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Orbit IMU Alinement

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Mission Planning and Analysis Division

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National Aeronautics and
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Lyndon B. Johnson Space Center
Houston, Texas 77058

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SHUTTLE PROGRAM

ORBIT IMU ALINEMENT

INTERPRETATION OF ONBOARD DISPLAY DATA

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CONTENTS

FIGURES

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1.0 SUMMARY

This document investigates the Space Shuttle inertial measurement unit (IMU) alinement algorithm to determine the most important alinement starpair selection criterion. Three crew-displayed parameters are considered: (1) the results of the separation angle difference (SAD) check for each starpair, (2) the separation angle of each starpair, and (3) the age of each star measurement. It was determined that the SAD for each pair cannot be used to predict the IMU alinement accuracy. If the age of each star measurement is less than approximately 30 minutes, time is a relatively unimportant factor and the most important alinement pair selection criterion is the starpair separation angle. Therefore, when there are three available alinement starpairs and all measurements were taken within the last 30 minutes, the pair with the separation angle closest to 90 degrees should be selected for IMU alinement.

2.0 INTRODUCTION

The purpose of this paper is to determine how the Shuttle IMU alinement accuracy can be predicted, using the star measurement data that are available to the crew via the star tracker/crew optical alinement sight (COAS) control monitor cathode-ray tube (CRT) display. The alinement algorithm will be analyzed in order to form conclusions relating to the crew and/or ground personnel interpretation of displayed measurement data for the purpose of alinement starpair selection. Several alinement-related characteristics of the current version of the star tracker software will be discussed briefly.

3.0 DISCUSSION

Alinement of the Space Shuttle IMU platforms to a desired reference orientation is a critical navigation procedure during onorbit operations. Nominally, each inertial platform is alined to its desired orientation by first determining its present orientation and then repositioning it to the desired orientation. The present orientation of each IMU platform is determined by taking directional measurements on a pair of stars. The star measurements are expressed as unit line-of-sight (LOS) vectors in a coordinate system that is fixed relative to the IMU platforms. Nominally, these star measurements will be taken using the Shuttle star trackers. These electro-optical tracking instruments are used because of their accuracy and their ability to be used in an automated system.

During the first two orbital flight test (OFT) missions, star tracker measurements will be taken manually by the crew. The majority of this discussion, however, concerns the automatic operation of the star trackers that will be employed after initial OFT missions. Under computer software control, each of the two trackers will acquire and track stars of opportunity. Measurement data for each star tracked are evaluated for validity by the software and then saved as a LOS vector in a star-sighting table. This sighting table is sized to store data on a maximum of three stars. After three stars have been saved, with each additional star tracked, the software determines which of the four stars is of least value for alinement computations and discards that star, leaving just three in the table.

Data for only two stars are needed for computing the present orientation of each IMU platform. Because there are inherent errors in the measurements, there exists an optimum starpair geometry which will result in the minimum IMU alinement error. This alinement error relation to the pair geometry is discussed in sections 3.1 and 3.2. Since stars are acquired in a more or less random fashion (rather than according to some optimum, deterministic pair selection criteria as is done in the manual sighting mode), the stars appearing in the sighting table will have random and unpredictable pair geometries. By increasing the number of stars saved from two to three, the number of available alinement pairs increases from one to three, thereby increasing the probability (at any one time) of having a pair with an acceptable geometry.

For each star that is saved in the sighting table, several related parameters are displayed to the crew on the star tracker/COAS control monitor CRT display. The identity number of each star that has been tracked and saved in the sighting table is displayed. Other information displayed includes the time since each star was sighted, and the separation angle of each of the three pairs. One final display item is an error parameter for each of the three starpairs. The SAD error (ERR on the display) is the absolute difference between the separation of the two measured LOS vectors and the actual or known separation between the two stars. When the software determines that there is at least one pair of stars existing in the sighting table that has an acceptable geometry for alinement, the software will select the two stars having the best pair geometry and flag these two stars on the crew display as its chosen pair for alinement computations. The crew may then execute the IMU alinement using the software selected stars. Alternatively, a crew or ground personnel review of the star

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tracker display parameters may result in the selection of a different pair of stars to ensure an optimum IMU alinement. The purpose of this discussion is to analyze the alinement algorithm in order to form conclusions relating to the interpretation of crew displayed star data for use in IMU alinements.

3.1 IMU ALINEMENT ERROR

The IMU alinement error relations will be derived geometrically by inspection. Each IMU will have some random orientation with respect to the measured starpair. The alinement errors, therefore, will be arbitrarily referenced relative to the coordinate system shown in figure 1. The X-Y plane is defined by the actual starpair LOS vectors S and T and will henceforth be referred to as the pair plane. The X-axis bisects the angle between the starpair LOS vectors. The Z-axis is perpendicular to the pair plane and the Y-axis completes the right angle triad. This orientation was selected to simplify the derivation of the error equations.

The measured star LOS vectors $\bar{S}$ and $\bar{T}$ are perturbed from the actual by small errors. The errors in the measured star LOS vectors are resolved into components parallel ($\parallel$) and perpendicular ($\perp$) to the pair plane. When an alinement is performed, the computed or measured orientation will be in error as a result of the sighting errors. The Shuttle alinement method assumes that the measured starpair lies in the same plane as the actual starpair (ref. 1). This method further assumes that the measured and actual directions of one of the pair are coincident. This direction vector will be referred to as the principal LOS of the pair. In this analysis the vector S will be the principal LOS. The alinement error, therefore, for this example is measured between the plane defined by $\underline{S}$ and $\underline{T}$ and having a reference direction S and the plane defined by $\bar{S}$ and $\bar{T}$ with the corresponding in-plane reference $\bar{S}$. An equation for each component of the alinement error vector ϕ will be derived.

The error about the X-axis, ϕ_x, is a result of the difference between $S_\perp$ and $T_\perp$, the out-of-plane errors. If both the in-plane and out-of-plane errors are small, then the tangent of ϕ_x is equivalent to the difference between $S_\perp$ and $T_\perp$ divided by the distance between the end points of S and T.

$$\phi_x \approx \tan \phi_x = \frac{S_\perp - T_\perp}{2 \sin {}^{\delta}/2}$$

$\delta \equiv$ starpair separation angle

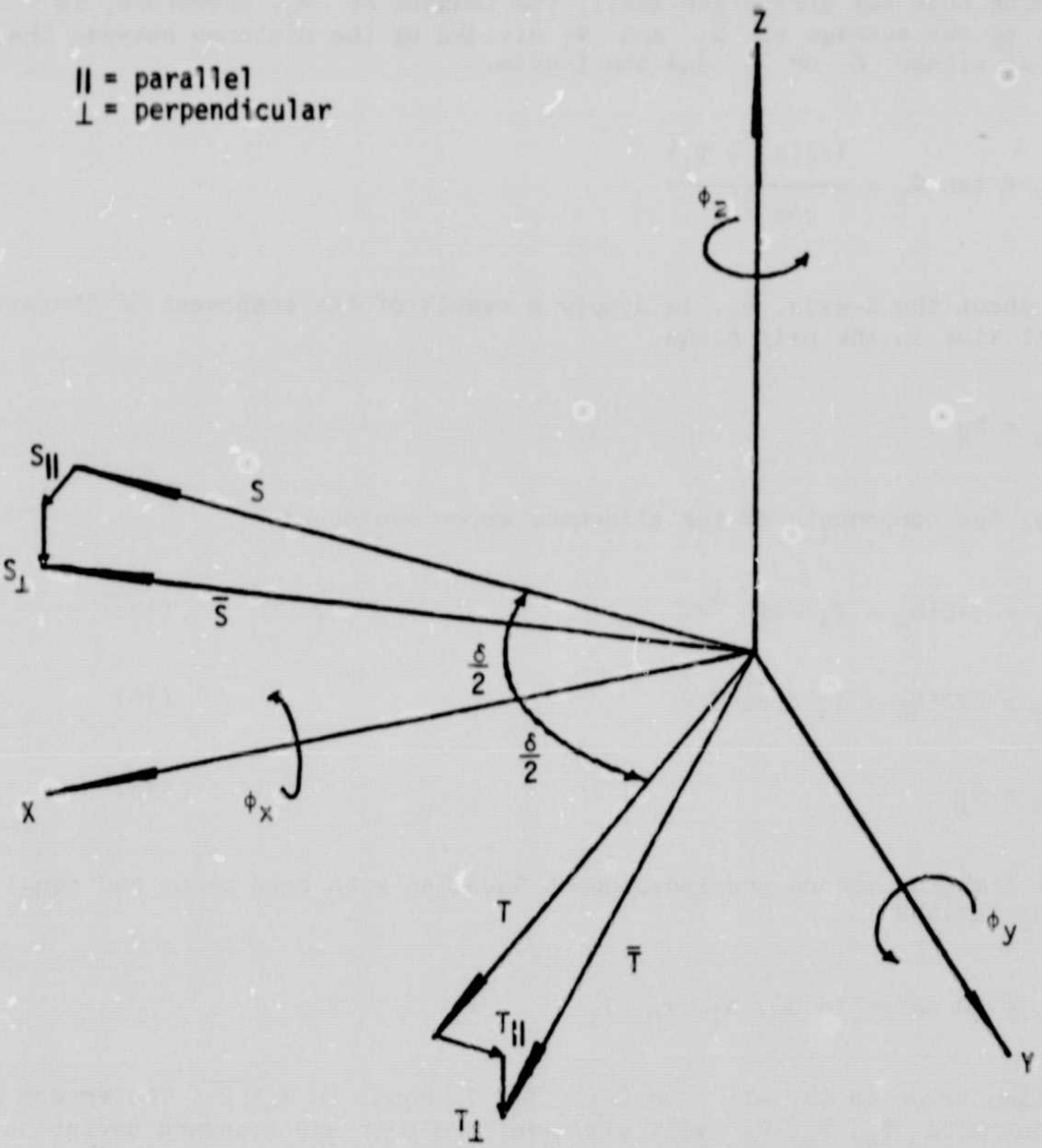

Figure 1.- Alinement starpair error components.

The error about the Y-axis, ϕ_y, is a result of the average magnitude of $S_\perp$ and $T_\perp$. Assuming that the errors are small, the tangent of ϕ_y, therefore, is equivalent to the average of $S_\perp$ and $T_\perp$ divided by the distance between the end point of either S or T and the Y-axis.

$$\phi_y \approx \tan \phi_y = \frac{1/2(S_\perp + T_\perp)}{\cos {}^{\delta}/2}$$

The error about the Z-axis, ϕ_z, is simply a result of the component of the error in S that lies in the pair plane.

$$\phi_z = S_{||}$$

In summary, the components of the alinement error vector are

$$\phi_x = 1/2(S_\perp - T_\perp)/\sin {}^{\delta}/2 \tag{1a}$$

$$\phi_y = 1/2(S_\perp + T_\perp)/\cos {}^{\delta}/2 \tag{1b}$$

$$\phi_z = S_{||} \tag{1c}$$

Assume all sighting errors are independent Gaussian with zero means and equal standard deviations.

$$\sigma_o \equiv 1\sigma \text{ error in } S_{||}, S_\perp, T_{||}, T_\perp$$

The one-sigma error in ($S_\perp - T_\perp$) or ($S_\perp + T_\perp$) is equal to $\sigma_o\sqrt{2}$. The errors in the misalinements ϕ_x, ϕ_y, ϕ_z will all have zero mean and standard deviation σ_x σ_y, σ_z respectively.

$$\sigma_x = 1/2\ \sigma_o \sqrt{2}/\sin {}^{\delta}/2 \tag{2a}$$

$$\sigma_y = 1/2\ \sigma_o \sqrt{2}/\cos {}^{\delta}/2 \tag{2b}$$

$$\sigma_z = \sigma_o \tag{2c}$$

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A total alinement error indicator will be defined as a root sum square (RSS) of each of the components ϕ_x, ϕ_y, and ϕ_z.

$$\phi = \sqrt{\phi_x^2 + \phi_y^2 + \phi_z^2}$$

Assuming this definition, it can be shown that the root mean square (RMS) of ϕ is equivalent to an RSS of the standard deviations of each of the components σ_x, σ_y, and σ_z.

$$\mathrm{RMS}_\phi \equiv \omega \equiv \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$

$$\omega = \sigma_o \sqrt{1 + 2 \csc^2\delta} \qquad (3)$$

It can also be shown that the standard deviation of the total sighting error (σ) is related to each axis standard deviation (σ_o) by:

$$\sigma = \sigma_o\sqrt{2}$$

The total RMS alinement error expressed as a function of the total system sighting error and the starpair separation, therefore, is:

$$\omega = \sigma \sqrt{1/2 + \csc^2\delta} \qquad (4)$$

The RMS indicator is equivalent to an RSS of the mean and standard deviation of the total alinement error. This equation for the RMS alinement error (eq. (4)) agrees with that derived by R. P. O'Donnell of the Massachusetts Institute of Technology (MIT) in reference 2. In figure 2 the total RMS alinement error, ω, is plotted as a function of both σ and δ. In figure 3 the standard deviation of each component of the alinement error is plotted separately.

The alinement error equation (eq. 4) was derived, assuming that both star sightings were taken at the same time and, thus, does not include the effects of errors due to IMU drift. If IMU drift error is added to each star measurement, the alinement error equation becomes quite complex. Rewriting equation (1) to include drift errors yields

$$\phi_x = 1/2\ (\bar{S}_\perp - {}_\perp\bar{T})/\sin {}^{\delta}/2$$

$$\phi_y = 1/2\ (\bar{S}_\perp + \bar{T}_\perp)/\cos {}^{\delta}/2$$

$$\phi_z = \bar{S}_{||}$$

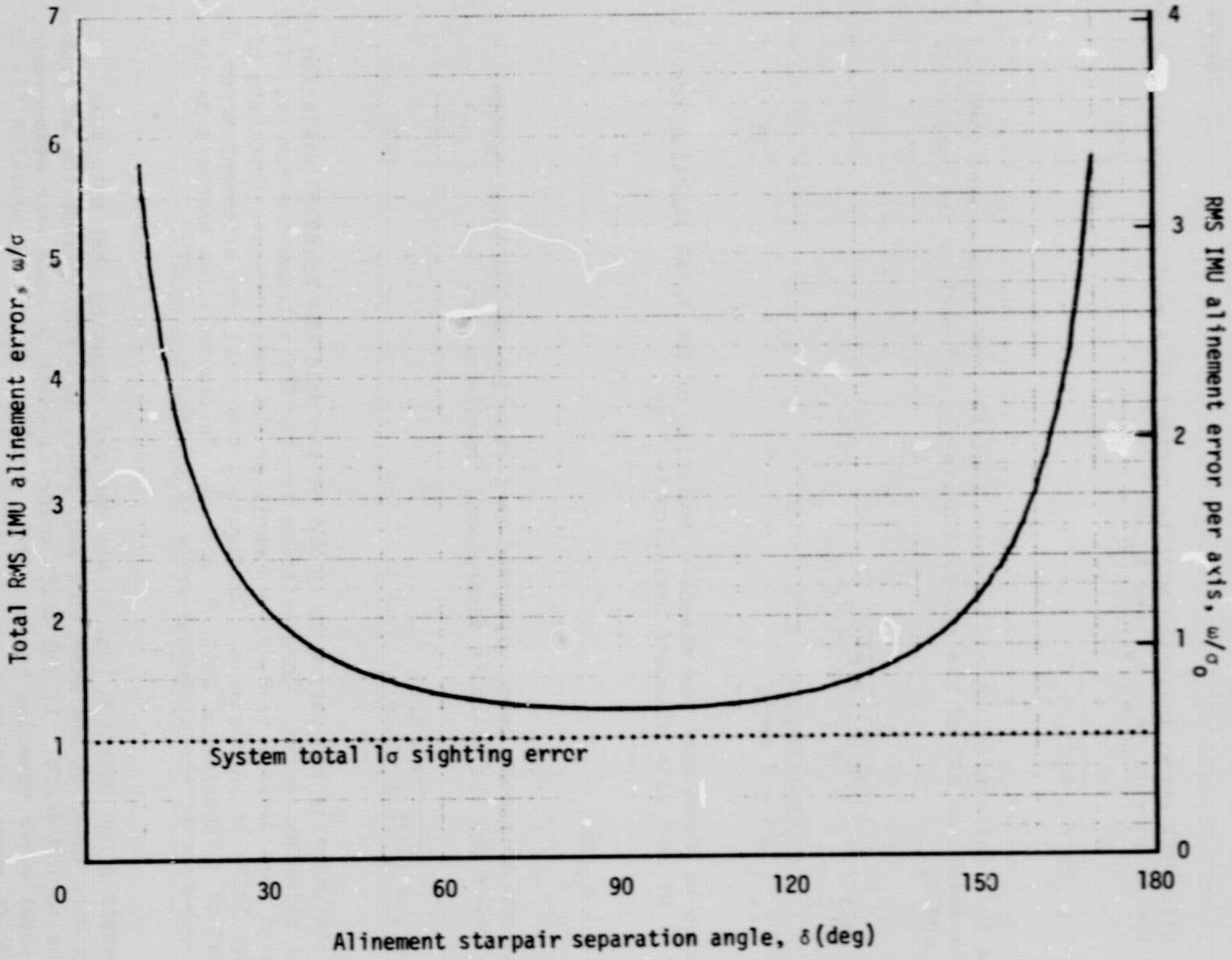

Figure 2.- IMU alinement error.

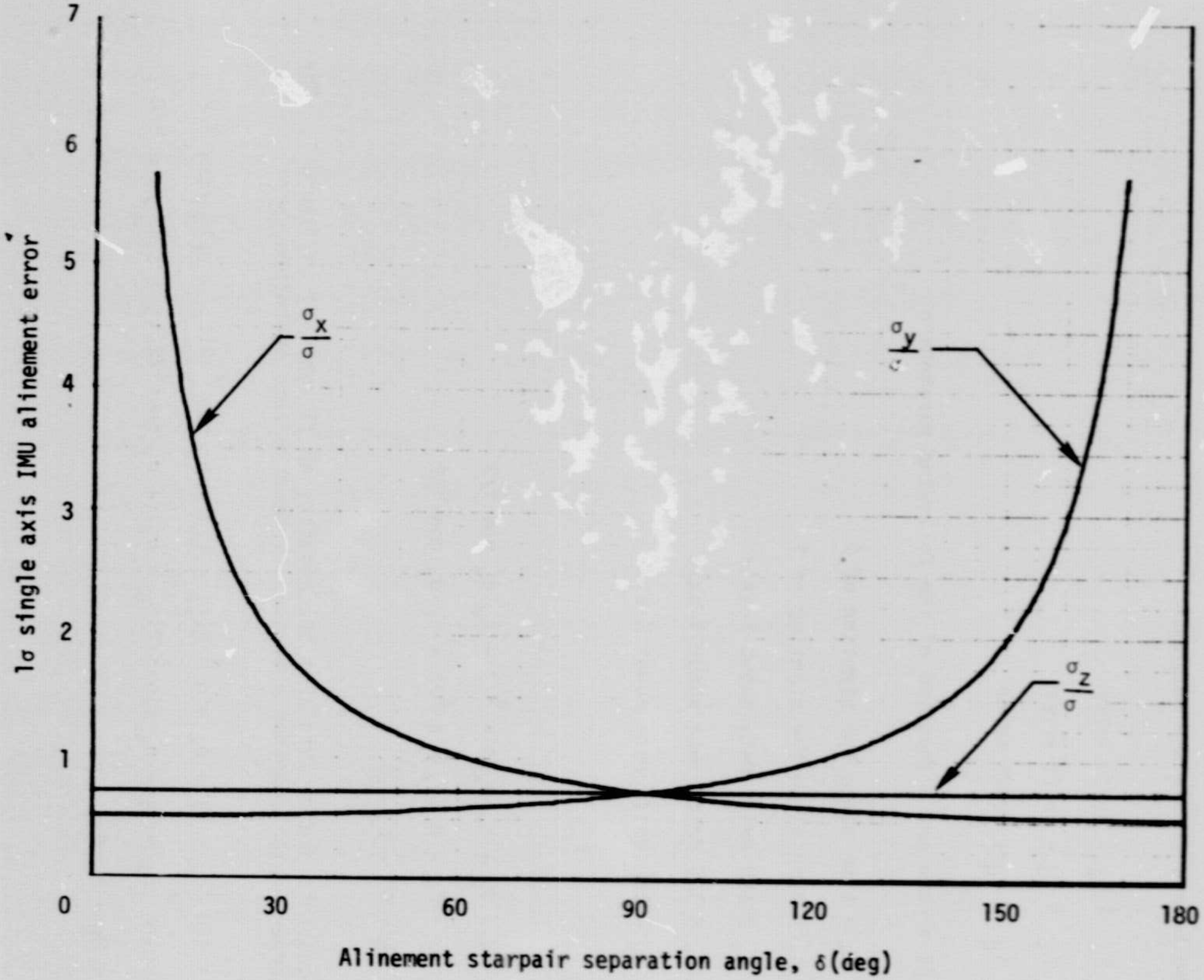

Figure 3.- Single axis IMU alinement error.

where the barred errors are defined:

$$\bar{S}_\perp \equiv S_\perp + \text{drift error}$$

$$\bar{T}_\perp \equiv T_\perp + \text{drift error}$$

$$\bar{S}_\parallel \equiv S_\parallel + \text{drift error}$$

The drift errors are a function of the following parameters:

t_s ≡ time since the sighting of S

t_t ≡ time since the sighting of T

d_x ≡ IMU drift rate about X-axis

d_y ≡ IMU drift rate about Y-axis

d_z ≡ IMU drift rate about Z-axis

Therefore,

$$\bar{S}_\perp = S_\perp + t_s\, d_x \sin {}^{\delta}/2 + t_s\, d_y \cos {}^{\delta}/2$$

$$\bar{T}_\perp = T_\perp - t_t\, d_x \sin {}^{\delta}/2 + t_t\, d_y \cos {}^{\delta}/2$$

$$\bar{S}_\parallel = S_\parallel + t_s\, d_z$$

These equations are derived from the geometry as illustrated in figure 1. The three perpendicular components of the alinement error, therefore, are:

$$\phi_x = 1/2 \left[S_\perp - T_\perp + d_x \sin {}^{\delta}/2(t_s + t_t) + d_y \cos {}^{\delta}/2(t_s - t_t)\right] \Big/ \sin {}^{\delta}/2$$

$$\phi_y = 1/2 \left[S_\perp + T_\perp + d_x \sin {}^{\delta}/2(t_s - t_t) + d_y \cos {}^{\delta}/2(t_s + t_t)\right] \Big/ \cos {}^{\delta}/2$$

$$\phi_z = S_\parallel + t_s\, d_z$$

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Now, computing the standard deviation of each component yields (again, the means of ϕ_x, ϕ_y, and ϕ_z can be shown to be zero):

$$\sigma_x = 1/2\sqrt{2\sigma_o^2 + (t_s^2 + t_t^2 - 2t_st_t \cos\delta)\sigma_d^2}\ /\sin{}^{\delta}/2$$

$$\sigma_y = 1/2\sqrt{2\sigma_o^2 + (t_s^2 + t_t^2 + 2t_st_t \cos\delta)\sigma_d^2}\ /\cos{}^{\delta}/2$$

$$\sigma_z = \sqrt{\sigma_o^2 + t_s\sigma_d^2}$$

Recall again that the standard deviation of the total sighting error is related to each axis standard deviation by:

$$\sigma = \sigma_o\sqrt{2}$$

The total RMS alinement error expressed as a function of the total system sighting error, the pair separation, and the age of each sighting, therefore, is:

$$\omega = \sigma\sqrt{1/2 + \csc^2\delta + 1/2\left[\frac{\sigma_d^2}{\sigma}\right]\left[t_s^2 + (t_s^2 + t_t^2)\csc^2\delta - 2t_st_t\cot{}^2\delta\right]} \quad (5)$$

3.2 STAR TABLE CRT DISPLAY DATA

If both star sightings were taken fairly recently, the alinement error contribution resulting from IMU drift will be small. This is because the ratio σ_d/σ (eq. (5)) is very small for the Shuttle system ($\approx 5 \times 10^{-4}$/sec). Furthermore, for the majority of the time the age of one of the sightings in each pair will be zero and the age of the other star will be less than 30 minutes. Simulations of the automatic acquisition mode have demonstrated that sighting data older than 30 minutes rarely appear in the table and most of the star data is less than 15 minutes old. Consider a pair of alinement stars with a separation angle of 90 degrees - the age of the most recent sighting is zero, and the other star measurement is either zero, 15, or 30 minutes old. The 30-minute case yields an RMS alinement error only 0.27σ greater than the optimum or the zero-minute case (eq. (5)). This is approximately 20 arc seconds for the Shuttle system. In the 15-minute case, the alinement error as predicted by equation (5) is only 0.07σ or approximately 5 arc seconds greater than the optimum case. For fairly recent sighting data, therefore, the pair separation angle affects alinement accuracy to a greater degree than sighting age does. (fig. 2).

Before concluding this discussion of the effect of IMU drift on alinement error, one final important point should be noted. The time since the measurement of the principal LOS of the starpair, t_s, appears in one term in equation (5) all by itself. The alinement error, therefore, is more strongly related to the IMU drift error in the principal star LOS. Since the principal LOS is favored by the alinement algorithm, this LOS should be the one that was most recently sighted. This procedure can result in an improvement in alinement accuracy of up to 20 percent.

The most obvious property of the curve in figure 2 demonstrates that the alinement error is significantly degraded for star separation angles close to zero or 180 degrees. It is desirable, therefore, to select alinement starpairs with near-right angle separations in order to guarantee minimum alinement error. Crew selection of an alinement pair should be driven by the separation angle displayed for each pair. The current version of the star tracker subsystem operating program (SOP) (ref. 3), however, already performs this selection based on pair separation, thereby relieving the crew of any decisive action if the data have been taken recently. The crew should, however, verify that all data in the sighting table are less than 30 minutes old before accepting the software selected pair.

There is one final item that is a candidate parameter for use in predicting alinement accuracy. This parameter, the SAD, is the absolute difference between the measured pair separation and the actual or known pair separation. The SAD error that is displayed to the crew is defined:

$$ERR = \left| S_{||} - T_{||} \right| \qquad (4)$$

The SAD error is clearly a function of only the in-plane errors. If the SAD gave any indication of the magnitudes of the in-plane errors, it would still be relatively invaluable because for angles not close to 90 degrees, the out-of-plane errors are the greater contributors to the alinement error (fig. 3). However, assuming that both stars of the pair were correctly identified, closer inspection of the equation for ERR reveals that knowledge of the SAD gives no clue to the magnitude of the in-plane alinement error. Solving equation (4) for ϕ_z, the in-plane alinement error

$$\phi_z = S_{||} = T_{||} \pm ERR$$

Since $T_{||}$ is a random variable, a better estimate of the in-plane alinement error, ϕ_z, cannot be determined using the known or measured value of ERR.

The displayed SAD error cannot be used to predict alinement error when both components of the starpair have been correctly identified. The SAD check is performed in the onboard Shuttle software to detect failure of the star tracker to acquire the desired (or software-selected star). The use of measurement data on the wrong star would result in very large alinement errors. To protect against this possibility, the star tracker software checks the SAD of each new star tracked and the previously tracked star (if available) for off-nominal values of ERR. If the value of ERR is very much greater than its nominal value ($\sigma_{ERR} = \sigma$), then at least one of the stars in the pair was falsely identified. If the test fails, the new star data is not saved in the star sighting table.

Futhermore, if the test fails and the previously sighted star is the only one in the sighting table, then it is deleted also. If the SAD check is passed, the new data is stored in the table. The software assumes that if a pair of stars passes this check, there is a high degree of certainty that both stars have been correctly identified. For this reason, each star is paired with only one other star for the purpose of performing the SAD check. It is erroneous to assume that the software SAD check provides absolute certainty of the star identities. This is because the SAD check is only sensitive to random errors that lie in the star plane (ref. 4). As stated previously, if the table is already full, the software determines which star in the table is of least value for alinement computations, and deletes that star to make room for the new star. Any one of the three stars in the table can be replaced by the new star. The SAD for each pair of stars in the sighting table is displayed on the CRT. When a new star is placed into a full table, the set of SAD angles may be reconfigured in such a way that star pairs previously tested by the software SAD check no longer exist. By this software sighting table maintenance scheme, therefore, it is possible that one, two, or all three of the displayed SAD angles have not been tested by the software against the allowable tolerance. Figure 4 illustrates how multiple pairs with common LOS vectors produce intersecting tolerance limits about each star LOS. Out-of-plane errors in a particular pair of stars can be seen by the SAD errors of other pairs with a single common LOS vector. It would be advantageous, therefore, for either the crew or ground personnel to check the displayed SAD angles for all pairs in the table that have not been checked by the software. This procedure also may isolate out-of-plane errors that were not detected by the software SAD check. If the SAD check is not done for the purpose of detecting out-of-plane errors, the tolerance windows will be bounded in the direction perpendicular to the pair plane by only the estimated/measured position check tolerance. This tolerance, illustrated by the circles in figure 4, however, is very coarse compared to the SAD error tolerance. The estimated/measured position check tolerance is currently set at 0.5 degrees for offset acquisition and 1.8 degrees for automatic acquisition.

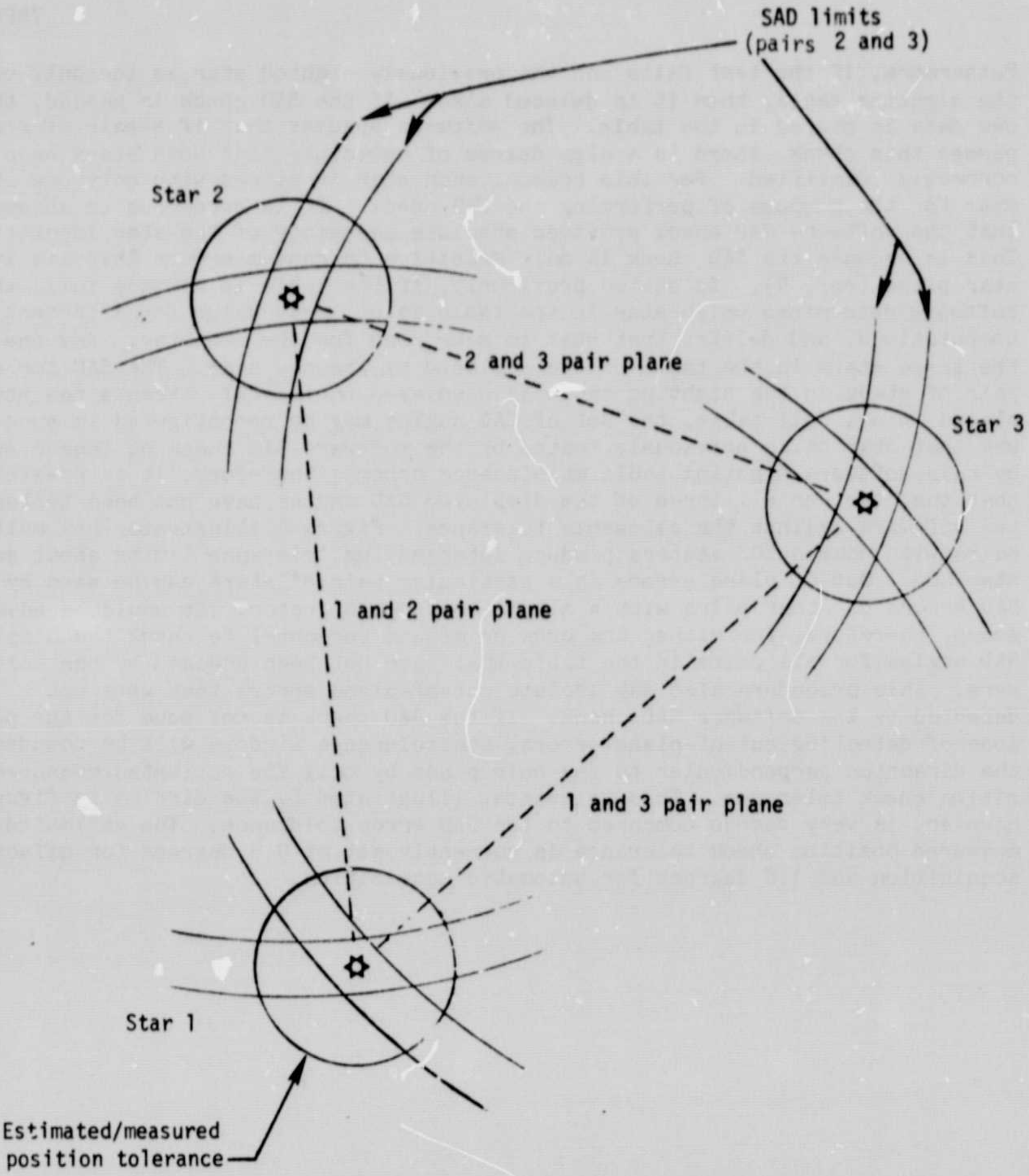

Figure 4.- Three pair SAD tolerance windows.

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4.0 CONCLUSIONS AND RECOMMENDATIONS

During the first two OFT missions, IMU alinements will be done manually. The nominal mode of operation will be to maneuver to an attitude in order to simultaneously track one star in each star tracker. The starpair separation angle will always be very close to 90 degrees and the age of both sightings will always be near zero. Therefore, the information presented here is not relevant to the initial OFT missions. During later missions however, when IMU alinements are done using the automatic acquisition mode, the following guideline is recommended whenever there is data for three stars stored in the sighting table: If all sighting data were taken within the last 30 minutes, the starpair separation angle is the most important alinement pair selection criterion. Pair selection based on separation is already implemented in software, and the crew should not override the software decision by selecting a different pair. If any single star in the table is older than 30 minutes, optimum pair selection becomes complex because of the interrelation of alinement accuracy, time, and pair separation. Development of a simple alinement accuracy indicator based on time and pair separation angle (for use in pair seletion) is a subject of future study.

Two alinement-related software characteristics that were discussed are mentioned again in summary. First of all, if the most recently sighted star is used as the principal LOS, this can result in a considerable improvement in the alinement accuracy over the converse case. This implementation is reflected by the IBM Corp. in their current interpretation of the star tracker SOP. Secondly, when three stars have been saved in the sighting table, a greater degree of certainty of the identity of each star sighted can be gained by having the crew or ground personnel monitor the SAD error for all pairs that have not been checked by the software. If the displayed value of ERR for any pair is greater than the I-load SAD tolerance (TOL10), then at least one star in the pair has been misidentified.

5.0 REFERENCES

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