delta d\delta + \pi_i di \tag{62}$$

Then, the following consists of a summary of those transversality conditions published in [1] which are altered when employing the new spacecraft model.

Equation (98) of [1] for π_{m_0} remains the same except the two terms containing j_p are eliminated. Equation (100) of [1] for $\pi_{V_{\infty n}}$ remains unchanged except the term $-j_{ps}m_{ps}$ is replaced by $-j_a m_a - j_{th}m_{th}$. Furthermore, there is a transcription error in that equation, such that the symbol v_c should be replaced by v_p.

As before, the arbitrary positive quantity λ_π is assigned a value which causes the transversality condition associated with the final mass ratio to be satisfied:

$$\lambda_\pi = - \lambda_{\nu_n}/\pi_{\nu_n} \tag{63}$$

This is defined as the negative of the symbol k in equation (144) of [1]; this sign is merely a matter of convention, so that the quantity (-k) is simply replaced by λ_π in the transversality equations appearing in [1]. (The form of λ_π expressed by (63) above is coded in the program.)

For open launch excess speed with m_0 being independent of δ and i, the transversality condition is

$$\frac{\lambda_\pi \pi_{V_{\infty 0}}}{\lambda_0} - 1 = 0 \quad . \tag{64}$$

For cases in which m_0 is a function of δ and/or i, the transversality conditions are written as follows: for open excess speed,

$$\lambda_\pi \pi_{m_0} \frac{\partial m_0}{\partial V_{\infty 0}} \left(\frac{V_{\infty 0}}{\Lambda_0 \cdot V_{\infty 0}}\right) - 1 = 0 \tag{65}$$

For open geocentric declination of $V_{\infty 0}$,

$$\lambda_\pi \pi_{m_0} \frac{\partial m_0}{\partial \delta} - \Lambda_0 \cdot \left[(V_{\infty 0} x \bar{n}_p) x V_{\infty 0} / v_{\infty 0} \cos\delta\right] = 0 \tag{66}$$

In addition, the factor f is dropped from equation (151) of [1]. No changes were required in the software for the conditions discussed in this paragraph, since they are coded in the form given here.

New transversality conditions arise due to the presence of the beam voltages V_I in the general transversality condition (58). For voltages V_I less than the maximum V_{Imax} , the transversality conditions (whose satisfaction generate optimal values of V_I) are, simply,

$$\lambda_{V_I}(t_n) = 0, \quad I = 1,2,3,\ldots \quad . \tag{67}$$

The constant beam voltages may be optimized individually or collectively, as specified by user program input. They may also be driven to specific values by the iterator.

A cautionary note is issued at this point to the program user wishing to optimize any or all of the beam voltages. Specifically, the quantities λ_{V_I} remain quite small (relative to unity) over any trajectory, so that their final values (equation (67)) are inherently small. This is due to the fact that $\lambda_{V_{Io}} = 0$, $I = 1,2,3,\ldots,$ and the derivatives $\dot{\lambda}_{V_I}$ are of the order of V_I^{-1} in magnitude. Furthermore, the independent variables V_I are relatively large compared to other independent variables such as Λ_0 and $\dot{\Lambda}_0$, which are of the order unity. Consequently, when attempting to optimize the beam voltages, the user must set the corresponding iterator independent variable weights to (e.g.,) $Xi(5) = 10^{-4}$ and dependent variable tolerances to (e.g.,) $Yi(3) = 10^{-8}$. However, since only limited experience has been gained in optimizing the beam voltages to date, the choice of best values for these weights and tolerances is not well understood.

Since V_{Imax} appears in the expression for the performance index π (through p_{to}), its transversality condition is given by

$$\lambda_{\pi}\pi_{V_{I\,max}} + \lambda_{V_{I_{max}}}(t_n) = 0 \tag{68}$$

When the power source is not matched to the thrust subsystem,

$$\pi_{V_{I_{max}}} = \alpha_t \eta_{max}\left(1-j_{th}j_r(1+k_{rt})e_x\right)\frac{\partial p_{to}}{\partial V_{I_{max}}} \quad . \tag{69}$$

When the power source is matched to the thrust subsystem (i.e., $p_{ao} = p_{a_{max}}$, where $p_{a_{max}}$ is given by (25)),

$$\pi_{V_{I_{max}}} = \eta_{max}\left\{(\alpha_a+\alpha_t)-j_r(1+k_{rt})e_x(j_a\alpha_a+j_{th}\alpha_t)\right\}\frac{\partial p_{to}}{\partial V_{I_{max}}} \quad . \tag{70}$$

In (69) and (70),

$$\frac{\partial p_{to}}{\partial V_{I_{max}}} = \frac{(1+C_5)I_{max}}{\eta_{pd}(C_6+C_7 I_{max})} + \left[\frac{p_{to}}{I_{max}} - \frac{C_7(1+C_5)(V_{I_{max}}+V_G-\Delta V_I)I_{max}}{\eta_{pd}(C_6+C_7 I_{max})^2}\right]\frac{\partial I_{max}}{\partial V_{I_{max}}} \tag{71}$$

When I_{max} is specified as constant,

$$\frac{\partial I_{max}}{\partial V_{I_{max}}} \equiv 0 \ , \tag{72}$$

and when the maximum power input to an individual thruster is specified via (25a),

$$\frac{\partial I_{max}}{\partial V_{I_{max}}} = - \frac{(1+C_5)I_{max}^2}{10^3 P_{max}} \quad . \tag{73}$$

2.3 Auxiliary Computations

This section presents equations employed in computations which are made after the iteration sequence involving the primary target is completed.

2.3.1 Additional Block Print Variables.

A standard print block is employed for printing information at various points along a trajectory. Each standard block contains a total of forty parameters, which are described in [1].

The standard block may now be augmented in a third way, in addition to the two ways (power degradation and target-relative coordinates) described in [1]. When the improved spacecraft model is invoked by program input NEW, two additional lines automatically appear as the sixth and seventh lines of the print block. The information contained in these lines is as follows:

NO. THR	Number of operating thrusters, n_t .
VOLTAGE	Beam voltage, V_I, in volts.
CURRENT	Beam current per thruster, I_B , in amps.
PHAN CUR	Beam current per thruster I_{BP} which would be realized if there were no limits imposed on the beam current; in amps. ("Phantom current", given by expression (26)).
SP IMP	Specific impulse, I_{sp}, given by expression (19), in seconds.

THRUST	Total thrust, f_t , as given by expression (18), but expressed in pounds.
BEAM POWER	Beam power per thruster, as given by expression (23) except divided by the number of operating thrusters n_t , in kilowatts.
DUMP POWER	Dumped power, in kilowatts; zero when $I_B \leq I_{max}$, otherwise computed from $V_I(I_{BP}-I_{max})$, where I_{BP} is the phantom current defined above.
UTIL EF	Propellant utilization efficiency, η_u , as given by expression (13) or (14).
THR RED	Thrust reduction factor, αF_t , as given by expression (12).
THR EF	Thruster efficiency, η_{TH} , as given by expression (11).
PPU EF	Power processor efficiency, η_{PPU} , as given by expression (16).
PTH EF	Efficiency η_{PTH} given by expression (15).
SS EF	Screen supply efficiency, η_s , given by expression (17).
ARRAY POWER	Total power output by the solar arrays (or other power source), p_a , as given by expression (6), in kilowatts.
PPU POWER	Power input to each PPU module, p_t , as given by expression (8), in kilowatts.

Whenever one of the triggers associated with the iterator independent variables X36 through X40 (the beam voltages) is turned on, the corresponding adjoint variables λ_{V_I} are integrated and printed as an additional line of the print block following the output quantities described above.

2.3.2 Additional Extremum Point Summary Print

When the improved spacecraft model is invoked by program input NEW, a second table of values is printed beneath the currently existing "Extremum Points of Selected Functions."

The usual extremum table contains a cross-reference column at the far right, which contains summary information relating to the second table if the entry (line) was caused by a parameter in the second table; otherwise the cross reference field is blank. The cross reference information consists of IMAX to denote points at which I_B attains the I_{max} threshold, IMIN to denote points at which I_B attains the I_{min} threshold, $\pm$ V to denote beam voltage switch points, and $\pm$ N to denote points of thruster staging.

The second table contains the time (repeated from the first table); the number of operating thrusters; the beam voltage (volts); the beam current per thruster I_B and phantom current I_{BP} (defined in the preceding section), both in amps; the propellant utilization efficiency η_u ; the specific impulse I_{SP} in seconds; the beam power per thruster p_b in kilowatts; the dumped power in kilowatts, as defined in the section immediately preceding, and the total thrust f_t in pounds.

If the engine state (number of thrusters and/or beam voltage) has switched, the corresponding table entry will be flagged with a plus sign or a minus sign, and the entire line will be repeated with values corresponding to after the switch has taken place. The plus and minus signs will also appear during coast phases, when the number of thrusters and beam voltage are printed as zero. Also, critical values of beam current are flagged with an asterisk(*).

3.0 PROGRAM INPUT

The following consists of a complete description of program inputs. With respect to the basic HILTOP report [1], many new input quantities necessary to characterize the more realistic propulsion system have been added. All new input names are flagged with a single asterisk (*), and inputs whose definitions have been modified are flagged with a double asterisk(**).

The new propulsion system logic is activated by a single program input key (NAMELIST input "NEW"); program modifications have been designed to retain the "old" HILTOP program within the framework of the new logic, so that old input data files (with no modifications required) will run the new program version and produce identical results as before.

3.1 Namelist

Inputs to HILTOP are given through the NAMELIST feature of the Fortran programming language. The input NAMELIST is named MINPUT, and every input required or used in the program is declared by name in the list. The general form for assigning an input value to a quantity is, simply,

NAME=VALUE

where NAME is the name assigned to the variable and is included in the NAME-LIST, and VALUE is a numerical or logical quantity consistent in form (i.e., logical, integer, or real) with NAME. Unless otherwise specified, all MINPUT names commencing with one of the letters I through N represent integers, whereas all names commencing with one of the letters A through H or O through Z are double precision floating point numbers. Each NAMELIST case must begin with the characters

&MINPUT

commencing in card column 2 and followed by at least one blank, and end with the characters

&END

preceded by at least one blank. Card column 1 is ignored on all NAMELIST input cards. Multiple data assignments on a single card are permissible if separated by commas. Blanks in the variable field, VALUE, are taken as zeroes. A comma following the last VALUE on a card is optional on the IBM system. The order of the input data assignments is arbitrary; i.e., they need not be in the same order as listed in the NAMELIST. In fact, there is no requirement that any specific input parameter be represented in the input data set. If no value is included in the inputs for a particular parameter, the default value is used (see Default Values). For other details regarding the NAMELIST feature, the reader is referred to, for example, the IBM System 360/Fortran IV Language manual. NAMELIST cases may be stacked back-to-back indefinitely. A single NAMELIST input error may cause the remaining NAMELIST inputs to be ignored.

3.2 Definitions of Input Parameters

Specific examples of the program inputs are given in the Sample Problems and Results section. Default-values of inputs are given in the next section.

The program inputs, in alphabetical order, are:

AAI	Desired final extra-ecliptic inclination, i. Related to AE, AR, and IOUT. [deg]
AE	Desired final extra-ecliptic eccentricity, e. Related to AAI, AR, and IOUT.
**ALPHAA	Specific mass of solar arrays, α_a . (See expression (2) of this report or expression (4) of [1].) [kg/kw]
**ALPHAT	Specific mass of power distribution, processing, and thruster subsystem, α_t . (See expression (3) of this report or expression (4) of [1].) [kg/kw]
ALTITU	This input variable is associated with program logic which has not been kept up-to-date, specifically, logic pertaining to optimum departure of a NERVA-type rocket from earth orbit. This variable should be ignored.

AN Trajectory-integration exponent n in expression (39) of [1].

AR Desired final extra-ecliptic perihelion distance, r_f. Related to AAI, AE, and IOUT. [AU]

ASOL Array of five elements consisting of the solar power law coefficients a_i in expression (18) of [1]. ASOL(1)≠0 tells the program to use the input coefficients rather than the internal coefficients. The coefficients are normalized internally, and the program executes the iterations to produce the required remarkable points of the power curve (which are printed).

BI Efficiency coefficient b in expression (16) of [1]. Related to DI and EI.

*BMASS Constant mass in expression (2), m_b. [kg]

B1, B2, B3 Launch vehicle coefficients b_1, b_2, and b_3 in expression (2) of [1]. Used only if MBOOST is negative. [kg, m/sec, kg]

CNI Inclination to ecliptic of primary-target orbit. Input only when MOPT3 = 11. Related to ECI, OMI, SAI, SOI, TPI, EMUODD, and RADODD. [deg]

CNIX Array of five elements, the first three of which may be currently used. Inclinations to ecliptic of intermediate-target orbits. Input CNIX(i) only when MOPTX(i) = 11. Related to ECIX, OMIX, SAIX, SOIX, TPIX, EMUODX, and RADODX. [deg]

*CSEP Array of seven elements, consisting of the quantities C_1, C_2, C_3, C_4, C_5, C_6, and C_7, respectively, found in equations (12) through (17).

CSTR Structural factor, k_s, in expression (8) of [1].

CTANK Propellant tankage factor, k_t, in expression (7) of [1].

CTRET Retro tankage factor, k_{rt}, in expression (11) of [1].

*CURMAX Maximum allowable beam current for an individual thruster, I_{max}. Related to POWMAX. [amps]

*CURMIN Minimum allowable beam current for an individual thruster, I_{min}. Related to TRATIO. [amps]

*CVOLT Neutralizer to beam coupling potential, V_G, in expressions (15) and (16) [volts]

*DEFFIC — Discharge supply efficiency, η_D , in expression (16).

DI — Efficiency coefficient d in expression (16) of [1]. Related to BI and EI. [km/sec]

*DLOSS — Discharge losses, ϵ_I , in expressions (15) and (16). [eV/ion]

DMRETR — Retro engine mass, m_{rs} , in expression (11) of [1]. [kg]

**DPOW — Ratio of housekeeping power p_h to reference power p_{ref}. Used only with the old spacecraft model. The power transmitted to the propulsion system is that generated by the arrays less housekeeping power which is constant along the trajectory. The power output of the arrays normal to the sun at 1 AU is $p_{ref} + p_h$. This option should not be invoked on missions during which large solar distances are encountered where the power developed is less than p_h . Erroneous results will be obtained.

*DVOLT — Discharge voltage, ΔV_I, in expressions (15) and (16). [volts]

ECI — Eccentricity of primary-target orbit. Must be less than unity. Input only when MOPT3 = 11. Related to CNI, OMI, SAI, SOI, TPI, EMMUODD, and RADODD.

ECIX — Array of five elements, the first three of which may be currently used. Eccentricities of intermediate-target orbits. Input ECIX(i) only when MOPTX(i) = 11. Related to CNIX, OMIX, SAIX, SOIX, TPIX, EMUODX, and RADODX.

EI — Efficiency coefficient e in expression (16) of [1]. Related to BI and DI.

EMUODD — Gravitational constant of primary-target. Input only when MOPT3 = 11. Related to ECI, CNI, OMI, SAI, SOI, TPI, and RADODD. [m^3/sec^2]

EMUODX — Array of five elements pertaining to the gravitational constants of intermediate-targets. These inputs must be ignored at present.

*ETAPD — Efficiency of the power distribution system, η_{pd}, in expression (7).

*ETCONV — DC-DC converter efficiency, η_{conv}, in expression (9).

GAMMAX — Maximum permissible value of the power function γ when MODE = 5. At solar distances less than the value for which γ = GAMMAX, the solar arrays are assumed to be tilted or shielded such that γ is maintained at the limiting value.

**GAP — Propulsion-corner proximity tolerance-interval, $\Delta\sigma$. See discussion in the section Avoiding Corners in the Propulsion-time Function in [1]. Whenever the thrust switch function σ grazes the zero-axis within the tolerance $|\Delta\sigma|$ on any trajectory, an internal counter is incremented, and the trajectory is considered to be in the neighborhood of a propulsion-time corner. Positive value of GAP causes forced-thrusting case to be inserted, negative value causes bypass to next case, whenever the internal counter reaches the related input variable NHUNG. Value is set negative when new spacecraft model is invoked.

HOUR — Hour-of-day of reference date (e.g., 17.352D0). Related to MYEAR, MONTH, and MDAY.

IBAL — Ballistic option indicator. Setting IBAL ≠ 0 invokes option 1 discussed in the section Ballistic Trajectory Option of [1].

INTPR — Indicator which specifies print-length when the iteration in subroutine INTERP fails. Value of 0 causes shortprint and 1 causes detailed-print.

IOUT — Extra-ecliptic mission indicator. IOUT = 1 or 2 indicates that extra-ecliptic target conditions are desired, in which the iterator dependent variable triggers Y1(2) through Y6(2) are set equal to 1, and for which the input LAUNCH (which see) should probably be set to 1, and parameters related to LAUNCH also set appropriately. Ordinarily MOPT2 = 3. No retro stage may be employed.

=1 i, e, r_p specified; $f_n = 0$.

=2 i, e, a specified; f_n optimized.

In the above, i = final extra-ecliptic inclination, e = final eccentricity, r_p = final perihelion distance, a = final semi-major axis, and f_n = true anomaly at the final time. Final Ω and ω are optimized in both cases. Related to AE, AR, and AAI.

IRK — Numerical integration option (currently not used).

IRL — Primer-origin-proximity step-size-control indicator. Value of zero causes the bypass of control, leaving the step-size Δu constant. See discussion in the section, Integration (Thrust) of [1].

IROT A non-zero value of IROT causes the input ecliptic projection of the primer vector and its time derivative to be rotated about the z-axis through an angle equal to the difference in longitudes of the spacecraft between the last trajectory of the previous case (or zero if no previous case) and the first trajectory of the current case. This feature permits one to use the initial adjoint variables from a 2-dimensional trajectory as the initial-guess inputs for a 3-dimensional trajectory using the ephemeris option.

ISPIN Spinner indicator. Not used at present.

ITF Provides normal termination conditions for runs which require more machine time than is estimated. The value specifies the number of machine-time seconds (CPU and I/O) required to execute the summary trajectory after halting the iteration-sequence. [sec] Does not apply if subroutine REMTIM is dummied.

ITPRNT Indicator for special print from MINMX3 iterator. Non-zero value invokes print.

*JA Jettison indicator j_a for solar arrays (or other power generation system) prior to primary-target retro-maneuver, as used in expression (5).

= 0 Solar arrays not jettisoned.

= 1 Solar arrays jettisoned prior to retro-maneuver.

**JPP Jettison indicator j_{th} for electric propulsion thrust subsystem prior to primary-target retro-maneuver, as used in expression (5). In the old spacecraft model, jettison indicator j_{ps} for entire electric propulsion system prior to primary-target retro-maneuver, as used in expression (9) of [1] .

= 0 Propulsion system not jettisoned.

= 1 Propulsion system jettisoned prior to retro maneuver.

JPRINT Unit 11 printout-length indicator. A value of zero causes the iterator independent and dependent variables to be output only for each summary-trajectory; a value of one causes the same output additionally at each iteration of an iteration sequence.

JT Jettison indicator j_t for electric propulsion tankage prior to primary-target retro-maneuver, as used in expression (5).

= 0 Tankage not jettisoned.

= 1 Tankage jettisoned prior to retro-maneuver.

KPART — Option for automatically selecting improved independent parameter perturbations for generating the iterator's partial derivative matrix. The option is invoked by setting KPART = N(N>0), where N is the maximum number of allowed steps, as discussed in the section, Perturbation Step Size Selector of [1]. KPART must be set back to zero if option is not desired on subsequent cases.

LAUNCH — Launch mode selector, pertaining to the optimization of the departure asymptote declination, invoked by LAUNCH = 1. Related to X10, Y10, X17, and Y17.

LOADX — Intermediate-target initial-guess feature. Should be used with NSET(5) = 1, and then set to zero on the subsequent case. A non-zero value of LOADX will invoke this feature, whereby the primer Λ and its derivative $\dot{\Lambda}$ will be loaded into the iterator independent-variable arrays at each intermediate-target provided that the trigger of the independent variable is on. The sole purpose of this capability is merely to generate an initial-guess for a multiple-target mission, where the values loaded into the iterator arrays represent continuous Λ and $\dot{\Lambda}$ at each target.

*MATCH — Logical indicator for maximum useable solar array (or other power source) power output under reference conditions (normal to sun at 1 AU distance) perfectly matched to maximum power acceptable by the power distribution system. Causes input PAO to be overridden.

= T Power is matched.

= F Power is not matched; input PAO is used.

MAXHAM — Maximum number of times that the program will print the warning message BAD HAMILTONIAN on any given computer run.

MBOOST — Launch vehicle selector.

=0 ATLAS (SLV3X)/CENTAUR
1 TITAN III C
2 TITAN III C (1207)
3 TITAN III X/CENTAUR
4 TITAN III X (1207)
5 TITAN III X (1207)/CENTAUR
6 SATURN IB/LM
7 SATURN IB/CENTAUR
8 SATURN IC/SIVB/CENTAUR
9 TITAN III X (1205)/CENTAUR
10 TITAN III B (CORE)/CENTAUR
11 TITAN III D (1205)/CENTAUR
12 DELTA
13 TITAN III D
14 TITAN III D (1205)/CENTAUR/TE364(2250)

(continued on next page)

15 TITAN III E/CENTAUR
16 SHUTTLE/TRANSTAGE
17 SHUTTLE/DELTA
18 SHUTTLE/AGENA
19 SHUTTLE/CENTAUR
20 SHUTTLE/CENTAUR/BURNER II (2300)
21 SHUTTLE/IUS
NEG Use input booster coefficients B1, B2, and B3.

MDAY — Day-of-month of reference date (e.g., 26). Related to MYEAR, MONTH and HOUR.

MODE — Power variation option selector. The value of MODE is equal to the option-number of the power-curve, discussed in the section, Electric Propulsion System of reference [1] (which see). Possibly related to ASOL and GAMMAX. MODE = 1 has been eliminated.

MONTH — Month-of-year of reference date (e.g., 8). Related to MYEAR, MDAY, and HOUR.

MOPT — Ballistic option indicator. Using MOPT invokes option 2, discussed in the section, Ballistic Trajectory Option of [1], as follows:

= 0 No action (use input Λ_0, $\dot{\Lambda}_0$, and $v_{\infty 0}$) .

= 1 Generate ballistic solution with flyby end conditions.

= 2 Generate ballistic solutuon with orbiter end conditions.

Related to REVS.

MOPTX — Array of five elements, the first three of which may be currently used. This array specifies the target-number, or planet-number, of the successive intermediate-targets, and a value of zero indicates absence of the intermediate-target. A zero-entry must not precede a non-zero entry. Planet selection is the same as for MOPT2. MOPTX(1) pertains to iterator parameters X41-X50 and Y41-Y50; MOPTX(2) pertains to X51-X60 and Y51-Y60; and MOPTX(3) pertains to X61-X70 and Y61-Y70. Times at the targets are X48, X58, and X68. Not to be used unless MOPT2 ≠ 0.

MOPT2 — Launch planet number and ephemeris-option indicator.

= 0 Analytical planetary ephemeris is not used.

≠ 0 Analytical planetary ephemeris is used and the specific launch planet is selected as follows:

(Continued on next page)

=1	Mercury	=29	Flora
2	Venus	30	Achilles
3	Earth	31	Amor
4	Mars	32	Hidalgo
5	Jupiter	33	Alinda
6	Saturn	34	Grigg-Skjellerup (1977)*
7	Uranus	35	Kopff
8	Neptune	36	Grigg-Skjellerup (1982)*
9	Pluto	37	Ganymed
10	Ceres	38	Ivar
11	Input Target**	39	Beira
12	D'Arrest (1982)*	40	Kepler
13	Encke (1980)*	41	Giacobini-Zinner (1985)*
14	Icarus (1978)*	42	Borrelly (1987)*
15	Eros	43	Tempel II (1988)*
16	Geographos (1983)*	44	Tempel II (1983)*
17	Encke (1977)*	45	Tuttle-Giacobini-Kresak
18	Encke (1984)*	46	Schaumasse
19	Encke (1987)*	47	Honda-Mrkos-Pajdusakova
20	Halley	48	Giacobini-Zinner (1979)*
21	Betulia	49	Icarus (1987)*
22	Toro (1983)*	50	Toro (1987)*
23	Pallas	51	Geographos (1987)*
24	Juno	52	Grigg-Skjellerup (1987)*
25	Vesta	53	Pons-Winnecke (1989)*
26	Astraea	54	Reinmuth-1 (1988)*
27	Hebe	55	Encke (1990)*
28	Iris		

MOPT3 Planet number of primary target. Planet selection is the same as for MOPT2. If ephemeris is not used, MOPT3 is used only for retro-stage mass computations.

MOPT4 Array of ten elements, specifying up to ten post-swingby targets. Planet selection is the same as for MOPT2, and a value of zero indicates the absence of a post-swingby target. A negative value in MOPT4(1) selects multiple ballistic swingbys, rather than a set of single swingbys in which case also set MAXHAM = 0. Negative values (in absolute value) produce planet selection the same as for MOPT2. When MOPT4(1) < 0, the remaining elements of MOPT4(i) may be positive or negative. See the section, Swingby Continuation Analysis of [1] for details and Sample Case H of [1] for an example-case. Should be used only for primary-target flyby missions. Related to T2, MSWING, NSWING and XSWING.

*Year-value indicates apparition for which internal orbital elements are most accurate.

**Input corresponding orbit elements (see CNI, CNIX). None are available for the launch planet.

MPOW Flag used in conjunction with the solar array degradation option. Value of zero results in the optimum orientation of the arrays relative to the sun line. A non-zero value forces the arrays to an orientation yielding the maximum power achievable at that instant. Related to TPOWER.

MPRINT Indicator for printing the summary-trajectory (final trajectory of a case) as a function of time or for invoking extra printout.

= 0 Small-size block print at thrust switch points only (SWITCH POINT SUMMARY page).

= 1 Same as = 0, except expands to become a standard print-block of parameters for each computed point along the trajectory, including the trajectory extension controlled by the input variable TGO.

= 2 Same as = 0, except each block contains extra lines consisting of target-relative coordinates and target magnitudes.

= 3 Combination of = 1 and = 2.

MPUNCH Punched-card and trajectory-tape generation control.

= 0 No special output.

1 Punch final values of independent parameters.

2 In addition, punch selected mission analysis parameters used for graphic documentation or other purposes.

<0 and>-100 Punch trajectory output used with the ASTEA program. The absolute value of MPUNCH determines the frequency of trajectory points output, e.g., -3 would result in the punching of every third integration point.

≤-101 Trajectory tape output used with the ASTEA program. The absolute value less 100 determines the frequency of trajectory points output. Related to NTAPE.

MREAD Card input option (iterator independent variables)

= 0 No special cards input.

= 1 The independent variables generated by a previous run by the MPUNCH = 1 or 2 option are input following the NAMELIST case, as discussed in the section, Program Output of [1].

MSWING Array of ten elements, used only when running multiple-target ballistic swingbys, such that MSWING(i) corresponds to MOPT4(i) and selects the type of swingby maneuver desired at the respective swingby target. Used only if MOPT4(1) < 0. The shooting method

(MINMX3 iterator) is used, and values of -1, -2, or -3 correspond to a swingby passage distance initial guess of $r_p = \infty$ (i.e., continuous heliocentric velocity). Each element MSWING(i) may have any of the following values:

= -1 Go* directly for unpowered swingby; if and only if it fails, go for powered swingby having flight time T2(i) = initial guess.

= -2 Go directly for powered swingby only, having T2(i) = flight time of post-swingby leg.

= -3 Go directly for unpowered swingby; then, whether it succeeds or not, go for powered swingby having T2(i) = flight time.

= -4 Go directly for unpowered swingby, but using initial velocity guess loaded into XSWING(j, i), j = 1,2,3, similar to MSWING(i) = - 1.

= -5 Same as = -2, except use initial guess as in = -4.

*"Go for" means "attempt to obtain (solution)".
Related to MOPT4, T2, XSWING, and NSWING.

MTMASS Mission-type selector pertaining to the primary target.

= 0 Flyby mission.

1 Orbiter (high-thrust retro-maneuver without velocity loss).

2 Orbiter (high-thrust retro-maneuver with velocity loss).

3 Specified arrival excess speed $v_{\infty n}$.

If $v_{\infty n} = 0$, rendezvous mission
If $v_{\infty n} > 0$, controlled flyby mission
No retro-maneuver in either case.

4 Orbiter (Electric propulsion system performs spiral maneuver. Arrival excess speed $v_{\infty n}$ must be specified as zero).

Other parameters which may be related to MTMASS are DMRETR, CTRET, RPER, RAP, THRET, SPIRET, JPP,JT, and JA.

MUPDAT Flag indicating whether iterator independent variables at end of one case are be updated for use as first guesses of next case.

= 0 Do not update independent parameters.

1 Update independent parameters for next case to be those obtained at end of iteration on the current case.

MYEAR Year of reference date (e.g., 1982). Related to MONTH, MDAY, and HOUR.

*NDELTA Increment Δn_t in number of thrusters which may be switched on or off at any given time. Related to NMIN and NMAX.

NDIST Identification number of celestial body to be used as the reference for the communication distance and angle measurement printed in the Extremum Point Summary Table. Identification code is the same as for MOPT2. Useful for determining minimum distance of spacecraft to other bodies in the solar system, including the primary target when attempting to generate a solution for the first time.

*NEW Master logical indicator for invoking the improved spacecraft model logic; related to all other inputs flagged by an asterisk(*).

= T Use new spacecraft model

= F Use old spacecraft model

NHUNG Maximum number of propulsion-corner-proximity occurrences allowed in a given iteration-sequence. Related to GAP.

*NMAX The maximum number of operating thrusters, n_{max}. Related to NMIN and NDELTA.

*NMIN The minimum number of operating thrusters, n_{min}; greater than zero -- pertains to thrust phases only. Related to NMAX and NDELTA.

NORMAL Automatic adjoint-variable scaling.

= 0 No action.

1 All Λ and $\dot{\Lambda}$ are scaled such that λ_{vo} becomes unity.

NPERF Identification number of end condition that is to be used as the performance index when employing the direct parameter optimization feature (Improve Mode). The identification code is the same as the i in the Yi end condition array.

NPRINT Print selection flag. Permits selection of amount of printout desired on each case.

= 0 Print only the case summary.

1 Print switching point summary of final trajectory.

2 Print MINPUT and case setup.

4 Print trajectory summary on each iteration.

8 Print partial derivative matrix each iteration.
(Continued on next page)

ORIGINAL PAGE IS OF POOR QUALITY

Combinations of options obtained by summing options desired. If NPRINT > 15, printout consistent with NPRINT = 0 is obtained. If the sign of NPRINT is reversed to negative, the iterator independent and dependent variables additionally are printed for every trajectory which HILTOP generates (including neighboring trajectories).

NSET — Iteration-sequence control array.

NSET(1) Not used for input.

NSET(2) Not used for input.

NSET(3) Maximum number of iterations permitted in attempting to satisfy constraints in satisfy mode. If zero, no upper limit imposed.

NSET(4) Flag indicating whether constraints are to be satisfied prior to entering improve mode.

= 0 Satisfy constraints first.

1 Proceed immediately to improve mode.

NSET(5) Maximum number of iterations permitted after entering improve mode. Setting NSET(5) = 1 causes iterator to be bypassed and computes single trajectory to obtain printout.

NSWING — Swingby continuation analysis option indicator. NSWING must be negative and has the same definition as MSWING (which see); NSWING must be used when MOPT4(1) > 0, and <u>may</u> be used when MOPT4(1) <0. If MOPT4(1) < 0 and MSWING(i) = 0, then MSWING(i) will be set to the value of NSWING. Related to MSWING, MOPT4 and T2.

NSWPAR — Iterator independent-variable perturbation-increment control.

= 0 No action.

1 Allows the iterator to vary a given independent-variable perturbation Δx whenever a neighboring trajectory is detected which has a different number of thrust switch points than the associated nominal trajectory. Δx is varied until the same number of switch points is achieved.

NTAPE — Specifies the unit-number for the ASTEA trajectory tape. Pertains to when MPUNCH $\leq$ - 101.

OMI — Ascending node angle (with respect to vernal equinox) of primary-target orbit. Input only when MOPT3 = 11. Related to CNI, ECI, SAI, SOI, TPI, EMUODD, and RADODD. [deg]

OMIX — Array of five elements, the first three of which may be currently used. Ascending node angles of intermediate-target orbits. Input OMIX(i) only when MOPTX(i) = 11. Related to CNIX, ECIX, SAIX, SOIX, TPIX, EMUODX, and RADODX. [deg]

*ONHDOT — Logical indicator which allows derivatives of hamiltonian switch functions to be monitored along each trajectory. The user should ignore this input.

*PAO — Maximum power output by solar arrays (or other power source) under reference conditions, p_{ao}, as used in expression (2). Related to MATCH. [kw]

*PDIU — Digital interface unit power requirement, p_{diu} , as used in expression (9). [kw]

*PLOSS — Thruster neutral propellant loss, $\dot{m}_{\ell}$, as used in expression (14). [equivalent amps]

*PLV — Power processor low voltage input power requirement, $p_{\ell v}$, as used in expression (9). [kw]

*PMM — Mission module power requirement, p_{mm}, as used in expression (9). [kw]

*PNFLOW — Neutralizer propellant flow rate, $\dot{m}_{n}$, as used in expression (14). [equivalent amps]

POWFIX — Launch-vehicle-independent (i.e., no launch vehicle) trajectory option in which the value of POWFIX is the spacecraft's reference power. [kw] Not available when NEW = T.

*POWMAX — Maximum input power to an individual thruster, p_{max}, as used in expression (25a). When non-zero, this input overrides CURMAX. [kw]

*PRAMP — Logical indicator used for special printout during implementation and debugging of the new spacecraft model. The user should ignore this input.

*PROOT — General logical indicator allowing more printout from the HILTOP trajectory function monitoring module.

- = F No printout from the function monitoring module.
- = T Generates summary printout from the function monitoring module along the summary trajectory of each case.

PSIGN	Flag defining the sense of the launch hyperbolic excess velocity relative to the initial primer vector. A value of +1. results in the assignment of the geocentric right ascension of the excess velocity equal to that of the initial primer vector. A value of -1. causes the geocentric right ascension of the excess velocity to be 180 degrees from that of the initial primer.
*PTS	Thrust subsystem power requirement, p_{ts}, as used in expression (9). [kw]
RADODD	Radius of primary target. Input only when MOPT3 = 11. Related to CNI, ECI, OMI, SAI, SOI, TPI, and EMUODD. [meters]
RADODX	Array of five elements pertaining to the radii of intermediate targets. These inputs are not used at present.
RAP	Apoapse distance of capture orbit about primary target. [planet radii]
REVS	Number of complete revolutions of the ballistic trajectory generated when the associated input MOPT is used. Must be a positive whole number.
RPER	Periapse distance of capture orbit about primary target. [planet radii]
SAI	Semi-major axis of primary-target orbit (must be positive). Input only when MOPT3 = 11. Related to CNI, ECI, OMI, SOI, TPI, EMUODD, and RADODD. [AU]
SAIX	Array of five elements, the first three of which may be currently used. Semi-major axes of intermediate-target orbits (must be positive). Input SAIX(i) only when MOPTX(i) = 11. Related to CNIX, ECIX, OMIX, SOIX, TPIX, EMUODX, and RADODX. [AU]
SOI	Argument of perihelion of primary-target orbit. Input only when MOPT3 = 11. Related to CNI, ECI, OMI, SAI, TPI, EMUODD, and RADODD. [deg]
SOIX	Array of five elements, the first three of which may be currently used. Arguments of perihelion of intermediate-target orbits. Input SOIX(i) only when MOPTX(i) = 11. Related to CNIX, ECIX, OMIX, SAIX, TPIX, EMUODX, and RADODX. [deg]
SPIRET	Retro-stage specific impulse (pertaining to the retro-maneuver at the primary target). [sec]
STATE	Array of six elements containing the Cartesian position and velocity components of the primary target. Use only when MOPT2 = 0 and the trigger settings of Y1(2) through Y6(2) are 0 or 1. [AU, AU/ tau] (tau = 58.132440991 days)
STEP1	Thrust-phase computation step size, Δu. Related to AN.

STEP2 Coast-phase computation step size, $\Delta\beta$.

TCOAST Array of twenty elements, consisting of the durations of the coast phases corresponding to the coast-phase start-times input in the associated array TOFF. [days]

TDV Time of occurrence of an impulsive deep space burn, in days from the start of the trajectory, which may be used only if the entire trajectory is ballistic (i.e., electric propulsion is not permitted with this option, nor is a third intermediate target). Iterator independent variables X64, X65, and X66 must be turned on, as these are used as the ΔV vector components of the deep space burn in EMOS. Also, set MAXHAM = 0. The following special feature is available regarding a first intermediate-target. If 1.D5 < TDV < 2.D5, then the burn occurs (TDV - 1.D5) days after passage of that target; if TDV > 2.D5, the burn occurs (TDV - 2.D5) days before passage of that target. [days]

TGO Ballistic trajectory-extension print option. When zero, no action. When positive, TGO = the number of days that the trajectory is to extend ballistically beyond the primary-target when no swingby-continuation is requested, and ballistically beyond the (last) post-swingby target when swingby-continuation is requested (in addition to the post-swingby trajectory segment itself). Any negative value will invoke printout of only the post-swingby trajectory segment or segments when swingby-continuation is requested. Applies also to trajectories with multiple swingbys. [days]

THRET Retro-stage thrust, f_r, used only when MTMASS = 2. [lbs]

TOFF Array of twenty elements, consisting of the times, in days from the start of the trajectory, at which imposed coast phases are to begin. Times must be in ascending order. Related to TCOAST. [days] First negative value indicates end of input.

TPI Time from reference date (MYEAR, etc.) to perihelion passage, for the primary target. Input only when MOPT3 = 11. Related to CNI, ECI, OMI, SAI, SOI, EMUODD, and RADODD. [days]

TPIX Array of five elements, the first three of which may be currently used. Times from reference date (MYEAR, etc.) to perihelion passages, for the intermediate targets. Input TPIX(i) only when MOPTX(i) = 11. Related to CNIX, ECIX, OMIX, SAIX, SOIX, EMUODX, and RADODX. [days]

TPOWER Solar-cell degradation characteristic-time; nuclear electric propulsion radioactive-decay characteristic-time. Related to MPOW. [days] The default value must be used when NEW = T.

*TRATIO Minimum throttling ratio, t_{ratio}, as used in expression (25b). When non-zero, this input overrides CURMIN.

TSCALE Iterator dependent-variable tolerance-interval scaling factor; scales all tolerances multiplicatively by the amount TSCALE.

T2 Array of ten elements consisting of initial estimates of swingby-continuation trajectory-segment flight-times, i.e., T2(i) corresponds to MOPT4(i). [days]

VOLTAGES The beam voltages are input as iterator independent variables X36 through X40 (which see).

XANG1 Latitude of the launch site. Used only if LAUNCH is non-zero. Related to XANG2. [deg]

XANG2 Maximum parking orbit inclination permitted by range safety considerations. Used only if LAUNCH is non-zero. Related to XANG1. [deg]

XSWING Array of velocity vectors consisting of initial velocity guesses of a given post-swingby trajectory segment. Used only when either NSWING or MSWING has a value of -4 or -5. See especially the description of MSWING = -4. Velocity consists of exactly the same values as found in the V1, V2, V3 locations of the trajectory block print (first block). Related to MSWING, NSWING, MOPT4, and T2. [AU/tau]

X0 Array of seven elements, the first six of which contain the Cartesian position and velocity components of the launch planet. The seventh element is not used for input. Used only when MOPT2 = 0. [AU, AU/tau]

The following describes the iterator independent and dependent variable arrays of the boundary value problem. Input pertaining to the individual independent parameters is contained in the arrays X1 through X70. The independent-parameter arrays have five elements for each variable, as follows (where i = 1, 2, 3, ..., 70):

Xi(1) Input value of parameter. Must be input regardless of trigger setting. If trigger is on (i.e., Xi(2) = 1), input value is used as initial guess of independent parameter and is varied at each subsequent iteration. If trigger is off, the parameter is not used as an independent parameter and is not changed.

Xi(2) Trigger indicating whether parameter is to be an independent parameter in boundary value problem.

Xi(2) = 0 Not an independent parameter. (Trigger is "off").
1 Use as independent parameter. (Trigger is "on").

Xi(3) Maximum change to parameter permitted in a single iteration. Should be a positive quantity. Used only if trigger is on. Units are same as that of the parameter.

Xi(4) Perturbation increment used to compute partial derivatives by finite differences. Used only if trigger is on. Units are same as that of the parameter.

Xi(5) Weighting factor. Should be a positive quantity. A value of 1. is generally recommended. The larger the weighting factor, the more the parameter is inhibited from varying. Used only if trigger is on.

The independent variables are as follows:

X1	$\Lambda_0(1)$	Initial primer vector.
X2	$\Lambda_0(2)$	
X3	$\Lambda_0(3)$	
X4	$\dot{\Lambda}_0(1)$	Initial primer derivative.
X5	$\dot{\Lambda}_0(2)$	
X6	$\dot{\Lambda}_0(3)$	
X7	$\lambda_{\nu 0}$	Initial mass-ratio adjoint-variable
X8	λ_τ	Propulsion-time adjoint-variable. Should be zero when NEW = T.
X9		Not used.
X10	δ	Geocentric declination of launch hyperbolic excess velocity. [deg]

There is no conversion from input to internal units for any of the adjoint variables.

X11	Reference thrust acceleration, g. [m/sec^2] Ignored when NEW = T.
X12	Electric propulsion system jet exhaust speed, c. [m/sec] Ignored when NEW = T.
X13	Launch hyperbolic excess speed, $v_{\infty 0}$. [m/sec]
X14	Hyperbolic excess speed at primary target, $v_{\infty n}$, [m/sec]
X15	Initial time, t_0, measured from the reference date (MYEAR, etc.). [days]

ORIGINAL PAGE IS
OF POOR QUALITY

X16		Time at the primary target, t_n, measured from the reference date (MYEAR, etc.). [days]
X17		Launch parking orbit inclination, i. Used only if LAUNCH = 1. Optimized internally by the program if both X17 and Y17 triggers are off. [deg]
X18	$\dot{x}_0$	Initial spacecraft heliocentric velocity. Not required unless one of the three triggers is on. [AU/tau] (tau = 58.132440991 days)
X19	$\dot{y}_0$	
X20	$\dot{z}_0$	
X21		Constant thrust cone-angle, ϕ . Non-zero value invokes the constant-ϕ constraint. $0 < \phi \leq 180°$. Zero-value implies that ϕ is optimized along the trajectory (variable ϕ). [deg]

X22 through X29 are currently not used (although some locations following X21 are reserved for additional constant thrust cone-angles).

X30	λ_s	Degradation-time adjoint-variable. Should be zero when NEW = T.

X31 through X35 are currently not used.

X36 through X40 are the constant (along the trajectory) beam voltages, in volts. When NEW = T, at least X36 must be greater than zero.

*X36	Beam voltages, V_I [volts]
*X37	
*X38	
*X39	
*X40	

Values of voltages must be ascending, starting with X36 as the lowest value. Value of zero indicates end of inputs, e.g., X39 = 0.D0 indicates there are three voltage levels to be simulated, X36, X37, and X38. X41 through X50 pertain to the first intermediate target X51 through X60 pertain to the second intermediate target, and X61 through X70 pertain to the third intermediate target. The corresponding intermediate-target parameters are ignored if the intermediate target is absent. Subscripts 1, 2, and 3 pertain to the first, second, and third intermediate targets, respectively. Intermediate targets are invo ked via the MOPTX array

X41	$\Lambda_1(1)$	Primer vector (at start of trajectory segment)
X42	$\Lambda_1(2)$	
X43	$\Lambda_1(3)$	
X44	$\dot{\Lambda}_1(1)$	Primer derivative (at start of trajectory segment)
X45	$\dot{\Lambda}_1(2)$	
X46	$\dot{\Lambda}_1(3)$	
X47		Encounter speed at first intermediate target, $v_{\infty 1}$. [m/sec]
X48		Time at the first intermediate target, t_1, measured from the reference date (MYEAR, etc.). [days]
X49		Sample-mass factor, $k_{samp\ 1}$, for sample-retrieval at first intermediate target.
X50		Drop-mass factor, $k_{drop\ 1}$, for instrument-package dropoff at first intermediate target.

The independent variables X51 through X60 and X61 through X70 are identical to X41 through X50 except that they pertain to the second and third intermediate targets, respectively. A third intermediate target may not be present when simulating ballistic missions having a deep space burn (See TDV), in which case X64, X65, and X66 are used as follows:

X64	$\Delta\dot{x}$	Deep-space velocity-increment. [AU/tau]
X65	$\Delta\dot{y}$	
X66	$\Delta\dot{z}$	

Inputs pertaining to the individual dependent parameters are contained in the arrays Y1 through Y70. The dependent-parameter arrays have three elements for each variable, as follows (where i = 1, 2, 3, ..., 70):

Yi(1)	Desired value of the dependent parameter.
Yi(2)	Trigger. If off (i.e., equal to zero), the parameter is ignored and is not considered a dependent parameter. Then the other two inputs pertaining to the parameter need not be input. If trigger is on, (i.e., not equal to zero), the parameter is considered to be a dependent parameter or constraint. Certain of the parameters may have up to three non-zero trigger settings. These will be discussed individually below.
Yi(3)	Tolerance of desired value (full interval width).

It should be noted that the transversality conditions, which comprise some of the parameters, are developed under the assumption that all constraints are of the point constraint type. Therefore, the satisfy-mode is sufficient in solving any optimization problems for which a complete set of transversality conditions is available.

The dependent-parameter arrays are as given below. T(x) represents "the transversality condition associated with x" and the function T(x) will have different values depending upon the constraints imposed on the problem. See NOMENCLATURE for definition of symbols and subscripts.

	Trigger 1		Trigger 2	Trigger 3
Y1	Δx_n[AU]	a[AU]	Solar distance*[AU]	$T(\sigma)$ (NERVA)
Y2	Δy_n[AU]	e	$T(\theta_t)$*	$T(\theta_t)$ (NERVA)
Y3	Δz_n[AU]	i[deg]		$T(t_n)$ (NERVA)
Y4	$\Delta \dot{x}_n$[AU/tau]	$T(\Omega)$	$T(\dot{x}_n)$ (optimal flyby)	$T(\xi)$ (NERVA)
Y5	$\Delta \dot{y}_n$[AU/tau]	$T(\omega)$	$T(\dot{y}_n)$ (optimal flyby)	$V_{\infty 0}$ (NERVA)
Y6	$\Delta \dot{z}_n$[AU/tau]	T(f)	$T(\dot{z}_n)$ (optimal flyby)	$T(\lambda)$

*Applicable only for two-dimensional motion in the xy plane. Also requires that MOPT2 = 0.

Under Trigger 1 above, the first set of conditions applies to ordinary targeting conditions for position and velocity, and also to extra-ecliptic conditions to be satisfied when IOUT = 1; the second set of conditions applies to extra-ecliptic missions when IOUT = 2. $T(\Omega)$, $T(\omega)$, and $T(f)$ are symbols for the transversality conditions yielding optimum final node angle, argument of perihelion, and true anomaly, respectively.

	Trigger 1	Trigger 2	Trigger 3
Y7	ν_n	$\lambda_{\nu n}$	m_{net}[kg]
Y8	$T(\tau)$	τ[days] ---	Not used when NEW = T.
Y9	Currently not used.		
Y10	$T(\delta)$	δ[deg]	Used only if LAUNCH ≠ 0.
Y11	$T(g)$	g[m/sec^2]	p_{ref}[kw]- Not used when NEW = T.
Y12	$T(c)$	c[m/sec] ---	Not used when NEW = T.
Y13	$T(v_{\infty o})$	$v_{\infty o}$[m/sec]	
Y14	$T(v_{\infty n})$	$v_{\infty n}$[m/sec]	extra-ecliptic inclination [deg]
Y15	$T(t_o)$	t_o[days]	
Y16	$T(t_n)$	t_n[days]	$t_n - t_o$ [days]*

*Time transversality with flight time fixed is assigned to Y15 under Trigger 1.

Y17	$T(i)$	i[deg], where i = parking orbit inclination.	Used only if LAUNCH ≠ 0.
Y18	$T(\dot{x}_o)$	$\dot{x}_o$[AU/tau]	
Y19	$T(\dot{y}_o)$	$\dot{y}_o$[AU/tau]	
Y20	$T(\dot{z}_o)$	$\dot{z}_o$[AU/tau]	
Y21	$T(\phi)$	ϕ[deg] for ϕ = constant with time.	

Y22 through Y29 are currently not used.

Y30	$T(s)$	s[days]	(Degradation time, not used when NEW = T.)

Y31 through Y35 are currently not used.

Y36 through Y40 pertain to the set of constant beam voltages:

	Trigger 1	Trigger 2
*Y36		
*Y37		
*Y38	$T(V_I)$	V_I [volts]
*Y39	(Caution: see discussion in section 2.2 before attempting to optimize the voltages)	
*Y40		

Y41 through Y50 pertain to the first intermediate target:

	Trigger 1	Trigger 2	
Y41	Δx_1 [AU]		
Y42	Δy_1 [AU]		
Y43	Δz_1 [AU]		
Y44	$\Delta \dot{x}_1$ [AU/tau]	$T(\dot{x}_1)$	optimal flyby
Y45	$\Delta \dot{y}_1$ [AU/tau]	$T(\dot{y}_1)$	optimal flyby
Y46	$\Delta \dot{z}_1$ [AU/tau]	$T(\dot{z}_1)$	optimal flyby
Y47		$v_{\infty 1}$ [m/sec]	
Y48	$T(t_1)$	t_1 [days]	
Y49	$m_{samp\ 1}$ [kg]		
Y50	$m_{drop\ 1}$ [kg]		

Y51 through Y60 and Y61 through Y70 are identical to Y41 through Y50 except that they pertain to the second and third intermediate targets, respectively.

3.3 Default Values of Input Parameters

The following is a complete, alphabetical list of the default values of program input quantities having non-zero (and non-false) default values, except for the iterator arrays. All other inputs are zeroed (or set false). The default values of the iterator arrays Xi(1), Xi(2), Yi(1), and Yi(2), for i = 1,2,3,...,70, are zero, and the default values of Xi(3) through Xi(5) and Yi(3) for the same range of i are listed in the listing of program inputs of Sample Case A. Exceptions to the setting of Xi(1) to zero are displayed below.

ALPHAA	15.	NDELTA	1
ALPHAT	15.	NDIST	3
AN	1.5	NHUNG	25
AR	1.	NMAX	4
BI	.76	NMIN	1
CSEP(1)	.987	NPRINT	7
CSEP(2)	-0.018	NSET(3)	300
CSEP(3)	.08	NSET(5)	300
CSEP(4)	2.3	NSWPAR	1
CSEP(5)	.002	NTAPE	17
CSEP(6)	.9316667	PAO	35.
CSEP(7)	.0033333	PDIU	.02
CTANK	.03	PLOSS	.267
CTRET	1/9	PLV	.03
CURMAX	2.	PMM	.4
CURMIN	.5	PNFLOW	.04
CVOLT	10.	POWFIX	-1.
DEFFIC	.89	PSIGN	1.
DI	13.	PTS	.2
DLOSS	198.	RADODD	1.
DVOLT	36.	RAP	38.
ETAPD	.95	RPER	2.
ETCONV	.95	SAI	1.
GAMMAX	1.	SPIRET	300.
GAP	.0001	STATE(1)	1.
HOUR	12.	STATE(5)	1.
IRK	1	STEP1	.03125
IRL	1	STEP2	.125
ITF	3	TDV	-1.
MAXHAM	5	TGO	-1.
MDAY	1	THRET	400.
MODE	4	TOFF	20*-1.
MONTH	1	TPOWER	10.**30
MOPT3	10	TSCALE	1.
MUPDAT	1	T2(i)	50*i
MYEAR	1975	XO(1)	1.

(Continued on next page)

X0(5)	1.0000015
X11	$4.\times10^{-4}$
X12	$3.\times10^{4}$
X36	1600.
X37	2000.
X38	2400.
X39	2800.
X40	3200.

4.0 SAMPLE PROBLEM

The sample problem consists of a Saturn orbiter mission simulation. The new HILTOP spacecraft model is employed. The assumptions, solution characteristics, and a graph depicting the optimal thruster subsystem operating time-history are displayed on the following pages, followed by a display of the complete program output.

The complete NAMELIST input data set used to generate this sample problem is reproduced below.

```
&MINPUT X1(2)=1.D0,X2(2)=1.D0,X3(2)=1.D0,X4(2)=1.D0,X5(2)=1.D0,X7=1.D0
X6(2)=1.D0,X13(2)=1.D0,X14(2)=1.D0,Y1(2)=1.D0,Y2(2)=1.D0,Y3(2)=1.D0
Y4(2)=1.D0,Y5(2)=1.D0,Y6(2)=1.D0,Y13(2)=1.D0,Y14(2)=1.D0
MOPT2=3,MOPT3=6,RPER=1.1D0,RAP=11.D0,THRET=6.D1,JPP=1,JT=1,JA=1
MYEAR=1985,MONTH=9,MDAY=25,X15=82.D0,X16=1982.D0,MBOOST=21,MTMASS=2
NEW=T,NMAX=8,MATCH=T,ALPHAT=22.5D0,PLV=0.D0,PMM=0.D0,PDIU=0.D0,PTS=0.D0
 X1=-9.4237522458 54D 01, X2= 5.5597404457 58D 00, X3= 2.3844702115 09D-01
 X4=-1.52578123261 1D 01, X5=-7.71564669199 0D 01, X6= 1.08182345716 5D-01
X13= 4.65093593920 0D 03,X14= 5.81401083432 4D 03 &END
```

PRECEDING PAGE BLANK NOT FILMED

SAMPLE PROBLEM

<u>1900 Day Saturn Orbiter</u>

<u>Assumptions</u>

- Launch on 16 December 1985
- Shuttle/IUS launch vehicle
- 1.1×11 Saturn radii capture orbit
- Retro stage with 300 seconds specific impulse and 60 pounds thrust; include velocity losses in calculations
- Allowable beam current range, 0.5-2.0 amps
- Permit 5 discrete values of beam voltage - 1600, 2000, 2400, 2800, and 3200 volts
- Assume maximum of 8 operating thrusters, minimum of 1 thruster, may be switched in increments of 1 thruster
- Solar array sized to accommodate 8 thrusters operating at maximum beam current and beam voltage at 1 AU
- α_a = 15 kg/kw; α_t = 22.5 kg/kw; k_t = .03, m_b = 0 ,

  m_{rs} = 0, k_{rt} = .111
- Entire SEP system jettisoned prior to capture orbit insertion
- C_1 = .987, C_2 = - .018, C_3 = .08, C_4 = 2.3, C_5 = .002,

  C_6 = .9316667, C_7 = .0033333
- V_G = 10, η_D = .89, ε_I = 198, ΔV_I = 36, η_{Pd} = .95, η_{conv} = .95
- $\dot{m}_\ell$ = .267, $\dot{m}_n$ = .04, $p_{diu} = p_{\ell v} = p_{pm} = p_{ts} = 0$
- Departure and arrival excess speeds are optimized.

SAMPLE PROBLEM

1900 Day Saturn Orbiter

Solution Characteristics

m_o = 4237 kg

m_a = 923 kg

m_{th} = 1384 kg

m_p = 853 kg

m_t = 26 kg

m_r = 655 kg

m_{net} = 396 kg

$V_{\infty o}$ = 4651 m/sec

$V_{\infty n}$ = 5814 m/sec

total burn time = 778 days

average thruster operating time = 7359 hours

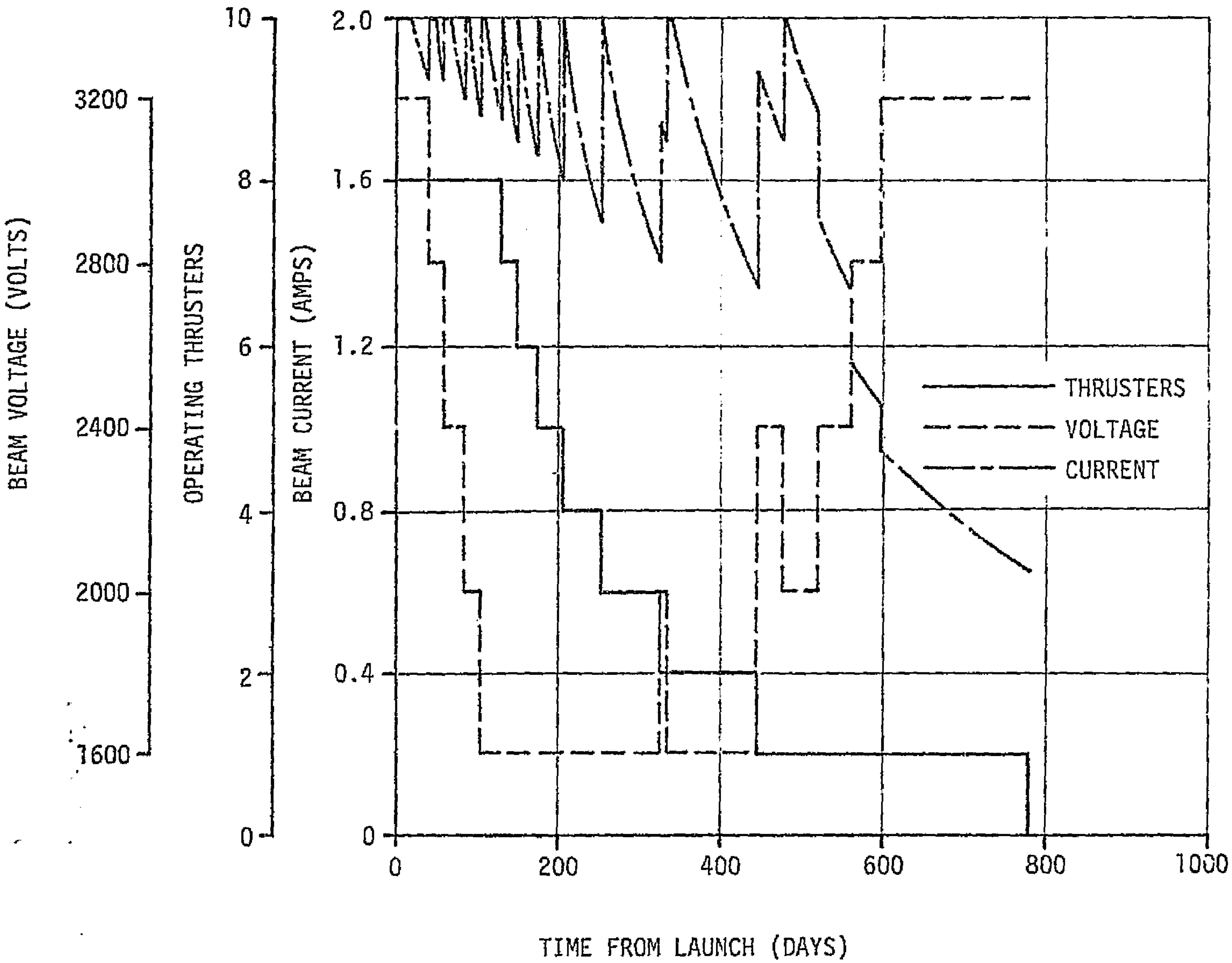
specific impulse = 4978 seconds @ I_B = 2 amps, V_I = 3200 volts

overall efficiency = .6965 @ I_B = 2 amps, V_I = 3200 volts

1900 DAY SEP SATURN ORBITER MISSION

OPTIMAL THRUSTER SUBSYSTEM OPERATING CHARACTERISTICS

LAUNCH DATE - 16 DECEMBER 1985



CASE 1 TIME TO GO CPU 59, I/O 42 SEC

PROGRAM INPUTS

X 1 =	-9.423752245854000D 01	1.000000000000000D 00	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X 2 =	5.555740445758000D 00	1.000000000000000D 00	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X 3 =	2.384470211509000D-01	1.000000000000000D 00	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X 4 =	-1.525781232611000D 01	1.000000000000000D 00	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X 5 =	-7.715646691990000D 01	1.000000000000000D 00	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X 6 =	1.081423457165000D-01	1.000000000000000D 00	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X 7 =	1.000000000000000D 00	0.0	3.000000000000000D 00	9.999999999999999D-07	1.000000000000000D 00
X 8 =	0.0	0.0	3.000000000000000D 00	9.999999999999999D-07	1.000000000000000D 00
X 9 =	0.0	0.0	3.000000000000000D 00	9.999999999999999D-07	1.000000000000000D 00
X10 =	0.0	0.0	9.000000000000000D 01	9.999999999999999D-07	1.000000000000000D 00
X11 =	3.999999999999999D-04	0.0	9.999999999999999D-04	1.000000000000000D-11	1.000000000000000D 00
X12 =	3.000000000000000D 04	0.0	2.000000000000000D 03	9.999999999999999D-04	1.000000000000000D 00
X13 =	4.650935939199999D 03	1.000000000000000D 00	5.000000000000000D 02	9.999999999999999D-05	1.000000000000000D 00
X14 =	5.814010834324000D 03	1.000000000000000D 00	5.000000000000000D 02	9.999999999999999D-04	1.000000000000000D 00
X15 =	8.200000000000000D 01	0.0	8.000000000000000D 00	9.999999999999999D-07	1.000000000000000D 00
X16 =	1.982000000000000D 03	0.0	1.000000000000000D 02	9.999999999999999D-07	1.000000000000000D 00
X17 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X18 =	0.0	0.0	1.000000000000000D 00	9.999999999999999D-05	1.000000000000000D 00
X19 =	0.0	0.0	1.000000000000000D 00	9.999999999999999D-05	1.000000000000000D 00
X20 =	0.0	0.0	1.000000000000000D 00	9.999999999999999D-05	1.000000000000000D 00
X21 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X22 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X23 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X24 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X25 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X26 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X27 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X28 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X29 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X30 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X31 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X32 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X33 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X34 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X35 =	0.0	0.0	1.000000000000000D 01	9.999999999999999D-05	1.000000000000000D 00
X36 =	1.600000000000000D 03	0.0	1.000000000000000D 02	9.999999999999999D-03	1.000000000000000D 00
X37 =	2.000000000000000D 03	0.0	1.000000000000000D 02	9.999999999999999D-03	1.000000000000000D 00
X38 =	2.400000000000000D 03	0.0	1.000000000000000D 02	9.999999999999999D-03	1.000000000000000D 00
X39 =	2.800000000000000D 03	0.0	1.000000000000000D 02	9.999999999999999D-03	1.000000000000000D 00
X40 =	3.200000000000000D 03	0.0	1.000000000000000D 02	9.999999999999999D-03	1.000000000000000D 00
X41 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X42 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X43 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X44 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X45 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X46 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X47 =	0.0	0.0	5.000000000000000D 02	9.999999999999999D-05	1.000000000000000D 00
X48 =	0.0	0.0	5.000000000000000D 01	9.999999999999999D-07	1.000000000000000D 00
X49 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X50 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X51 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X52 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X53 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X54 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X55 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X56 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X57 =	0.0	0.0	5.000000000000000D 02	9.999999999999999D-05	1.000000000000000D 00
X58 =	0.0	0.0	5.000000000000000D 01	9.999999999999999D-07	1.000000000000000D 00
X59 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00
X60 =	0.0	0.0	3.000000000000000D 00	1.000000000000000D-08	1.000000000000000D 00

X60 =	0.0	, [illegible]	, [illegible]	[illegible]	[illegible]
X61 =	0.0	, 0.0	, 3.0000000000000000D 00,	1.0000000000000000D-08,	1.0000000000000000D 00,
X62 =	0.0	, 0.0	, 3.0000000000000000D 00,	1.0000000000000000D-08,	1.0000000000000000D 00,
X63 =	0.0	, 0.0	, 3.0000000000000000D 00,	1.0000000000000000D-08,	1.0000000000000000D 00,
X64 =	0.0	, 0.0	, 3.0000000000000000D 00,	1.0000000000000000D-08,	1.0000000000000000D 00,
X65 =	0.0	, 0.0	, 3.0000000000000000D 00,	1.0000000000000000D-08,	1.0000000000000000D 00,
X66 =	0.0	, 0.0	, 3.0000000000000000D 00,	1.0000000000000000D-08,	1.0000000000000000D 00,
X67 =	0.0	, 0.0	, 5.0000000000000000D 02,	9.9999999999999999D-05,	1.0000000000000000D 00,
X68 =	0.0	, 0.0	, 5.0000000000000000D 01,	9.9999999999999999D-07,	1.0000000000000000D 00,
X69 =	0.0	, 0.0	, 3.0000000000000000D 00,	1.0000000000000000D-08,	1.0000000000000000D 00,
X70 =	0.0	, 0.0	, 3.0000000000000000D 00,	1.0000000000000000D-08,	1.0000000000000000D 00,
Y 1 =	0.0	, 1.0000000000000000D 00,	9.9999999999999999D-05,		
Y 2 =	0.0	, 1.0000000000000000D 00,	9.9999999999999999D-05,		
Y 3 =	0.0	, 1.0000000000000000D 00,	9.9999999999999999D-05,		
Y 4 =	0.0	, 1.0000000000000000D 00,	9.9999999999999999D-05,		
Y 5 =	0.0	, 1.0000000000000000D 00,	9.9999999999999999D-05,		
Y 6 =	0.0	, 1.0000000000000000D 00,	9.9999999999999999D-05,		
Y 7 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y 8 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y 9 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y10 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y11 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y12 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y13 =	0.0	, 1.0000000000000000D 00,	9.9999999999999999D-05,		
Y14 =	0.0	, 1.0000000000000000D 00,	9.9999999999999999D-05,		
Y15 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y16 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y17 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y18 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y19 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y20 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y21 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y22 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y23 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y24 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y25 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y26 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y27 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y28 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y29 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y30 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y31 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y32 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y33 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y34 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y35 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y36 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y37 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y38 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y39 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y40 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y41 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y42 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y43 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y44 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y45 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y46 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y47 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y48 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y49 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y50 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y51 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y52 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y53 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y54 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y55 =	0.0	, 0.0	, 9.9999999999999999D-05,		
Y56 =	0.0	, 0.0	, 9.9999999999999999D-05,		

```
Y57 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y58 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y59 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y60 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y61 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y62 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y63 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y64 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y65 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y66 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y67 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y68 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y69 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
Y70 =  0.0                 ,    0.0                    ,    9.9999999999999990-05,
&OUTPUT
AAI= 0.0               ,AE= 0.0               ,ALPHAA=  15.0000000000000    ,ALPHAT=  22.5000000000000    ,ALTITU=
 0.0              ,AN=  1.50000000000000    ,AR=  1.00000000000000    ,ASOL= 0.0                    , 0.0          ,
 0.0              , 0.0                , 0.0                ,BI= 0.700000000000000    ,BMASS= 0.0              ,
B1= 0.0            ,B2= 0.0             ,B3= 0.0              ,CNI = 0.0              ,CNIX=
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,CSEP=
 0.987000000000000   ,-0.1800000000000000D-01, 0.8000000000000000D-01,  2.30000000000000    , 0.1999999999999999D-02,
 0.931666700000000   , 0.3333299999999999D-02,CSTR= 0.0             ,CTANK= 0.2999999999999999D-01,CTRET=
 0.111111111111111   ,CURMAX=  2.00000000000000    ,CURMIN= 0.500000000000000    ,CVOLT=  10.0000000000000    ,DEFFIC=
 0.890000000000000   ,DI=  13.0000000000000    ,DLOSS=  198.000000000000    ,DMRETR= 0.0                ,DPOW=
 0.0              ,DVOLT=  36.0000000000000    ,ECI= 0.0              ,ECIX= 0.0            ,
 0.0              , 0.0                , 0.0                , 0.0                ,EI= 0.0              ,EMUODD=
 0.0              ,EMUODX= 0.0           , 0.0                , 0.0                , 0.0           ,
 0.0              ,ETAPD= 0.950000000000000    ,ETCONV= 0.950000000000000    ,GAMMAX=  1.00000000000000    ,GAP=
 0.9999999999999999D-04,HOUR=  12.0000000000000    ,OHI= 0.0              ,OHIX= 0.0          ,
 0.0              , 0.0                , 0.0                , 0.0                ,PAO=  35.0000000000000    ,PDIU=
 0.0              ,PLOSS= 0.267000000000000    ,PLV= 0.0              ,PMM= 0.0               ,PNFLOW=
 0.3999999999999999D-01,POWFIX= -1.00000000000000    ,POWMAX= 0.0              ,PSIGN=  1.00000000000000    ,PTS=
 0.0              ,RADODD=  1.00000000000000    ,RADODX= 0.0              , 0.0          ,
 0.0              , 0.0                , 0.0                ,RAP=  11.0000000000000    ,REVS= 0.0          ,
RPER=  1.10000000000000    ,SAI=  1.00000000000000    ,SAIX= 0.0              , 0.0          ,
 0.0              , 0.0                , 0.0                ,SOI= 0.0              ,SOIX= 0.0           ,
 0.0              , 0.0                , 0.0                , 0.0                ,SPIRET=  300.000000000000    ,
STATE=  1.00000000000000    , 0.0            , 0.0              , 0.0              ,  1.00000000000000    ,
 0.0              ,STEP1= 0.3125000000000000D-01,STEP2= 0.125000000000000    ,TCOAST= 0.0           ,
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,
 0.0              , 0.0                , 0.0                , 0.0                ,TCV= -1.00000000000000    ,TGO=
 -1.00000000000000   ,TMRET=  60.0000000000000    ,TOFF= -1.00000000000000    , -1.00000000000000    , -1.00000000000000    ,
 -1.00000000000000   , -1.00000000000000    , -1.00000000000000    , -1.00000000000000    , -1.00000000000000    ,
 -1.00000000000000   , -1.00000000000000    , -1.00000000000000    , -1.00000000000000    , -1.00000000000000    ,
 -1.00000000000000   , -1.00000000000000    , -1.00000000000000    , -1.00000000000000    , -1.00000000000000    ,
 -1.00000000000000   , -1.00000000000000    ,TP1= 0.0              ,TPIX= 0.0             , 0.0          ,
 0.0              , 0.0                , 0.0                ,TPOWER= 0.1000000000000000D 31,TRATIO=
 0.0              ,TSCALE=  1.00000000000000    ,T2=  50.0000000000000    ,  100.000000000000    ,  150.000000000000    ,
  200.000000000000    ,  250.000000000000    ,  300.000000000000    ,  350.000000000000    ,  400.000000000000    ,
  450.000000000000    ,  500.000000000000    ,XANG1= 0.0             ,XANG2= 0.0             ,XSWING=
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,
 0.0              , 0.0                , 0.0                , 0.0                , 0.0               ,X0=
  1.00000000000000    , 0.0                , 0.0                , 0.0                ,  1.00001500000000    ,
 0.0              , 0.0                ,IBAL=          0,INTPR=          0,IOUT=          0,IRK=          1,IRL=          1,
IRCT=          0,ISPIN=          0,ITF=          3,ITPRNT=          0,JA=          1,JPP=          1,JPRINT=          0,JT=
          1,KPART=          0,LAUNCH=          0,LOADX=          0,MAXHAM=          5,MBOOST=          21,MDAY=          25,MODE=
          4,MONTH=          9,MCPT=          0,MOPTX=          0,          0,          0,          0,MOPT2=          3,
MOPT3=          6,MOPT4=          0,          0,          0,          0,          0,          0,          0,          0,          0,
          0,MPOW=          0,MPRINT=          0,MPUNCH=          0,MREAD=          0,MSWING=          0,          0,          0,
          0,          0,          0,          0,          0,          0,MTMASS=          2,MUPDAT=          1,MYEAR=
       1985,NDELTA=          1,NDIST=          3,NHUNG=          25,NMAX=          8,NMIN=          1,NORMAL=          0,NPERF=
          0,NPRINT=          7,NSCT=          0,          0,        300,          0,        300,NSWING=          0,NSWPAR=
          1,NTAPE=          17,MATCH=T,NEW=T,ONHDOT=F,PRAMP=F,PROOT=F
&END
```


ORIGINAL PAGE IS
OF POOR QUALITY

CASE 1

ITERATON PARAMETERS

INDEPENDENT VARIABLES

NO.	INDEX	VALUE	STEP LIMIT	PERTURBATION	WEIGHT
1	1	-9.4237522458540000D 01	3.0000000000000000D 00	1.0000000000000000D-08	1.0000000000000000D 00
2	2	5.5597404457580000D 00	3.0000000000000000D 00	1.0000000000000000D-08	1.0000000000000000D 00
3	3	2.3844702115050000D-01	3.0000000000000000D 00	1.0000000000000000D-08	1.0000000000000000D 00
4	4	-1.5257812326110000D 01	3.0000000000000000D 00	1.0000000000000000D-08	1.0000000000000000D 00
5	5	-7.7156466919900000D 01	3.0000000000000000D 00	1.0000000000000000D-08	1.0000000000000000D 00
6	6	1.0818234571650000D-01	3.0000000000000000D 00	1.0000000000000000D-08	1.0000000000000000D 00
7	13	4.6509355391999900 03	5.0000000000000000D 02	9.9999999999999990D-05	1.0000000000000000D 00
8	14	5.8140108343240000D 03	5.0000000000000000D 02	9.9999999999999990D-04	1.0000000000000000D 00

DEPENDENT VARIABLES

NO.	INDEX	VALUE	TOLERANCE
1	1	0.0	9.9999999999999990D-05
2	2	0.0	9.9999999999999990D-05
3	3	0.0	9.9999999999999990D-05
4	4	0.0	9.9999999999999990D-05
5	5	0.0	9.9999999999999990D-05
6	6	0.0	9.9999999999999990D-05
7	13	0.0	9.9999999999999990D-05
8	14	0.0	9.9999999999999990D-05

```
---------------------------------------------- -- MODULAR SPACECRAFT PARAMETERS ---------------------------------------------
(PRE-ITERATION)

5 DISCRETE VOLTAGES (VOLTS) =    1600.0    2000.0    2400.0    2800.0    3200.0
COMPUTED MAXIMUM INPUT POWER TO AN INDIVIDUAL THRUSTER =   6.8287 KW
COMPUTED MINIMUM THROTTLING RATIO = 0.2500000
UNIT THRUSTER REFERENCE POWER =    7.689 KW
MAXIMUM USABLE ARRAY POWER OUTPUT =    61.512 KW   = 100.00 PERCENT OF AVAILABLE POWER (  MATCHED)
HOUSEKEEPING POWER (KW)     MINIMUM =  0.0       MAXIMUM =  0.0      INCREMENT =  0.0
REFERENCE SPECIFIC IMPULSE =  4578.4 SEC

-------------------------------------------------------------------------------------------------------------------------------

THIS CASE IS CONVERGED.

     1 TRAJECTORIES WITHOUT PARTIAL DERIVATIVES AND   0 TRAJECTORIES WITH PARTIAL DERIVATIVES REQUIRED FOR THIS CASE.
```

CASE 1

SWITCH POINT SUMMARY

TIME	SEMI-MAJOR AXIS	ECCENTRICITY	INCLINATION	NODE	ARG POS	RMAG	TRAVEL
R1	R2	R3	V1	V2	V3	MASS RATIO	THRUST ACC
L1	L2	L3	L4	L5	L6	L7	HAM
LG	LC	LPHI	CONE	CLOCK	HMAG	POWER FNCT	SWITCH FNCT
PSI	THETA	PHI	LATITUDE	LONGITUDE	FLT PTH ANGLE	VMAG	PROP TIME
NO. THR	VOLTAGE	CURRENT	PHAN CUR	SP IMP	THRUST	BEAM POWER	DUMP POWER
UTIL EF	THR RED	THR EF	PPU EF	PTH EF	SS EF	ARRAY POWER	PPU POWER
EARTH			START OF TRAJECTORY, THRUST ON				
0.0	1.51866580D 00	3.52110368D-01	1.92809443D-02	8.44891452D 01	-2.54444375D-14	9.84095462D-01	0.0
9.45069497D-02	9.79546995D-01	0.0	-1.16770458D 00	1.01569217D-01	3.94417451D-04	1.00000000D 00	6.98589118D-02
-9.42375225D 01	5.55974045D 00	2.38447021D-01	-1.52578123D 01	-7.71564669D 01	1.08182346D-01	1.00000000D 00	2.02802145D-01
0.0	0.0	0.0	1.03294198D 02	8.52093606D 01	1.15342057D 00	1.02043234D 00	9.37916096D 01
1.25454664D-01	9.21344821D 01	9.21344770D 01	0.0	8.44891452D 01	-5.39668312D-01	1.17211366D 00	0.0
8.00000000D 00	3.20000000D 03	2.00000000D 00	2.04114149D 00	4.97842449D 03	3.94573195D-01	5.35558148D 00	1.05322293D 00
9.25263911D-01	9.51000000D-01	7.84275867D-01	9.34853869D-01	9.37221396D-01	9.38333300D-01	6.27689187D 01	7.45380909D 00
			SWITCH THRUST OFF				
7.77821993D 02	6.10499196D 00	7.83752410D-01	3.86439733D-01	8.16003787D 01	1.92050431D 02	6.17663077D 00	1.89161264D 02
3.93239018D-01	-6.16409402D 00	-8.69722912D-03	2.67895090D-01	-2.97032342D-01	-2.08016813D-03	7.98569193D-01	0.0
5.94961045D 00	-1.16532724D 01	-1.05698418D 00	3.88783715D-01	2.09362431D 00	-5.76650288D-02	2.05581671D 01	2.02795781D-01
0.0	0.0	0.0	7.61734291D 01	8.13984882D 01	1.53456565D 00	4.05526026D-02	1.06581410D-13
-4.39432318D 00	2.34210083D 01	2.38068389D 01	-8.06774284D-02	-8.63497564D 01	5.16023119D 01	4.00000933D-01	7.77821993D 02
0.0	0.0	0.0	6.45960285D-01	0.0	0.0	0.0	2.49453655D 00
0.0	9.87000000D-01	0.0	0.0	0.0	9.31666700D-01	2.49453655D 00	0.0
SATURN			END OF TRAJECTORY, THRUST OFF				
1.90000000D 03	6.10499196D 00	7.83752410D-01	3.86439733D-01	8.16000787D 01	2.17393847D 02	1.00255430D 01	2.14504681D 02
4.85941314D 00	-8.76903244D 00	-4.10639064D-02	1.87550182D-01	-2.26575508D-02	-1.27373292D-03	7.98569193D-01	0.0
1.05170114D 01	2.42227923D 01	-1.62685780D 00	3.58851325D-03	1.91880028D 00	-8.30928371D-03	2.05581671D 01	2.02795781D-01
0.0	0.0	0.0	7.65287178D 01	8.17669545D 01	1.53456565D 00	1.49768777D-02	5.20184162D 00
-3.42477003D 00	1.27513974D 02	1.27435459D 02	-2.34680068D-01	-6.10067025D 01	3.58823940D 01	1.88918137D-01	7.77821993D 02
0.0	0.0	0.0	4.48120793D-01	0.0	0.0	0.0	9.21258939D-01
0.0	9.87000000D-01	0.0	0.0	0.0	9.31666700D-01	9.21258939D-01	0.0

```
CASE  1                                         ITERATOR SUMMARY

                                            INDEPENDENT PARAMETERS

  1.PRIM1(-5.4237522D 01)  2.PRIM2( 5.5597404D 00)  3.PRIM3( 2.3844702D-01)  4.PDOT1(-1.5257812D 01)  5.PDOT2(-7.7156467D 01)
  6.PDOT3( 1.0818235D-01)  7.LMASS( 1.0000000D 00)  8. LTAU( 0.0          )  9.      ( 0.0          ) 10.DECLN( 0.0          )
 11.ACCEL( 4.0000000D-04) 12.V JET( 3.0000000D 04) 13.VINFO( 4.6509359D 03) 14.VINFN( 5.8140108D 03) 15.TIMEO( 8.2000000D 01)
 16.TIMEN( 1.9820000D 03) 17.IPARK( 0.0          ) 18.VELO1( 0.0          ) 19.VELO2( 0.0          ) 20.VELO3( 0.0          )
 21.THET1( 0.0          ) 22.     ( 0.0          ) 23.     ( 0.0          ) 24.     ( 0.0          ) 25.     ( 0.0          )
 26.     ( 0.0          ) 27.     ( 0.0          ) 28.     ( 0.0          ) 29.     ( 0.0          ) 30.LDEGR( 0.0          )
 31.     ( 0.0          ) 32.     ( 0.0          ) 33.     ( 0.0          ) 34.     ( 0.0          ) 35.     ( 0.0          )
 36.VOLT1( 1.6000000D 03) 37.VOLT2( 2.0000000D 03) 38.VOLT3( 2.4000000D 03) 39.VOLT4( 2.8000000D 03) 40.VOLT5( 3.2000000D 03)
 41.PR1-A( 0.0          ) 42.PR2-A( 0.0          ) 43.PR3-A( 0.0          ) 44.PD1-A( 0.0          ) 45.PD2-A( 0.0          )
 46.PD3-A( 0.0          ) 47.VINFA( 0.0          ) 48.TIMEA( 0.0          ) 49.KSAMP( 0.0          ) 50.KDROP( 0.0          )
 51.PR1-B( 0.0          ) 52.PR2-B( 0.0          ) 53.PR3-B( 0.0          ) 54.PD1-B( 0.0          ) 55.PD2-B( 0.0          )
 56.PD3-B( 0.0          ) 57.VINFB( 0.0          ) 58.TIMEB( 0.0          ) 59.KSAMP( 0.0          ) 60.KDROP( 0.0          )
 61.PR1-C( 0.0          ) 62.PR2-C( 0.0          ) 63.PR3-C( 0.0          ) 64.PD1-C( 0.0          ) 65.PD2-C( 0.0          )
 66.PD3-C( 0.0          ) 67.VINFC( 0.0          ) 68.TIMEC( 0.0          ) 69.KSAMP( 0.0          ) 70.KDROP( 0.0          )

                                             DEPENDENT PARAMETERS

  1.DELTA X(-1.36581D-08)  2.DELTA Y( 2.93298D-08)  3.DELTA Z( 2.44313D-09)  4.DELT XD(-7.06402D-09)  5.DELT YD( 3.43426D-09)
  6.DELT ZD( 1.36471D-09)  7.       ( 0.0        )  8.       ( 0.0        )  9.       ( 0.0        ) 10.       ( 0.0        )
 11.       ( 0.0        ) 12.       ( 0.0        ) 13.T.VINFO(-1.24087D-08) 14.T.VINFN(-3.01457D-08) 15.       ( 0.0        )
 16.       ( 0.0        ) 17.       ( 0.0        ) 18.       ( 0.0        ) 19.       ( 0.0        ) 20.       ( 0.0        )
 21.       ( 0.0        ) 22.       ( 0.0        ) 23.       ( 0.0        ) 24.       ( 0.0        ) 25.       ( 0.0        )
 26.       ( 0.0        ) 27.       ( 0.0        ) 28.       ( 0.0        ) 29.       ( 0.0        ) 30.       ( 0.0        )
 31.       ( 0.0        ) 32.       ( 0.0        ) 33.       ( 0.0        ) 34.       ( 0.0        ) 35.       ( 0.0        )
 36.       ( 0.0        ) 37.       ( 0.0        ) 38.       ( 0.0        ) 39.       ( 0.0        ) 40.       ( 0.0        )
 41.       ( 0.0        ) 42.       ( 0.0        ) 43.       ( 0.0        ) 44.       ( 0.0        ) 45.       ( 0.0        )
 46.       ( 0.0        ) 47.       ( 0.0        ) 48.       ( 0.0        ) 49.       ( 0.0        ) 50.       ( 0.0        )
 51.       ( 0.0        ) 52.       ( 0.0        ) 53.       ( 0.0        ) 54.       ( 0.0        ) 55.       ( 0.0        )
 56.       ( 0.0        ) 57.       ( 0.0        ) 58.       ( 0.0        ) 59.       ( 0.0        ) 60.       ( 0.0        )
 61.       ( 0.0        ) 62.       ( 0.0        ) 63.       ( 0.0        ) 64.       ( 0.0        ) 65.       ( 0.0        )
 66.       ( 0.0        ) 67.       ( 0.0        ) 68.       ( 0.0        ) 69.       ( 0.0        ) 70.       ( 0.0        )

THRUST SWITCHING TIMES (DAYS)        0.0    ON     777.822  OFF   1900.000  OFF

                                         ELECTRIC PROPULSION PARAMETERS
    POWER                EFFICIENCY          PROP TIME              J              PROP TIME RATIO         AVE ACCEL
    61.5120826217        0.6965241624        777.8219925092         5.1016639114        0.4093799961        0.0004635853

                                          MASS COMPONENT BREAKDOWN
    INITIAL              PROPULSION          PROPELLANT             TANKAGE             STRUCTURE              PAYLOAD
   4236.7038803202       2306.7030983135     853.4026832934         25.6020804988       0.0                    396.4836542323


   126 THRUST COMPUTE STEPS;    22 COAST COMPUTE STEPS
```

CASE 1

EXTREMUM POINTS OF SELECTED FUNCTIONS

I	TIME	ECLIPTIC LONGITUDE		SOLAR DISTANCE		COMMUNICATION ANGLE		COMMUNICATION DISTANCE		SWITCH FUNCTION
0	0.0	0.0		0.984		87.9		0.0	ON	9.38D 01
1	1.764	2.1	MIN	0.984		89.7		0.00		9.40D 01
12	18.672	21.9		1.000		107.0		0.05		9.41D 01
5	28.131	32.7		1.023		117.2		0.08		9.28D 01
6	37.494	42.9		1.056		128.1		0.11		9.07D 01
6	47.082	52.7		1.098		139.9		0.14		8.78D 01
6	59.600	64.5		1.166		155.8		0.19		8.35D 01
4	67.505	71.3		1.214		165.8		0.23		8.02D 01
6	79.424	80.6		1.293	MAX	179.9		0.30		7.56D 01
4	80.251	81.2		1.299		179.1		0.31		7.53D 01
4	88.127	86.7		1.355		170.5		0.36		7.18D 01
6	103.399	96.3		1.471		155.5		0.50		6.61D 01
5	111.938	101.1		1.538		148.1		0.59		6.23D 01
14	128.194	109.2		1.669		135.3		0.80		5.68D 01
4	128.226	109.2		1.669		135.3		0.80		5.70D 01
9	147.608	117.4		1.829		121.8		1.08		5.11D 01
4	147.777	117.5		1.830		121.7		1.08		5.13D 01
4	171.968	126.0		2.032		106.4		1.50		4.49D 01
4	172.405	126.2		2.036		106.2		1.59		4.52D 01
9	204.451	135.3		2.303		87.2		2.12		3.80D 01
5	205.481	135.6		2.312		86.6		2.14		3.83D 01
8	251.671	145.8		2.690		59.6		3.06		3.02D 01
4	254.283	146.3		2.711		58.1		3.11		3.05D 01
9	325.768	157.5		3.272		13.4		4.23		2.12D 01
8	330.911	158.2		3.311		10.0		4.28		2.27D 01
4	338.746	159.2		3.370		4.7		4.35		2.09D 01
5	345.607	160.1		3.422	MIN	0.0		4.41		2.02D 01
5	383.759	164.4		3.704		27.5	MAX	4.55		1.67D 01
6	445.356	170.2		4.142		77.5		4.24		1.18D 01
7	473.353	172.5		4.334		103.6		3.99		1.35D 01
6	476.826	172.7		4.358		107.1		3.96		1.26D 01
4	517.853	175.7		4.631		150.5		3.72		1.01D 01
5	523.088	176.1		4.665		156.4	MIN	3.72		1.06D 01
5	543.992	177.5		4.801	MAX	180.0		3.79		9.41D 00
8	561.027	178.5		4.909		160.9		3.94		8.51D 00
9	593.995	180.5		5.116		126.1		4.45		7.39D 00
8	595.123	180.5		5.123		124.9		4.47		7.76D 00
6	599.255	180.8		5.148		120.8		4.55		7.56D 00
5	737.874	187.5		5.958	MIN	0.1		6.94		1.51D 00
5	760.592	188.5		6.083		19.2	MAX	7.00		6.39D-01
5	777.822	189.2		6.177		33.9		6.97	OFF	1.07D-13
6	911.527	193.9		6.857		163.3	MIN	5.88		-4.27D 00
5	927.002	194.4		6.931	MAX	179.9		5.91		-4.68D 00
7	1115.061	199.7		7.753	MIN	0.1		8.74		-8.11D 00
6	1139.656	200.1		7.816		13.8	MAX	8.77		-8.14D 00
5	1290.957	203.8		8.412		168.0	MIN	7.41		-7.54D 00
6	1302.302	204.0		8.451	MAX	179.8		7.43		-7.38D 00
5	1488.139	207.7		9.040	MIN	0.2		10.02		-4.77D 00
6	1499.225	207.9		9.072		10.0	MAX	10.04		-4.54D 00
5	1666.650	210.8		9.518		171.4	MIN	8.51		-7.36D-01
7	1674.821	210.9		9.538	MAX	179.8		8.52		-5.40D-01
7	1859.478	213.9		9.946	MIN	0.2		10.93		4.13D 00
6	1867.135	214.0		9.962		7.0	MAX	10.94		4.33D 00
4	1900.000	214.5		10.026		37.0		10.80	OFF	5.20D 00

I		THRUST ANGLES PSI		THRUST ANGLES THETA		PHI		INPUT POWER		ARRAY ANGLE	OTHER TABLE
0		0.1		92.1		92.1		62.8	ON	0.0	
1		0.1		91.5		91.5	MAX	62.8		0.0	
12		0.1		85.0		85.0		61.5		0.0	IMAX
5	MAX	0.1		81.3		81.3		59.7		0.0	
6		0.1		77.8		77.8		57.3		0.0	-V
6		0.1		74.3		74.3		54.3		0.0	IMAX
6		0.1		70.0		70.0		50.0		0.0	-V
4		0.1		67.5		67.5		47.1		0.0	IMAX
6		0.1		63.9		63.9		42.9		0.0	
4		0.1		63.7		63.7		42.6		0.0	-V
4		0.1		61.5		61.5		39.9		0.0	IMAX
6		0.1		57.6		57.6		35.2		0.0	-V
5		0.1		55.6		55.6		32.7		0.0	IMAX
14		0.0		52.1		52.1		28.7		0.0	-N
4		0.0		52.1		52.1		28.6		0.0	IMAX
9		-0.0		48.4		48.4		24.6		0.0	-N
4		-0.0		48.4		48.4		24.6		0.0	IMAX
4		-0.1		44.4		44.4		20.5		0.0	-N
4		-0.1		44.3		44.3		20.5		0.0	IMAX
9		-0.2		39.8		39.8		16.5		0.0	-N
5		-0.2		39.6		39.6		16.4		0.0	IMAX
8		-0.4		34.4		34.4		12.5		0.0	-N
4		-0.4		34.1		34.1		12.3		0.0	IMAX
9		-0.7		28.0		28.0		8.7		0.0	-N+V
8		-0.7		27.7		27.7		8.5		0.0	-V
4		-0.7		27.1		27.1		8.2		0.0	IMAX
5		-0.8		26.7		26.7		8.0		0.0	
5		-1.0		24.4		24.4		6.8		0.0	
6		-1.3		21.6		21.6		5.5		0.0	-N+V
7		-1.5		20.6		20.7		5.0		0.0	-V
6		-1.5		20.5		20.6		5.0		0.0	IMAX
4		-1.8		19.5		19.6		4.4		0.0	+V
5		-1.8		19.4		19.5		4.4		0.0	
5		-2.0		19.0		19.1		4.1		0.0	
8		-2.1		18.8		19.0		3.9		0.0	+V
9		-2.4		18.7		18.8		3.6		0.0	+V
8		-2.4		18.7	MIN	18.8		3.6		0.0	
6		-2.4	MIN	18.6		18.8		3.6		0.0	
5		-3.9		21.4		21.7		2.7		0.0	
5		-4.2		22.5		22.8		2.6		0.0	
5		-4.4		23.4		23.8		2.5		0.0	
6		*****		******		*****		0.0		90.0	
5		*****		******		*****		0.0		90.0	
7		*****		******		*****		0.0		90.0	
6		*****		******		*****		0.0		90.0	
5		*****		******		*****		0.0		90.0	
6		*****		******		*****		0.0		90.0	
5		*****		******		*****		0.0		90.0	
6		*****		******		*****		0.0		90.0	
5		*****		******		*****		0.0		90.0	
7		*****		******		*****		0.0		90.0	
7		*****		******		*****		0.0		90.0	
6		*****		******		*****		0.0		90.0	
4		*****		******		*****		0.0	ON	90.0	

TIME	NO. OF THRUSTERS	BEAM VOLTAGE	BEAM CURRENT	PHANTOM CURRENT	UTILIZ EFFIC	SPECIFIC IMPULSE	THRUSTER POWER	EXCESS POWER	THRUST (LBS)
0.0	8	3200	2.0000	2.0411	0.9253	4978	5.35	1.05	0.395
1.764	8	3200	2.0000	2.0416	0.9253	4978	5.36	1.06	0.395
19.672	8	3200	* 2.0000	2.0000	0.9253	4978	5.36	0.0	0.395
28.131	8	3200	1.9414	1.9414	0.9198	4955	5.18	0.0	0.383
37.494	8	- 3200	1.8622	1.8622	0.9121	4920	4.94	0.0	0.368
---REPEAT	8	2800	2.0000	2.1106	0.9253	4657	4.69	2.48	0.369
47.082	8	2800	* 2.0000	2.0000	0.9253	4657	4.69	0.0	0.369
59.600	8	- 2800	1.8389	1.8389	0.9097	4593	4.26	0.0	0.340
---REPEAT	8	2400	2.0000	2.1216	0.9253	4311	4.02	2.34	0.342
67.505	8	2400	* 2.0000	2.0000	0.9253	4311	4.02	0.0	0.342
79.424	8	2400	1.8192	1.8192	0.9077	4244	3.61	0.0	0.312
80.251	8	- 2400	1.8069	1.8069	0.9364	4239	3.58	0.0	0.310
---REPEAT	8	2000	2.0000	2.1347	0.9253	3936	3.35	2.15	0.312
88.127	8	2000	* 2.0000	2.0000	0.9253	3936	3.35	0.0	0.312
103.399	8	- 2000	1.7593	1.7593	0.9013	3851	2.89	0.0	0.276
---REPEAT	8	1600	2.0000	2.1485	0.9253	3520	2.68	1.90	0.279
111.938	8	1600	* 2.0000	2.0000	0.9253	3520	2.68	0.0	0.279
128.194	- 8	1600	1.7491	1.7491	0.9002	3441	2.39	0.0	0.245
---REPEAT	7	1600	2.0000	2.0005	0.9253	3520	2.68	0.01	0.244
128.226	7	1600	* 2.0000	2.0000	0.9253	3520	2.68	0.0	0.244
147.608	- 7	1600	1.7150	1.7150	0.8963	3429	2.25	0.0	0.210
---REPEAT	6	1600	2.0000	2.0026	0.9253	3520	2.68	0.03	0.209
147.777	6	1600	* 2.0000	2.0000	0.9253	3520	2.68	0.0	0.209
171.968	- 6	1600	1.6702	1.6702	0.8912	3412	2.18	0.0	0.176
---REPEAT	5	1600	2.0000	2.0063	0.9253	3520	2.68	0.05	0.174
172.405	5	1600	* 2.0000	2.0000	0.9253	3520	2.68	0.0	0.174
204.451	- 5	1600	1.6086	1.6086	0.8837	3387	2.09	0.0	0.141
---REPEAT	4	1600	2.0000	2.0133	0.9253	3520	2.68	0.09	0.140
205.481	4	1600	* 2.0000	2.0000	0.9253	3520	2.68	0.0	0.140
251.671	- 4	1600	1.5193	1.5193	0.8723	3349	1.95	0.0	0.107
---REPEAT	3	1600	2.0000	2.0289	0.9253	3520	2.68	0.14	0.105
254.283	3	1600	* 2.0000	2.0000	0.9253	3520	2.68	0.0	0.105
325.768	- 3	+ 1600	1.4076	1.4076	0.8569	3297	1.78	0.0	0.074
---REPEAT	2	2000	1.7327	1.7327	0.8983	3841	2.84	0.0	0.068
330.911	2	- 2000	1.6940	1.6940	0.8939	3825	2.77	0.0	0.066
---REPEAT	2	1600	2.0000	2.0687	0.9253	3520	2.68	0.22	0.070
338.746	2	1600	* 2.0000	2.0000	0.9253	3520	2.68	0.0	0.070
365.607	2	1600	1.9428	1.9428	0.9200	3504	2.59	0.0	0.068
383.759	2	1600	1.6690	1.6690	0.8910	3411	2.18	0.0	0.059
445.356	- 2	+ 1600	1.3439	1.3439	0.8474	3264	1.69	0.0	0.047
---REPEAT	1	2400	1.8701	1.8701	0.9129	4264	3.72	0.0	0.040
473.353	1	- 2400	1.7111	1.7111	0.8959	4197	3.36	0.0	0.037
---REPEAT	1	2000	2.0000	[illegible]	0.9253	3936	3.35	0.04	0.039
476.826	1	2000	* 2.0000	[illegible]	0.9253	3936	3.35	0.0	0.039
517.853	1	+ 2000	1.7740	1.7740	0.9029	3657	2.92	0.0	0.035
---REPEAT	1	2400	1.5018	1.5018	0.8700	4092	2.89	0.0	0.032
523.088	1	2400	1.4801	1.4801	0.8670	4080	2.84	0.0	0.032
543.992	1	2400	1.3986	1.3986	0.8556	4032	2.65	0.0	0.030
561.027	1	+ 2400	1.3378	1.3378	0.8465	3994	2.52	0.0	0.029
---REPEAT	1	2800	1.1599	1.1599	0.8171	4178	2.48	0.0	0.027
593.995	1	+ 2800	1.0687	1.0687	0.8009	4097	2.24	0.0	0.025
---REPEAT	1	3200	0.9433	0.9433	0.7736	4246	2.20	0.0	0.024
598.123	1	3200	0.9407	0.9407	0.7730	4243	2.19	0.0	0.024
599.255	1	3200	0.9315	0.9315	0.7709	4232	2.16	0.0	0.023
737.874	1	3200	0.6948	0.6948	0.7054	3889	1.49	0.0	0.018
760.592	1	3200	0.6662	0.6662	0.6956	3837	1.41	0.0	0.017
777.822	1	3200	0.6460	0.6460	0.6883	3799	1.35	0.0	0.016
911.527	0	0	0.0	0.5223	0.6371	3524	0.0	0.0	0.0
927.002	0	0	0.0	0.5111	0.6317	3495	0.0	0.0	0.0
1115.061	0	0	0.0	0.5303	0.6408	3069	0.0	0.0	0.0

1130.656	0	0	0.0	0.5216	0.6368	3050	0.0	0.0	0.0
1290.957	0	0	0.0	0.5291	0.6433	2799	0.0	0.0	0.0
1302.302	0	0	0.0	0.5240	0.6379	2789	0.0	0.0	0.0
1488.139	0	0	0.0	0.5561	0.6524	2559	0.0	0.0	0.0
1499.225	0	0	0.0	0.5520	0.6506	2543	0.0	0.0	0.0
1666.650	0	0	0.0	0.4995	0.6261	2450	0.0	0.0	0.0
1674.821	0	0	0.0	0.4973	0.6250	2445	0.0	0.0	0.0
1859.478	0	0	0.0	0.4556	0.6031	2362	0.0	0.0	0.0
1867.135	0	0	0.0	0.4541	0.6023	2359	0.0	0.0	0.0
1900.000	0	0	0.0	0.4481	0.5990	2346	0.0	0.0	0.0

MISSION SCHEDULE

DECEMBER 16, 1985 1.20000000D 01 G.M.T.
2446416.0000 00 JULIAN DATE — DEPART EARTH

	X	Y	Z	XDOT	YDOT	ZDOT	RADIUS	LAT.	LONG.
PLANET	9.45069500D-02	9.79546990D-01	0.0	-1.01182540 00	9.23727990D-02	0.0	9.84095460D-01	0.0	84.489
S/C	9.45069500D-02	9.79546990D-01	0.0	-1.16770460 00	1.01569220D-01	3.94417450D-04	9.84095460D-01	0.0	84.489

FEBRUARY 28, 1991 1.20000000D 01 G.M.T.
2448316.0000 00 JULIAN DATE — PASS SATURN AT 5.811 KM/SEC

	X	Y	Z	XDOT	YDOT	ZDOT	RADIUS	LAT.	LONG.
PLANET	4.85941310 00	-8.76903250 00	-4.10639090D-02	2.65143350D-01	1.56055000D-01	-1.32764820D-02	1.00255430 01	-0.235	-61.007
S/C	4.85941310 00	-8.76903240 00	-4.10639060D-02	1.87550180D-01	-2.26576510D-02	-1.27373290D-03	1.00255430 01	-0.235	-61.007

TWO-BODY TRANSFER ANGLE BETWEEN EARTH AND SATURN IS 214.5049 DEGREES.

CASE 1 (CONVERGED)

PERFORMANCE SUMMARY

EARTH TO SATURN RENDEZ WITH HIGH THRUST CAPTURE MANEUVER.

LAUNCH VEHICLE IS SHUTTLE/IUS (COEFFICIENTS = 209698.4200 3661.6300 3757.8797)

LD = DEC 16, 1985, 12.0000 HOURS GMT JULIAN DATE 46416.0000
AD = FEB 28, 1991, 12.0000 HOURS GMT JULIAN DATE 48316.0000
FLIGHT TIME = 1900.0000 DAYS.

ELECTRIC PROPULSION SYSTEM PARAMETERS

ALPHA A (KG/KW)	ALPHA T (KG/KW)	TANKAGE FACTOR	STRUCTURE FACTOR	EFFICIENCY COEFFICIENTS B	D (KM/SEC)	E
15.0000	22.5000	0.0300	0.0	0.76000	13.00000	0.0

ELECTRIC PROPULSION SYSTEM MASS SUMMARY (KG)

INITIAL	POWER SOURCE	ENGINE	PROPELLANT	TANKAGE	STRUCTURE	NET MASS
4236.7039	922.6812	1384.0219	853.4027	25.6021	0.0	396.4837

ELECTRIC PROPULSION SYSTEM PERFORMANCE SUMMARY

P(1 AU) (KW)	P(HSKP) (KW)	P(TARG) (KW)	THR(1 AU) (N)	ACC(1 AU) (M/SEC**2)	ISP (SEC)	EFFIC	CHAR DEG (DAYS)
61.5121	0.0	0.9213	1.755149	4.1427230-04	4578.424	0.69652	1.00000000D 30

EXTREME TRAJECTORY AND PERFORMANCE CONDITIONS

MAX DIST (AU)	MIN DIST (AU)	MAX POWER (KW)	MAX THRUST (N)	BURN TIME (DAYS)	DEGRD TIME (DAYS)	TRAV ANG (DEG)
10.0255430	0.9839275	62.782322	1.75514909	777.82199	148.50263	214.50468

POWER SOURCE JETTISONED PRIOR TO RETRO MANEUVER
ENGINE MASS JETTISONED PRIOR TO RETRO MANEUVER
TANKAGE MASS JETTISONED PRIOR TO RETRO MANEUVER

DEPARTURE AND ARRIVAL CONDITIONS

DEP DECL (DEG)	PARK INC (DEG)	DEP VINF (M/SEC)	C3 (KM**2/SEC**2)	ARR VINF (M/SEC)	C4 (KM**2/SEC**2)
1.4758	28.5000	4650.93594	21.631205	5814.01083	33.802722

HIGH THRUST CAPTURE MANEUVER STAGE AND ORBIT SUMMARY

STRUCTURE (KG)	PROPELLANT (KG)	THRUST (LBS)	ISP (SEC)	BURNING TIME (SEC)
65.4512	589.0611	60.0000	300.0000	662.1310

PERIAPSE (RADII)	APOAPSE (RADII)	ORBIT VEL (M/SEC)	DEL VEL (M/SEC)	VEL LOSS (M/SEC)
1.1000	11.0000	33003.4812	2418.5247	322.7827

5.0 PROGRAM INSTALLATION AT LeRC

The new version of HILTOP is stored on the UNIVAC system at Lewis in terms of individual source and object subroutine modules, under the name of MASTERS in the general disk file MFILE. A complete alphabetical cross-reference list between HILTOP subroutine names and source module names (disk file MFILE) on the UNIVAC system at Lewis is displayed on the next page.

Implementation of the more sophisticated spacecraft model in the HILTOP computer program has enlarged the program approximately 40K bytes. Although the older version of HILTOP could execute within the 65K word capacity of the UNIVAC machine (with a rather simple overlay scheme), the newer version will not fit on the machine at all unless certain size-saving steps are first taken. In order to construct a viable overlay scheme for the HILTOP program, it was deemed necessary to adopt the ground rule that no overlay action be allowed during a HILTOP iteration sequence (solution of the two-point boundary value problem). It is felt that if this ground rule were not adhered to, i.e., if overlay action occurred during an iteration sequence, the execution-time properties of the HILTOP program would be significantly degraded. (Indeed, it is not certain, at this point, if significant core savings could be obtained by ignoring this ground rule.)

The following necessary size-saving steps were taken at LeRC:

(1) A sizeable chunk of code associated with ballistic swingby continuation and ballistic trajectory extension was dummied-out. This consisted of writing a dummy subroutine MØRE which, if referenced during execution, writes a simple message to the user explaining the non-availability of the attempted program option(s). The dummy MØRE is then used in the overlay structure, and this allows the deletion of subroutines SWING, SWTRAJ, TAPSET, and CØNVER from the overlay tree. It is emphasized that all dummy-replacement operations have not destroyed the associated program

Cross reference list between HILTOP subroutine names and source module names (MFILE) on the UNIVAC system at the Lewis Research Center.

HILTOP Subroutine Name	MFILE Module Name	Overlay Segment Name	HILTOP Subroutine Name	MFILE Module Name	Overlay Segment Name
AEINWT	A	RØØT	ØMASS	AN	RØØT
ALBEDØ	ALBEDØ	TRAJEC	PARINC	AØ	LIMBØ
ANSTEP	C	RØØT	PDATE	AP	FINAL
BEGIN	D	SBEGIN	PMPINT	AQ	RØØT
BHAM	E	TRAJEC	PRINT	AR	RØØT
CARKEP	F	TRAJEC	PRINTR	AS	RØØT
CDERIV	G	TRAJEC	PRIØR	AT	TRAJEC
CHECK	H	TRAJEC	PUNCH	AU	LIMBØ
CØNVER	I	absent	QPRINT	AV	SEGQPR
CØNVRT	J	TRAJEC	QSTART	AW	INITAL
CØRNER	DUMCØR/K	RØØT/absent	RADAR	AX	RØØT
DATE1	L	RØØT	RECHEC	AY	TRAJEC
DECLIN	M	TRAJEC	REMTIM	REMTIM	RØØT
DERIV	N	TRAJEC	RETINJ	AZ	TRAJEC
EFM	Ø	RØØT	RKSTEP	BA	TRAJEC
EFMPRT	P	FINAL	SCØMP	BB	RØØT
ETA	Q	RØØT	SETUP	BC	INITAL
ETAU	R	RØØT	SMQINT	BD	RØØT
EXTAB	S	RØØT	SØLAR	BE	RØØT
FIND	T	TRAJEC	SPRINT	BF	TRAJEC
FINISH	U	RØØT	STEP	BG	TRAJEC
FUNCT	V	TRAJEC	STØRE	BH	TRAJEC
GETAMP	W	TRAJEC	SUMMRY	DUMSUM/BI	RØØT/absent
GETHAM	X	TRAJEC	SWING	BJ	absent
GETHOO	Y	TRAJEC	SWSTØ	BK	TRAJEC
GETI	Z	TRAJEC	SWTRAJ	BL	absent
GETQ	AA	TRAJEC	TAP	BM	RØØT
GETRV	AB	TRAJEC	TAPSET	BN	absent
GUESS	AC	BALLIS	THANGD	BØ	TRAJEC
GUNTHR	AD	TRAJEC	TIKTØK	BP	RØØT
IMPRNT	AE	BALLIS	TIMLFT	TIMLFT	RØØT
IMPULS	AF	BALLIS	TRAJ	BQ	RØØT
INCØND	AG	TRAJEC	TRAJI	BR	TRAJEC
INPUT	AH	INITAL	TRAVEL	BS	TRAJEC
INTERP	AI	TRAJEC	TWINKL	BT	INITAL
LØAD	AJ	TRAJEC	VDØT	VDØT	RØØT
MAIN	AK	RØØT	VMAG	VMAG	RØØT
MINMX3	AL	RØØT	VPRINT	BV	LIMBØ
MØRE	DUMØRE/AM	FINAL/absent	VSCAL	BW	RØØT
NEWINT	NEWINT	INITAL	WRAPUP	BX	TRAJEC

ORIGINAL PAGE IS
OF POOR QUALITY

capabilities. Both the full source and object codes remain available for each subroutine on the UNIVAC disk pack, but the dummied object code is specified in the overlay tree which is input to the linkage editor program. The absence of the ballistic trajectory extension capabilities from the UNIVAC 1100 version of HILTOP in no way degrades the program's capability to perform electric propulsion performance investigations.

(2) Subroutine CØRNER was dummied-out. This only affects simulations involving the "old" spacecraft model, and only when a trajectory is "hung" on a propulsion-time corner (as explained in the primary HILTOP program document). The program will no longer automatically attempt to avoid this numerical difficulty by varying the propulsion time adjoint variable λ_τ , but the user can still do this manually. Or, when simulating the old spacecraft model, the user could use the older version of HILTOP which is currently being maintained on the UNIVAC 1100 at Lewis. Since there is no λ_τ in the new model, this change does not degrade the program's capability when simulating the new spacecraft model.

(3) Subroutine SUMMRY was dummied-out. This subroutine prints a very brief (one line per case) summary at the end of each computer run, which can be effectively deleted without degrading any program capabilities. This deletion eliminates some significant storage arrays.

(4) To obtain the capability of receiving full program output in the event of execution time-out, the UNIVAC 1100 sponsored subroutine TIMLFT was added to the program. Also added was subroutine REMTIM, which acts as an interface between the HILTOP code and TIMLFT. The associated program input variable is ITF, which has a default value of 3 (seconds) pertaining to the IBM 360/91, and it may be necessary to increase this value (e.g., ITF = 10) on the UNIVAC 1100.

(5) Overlay segment LIMBØ was created for three subroutines (PARINC, PUNCH, and VPRINT) associated with program options which are rarely exercised. These options, which are then effectively dummied-out on the UNIVAC 1100, are described in the HILTOP program report by the Namelist inputs (respectively) KPART, MPUNCH and ALTITU. Suppression of these options does not degrade the program's basic electric propulsion mission analysis capability. Attempt to use any of these options on the UNIVAC could cause an abrupt execution halt due to an invalid overlay occurrence.

The card images representing the HILTOP overlay tree on the UNIVAC 1100 are displayed on the next page.

HILTOP Overlay Tree for Univac 1100

```
SEG RØØT
IN MFILE.A,.C,.DUMCØR,.L,.Ø,.Q,.R,.S,.U,.AK,.AL,.AN
IN MFILE.AQ,.AR,.AS,.AX,.REMTIM,.BB,.BD,.BE,.DUMSUM,.BM
IN MFILE.BP,.TIMLFT,.BQ,.VDØT,.VMAG,.BW
SEG SEGQPR*, (RØØT)
IN MFILE.AV
SEG INITAL*, (SEGQPR)
IN MFILE.AH,.NEWINT,.AW,.BC,.BT
SEG FINAL*, (SEGQPR)
IN MFILE.P,.DUMØRE,.AP
SEG BALLIS*, (SEGQPR)
IN MFILE.AC,.AE,.AF
SEG TRAJEC*, (RØØT)
IN MFILE.ALBEDØ,.E,.F,.G,.H,.J,.M,.N,.T,.V,.W,.X,.Y,.Z
IN MFILE.AA,.AB,.AD,.AG,.AI,.AJ,.AT,.AY,.AZ,.BA,.BF,.BG,.BH
IN MFILE.BK,.BØ,.BR,.BS,.BX
SEG SBEGIN*, (RØØT)
IN MFILE.D
SEG LIMBØ*, (RØØT)
IN MFILE.AØ,.AU,.BV
```

ORIGINAL PAGE IS
OF POOR QUALITY

6.0 REFERENCES

[1] F.I. Mann and J.L. Horsewood, "Program Manual for HILTOP, A Heliocentric Interplanetary Low Thrust Trajectory Optimization Program." Analytical Mechanics Associates, Inc., Report No. 74-34, December 1974.

Part I - User's Guide (287 pages)

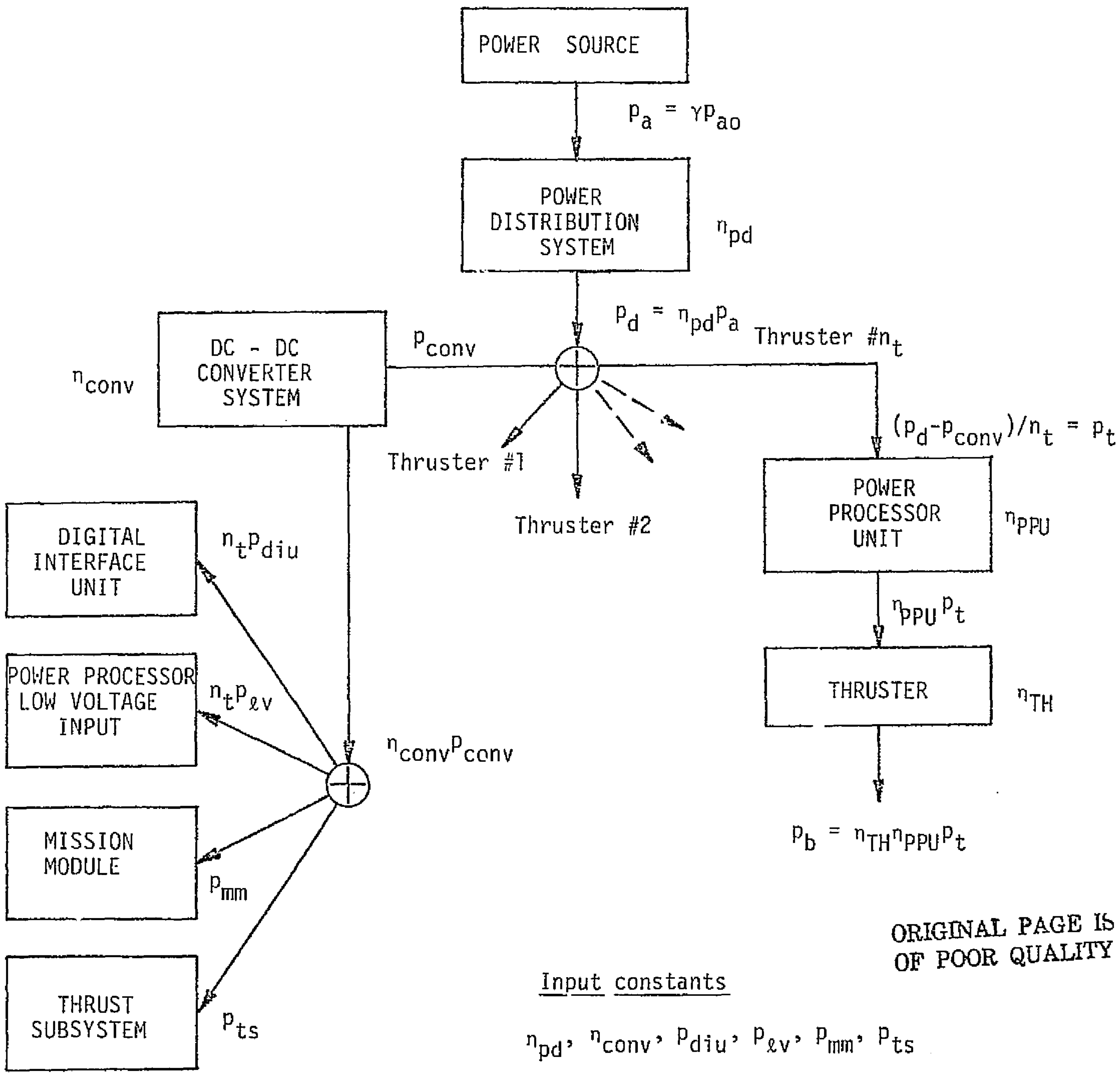
Part II - Subroutine Descriptions (854 pages)

[2] Private Communications, Mr. Phillip A. Masters, NASA Lewis Research Center, Cleveland, Ohio, May 1977 through February 1978.

APPENDIX A

HILTOP FORMULATION

Electric Propulsion System Power Flow Schematic

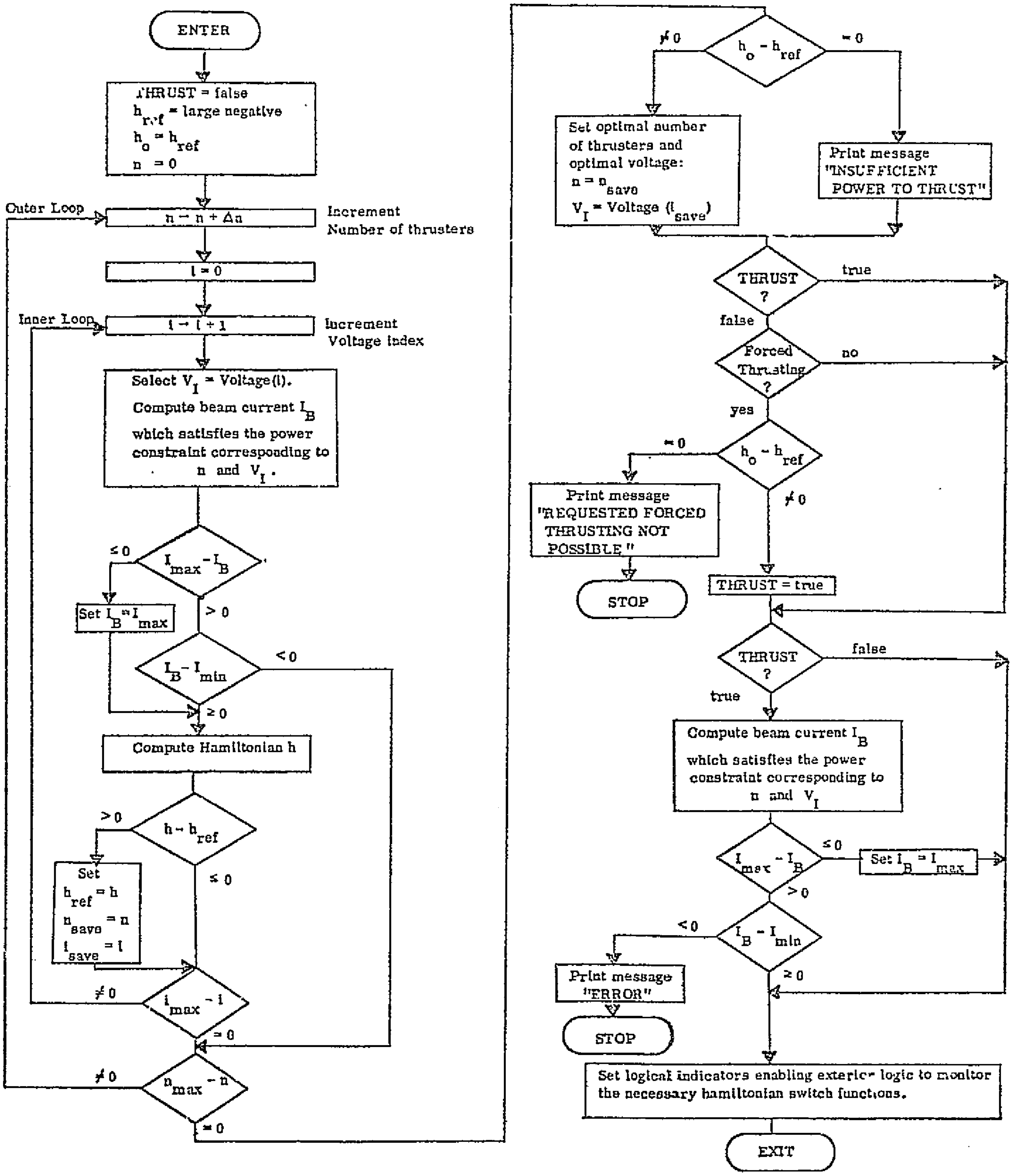


Input constants

η_{pd}, η_{conv}, P_{diu}, $P_{\ell v}$, P_{mm}, P_{ts}

C-2

APPENDIX B



Selection of Optimal Voltage and Number of Thrusters at any Point in Time

PRECEDING PAGE BLANK NOT FILMED