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Robert W. Herr
*Langley Research Center*
*Hampton, Virginia*

NASA

National Aeronautics
and Space Administration

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## SUMMARY

An experimental study was undertaken to determine the effects of tilt and structural asymmetry on the vibration characteristics of partly liquid-filled thin-wall cylinders. It was found that tilting the longitudinal axis of a partly filled axisymmetric cylinder from the vertical could markedly reduce its resonant frequencies and change significantly the shape of the circumferential modes. For the minimum frequency modes, vibratory motion occurred only on that side of the cylinder where the liquid was deepest. An empirical equation was derived that gives the equivalent liquid depth of an untilted cylinder having the same minimum resonant frequency as a tilted, partly filled cylinder.

Circumferential mode shapes of an untilted asymmetric cylinder were similar to those of the tilted partly filled axisymmetric cylinder. Vibratory motion in the minimum frequency modes occurred in most instances only on the side of minimum thickness. Correlation between test data and results from a reformulated NASTRAN[1] hydroelastic analysis was excellent.

## INTRODUCTION

Until the advent of the space shuttle, propellant tanks of typical vertical launch vehicles were generally axisymmetric with the thrust vector coincident with the longitudinal axis and the free surface of the liquid propellant normal to the longitudinal axis. Under these symmetric conditions, a number of analysis methods were available to predict propellant-tank hydroelastic modes with reasonable accuracy. For the space shuttle, however, the canted thrust axis results in the free liquid surface being inclined by as much as 13° during launch. Structural asymmetry of the propellant tanks was also introduced by the mounting of the orbiter to one side of the tanks and the nature and magnitude of the effects of such asymmetry on vibration mode shapes and frequencies of propellant tanks were unknown.

An experimental study was undertaken at Langley Research Center to ascertain the effects of these asymmetries and to provide data for analysis-test correlation studies. For these experiments, the vibration mode shapes and frequencies of an axisymmetric cylinder and a similar asymmetric cylinder whose wall thickness varied sinusoidally with circumference were measured for various conditions of tilt, liquid fill level, and internal pressure.

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[1]NASTRAN: Registered trademark of the National Aeronautics and Space Administration.

## SYMBOLS

| | |
|---|---|
| $b$ | liquid depth of untilted cylinder, m |
| $\bar{b}$ | liquid depth at center line (average) of tilted cylinder, m |
| $b_{eqv}$ | equivalent liquid depth of untilted cylinder, m |
| $b_{max}$ | liquid depth on "deep" side of tilted cylinder, m |
| $b_{min}$ | liquid depth on "shallow" side of tilted cylinder, m |
| $f$ | resonant frequency of vibration, Hz |
| $k$ | location of $b_{eqv}$ |
| $L$ | length of cylindrical shell, 50.8 cm |
| $m$ | number of axial half waves in vibration mode |
| $n$ | number of circumferential waves in vibration mode |
| $n_d$ | circumference divided by wave length on "deep" side of tilted axisymmetric cylinder |
| $n_s$ | circumference divided by wave length on "shallow" side of tilted axisymmetric cylinder |
| $n_1$ | circumference divided by wave length at $t_{max}$ of asymmetric cylinder |
| $n_2$ | circumference divided by wave length at $t_{min}$ of asymmetric cylinder |
| $p$ | internal static air pressure, $N/cm^2$ |
| $r$ | radius of cylinder, 25.4 cm |
| $t_{max}$ | maximum wall thickness of asymmetric cylinder, 1.016 mm |
| $t_{min}$ | minimum wall thickness of asymmetric cylinder, 0.508 mm |
| $\theta$ | angle of tilt of cylinder from vertical, deg |

## APPARATUS

### Test Cylinder

Each of the two cylinders used in the experiments consisted of a circular cylindrical aluminum shell 50.8 cm in length and 50.8 cm in diameter welded to heavy aluminum end plates (figs. 1 and 2). The axisymmetric cylinder had a constant wall thickness of 0.813 mm. The asymmetric cylinder had a wall thickness that varied sinusoidally with circumference from a maximum of 1.016 mm on one side to a minimum of 0.508 mm on the opposite side. The sinusoidal variation in wall thickness was achieved by a combination of chemical etching and hand grinding. The thickness of the completed cylinder was measured at 168 locations. Average deviation from the desired thickness was 0.0035 mm and the maximum deviation was 0.028 mm. Each of the cylinders contained one longitudinal butt weld ground down to the thickness of the wall.

### Instrumentation and Test Procedure

Sinusoidal excitation of the cylinders was provided by a small servo-controlled electrodynamic exciter having a maximum force capability of 4.4 N. A force gage used to monitor the input force was installed between the exciter and the specimen. A servo control and oscillator were used to maintain a constant exciter force. The vibration response was detected by a motorized non-contacting displacement transducer capable of traversing the cylinders either circumferentially or longitudinally (fig. 1). A co-quad analyzer was used to determine the quadrature component of the displacement ($90^{\circ}$ out of phase with the input force). Resonance was determined by manually adjusting the frequency at a given level of input force until a maximum quadrature component was obtained. The mode shape was then recorded by inputting the quadrature component of the displacement into an x-y plotter as the transducer traversed the cylinder. The vibration exciter was in all cases located below the liquid surface and, in cases of unsymmetric response, on the side of the cylinder having maximum response.

The cylinders, along with the mode mapping mechanism, were mounted on a common base which permitted angles of tilt up to $30^{\circ}$ in increments of $5^{\circ}$. The liquid (water) depth was determined with an external sight glass (fig. 1) and was measured as indicated in figure 2(a). The level of internal static pressure was controlled by a pressure regulator valve in the air supply line.

## RESULTS AND DISCUSSION

Experimental data for the axisymmetric cylinder are presented in tables I to III and figures 3 to 9, and data for the asymmetric cylinder in tables IV to VII and figures 10 to 15.

### Axisymmetric Cylinder

Vibration modes of the untilted axisymmetric cylinder are identified in table I for ratios of liquid depth $b$ to cylinder length $L$ of 0, 1/8, 1/5, 1/4, 1/3, 1/2, 2/3, and 1 and internal pressures of 0 and 5.516 N/cm$^2$. The number of circumferential waves $n$ ranged from 2 to 18 while the number of axial half waves $m$ was either 1 or 2. Vibration modes are identified in table II for the axisymmetric cylinder at two angles of tilt ($\theta = 15^{\circ}$ and $30^{\circ}$) and at pressures of 0 and 5.516 N/cm$^2$. Ratios of average liquid depth $\bar{b}$

(depth at the center line of the cylinder) to cylinder length L were 1/5, 1/4, 1/3, 1/2, and 2/3. In table III minimum resonant frequencies, for a given set of experimental conditions, are presented for the above liquid and pressure levels and for tilt angles to 30° by 5° increments.

Untilted cylinder.- Resonant vibration frequencies of the untilted axisymmetric unpressurized cylinder are plotted in figure 3(a) as a function of the number of circumferential waves n for ratios of liquid depth to cylinder length (b/L) of 0, 1/8, 1/5, 1/4, 1/3, 1/2, 2/3, and 1. Only the modes having one axial half wave (m = 1) are plotted. The trends are similar for all liquid depths with the frequencies increasing with decreasing depth. For most depths the minimum frequency occurs at n = 7. Frequencies for the same liquid depths are plotted in figure 3(b) for an internal pressure of 5.516 $N/cm^2$. The trends resemble those of the unpressurized cylinder but corresponding modes are at higher frequencies and the minimum frequency modes occur at a lower value of n.

The effects of circumferential wave number on the longitudinal mode shapes of the untilted axisymmetric cylinder are shown in figures 4(a) and 4(b) for b/L = 1/3 and 2/3, respectively. At the higher values of n, the vibration amplitude above the liquid surface was small relative to the motion below the surface. The relative amplitude of the motion above the liquid surface increases with decreasing values of n. Although these results are for an untilted and unpressurized cylinder, they are typical of the longitudinal mode shapes obtained under all test conditions. Circumferential mode shapes of the untilted axisymmetric cylinders are not presented as they were of characteristic sinusoidal wave shape about the entire circumference.

Tilted cylinder.- Typical circumferential mode shapes of a tilted, partly filled axisymmetric cylinder are shown in figure 5. The circumferential traverses were made in each case at the longitudinal station of maximum response. The first 11 circumferential modes are plotted for $\bar{b}/L = 1/3$ and $\theta = 15^\circ$. The two vertical dashed lines indicated those points on the circumference of the cylinder where the liquid depth is a maximum or minimum. The lower frequency modes exhibited unusual behavior in that the vibratory motion occurred only on the "deep" side of the cylinder. As the frequency of the mode increased, vibratory motion extended progressively farther around the circumference until, at sufficiently high frequency, vibratory motion of relatively uniform amplitude occurred around the entire circumference. The wave length of the vibratory motion was generally shorter on the deep side of the cylinder as compared to the shallow side and may be noted in table II by comparing the entries in the columns under the heading $n_d$ and $n_s$. The quantities $n_d$ and $n_s$ were obtained by dividing the circumference of the cylinder by the wave lengths measured on the deep and shallow sides of the cylinder, respectively.

Minimum-frequency circumferential mode shapes are relatively insensitive to large variations of liquid depth and tilt angle as illustrated in figure 6. For average liquid depths of one-third the cylinder length (or less), the mode shapes are nearly identical for $\theta = 5^\circ$, $15^\circ$, and $30^\circ$. The frequencies, however, vary appreciably with the angle of tilt for a given liquid depth. Since most of the motion occurs on the deep side of the cylinder, the liquid depth on this side should have a controlling influence on the resonant frequency.

In figure 7, the minimum resonant frequency is plotted as a function of $\bar{b}/L$ for several angles of tilt of the unpressurized cylinder. For $\bar{b}/L > 2/3$, tilt has only a minor effect on the resonant frequencies. At a value $\bar{b}/L = 1/5$, however, tilting the tank 30° reduces the resonant frequency by nearly a half. For an internal pressure of 5.516 N/cm², resonant frequencies were appreciably higher than these for no internal pressure but otherwise the trends were similar to those shown in figure 7.

To provide a simple way to estimate the effect of tilt on resonant frequencies, an equivalent water depth for an untilted cylinder $b_{eqv}$ has been identified which gives the same frequency as a tilted cylinder. The quantity $b_{eqv}$ lies at some fraction $k$ of the distance between the average water depth and the maximum water depth of the tilted cylinder (see sketch (a)). In equation form

$$b_{eqv} = \bar{b} + k(b_{max} - \bar{b})$$

where $k$ is an experimentally determined factor. From the geometry involved it is seen that

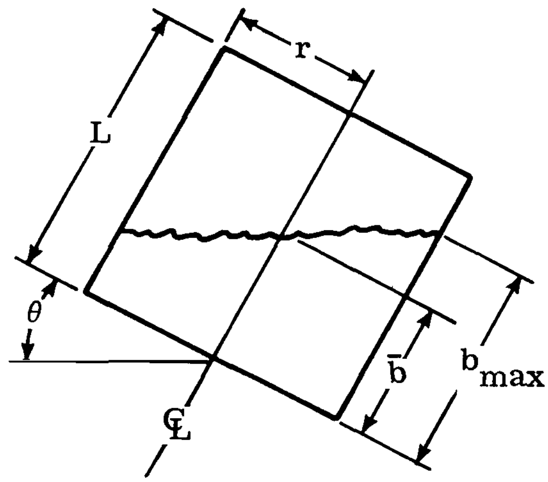
$$b_{max} = \bar{b} + r \tan \theta$$

therefore,

$$b_{eqv} = \bar{b} + kr \tan \theta$$

or

$$\frac{b_{eqv}}{L} = \frac{\bar{b}}{L} + \frac{rk \tan \theta}{L} \qquad (1)$$




Sketch (a)

From the data presented in figure 7, it may be seen that the minimum resonant frequency of a cylinder tilted 30° with a depth-length ratio $\bar{b}/L$ of 1/5 was the same as that of an untilted tank ($\theta = 0°$ and $b = \bar{b}$) with $\bar{b}/L = 0.42$. Substituting $r/L = 1/2$, $\bar{b}/L = 1/5$, $\theta = 30°$, and $b_{eqv}/L = 0.42$ into equation (1) and solving for $k$ yields a value of 0.76.

Similar substitution of several data sets from figure 7 into equation (1) results in an average value of $k$ of 0.71. In figure 8, the frequencies presented in figure 7 have been replotted as a function of the equivalent liquid depth ($k = 0.71$) rather than the average liquid depth with the result that the data for all tilt angles are superimposed on the curve for the untilted, unpressurized cylinder. Results were equally as good for an internal pressure of 5.516 N/cm². It should be possible, therefore, using any of several proven computational methods, to predict the minimum resonant frequency of a tilted, partly filled cylinder by computing the resonant frequencies of an untilted cylinder of "equivalent" liquid depth $b_{eqv}$.

The resonant frequencies of the unpressurized cylinder filled to a depth of 1/5 the cylinder length are plotted in figure 9 as a function of circumferential wave number for tilt angles of $0^{\circ}$, $15^{\circ}$, and $30^{\circ}$. The vibratory motion of the cylinder did not always extend completely around the cylinder; thus, an integer value of $n$ did not exist for each resonance. In such cases, $n_d$ (circumference divided by wave length on the deep side of the cylinder) from table II was used in plotting these data. The results indicate that tilt affects not only the minimum-frequency modes ($n \approx 7$) but also the frequency of modes occurring at other values of $n$. It was found that the equation for equivalent liquid depth is applicable for these other values of $n$ as well. Use of the equivalent-depth equation indicates a value of $b_{eqv}/L$ for the untilted cylinder of 0.30 for $\theta = 15^{\circ}$ and 0.40 for $\theta = 30^{\circ}$. The frequencies of the untilted cylinder filled to these levels were found for each integer value of $n$ from cross plots of figure 3(a) and are indicated in figure 9 by the dashed curves. Although the fit is excellent, as it was for all combinations of fill level, tilt, and internal pressure, these equivalent-depth curves cannot be used to predict the discrete frequencies of the tilted cylinder as it is not known a priori at what values of $n_d$ (noninteger) the tilted cylinder resonates.

## Asymmetric Cylinder

Vibration modes of the asymmetric cylinder are identified in tables IV to VII for depth-length ratios of 0, 1/8, 1/5, 1/4, 1/3, 1/2, and 1, internal pressures of 0 and 5.516 N/cm$^2$, and tilt angles of $0^{\circ}$ and $15^{\circ}$. Modes for $m > 1$ were not recorded.

Modes of empty and full cylinder.- The first five circumferential modes for the empty, unpressurized cylinder are shown in figure 10(a) and are similar to the mode shapes of the tilted, partly filled symmetric cylinder (fig. 5). For the first mode, vibratory motion is present only on the side of the cylinder of minimum thickness. For the higher modes, vibratory motion of nearly uniform amplitude occurs around the entire circumference; however, the wave length of the motion is generally shorter on the thinner side of the cylinder. This may be noted in tables IV and VII by comparing the values of $n_1$ and $n_2$. The quantities $n_1$ and $n_2$ were derived by dividing the circumference of the cylinder by the wave lengths measured at $t_{max}$ and $t_{min}$, respectively.

The circumferential modes of the empty cylinder with an internal pressure of 5.516 N/cm$^2$ are given in figure 10(b) and display an anomalous behavior. The maximum response in the first mode occurs at $t_{max}$ while, for the higher modes, the maximum wave length occurs at $t_{min}$ ($n_1 > n_2$ in table V). For all other test conditions of the untilted asymmetric cylinder, the opposite is true.

Circumferential modes for the unpressurized liquid-filled cylinder are shown in figure 11(a). These modes resemble the empty-cylinder modes but occur at much lower frequencies. The number of circumferential waves is the same in the second and third modes but the wave motion of the third mode is symmetric about $t_{min}$ and antisymmetric for the second mode. The first five modes of the pressurized liquid-filled cylinder are given in figure 11(b).

Modes of partly filled cylinder.- Circumferential sweeps of the partly filled asymmetric cylinder were made with the displacement probe at nine equally spaced stations along the length of the cylinder. The resulting mode shapes for the untilted cylinder are shown in figure 12 for $p = 0$ and 5.516 N/cm$^2$ and $b/L = 1/5$ and 1/2. The maximum response in the first mode in all cases occurred circumferentially at $t_{min}$. Longitudinally the major response generally occurred below the liquid surface. For the pressurized cylinder, the decrease in response above the liquid surface became more abrupt with increasing frequency as typified by the fifth and sixth modes of figure 12(d).

The modes shown in figure 13 were obtained under conditions identical to those of figure 12 with the exception that the cylinder was tilted 15$^{\circ}$ toward the side of maximum thickness. The intersection of the liquid surface with the cylinder wall is denoted in the figures by the sinusoidal dashed line. In most cases the maximum response in the first mode now occurs on the side of the cylinder where the liquid is deepest rather than the thinner side. In the fifth and sixth modes of figure 13(a) ($p = 0$ and $\bar{b}/L = 1/5$), there appears to be a strong coupling of the response below the liquid surface on the deep side with the response above the liquid surface on the shallow side. It is likely that the strong response on the side of $t_{min}$ corresponds to the lower frequency modes of the empty cylinder (fig. 10(a)) as the resonant frequencies are comparable and the liquid depth is very shallow. The results obtained for $p = 5.516$ N/cm$^2$ and $\bar{b}/L = 1/5$ (fig. 13(c)) are somewhat different in that no response was obtained at $t_{min}$ throughout the first six modes.

Because of the complexity of the space shuttle vehicle and deficiencies in computational economy of available hydroelastic analysis methods, Coppolino has reformulated the NASTRAN hydroelastic analysis (ref. 1). The unusual character of the mode shapes of the asymmetric cylinder made it attractive as a test vehicle to verify the adequacy of his analysis. In figures 14(a) and 14(b) the first two analytical mode shapes are compared with test results for the half-full cylinder with internal pressures of 0. Similar results are shown in figure 15 for an internal pressure of 5.516 N/cm$^2$. The excellent correlation of analysis and test lends credence to the analysis and the unusual observed vibration modes.

## CONCLUSIONS

Experimental studies were undertaken to determine the effects of tilt and structural asymmetry on the vibration characteristics of thin-wall cylinders partly filled with liquid. Tests were made using axisymmetric and asymmetric cylinders with various levels of fill and tilt in both pressurized and unpressurized conditions.

Tests with a partly filled axisymmetric cylinder indicated that tilting the cylinder from the vertical resulted in the following conclusions:

1. Resonant frequencies were markedly reduced at the lower fill levels.

2. Mode shapes were unusual in that the vibratory motion of the minimum-frequency modes occurred only on that side of the cylinder where the liquid was deepest.

3. Vibratory motion extended progressively farther around the circumference as the frequency of the mode increased until vibratory motion of relatively uniform amplitude occurred around the entire circumference. For these higher frequency modes the wave length of the vibratory motion was generally shorter on the deep side of the cylinder.

4. Resonant frequencies for an internal pressure of 5.516 N/cm$^2$ were appreciably higher than those for no internal pressure but otherwise trends were similar.

An empirical equation was derived that gives the equivalent liquid depth of an untilted cylinder having the same minimum resonant frequency as a tilted, partly filled cylinder.

For the asymmetric cylinder tested, the following conclusions can be made:

1. Circumferential mode shapes of the untilted asymmetric cylinder were similar for all fill conditions to those of the tilted, partly filled axisymmetric cylinder.

2. In most instances vibratory motion in the first mode of the untilted, partly filled asymmetric cylinder was present only on the side of minimum thickness.

3. For the higher modes of the untilted cylinder, vibratory motion of nearly uniform amplitude was present around the entire circumference, the wave length being generally shorter on the thinner side of the cylinder.

Excellent correlation was obtained between test data and results from a reformulated NASTRAN hydroelastic analysis.

Langley Research Center
National Aeronautics and Space Administration
Hampton, VA 23665
May 4, 1978

## REFERENCE

1. Coppolino, Robert N.: Numerically Efficient Finite Element Hydroelastic Analysis. Volume 1: Theory and Results. NASA CR-2662, 1976.

TABLE I.- RESONANT FREQUENCIES OF UNTILTED AXISYMMETRIC CYLINDER

| m | b/L | f, Hz, for n of - | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| | | p = 0 N/cm$^2$ | | | | | | | | | | | | | | | | |
| 1 | 0 | | | 417 | 317 | 268 | 246 | 253 | 280 | 324 | 376 | 447 | | 598 | | | | |
| | 1/8 | | | 359 | 289 | 247 | 230 | 239 | 264 | 298 | 329 | | | | | | | |
| | 1/5 | | | 273 | 222 | 195 | 186 | 189 | 202 | 215 | 231 | 249 | 269 | 298 | | | | |
| | 1/4 | 385 | 294 | 226 | 186 | 163 | 154 | 157 | 167 | 179 | 194 | 213 | 236 | 264 | 296 | 332 | 372 | 416 |
| | 1/3 | | | 171 | 138 | 122 | 116 | 120 | 129 | 142 | 159 | 179 | 203 | 232 | 265 | 300 | 345 | |
| | 1/2 | | | 121 | 97.6 | 87.4 | 84.2 | 89.3 | 99.5 | 114 | 133 | 156 | 182 | 213 | 248 | 286 | 328 | 374 |
| | 2/3 | | 129 | 97.0 | 78.7 | 71.0 | 69.5 | 75.0 | 86.0 | 102 | 122 | 146 | 174 | 205 | 240 | | | |
| | 1 | | 107 | 83.0 | | 62.9 | | 69.5 | 80.2 | 96.6 | 117 | 142 | 169 | 201 | 236 | | | |
| 2 | 1/8 | | | | | | 414 | 382 | | | 398 | | | | | | | |
| | 1/4 | | | | | 404 | 349 | | 324 | 339 | 382 | 440 | | | | | | |
| | 1/3 | | | | | 390 | 339 | 313 | | 318 | 331 | | 356 | 372 | 395 | | | |
| | 1/2 | | | | | 292 | | 234 | 223 | 221 | 229 | 240 | 255 | 280 | 309 | 342 | 379 | |
| | 2/3 | | | | | | 189 | 172 | 165 | 167 | 177 | 194 | 217 | 242 | | | | |
| | 1 | | | | | 160 | 143 | 132 | 128 | 134 | 147 | 165 | 188 | 218 | | | | |
| | | p = 5.516 N/cm$^2$ | | | | | | | | | | | | | | | | |
| 1 | 0 | | | 447 | | 392 | 414 | 466 | 521 | 590 | | 760 | 833 | | | | | |
| | 1/8 | | | 384 | | | 365 | 384 | 406 | 423 | 442 | 464 | 491 | 521 | 556 | 594 | 638 | 690 |
| | 1/5 | | 363 | 293 | | 267 | 271 | 278 | 290 | 304 | 323 | 346 | 375 | 406 | 441 | | | 575 |
| | 1/4 | | | 244 | | 216 | 223 | 232 | 244 | 260 | 281 | 306 | 328 | 367 | 408 | 446 | 486 | |
| | 1/3 | | 226 | 181 | | 167 | 173 | 185 | 201 | 219 | 243 | 270 | 301 | 336 | 375 | 418 | 465 | 516 |
| | 1/2 | 227 | 160 | 129 | 119 | 122 | 133 | 148 | 167 | 190 | 216 | 246 | 280 | 317 | 358 | 401 | 449 | |
| | 2/3 | | 128 | 104 | 97.0 | 102 | 115 | 132 | 153 | 178 | 206 | 238 | 272 | 311 | 352 | 396 | 445 | 496 |
| | 1 | | 110 | 90.7 | 86.6 | 92.7 | 106 | 124 | 146 | 172 | 201 | 233 | 268 | 309 | 348 | 393 | 441 | 492 |

TABLE II.- RESONANT FREQUENCIES OF TILTED AXISYMMETRIC CYLINDER

| $\bar{b}/L$ | f, Hz | n | $n_d$ | $n_s$ |
|---|---|---|---|---|
| $\theta = 15^\circ$; $p = 0$ N/cm$^2$ | | | | |
| 1/5 | 132 | --- | 7.7 | ---- |
| | 151 | --- | 9.8 | ---- |
| | 171 | --- | 11.1 | ---- |
| | 189 | --- | 12.1 | ---- |
| | 213 | --- | 12.8 | ---- |
| | 231 | 10 | 13.5 | 5.8 |
| 1/4 | 115 | --- | 7.4 | ---- |
| | 132 | --- | 9.5 | ---- |
| | 149 | --- | 10.6 | ---- |
| | 164 | --- | 11.5 | ---- |
| | 184 | --- | 12.2 | ---- |
| | 202 | 10 | 12.9 | 5.7 |
| | 218 | 11 | 13.6 | 7.8 |
| | 240 | 12 | 14.3 | 9.1 |
| | 266 | 13 | 15.0 | 9.5 |
| | 295 | 14 | 15.9 | 9.4 |
| | 327 | 15 | 16.3 | 9.2 |
| 1/3 | 94.1 | --- | 7.3 | ---- |
| | 107 | --- | 8.7 | ---- |
| | 120 | --- | 9.7 | ---- |
| | 133 | --- | 10.8 | ---- |
| | 147 | --- | 11.3 | ---- |
| | 159 | --- | 11.8 | ---- |
| | 175 | 11 | 12.5 | 8.5 |
| | 184 | 11 | 13.0 | 6.3 |
| | 195 | 12 | 13.2 | 10.0 |
| | 216 | 13 | 13.9 | 11.4 |
| | 244 | 14 | 14.8 | 12.9 |
| 1/2 | 75.1 | --- | 6.9 | ---- |
| | 83.9 | --- | 8.3 | ---- |
| | 91.7 | 8 | 9.1 | 6.9 |
| | 101 | 9 | 9.8 | 9.1 |
| | 118 | 10 | 10.4 | 8.3 |
| | 136 | 11 | 11.5 | 10.3 |
| | 158 | 12 | 12.3 | 11.5 |
| | 185 | 13 | 13.2 | 12.8 |
| 2/3 | 66.6 | --- | 6.8 | ---- |
| | 71.9 | 7 | 7.9 | 5.6 |
| | 76.8 | 8 | 8.7 | 7.5 |
| | 81.8 | 5 | 4.4 | 6.1 |
| | 87.6 | 9 | 9.3 | 8.4 |
| | 98.6 | 4 | 3.4 | 4.8 |
| | 103 | 10 | 10.3 | 9.8 |
| | 123 | 11 | 11.3 | 10.8 |
| | 147 | 12 | 12.3 | 11.9 |
| | 174 | 13 | 13.2 | 12.9 |
| | 206 | 14 | 14.1 | 14.0 |
| | 237 | 15 | 15.0 | 15.0 |
| $\theta = 30^\circ$; $p = 0$ N/cm$^2$ | | | | |
| 1/5 | 97.0 | --- | 7.4 | ---- |
| | 118 | --- | 9.5 | ---- |
| | 140 | --- | 10.8 | ---- |
| | 163 | --- | 11.9 | ---- |
| | 190 | --- | 12.9 | ---- |
| | 221 | --- | 13.9 | ---- |
| 1/3 | 78.8 | --- | 7.3 | ---- |
| | 93.4 | --- | 9.1 | ---- |
| | 110 | --- | 10.1 | ---- |
| | 127 | --- | 10.7 | ---- |
| | 148 | --- | 11.7 | ---- |
| | 169 | --- | 12.6 | ---- |
| | 192 | --- | 13.2 | ---- |
| | 214 | --- | 14.2 | ---- |
| | 239 | 12 | 14.8 | 6.9 |
| | 260 | 13 | 15.2 | 8.3 |
| | 293 | 14 | 16.0 | 11.8 |
| | 324 | 15 | 16.5 | 9.7 |
| $\theta = 15^\circ$; $p = 5.516$ N/cm$^2$ | | | | |
| 1/5 | 185 | --- | 6.5 | ---- |
| | 208 | --- | 8.5 | ---- |
| | 232 | --- | 9.8 | ---- |
| | 257 | --- | 11.0 | ---- |
| | 283 | --- | 11.8 | ---- |
| | 308 | --- | 12.8 | ---- |
| 1/3 | 135 | --- | 5.8 | ---- |
| | 152 | --- | 7.6 | ---- |
| | 169 | --- | 8.5 | ---- |
| | 186 | --- | 9.6 | ---- |
| | 204 | --- | 10.3 | ---- |
| | 220 | 9 | 10.9 | 6.1 |
| $\theta = 30^\circ$; $p = 5.516$ N/cm$^2$ | | | | |
| 1/5 | 140 | --- | 5.9 | ---- |
| | 167 | --- | 8.2 | ---- |
| | 197 | --- | 10.0 | ---- |
| | 228 | --- | 11.2 | ---- |
| | 263 | --- | 12.4 | ---- |
| | 300 | --- | 13.5 | ---- |
| | 339 | --- | 14.4 | ---- |
| | 379 | --- | 15.3 | ---- |
| | 419 | 12 | 16.6 | 6.9 |
| | 457 | 13 | 16.9 | 7.8 |
| 1/3 | 114 | --- | 5.7 | ---- |
| | 135 | --- | 7.3 | ---- |
| | 158 | --- | 8.4 | ---- |
| | 182 | --- | 9.9 | ---- |
| | 208 | --- | 10.8 | ---- |
| | 235 | --- | 11.7 | ---- |
| | 263 | --- | 12.6 | ---- |
| | 296 | --- | 13.6 | ---- |
| | 330 | --- | 14.1 | ---- |
| | 358 | 12 | 14.9 | 6.0 |
| | 393 | 13 | 15.7 | 6.6 |
| | 469 | (a) | 17.5 | 7.9 |

[a]Undefinable due to irregularities in mode shape.

TABLE III.- MINIMUM RESONANT FREQUENCIES OF AXISYMMETRIC CYLINDER

| $\bar{b}/L$ | θ, deg | $b_{eqv}/L$ | f, Hz | $n_d$ | f, Hz | $n_d$ |
|---|---|---|---|---|---|---|
| | | | p = 0 N/cm$^2$ | | p = 5.516 N/cm$^2$ | |
| 1/5 | 0 | 0.200 | 186 | 7 | 267 | 6 |
| ↓ | 5 | .231 | 167 | 7.6 | 230 | 6.5 |
| ↓ | 10 | .262 | 147 | 7.6 | 204 | 6.5 |
| ↓ | 15 | .295 | 132 | 7.7 | 183 | 6.5 |
| ↓ | 20 | .329 | 117 | 7.6 | 165 | 6.5 |
| ↓ | 25 | .365 | 106 | 7.6 | 150 | 6.2 |
| ↓ | 30 | .404 | 97.2 | 7.7 | 138 | 6.1 |
| 1/4 | 0 | .250 | 154 | 7 | 216 | 6 |
| ↓ | 5 | .281 | 140 | 7.4 | 194 | 6.2 |
| ↓ | 10 | .312 | 126 | 7.5 | 176 | 6.2 |
| ↓ | 15 | .345 | 115 | 7.5 | 162 | 6.2 |
| ↓ | 20 | .379 | 104 | 7.7 | 148 | 6.1 |
| ↓ | 25 | .415 | 95.5 | 7.4 | 137 | 6.0 |
| ↓ | 30 | .454 | 88.7 | 7.3 | 127 | 5.9 |
| 1/3 | 0 | .333 | 116 | 7 | 167 | 6 |
| ↓ | 5 | .364 | 109 | 7.1 | 153 | 5.9 |
| ↓ | 10 | .395 | 101 | 7.4 | 143 | 5.9 |
| ↓ | 15 | .428 | 94.0 | 7.2 | 134 | 5.9 |
| ↓ | 20 | .462 | 87.8 | 7.2 | 127 | 5.9 |
| ↓ | 25 | .498 | 82.7 | 7.1 | 119 | 5.7 |
| ↓ | 30 | .537 | 78.1 | 7.1 | 112 | 5.5 |
| 1/2 | 0 | .500 | 84.2 | 7 | 119 | 5 |
| ↓ | 5 | .531 | 81.1 | 7.0 | 117 | 5.5 |
| ↓ | 10 | .562 | 78.8 | 7.1 | 113 | 5.5 |
| ↓ | 15 | .595 | 75.6 | 7.0 | 107 | 5.5 |
| ↓ | 20 | .629 | 73.0 | 6.9 | 104 | 5.5 |
| ↓ | 25 | .665 | 70.7 | 6.9 | 101 | 5.5 |
| ↓ | 30 | .704 | 68.8 | 6.9 | 98.0 | 5.5 |
| 2/3 | 0 | .667 | 69.5 | 7 | 97.0 | 5 |
| ↓ | 5 | .697 | 68.7 | 6.9 | 98.0 | 5.2 |
| ↓ | 10 | .729 | 67.9 | 6.8 | 96.0 | 5.2 |
| ↓ | 15 | .762 | 66.4 | 6.7 | 94.3 | 5.2 |
| ↓ | 20 | .796 | 66.0 | 6.7 | 93.0 | 5.2 |
| ↓ | 25 | .832 | 65.4 | 6.7 | 92.0 | 5.2 |
| ↓ | 30 | .871 | 64.9 | 6.7 | 91.0 | 5.2 |

TABLE IV.- RESONANT FREQUENCIES OF UNTILTED, UNPRESSURIZED ASYMMETRIC CYLINDER

| b/L | f, Hz | n | $n_1$ | $n_2$ |
|---|---|---|---|---|
| 0 | 220 | --- | ---- | 8.6 |
| | 242 | 8 | 6.2 | 12.5 |
| | 258 | 9 | (a) | (a) |
| | 298 | 10 | 7.9 | 11.4 |
| | 314 | 5 | 5.0 | 5.0 |
| | 344 | 11 | 9.1 | 12.3 |
| | 402 | 12 | 9.5 | 15.6 |
| | 469 | 13 | 10.4 | 15.2 |
| 1/8 | 203 | --- | ---- | 8.3 |
| | 224 | 8 | 6.4 | 11.6 |
| | 239 | 8 | 4.8 | 11.9 |
| | 249 | 9 | 6.7 | 12.5 |
| | 269 | 10 | 8.0 | 12.8 |
| | 282 | 5 | 5.0 | 5.0 |
| | 346 | 4 | 4.0 | 4.0 |
| | 419 | 12 | 9.6 | 14.3 |
| 1/5 | 152 | --- | ---- | 9.3 |
| | 165 | --- | ---- | 12.8 |
| | 173 | --- | ---- | 13.9 |
| | 190 | 10 | 6.7 | 16.1 |
| | 213 | 5 | 5.0 | 5.0 |
| | 218 | 12 | 8.8 | 16.7 |
| | 236 | 13 | 9.6 | 17.9 |
| | 314 | 16 | 12.8 | 20 |
| | 387 | 18 | 14.7 | 22.7 |

| b/L | f, Hz | n | $n_1$ | $n_2$ |
|---|---|---|---|---|
| 1/4 | 124 | --- | ---- | 8.9 |
| | 127 | --- | ---- | 9.5 |
| | 138 | --- | ---- | 12.5 |
| | 147 | --- | ---- | 12.5 |
| | 156 | 9 | 5.0 | 14.3 |
| | 163 | 10 | 6.7 | 15.0 |
| | 176 | 11 | 7.4 | 15.6 |
| | 190 | 12 | 9.1 | 16.7 |
| | 213 | 13 | 9.4 | 16.7 |
| | 213 | 4 | (a) | (a) |
| | 235 | 14 | 10.9 | 17.9 |
| | 259 | 15 | 11.9 | 20.0 |
| | 285 | 3 | (a) | (a) |
| | 320 | 17 | 13.5 | 21.7 |
| | 359 | 18 | 14.7 | 21.7 |
| 1/3 | 95 | --- | ---- | 8.3 |
| | 105 | --- | ---- | 11.6 |
| | 114 | --- | ---- | 12.2 |
| | 123 | 9 | 5.7 | 13.9 |
| | 130 | 10 | 6.9 | 13.9 |
| | 139 | 5 | (a) | (a) |
| | 145 | 11 | 7.5 | 14.3 |
| | 159 | 12 | 8.8 | 15.6 |
| | 164 | 4 | (a) | (a) |
| | 205 | 14 | 11.1 | 17.2 |
| | 233 | 15 | 11.4 | 17.9 |
| | 263 | 16 | 12.5 | 19.2 |
| | 297 | 17 | 12.5 | 21.7 |

| b/L | f, Hz | n | $n_1$ | $n_2$ |
|---|---|---|---|---|
| 1/2 | 66 | --- | ---- | 7.8 |
| | 74 | --- | ---- | 10.9 |
| | 82 | 8 | 5.3 | 10.9 |
| | 89 | 9 | 6.8 | 11.9 |
| | 94 | 5 | 4.2 | 6.5 |
| | 101 | 10 | 7.1 | 12.5 |
| | 117 | 11 | 8.3 | 13.2 |
| | 135 | 12 | 9.1 | 15.2 |
| | 148 | 3 | (a) | (a) |
| | 161 | 13 | 9.8 | 14.7 |
| | 184 | 14 | 10.9 | 16.7 |
| | 244 | 16 | 12.5 | 19.2 |
| | 283 | 17 | 13.2 | 20 |
| | 321 | 18 | 14.3 | 20.8 |
| 1 | 51 | --- | ---- | 7.6 |
| | 58 | 7 | 5.0 | 10.0 |
| | 64 | 8 | 6.7 | 10.4 |
| | 68 | 8 | 5.4 | 11.4 |
| | 74 | 9 | 7.1 | 10.9 |
| | 79 | 4 | (a) | (a) |
| | 88 | 10 | 7.9 | 12 |
| | 102 | 11 | 9.4 | 12.5 |
| | 179 | 14 | 11.6 | 16.1 |

[a]Undefinable due to irregularities in mode shape.

TABLE V.- RESONANT FREQUENCIES OF UNTILTED, PRESSURIZED ASYMMETRIC CYLINDER

[p = 5.516 N/cm$^2$]

| b/L | f, Hz | n | $n_1$ | $n_2$ |
|---|---|---|---|---|
| 0 | 381 | --- | 5.6 | ---- |
| | 399 | 6 | 7.9 | 4.8 |
| | 429 | 7 | 7.7 | 5.9 |
| | 448 | 4 | 3.8 | 5.0 |
| | 476 | 8 | 8.1 | 6.8 |
| | 555 | 9 | 9.4 | 7.9 |
| | 611 | 10 | 10.4 | 9.1 |
| | 678 | 11 | 11.1 | 10.0 |
| | 755 | 12 | 12.0 | 12.0 |
| | 843 | 13 | 13.0 | 13.0 |
| 1/8 | 343 | --- | 5.6 | ---- |
| | 353 | 6 | 7.4 | 5.2 |
| | 371 | 7 | 7.5 | 6.0 |
| | 381 | 8 | (a) | (a) |
| | 391 | 4 | 4.0 | 4.0 |
| | 402 | 9 | 9.0 | 9.0 |
| | 416 | 10 | 8.6 | 10.6 |
| | 435 | 11 | 9.3 | 12.5 |
| | 456 | 12 | 10.0 | 13.9 |
| | 480 | 13 | 10.9 | 14.3 |
| 1/5 | 249 | --- | ---- | 6.2 |
| | 257 | --- | ---- | 8.7 |
| | 261 | 7 | 5.9 | 9.3 |
| | 268 | 8 | 6.3 | 10.0 |
| | 278 | 9 | 7.5 | 10.6 |
| | 294 | 10 | 8.3 | 11.4 |
| | 311 | 11 | 10.0 | 12.5 |

| b/L | f, Hz | n | $n_1$ | $n_2$ |
|---|---|---|---|---|
| 1/5 | 334 | 12 | 10.0 | 13.3 |
| | 348 | 3 | (a) | (a) |
| | 389 | 14 | 11.9 | 15.4 |
| | 425 | 15 | 13.5 | 16.7 |
| | 458 | 16 | 13.9 | 18.2 |
| | 542 | 18 | 15.4 | 20.8 |
| 1/4 | 203 | --- | ---- | 6.5 |
| | 211 | --- | ---- | 8.5 |
| | 216 | 7 | 5.7 | 9.1 |
| | 225 | 8 | 6.6 | 9.5 |
| | 233 | 4 | (a) | (a) |
| | 236 | 9 | 8.2 | 10.2 |
| | 253 | 10 | 8.5 | 11.1 |
| | 272 | 11 | 9.4 | 12.5 |
| | 297 | 12 | 10.6 | 13.3 |
| | 323 | 13 | 11.6 | 14.3 |
| | 354 | 14 | 12.2 | 16.1 |
| | 389 | 15 | 13.2 | 16.7 |
| | 427 | 16 | 13.9 | 18.5 |
| 1/3 | 152 | --- | ---- | 5.7 |
| | 161 | 7 | 6.3 | 8.3 |
| | 176 | 4 | (a) | (a) |
| | 180 | 8 | 6.5 | 9.3 |
| | 194 | 9 | 8.0 | 10.0 |
| | 213 | 10 | 9.1 | 10.6 |
| | 236 | 11 | 9.6 | 11.9 |
| | 262 | 12 | 10.5 | 13.5 |

| b/L | f, Hz | n | $n_1$ | $n_2$ |
|---|---|---|---|---|
| 1/3 | 289 | 13 | 11.6 | 14.3 |
| | 324 | 14 | 12.5 | 16.7 |
| | 359 | 15 | 13.9 | 16.7 |
| | 444 | 16 | 14.7 | 18.5 |
| | 490 | 17 | 15.6 | 20.0 |
| | 538 | 18 | 16.1 | 21.7 |
| 1/2 | 110 | --- | ---- | 5.3 |
| | 118 | 6 | 5.7 | 7.5 |
| | 130 | 7 | 6.1 | 7.7 |
| | 145 | 8 | 7.4 | 8.6 |
| | 164 | 9 | 8.3 | 9.8 |
| | 186 | 10 | 9.1 | 10.9 |
| | 210 | 11 | 10.0 | 11.6 |
| | 239 | 12 | 10.9 | 13.5 |
| | 344 | 15 | 13.3 | 16.7 |
| 1 | 82 | --- | ---- | 4.9 |
| | 90 | 5 | 3.4 | 6.9 |
| | 104 | 7 | 6.3 | 7.2 |
| | 109 | 3 | 3.2 | 2.6 |
| | 123 | 8 | 7.1 | 8.3 |
| | 145 | 9 | 8.3 | 9.2 |
| | 170 | 10 | 8.8 | 10.2 |
| | 379 | 16 | 13.5 | 17.9 |

[a]Undefinable due to irregularities in mode shape.

TABLE VI.- RESONANT FREQUENCIES OF UNPRESSURIZED ASYMMETRIC

CYLINDER TILTED 15°

| $\bar{b}/L$ | f, Hz | n | $n_1$ | $n_2$ |
|---|---|---|---|---|
| 1/8 | 188 | --- | 7.4 | ---- |
| | 209 | 8 | 8.0 | 8.0 |
| | 243 | 10 | 10.0 | 10.0 |
| | 262 | 5 | (a) | (a) |
| | 300 | 12 | 13.2 | 11.7 |
| | 327 | 4 | (a) | (a) |
| | 374 | 14 | 16.1 | 11.6 |
| 1/5 | 146 | --- | 7.1 | ---- |
| | 163 | --- | 9.0 | ---- |
| | 180 | --- | 10.4 | ---- |
| | 197 | 9 | 10.9 | 7.4 |
| | 209 | 10 | 11.4 | 8.6 |
| | 250 | 12 | 12.5 | 10.5 |
| | 277 | 13 | 13.5 | 12.5 |
| | 305 | 14 | 14.3 | 13.5 |
| 1/4 | 130 | --- | 6.7 | ---- |
| | 143 | --- | 9.4 | ---- |
| | 158 | --- | 8.9 | ---- |
| | 171 | 9 | 11.1 | 6.8 |
| | 184 | 10 | 11.1 | 8.9 |
| | 203 | 11 | 11.6 | 8.8 |
| | 222 | 12 | 12.3 | 10.8 |
| | 244 | 13 | 13.0 | 13.0 |
| | 265 | 14 | 13.2 | 13.2 |
| | 324 | 16 | 12.8 | 15.6 |
| | 352 | 17 | 14.7 | 17.9 |
| | 389 | 18 | 14.7 | 20.8 |
| 1/3 | 101 | --- | 5.9 | ---- |
| | 113 | 8 | 8.0 | 8.0 |
| | 129 | 9 | 8.9 | 7.9 |
| | 137 | 10 | 10.0 | 10.0 |
| | 149 | 11 | 10.0 | 11.4 |
| | 163 | 12 | 10.5 | 13.9 |
| | 185 | 13 | 11.1 | 15.0 |
| | 206 | 14 | 11.6 | 15.9 |
| | 222 | 3 | (a) | (a) |
| | 238 | 15 | 12.5 | 17.9 |
| | 266 | 16 | 13.9 | 18.5 |
| 1/2 | 74 | 7 | 6.3 | 8.0 |
| | 83 | 8 | 6.7 | 9.6 |
| | 90 | 9 | 7.6 | 10.2 |
| | 96 | 5 | (a) | (a) |
| | 102 | 10 | 8.3 | 11.8 |
| | 115 | 4 | (a) | (a) |
| | 117 | 11 | 9.3 | 12.8 |
| | 137 | 12 | 10.0 | 14.7 |
| | 151 | 3 | (a) | (a) |
| | 162 | 13 | 10.0 | 15.4 |
| | 185 | 14 | 11.6 | 16.7 |
| | 246 | 16 | 12.5 | 18.5 |

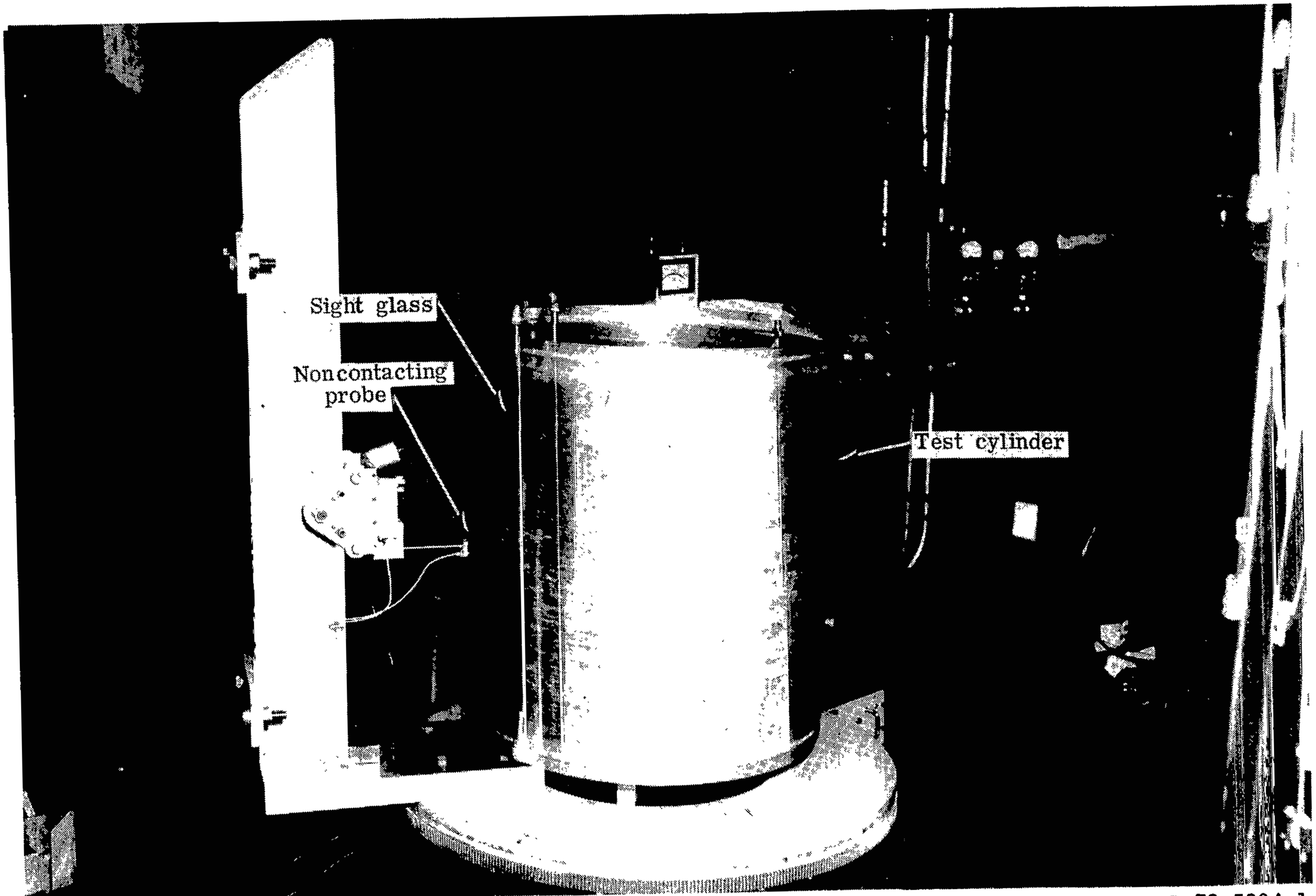
[a]Undefinable due to irregularities in mode shape.

## TABLE VII.- RESONANT FREQUENCIES OF PRESSURIZED ASYMMETRIC CYLINDER TILTED 15°

$\left[p = 5.516 \text{ N/cm}^2\right]$

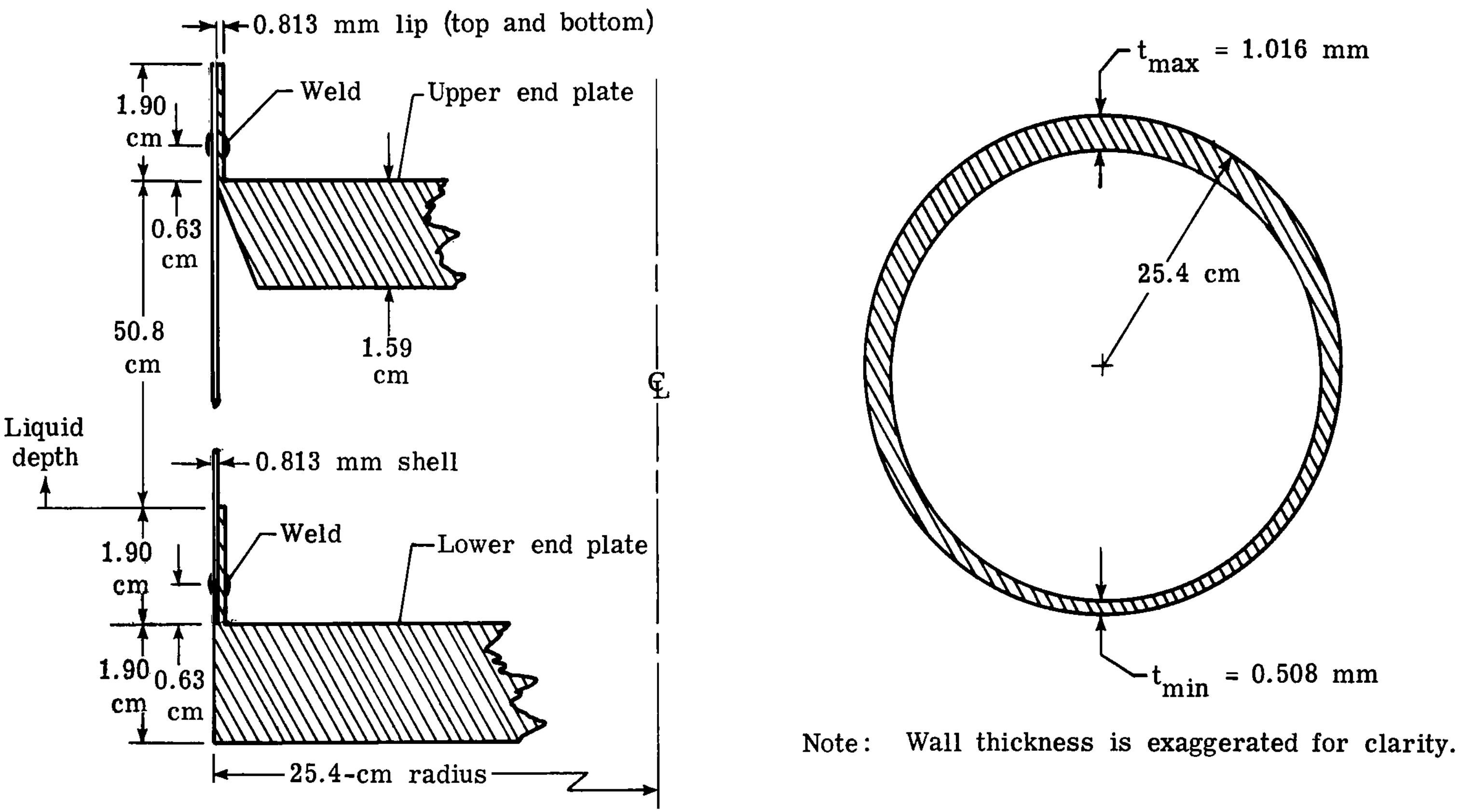
| $\bar{b}/L$ | f, Hz | n | $n_1$ | $n_2$ |
|---|---|---|---|---|
| 1/8 | 275 | --- | 10.0 | ---- |
| | 304 | --- | 10.6 | ---- |
| | 342 | --- | 11.1 | ---- |
| | 373 | --- | 12.5 | ---- |
| | 402 | 9 | 13.5 | 4.8 |
| | 427 | 10 | 14.3 | 5.7 |
| | 452 | 11 | 14.3 | 6.0 |
| 1/5 | 189 | --- | 6.3 | ---- |
| | 214 | --- | 8.5 | ---- |
| | 240 | --- | 9.4 | ---- |
| | 263 | --- | 10.4 | ---- |
| | 285 | --- | 11.4 | ---- |
| | 293 | --- | 11.6 | ---- |
| | 315 | --- | 12.2 | ---- |
| | 321 | --- | 12.5 | ---- |
| | 344 | --- | 12.8 | ---- |
| | 373 | --- | 13.6 | ---- |
| | 392 | 11 | 14.5 | ---- |
| | 417 | 12 | 14.8 | 5.1 |
| | 443 | 13 | 15.4 | 6.5 |
| | 473 | 14 | 15.9 | 7.7 |
| 1/4 | 167 | --- | 6.2 | ---- |
| | 190 | --- | 8.3 | ---- |
| | 210 | --- | 10.2 | ---- |
| | 228 | --- | 10.0 | ---- |
| | 231 | --- | 10.0 | ---- |
| | 248 | --- | 10.6 | ---- |
| | 272 | --- | 11.1 | ---- |
| | 295 | --- | 11.9 | ---- |
| | 312 | --- | 12.5 | ---- |
| | 330 | --- | 13.3 | ---- |
| | 345 | --- | 13.9 | ---- |
| | 388 | 13 | 15.0 | 11.4 |
| | 419 | 14 | 15.0 | 12.5 |
| | 455 | 15 | 16.7 | 14.3 |
| | 492 | 16 | 17.2 | 15.6 |
| 1/3 | 144 | --- | 5.7 | ---- |
| | 160 | --- | 8.3 | ---- |
| | 174 | --- | 8.8 | ---- |
| | 191 | --- | 9.3 | ---- |
| | 194 | --- | 9.3 | ---- |
| | 207 | --- | 10.2 | ---- |
| | 224 | 9 | 10.5 | 6.3 |
| | 236 | 10 | 10.9 | 7.9 |
| | 256 | 11 | 11.5 | 9.4 |
| | 278 | 12 | 12.0 | 11.4 |
| | 307 | 13 | 13.0 | 13.0 |
| | 339 | 14 | 14.0 | 14.0 |
| | 372 | 15 | 13.9 | 15.6 |
| | 409 | 16 | 14.3 | 16.7 |
| | 459 | 17 | 14.7 | 17.9 |
| 1/2 | 112 | --- | 5.0 | ---- |
| | 122 | 6 | 7.1 | 5.2 |
| | 130 | 6 | 7.7 | 4.0 |
| | 136 | 7 | 7.9 | 5.7 |
| | 149 | 8 | 8.3 | 7.5 |
| | 160 | 3 | (a) | (a) |
| | 168 | 9 | 9.0 | 9.0 |
| | 188 | 10 | 10.0 | 10.0 |
| | 214 | 11 | 10.5 | 11.1 |
| | 241 | 12 | 11.6 | 12.5 |
| | 273 | 13 | 13.0 | 13.0 |
| | 485 | 18 | 16.0 | 19.2 |

[a]Undefinable due to irregularities in mode shape.




L-73-5394.1

Figure 1.- Photograph of test apparatus.

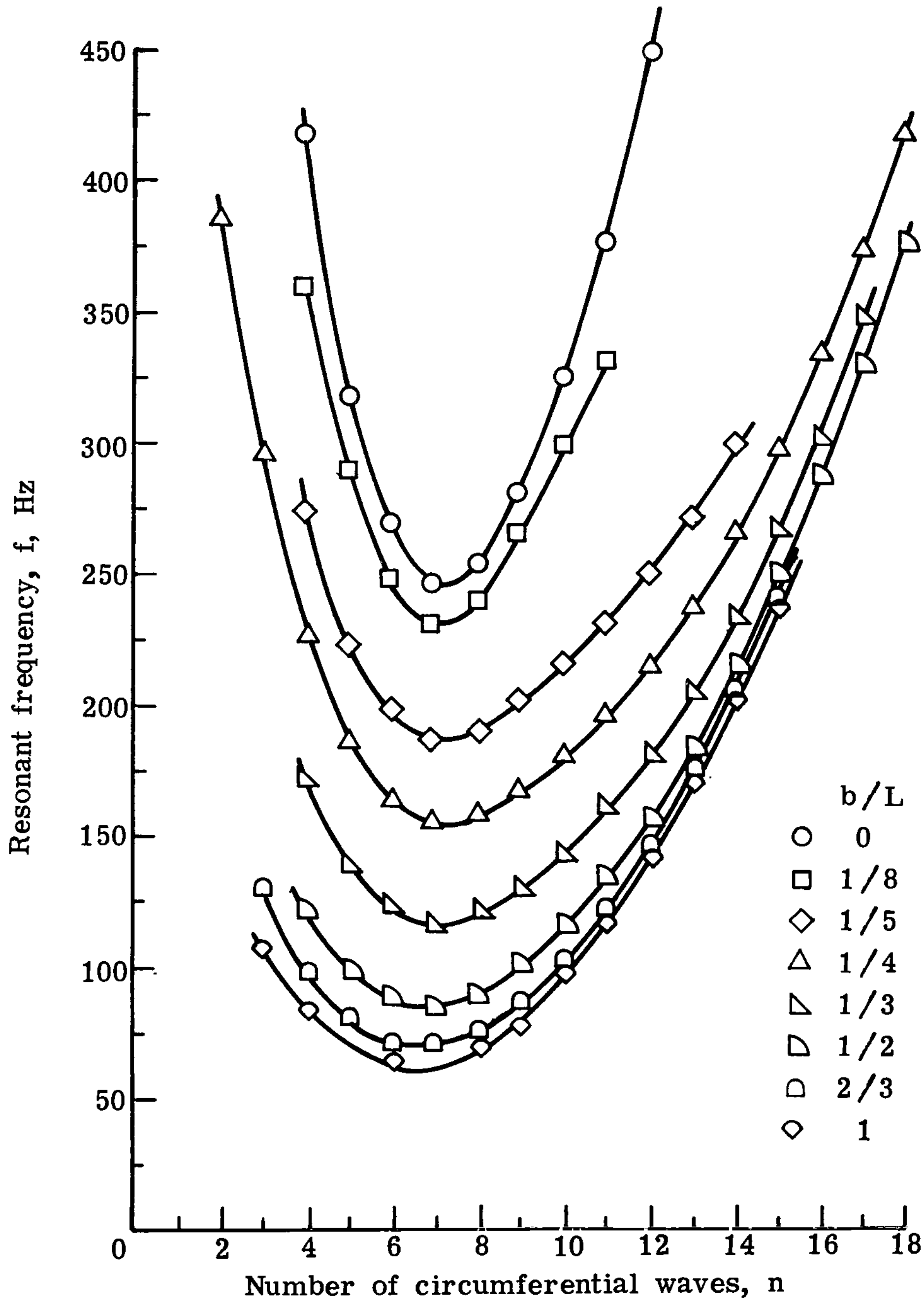



Note: Wall thickness is exaggerated for clarity.

(a) Axisymmetric cylinder.

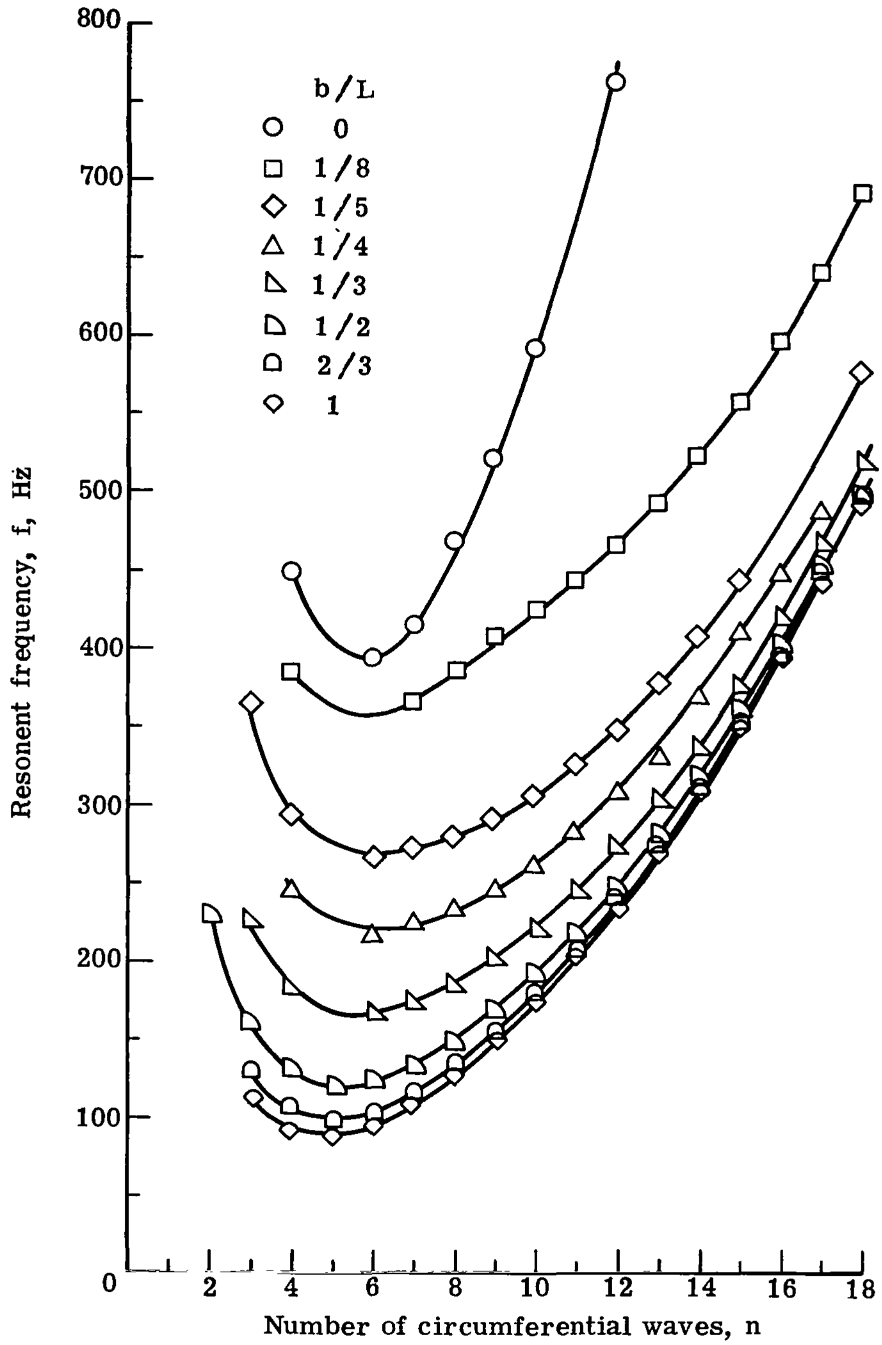
(b) Cross section of asymmetric cylinder.

Figure 2.- Construction details of test cylinders. Material: 6061 aluminum.



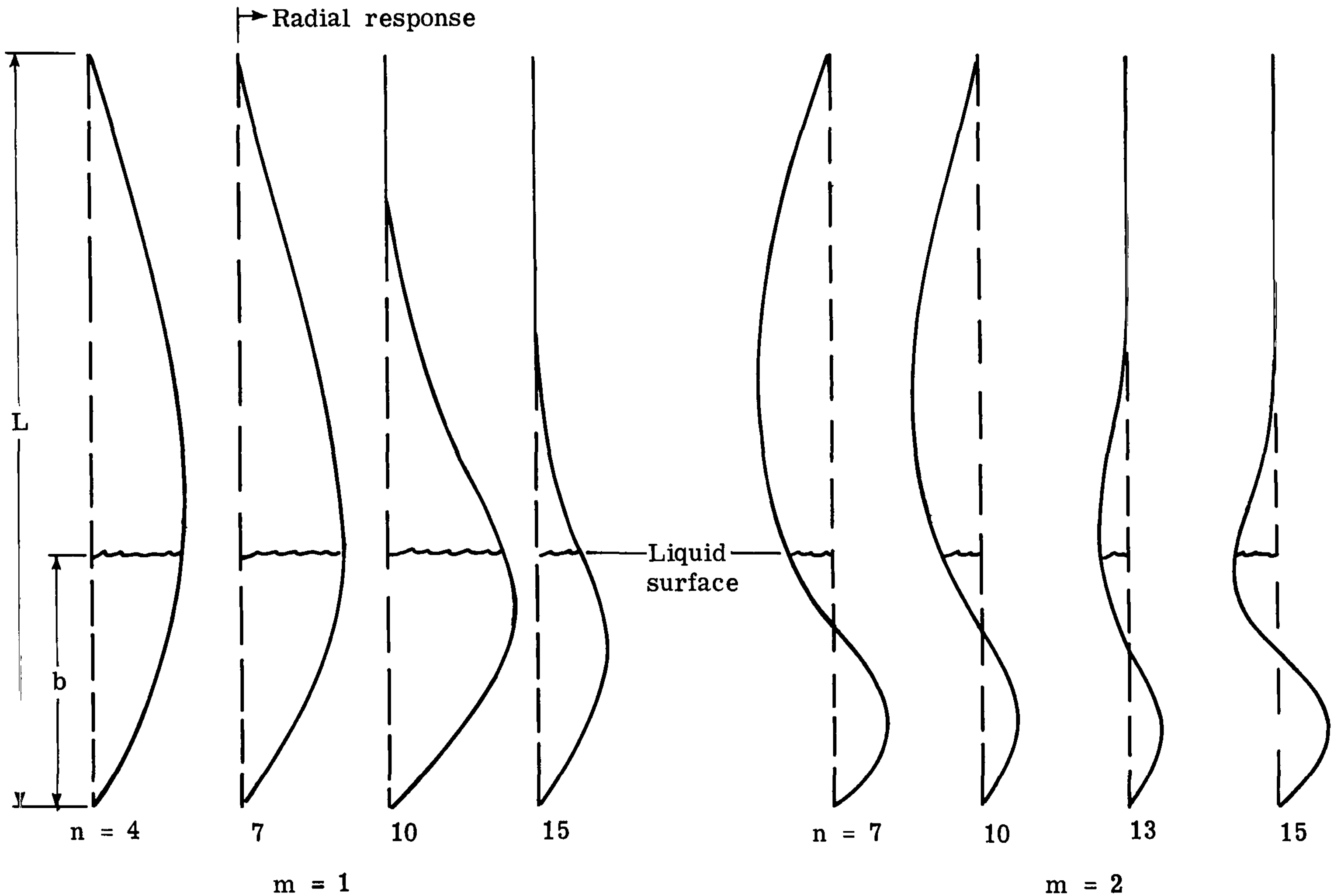

(a) p = 0.

Figure 3.- Resonant frequencies of untilted axisymmetric cylinder as a function of the number of circumferential waves for several liquid depths. m = 1.



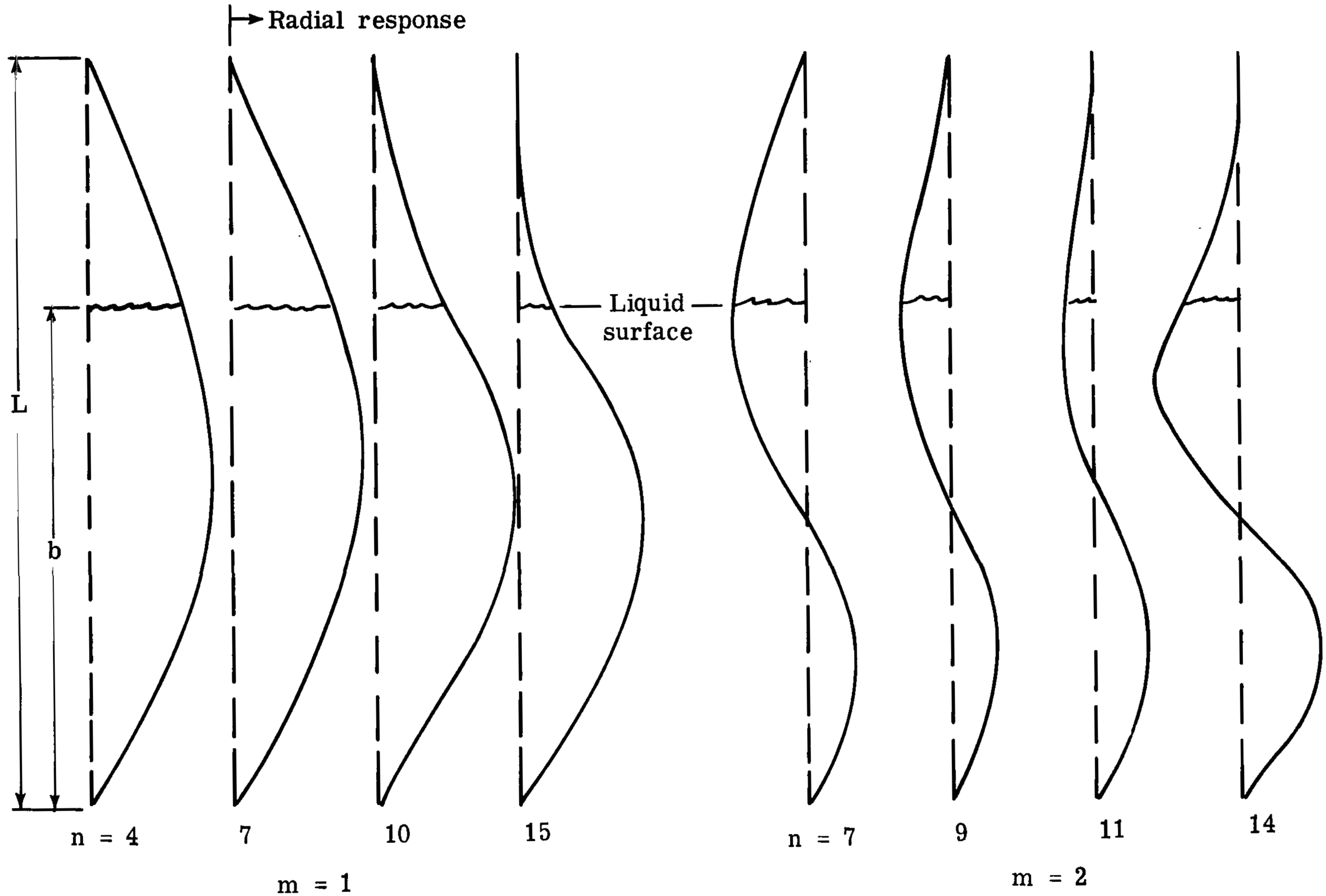

(b) p = 5.516 N/cm$^2$.

Figure 3.- Concluded.




(a) $b/L = 1/3$.

Figure 4.- Effect of circumferential wave number $n$ on the longitudinal mode shape of axisymmetric cylinder. $\theta = 0^{\circ}$; $p = 0$.




(b) b/L = 2/3.

Figure 4.- Concluded.

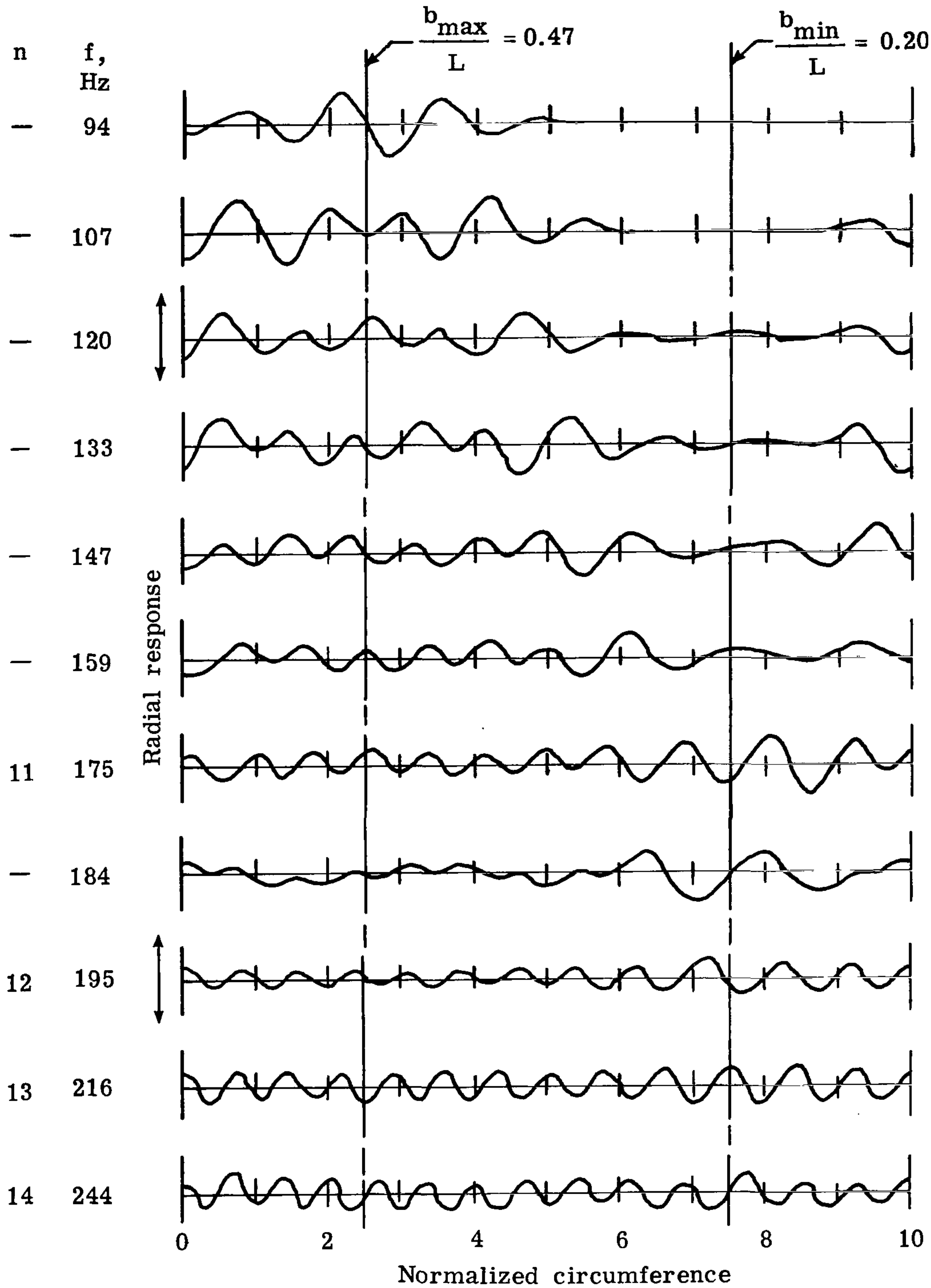
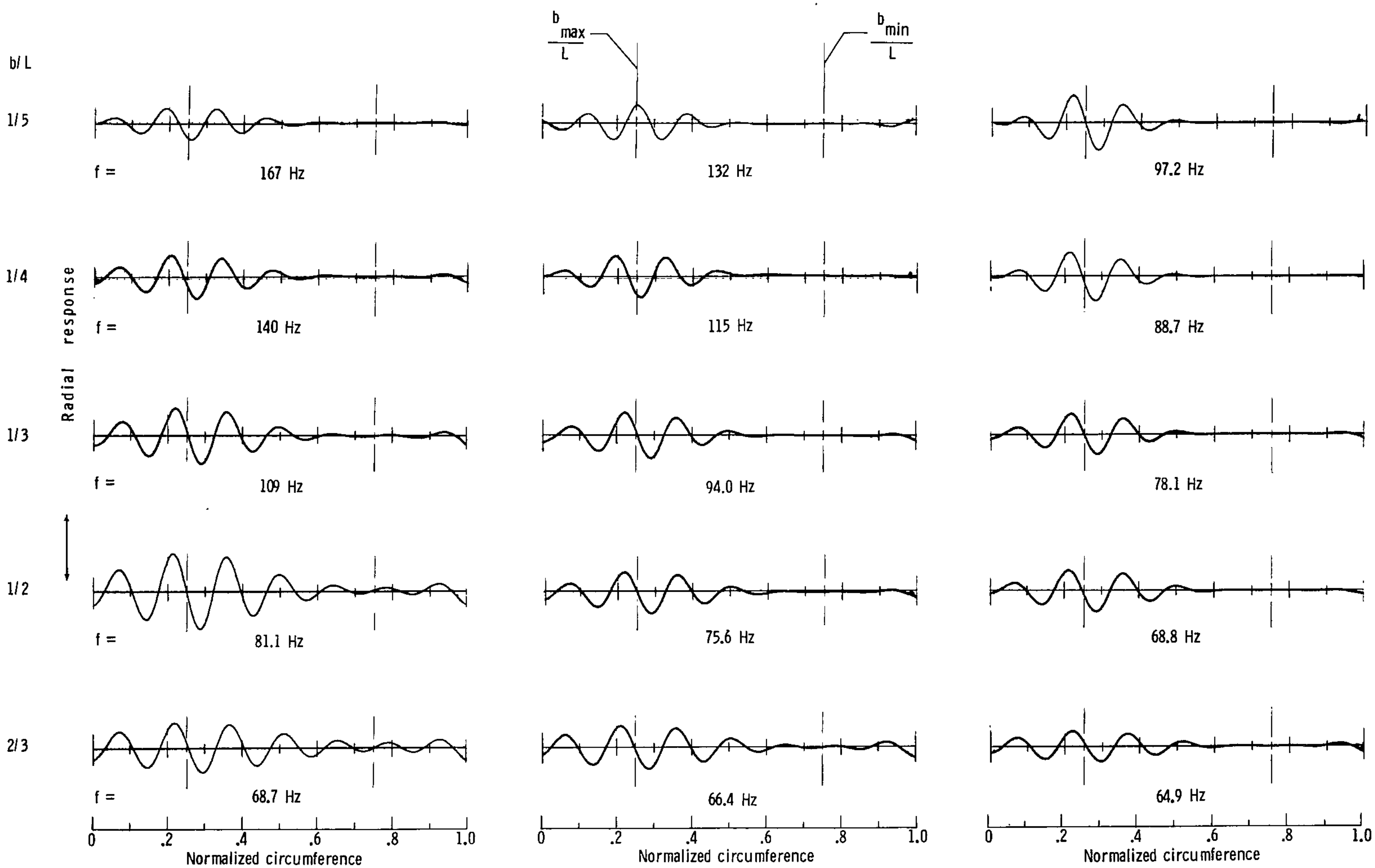



Figure 5.- Typical circumferential mode shapes of tilted, partly filled axisymmetric cylinder. $\bar{b}/L = 1/3$; $\theta = 15^{\circ}$; $p = 0$.




(a) $\theta = 5^{\circ}$.  (b) $\theta = 15^{\circ}$.  (c) $\theta = 30^{\circ}$.

Figure 6.- Comparison of minimum-frequency circumferential mode shapes of axisymmetric cylinder for several depths of liquid and angles of tilt. $p = 0$; $m = 1$.

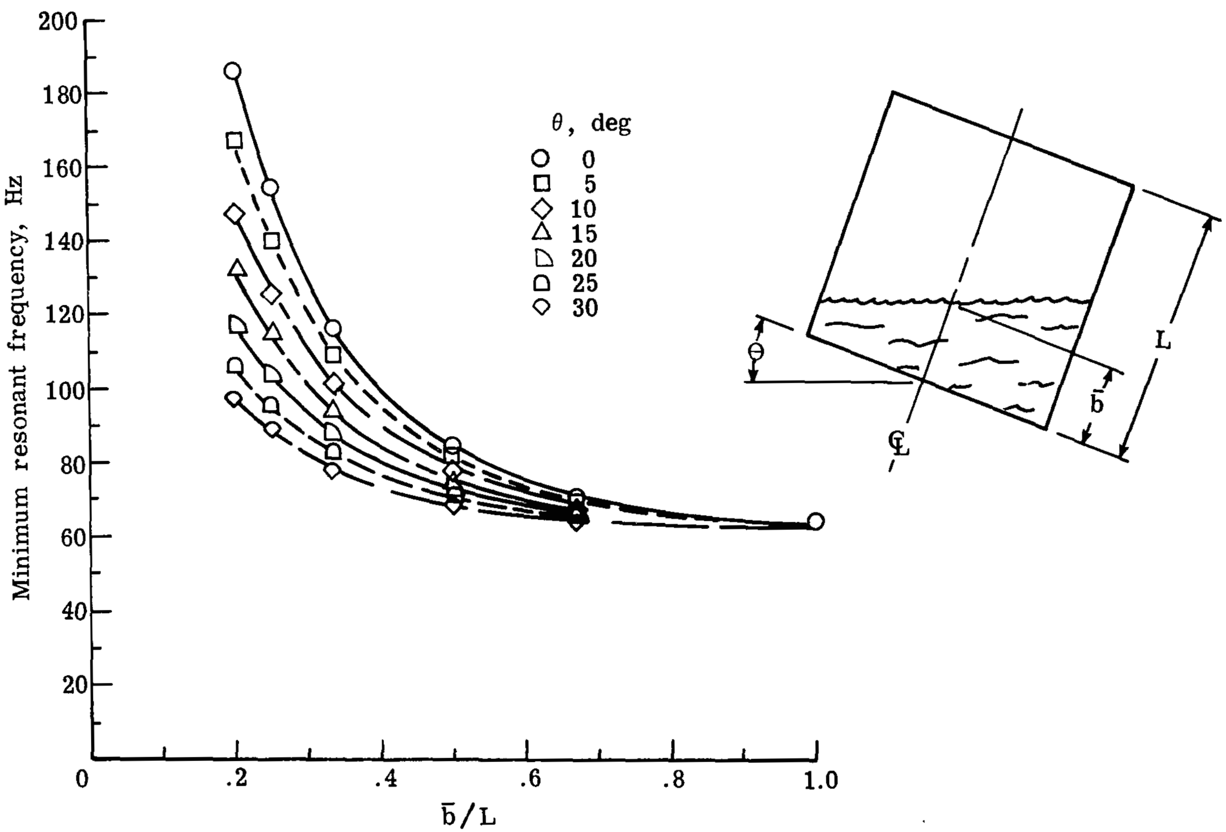



Figure 7.- Minimum resonant frequency of axisymmetric cylinder as a function of average liquid depth for various angles of tilt. $p = 0$.

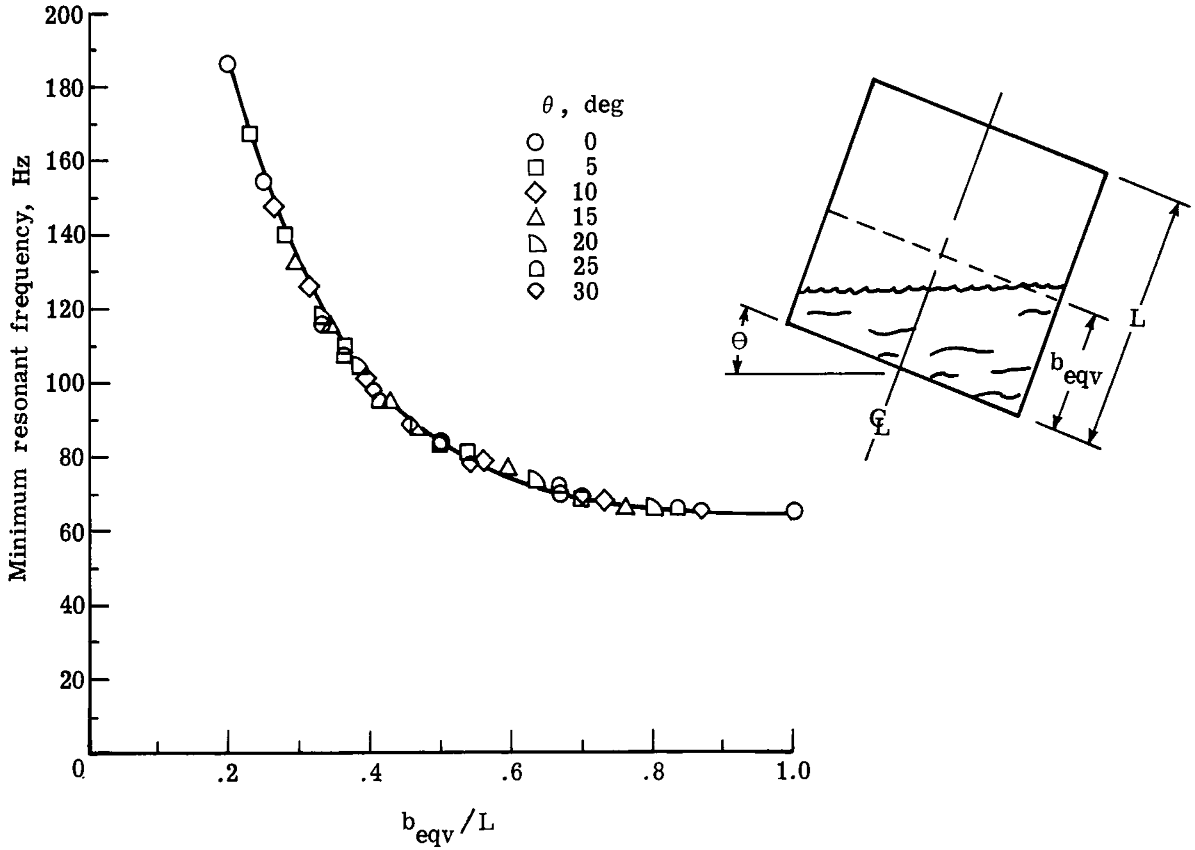



Figure 8.- Minimum resonant frequency of axisymmetric cylinder as a function of equivalent liquid depth for various angles of tilt. $k = 0.71$; $p = 0$.

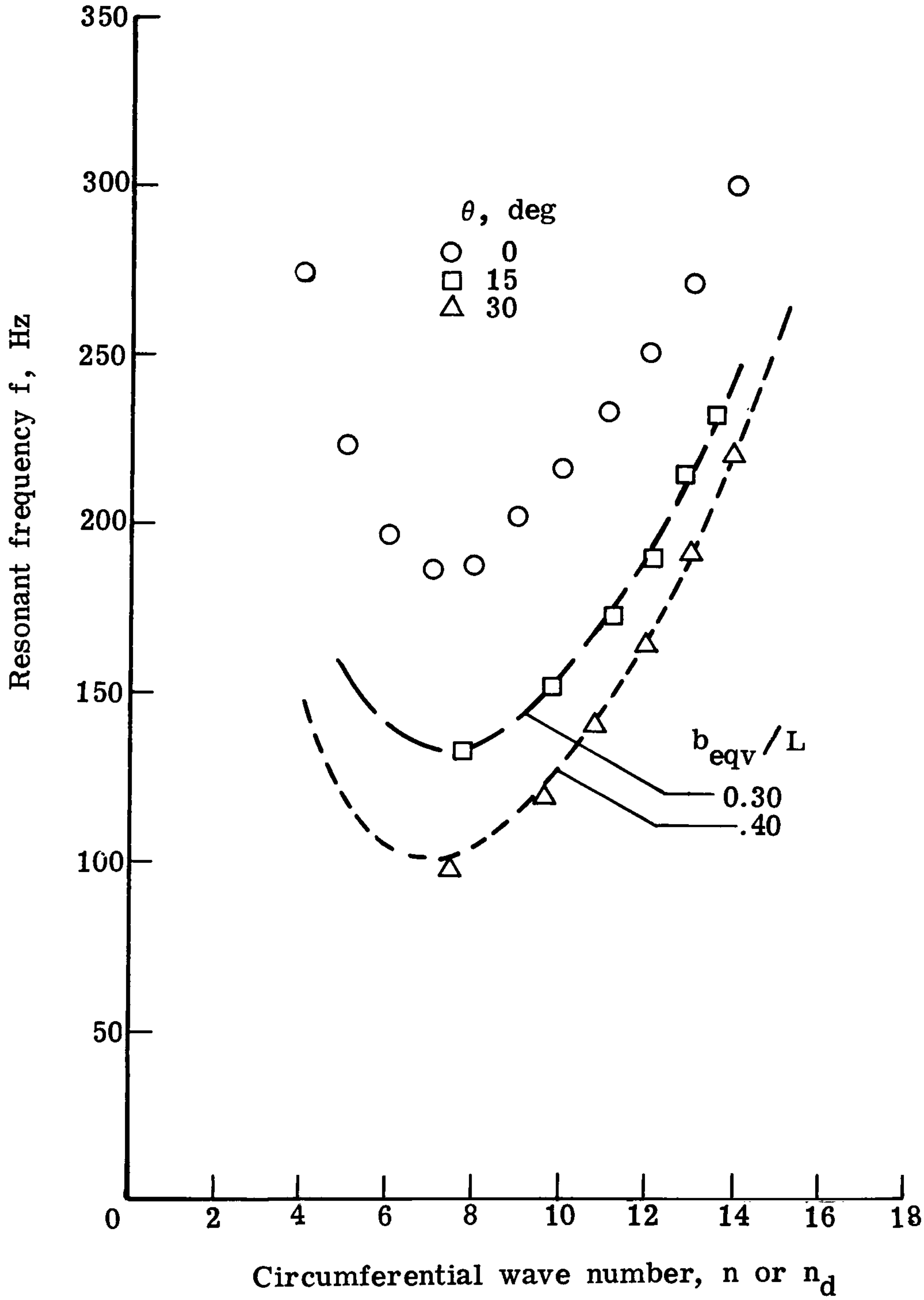
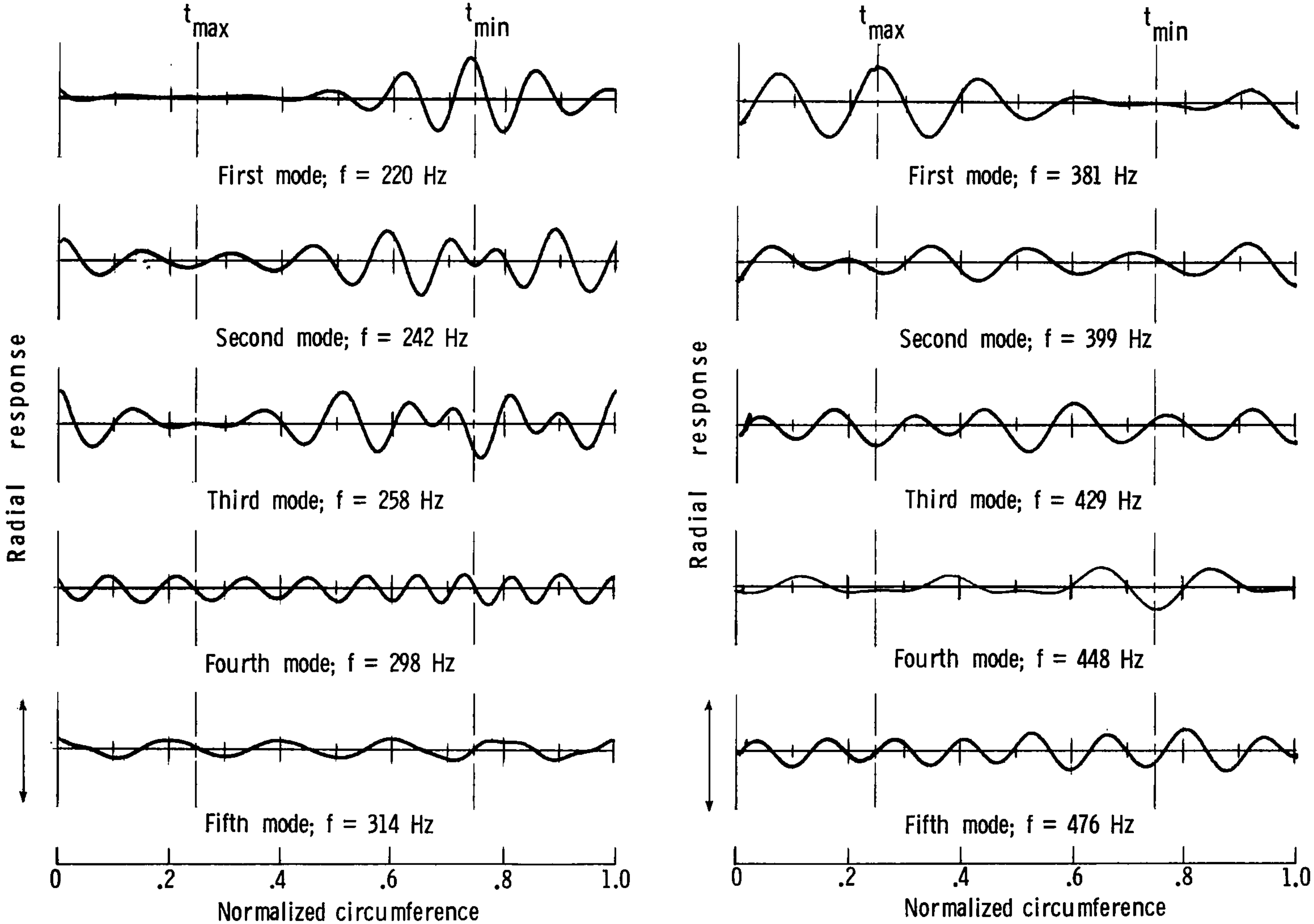


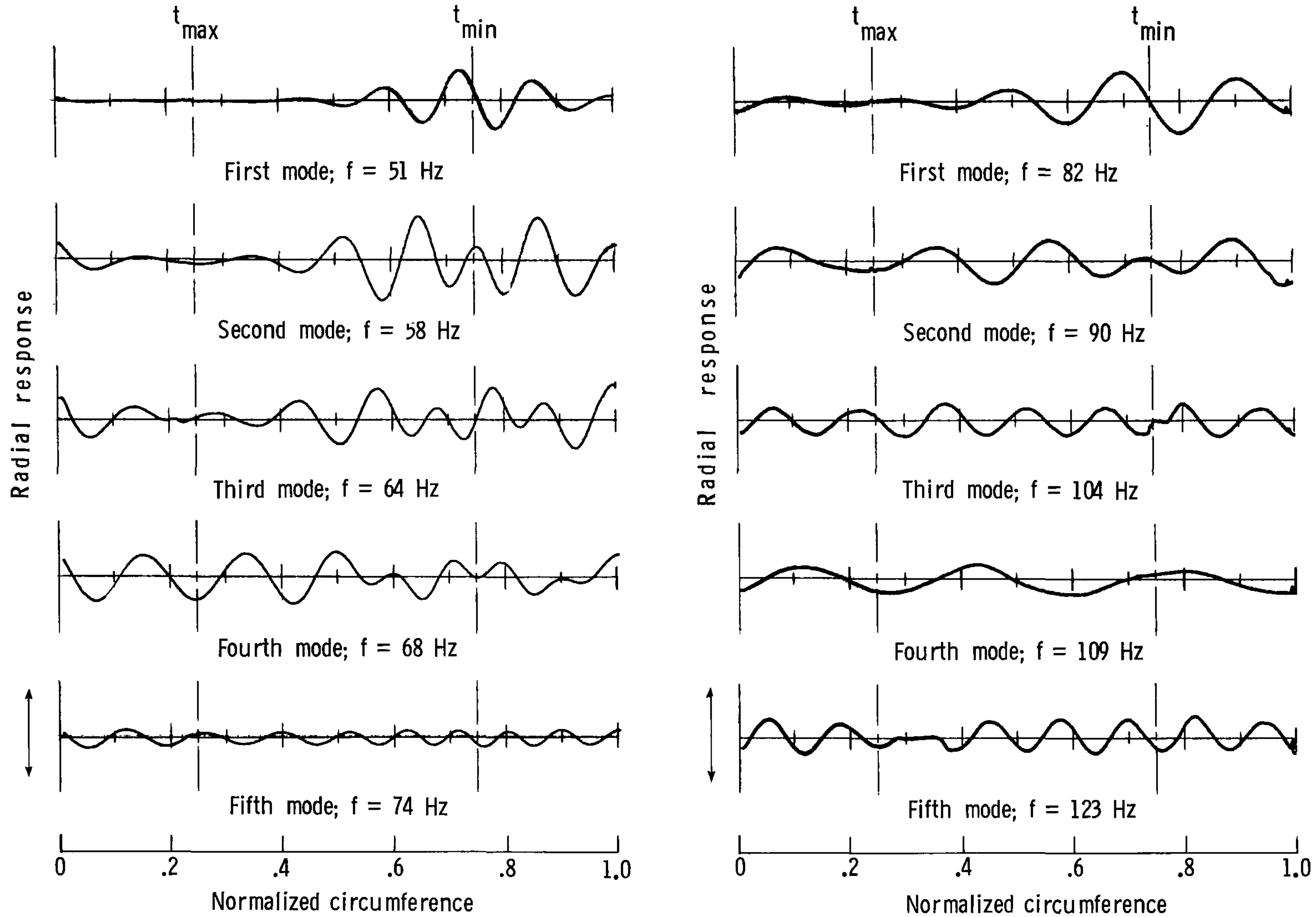

Figure 9.- Variation of resonant frequency of partly filled axisymmetric cylinders with circumferential wave number. $\bar{b}/L = 1/5$; $p = 0$.




(a) p = 0.

(b) p = 5.516 N/cm$^2$.

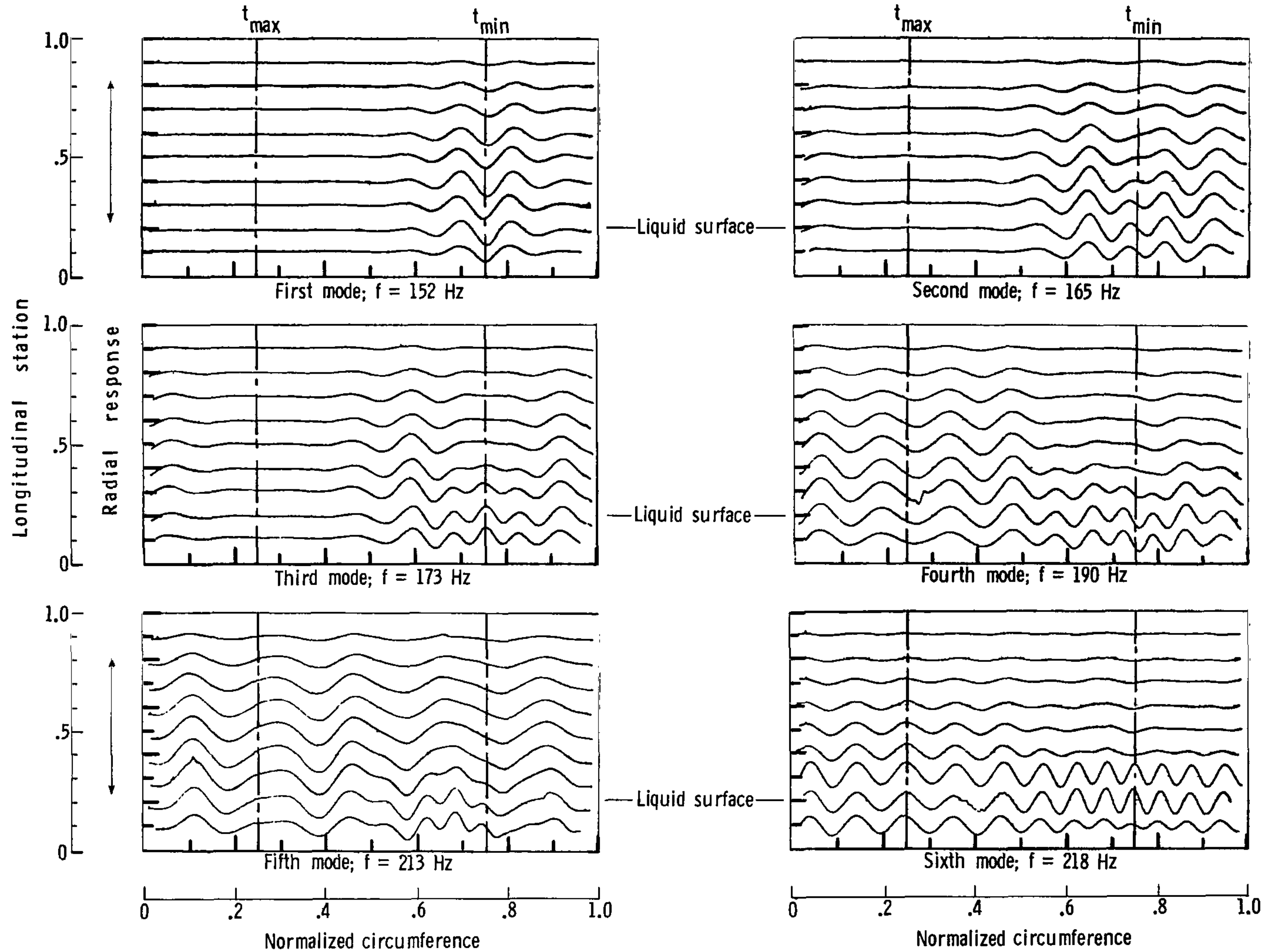
Figure 10.- Circumferential mode shapes of empty asymmetric cylinders. $\theta = 0^{\circ}$.




(a) p = 0.

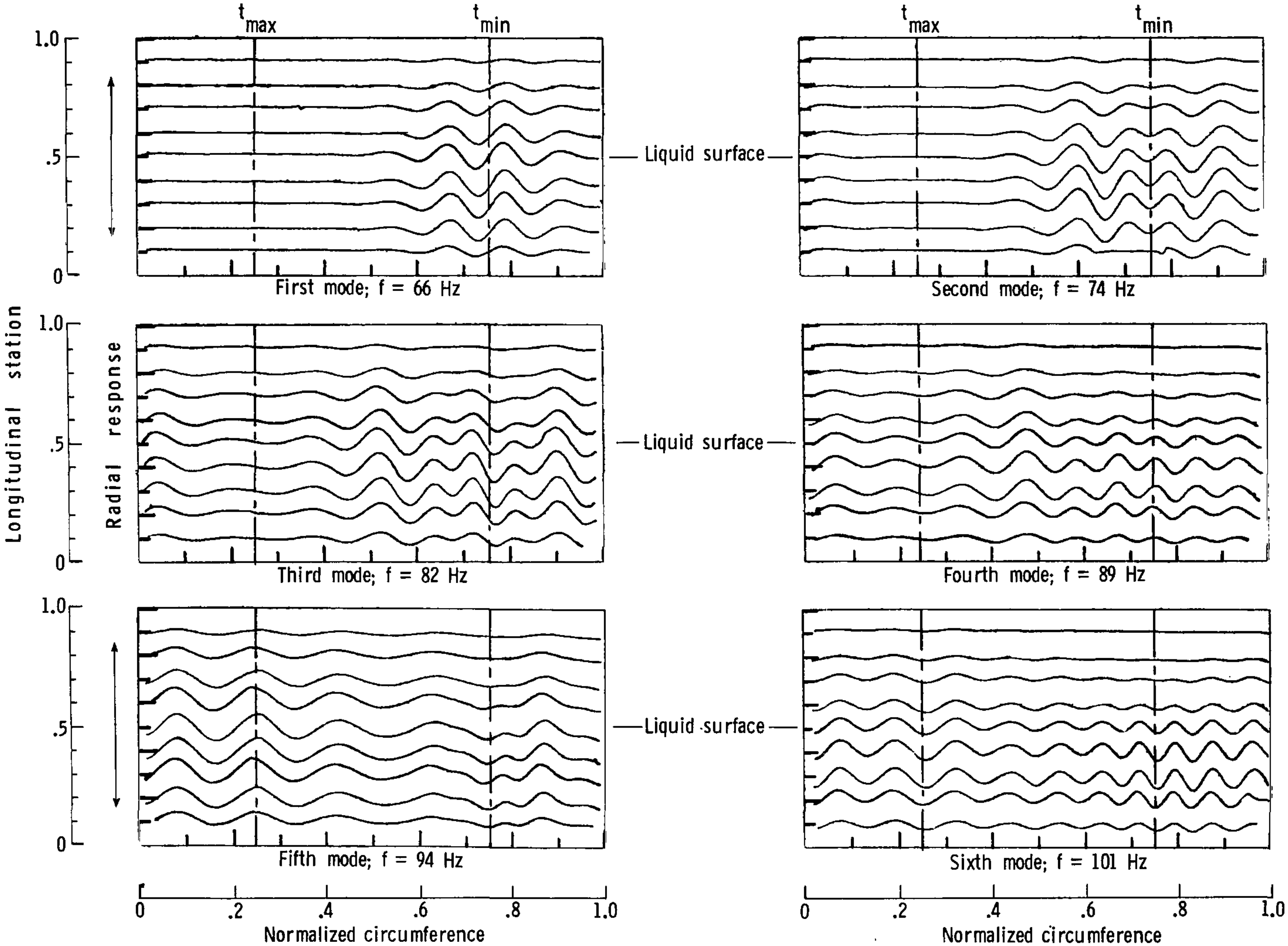
(b) p = 5.516 N/cm$^2$.

Figure 11.- Circumferential mode shapes of liquid-filled asymmetric cylinder. $\theta = 0^{\circ}$.



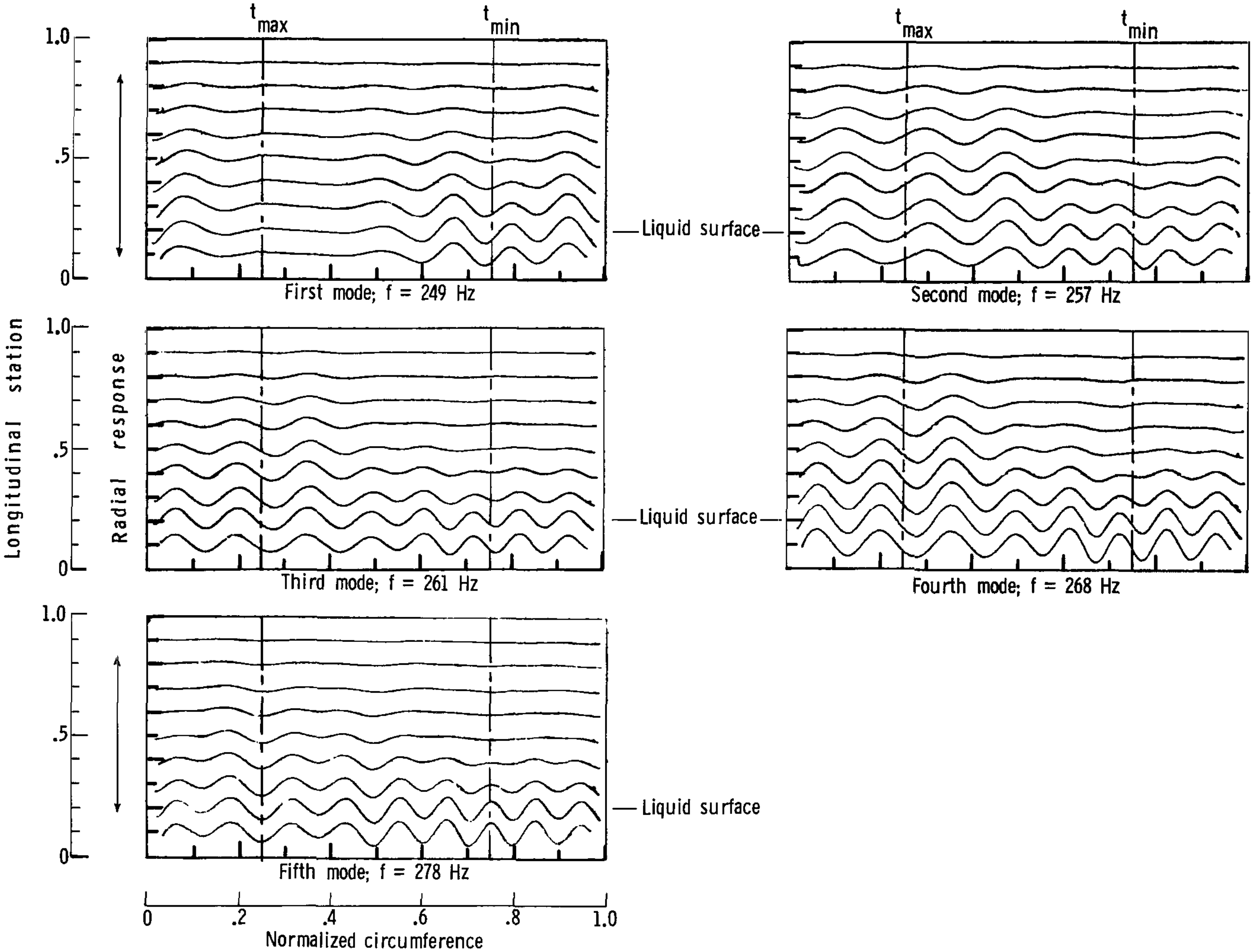

(a) p = 0; b/L = 1/5.

Figure 12.- Mode shapes of untilted, partly filled asymmetric cylinder at several longitudinal stations.



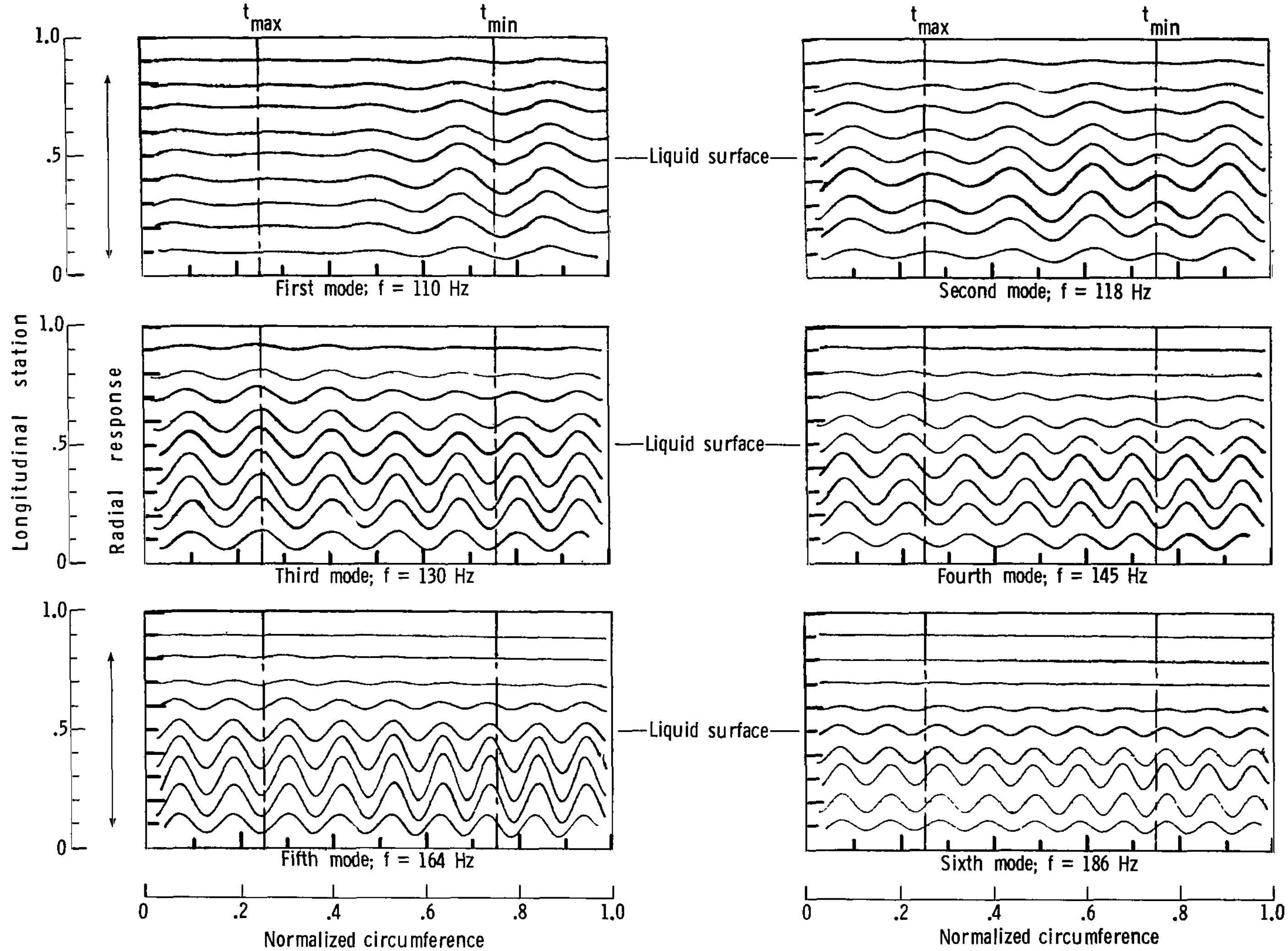

(b) p = 0; b/L = 1/2.

Figure 12.- Continued.



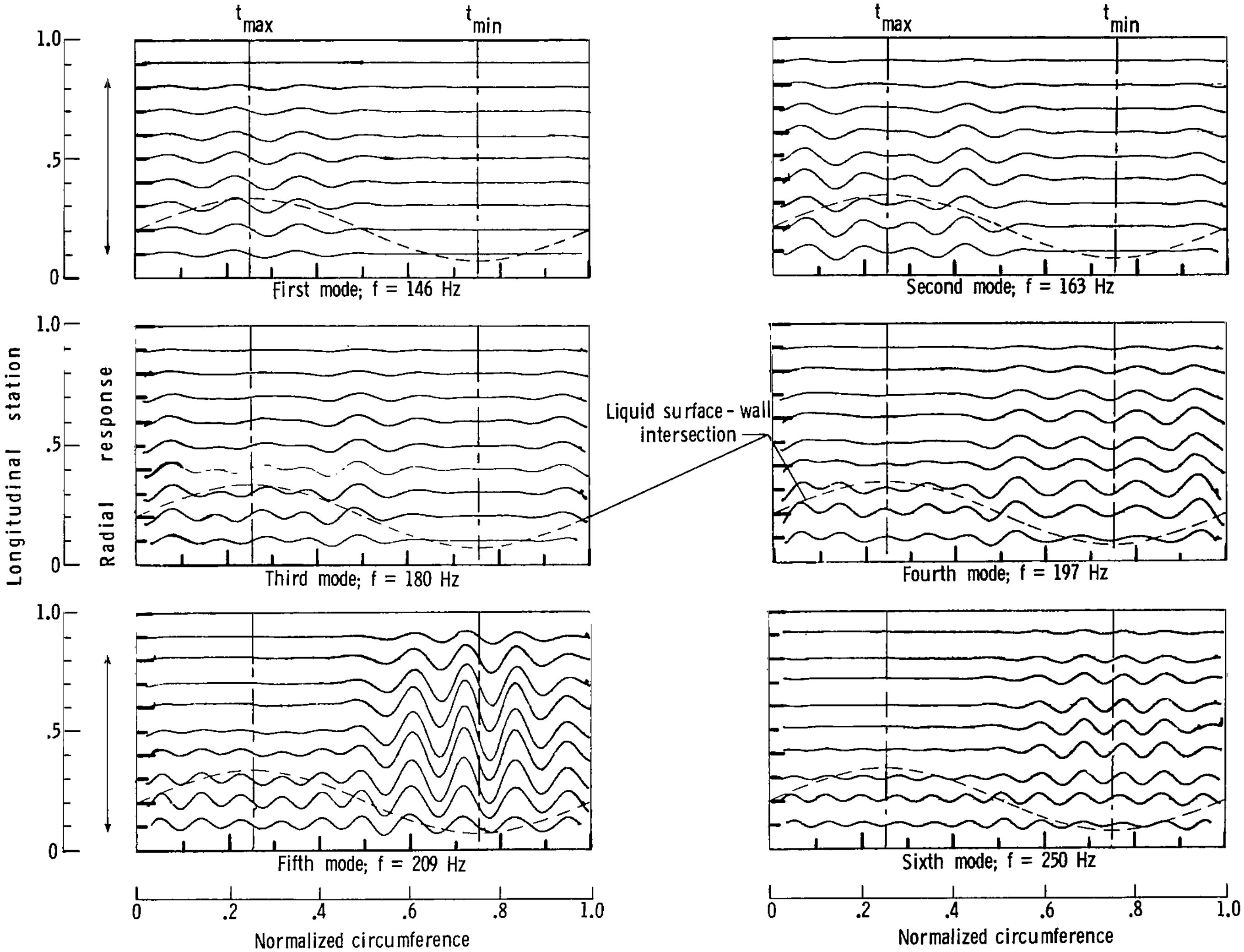

(c) p = 5.516 N/cm$^2$; b/L = 1/5.

Figure 12.- Continued.



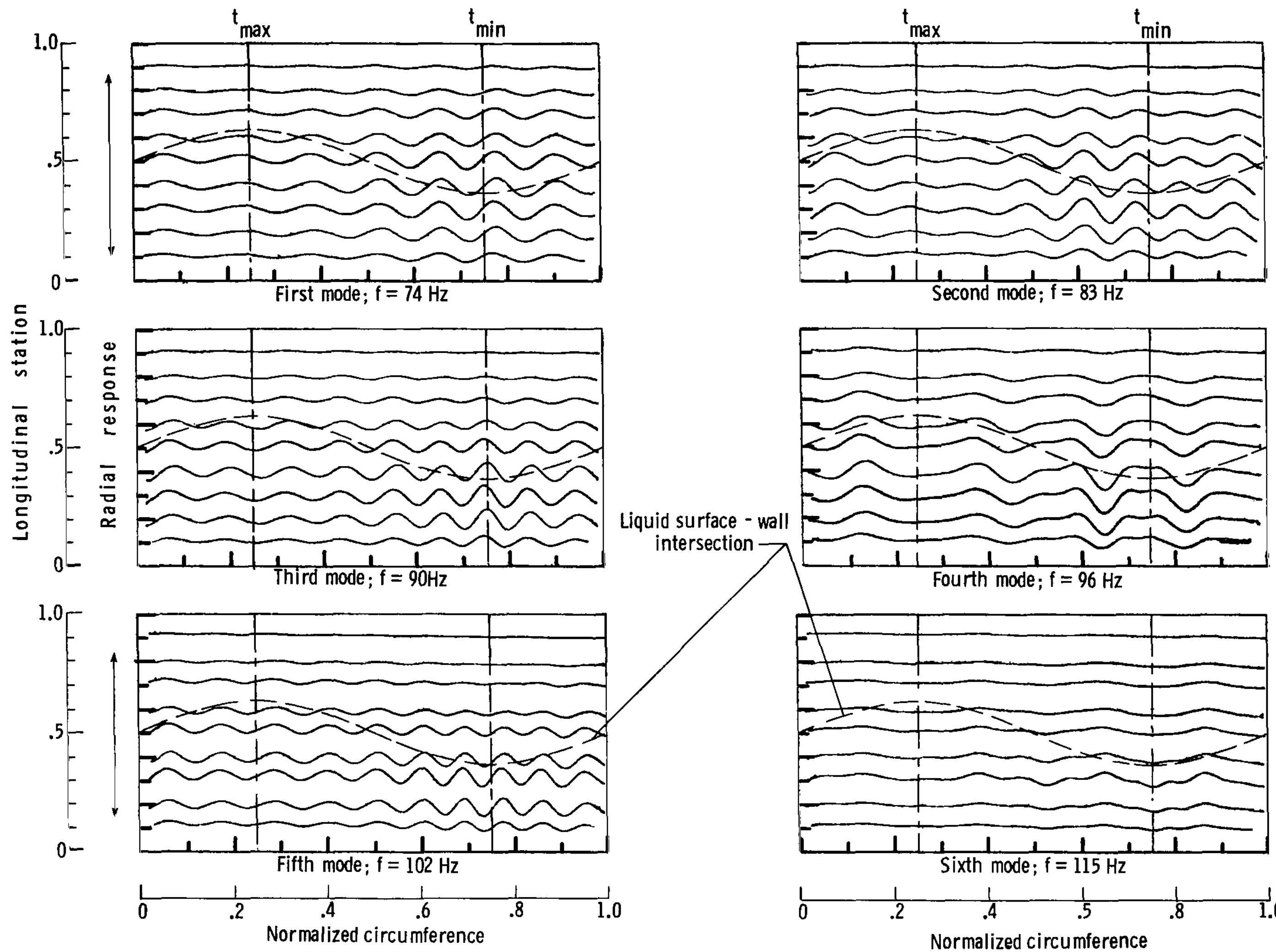

(d) $p = 5.516\ N/cm^2$; $b/L = 1/2$.

Figure 12.- Concluded.



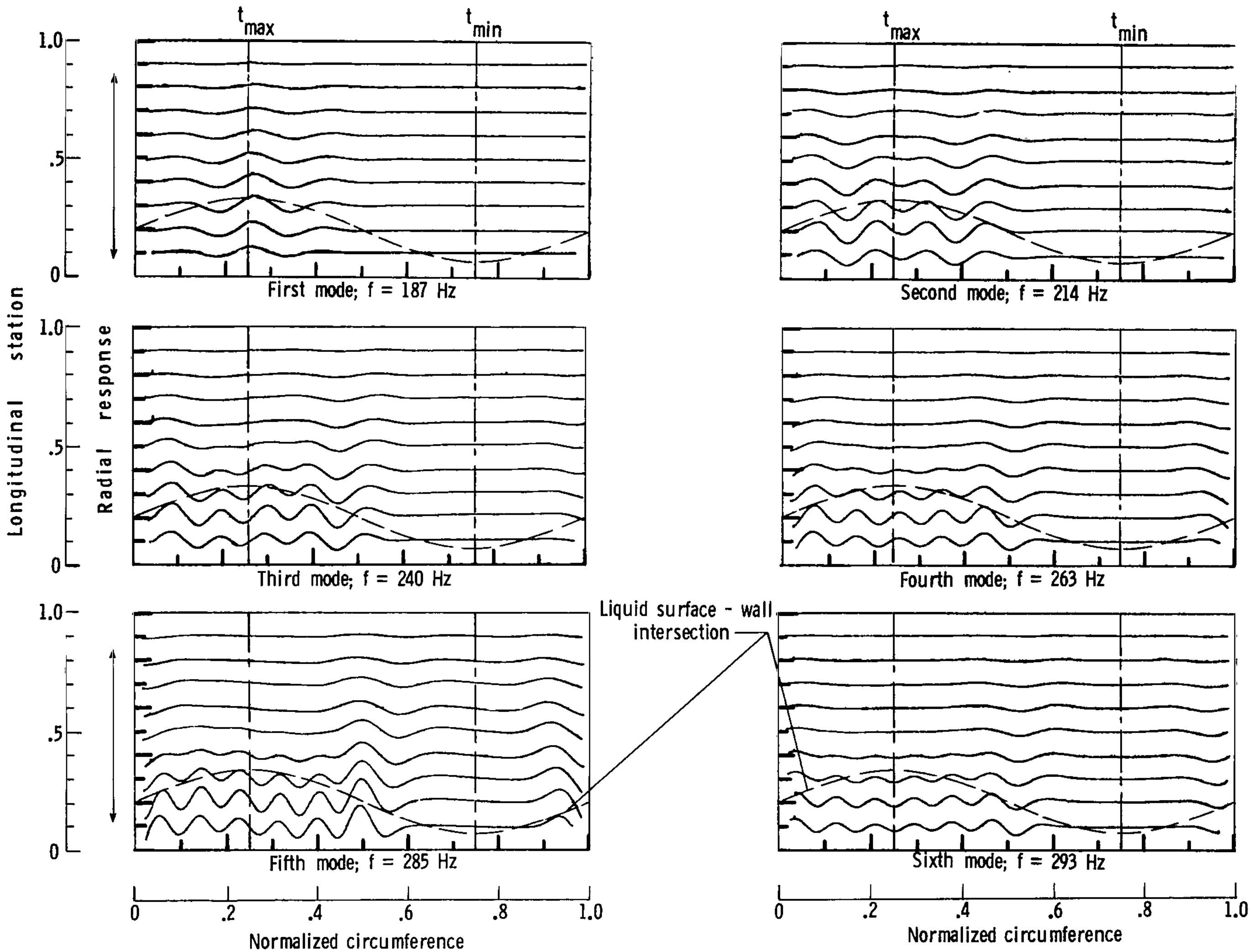

(a) p = 0; $\bar{b}/L = 1/5$.

Figure 13.- Mode shapes of tilted, partly filled asymmetric cylinder at several longitudinal stations. $\theta = 15^{\circ}$ (toward side of maximum thickness).



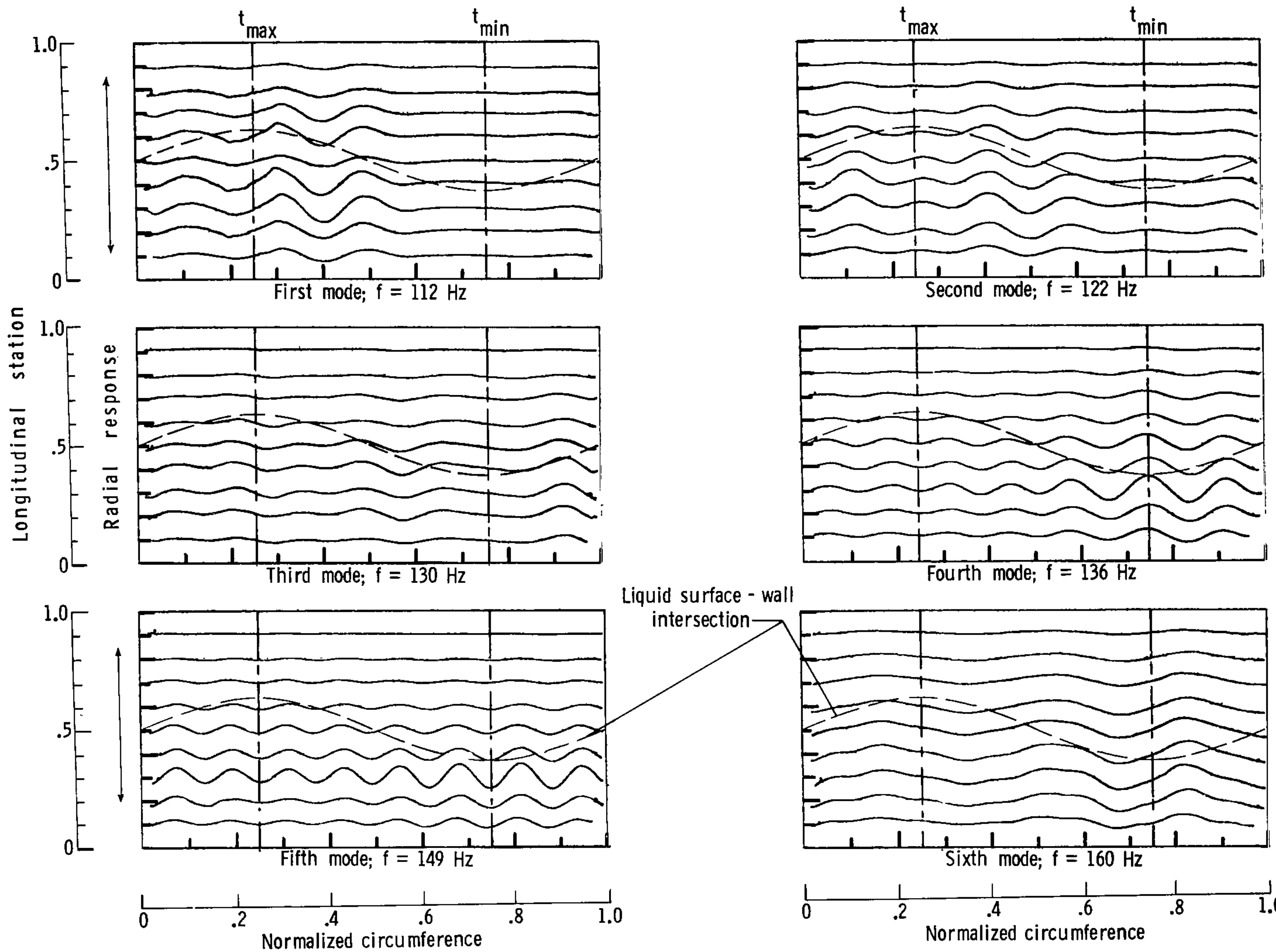

(b) $p = 0$; $\bar{b}/L = 1/2$.

Figure 13.- Continued.



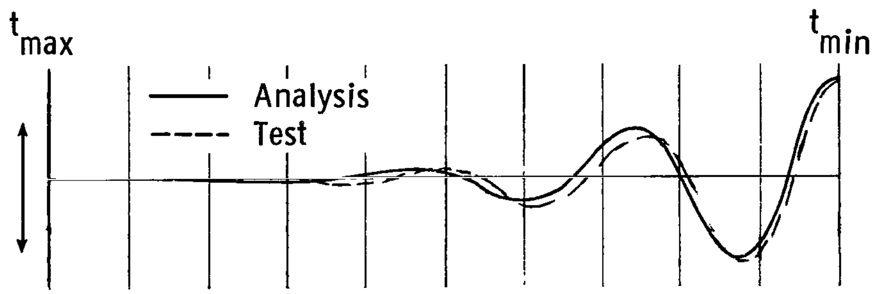

(c) $p = 5.516\ N/cm^2$; $\bar{b}/L = 1/5$.

Figure 13.- Continued.

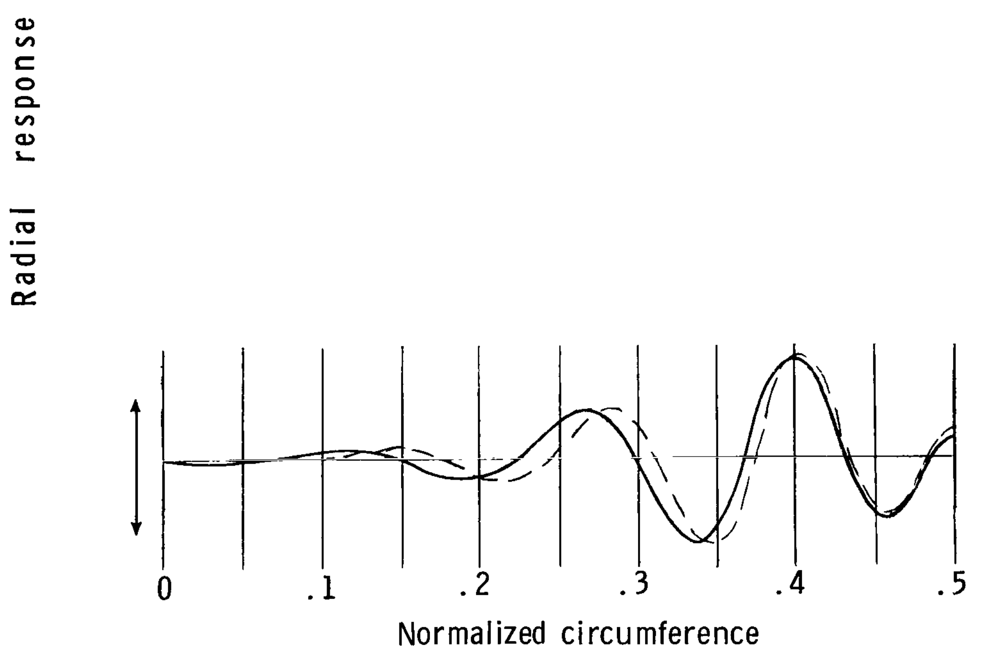



(d) $p = 5.516\ N/cm^2$; $\bar{b}/L = 1/2$.

Figure 13.- Concluded.

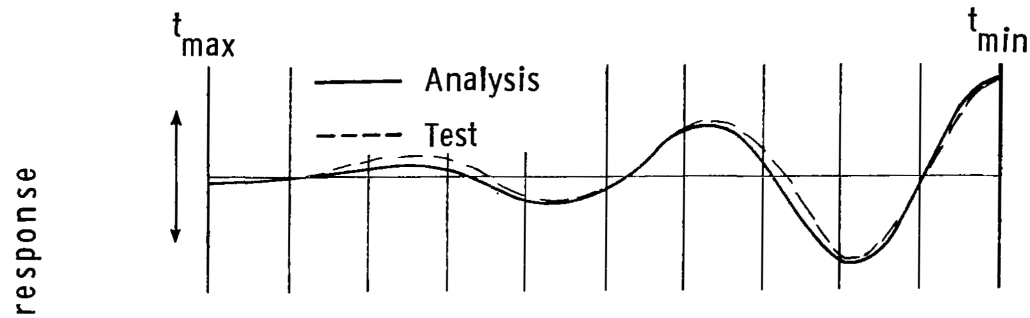



(a) First mode; $f_{analysis}$ = 67.4 Hz; $f_{test}$ = 66.0 Hz.

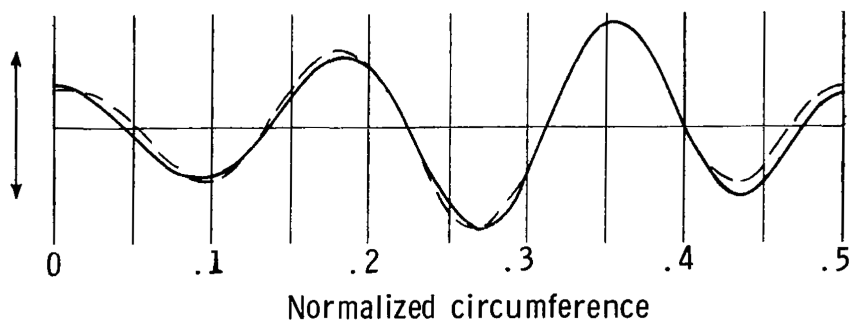



(b) Second mode; $f_{analysis}$ = 78.5 Hz; $f_{test}$ = 74.0 Hz.

Figure 14.- Comparison of analytical and test modes of unpressurized asymmetric cylinder. b/L = 1/2; θ = 0°.




(a) First mode; $f_{analysis}$ = 117 Hz; $f_{test}$ = 110 Hz.




(b) Second mode; $f_{analysis}$ = 128 Hz; $f_{test}$ = 118 Hz.

Figure 15.- Comparison of analytical and test modes of pressurized asymmetric cylinder. $p = 5.516\ N/cm^2$; $b/L = 1/2$; $\theta = 0^{\circ}$.

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15. Supplementary Notes

16. Abstract

Experimental studies to determine the effects of tilt and structural asymmetry on the vibration characteristics of partly liquid-filled thin-wall cylinders found that tilting the longitudinal axis of a partly filled axisymmetric cylinder from the vertical could markedly reduce its resonant frequencies and change significantly the shape of the circumferential modes. For the minimum frequency modes, vibratory motion occurred only on that side of the cylinder where the liquid was deepest. An empirical equation was derived that gives the equivalent liquid depth of an untilted cylinder having the same minimum resonant frequency as a tilted, partly filled cylinder.

Circumferential mode shapes of an untilted asymmetric cylinder were similar to those of the tilted, partly filled axisymmetric cylinder. Vibratory motion in the minimum frequency modes occurred in most instances only on the side of minimum thickness. Correlation between test data and results from a reformulated NASTRAN® hydroelastic analysis was excellent.

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