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MEASUREMENT OF CONTROL SYSTEM RESPONSE USING AN ANALOG OPERATIONAL CIRCUIT

by Vincent R. Lalli
Lewis Research Center
Cleveland, Ohio 44135

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NASA Lewis Research Center

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ABSTRACT

Ten basic steps are established for an analog method that measures control system response parameters. An example shows how these steps were used on a speed control portion of an auxiliary power unit. The equations and calculations necessary to describe this subsystem are given. The mechanization schematic and simulation diagram for obtaining the measured response parameters of the control system using an analog circiut are explained. Methods for investigating the various effects of the control parameters are described. It is concluded that the optimum system should be underdamped enough to be slightly oscillatory during transients.

INTRODUCTION

Control system response parameters can be obtained by various techniques, some of which are listed below:

1. Check out of subsystems with analytical system studies.
2. Test equipment and physical duplication of the controlled process.
3. Digital simulation of the controlled process.
4. Analog simulation of the controlled process.

Method one is simple and does not require complex apparatus or setup time. Its major disadvantage is customer dissatisfaction. System startup is quite difficult when no measurements have been made on the entire system prior to setup in the operating site. Much time is required to "debug" the controlled process and get it to meet the customer specifications

Method two is complex and requires working with complex apparatus usually not available. It is prohibitively expensive to set up a complete system test for all possible combinations of processes and equipment. For these reasons many manufactures make measurements on as much of the controlled process as possible and hope that no insurmountable problem occur at the job site.

Method three is simple and suitable for solution of nontrivial mathmatical equations. Several software packages are currently available with which controlled processes can be analyzed (Ref. 1). Its major disadvantage is that a digital computer is required to perform the analysis which may very well not be available to many designers. If you have access to a digital computer, by all means look further into this method of analyzing controlled processes.

Method four, the subject of this tutorial type paper, is generally a good compromise between the other methods. Analog simulation is not complicated after one becomes familiar with how it is done. Circuit components are available either at a reasonable cost or by renting.

There are two basic approaches to electronic analog simulations. The conventional approach is to use a general purpose analog computer. In this case the equations would be written in a form that contains only integrators, summers and gain elements. Direct identity of physical parameters and transfer functions are often lost in the changes necessary to construct a simulation that contains only these elements. The second approach presented here shows how to produce a simulation with an electronic circuit constructed with operational amplifiers for minimum cost. This approach could be used even if your facility is very limited in available funds.

There are ten basic steps in developing analog simulation for measuring control system response. The steps are as follows:

1. Study the control process
2. Draw a system diagram of the process
3. Obtain transfer functions for each block
4. Estimate the process constants
5. Calculate network simulation constants
6. Arrange equations to avoid differentiators

7. Derive the network simulation schematic
8. Determine magnitude and time scaling for the variables
9. Build the specialized simulation circuit
10. Obtain simulated system response data

DESIGN EXAMPLE

As an example of how the steps produce solutions to practical problems consider the speed control portion of an auxiliary power unit (APU). APU control systems generally sense output voltage, current, phase and frequency. These variables are then compared with fixed references to determine the power systems electrical performance. In our sample portion of these larger systems, a throttle valve simply controls the rotating shaft speed of the alternator by regulating the rate of fuel consumption. Fig. 1 shows the speed control subsystem diagram. From a study of the control processes involved in this portion of the hardware or from their specifications, one can determine how the various blocks are connected together and prepare a diagram of the process. This simplified speed control shows a turbine-driven alternator whose output ac waveform is controlled by a combination of three feedback loops - phase, frequency and throttle valve position.

The transfer function diagram is derived from the subsystem diagram and from knowledge of the component dynamics. For example, the self-saturating magnetic amplifier in the subsystem diagram, block number 3, is known to have an approximate transfer function of the form shown in Eq. 1 (see refs. 2, 3, and 4).

$$E_o(s)/I_c(s) = K_3/(\tau_3 s + 1) \text{ V/mA} \qquad (1)$$

The constants K_3 and τ_3 are obtained from specifications, calculated or measured experimentally. Considerable work has been done to develop appropriate transfer function for various devices. Refs. 5, 6, 7, and 8 should give the reader a starting point from which to obtain transfer functions for most control devices. One can, however, always go back to the original differential equations and derive whatever is required.

Fig. 2 shows the transfer function diagram for the speed control process. An index is used to identify the control process components by number. For example, K_1 represents the gain of the 50 Hz phase network and K_2 represents tha gain of the pulse time modulator.

The fifth step in the technique is to obtain the process constants. Again, these constants may be obtained from the control process by measurements on the components, from calculations or from the specifications. For our example, consider transfer function nine which represents the subsystem circuit shown in Fig. 3. This lead-lag network has the transfer function

$$E_2(s)/E_1(s) = K_9(1 + \tau_9' s)/(1 + \tau_9 s) \qquad (2)$$

Source and load impedance have been neglected on the assumption that $R_s << R_1 << R_L$. The equations and calculations necessary to define K_9, τ_9', and τ_9 are also derived in this figure. The network constants are given by the initial specifications to be $R_1 = 300$ KΩ, $C_1 = 0.18$ uF and $R_2 = 100$ KΩ. Using the equations derived in Fig. 3, calculations for the lead-lag network constants result in $K_9 = 2.5\times10^{-1}$ V/V, $\tau_9' = 5.4\times10^{-2}$ s, and $\tau_9 = 1.3\times10^{-2}$ s. Later on in this technique we may want to vary these constants, if it turns out that the position loop is a cause for improper performance. The remaining process constants are shown in Table I and were obtained in a similar manner.

BUILDING THE SIMULATION NETWORK

The analog simulation schematic diagram can be prepared from the transfer function diagram by combining active networks. Fig. 4 shows the input-output voltage relations for six basic circuits that could be applied to some problem. Additional transfer functions simulation networks can be found in Refs. 9 to 12.

Let us explain how transfer functions would be mechanized by first modeling the servomotor and gear train. These components have a transfer function as given in Eq. 3.

$$\theta_4/I_3 = K_4/s(1 + \tau_4 s) \qquad (3)$$

Combining transfer functions number 3 and 4 from Fig. 4 gives the required simulation as shown in Fig. 5. The resistor and capacitor values are obtained using the equations given in Fig. 4 and the constants from Table II. In this manner all simulation constants are calculated for the complete schematic.

In preparing this diagram one should also give consideration to the variables of the control process. The magnitude and frequency range of these variables are scaled to fit within the ±15 V range and the 0 to 100 Hz frequency range allowed by the pen recorders. The APU performance is slow enough that it was studied in real time and the variables were all limited and scaled as required to correspond with physical control boundary conditions. Each monitored variable will require an additional channel of recording equipment so some care must be used in selecting the variables which require monitoring. For our example, five variables defined below required monitoring.

Monitored Process Variables

Symbol	Definition
θ_4	Valve position, degrees
P_5	Decomposition chamber pressure, psig
T_L	Load torque, lb-ft
ψ_8	Phase shift, radians
$d\psi_8/dt$	Frequency, radians/second

The frequency network (transfer function thirteen) suggests that ψ_8 be differentiated. Differentiators are normally avoided in simulation problems as they are peaking circuits tending to overdrive (saturate) amplifiers. Fortunately, all stable linear systems can be simulated without differentiators, see Ref. 13. Nonlinear simulations which demand differentiation can be approximated by the circuit in Fig. 4 if phase lead and gain limitations at high frequencies are acceptable.

Since our simple example is linear, we avoid this complication by rearranging the circuit. We are able to obtain $d\psi_8/dt$ directly by combining K_7 and K_8 with a simple lag as shown in Fig. 2. Integrating this signal gives us ψ_8.

Analog simulation has several known sources of error (Ref. 15). Methods described in the literature can be used to identify how accurate the simulation will be - typically around 90 percent. Even with accuracy limitations analog simulation is an excellent method for predicting where problems might occur and for planning methods by which corrective action may be implemented.

Its low cost is also a key advantage. We could buy the simulation network components for a reasonable cost estimated below. Notice that these parts could easily be purchased for less than \$250.00. To make permanent records for each condition studied in this problem, we could rent a five channel recorder for a few months for less than \$500.00. This whole problem could be run for less than \$1000.00 - a small price to pay for all that is being learned.

	Number	Cost
1. Summer Amplifiers	5	\$50
2. Integrator Amplifiers	8	80
3. Potentiometers	5	25
4. Power Supplies (dual)	1	80
		235

ANALYZING THE DATA

By applying a step-change in load torque to the simulation network through an amplifier we can observe how control variables will respond for various conditions in the network simulation. Fig. 6 shows transients of several variables when a 40 hp step load is applied to the speed control. The top graph in this figure plots valve position as a function of time. The valve would hit the position limit at 40° in about 0.4 s when the load is switched on. Thus protection against damage due to rapid slamming will be required. After a 1 second oscillatory condition, the valve is open about 32° within the design operating range. Chamber pressure, P_5, reaches a 320 psig peak during transient periods. Reliable design practice requires that the decomposition chamber be designed with a safety margin of 5 for these peak stress conditions. The steady state chamber pressure would be about 230 psig. Graph 4 shows that the peak frequency error, $d\psi_8/dt$, is about 3.75 rad/sec. and occurs at about 0.25 sec after the load is switched on or off. The frequency error goes to zero in about 2 sec. The maximum phase error, ψ_8, is about 17 radians as can be seen in graph 5. The speed control with the assigned constants does not exceed the allowable phase error, approximately 25 radians, and therefore, would not drop out of synchronization for this load change.

Individual run can be made to determine the effects of varying gains, process constants, environmental factors or aging on the speed control performance. Variations of any of the network constants in the simulation can be accomplished by inserting new values for the resistors and capacitors as required. Potentiometers make these variations much easier if your circuit design allows for their use. For each set of conditions a recording like Fig. 6 is made and evaluated. The response shown in Fig. 6 is underdamped enough to be slightly oscillatory. Further investigation showed, however, that this was about optimum for this speed control with its inherent decomposition lags, see Ref. 16.

CONCLUDING REMARKS

The analog method of system design was accomplished through ten basic steps. This design process was illustrated by analyzing a speed control portion of an auxiliary power unit. The equations and calculations necessary to describe the subsystem are given. The simulation schematic and diagram for obtaining measurements of the control system response using an analog circuits are explained. Once the simulated control process is setup in network form, individual runs can easily be made to evaluate the effects of varying gains, process constants, environmental factors or aging factors on the control system performance. Variations of any of the network constants in the simulation can be accomplished by inserting new values for the resistors and capacitors. For each set of conditions a time response recording is made and evaluated. The system should be underdamped enough to be slightly oscillatory during transients to optimize the control process.

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TABLE I. - PROCESS CONSTANTS

Symbol	Nominal magnitude	Range	Units
K_1	1	0 to 2	Radian/radian
K_2	Reference, 1.43	0 to 1.91	mA/radian
	Phase, 1.43		
K_3	Position, 20.9		V/mA
	Frequency, 7.1		
	Phase, 0.028		
τ_3	0.01		Sec
K_4	Motor, 0.0015		Dyne-cm/rms V
	Gear train, 0.021		Radian/radian
D_4	200		Dyne-cm/radian/sec
J_4	3.6		Gm-cm^2
K_5	7.5		Psi/degree
τ_5	75	50 to 100	Msec
K_6	0.0114	0.01 to 0.0128	lb-ft/psi
$1/D_7$	3580	1920 to 5240	Rad/sec/lb-ft
T_L	1.375	0 to 2.75	lb-ft
J_7	0.00235	0.001 to 0.0037	Slug-ft^3
K_8	0.314		Radian/radian
K_9	0.25		V/V
τ'_9	0.054		Sec
τ_9	0.0135		Sec
K_{10}	1.81	1.08 to 1.81	mA/rms V
K_{11}	0.20		Rms V/degree
K_{12}	0.154		V/V
τ'_{12}	0.25		Sec
τ_{12}	0.01		Sec
K_{13}	0.995	0 to 1.99	mA-sec/radian
K_{14}	0.125		Radian/radian

TABLE II. - COMPUTER CONSTANTS

Symbol	Nominal magnitude	Pot Range	Units	Schematic Parts
K_1	1	0 to 0.25	Radian/radian	P_1
K_2	Reference, 1.43	0 to 0.76	mA/radian	P2 R4/R2 R4/R3
K_3	Phase, 1.43		V/mA	
K_3	Modulator, 9.3		V/mA	K8/R5R9C2 R8/R6R9C2
	Position, 9.3			R8/R7R9C2
	Frequency, 37.2			
τ_3	0.01		Sec	C1R8
K_4	9.25		Degree/sec Rms V	R11/R10
τ_4	0.018		Sec	R11C3
K_5	7.5		Psi/degree	R18/R17
τ_5	0.075		Sec	R18C6
K_6	0.011	0.10 to 0.16	lb-ft/psi	P_3
K_7K_8	3580	0.30 to 0.83	Rad/sec lb-ft	R21/R19, R21/R20
τ_7	13.25	0.30 to 0.83	Sec	R21C7
K_9K_{10}	0.453		mA/rms V	R16/R15
τ'_9	0.054		Sec	R13C4
τ_9	0.013		Sec	R14C5
K_{11}	0.20		Rms V/degree	R14/R13
K_{12}	0.038		mA/mA	R27/R25
τ'_{12}	0.25		Sec	R25C9
τ_{12}	0.01		Sec	R27C10
K_{13}	0.995	0 to 0.99	mA/mA	R24/R23
K_{14}	0.125		Radian/radian	R31/R30

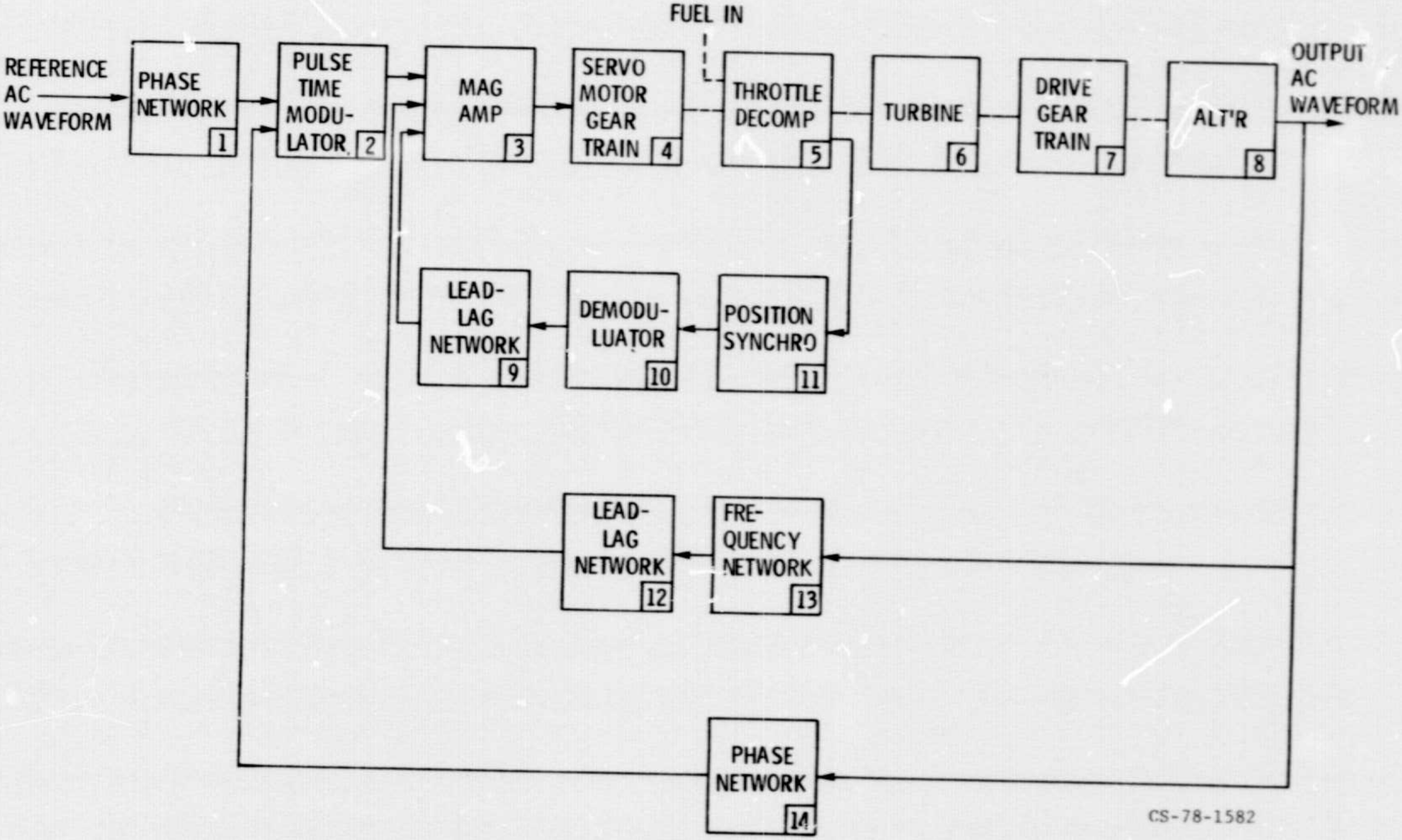

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Figure 1. - APU speed control subsystem diagram.

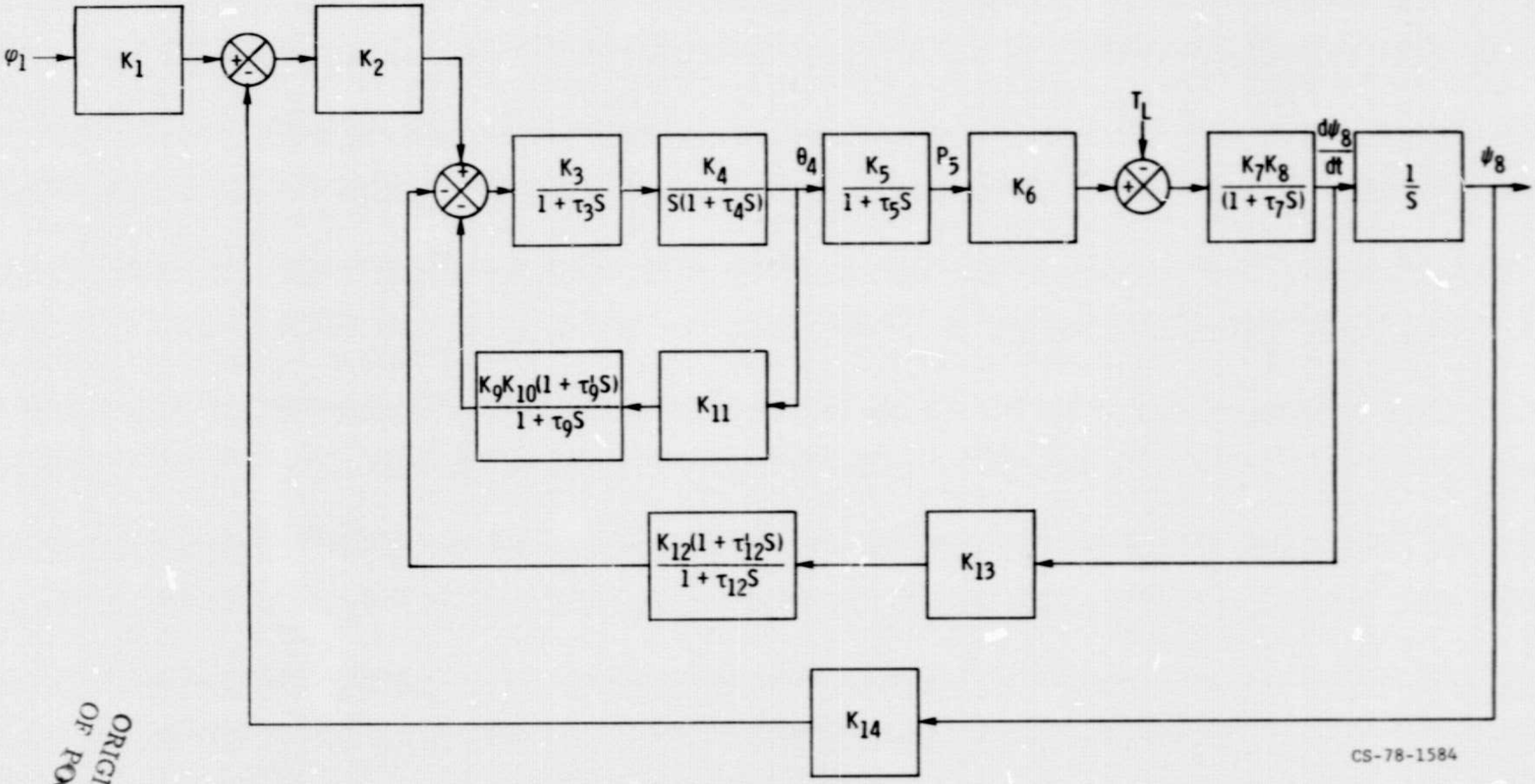
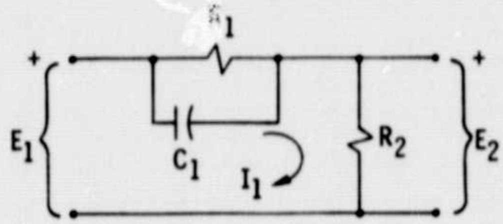


Figure 2. - Speed control transfer function diagram.



$$E_1 = \frac{R_1 + R_2 + R_1R_2C_1S}{1 + R_1C_1S} I_1$$

$$E_2 = I_1R_2$$

$$\frac{E_2}{E_1} = \frac{R_2(1 + R_1C_1S)}{R_1 + R_2 + R_1R_2C_1S}$$

$$\frac{E_2}{E_1} = \frac{\left(\frac{R_2}{R_1 + R_2}\right)(1 + R_1C_1S)}{1 + \left(\frac{R_1R_2}{R_1 + R_2}\right)C_1S}$$

HENCE

$$K_9 = \frac{R_2}{R_1 + R_2} \qquad R_g \simeq 100\ \Omega,\ R_L \simeq 1\ \mathrm{M}\Omega$$

$$\tau_9' = R_1C_1$$

$$\tau_9 = \left(\frac{R_1R_2}{R_1 + R_2}\right)C_1$$

Figure 3. - Lead-lag network circuit constants.

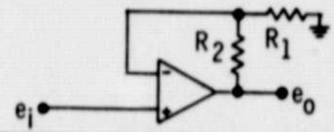
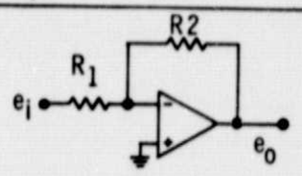
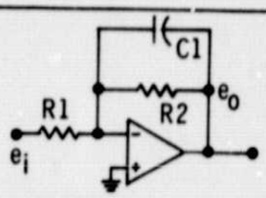
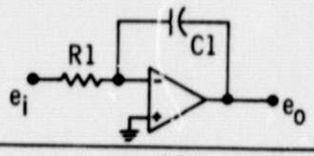
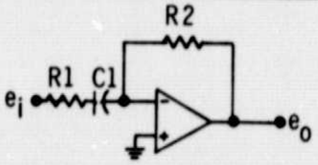
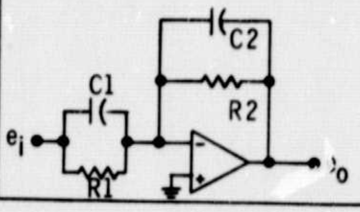
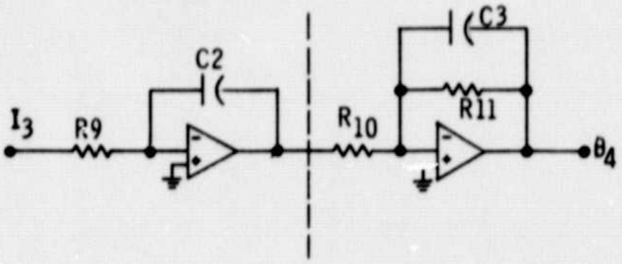
TRANSFER FUNCTION	SCHEMATIC	CONSTANTS
1. K		$K = \frac{R_1 + R_2}{R_1}$
2. K		$K = -\frac{R_2}{R_1}$
3. $\frac{K}{1 + \tau S}$		$K = -\frac{R_2}{R1}$ $\tau = R2C1$
4. $\frac{K}{S}$		$K = -\frac{1}{R1C1}$
5. $\frac{KS}{1 + \tau S}$		$K = -R2C1$ $\tau = R1C1$
6. $\frac{K(1 + \tau' S)}{(1 + \tau S)}$		$K = -R2/R1$ $\tau' = R1C1$ $\tau = R2C2$

Figure 4. - Basic operational amplifier circuits.



$K_4 = 9.25$

$= \frac{R11}{R10}$

$\tau_4 = 1.8 \times 10^{-2}$ sec

$= R11C3$

CHOOSE R10 = 100 KΩ

R11 = 9.25 x 100 K = 925 KΩ

$$C3 = \frac{1.8 \times 10^{-2}\ \text{sec}}{9.25 \times 10^{5}\ \Omega}$$

$$= 0.0194 \times 10^{-6}\ \text{F}$$

Figure 5. - Network and constants for transfer function 4.

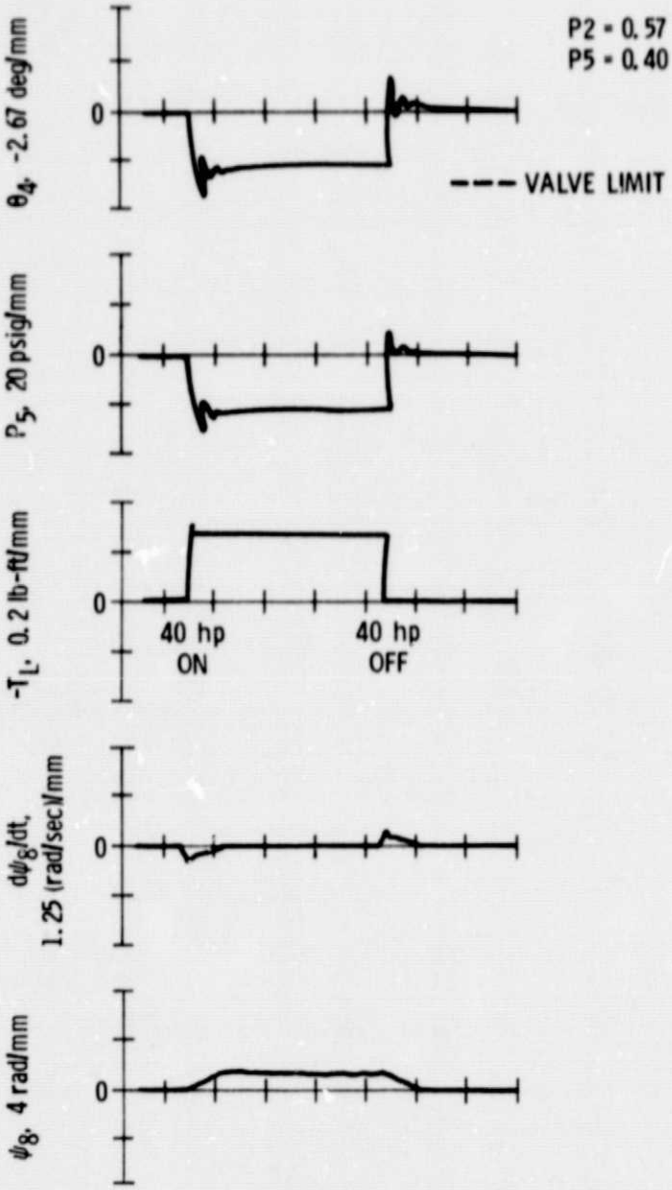


Figure 6. - Typical variables response recording.

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15. Supplementary Notes

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