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Long Range Orbital Error Estimation For Applications Satellites

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LONG RANGE ORBITAL ERROR
ESTIMATION FOR APPLICATIONS SATELLITES

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ABSTRACT

In this report, we estimate by means of an optimum averaging technique, the orbital tracking errors for polar orbiting NASA applications satellites. Accurate long range estimations of orbital tracking errors are important to users in the field as an apriori condition for orbit updating. The approach we use is also called the method of precise conversion of elements, and utilizes results of a two week ephemeris produced by a double precision Cowell numerical integration calculation, starting with a nominal set of initial orbital elements. The gravitational field representation in the numerical integration is complete through both order and degree twenty. Included in this model are the effects of solar radiation pressure, lunisolar perturbations, and atmospheric drag. Periodic values of inertial Cartesian coordinates of the satellite state are taken from the first week of the ephemeris and used as observational data in an orbit determination calculation. This data is then used to find the boundary conditions or epoch vectors, through a least square processing, of the equations

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of motion representing analytic solutions of two separate differential equations of an orbit; specifically, those of Vinti and Brouwer-Lyddane. These latter contain only a gravitational field representation through the fourth and fifth zonal harmonics respectively, and are well adapted to applications satellite orbits, namely low eccentricity, low semi-major axis, and with little or no air drag. Both the Vinti and Brouwer-Lyddane methods are then used separately to predict an orbit over a period of eleven to thirteen days beyond epoch, and are compared with the Cowell solution during this time by calculating the in-track errors, using Cowell as a standard. The Vinti and Brouwer-Lyddane methods give from eight to thirteen kilometers in-track error respectively for the first week of free propagation, and approximately twenty-two to twenty-seven kilometers for the in-track component after thirteen days.

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I. INTRODUCTION

One of the more important tasks associated with the applications satellite support systems is pre-flight orbital error estimation. Accurate long term orbit prediction is useful as an apriori condition at the field sites for updating the orbit. Crucial to the results of such an analysis are not only the particulars of the estimation algorithm, but also the physical model used to describe the problem under investigation. How meaningful these results are will depend to a great extent upon how fundamental or comprehensive the system is, as well as the accuracy of the estimated initial conditions. A practical test is to make a direct comparison with real data from missions of a similar nature, being careful to consider the essential differences between the systems.

In this report, we shall employ a method of optimum orbital averaging or precise conversion of orbital elements to study the long range accuracy potential of polar orbiting applications satellites such as Landsat and TIROS-N. The essence of this approach involves the determination of the boundary conditions of one set of differential equations of motion, in this case, those describing the orbital motion of an artificial earth satellite, by adjusting the initial conditions or constants of integration in a least square sense, with the

use of 'data' generated by another set of differential equations of motion. In theory, so that the model might describe a reasonable approximation to the expected behavior of a system such as TIROS-N, one needs at the very least, to include a data sample commensurate with real cases. For example, one observational pair of radio direction cosine data at a specific time is insufficient to determine an orbit, since an indefinite number of ellipses corresponding to this piece of information can pass through this point. On the other hand, one Cartesian coordinate and velocity set will completely determine the spacecraft trajectory. As a result, it is necessary to process several distinct direction cosine pairs in order to fix the orbital elements of the satellite. However, the corresponding arc of observations will virtually never exhibit the quality and degree of uniformity possible by interrogating a numerical integration program. Likewise, the kinds of uncertainties such as noise, bias, and forces of nature, that are inherent in real observational data, cannot be fully and precisely reproduced in a simulation. For the case in which one wishes to transfer orbital elements from a theory that was fitted to real data to any other, the method employed in this article then becomes a powerful method of calculation. Uniformly spaced, high density data, of the type provided by complete sets of Cartesian coordinates would offer one of the best possibilities for exchange of information from one set of boundary conditions to those of another orbit determination algorithm.

In this paper, we fit analytic calculations to a uniform span of data, spaced
at hourly intervals, covering periods of one week and ten days respectively.
The analytic methods used here are those of Brouwer with modifications for
small eccentricity and inclination by Lyddane (1), and that of Vinti (2).
The fitted analytic methods are then used to calculate the orbit for a period of
two weeks from its epoch, and a comparison with the numerical integration
is made by calculating the tracking error along the direction of motion up to
thirteen days.

II. STATEMENT OF THE PROBLEM

The Satellite Aided Search & Rescue Demonstration will be flown on a TIROS-N

spacecraft. In this demonstration, field sites with only minicomputer capabil-

ity may, under certain adverse contingencies, be required to predict orbits

for relatively long periods of time. This in turn will directly affect the

"distress beacon" position location accuracy. In the following calculation we

consider a nominal TIROS-N orbit. The same general conclusions however,

apply to other applications spacecraft such as NIMBUS, GEOS, Landsat, etc.

These nominal initial conditions for the TIROS-N satellite are as follows:

a = semi-major axis = 7200886.36 meters

e = eccentricity = zero

i = inclination = 98.70 degrees

M = mean anomaly = zero degrees

ω = argument of perigee = zero degrees

Ω = longitude of right ascending node = zero degrees

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The epoch time is taken to be January 1, 1977, at zero hours, zero minutes, and

zero seconds, and the trajectory time span covers twenty-one days. The effec-

tive cross-sectional diameter and the mass of the spacecraft is taken to be

3.489 meters and 711.687 kilograms respectively. The reflectivity constant

is 1.2, while the area for radiation pressure is 9.560 square meters, and the

solar pressure constant 4.50×10^{-3} (kilograms/(second)^2)/kilometer. In-

cluded in the force model of the numerical integration method is the effect of

atmospheric drag. The corresponding atmospheric density model is that of Harris-Priester, 1964, with a profile range between one hundred and one thousand kilometers. Lunisolar perturbations are also modelled. The gravitational field coefficients are those of the Goddard Earth Model-I (GEM-I), (3).

The integration phase was performed using a twelfth order Cowell-Adams system of fixed stepsize, 24 seconds in duration, since it was anticipated that during the compare interval with the analytic theories, the integration method might degrade appreciably as opposed to effects of the gravitational field variations. The base inertial system employed in all calculations is that of the mean equator and equinox of 1950.

The method of compare used is described as follows: R_N, V_N, and $L_N = R_N \times V_N$, are the position, velocity and total orbital angular momentum vectors of the spacecraft as functions of time determined by the numerical integration, R_A is the corresponding satellite position vector at the same time, and determined by the analytic theory, while the total position error of the analytic theory at the given time is the magnitude of the vector difference, $\Delta R = R_N - R_A$. Then with respect to the numerically integrated orbit plane, the instantaneous components of this total position error, along the radial direction $\hat{h}$, perpendicular to the orbit plane $\hat{c}$, and along the direction of the velocity vector $\hat{\ell}$, are given by,

$$h = \hat{h} \cdot \Delta R = \frac{R_N}{|R_N|} \cdot \Delta R$$

$$-\hat{\mathcal{C}} \cdot \Delta \underline{R} = \frac{L_N}{|L_N|} \cdot \Delta \underline{R}$$

and

$$\ell = \hat{\mathcal{C}} \cdot \Delta \underline{R} = \frac{V_N}{|V_N|} \cdot \Delta \underline{R}$$

respectively.

The convergence criterion for the orbit differential correction least squares solution process is as follows: For a set of m observation residuals, the current weighted root mean square value of the observation residual column vector at the ith iteration is $\frac{\|\Delta \bar{y}_i\|}{m}$. The corresponding predicted weighted root mean square is $\frac{\|\Delta y_i - A_i \Delta \hat{x}_{i+1}\|}{m}$; where A_i is the coefficient matrix of the equations of condition, and $\Delta \hat{x}_{i+1}$ is the (i + 1st) correction to the state vector. From this, we have that when,

$$\left| \frac{\dfrac{\|\Delta \bar{y}_i\|}{m} - \dfrac{\|\Delta \bar{y}_i - A_i \Delta \hat{x}_{i+1}\|}{m}}{\dfrac{\|\Delta \bar{y}_i\|}{m}} \right| < \epsilon,$$

where $\epsilon = 10^{-4}$, the differential correction has converged.

For the first orbit fitting calculation, both analytic methods use only a gravitational field representation derived from GEM-1, with Brouwer-Lyddane retaining those coefficients through the fifth zonal harmonic, and Vinti through the fourth zonal harmonic. In the second case, both analytic methods employ the Goddard Earth Model-5 (GEM-5), (4), representation during the correction process, through the same zonal harmonics specified in the first calculation.

III. RESULTS AND CONCLUSIONS

In Tables 1 and 2 we list the results of post differential correction comparisons of in-track calculations of twenty-four hour intervals, for both the Brouwer-Lyddane and Vinti analytic orbit calculation methods. Table 1 represents the case in which the analytic theories utilize the GEM-1 gravitational field representation and Table 2 gives results involving the use of the GEM-5 field in the analytic theories. In both tables, time is measured in days, and the in-track error is measured in kilometers. From Table 1, the comparison over the period of eleven days shows an in-track error growth of approximately 2.3 kilometers per day. After one week, this error is approximately 13 kilometers. With use of the GEM-5 field the results are somewhat improved (Table 2). The in-track error growth for Brouwer-Lyddane is approximately 2.1 kilometers per day and for Vinti, 1.7 kilometers per day. After one week, Brouwer-Lyddane shows an 11 kilometer error, and Vinti, 8.5 kilometers. After thirteen days, the in-track errors are approximately 27 kilometers and 21.5 kilometers for Brouwer-Lyddane and Vinti respectively. Both analytic methods appear to give consistent results.

Table 1

Estimated In Track Error For TIROS-N Using

The GEM-1 Field

Time (Days)	Brouwer–Lyddane In-Track Error (kilometers)	Vinti In-Track Error (kilometers)
1	2.3865	2.8083
2	3.7461	3.4563
3	5.4258	5.2077
4	7.5962	7.7914
5	9.7219	9.9300
6	11.3436	11.0218
7	13.4276	12.8735
8	16.4265	16.4257
9	19.6.57	20.0096
10	22.3330	22.3113
11	24.7967	24.0057

Table 2

Estimated In Track Error For TIROS-N

Using The GEM-5 Field

Time (Days)	Brouwer-Lyddane In-Track Error (kilometers)	Vinti In-Track Error (kilometers)
1	1.3907	..0096
2	2.5823	1.8987
3	3.5839	2.7664
4	4.5635	3.2121
5	6.3790	4.9396
6	8.7123	6.7720
7	10.9978	8.5331
8	12.7566	10.6874
9	14.9692	12.4755
10	18.1158	14.0599
11	21.4816	16.6674
12	24.3299	19.2313
13	26.9314	21.5138

In addition, since few nonconservative forces are expected to greatly perturb the orbit, an in-track error growth of approximately 1.7 to 2.3 kilometers per day for an orbit of the TIROS-N class, would appear to be a reasonable a priori estimate. Furthermore, since this growth appears almost linear over eleven to thirteen days, one might expect a rapid convergence during subsequent orbit improvement calculations.

As a final consideration in orbit error estimation, let us consider two important error sources. If we compare the in-track behavior of a TIROS-N type satellite, with no nonconservative perturbations present, using a Brouwer orbit calculation with and without the second-order short period terms in the semi-major axis, we will get an in-track error growth of approximately 320 meters per day. On the other hand, if we assume an uncertainty of one part in 10^6 for μ, the product of the planetary mass and the gravitational constant, we obtain a value for this error growth of slightly more than 300 meters per day. Lyddane and Cohen (5), have shown that failure to initially adjust the semi major axis in the second order short period terms may immediately show as a serious secular like in-track error manifest through the mean anomaly for which 320 meters per day would represent a lower bound. This, together with a built in 'boundary uncertainty' for the parameter μ, might seem to account for the secular in-track error growth pattern of Tables I and II.

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V. REFERENCES

1. Lyddane, R. H., 'Small Eccentricities or Inclinations in the Brouwer Theory of Artificial Satellite Motion,' The Astronomical Journal, Vol. 68, June 1963. pp. 555-558.

2. Vinti, J. P., 'Inclusion of the Third Zonal Harmonic in an Accurate Reference Orbit of an Artificial Satellite,' Journal of Research of The National Bureau of Standards, Vol. 70B No. 1, January-March 1966.

3. Lerch, Francis J., et al., 'Gravitational Field Models for the Earth (GEM 1 and 2),' NASA/GSFC X-553-72-146, May 1972.

4. Lerch, Francis, J., et al., 'Goddard Earth Models (GEM 5 and 6).' NASA/GSFC X-921-74-145. December 1974.

5. Lyddane, R. H., and Cohen, C. J., 'Numerical Comparison between Brouwer's Theory and Solution by Cowell's Method for the Orbit of an Artificial Satellite,' The Astronomical Journal, Vol. 67, No. 5, April 1962, pp. 176-177.