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EXTRAVEHICULAR ACTIVITY TRANSLATION ARM (EVATA) STUDY

FINAL REPORT

ARC-TN-1064

by

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SECTION 1

INTRODUCTION

This report documents the preliminary design of a deployable Extravehicular Activity Translation Arm (EVATA) assembly which will allow an EVA crewman to perform tasks in the vicinity of the External Tank (ET) umbilical doors and to inspect most of the underside of the Shuttle spacecraft.

As conceived, the EVATA would stow in the last 48 inches of the Shuttle bay. The stowed package dimensions for this analysis were 40 inches wide, 58 inches deep, and 43 inches long. The deployed lengths as shown on NASA Drawing PD78-33600 were 46 feet for the upper boom and 63 feet of the lower boom.

The concept chosen for the boom structure was the Astro Extendable Support Structure (ESS) which formed the main structure for the Synthetic Aperture Radar (SAR) Antenna System on the SEASAT A spacecraft. The antenna system was successfully deployed in late June and the SAR was successfully operated in early July. This structure is a deployable triangular truss as shown in Figure 1.1. A comparison of the EVATA and the SEASAT A ESS is shown in Table 1-1. The development status of the ESS is shown in Table 1-2. The satellite configuration, the stowed truss load path, and the envelope deployment sequence for the ESS are shown in Figure 1-2. Figures 1-3, 1-4, and 1-5 show the actual SAR stowed, partially deployed, and deployed, respectively.

TABLE 1-1. COMPARISON OF EVATA AND SEASAT ESS STRUCTURE

	EVATA	ESS
OVERALL LENGTH	63 AND 46 FEET	35 FEET
PANEL LENGTH	34.5 INCHES	52.9 INCHES
PANEL WIDTH	39.1 INCHES	40 INCHES
TRUSS HEIGHT	19 INCHES	17.9 INCHES
TUBE MATERIAL	GRAPHITE/EPOXY	GRAPHITE/EPOXY
FITTING MATERIAL	6061-T6 ALUMINUM	6AL4V TITANIUM
TUBE DIAMETER	1.125 INCHES	0.5 INCH
TRUSS WEIGHT	TOTAL 2 BOOMS GR = 389 POUNDS AL = 767 POUNDS	30 POUNDS
PACKAGE SIZE	40 x 43 x 58 (10 INCHES FOR ELBOW)	55.5 x 55 x 8

TABLE 1-2. DEVELOPMENT STATUS OF EXTENDABLE SUPPORT STRUCTURE

One-half scale model demonstrated	December 1975
Full-scale model demonstrated	September 1976
SEASAT A SAR ESS tested and delivered	July 1977
SEASAT A launch	26 June 1978

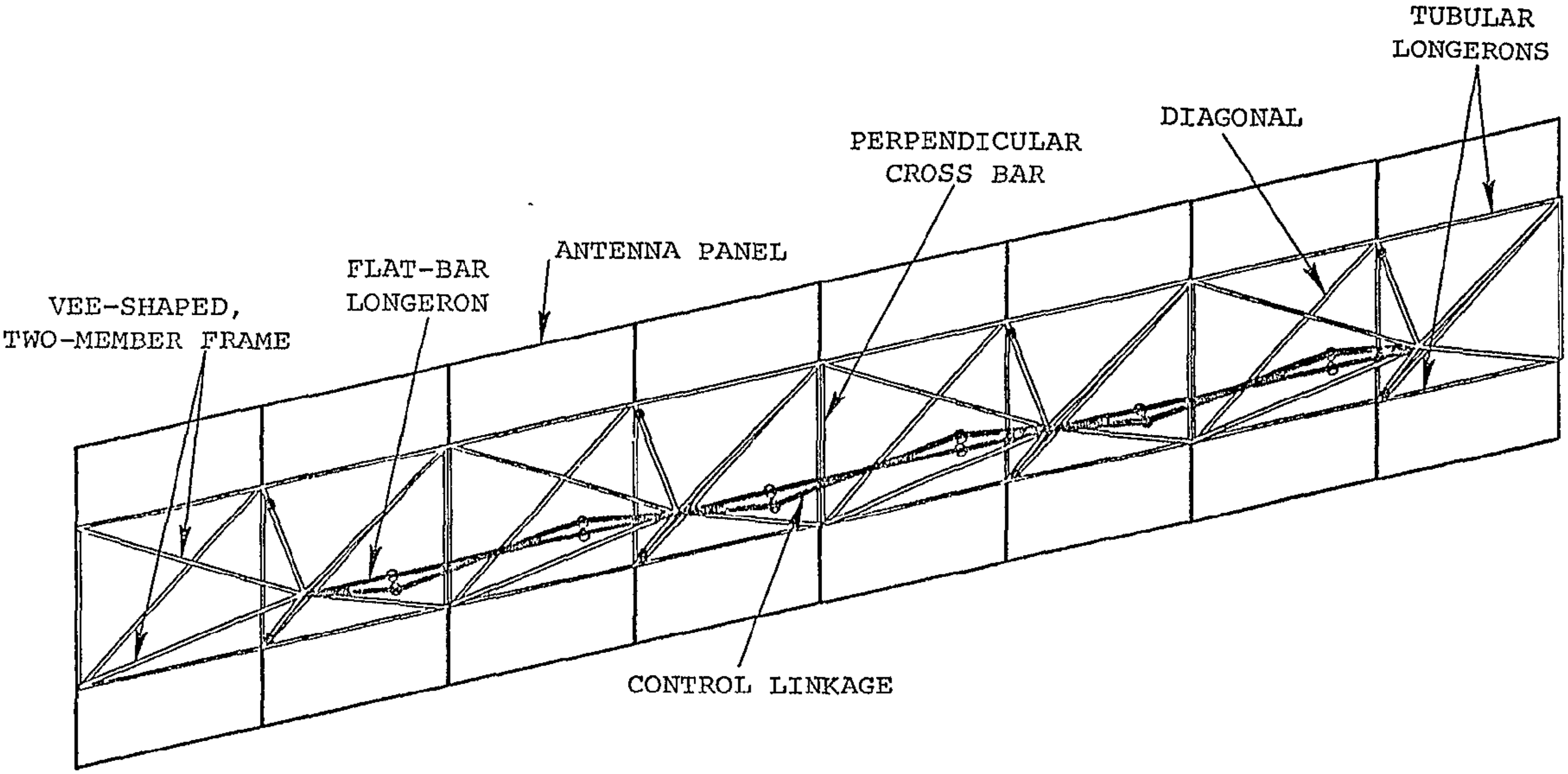
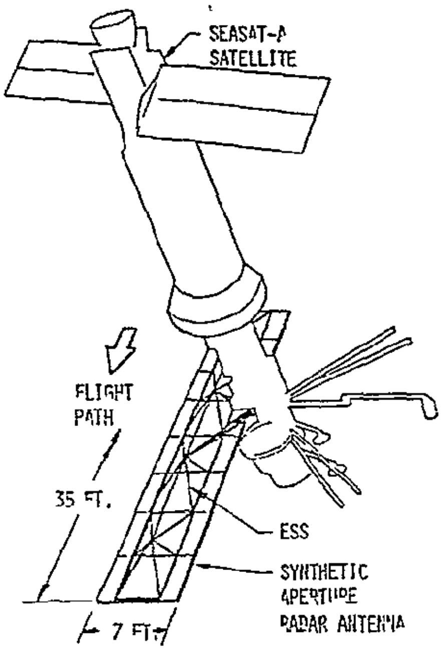
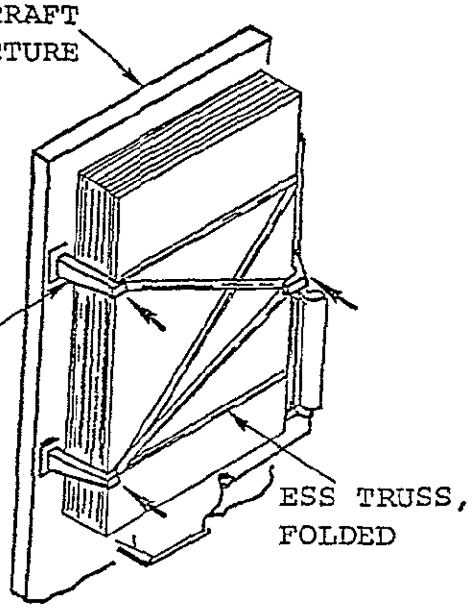


Figure 1-1. Deployed ESS.



Satellite Configuration.



Stowed Truss Load Paths.

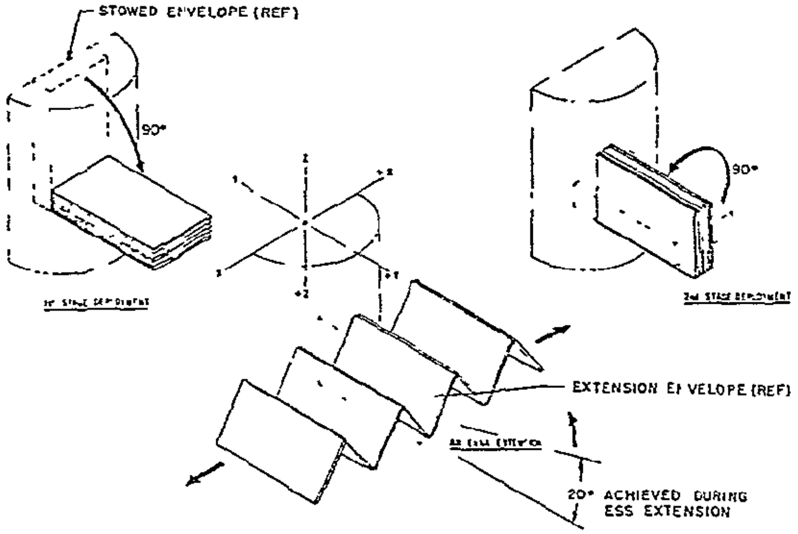


Figure 1-2. Envelope deployment sequence.

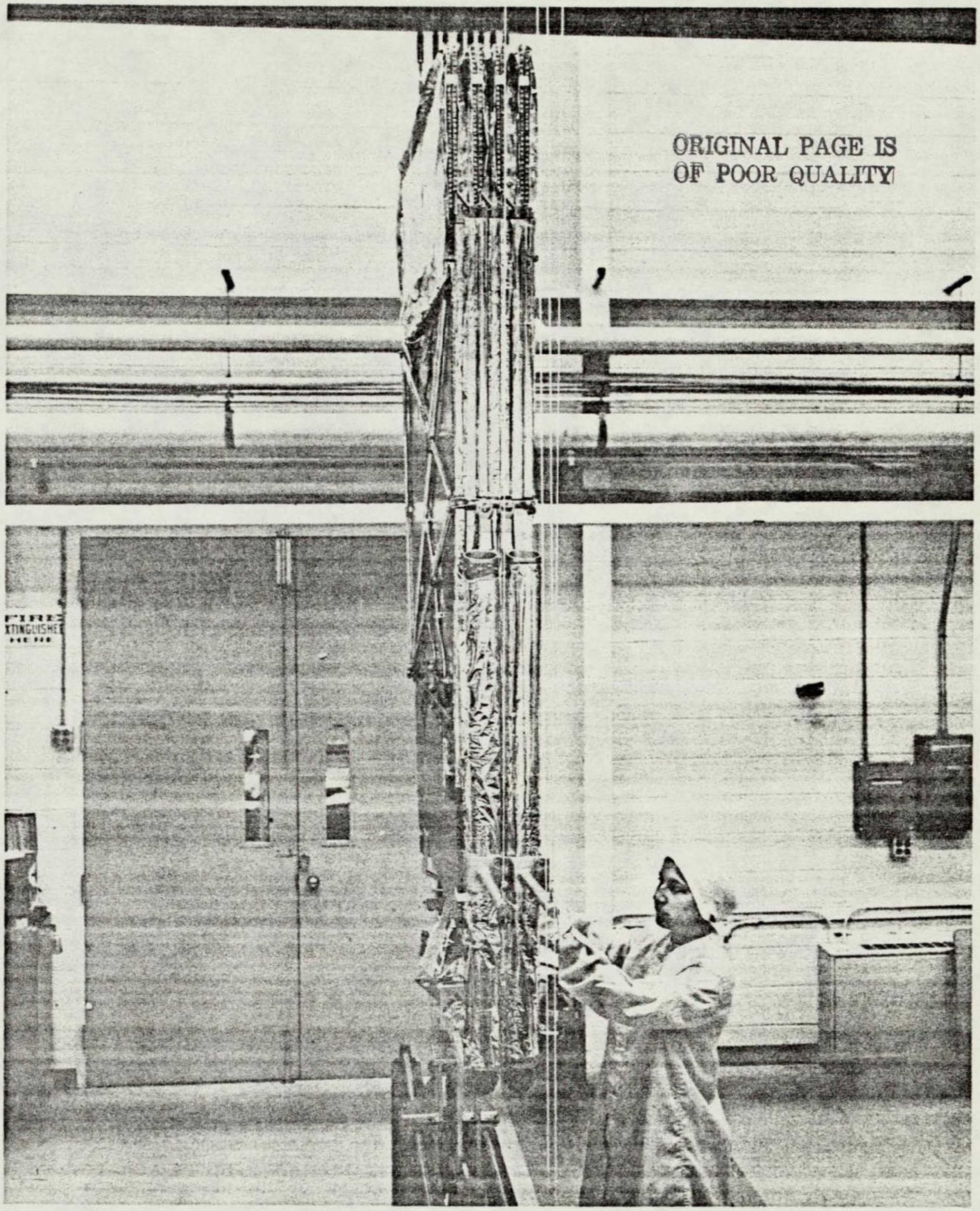


Figure 1-3. SAR stowed.

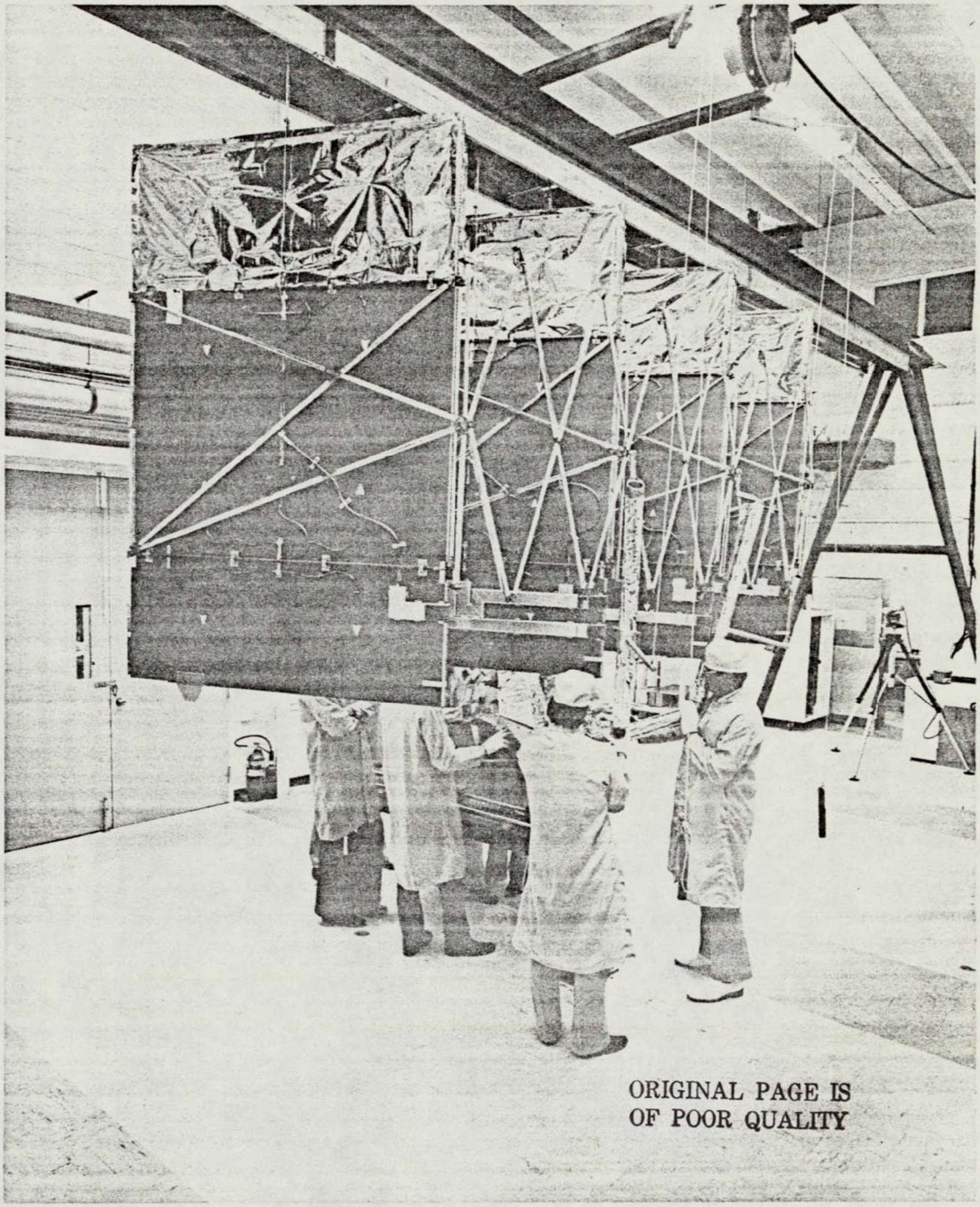


Figure 1-4. SAR partially deployed.

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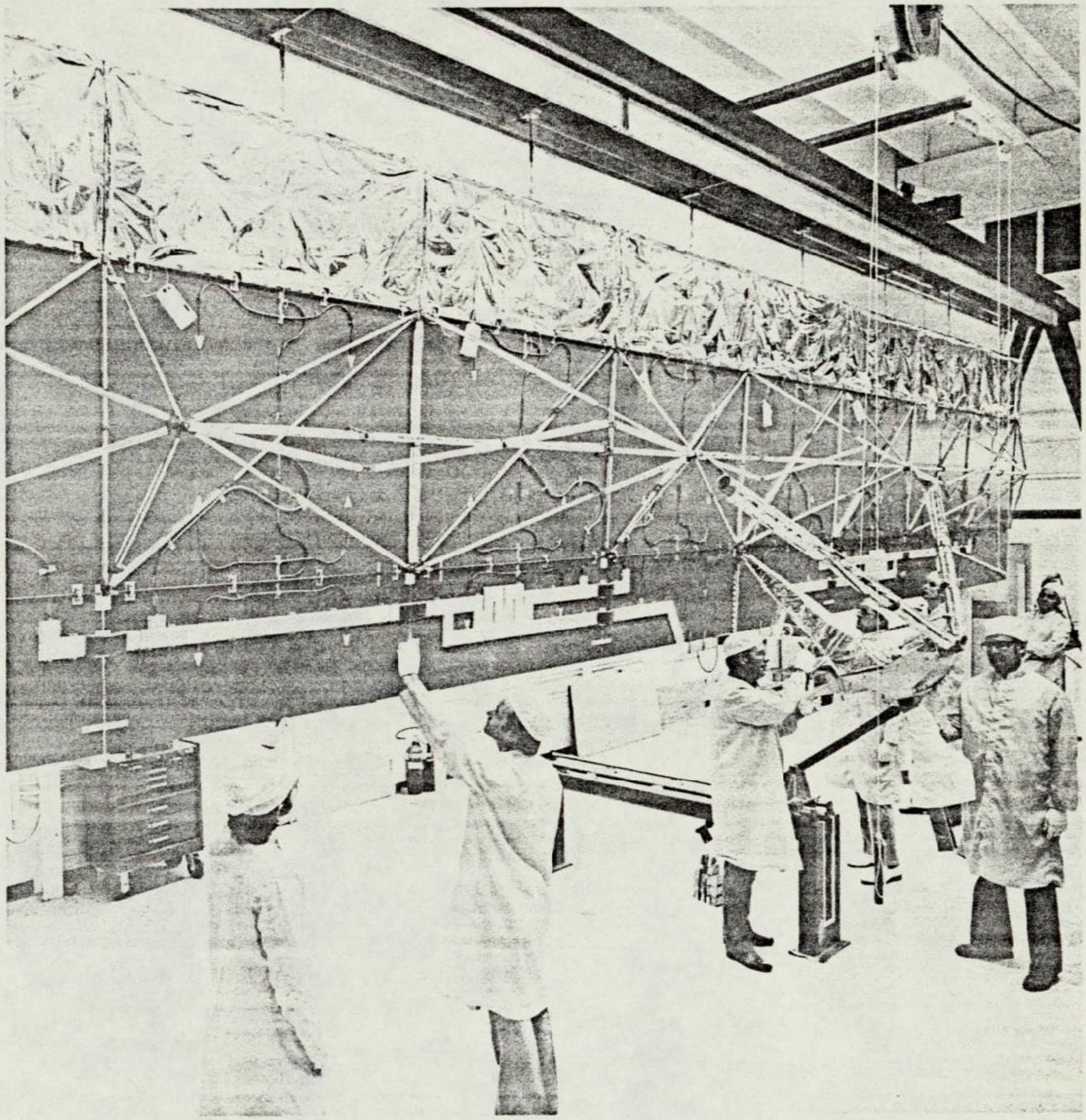


Figure 1-5. SAR deployed.

SECTION 2

GEOMETRY AND KINEMATICS OF THE TRUSS

2.1 CONSTRAINTS AND ASSUMPTIONS

The configuration of the truss is determined by the following constraints and assumptions:

- Package limitations and the length requirement for the extended truss confine the panel length to certain ranges, related to the number of panels, and limit the cross section of the longitudinal members. The lowest number of panels possible allows the largest truss depth and the biggest width of the longitudinal members; therefore, the stiffest configuration.
- The package space is best utilized when the width of the main scissor member equals the height of the longitudinal panel members. For practical reasons, all members are assumed to be cylindrical tubing of circular cross section. For additional simplification, all members except the control linkage of the scissor are of the same outer diameter.
- Again, for best package utilization, the knee braces fold into a position where both legs lie flat against each other and parallel to the cross braces. The gap shown in Figure 2-1 between upper knee brace and main scissor member is reduced to zero for the same reason.
- A highly efficient truss avoids load eccentricities at the joints, i.e., all axes of truss members pass through a common point in each joint. For simplification, this rule was broken only where the A-frame joins the front panels of the truss. There the same distance between truss-member axis and hinge axis was used for the panel members as well as the A-frame member. The resulting eccentricity is less than 10 percent of the member diameter.

2.2 SYSTEM OF CONDITIONS AND EQUATIONS

With the above restraints and the geometric relations depicted in Figures 2-1 and 2-2, the following system of conditions and equations are formulated.

$N_u \ell \approx 550$ in.	length of upper arm
$N_\ell \ell \approx 710$ in.	length of lower arm
$(N_u + N_\ell) d_\ell \leq 58 - 10$ in.	length of package allowing room for elbow joint
$\ell + 2q \leq w_p$	width of package
$d_\ell = d_{s1} = d_a = d_b = d_d = d_k$	tubular members of same o.d.
$d_p + h_a + c + \frac{d_{s1}}{2} + d_{s2} = h_p$	package height
$d_p + \ell + \frac{d_\ell}{2} + (k_3 + k_4)\cos\gamma + d_k + \frac{d_{s1}}{2} + d_{s2} = h_p$	package height
$b + 2(k_1 - k_3 \sin\gamma) + 2 d_{pk} = w_p$	package width
$2(k_2 + k_4 \sin\gamma) + 2 d_{pk} = w_p$	package width
$k_4 \cos\gamma - \frac{d_k}{2} - \frac{d_{s1}}{2} = 0$	tight package
$e + f - \frac{d_a}{2} = 0$	tight package

$$c \tan\alpha - f \frac{1}{\cos\alpha} - e = 0$$

$$2(k_1 + k_2 + k_3 + k_4)\sin\gamma - b = 0$$

$$\frac{d_a}{2}\cos\alpha + h_a \sin\alpha + f \cos\alpha + e - \ell = 0$$

$$\frac{d_\ell}{2} - \frac{d_a}{2}\sin\alpha + h_a \cos\alpha - f \sin\alpha + c - (k_1 + k_2 + k_3 + k_4)\cos\gamma = 0$$

The first four equations allow the choice of N_u, N_ℓ, ℓ, and d_ℓ when the package width is given and the space needed for the special scissor hinges, g, is estimated.

The remaining 15 equations contain 23 variables of which two already have been determined, two (h_p and w_p) are either given or used as parameters, leaving another four as parameters. In this study, the hinge pin diameters, d_p and d_{pk}, were fixed as 0.1875 and 0.125, respectively, the thickness or diameter of the scissor control linkage was assumed as 0.5 inch, and the hinge location, e, was used as the variable parameter.

The following values were used for the configuration with the ladder (truss-face panel) in vertical and horizontal position, respectively.

Variable	Vertical Ladder	Horizontal Ladder
N_u	16	15
N_ℓ	22	20
ℓ	34.500	37.500
d_ℓ	1.125	1.250
w_p	43.00	40.00
h_p	40.00	Parameter
d_p	0.1875	0.1875
d_{pk}	0.125	0.125
d_{s2}	0.500	0.500

Three additional dimensions of the deployed truss are calculated once the above mentioned variables are all determined:

$$\text{truss height, } h = \sqrt{(k_1 + k_2 + k_3 + k_4)^2 - \frac{\ell^2}{4}}$$

theoretical length of A-frame member, a $= \sqrt{(k_1 + k_2 + k_3 + k_4)^2 + \ell^2}$

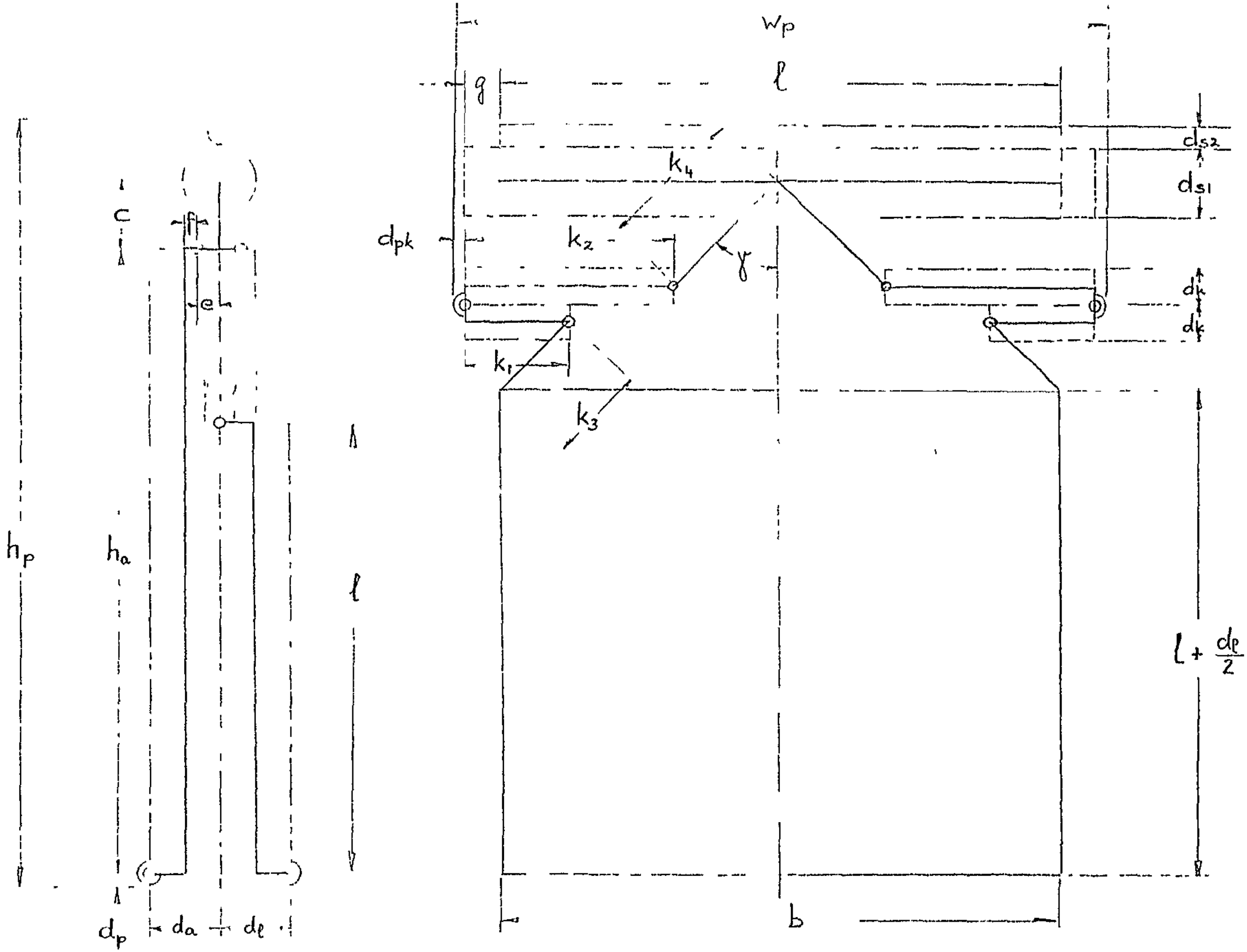
theoretical length of panel diagonal, D $= \sqrt{b^2 + \ell^2}$

2.3 BASELINE TRUSS DESIGN

The values for the baseline truss design, thus derived, are presented in Table 2-1.

TABLE 2-1. BASELINE TRUSS CONFIGURATION

Parameter	Vertical Ladder	Horizontal Ladder
Truss width, b	39.088	34.867
Truss height, h	18.933	30.194
Panel length, ℓ	34.500	37.500
Diagonal length, D	52.136	51.205
A-frame member, a	43.940	51.204
Knee brace length, Σ_k	27.211	34.866
A-frame angle, α	61.141°	50.994°
Knee brace angle, γ	45.910°	30.001°
Package width, w_p	43.00	40.00
Package height, h_p	40.00	48.766
Package depth		
Upper arm $N_u \times d$	18.00	18.75
Lower arm $N_\ell \times d$	24.75	25.00

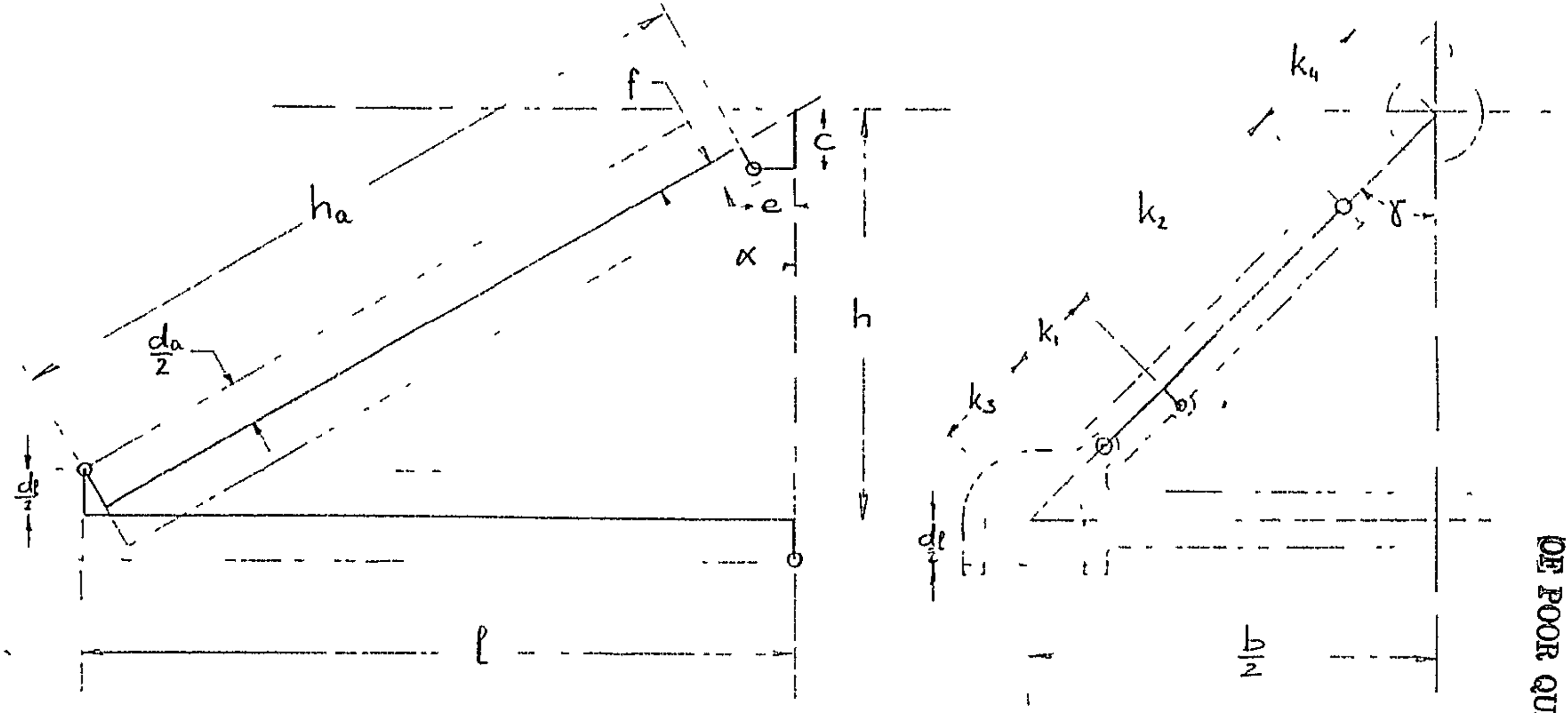


SIDE VIEW
SINGLE BAY

CROSSVIEW

TRUSS
STOWED

Figure 2-1. Schematic Geometry of Stowed Truss



SIDE VIEW
HALF BAY

CROSS VIEW
HALF BAY

Figure 2-2. Schematic geometry of deployed truss.

SECTION 3

STRUCTURAL CHARACTERISTICS OF TRUSS

3.1 NOTATION AND CONVENTION OF FORCES

In order to determine strength and compliance of the truss with respect to external loads, a convention for the representation of external forces is established as shown in the lower right corner of Figure 3-1. The same figure shows the relations among internal forces as found from equilibrium considerations for each joint. A cut at distance x from the local coordinate origin provides the relations between external and internal forces:

$$P = P_1 - \frac{l}{d} P_d + P_2 + P_3 + \frac{l}{a}(P_A + P_B) + \frac{l}{d} P_d - \frac{l}{a}(P_A + P_B)$$

$$V_2 = -\frac{b}{d} P_d + \frac{b}{2a}(P_A - P_B)$$

$$V_1 = \frac{h}{a}(P_A + P_B)$$

$$Q = \frac{h}{a}(P_A - P_B)\frac{b}{2}$$

$$M_{10} - V_1 x = -\left(P_3 + \frac{l}{a}(P_A + P_B)\right)h + \frac{l}{a}(P_A + P_B)h\left(1 - \frac{x}{l}\right)$$

$$M_{20} - V_2 x = \left(P_1 - \frac{l}{d}P_d - P_2\right)\frac{b}{2} - \frac{l}{d}P_d\frac{b}{2}\left(1 - \frac{2x}{l}\right) - \frac{l}{a}(P_A - P_B)\frac{b}{2}\frac{x}{l}$$

Inversion of this equation matrix leads to the following system of equations for the internal forces. For details, see Appendix A.

$$\begin{aligned}
P_1 &= \frac{P}{2} - \frac{\ell}{b}V_2 + \frac{\ell}{bh}Q + \frac{M_{10}}{2h} + \frac{M_{20}}{b} \\
P_2 &= \frac{P}{2} + \frac{\ell}{b}V_2 - \frac{\ell}{bh}Q + \frac{M_{10}}{2h} - \frac{M_{20}}{b} \\
P_3 &= - \frac{M_{10}}{h} \\
P_A &= \frac{a}{2h}V_1 + \frac{a}{bh}Q \\
P_B &= \frac{a}{2h}V_1 - \frac{a}{bh}Q \\
P_d &= -\frac{d}{b}V_2 + \frac{d}{bh}Q
\end{aligned}$$

3.2 DETERMINATION OF FORCES

With these equations and Figure 3-1, the force in any of the truss members can be determined simply by reducing the external forces to the load vector in the symmetry plane of the module in question.

For a given external load condition, the strength of the truss is determined by comparing the loads of each truss member with its load capacity which may be limited either by yielding or buckling. It is assumed that the shear strength of the joint pins and the bearing strength of the joints are considerably higher than the strength of the truss members; but, in any case, this must be proven.

In order to establish the stiffness and compliance characteristics of the structure, the truss is considered equivalent to a beam of constant cross-sectional properties, namely bending stiffness about the two neutral axes, EI_1 and EI_2; center of gyration, e_1 and e_2; shear stiffnesses, GA_{S1} and GA_{S2}; shear center, e_3 and e_4;

and torsional stiffness, GJ. These features are determined by comparing the strain energy of one truss module with that of an equally long (2ℓ) beam section. When expressed as functions of the external loads, the truss properties can be found by comparing corresponding terms. Thus, as developed in Appendix B,

$$\frac{1}{EI_1} = \frac{1}{h^2}\left(\frac{1}{2(EA)_\ell} + \frac{1}{(EA)_s}\right)$$

$$\frac{1}{EI_2} = \frac{2}{b^2}\,\frac{1}{(EA)_\ell}$$

$$e_1 = \frac{h}{\left(1 + 2\frac{(EA)_\ell}{(EA)_s}\right)}$$

$$e_2 = 0$$

$$\frac{1}{GA_{S1}} = \frac{\ell^2}{h^2}\,\frac{1}{6(EA)_\ell}\left[4\frac{(EA)_\ell}{(EA)_s} + 3\frac{a^3}{\ell^3}\,\frac{(EA)_\ell}{(EA)_a} - 1\right]$$

$$\frac{1}{GA_{S2}} = \frac{2}{3}\,\frac{\ell^2}{b^2}\,\frac{1}{(EA)_\ell}\,\frac{\left[\left(\frac{a^3}{\ell^3}\frac{(EA)_\ell}{(EA)_a} - 1\right) + \left(3\frac{a^3}{\ell^3}\frac{(EA)_\ell}{(EA)_a} - 1\right)\left(\frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right)\right]}{\left[1 + 2\frac{a^3}{\ell^3}\frac{(EA)_\ell}{(EA)_a} + \frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right]}$$

$$e_3 = h\,\frac{\left[1 + \frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right]}{\left[1 + 2\frac{a^3}{\ell^3}\frac{(EA)_\ell}{(EA)_a} + \frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right]}$$

$$e_4 = 0$$

$$\frac{1}{GJ} = \frac{\ell^2}{b^2 h^2} \frac{1}{(EA)_\ell} \left[1 + 2 \frac{a^3}{\ell^3} \frac{(EA)_\ell}{(EA)_a} + \frac{d^3}{\ell^3} \frac{(EA)_\ell}{(EA)_d} + \frac{1}{2} \frac{b^3}{\ell^3} \left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}} \right) \right]$$

3.3 COMPLIANCE

The compliance of the EVATA is determined by formulating the force vector of the external loads for both upper and lower arms and, then, by applying the principle of Castigliano. For simplification, the shear terms are neglected because the truss can be regarded as a slender beam. As a further simplification, it was assumed that the external forces do not include couples. Thus, the compliance matrix can be reduced to half size. And finally, since the main interest focuses on the displacement vector at the place of the load applications, only that part of the compliance matrix was determined (see Figure 3-2). Then, each compliance, C_{ij}, is composed of contributing factors representing the influence of both bending stiffnesses, torsional stiffness, and thus, for the lower and upper arm as shown in Figure 3-3. A numerical example is presented in Table 3-1.

The transformation of the external forces from the lower to upper arm is detailed in Appendix C. Details of the Castigliano method applied to the truss are presented in Appendix D.

TABLE 3-1. COMPLIANCES AND THEIR CONTRIBUTORS (POSITION I)

ij	C_{ij}	K_{f1}	K_{u1}	$\frac{10^6}{EI_1}\{K_{f1}+K_{u1}\}$	K_{f2}	K_{u2}	$\frac{10^6}{EI_2}\{K_{f2}+K_{u2}\}$	K_{fQ}	K_{uQ}	$\frac{10^6}{GJ}\{K_{fQ}+K_{uQ}\}$
11	0.2249	66.505	35.268	53.521×10^{-3}	0	0.1506	0.0253×10^{-3}	0.379	64.2	171.34×10^{-3}
12	-0.0133	0	1.499	0.788×10^{-3}	0	2.8337	0.4753×10^{-3}	-0.162	-5.330	-14.572×10^{-3}
13	0.00467	1,0844	-0.671	0.2176×10^{-3}	0	-2,003	-0.3359×10^{-3}	0	1,8057	4.791×10^{-3}
22	0.0261	0	0.1288	0.06775×10^{-3}	66.51	80.512	24.661×10^{-3}	0.069	0.44256	1.3578×10^{-3}
23	-0.0062	0	-0.1349	-0.0709×10^{-3}	0	-34.238	-5.743×10^{-3}	0	-0.14992	-0.3978×10^{-3}
33	-0.00478	0.0236	0.1871	0.11077×10^{-3}	0	27.063	$4,54\times10^{-3}$	0	0.05079	0.1348×10^{-3}

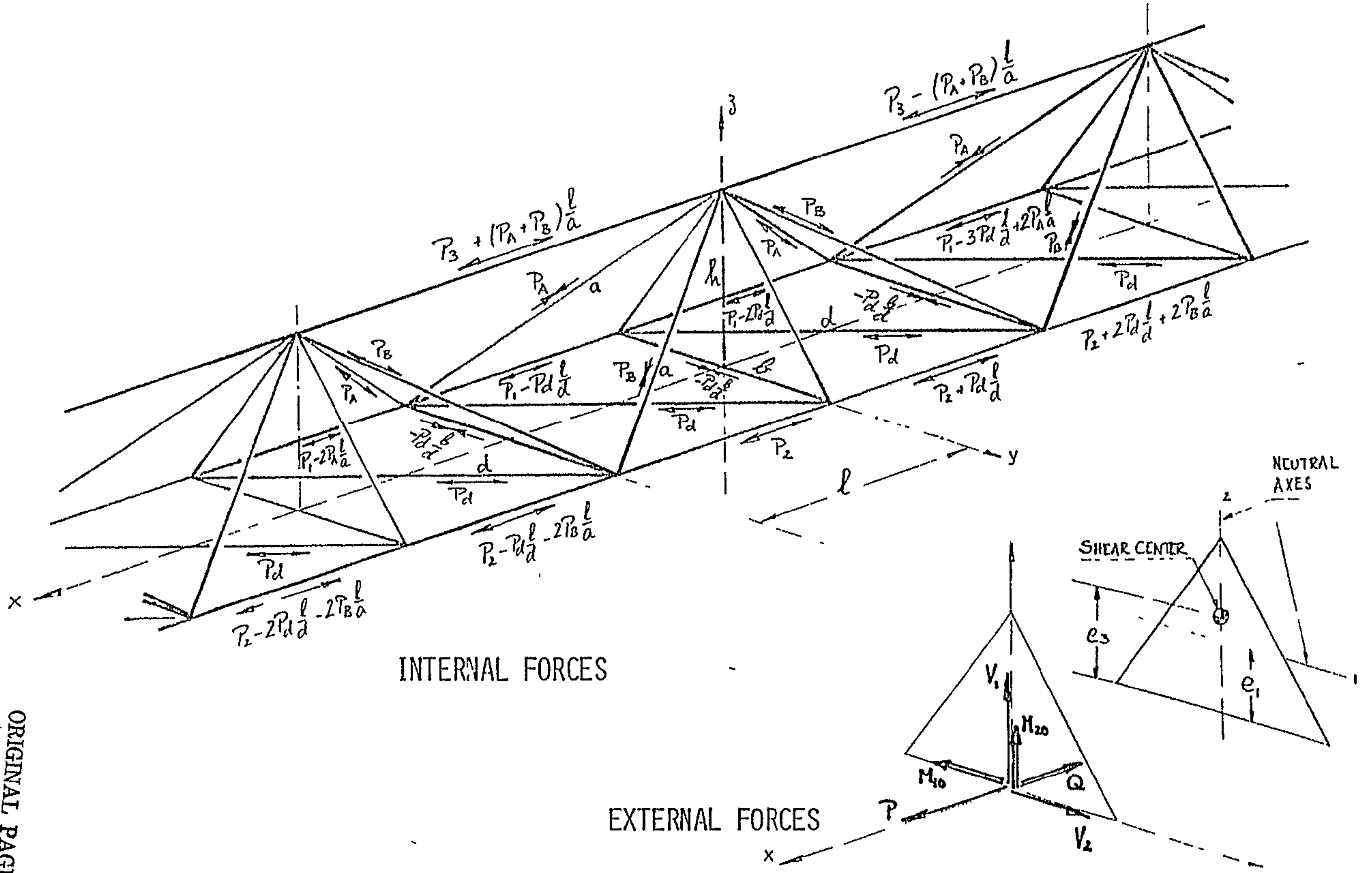


Figure 3-1. Notation of forces.

$$\begin{vmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} \times \begin{vmatrix} V_1 \\ V_2 \\ P_\ell \end{vmatrix}$$

Figure 3-2. General compliance matrix.

CONFIGURATION: VERTICAL LADDER MATERIAL: C/EP

$$C_{ij} = \frac{10^6}{EI_1}\left\{K_{l1ij} + K_{u1ij}\right\} + \frac{10^6}{EI_2}\left\{K_{l2ij} + K_{u2ij}\right\} + \frac{10^6}{GJ}\left\{K_{lQij} + K_{uQij}\right\}$$

$e_1 = 6.35$

$e_3 = 10.884$ $\qquad \frac{10^6}{EI_1} = 0.5289 \times 10^{-3} \qquad \frac{10^6}{EI_2} = 0.16774 \times 10^{-3}$

$l_l = 584.33$

$$\frac{10^6}{GJ} = 2.653 \times 10^{-3}$$

Figure 3-3. Compliances and their contributors.

SECTION 4

APPLIED LOADS AND RESULTING TRUSS FORCES

4.1 APPLIED LOADS

The forces and deformations were calculated for applied loads consisting of a 50-pound component in the Z-direction and 20 pounds in the X - Y plane and perpendicular to the axis of the lower arm. For the truss configuration with the face panels in a vertical position (vertical ladder), the forces are applied at a point located 6 inches above the upper longeron and in the panel plane, while for the horizontal ladder the application point lies 6 inches above the panel plane in the symmetry plane of the truss cross section (see Figure 4-1).

Several arm positions were investigated: the upper arm was kept fixed at an angle of 45 degrees away from the Orbiter X-axis and an inclination of 22.5 degrees downwards; the lower arm, with its origin 63 inches below the intersection of upper-arm axis and elbow rotary axis, was rotated forward and upwards according to the following table:

Position	I	II	III	IV
Out-of-plane angle between lower and upper arm	3.93°	-30°	-70°	-102.5°
Downwards inclination of lower arm	8° - 50'	6°	2° - 42'	0°
l_1	584.3	628	565	715

The distance of the load application from the elbow in the local X-axis of the lower arm, l_1, represents the intersection of the lower-arm axis with the symmetry plane of the Orbiter in Positions I and IV while for Positions II and III, loading points beneath the leading edge of the wing were assumed. For details see Appendix E.

4.2 FORCES AND DEFORMATIONS

The truss forces and deformations were calculated for trusses fabricated from aluminum, stainless steel, and carbon/epoxy composite tubes. The maximum forces encountered in Position I were used to determine the wall thicknesses of the members as shown in Table 4-1 and, thus, the allowable compression loads for a safety factor of 3.0 listed in Table 4-2.

Table 4-3 shows a comparison of the load capacity of the truss in its four positions as fractions of the 50-/20-pound load vector and indicates in which quadrant the applied critical force lies as defined by Figure 4-2. The magnitude of the deflection vector when the load is applied in different quadrants is shown in Table 4-4. A comparison of the margin of safety and maximum deflection for the truss in Position I is shown in Table 4-5 for various materials and lists, as well as, the corresponding truss weight.

See Appendix F for detailed load analyses and resulting truss forces and margins of safety. Appendix G is a collection of the compliance matrices, their contributing factors, and the deformations resulting from the standard forces as determined for various positions and materials. Appendix H contains the analyses for the configuration with the truss panels, horizontal rather than vertical.

TABLE 4-1. CROSS SECTIONS OF TRUSS MEMBERS

CONFIGURATION	VERTICAL LADDER			HORIZONTAL LADDER
MATERIAL	AL	CRES	G/EPOXY	G/EPOXY
MAIN SCISSOR (REAR LONGERON)				
O.D. (IN.)	1.125	1.125	1.125	1.25
I.D. (IN.)	(FULL)	0.935	0.875	0.97
A (IN^2)	0.994	0.3074	0.3927	0.4882
FRONT LONGERONS (A-FRAME, DIAGONAL, CROSS BRACES)				
O.D. (IN.)	1.125	1.125	1.125	1.25
I.D. (IN.)	0.75	0.935	0.875	1.03
A (IN^2)	0.552	0.3074	0.3927	0.3940
KNEE BRACES				
O.D. (IN.)	1.125	1.125	1.125	1.25
I.D. (IN.)	0.995	1.055	1.000	1.125
A (IN^2)	0.2165	0.1199	0.2086	0.2332
AUXILIARY SCISSOR (ACTUATOR)				
O.D. (IN.)	0.50	0.50	0.50	0.50
I.D. (IN.)	0.375	0.416	0.40	0.40
A (IN^2)	0.0859	0.06043	0.07068	0.07068
TRUSS WEIGHT, (LBS)	767	1062	392	389

TABLE 4-2. ALLOWABLE COMPRESSION LOADS FOR TRUSS MEMBERS (S.F. = 3.0)

ARM POSITION	I	II	III	IV
CRITICAL LOADING	Q_2	Q_4	Q_3	Q_3
FRACTIONAL LOAD CAPACITY WITH S.F. = 3				
VERTICAL LADDER				
ALUMINUM	1.00	0.29	0.46	0.31
GRAPHITE/EPOXY	1.27	0.40	0.65	0.44
(CRITICAL MEMBER)	LOWER NO. 2 SCISSORS	UPPER ARM DIAGONALS	UPPER ARM DIAGONALS	UPPER ARM DIAGONALS
HORIZONTAL LADDER				
GRAPHITE/EPOXY	1.05			
(CRITICAL MEMBER)	UPPER NO. 14 SCISSORS			

TABLE 4-3. SUMMARY OF FRACTIONAL LOAD CAPACITY (Pounds)

	VERTICAL LADDER		HORIZONTAL LADDER
	ALUMINUM	G/EPOXY	G/EPOXY
LONGERONS	1744	2756	3022
REAR LONGERON (SCISSOR)	543	689	894
A-FRAME MEMBER	1112	1764	1621
DIAGONALS	763	1207	1621
BATTENS	1358	2147	3496

TABLE 4-4. DEFLECTIONS (ABSOLUTE) = $\sqrt{\delta_1^2 + \delta_2^2 + \delta_3^2}$ UNDER 20/50 LOAD

Configuration	Material	Position	δ (in.) (no joint knockdown)	
			Q_1, Q_3	Q_2, Q_4
Vertical ladder	Al	I	7.20	5.23
		II	8.90	19.82
		III	26.35	30.31
		IV	49.13	48.24
	G/epoxy	I	5.35	4.00
		IV	34.62	34.46
Horizontal ladder	G/epoxy	I	3.65	2.53

TABLE 4-5. PERFORMANCE/WEIGHT COMPARISON

CONFIGURATION	VERTICAL LADDER			HORIZONTAL LADDER
MATERIAL	AL	CRES	G/EPOXY	G/EPOXY
MINIMUM MARGIN OF SAFETY (S.F.=3) IN POSITION I WITH Q_2 LOAD	0.004	0.36	0.27	0.075
DISPLACEMENT AT LOADING POINT WITH Q_1 LOAD (NO JOINT KNOCKDOWN)	7.20 IN.	5.26 IN.	5.35 IN.	3.65 IN.
TRUSS WEIGHT	767 LBS	1062 LBS	392 LBS	389 LBS

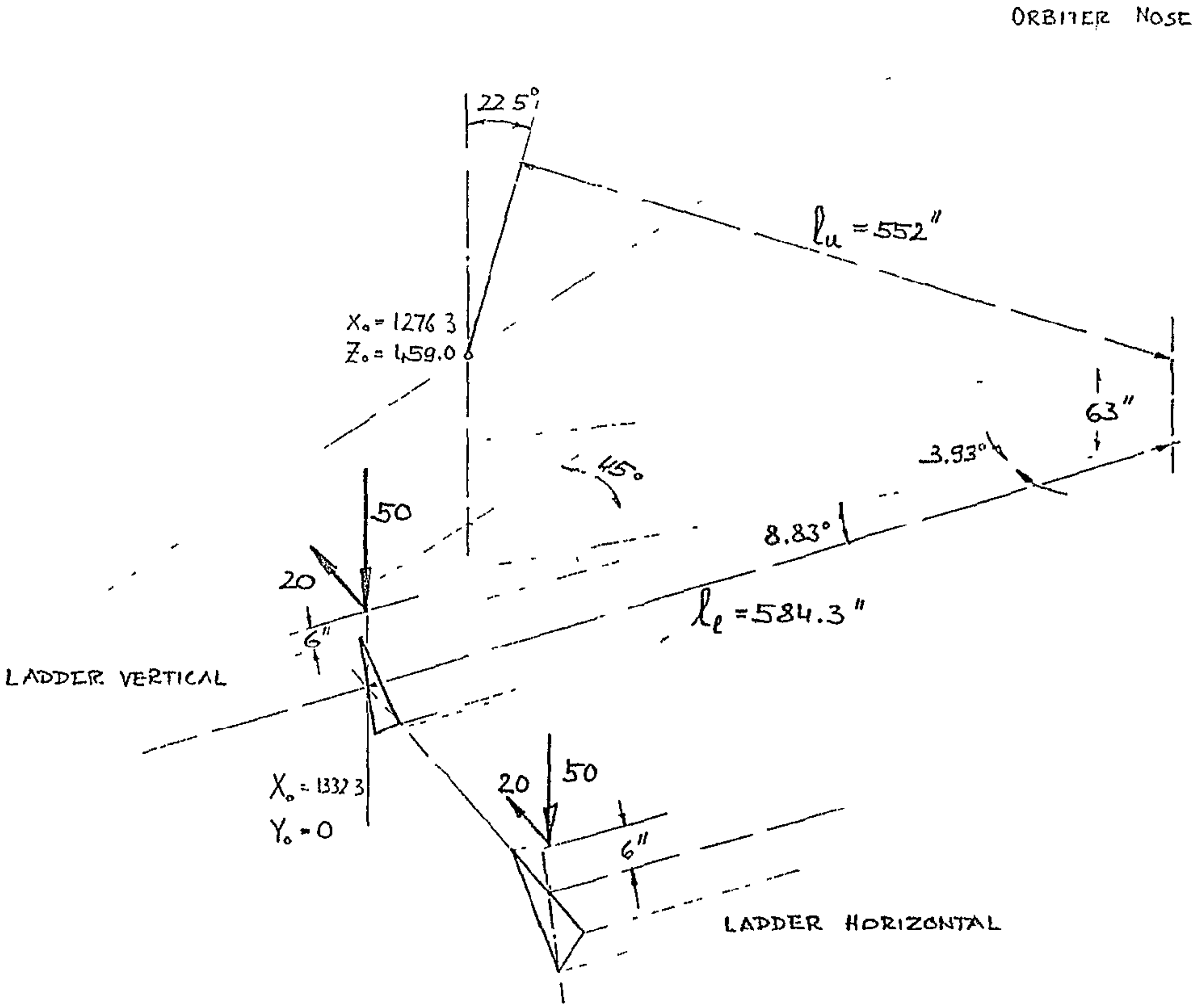
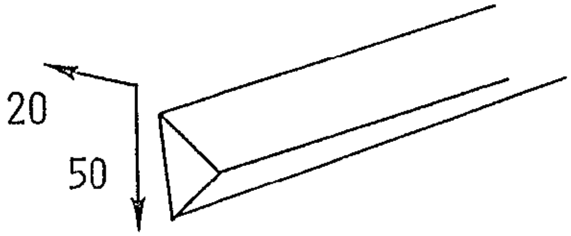


Figure 4-1. Standard applied load.

SENSE OF LOAD



50 LB IN -Z DIRECTION / 20 LB IN X-Y PLANE FORWARD

Q_1 50/20 (AS ILLUSTRATED ABOVE)

Q_2 50/-20

Q_3 -50/-20

Q_4 -50/20

Figure 4-2. Loading combinations.

SECTION 5

DEPLOYMENT

5.1 DEPLOYMENT SEQUENCE

The deployment sequence will be governed by the design of the cradle and shoulder which mount the booms to the Shuttle. The ESS was mounted at the center of its 35-foot length and deployed outward in both directions simultaneously. This is shown in Figure 1-2, Section 1. The center face hinge was hard mounted to the spacecraft and the actuating arms forced the synchronized scissor joints apart, thereby deploying the truss.

The deployment sequence envisioned for the EVATA starts with both booms and the elbow stowed and the entire package angled over at the required upper boom angle by the shoulder. The two face hinges at the end of the upper booms are hard mounted to the shoulder and the deployment mechanism, which could be a ball screw assembly, operates on the first synchronized scissor joint. The upper boom would be fully deployed and locked by an astronaut from the Shuttle bay. The spring-loaded latches in the scissors and knees will automatically lock when the boom is fully deployed. The astronaut would then translate along the upper boom to the elbow. If he wished to proceed further, he would deploy and lock the elbow. The deployment of the lower boom would be identical to the deployment of the upper boom.

5.2 DEPLOYMENT RATE

The deployment rate chosen for each boom is a tradeoff between spacecraft and boom dynamics and astronaut fatigue. Assuming enough mechanical advantage could be obtained with a small number of turns of the deployment device, the faster deployment rate will lower the EVA time and lessen astronaut fatigue. However, the deployment of the 46- and 63-foot booms will have an impact on spacecraft dynamics and may dictate an acceptable deployment rate. Also, the faster deployment rate results in lead screw angles that have poor back-driving characteristics. This is true for most of the common devices used to convert rotary motion to linear motion. With faster deployment rates, it may be necessary to have secondary locking devices to enable the astronaut to stop in a partially deployed position.

SECTION 6

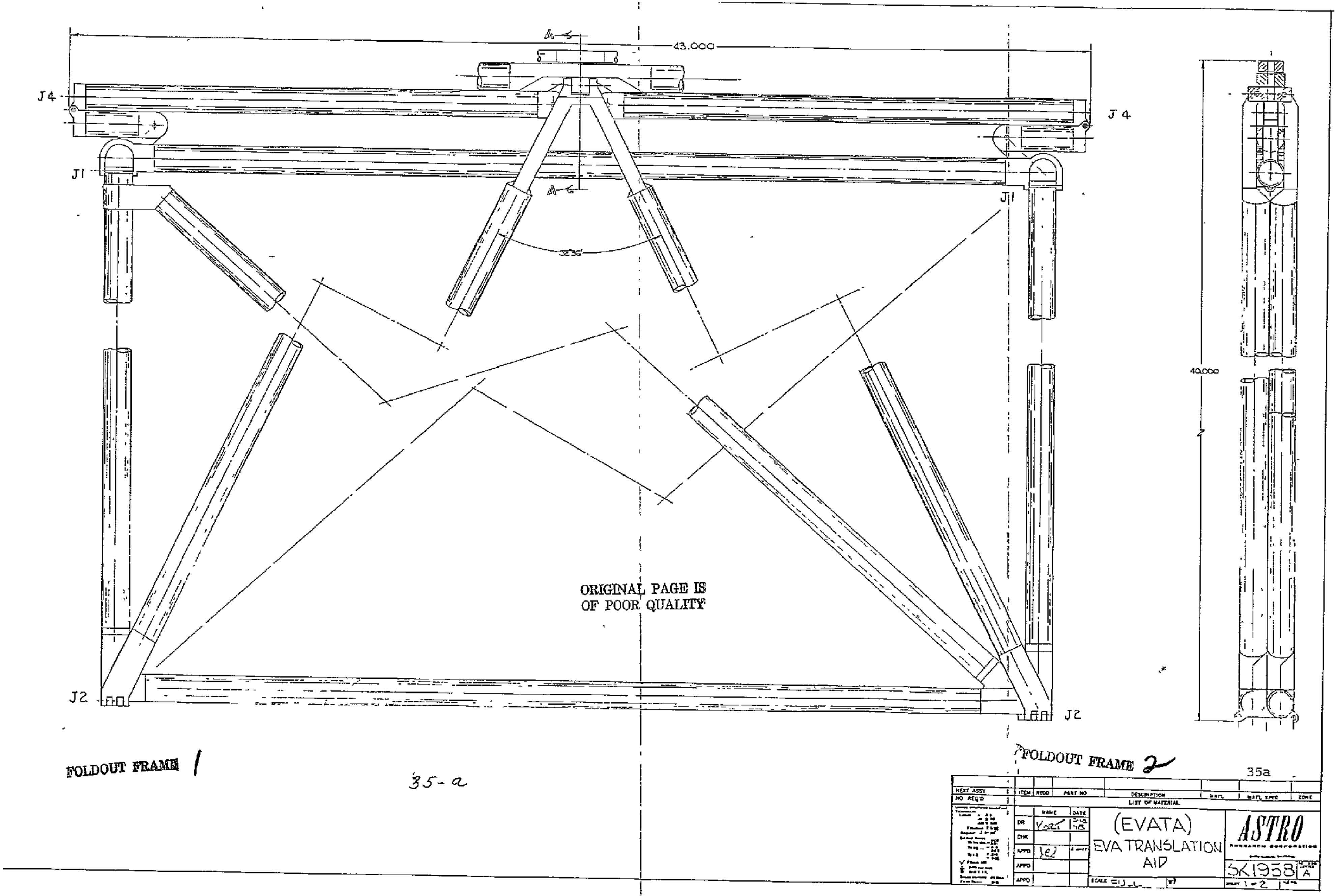
INTERFACES

6.1 ORBITER INTERFACES

The cradle and shoulder assemblies, as designed by JSC, interface directly with the Shuttle payload mounts. As described in Section 5.1, the upper boom is attached to the shoulder through the two face hinges and through the deployment actuator at the synchronized scissor joint. The configuration of the boom at the last bay can be seen in Astro SK 1958.

6.2 WORK STATION

The work station designs included in the Statement of Work (NASA-S-78-11292 and NASA-S-78-11293) were reviewed and found to be inadequate for transferring loads from the working astronaut to the truss structure efficiently. During the working sessions at NASA JSC, Astro proposed the use of a translatable roller-mounted structure which would pick up the sides of the face longerons of the truss and apply loads into the truss over four widely spaced contact points. NASA JSC has developed this concept as a preliminary design.



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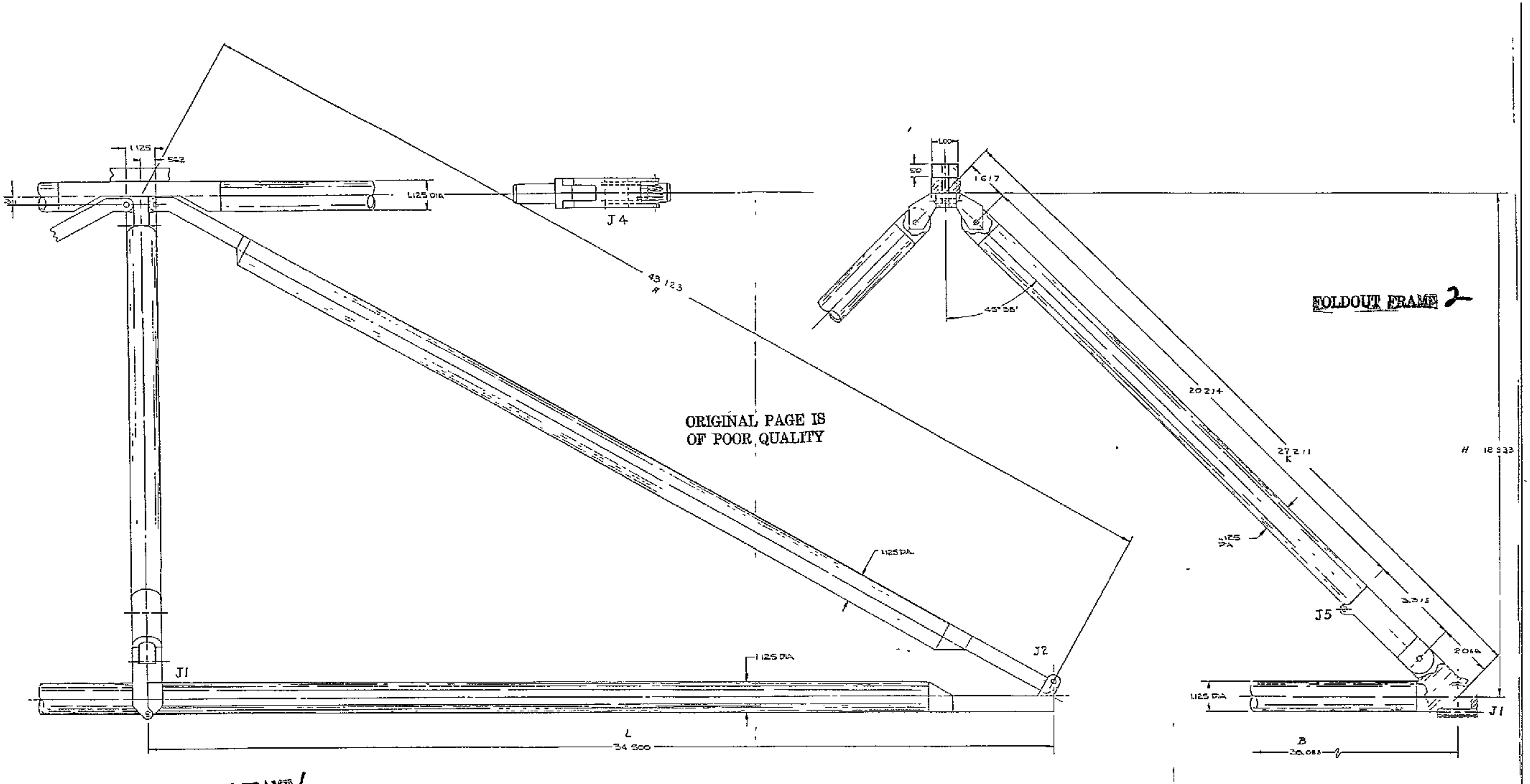

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35-b

NEXT ASSY | ITEM | REQD | PART NO | DESCRIPTION | MATL | MATL SPEC | ZONE
NO REQ D
LIST OF MATERIAL

	NAME	DATE
DR	KORT	5/18/76
CHK		
APPD	JCL	6-?-76
APPD		
APPD		

(EVATR)
EVR TRANSLATION
AID

ASTRO
RESEARCH CORPORATION

SK 1958
A

SCALE
WT
SHEET 2 of 2

SECTION 7

RETRACTION AND JETTISON

7.1 RETRACTION

The ESS, as developed and flown on SEASAT, is required to deploy only. The latches and the spring-loaded hinges were designed to deploy and lock. The EVATA structure has to retract as well as deploy. For retraction, the spring-loaded knee hinges and the spring-biased scissor latches have to be forced open. Figures 7-1 and 7-2 show the ESS latches and joints. Rather than add secondary mechanisms to force the hinges and latches open, Astro investigated using the existing synchronizer assembly (see Figure 7-3) to initiate and control the retraction. In the ESS, the synchronized scissor joint controls the deployment of each bay of the truss by operating on the A-frame assembly through a linkage system with a fixed relationship between scissor rotation and A-frame angle. If the synchronizer attachment point is moved from the A-frame to the knee strut, a reversal of scissor force will act to force open the spring-loaded knee hinges directly. The synchronizer linkage arrangement for the ESS was tailored to match the relationship between the A-frame angle and the scissor angle. Appendix I shows the knee brace synchronizer analysis necessary to tailor the rotation of the scissors to the required knee strut angle. The analysis was performed early in the program before the kinematic analysis determined the final dimensions for panel length and truss depth. The numbers chosen, while different from the proposed design, indicate the shape of the required synchronizer curve and the equations which were set up apply to the final design as proposed.

Astro has modified the half-scale ESS model with a knee brace synchronizer of the type proposed. The model proved the concept of initiating and controlling the retraction. The accuracy of synchronizing with the knee brace synchronizer was not determined due to the use of existing components which prevented the use of the ideal theoretical linkage dimensions.

7.2 JETTISON

The jettison problem was not considered as part of this study due to the jettison plane being moved to the NASA JSC portion of the structure.

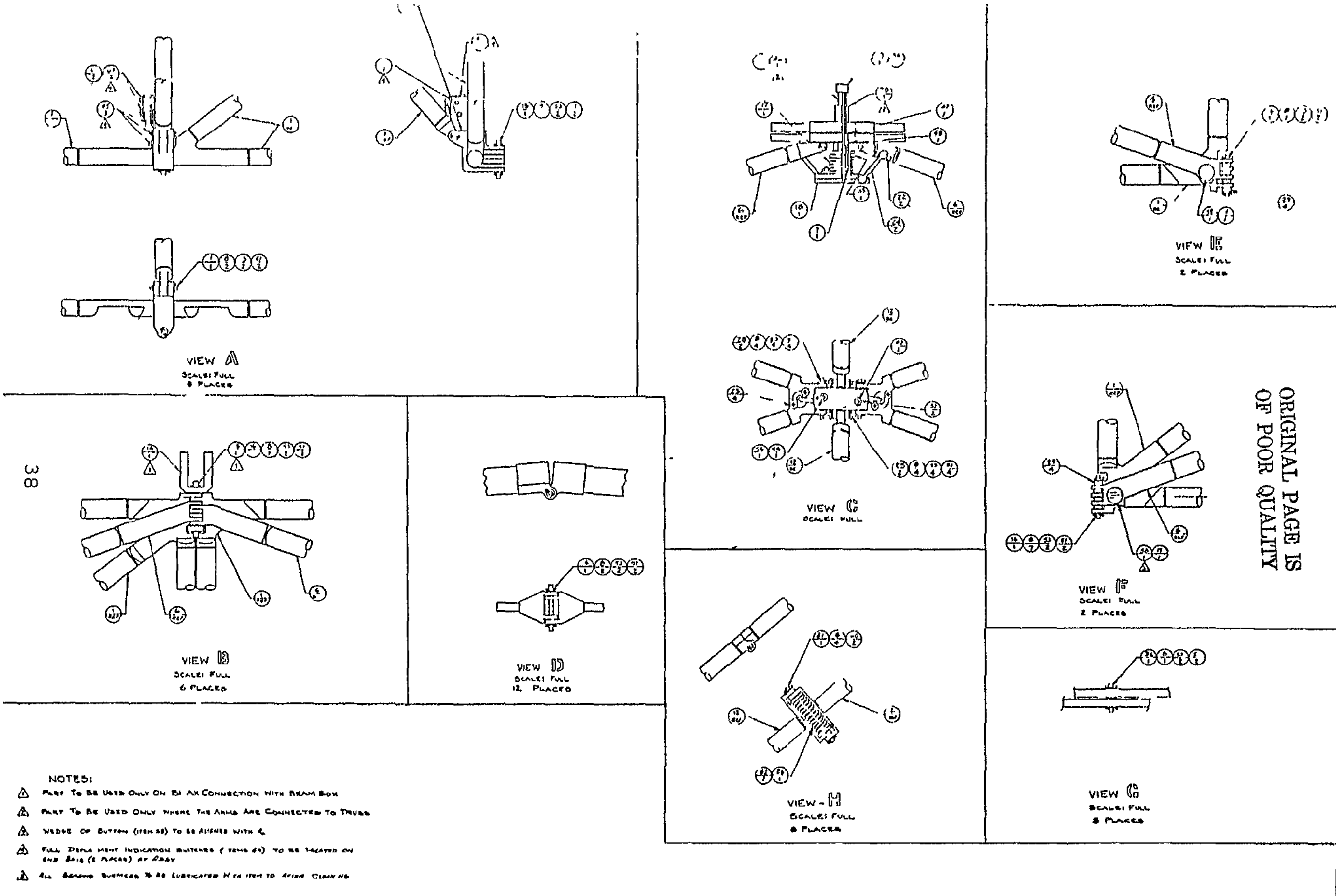

Figure 7-1. Existing ESS joints.

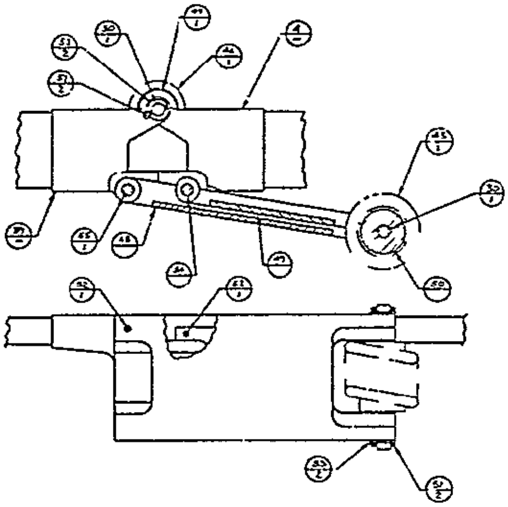

Figure 7-2. Longeron hinge.

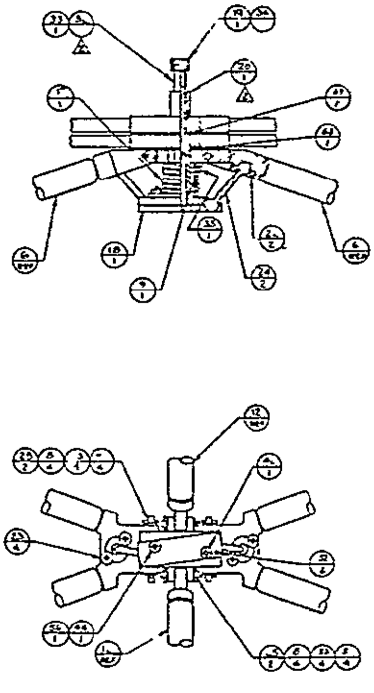

Figure 7-3. Synchronized joint.

SECTION 8

DEVELOPMENT PLAN AND SCHEDULE

8.1 PRELIMINARY MATERIAL SELECTION

The only truss material analyzed which meets the baseline requirements for weight (400 pounds) and deflection (6-inch system deflection at ET door) is graphite/epoxy tube. Aluminum alloy fittings were chosen due to cost and schedule advantages over titanium fittings. The materials combination used on the SEASAT ESS were graphite/epoxy tube and titanium fittings. The joints would be bonded using techniques developed on the SEASAT and JPL Voyager programs.

8.2 GRAPHITE/EPOXY EXPERIENCE

Astro Research Corporation has flight system experience on the SEASAT program and has successfully completed programs for the development of bonded joints between graphite/epoxy tubes and titanium fittings. The SEASAT program also included in-house preparation and control of graphite/epoxy tube material procurement, fabrication, and bonding specifications.

Through in-house Research and Development programs, Astro developed experimental Astromasts using in-house fabricated graphite/epoxy longerons. Table 8-1 summarizes the major parameters controlled by the graphite/epoxy procurement specifications.

8.3 INDUSTRY EXPERIENCE - GRAPHITE/EPOXY/ALUMINUM ALLOY FITTINGS

NASA JPL concluded a development program with the selection of graphite/epoxy tube and aluminum alloy fittings as the main structural members for the Voyager science boom and the antenna support structure. As part of the Voyager development program, JPL developed a bonded joint using a fiberglass interliner to compensate for the different thermal expansion between aluminum and graphite/epoxy. The joint was qualified by comparing room temperature joint strength before and after plunging the joints in LN_2.

8.4 REQUIRED EVATA COMPONENT DEVELOPMENT TESTS

Using the development history of the SEASAT ESS as a guide, the list of component development tests shown on Table 8-2 was generated. All the tests listed would be performed by Astro except the thermal coating evaluation.

8.5 REQUIRED EVATA SYSTEM TESTS

Table 8-3 shows the system tests required for the full-scale model and the flight deliverable EVATA. The full-scale model tests would be performed at Astro. The flight deliverable environmental testing and the deployment and retraction on an air bearing floor would be at NASA JSC.

8.6 SCHEDULE

The program schedule in Figure 8-1 shows a 24-week development program and a 14-week flight hardware production program. This program schedule was generated using the SEASAT development and production program as a guide.

TABLE 8-1. SEASAT A ESS
GRAPHITE/EPOXY COMPOSITE SPECIFICATIONS

Tubing	
Diameter:	0.500 in ± 0.005 in.
Wall thickness (approx.):	0.030 in.
Length:	72 in. ± 3 in.
Roundness (within):	0.005 in. on diameter
Bow:	Less than 0.1 in. over 6 ft
Strip	
Rectangular cross section:	0.25 in. x 0.75 in.
Flatness:	Less than 0.03 in.
Mechanical Properties	
Tensile modulus:	$>15 \times 10^6$ psi
Thermal Properties	
Coefficient of thermal expansion:	$\pm 0.5 \times 10^{-6}$ in/in/°F
Void Content:	<1%

TABLE 8-2. EVATA REQUIRED COMPONENT DEVELOPMENT TESTS

- HINGE DEVELOPMENT
  - 500 CYCLES UNDER LOAD FOR WEAR
- GRAPHITE/EPOXY ALUMINUM BOND JOINT TESTS
  - BONDS FOR NOMINAL STRENGTH
  - BONDS FOR DEGRADATION AFTER THERMAL SHOCK
- SINGLE BAY MODEL (INCLUDING DEPLOYMENT ACTUATOR)
  - DEPLOYMENT TESTS
  - ORIENTATION AND ALIGNMENT TESTS
  - OPERATION IN THERMAL VACUUM CHAMBER
- SYNCHRONIZER MODEL TESTS
  - SYNCHRONIZATION
  - MECHANICAL EFFICIENCY
- THERMAL COATING EVALUATION (JSC)

TABLE 8-3. REQUIRED EVATA SYSTEM TESTS

- FULL SCALE MODEL TESTS - LOWER ARM
  - DEPLOYMENT AND RETRACTION, 1G - AIR BEARING SUPPORTED
  - ORIENTATION AND ALIGNMENT
  - COMPLIANCE
  - WEIGHT AND CENTER OF GRAVITY - STOWED
  - NATURAL FREQUENCY
  - CENTER OF GRAVITY - DEPLOYED

- FLIGHT DELIVERABLE - EVATA
  - LAUNCH AND BOOST, VIBRATION, AND ACCELERATION - STOWED
  - DEPLOYMENT AND RETRACTION, 1G - AIR BEARING SUPPORTED
  - SYSTEM WEIGHT AND CENTER OF GRAVITY - STOWED

PLANNING SHEET ASTRO RESEARCH CORPORATION SANTA BARBARA, CALIF.

Figure 8-1 Program schedule.

- EVATA -

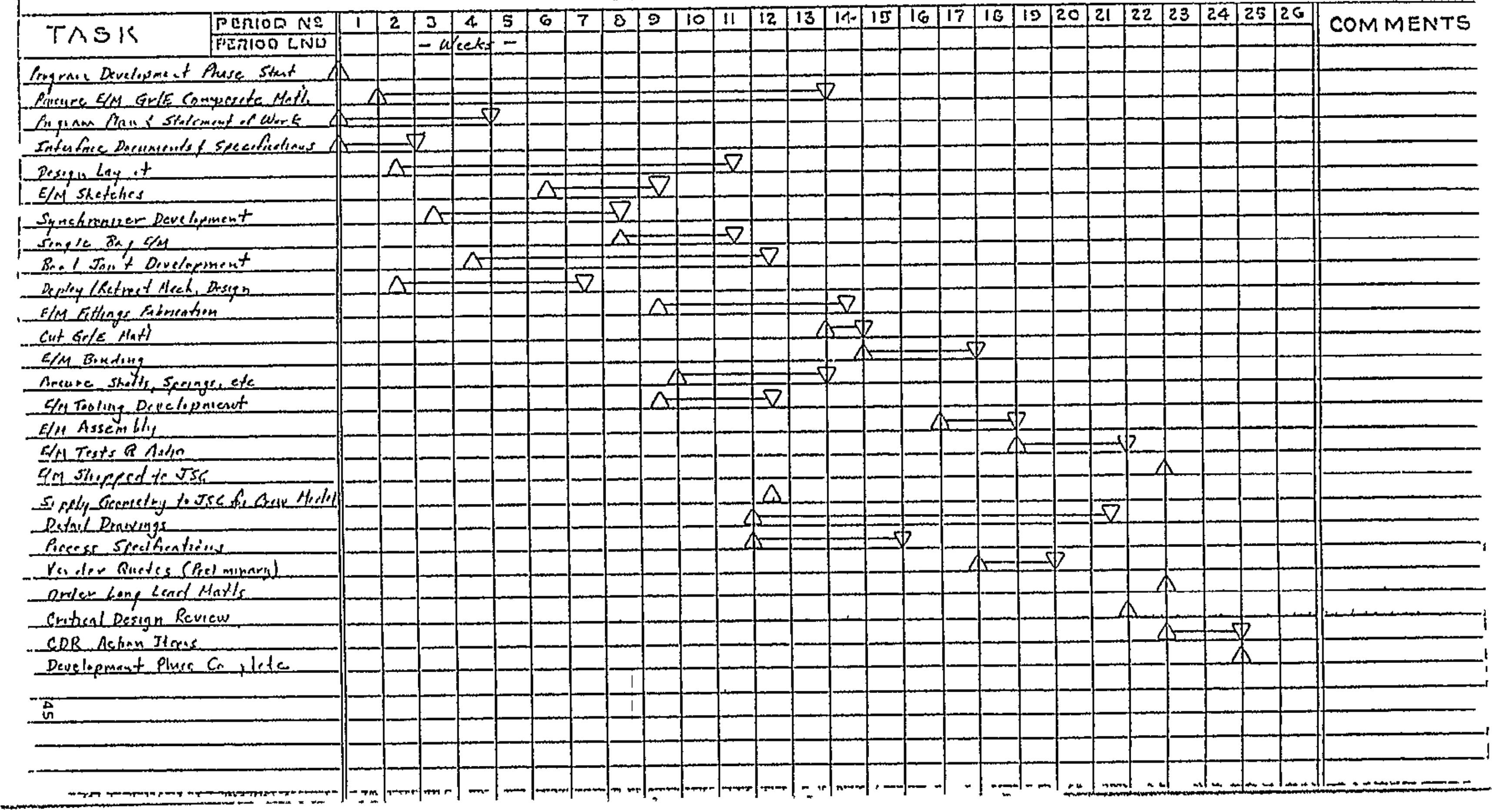

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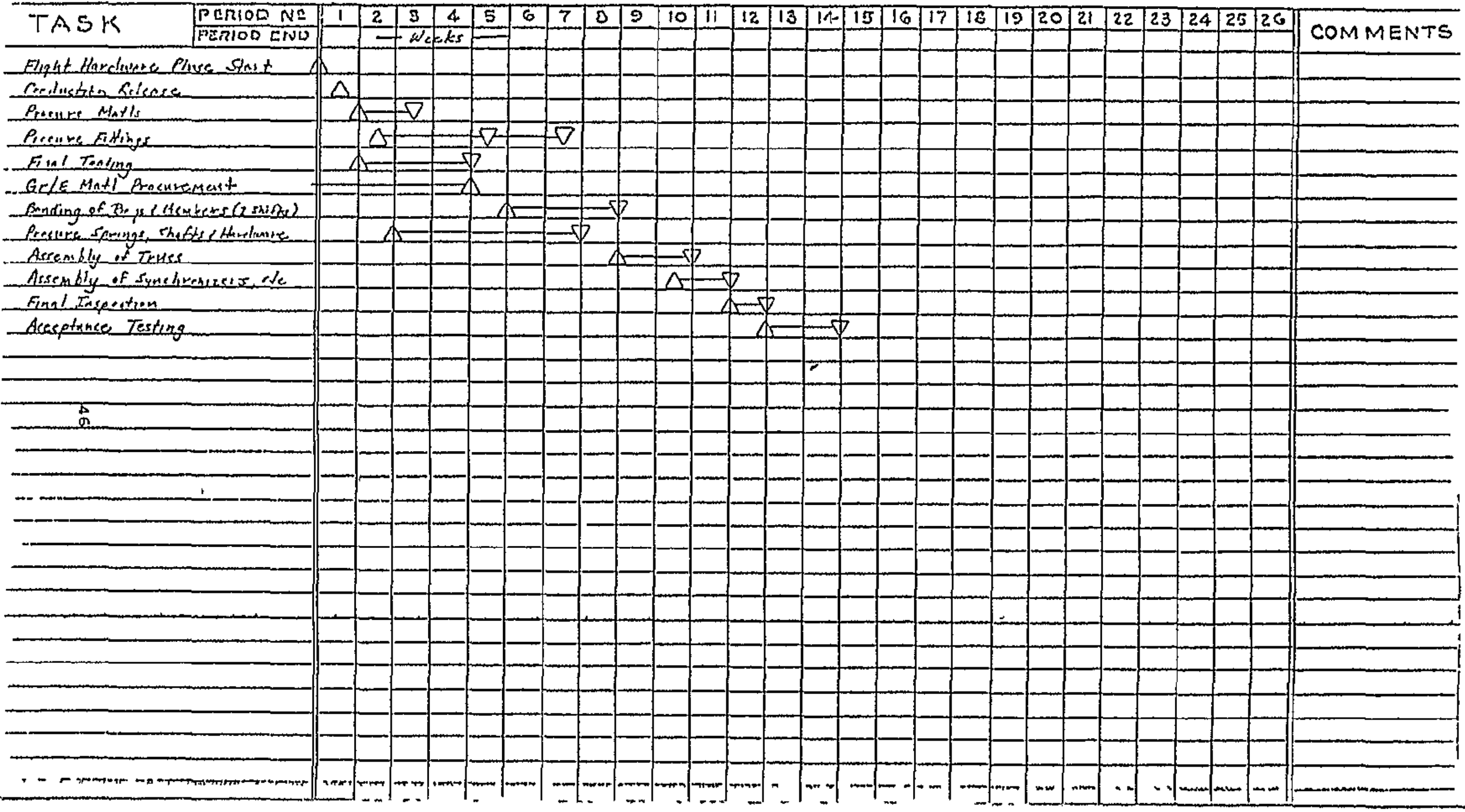
Figure 8-1 Continued

- EVATA -



SECTION 9

EVATA COSTS

Table 9-1 details the budgetary cost estimate for each phase of the EVATA development and production program. It also gives a comparison of costs using graphite/epoxy tubing or aluminum tubing as the basic truss material.

TABLE 9-1. ESTIMATED EVATA PROGRAM COSTS

PHASE I	DESIGN, DEVELOPMENT, FABRICATION, AND TESTING OF ENGINEERING MODEL UNIT	
	A. USING GRAPHITE/EPOXY TUBING (INCLUDES PROCUREMENT OF GRAPHITE/EPOXY FOR FLIGHT HARDWARE)	$1,075,801
	B. USING ALUMINUM TUBING	418,654
PHASE II	FABRICATION AND TEST OF FLIGHT UNIT	
	A. USING GRAPHITE/EPOXY TUBING (PURCHASED IN PHASE I)	278,654
	B. USING ALUMINUM TUBING	285,039
TOTAL PROGRAM COSTS, GRAPHITE/EPOXY TUBING		1,354,455
TOTAL PROGRAM COSTS, ALUMINUM TUBING		703,693

SECTION 10

CONCLUSIONS AND RECOMMENDATIONS

The primary purpose of this study contract was to perform a preliminary design of a deployable structure to allow an astronaut to perform tasks in the vicinity of the ET umbilical doors. As a secondary goal, the EVATA allows inspection of the entire underside of the Shuttle spacecraft. This secondary goal requires that the boom lengths be increased to reach the ET doors from the center of the spacecraft. As shown in the analysis sections of this report, the length of the booms is the major driver in terms of system weight and cost. Because of this length effect, the secondary function of the EVATA has dictated a very large and expensive structure.

If the system was designed solely for the primary function of ET door inspection and repair, the original NASA JSC concept of using two short arms over the trailing edge of the wing is the optimum from a standpoint of weight and cost. Going over the trailing edge of the wing results in deployable booms similar in length to the SEASAT A ESS.

It is Astro's recommendation that the original concept of going over the trailing edge be investigated to fulfill the primary mission of ET door inspection and repair. A rough-order-of-magnitude analysis shows that an all aluminum deployable truss-type structure can be designed and built within the weight and cost budgets of 800 pounds and $500,000, respectively. This would include upper and lower booms, the elbow, and all the structure necessary to mount the booms to the Shuttle keel and longeron payload mounting points. The entire structure would stow within the last 48 inches of the Shuttle payload bay. The design, fabrication, and testing of this structure could be handled entirely by Astro with little or no impact on NASA JSC in-house programs and personnel.

APPENDIX A

RELATIONS BETWEEN INTERNAL AND EXTERNAL FORCES

Relations between Internal and External Forces

$$P_1 \cancel{- \frac{l}{d}P_d} + P_2 + P_3 \cancel{\pm \frac{l}{a}(P_A + P_B)} \cancel{\pm \frac{l}{d}P_d} \cancel{- \frac{l}{a}}(P_A - P_B) = P$$

$$-\frac{b}{d}P_d \pm \frac{b}{2a}(P_A - P_B) = V_2$$

$$\frac{h}{a}(P_A + P_B) = V_1$$

$$\frac{bh}{2a}(P_A - P_B) = Q$$

$$-\left(P_3 \cancel{+ \frac{l}{a}(P_A + P_B)}\right)h \qquad -\frac{l}{a}(P_A + P_B)\frac{h}{l}\left(1 - \frac{x}{l}\right) = M_1 = M_{10} - V_1 x$$

$$\left(P_1 - \frac{l}{d}P_d - P_2\right)\frac{l}{2} \qquad -\frac{b}{d}P_d\frac{l}{2}\left(1 - 2\frac{x}{l}\right) - \frac{l}{a}(P_A - P_B)\frac{b}{2}\frac{x}{l} = M_2 = M_{20} - V_2 x$$

Thus,

$$P_1 + P_2 - P_3 = P$$

$$-\frac{b}{d}P_d + \frac{b}{2a}(P_A - P_B) = V_2$$

$$\frac{h}{a}(P_A + P_B) = V_1$$

$$\frac{bh}{2a}(P_A - P_B) = Q$$

$$-P_3 h = M_{10}$$

$$(P_1 - P_2)\frac{l}{2} \qquad -\frac{bl}{d}P_d = M_{20}$$

Then,

$$-\frac{b}{d}P_d = V_2 - \frac{Q}{h}$$

$$\text{or } P_d = \frac{d}{b}\left(\frac{Q}{h} - V_2\right)$$

$$P_3 = -\frac{M_{10}}{h}$$

$$P_1 + P_2 = P + \frac{M_{10}}{h}$$

$$P_1 - P_2 = \frac{2}{b}\left[M_{20} + \ell\left(\frac{Q}{h} - V_2\right)\right]$$

$$P_A - P_B = \frac{2a}{bh} Q$$

$$P_A + P_B = \frac{a}{h} V_1$$

$$P_1 = \frac{P}{2} + \frac{M_{10}}{2h} + \frac{M_{20}}{b} + \frac{\ell}{bh} Q - \frac{\ell}{b} V_2$$

$$P_2 = \frac{P}{2} + \frac{M_{10}}{2h} - \frac{M_{20}}{b} - \frac{\ell}{bh} Q + \frac{\ell}{b} V_2$$

$$P_A = \frac{a}{2h} V_1 + \frac{a}{bh} Q$$

$$P_B = \frac{a}{2h} V_1 - \frac{a}{bh} Q$$

Also, for S E

$$2P_1^2 + 2P_2^2 = \left(P + \frac{M_{10}}{h}\right)^2 + \left(\frac{2M_{20}}{b} + \frac{2\ell}{bh} Q - 2\frac{\ell}{b} V_2\right)^2$$

and $2P_A^2 + 2P_B^2 = \left(\frac{2a}{bh} Q\right)^2 + \left(\frac{a}{h} V_1\right)^2$

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APPENDIX B

CHARACTERIZATION OF TRUSS BY CONTINUOUS BEAM

Characterization of Truss by Continuum Beam

Strain Energy for Length $2l$

1 Truss

$$SE/\text{module} = \frac{l}{2(EA)_l}\left[\left(P_1 - \frac{l}{d}P_d\right)^2 + \left(P_1 - 2\frac{l}{d}P_d\right)^2 + P_2^2 + \left(P_2 + \frac{l}{d}P_d\right)^2\right]$$

$$+ \frac{l}{2(EA)_s}\left[\left(P_3 + \frac{l}{a}(P_A + P_B)\right)^2 + \left(P_3 - \frac{l}{a}(P_A + P_B)\right)^2\right]$$

$$+ \frac{a}{2(EA)_a}\left[2P_A^2 + 2P_B^2\right]$$

$$+ \frac{d}{2(EA)_d}\, 2P_d^2$$

$$+ \frac{b}{2(EA)_{b_1}}\left[\frac{l}{d}P_d\right]^2$$

$$+ \frac{b}{2(EA)_{b_2}}\left[\frac{l}{d}P_d\right]^2$$

$$= \frac{l}{2(EA)_l}\left[2P_1^2 - 6\frac{l}{d}P_1P_d + 6\left(\frac{l}{d}\right)^2 P_d^2 + 2P_2^2 + 2\frac{l}{d}P_2P_d\right]$$

$$+ \frac{l}{2(EA)_s}\left[2P_3^2 + 2\left(\frac{l}{a}\right)^2(P_A + P_B)^2\right]$$

$$+ \frac{a}{2(EA)_a}\left[2P_A^2 + 2P_B^2\right]$$

$$+ \frac{d}{2(EA)_d}\, 2P_d^2$$

$$+ \frac{l}{2}\left(\frac{1}{(EA)_{b_1}} + \frac{1}{(EA)_{b_2}}\right)\left(\frac{l}{d}\right)^2 P_d^2$$

$$S\ E/\text{module} = \frac{\ell}{2(EA)_\ell}\Big[\Big(P + \frac{M_{10}}{h}\Big)^2 + \Big(\frac{2M_{20}}{b} + \frac{2\ell}{bh}Q - 2\frac{\ell}{b}V_2\Big)^2 + 6\Big(\frac{\ell}{b}\Big)^2\Big(\frac{Q}{h} - V_2\Big)^2$$

$$+ 2\frac{\ell}{b}\Big(\frac{Q}{h} - V_2\Big)\Big\langle \frac{P}{2} + \frac{M_{10}}{2h} - \frac{M_{20}}{b} - \frac{\ell}{bh}Q + \frac{\ell}{b}V_2$$

$$- 3\frac{P}{2} - 3\frac{M_{10}}{2h} - 3\frac{M_{20}}{b} - 3\frac{\ell}{bh}Q + 3\frac{\ell}{b}V_2\Big\rangle\Big]$$

$$+ \frac{\ell}{2(EA)_s}\Big[2\frac{M_{10}^2}{h^2} + 2\frac{\ell^2}{h^2}V_1^2\Big]$$

$$+ \frac{a}{2(EA)_a}\Big[4\frac{a^2}{b^2h^2}Q^2 + \frac{a^2}{h^2}V_1^2\Big]$$

$$+ \frac{d}{2(EA)_d}\Big[2\frac{d^2}{b^2}\Big(\frac{Q}{h} - V_2\Big)^2\Big]$$

$$+ \frac{b}{2}\Big(\frac{1}{(EA)_{b_1}} + \frac{1}{(EA)_{b_2}}\Big)\Big(\frac{Q}{h} - V_2\Big)^2$$

$$= \frac{\ell}{2(EA)_\ell}\Big[P^2 + \frac{M_{10}^2}{h^2} + 4\frac{M_{20}^2}{b^2} + 4\frac{\ell^2}{b^2h^2}Q^2 + 4\frac{\ell^2}{b^2}V_2^2 + 2P\frac{M_{10}}{h} + 8\frac{\ell}{b^2h}M_{20}Q - 8\frac{\ell}{b^2}M_{20}V_2 - 8\frac{\ell^2}{b^2h}QV_2$$

$$+ 6\frac{\ell^2}{b^2h^2}Q^2 + 6\frac{\ell^2}{b^2}V_2^2 \qquad - 12\frac{\ell^2}{b^2h}QV_2$$

$$- 8\frac{\ell^2}{b^2h^2}Q^2 - 8\frac{\ell^2}{b^2}V_2^2 \qquad - 8\frac{\ell}{b^2h}M_{20}Q + 8\frac{1}{\ell^2}M_{20}V_2 + 16\frac{\ell^2}{b^2h}$$

$$- 2\frac{\ell}{bh}PQ - 2\frac{\ell}{bh^2}M_{10}Q + 2\frac{\ell}{bh}V_2M_{10} - 2\frac{\ell}{b}PV_2$$

+

$$S\,E\ /\text{module} = M_{10}^2\left(\frac{\ell}{2h^2(EA)_\ell} + \frac{\ell}{h^2(EA)_s}\right)$$

$$+ M_{20}^2\,\frac{2\ell}{b^2(EA)_\ell}$$

$$+ Q^2\left[\frac{\ell^3}{b^2h^2(EA)_\ell} + 2\frac{a^3}{b^2h^2(EA)_a} + \frac{d^3}{b^2h^2(EA)_d} + \frac{b^3}{b^2h^2}\frac{1}{2}\left(\frac{1}{(EA)_{b_1}} + \frac{1}{(EA)_{b_2}}\right)\right]$$

$$+ P^2\,\frac{\ell}{2(EA)_\ell}$$

$$+ V_1^2\left[\frac{\ell^3}{h^2(EA)_s} + \frac{1}{2}\frac{a^3}{h^2(EA)_a}\right]$$

$$+ V_2^2\left[\frac{\ell^3}{b^2(EA)_\ell} + \frac{d^3}{b^2(EA)_d} + \frac{\ell}{2}\left(\frac{1}{(EA)_{b_1}} + \frac{1}{(EA)_{b_2}}\right)\right]$$

$$+ P M_{10}\,\frac{\ell}{h\,(EA)_\ell}$$

$$- P Q\,\frac{\ell^2}{bh(EA)_\ell}$$

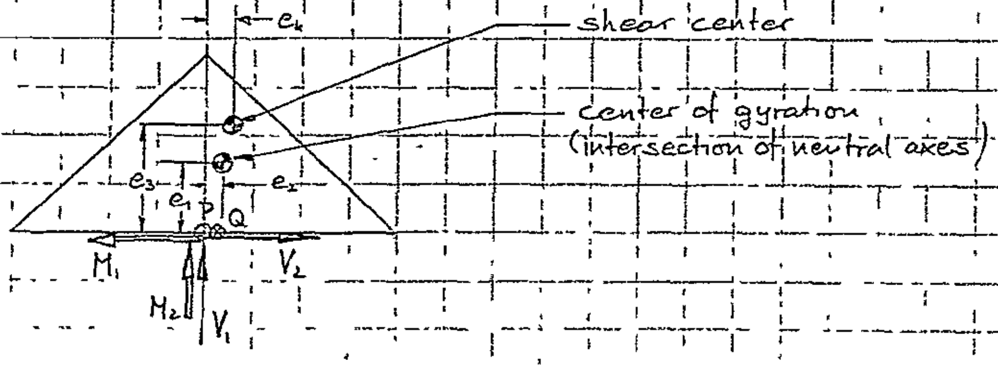
$$+ P V_2\,\frac{\ell^2}{h\,(EA)_\ell}$$

$$+ V_2 M_{10}\,\frac{\ell^2}{\ell h\,(EA)_\ell}$$

$$- V_2 Q\left[2\frac{\ell^3}{b^2h\,(EA)_\ell} + 2\frac{d^3}{b^2h\,(EA)_d} + \frac{b^3}{\ell^2 h}\left(\frac{1}{(EA)_{b_1}} + \frac{1}{(EA)_{b_2}}\right)\right]$$

$$- M_{10} Q\,\frac{\ell^2}{b\,h^2(EA)_\ell}$$

2. Equivalent Continuous Beam



$$S\,E/2\ell = \frac{1}{2}\int_{-\ell}^{\ell} dx \left[\frac{(M_{10} - V_1 x + P e_1)^2}{EI_1} + \frac{(M_{20} - V_2 x + P e_2)^2}{EI_2} + \frac{(Q + V_1 e_4 - V_2 e_3)^2}{GJ} \right.$$

$$\left. + \frac{P^2}{EA} + \frac{V_1^2}{GA_{s1}} + \frac{V_2^2}{GA_{s2}} + \text{“asymmetrical” terms} \right]$$

$$= \ell \left\{ \frac{(M_{10} + P e_1)^2}{EI_1} + \frac{\ell^2}{3}\frac{V_1^2}{EI_1} + \frac{(M_{20} + P e_2)^2}{EI_2} + \frac{\ell^2}{3}\frac{V_2^2}{EI_2} + \frac{(Q + V_1 e_4 - V_2 e_3)^2}{GJ} \right.$$

$$\left. + \frac{P^2}{EA} + \frac{V_1^2}{GA_{s1}} + \frac{V_2^2}{GA_{s2}} + \text{“asymmetrical” terms} \right]$$

$$= M_{10}^2 \frac{\ell}{EI_1} + M_{20}^2 \frac{\ell}{EI_2} + Q^2 \frac{\ell}{GJ} + P M_{10} \frac{2\ell e_1}{EI_1} + P M_{20} \frac{2\ell e_2}{EI_2}$$

$$+ P^2 \left(\frac{e_1^2 \ell}{EI_1} + \frac{e_2^2 \ell}{EI_2} + \frac{\ell}{EA} \right)$$

$$+ V_1^2 \left(\frac{\ell^3}{3EI_1} + \frac{\ell}{GA_{s1}} + \frac{\ell e_4^2}{GJ} \right)$$

$$+ V_2^2 \left(\frac{\ell^3}{3EI_2} + \frac{\ell}{GA_{s2}} + \frac{\ell e_3^2}{GJ} \right)$$

$$+ V_1 Q \frac{2\ell e_4}{GJ} - V_2 Q \frac{2\ell e_3}{GJ} - V_1 V_2 \frac{2\ell e_3 e_4}{GJ} + \text{“asymmetric” terms}$$

3 Comparison of Terms   Truss ⟷ Equivalent Beam

$$M_{10}^2 \quad \frac{\ell}{h^2}\left(\frac{1}{2(EA)_\ell} + \frac{1}{(EA)_1}\right) = \frac{\ell}{EI_1}$$

$$M_{20}^2 \quad \frac{2\ell}{b^2}\frac{1}{(EA)_\ell} = \frac{\ell}{EI_2}$$

$$Q^2 \quad \frac{\ell^3}{b^2 h^2}\left[\frac{1}{(EA)_\ell} + 2\frac{a^3}{\ell^3}\frac{1}{(EA)_a} + \frac{d^3}{\ell^3}\frac{1}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{1}{(EA)_{b_1}} + \frac{1}{(EA)_{b_2}}\right)\right] = \frac{\ell}{GJ}$$

$$PM_{10} \quad \frac{\ell}{h(EA)_\ell} = \frac{2\ell e_1}{EI_1}$$

$$PM_{20} \quad 0 = \frac{2\ell e_2}{EI_2}$$

$$P^2 \quad \frac{\ell}{2(EA)_\ell} = \ell\left(\frac{e_1^2}{EI_1} + \frac{e_2^2}{EI_2} + \frac{1}{EA}\right)$$

$$V_1^2 \quad \frac{\ell^3}{h^2}\left(\frac{1}{(EA)_s} + \frac{1}{2}\frac{a^3}{\ell^3}\frac{1}{(EA)_a}\right) = \ell\left(\frac{\ell^2}{3EI_1} + \frac{1}{GA_{s1}} + \frac{e_4^2}{GJ}\right)$$

$$V_2^2 \quad \frac{\ell^3}{b^2}\left(\frac{1}{(EA)_\ell} + \frac{d^3}{\ell^3}\frac{1}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{1}{(EA)_{b_1}} + \frac{1}{(EA)_{b_2}}\right)\right) = \ell\left(\frac{\ell^2}{3EI_2} + \frac{1}{GA_{s2}} + \frac{e_3^2}{GJ}\right)$$

$$V_1 Q \quad 0 = \frac{2\ell e_4}{GJ}$$

$$V_2 Q \quad 2\frac{\ell^3}{b^2 h}\left[\frac{1}{(EA)_\ell} + \frac{d^3}{\ell^3}\frac{1}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{1}{(EA)_{b_1}} + \frac{1}{(EA)_{b_2}}\right)\right] = 2\frac{\ell e_3}{GJ}$$

$$V_1 V_2 \quad 0 = -2\frac{\ell e_3 e_4}{GJ}$$

thus,

$$\frac{1}{EI} = \frac{1}{h^2}\left(\frac{1}{2(EA)_\ell} + \frac{1}{(EA)_s}\right)$$

$$\frac{1}{EI_2} = \frac{2}{b^2}\frac{1}{(EA)_\ell}$$

$$\frac{1}{GJ} = \frac{\ell^2}{b^2 h^2}\frac{1}{(EA)_\ell}\left[1 + 2\frac{a^3}{\ell^3}\frac{(EA)_\ell}{(EA)_a} + \frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right]$$

$$e_1 = \frac{EI_1}{2h(EA)_\ell} = \frac{h}{2}\frac{\frac{1}{(EA)_\ell}}{\left(\frac{1}{2(EA)_\ell} + \frac{1}{(EA)_s}\right)} = h\frac{1}{\left(1 + 2\frac{(EA)_\ell}{(EA)_s}\right)}$$

$$e_2 = 0$$

$$\frac{1}{EA} = \frac{1}{2(EA)_\ell} - \frac{e_1^{\,2}}{EI_1} = \frac{1}{2(EA)_\ell} - \frac{\cancel{h^2}}{\left(1+2\frac{(EA)_\ell}{(EA)_s}\right)^{\cancel{2}}}\frac{1}{\cancel{h^2}}\frac{1}{2(EA)_\ell}\cancel{\left(1 + 2\frac{(EA)_\ell}{(EA)_s}\right)}$$

$$= \frac{1}{2(EA)_\ell}\left(1 - \frac{1}{1 + 2\frac{(EA)_\ell}{(EA)_s}}\right) = \frac{1}{(EA)_s + 2(EA)_\ell}$$

$$e_3 = \frac{\frac{\ell^2}{b^2 h}\cancel{\frac{1}{(EA)_\ell}}\left[1 - \frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right]}{\frac{\ell^2}{b^2 h^{\cancel{2}}}\cancel{\frac{1}{(EA)_\ell}}\left[1 + 2\frac{a^3}{\ell^3}\frac{(EA)_\ell}{(EA)_a} + \frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right]}$$

$$= h\frac{\left[1 + \frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right]}{\left[1 + 2\frac{a^3}{\ell^3}\frac{(EA)_\ell}{(EA)_a} + \frac{d^3}{\ell^3}\frac{(EA)_\ell}{(EA)_d} + \frac{1}{2}\frac{b^3}{\ell^3}\left(\frac{(EA)_\ell}{(EA)_{b_1}} + \frac{(EA)_\ell}{(EA)_{b_2}}\right)\right]}$$

$$e_4 = 0$$

abbreviations $\frac{a^3}{l^3}\frac{(EA)_l}{(EA)_a} = k_a$

$$\frac{d^3}{l^3}\frac{(EA)_l}{(EA)_d} = k_d$$

$$\frac{b^3}{l^3}\left(\frac{(EA)_l}{(EA)_{b_1}}+\frac{(EA)_l}{(EA)_{b_2}}\right) = k_b$$

$$\frac{(EA)_l}{(EA)_s} = k_s$$

then,

$$\frac{1}{GA_{s1}} = \frac{l^2}{h^2}\left(\frac{1}{(EA)_s}+\frac{1}{2}\frac{a^3}{l^3}\frac{1}{(EA)_a}\right)-\frac{l^2}{3EI_1} = \frac{l^2}{h^2(EA)_l}\left[k_s+\frac{1}{2}k_a-\frac{1}{6}-\frac{1}{3}k_s\right]$$

$$= \frac{l^2}{6h^2(EA)_l}\left[4k_s+3k_a-1\right]$$

$$\frac{1}{GA_{s2}} = \frac{l^2}{b^2(EA)_l}\left[1+k_d+\frac{1}{2}k_b\right]-\frac{l^2}{3EI_2}-\frac{e_3^2}{GJ}$$

$$= \frac{l^2}{b^2(EA)_l}\left[1+k_d+\frac{1}{2}k_b-\frac{2}{3}\right]-\frac{h^2l^2}{b^2h^2(EA)_l}\frac{\left[1+k_d+\frac{1}{2}k_b\right]^2}{\left[1+2k_a-k_d+\frac{1}{2}k_b\right]^{\cancel{2}}}\left[1+\cancel{2k_a+k_d}+\frac{1}{2}k_b\right]$$

$$= \frac{l^2}{b^2(EA)_l}\left\{\left[\frac{1}{3}+k_d-\frac{1}{2}k_b\right]-\frac{\left[1+k_d+\frac{1}{2}k_b\right]^2}{\left[1+2k_a+k_d+\frac{1}{2}k_b\right]}\right\}$$

$$= \frac{l^2}{l^2(EA)_l}\frac{\frac{1}{3}(1+2k_a)+(\frac{1}{3}+1+2k_a+k_d+\frac{1}{2}k_b)-1-2k_d+\frac{1}{2}k_b)}{1+2k_a+k_d+\frac{1}{2}k_b}$$

$$= \frac{l^2}{b^2(EA)_l}\frac{\frac{2}{3}k_a-\frac{2}{3}+(k_d+\frac{1}{2}k_b)(2k_a-\frac{2}{3})}{1+2k_a+k_d+\frac{1}{2}k_b}$$

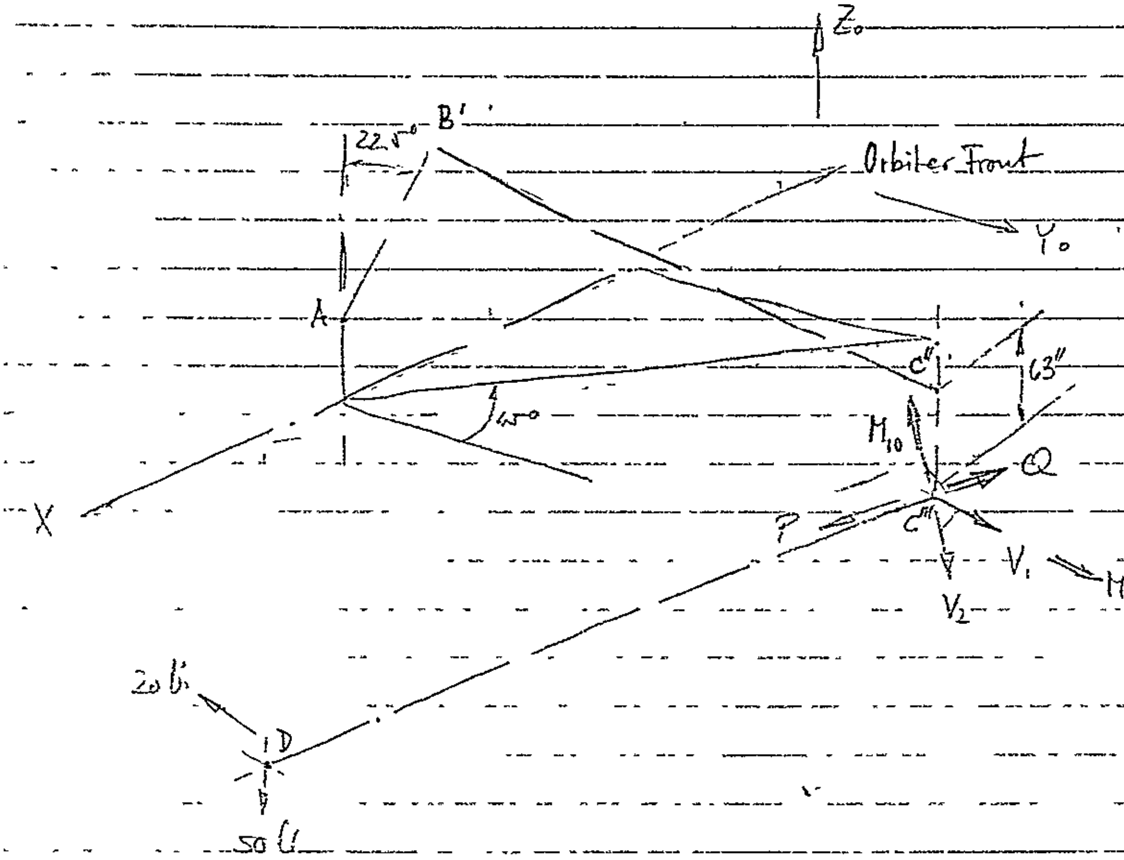
$$= \frac{2}{3}\frac{l^2}{b^2(EA)_l}\frac{\left[(k_a-1)+(3k_a-1)(k_d+\frac{1}{2}k_b)\right]}{\left[1+2k_a+k_d+\frac{1}{2}k_b\right]}$$

APPENDIX C

EXTERNAL FORCES ALONG LOWER AND UPPER ARMS

External Forces along Lower and Upper Arms

(17)
5-12-78



unit vector in V_2 direction

$$(-\sin 8°50' \sin 48.93°,\ \sin 8°50' \cos 48.93°,\ -\cos 8°50')$$

$$\bar{v}_2 = (-.1158,\ .1009,\ -.9881)$$

in V direction $= \left(\bar{v}_3 \times \bar{v}_2\right) \frac{1}{|\bar{v}_3 \times \bar{v}_2|} = (.6570,\ .7539,\ 0)$

$$\bar{v}_1 = (\cos 48.93°,\ \sin 48.93°,\ 0)$$

Thus, $V_{1\ell} = \bar{F} \cdot \bar{v}_1 = -20 \cdot \bar{F}_x \cos 48.93 + \bar{F}_y \sin 48.93$ $\quad \bar{F} = (-20 \cos 48.93°, -20 \sin 48.93°, -50)$

$V_{2\ell} = \bar{F} \cdot \bar{v}_2 = 60.4$ [illegible]

$P_\ell = \bar{F} \cdot \bar{v}_3 = 7.7$ [illegible]

[illegible] $Q_\ell = -20\left(\frac{h}{2} + 6\right)$ $\quad X = h_\ell = 584.33$

$M_{1\ell} = 0$

$M_{2\ell} = 0$

5-12-72

at C''' $V_{10\ell} = -20$ $X_{lower} = 0$

$V_{20\ell} = 49.4$

$P_{0\ell} = 7.7$

$M_{10\ell} = V_{1\ell} \times 584.33 = -11,687$ in lb

$M_{20\ell} = 49.4 \times 584.33 = 28,866$ in lb

$Q_{0\ell} = -20\left(\frac{b}{2} + 6\right) = -20 \times 25.48 \approx -510$

transformation to upper arm

rotate up 8°50'

$V_{10\ell} \rightarrow V_{10\ell}$

$V_{20\ell} \rightarrow V_{20\ell} \cos 8°50' + P_{0\ell} \sin 8°50'$

$P_{0\ell} \rightarrow -V_{20\ell} \sin 8°50' + P_{0\ell} \cos 8°50'$

$M_{10\ell} \rightarrow M_{10\ell} \cos 8°50' + Q_{0\ell} \sin 8°50'$

$M_{20\ell} \rightarrow M_{20\ell}$

$Q_{0\ell} \rightarrow -M_{10\ell} \sin 8°50' + Q_{0\ell} \cos 8°50'$

(10)

5-12-73

rotate 3.93° about new M_{10}/V_{20} axes

$V_{10\ell} \rightarrow V_{10\ell}\cos 3.93^\circ + (-V_{20\ell}\sin 8^\circ 50' - P_{0\ell}\cos 8^\circ 50')\sin 3.93^\circ$

$V_{20\ell} \rightarrow V_{2\ell}\cos 8^\circ 50' + P_{0\ell}\sin 8^\circ 50'$

$P_{0\ell} \rightarrow (-V_{20\ell}\sin 8^\circ 50' + P_{0\ell}\cos 8^\circ 50')\cos 3.93 - V_{10\ell}\sin 3.93^\circ$

$M_{10\ell} \rightarrow M_{10\ell}\cos 3.93 + Q_{0\ell}\sin 3.93$

$M_{20\ell} \rightarrow M_{20\ell}\cos 3.93 - (-M_{10\ell}\sin 8^\circ 50' + Q_{0\ell}\cos 8^\circ 50')\sin 3.93$

$Q_{0\ell} \rightarrow M_{20\ell}\sin 3.93 + (-M_{10\ell}\sin 8^\circ 50' + Q_{0\ell}\cos 8^\circ 50')\cos 3.93$

translate up to C'' (.63 in in + Z_0 dir)

no change in V_1, V_2, P, M_{10}

$M_{20} \rightarrow M_{20\ell}\cos 3.93^\circ + (M_{10\ell}\sin 8^\circ 50' - Q_{0\ell}\cos 8^\circ 50')\sin 3.93 -$

$- .63\left[(P_{0\ell}\cos 8^\circ 50' - V_{20\ell}\sin 8^\circ 50')\cos 3.93 - V_{10\ell}\sin 3.93'\right]$

$Q_0 \rightarrow M_{20\ell}\sin 3.93^\circ - (M_{10\ell}\sin 8^\circ 50' - Q_{0\ell}\cos 8^\circ 50')\cos 3.93$

$- .63\left[(P_{0\ell}\cos 8^\circ 50' - V_{20\ell}\sin 8^\circ 50')\sin 3.93^\circ + V_{10\ell}\cos 3.93^\circ\right]$

(20)

5-12-78

rotate up 22.5° about V_1, H_{20} axis

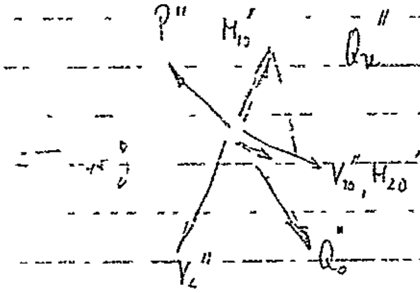
$$V_{10}'' = V_{10\ell}\cos 3.93 + (P_{0\ell}\cos 8°50' - V_{20\ell}\sin 8°50')\sin 3.93$$

$$V_{20}'' = (V_{20\ell}\cos 8°50' + P_{0\ell}\sin 8°50')\cos 22.5° + [(P_{0\ell}\cos 8°50' - V_{20\ell}\sin 8°50')\cos 3.93 - V_{1}$$

$$P_0'' = -(V_{20\ell}\cos 8°50' + P_{0\ell}\sin 8°50')\sin 22.5° + [(P_{0\ell}\cos 8°50' - V_{20\ell}\sin 8°50')\cos 3.93 - V_{1\ell}\sin 3.93]$$

$$H_{10}' = (H_{10\ell}\cos 3.93 + Q_{0\ell}\sin 3.93)\cos 22.5° +$$
$$+ \sin 22.5\{H_{20\ell}\sin 3.93 - (H_{10\ell}\sin 8°50' - Q_{0\ell}\cos 8°50')\cos 3.93 -$$
$$- .63[(P_{0\ell}\cos 8°50' - V_{20\ell}\sin 8°50')\sin 3.93 + V_{10\ell}\cos 3.93]\}$$

$$H_{20}' = H_{20\ell}\cos 3.93 + (H_{10\ell}\sin 8°50' - Q_{0\ell}\cos 8°50')\sin 3.93 -$$
$$- .63[(P_{0\ell}\cos 8°50' - V_{20\ell}\sin 8°50')\cos 3.93 - V_{1\ell}\sin 3.93]$$



$$Q_u'' = -(H_{10\ell}\cos 3.93 + Q_{0\ell}\sin 3.93)\sin 22.5°$$
$$+ \{H_{20\ell}\sin 3.93 - (H_{10\ell}\sin 8°50' - Q_{0\ell}\cos 8°50')\cos 3.93$$
$$- .63[(P_{0\ell}\cos 8°50' - V_{20\ell}\sin 8°50')\sin 3.93 + V_{1\ell}\cos 3.93]\}\cos$$

normalization for upper arm

at C'':

$V_{1u} = V_{10}''$
$V_{2u} = -V_{20}''$
$P_u = -P_0''$
$M_{1u} = -M_{10}''$
$M_{2u} = M_{20}''$
$Q_u = -Q_u'$

at B':

$V_{10u} = V_{10}''$
$V_{20u} = -V_{20}''$
$P_{0u} = -P_0''$
$H_{10u} = -H_{10}'' + V_{10}'' \ell_u$
$H_{20u} = H_{20}'' - V_{20}'' \ell_u$
$Q_{0u} = -Q_u''$

APPENDIX D

COMPLIANCE (CASTIGLIANO)

5-17-75
(30)

Compliance use Castigliano (without shear!)

- define external loads
- define moments along lower and upper arm (transformation at elbow) in terms of V_1', V_2, P
- determine strain energy by integrating along both upper and lower arm
- determine deflection

note: p ⑰ $V_{1\ell} = -20$ $x = 534.33$

$V_{2\ell} = 494$

$P_\ell = 77$

$M_{1\ell}' = 0$

$M_{2\ell} = 0$

$Q_\ell = V_{1\ell} \times 2548 = -510$

note: these loads are with reference to ℄ of panel not neutral axis

for neutral axis

$M_1 (x=534.33) = P_\ell e_1 = 77 e_1$

$Q (x=534.33) = -510 - V_2 e_3 = -(510 + 494 e_3)$

5-17-76

at root of lower arm (elbow)

forces same

moment: $M_{10}' = V_{1\ell} \times \ell_\ell = -20 \times 584.33 = -11{,}687$

$M_{20} = V_{2\ell} \times \ell_\ell = 494 \times 584.33 = 28{,}866$

$Q_0 = Q_\ell = -510$

for integration $M_{10} = P_\ell\, e_1 + M_{1\ell}' = P_\ell e_1 + V_{1\ell}\, \ell_\ell$

moments along neutral axis (x origin at elbow)

$$M_1(x) = M_{10} - V_{1\ell}\, x = P_\ell e_1 + V_{1\ell}(\ell_\ell - x)$$

$$M_2(x) = M_{20} - V_{2\ell}\, x = V_{2\ell}(\ell_\ell - x)$$

$$Q(x) = Q_0 = Q_\ell = V_{1\ell} \times 2548 - V_{2\ell} e_3$$

$$U_\ell = (SE)_{\text{lower arm}} = \int_0^{\ell_\ell} \left[\frac{M_1^2(x)}{2EI_1} + \frac{M_2^2(x)}{2EI_2} + \frac{Q^2(x)}{2GJ} \right] dx$$

$$\frac{\partial U_\ell}{\partial V_{1\ell}} = \int_0^{\ell_\ell} \left[\frac{M_1(x)\frac{\partial M_1}{\partial V_{1\ell}}}{EI_1} + \frac{M_2(x)\frac{\partial M_2}{\partial V_{1\ell}}}{EI_2} + \frac{Q(x)\frac{\partial Q}{\partial V_{1\ell}}}{GJ} \right] dx$$

$$= \int_0^{\ell_\ell} \left[\frac{(P_\ell e_1 + V_{1\ell}(\ell - x))(\ell - x)}{EI_1} + 0 + \frac{(V_{1\ell}\, 2548 - V_{2\ell} e_3)\, 2548}{GJ} \right] dx$$

$$= V_{1\ell} \left[\frac{1}{EI_1}\frac{\ell_\ell^3}{3} + \frac{2548^2 \ell_\ell}{GJ} \right] - V_{2\ell} \left[\frac{e_3 \times 2548 \times \ell_\ell}{GJ} + P_\ell \frac{e_1 \ell_\ell^2}{2EI_1} \right]$$

$$\frac{\partial U}{\partial V_{2\ell}} = \int_0^{\ell_\ell} \left[0 + \frac{V_{2\ell}(\ell_\ell - x)^2}{EI_2} - \frac{(V_{1\ell}\, 25\,48 - V_2 e_3)\, e_3}{GJ} \right] dx$$

$$= V_{1\ell}\left(- \frac{25\,48\, e_3 \ell_\ell}{GJ} \right) + V_{2\ell}\left(\frac{\ell_\ell^3}{3EI_2} + \frac{e_3^2 \ell_\ell}{GJ} \right) + P_\ell \cdot 0$$

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OF POOR QUALITY

$$\frac{\partial U}{\partial P} = \int_0^{\ell_\ell} \frac{P_\ell e_1 + V_{1\ell}(\ell_\ell - x)}{EI_1} e_1 \, dx$$

$$= V_{1\ell}\left(\frac{e_1 \ell_\ell^2}{2EI_1} \right) + V_{2\ell} \; 0 + P_\ell \left(\frac{e_1^2 \ell_\ell}{EI_1} \right)$$

now transform $\overline{\overline{T}}_{\ell 0}$ into $\overline{\overline{T}}_{u}(x = 552)$ see ⑱ to ⑳

$$\begin{bmatrix} V_{1u} \\ V_{2u} \\ P_u \\ M'_{1u} \\ M_{2u} \\ Q_u \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14}=0 & T_{15}=0 & T_{16}=0 \\ T_{21} & T_{22} & \cdot & 0 & 0 & 0 \\ T_{31} & & T_{33} & 0 & 0 & 0 \\ T_{41} & & \cdot & T_{44} & \cdot & \cdot \\ T_{51} & & & & T_{55} & \cdot \\ T_{61} & & & & & T_{66} \end{bmatrix} \begin{bmatrix} V_{1\ell} \\ V_{2\ell} \\ P_\ell \\ M'_{10\ell} \\ M_{20\ell} \\ Q_{0\ell} \end{bmatrix}$$

5-17-76

$T_{11} = \cos 3.93$ .99765

$T_{12} = -\sin 3.93 \sin 8^{50}$ -.010525

$T_{13} = \sin 3.93 \cos 8^{50}$ .067725

T_{14} to $T_{16} = 0$

$T_{21} = \sin 3.93 \sin 22.5$ .026223

$T_{22} = -\cos 8^{50} \cos 22.5 + \cos 3.93 \sin 8^{50} \sin 22.5$ -.85429

$T_{23} = -\sin 8^{50} \cos 22.5 - \cos 3.93 \cos 8^{50} \sin 22.5$ -.51913

T_{24} to $T_{26} = 0$

$T_{31} = \sin 3.93 \cos 22.5$ .0633205

$T_{32} = \cos 8^{50} \sin 22.5 + \cos 3.93 \sin 8^{50} \cos 22.5$ .51968

$T_{33} = \sin 8^{50} \sin 22.5 - \cos 3.93 \cos 8^{50} \cos 22.5$ -.85201

T_{34} to $T_{36} = 0$

$T_{41}/T_{41}' = 63 \cos 3.93 \sin 22.5$ 24.052 / -440.37 [redacted] 5

$T_{42}/T_{42}' = -63 \sin 3.93 \sin 8^{50} \sin 22.5$ -.2537 , -44.70 [redacted]

$T_{43} = .63 \sin 3.93 \cos 8^{50} \sin 22.5$ 1.6328

$T_{44} = -\cos 3.93(\cos 22.5 - \sin 8^{50} \sin 22.5)$ -.86308

$T_{45} = -\sin 3.93 \sin 22.5$ -.026228

$T_{46} = -\sin 3.93 \cos 22.5 + \cos 3.93 \cos 8^{50} \sin 22.5$ .31393

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$T_{51}/T_{51}' = 63 \sin 3.93$ 4 3179 / 8 ~~40~~10 874

$T_{52}/T_{52}' = 63 \cos 3.93 \sin 8^{50}$ 9.6516 / 5~~60~~35 55

$T_{53} = -63 \cos 3.93 \cos 8^{50}$ −62 106

$T_{54} = \sin 3.93 \sin 8^{50}$ 010525

$T_{55} = \cos 3.93$ 99765

$T_{56} = -\sin 3.93 \cos 8^{50}$ − 067765

$T_{61}/T_{61}' = 63 \cos 3.93 \cos 22.5$ 58 0675 / 3~~24~~40 341

$T_{62}/T_{62}' = -63 \sin 3.93 \sin 8^{50} \cos 22.5$ − 61258 / ~~−35 566~~ −3

$T_{63} = 63 \sin 3.93 \cos 8^{50} \cos 22.5$ 3.9419

T_{64} $\cos 3.93 (\sin 22.5 + \sin 8^{50} \cos 22.5)$ 52332

$T_{65} = -\sin 3.93 \cos 22.5$ − 063320

$T_{66} = \sin 3.93 \sin 22.5 - \cos 3.93 \cos 8^{50} \cos 22.5$ − 88454

note because the moments are functions of V_{1c}, V_{2c}, $\dot{\nu}_c$

this matrix can be reduced to a 6×3 size

for forces at shoulder (root of upper arm see p. ⑳)

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reduce matrix $M'_{10e} = V_{1e}\, l_e$

$$M_{20e} = V_{2e}\, l_e$$

$$Q_{0e} = V_{1e}\, 2548$$

Thus

$$\begin{bmatrix} V_{1u} \\ V_{2u} \\ P_u \\ M'_{1u} \\ M_{2u} \\ Q_u \end{bmatrix} = \begin{Bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \\ \overbrace{\left(T_{41}+T_{44}l_e+T_{46}\,2548\right)}^{T_{41}'} & \overbrace{T_{42}+T_{45}l_e}^{T_{42}'} & T_{43} \\ \overbrace{\left(T_{51}+T_{54}l_e+T_{56}\,2548\right)}^{T_{51}'} & \overbrace{T_{52}+T_{55}l_e}^{T_{52}'} & T_{53} \\ \overbrace{\left(T_{61}+T_{64}l_e+T_{66}\,2548\right)}^{T_{61}'} & \overbrace{T_{62}+T_{65}l_e}^{T_{62}'} & T_{63} \end{Bmatrix} \begin{bmatrix} V_{1e} \\ V_{2e} \\ P_e \end{bmatrix}$$

moment distribution in upper arm

$$M_{1u}(x) = M'_{1u} + P_u e_1 + V_{1u}(l_u - x)$$

$$M_{2u}(x) = M_{2u} + V_{2u}(l_u - x)$$

$$Q_u(x) = Q_u - V_{2u} e_3$$

$$U_u = \int_0^{l_u} \left[\frac{M_{1u}^2}{2EI_1} + \frac{M_{2u}^2}{2EI_2} + \frac{Q_u^2}{2GJ}\right] dx$$

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$$\mathcal{J}_{um}\ \frac{\partial U_u}{\partial V_{1L}} = \int_0^{l_u} \Big\{ \frac{1}{EI_1}\left[M_{1u}' + P_u e_1 + V_{1u}(l_u - x)\right]\left[T_{41} + T_{44} l_t + T_{46}\cdot 25.48 + T_{31} e_1 + T_{11}(l_u - x)\right]$$

$$+ \frac{1}{EI_2}\left[M_{2u} + V_{2u}(l_u - x)\right]\left[T_{51} + T_{54} l_t' + T_{56}\cdot 25.48 + T_{21}(l_u - x)\right]$$

$$+ \frac{1}{GJ}\left[Q_u - V_{2u} e_3\right]\left[T_{61} + T_{64} l_t + T_{66}\, 25.48 - T_{21} e_3\right]\Big\}\, dx$$

$$= \frac{1}{EI_1}\Big\{ (M_{1u}' + P_u e_1)(\overbrace{T_{41} + T_{44} l_t + T_{46}\cdot 25.48}^{T_{41}'} + T_{31} e_1)\, l_u$$

$$+ \left[(M_{1u}' + P_u e)T_{11} + V_{1u}(\ \ ')\right]\frac{l_u^2}{2} + V_{1u} T_{11}\frac{l_u^3}{3}\Big\}$$

$$+ \frac{1}{EI_2}\Big\{ M_{2u}(\underbrace{T_{51} + T_{54} l_t + T_{56}\, 25.48}_{})\, l_u +$$

$$\left[M_{2u} T_{21} + V_{2u}(\underset{T_{51}'}{\ \ })\right]\frac{l_u^2}{2} + V_{2u} T_{21}\frac{l_u^3}{3}\Big\}$$

$$+ \frac{l_u}{GJ}\left[Q_u - V_{2u} e_3\right]\left[\underbrace{T_{61} + T_{64} l_t + T_{66}\, 25.48}_{T_{61}'} - T_{21} e_3\right] =$$

$$= V_{1L}\Big\{ \frac{1}{EI_1}\left[(T_{41}' + T_{31} e_1)^2 l_u + 2(T_{41}' + T_{31} e_1) T_{11}\frac{l_u^2}{2} + T_{11}^2\frac{l_u^3}{3}\right]$$

$$+ \frac{1}{EI_2}\left[T_{51}'^2 l_u + 2 T_{51}' T_{21}\frac{l_u^2}{2} + T_{21}^2\frac{l_u^3}{3}\right]$$

$$+ \frac{l_u}{GJ}(T_{61}' - T_{21} e_3)^2\Big\}$$

$$+ V_{2L}\Big\{\frac{1}{EI_1}\Big\langle (T_{42}' + T_{32} e_1)(T_{41}' + T_{31} e_1)\, l_u + \left[(T_{42}' + T_{32} e_1)T_{11} + T_{12}(T_{41}' + T_{31} e_1)\right]\frac{l_u^2}{2} + T_{12} T_{11}\frac{l_u^3}{3}\Big\rangle$$

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$$+\frac{1}{EI_2}\left[T_{52}'T_{51}'\,l_u+\left(T_{52}'T_{21}+T_{22}T_{51}'\right)\frac{l_u^2}{2}+T_{22}T_{21}\frac{l_u^3}{3}\right]$$

$$+\frac{l_u}{GJ}\left[\left(T_{62}'-T_{22}e_3\right)\left(T_{61}'-T_{21}e_3\right)\right]\Bigg\}$$

$$+P_l\left\{\frac{1}{EI_1}\left\langle\left(T_{43}+T_{33}e_1\right)\left(T_{41}'+T_{31}e_1\right)l_u+\left[\left(T_{43}+T_{33}e_1\right)T_{11}+T_{13}\left(T_{41}'+T_{31}e_1\right)\right]\frac{l_u^2}{2}+T_{13}T\right.\right.$$

$$+\frac{1}{EI_2}\left\langle T_{53}T_{51}'\,l_u+\left(T_{53}T_{21}+T_{23}T_{51}'\right)\frac{l_u^2}{2}+T_{23}T_{21}\frac{l_u^3}{3}\right\rangle$$

$$+\frac{l_u}{GJ}\left(T_{63}-T_{23}e_3\right)\left(T_{61}'-T_{21}e_3\right)\Bigg\}.$$

$$\frac{\partial U_u}{\partial V_{2l}}=\int_0^{l_u}\left\{\frac{1}{EI_1}\left(M_{1u}'+P_ue_1+V_{1u}'(l_u-x)\right)\left(T_{42}'+T_{32}e_1+T_{12}(l_u-x)\right)\right.$$

$$+\frac{1}{EI_2}\left(M_{2u}+V_{2u}(l_u-x)\right)\left(T_{52}'+T_{22}(l_u-x)\right)$$

$$\left.+\frac{1}{GJ}\left(Q_u-V_{2u}e_3\right)\left(T_{62}'-T_{22}e_3\right)\right\}dx$$

$$=\frac{1}{EI_1}\left\{\left(M_{1u}'+P_ue_1\right)\left(T_{42}'+T_{22}e_1\right)l_u+\left[\left(M_{1u}'+P_ue_1\right)T_{12}+V_{1u}\left(T_{42}'+T_{22}e_1\right)\right]\frac{l_u^2}{2}+V_{1u}T_{12}\frac{l_u^3}{3}\right\}$$

$$+\frac{1}{EI_2}\left\{M_{2u}T_{52}'\,l_u+\left(M_{2u}T_{22}+V_{2u}T_{52}'\right)\frac{l_u^2}{2}+V_{2u}T_{22}\frac{l_u^3}{3}\right\}$$

$$+\frac{1}{GJ}\left(Q_u-V_{2u}e_3\right)\left(T_{62}'-T_{22}e_3\right)l_u$$

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$$= V_{1L}\left\{\frac{1}{EI_1}\left[(T_{41}' + T_{31}e_1)(T_{42}' + T_{32}e_1)\,l_u + \left\{(T_{41}' + T_{31}e_1)T_{12} + T_{11}(T_{42}' + T_{32}e_1)\right\}\frac{l_u^2}{2} + T_{11}T_{12}\frac{l_u^3}{3}\right]\right.$$

$$+ \frac{1}{EI_2}\left[T_{51}'T_{52}'\,l_u + (T_{51}'T_{22} + T_{21}T_{52}')\frac{l_u^2}{2} + T_{21}T_{22}\frac{l_u^3}{3}\right]$$

$$\left. + \frac{l_u}{GJ}(T_{61}' - T_{21}e_3)(T_{62}' - T_{22}e_3)\right\}$$

$$+ V_{2L}\left\{\frac{1}{EI_1}\left[(T_{42}' + T_{32}e_1)^2 l_u + 2(T_{42}' + T_{32}e_1)T_{12}\frac{l_u^2}{2} + T_{12}^2\frac{l_u^3}{3}\right]\right.$$

$$+ \frac{1}{EI_2}\left[T_{52}'^2\,l_u + 2T_{52}'T_{22}\frac{l_u^2}{2} + T_{22}^2\frac{l_u^3}{3}\right]$$

$$\left. + \frac{l_u}{GJ}(T_{62}' - T_{22}e_3)^2\right\}$$

$$+ V_{3L}\left\{\frac{1}{EI_1}\left((T_{43} + T_{33}e_1)(T_{42}' + T_{32}e_1)\,l_u + \left[(T_{43} + T_{33}e_1)T_{12} + T_{13}(T_{42}' + T_{32}e_1)\right]\frac{l_u^2}{2} + T_{13}T_{12}\frac{l_u}{3}\right)\right.$$

$$+ \frac{1}{EI_2}\left[T_{53}T_{52}'\,l_u + (T_{53}T_{22} + T_{23}T_{52}')\frac{l_u^2}{2} + T_{23}T_{22}\frac{l_u^3}{3}\right]$$

$$\left. + \frac{l_u}{GJ}(T_{63} - T_{23}e_3)(T_{62}' - T_{22}e_3)\right\}$$

$$\frac{\partial U_u}{\partial P_\ell} = \int_0^{l_u} \Big\{ \frac{1}{EI_1}\big(M_{1u}' + P_u e_1 + V_{1u}(l_u - x)\big)\big(T_{43} + T_{33} e_1 + T_{13}(l_u - x)\big)$$

$$+ \frac{1}{EI_2}\big(M_{2u} + V_{2u}(l_u - x)\big)\big(T_{53} + T_{23}(l_u - x)\big)$$

$$+ \frac{1}{GJ}\big(Q_u - V_{2u} e_3\big)\big(T_{63} - T_{23} e_3\big) \Big\} dx$$

$$= \frac{1}{EI_1}\Big\{ (M_{1u}' + P_u e_1)(T_{43} + T_{33} e_1) l_u + \big[(M_{1u}' + P_u e_1) T_{13} + V_{1u}(T_{43} + T_{33} e_1)\big]\frac{l_u^2}{2} + V_{1u} T_{13} \frac{l_u^3}{3} \Big\}$$

$$+ \frac{1}{EI_2}\Big\{ M_{2u} T_{53} l_u + (M_{2u} T_{23} + V_{2u} T_{53})\frac{l_u^2}{2} + V_{2u} T_{23} \frac{l_u^3}{3} \Big\}$$

$$+ \frac{l_u}{GJ}(Q_u - V_{2u} e_3)(T_{63} - T_{23} e_3)$$

$$= V_{1\ell} \Big\{ \frac{1}{EI_1}\Big\langle (T_{41}' + T_{31} e_1)(T_{43} + T_{33} e_1) l_u + \big[(T_{41}' + T_{31} e_1) T_{13} + T_{11}(T_{43} + T_{33} e_1)\big]\frac{l_u^2}{2} + T_{11} T_{13} \frac{l_u^3}{3} \Big\rangle$$

$$+ \frac{1}{EI_2}\Big[T_{51}' T_{53} l_u + (T_{51}' T_{23} + T_{21} T_{53})\frac{l_u^2}{2} + T_{21} T_{23} \frac{l_u^3}{3} \Big]$$

$$+ \frac{l_u}{GJ}(T_{61}' - T_{21} e_3)(T_{63} - T_{22} e_3) \Big\}$$

$$+ V_{2\ell} \Big\{ \frac{1}{EI_1}\Big\langle (T_{42}' + T_{32} e_1)(T_{43} + T_{33} e_1) l_u + \big[(T_{42}' + T_{32} e_1) T_{13} + T_{12}(T_{43} + T_{33} e_1)\big]\frac{l_u^2}{2} + T_{12} T_{13} \frac{l_u^3}{3} \Big\rangle$$

$$+ \frac{1}{EI_2}\Big[T_{52}' T_{53} l_u + (T_{52}' T_{23} + T_{22} T_{53})\frac{l_u^2}{2} + T_{22} T_{23} \frac{l_u^3}{3} \Big]$$

$$+ \frac{l_u}{GJ}(T_{62}' - T_{22} e_3)(T_{63} - T_{23} e_3) \Big\}$$

$$+ P_\ell \Big\{ \frac{1}{EI_1}\Big[(T_{43} + T_{33} e_1)^2 l_u + 2(T_{43} + T_{33} e_1) T_{13} \frac{l_u^2}{2} + T_{13}^2 \frac{l_u^3}{3} \Big]$$

$$+ \frac{1}{EI_2}\Big[T_{53}^2 l_u + 2 T_{53} T_{23} \frac{l_u^2}{2} + T_{23}^2 \frac{l_u^3}{3} \Big]$$

$$+ \frac{l_u}{GJ}(T_{63} - T_{23} e_3)^2 \Big\}$$

APPENDIX E

VARIATION OF ARM POSITION

Variation of Arm Position

calculate margin when forces remain 50 lb in-Z direction
±20 lb perpendicular to lower arm pair

		position II	III	IV
angles	22.5 remains			
	3.93 change to	-30	-70	-102.5
	8 5/6 change to	6°	2.7°	0
distance X_E	584.32	628	565	715

Force $\overline{F}_E$	V_1	∓20	∓20	∓20
	V_2	49.7	49.9	50
	P	5.2	2.4	0
	Q_t	∓510	∓510	∓510

F_c'''	V_1	∓20	∓20	∓20
	V_2	49.7	49.9	50
	P	5.2	2.4	0
	M_{10}'	∓12560	∓11300	-14300
	M_{20}	31,230	28,220	35,750
	Q	∓510	∓510	∓510

		q_1	q_2	q_1	q_2	q_1	q_2
F_c''	V_1	-17.31	17.33	-6.88	6.90	-4.33	-.55
	V_2	-42.33	-45.99	-32.97	-46.51	-38.72	-53.61
	P	28.38	9.50	36.47	12.11	37.17	1.09
	M_{10}'	14,769	-2817	12,973	-2355	10,184	16,529
	M_{20}	28,680	26,015	10,856	15,116	-7006	-8470
	Q	8,709	36,949	22,796	18,313	33,771	30,121

(56.
5-2[illegible]

Transformation Matrix $[T_{ij}]$

Position	I	II	III	IV
β	3 93°	-30°	-70°	-102 5°
ε	8½°	6°	27°	0°
T_{11}	.99765	86603	34202	-.21644
T_{12}	-.010525	052264	044266	0
T_{13}	067725	-49726	-93865	-97630
T_{21}	026228	-19134	-35960	-37361
T_{22}	-85425	-88418	-91669	-92388
T_{23}	-51913	-42617	-17426	082828
T_{31}	0632205	-46194	-.86816	-.90198
T_{32}	51863	46422	39714	.38268
T_{33}	-85201	-.75572	-29761	19996
T_{41} T'_{41}	24.052 -472 273	20 879 /-439 661	8 2458 -141 351	-5 2132, 158 62[illegible]
T_{42} T'_{42}	-.2537 -15580	1 2600 /121 422	1 0672 201 241	0 / 267 131
T_{43}	1 6323	-11.988	-22 630	-23 538
T_{44}	-86303	-76546	-30982	19996
T_{45}	-02622	19134	35960	37261
T_{46}	[illegible]	79154	48890	81915
T_{51} T'_{5}	4 3170 8 742	-31 5, -51.652	-59 201 -10 224	-61 507 -36 [illegible]
T_{52} T'	5.6516, 52 106	5 7030 549 570	1.0150, 194 256	0, -154 755
T_{53}	-62 106	-54.261	-21 523	+13 636
T_{54}	-010525	-052264	-044266	0
T_{55}	.99765	.86603	34202	-2'644
T_{56}	-.067725	49726	.93865	97630
T_{61}/T'_{61}	58 0675/341 321	50.4065 285 968	19 907 85 062	-12 598 -17 043
T_{62}/T'	-61253 -37 [illegible]	3 0420, 293 140	2 5765 493 087	0, 644 911
T	3.9412	-28 943	-54 634	-56 825
T	[illegible]	41505	.14577	-032828
T	-0632205	46194	86816	90198
T_{66}	-.82454	-92703	-07524	-1753[illegible]

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		II	I	IV
$\overline{\overline{F}}(x_u = 32\text{-}34.5)$	V_1	-17 31 17 33	-6.88 6 80	4 33 -4 33
	V_2	-42 33 -49 59	-38 97 -48 97	-38 72 -53 67
	P	28 38 9 30	36 47 12 11	37 17 1 09
	M_{10}'	14 172 -2219	12 736 -2160	10 333 16380
	M_{20}	26 620 24,290	9 512 13471	-8 342 -10 322
	Q_1	8 709 36,949	22 796 18313	33 771 30 721
$\overline{F}(x_u = 34.5)$	V_1	-17.31 17 33	-6.88 6.80	4 33 -4.33
	V_2	-42 33 -49 55	-38 97 -48 97	-38 72 -53 67
	P	26 38 9 50	36 47 12.11	37 17 1 09
	M_{10}'	6 330 5631	9 619 920	12 295 14 418
	M_{20}	7444 1645	-8162 -2713	-25 882 -34 634
	Q_0	8 709 36 949	22 796 18313	33 771 30 721
$\overline{F}(x_l = 34.5)$	V_1	∓20	∓20	∓20
	V_2	49.7	49.9	50
	P	5 2	2.4	0
	M_{10}'	∓11 870	∓10 610	∓13 610
	M_{20}	29 515	26 498	34 025
	Q_0	∓510	∓510	∓510

Note second number is for second quadrant (20 lb. force reversed)

APPENDIX F

FORCES IN TRUSS MEMBERS (NUMERICAL) AND MARGINS OF SAFETY

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Forces in Truss Members and Margins of Safety (Vertical Ladder)

Applied load 20 lb in X-Y plane of Orbiter
50 lb in -Z direction

Quadrant 1 $V_1 = -20$
" 2 $V_1 = +20$
" 3 $V_1 = +20$ 50 lb reversed
" 4 $V_1 = -20$ 50 lb reversed

Euler buckling of members $\frac{\pi^2 E I_d}{L^2}$ where $L = l, a, b, d$

	Aluminum	CRES	C/epoxy
Longeron 1 & 2	5,232	8,864	8,268
Main Scissor (2 l)	1,475$^{1)}$ 1,630$^{2)}$	2,216	2,067
A-Frame	3,222	8,614	5,092
Crossbrace, single	4,102	6,949	6,432
Diagonal brace	2,300	3,895	3,635

$^{1)}$ 625 ID $^{2)}$ full cross section

Yield Limits

	Aluminum (35 ksi)	CRES (58 ksi)
Main Scissor	24,050$^{1)}$ 34,800$^{2)}$	17,215
Other Members	19,350	17,215

$^{1)}$ 625 ID $^{2)}$ full cross section

Bearing Loads in Aluminum joints

pin dia = 1875 bearing width = 25 σ_{bu} = 56 ksi

bearing load max = 2,625 lb

Margin = $\frac{P_{ul}}{3P} - 1$ q_1 quadrant 5-24-7

	member force \ position	I		II		III		IV	
	P_{Au}	-535 80	1.00	495.7	1.17	1341 9	-.20	2004 6	-.46
	P_{Bu}	489.74	1.19	-535 6	1.00	-1357 7	-.21	-1994 7	-.46
	P_{du}	-544 14	.41	666 9	.15	1649 8	-.54	2418 7	-.68
	P_{bu}	407 39	2.36	-499 3	1.74	-1235.2	.11	-1810.8	-.24
	P_{Al}	-53 27		-53 27		-53.27		-53 3	
	P_{Bl}	7 13		7 13		7 13		7 13	
	P_{dl}	-101.73		-102 13		-102.40		-102.53	
	P_{l}	76 16		76 46		76 66		76 76	
x_u -34 5	P_1	229.0		813.5		1155 3		1211.7	
	P_2	192 4		-452.9		-614 1		-529 4	
	P_3	54.52		-332.2		-504.7		-654 2	
λ_u - 52-34 5	P_1	551 32		1551 4		1690 2		1679 0	
	P_2	-113.67		-739 3		-985 4		-1099 6	
	P_3	-419 8		-743 7		-668 3		-542 2	
λ_l = 34 5	P_1	368.62		381 0		335 0		448 2	
	P_2	-938.14		-998.6		-889 4		-1162 4	
	P_3	577 22		622.9		556 8		714 2	
lower // arm 1st panel	P_{L1}	492 6		-70 7		61 6		-391 7	
	P_{L2}	-168 3		-10 8		479 6		1074 0	
	P_S	90 7		-300 9		-492 3		-662 0	
2nd panel	P_{L1}	131 7		371 4		-1032 1	.69	-1995 1	-.13
	P_{L2}	192.4		-452 9		-614 1		-529 4	
	P_S	18 4		-363 5		-517 1		-646 4	
15th panel	P_{L1}	1272 8	.37	1109 3	.57	596 5	(1.92)	75 6	
	P_{L2}	-474 4		-297 2		108 3		503.8	
	P_{L3}	-383 7		-775 0	.37	-655 9	-.25	-550	
16th panel	P_{L1}	912 0		667 2		-497 2		-1527 6	
	P_{L2}	-113.7		-735 3		-925 4		-1099 6	
	P_{L3}	[illegible]		712 4		-680.7		-531 1	

5-24-78

		I	II	III	IV
Lower arm 1st panel	P_{c_1}	503.5	516.4	470.8	584.1
	P_{c_2}	-1005.6	-1066.3	-957.3	-1230.4
	P_s	613.4	659.1	593.0	750.4
2nd panel	P_{c_1}	436.1	448.7	402.9	516.2
	P_{c}	-938.1	* -998.6	-889.4	-1162.4
	P_s	541.0 -.09	586.7	520.6	678.0 -.27

loads producing min. margins are circled

* CC4 for full cross section

margin 20 lb force reversed g_2 (50 a)

member forces	I		II		III		IV	
P_{Au}	319.0	2.37	2207.8	-.51	1092.2	-.017	1814.1	-.41
P_{Bu}	-273.0	2.93	-2167.8	-.50	-1076.5	-.002	-1824.0	-.41
P_{du}	411.4	.86	2656.5	-.71 fail	1348.9	-.43	2224.9	-.66 near fail.
P_{bu}	-308.0	3.44	-1988.9	-.31	-1010.0	.35	-1665.7	-.18
P_{Al}	53.3		53.3		53.3		53.3	
P_{Bl}	-7.1		-7.1		-7.1		-7.1	
P_{dl}	-30.2		-30.6		-30.9		-31.0	
P_{bl}	22.6		22.9		23.1		23.2	

$x_u = 24.5$	P_1	423.1	1956.0	700.8	864.9
	P_2	-396.3	-1650.6	-642.4	-207.2
	P_3	-6.4	-295.5	-48.3	-756.6
$x_u = 552-36.5$	P_1	736.4	2331.4	1189.3	1640.3
	P_2	-1215.3	-2437.5	-1290.6	-779.7
	P_3	499.3	115.9	113.3	-859.5
$x_l = 36.5$	P_1	969.5	428.4	939.2	1209.8
	P_2	-384.6	-1046.0	-380.0	-495.6
	P_3	-577.2	622.9	-556.8	-714.2

42

59a
5-26-71

margin			I	II	III	IV
upper arm	1st panel	P_{e_1}	-122.4	-1566.1	-1087 6	-2526 2 -.31
		P_{e_2}	-123.6	110 5	253 8	1538 7 .091
		P_s	-42.5	264 1	-60 6	-748 8
	2nd panel	P_{e_1}	150.4	194 9	-193 4	-781 0
		P_{e_2}	-396.3	-1650 6	-640 4	-207 2
		P_s	29 7	326.9	-36.0	764 4
	15th panel	P_e	190 9	-1190 7	-599.1	-1851 4
		P_{e_2}	-942 6	-676.4	-396.4	966 2
		P_s	463 2	24 5	101 0	-851 7 -.42 -36 -19
	16th panel	P_{e_1}	263 7	570.3	255 1	-105 6
		P_{e_2}	-1215 3 .43	-2437 5 -.28	-1290 6 .35	-779 7
		P_s	535 4 1.26	147 3 .13	125 6 1.14	-867 3
lower arm	1st panel	P_{e_1}	1009.5	488 3	999.1	1250.9
		P_{e_2}	-404 6	-1075 9	-350 1	-516 2
		P_s	-613 5 ~~-.120~~ ~~-.114~~ .12	586 6 ~~-.16~~ ~~-.074~~ .175	-553 1 -.17 -.084 .16	-750 5
	2nd panel	P_{e_1}	989 5	458 3	969 1	1230 4
		P_{e_2}	-384 6	-1066 0	-380 0	-495 6
		P_s	-540 9 -.09 +.005 .27	659 2 -.254 -.176 .045	-520.5	-677.9

margin with full Al scissor in black

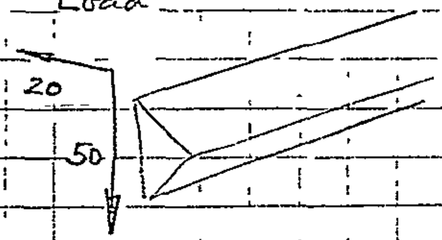
margin for Clep tubes 1115/875

(6b)

6-9-7

Normalization of Load Limits

Code of Load



50 lb in -Z" direction / 20 lb in X-Y plane forward

q1 as given 50/20
q2 -- 50/-20
q3 -50/-20
q4 -50/20

Arm Position	I	II	III	IV
Vertical Ladder				
Critical Member	S	D	D	D
Critical panel	lower No. 2	upper arm	upper arm	upper arm
Loading quadrant	q2	q4	q3	q3
Load Capacity with S.F.=3 % normalized load				
Al Barbus	100%	29%	46%	31%
C/Epoxy	127%	40%	65%	44%

Horizontal Ladder, C/Epoxy

Loading Quadrant	q2
Load Capacity with S.F=3 % normalized load	105%

APPENDIX G

COMPLIANCE MATRICES (NUMERICAL)

Compliances and Their Contributors

Configuration: Vertical Ladder, Position I   Material: Aluminum

$$C_{ij} = \frac{10^6}{EI_1}\{K_{\ell 1ij} + K_{u1ij}\} + \frac{10^6}{EI_2}\{K_{\ell 2ij} + K_{u2ij}\} + \frac{10^6}{GJ}\{K_{\ell Qij} + K_{uQij}\}$$

$\ell_\ell = 584\ 33$ $e_1 = 9\ 027$ $\frac{10^6}{EI_1} = 5263\times10^{-3}$ $\frac{10^6}{EI_2} = 2386\times10^{-3}$ $\frac{10^6}{GJ} = 3\ 7733\times10^{-3}$

$\ell_u = 552$ $e_3 = 10\ 884$

ij	C_{ij}	K_{ℓ_1}	K_{u_1}	$\frac{10^6}{EI_1}\{K_{\ell_1}+K_{u_1}\}$	K_{ℓ_2}	K_{u_2}	$\frac{10^6}{EI_2}\{K_{\ell_2}+K_{u_2}\}$	$K_{\ell Q}$	K_{uQ}	$\frac{10^6}{GJ}\{K_{\ell Q}+K_{uQ}\}$
11	2973	66 505	35 231	$53\ 544\times10^{-3}$	0	15062	03594×10^{-3}	3794	64 200	243 677×10
12	-01935	0	-2 0294	7093×10^{-3}	0	2 8337	6761×10^{-3}	-16409	-5 330	-20 731×10
13	006925	1 5411	-4223	5288×10^{-3}	0	-2 0027	-4778×10^{-3}	0	1 8058	6 8138×10
22	03707	0	1680	05609×10^{-3}	66 505	30 512	$35\ 078\times10^{-3}$	06923	44255	1 9311×10
23	-008679	0	1111	05589×10^{-3}	0	-34 238	$-8\ 169\times10^{-3}$	0	-14093	-5657×
33	006754	04762	15238	1053×10^{-3}	0	27 063	$6\ 457\times10^{-3}$	0	05079	1916×10

Deflections

Q_1 $\bar{F} = (-20, 494, 77)$ $\bar{\delta} = (-6\ 849, 2\ 151, -515)$ $|\delta| = 7\ 197$

Q_2 $\bar{F} = (20, 494, 77)$ $\bar{\delta} = (5\ 043, 1\ 377, -238)$ $|\delta| = 5\ 234$

Compliances and Their Contributors

Configuration: Vertical Ladder, Position II Material: Aluminum

$$C_{ij} = \frac{10^6}{EI_1}\left\{K_{\ell 1ij} + K_{u1ij}\right\} + \frac{10^6}{EI_2}\left\{K_{\ell 2ij} + K_{u2ij}\right\} + \frac{10^6}{GJ}\left\{K_{\ell\theta ij} + K_{u\theta ij}\right\}$$

$\ell_\ell = 628$ $e_1 = 9\ 027$ $\frac{10^6}{EI_1} = 5263\times10^{-3}$ $\frac{10^6}{EI_2} = 2386\times10^{-3}$ $\frac{10^6}{GJ} = 3\ 7733\times10^{-3}$

$\ell_u = 522$ $e_3 = 10\ 884$

ij	C_{ij}	K_{ℓ_1}	K_{u_1}	$\frac{10^6}{EI_1}\{K_{\ell_1}+K_{u_1}\}$	K_{ℓ_2}	K_{u_2}	$\frac{10^6}{EI_2}\{K_{\ell_2}+K_{u_2}\}$	$K_{\ell\theta}$	$K_{u\theta}$	$\frac{10^6}{GJ}\{K_{\ell\theta}+K_{u\theta}$
11	2366	82 558	33 666	$61\ 172\times10^{-3}$	0	4 671	$1\ 115\times10^{-3}$	4077	45 782	174 288 ×
12	1693	0	-15 197	$-7\ 999\times10^{-3}$	0	-15 245	$-3\ 637\times10^{-3}$	-1742	48 131	180 955 ×
13	005664	1 7801	11 606	$18\ 598\times10^{-3}$	0	6 897	$1\ 646\times10^{-3}$	0	-3 864	-14 580 ×
22	2315	0	10 862	$5\ 717\times10^{-3}$	82 558	62 488	$34\ 608\times10^{-3}$	0744	50 529	191 206 ×
23	- 02164	0	-12 427	$-\ 6541\times10^{-3}$	0	-23 708	$-5\ 657\times10^{-3}$	0	-4 062	-15 327
33	01466	05117	16 909	$8\ 927\times10^{-3}$	0	18 854	$4\ 499\times10^{-3}$	0	3261	1 230 ×

Deflections

Q_1 $\bar{F} = (-20, 49\ 7, 5\ 2)$ $\bar{\delta} = (3\ 712, 8\ 007, -1\ 113)$ $|\delta| = 8\ 90$

Q_2 $\bar{F} = (20, 49\ 7, 5\ 2)$ $\bar{\delta} = (-13\ 176, -14\ 779, 886)$ $|\delta| = 19\ 82$

Compliances and Their Contributors

Configuration: Vertical Ladder Position III Material: Aluminum

$$C_{ij} = \frac{10^6}{EI_1}\left\{K_{\ell 1ij} + K_{u1ij}\right\} + \frac{10^6}{EI_2}\left\{K_{\ell 2ij} + K_{u2ij}\right\} + \frac{10^6}{GJ}\left\{K_{\ell\theta ij} + K_{u\theta ij}\right\}$$

$\ell_\ell = 565$ $e_1 = 9\ 027$ $\frac{10^6}{EI_1} = 5263\times 10^{-3}$ $\frac{10^6}{EI_2} = 2386\times 10^{-3}$ $\frac{10^6}{GJ} = 3\ 7733\times 10^{-3}$

$\ell_u = 552$ $e_3 = 10\ 884$

ij	C_{ij}	K_{ℓ_1}	K_{u_1}	$\frac{10^6}{EI_1}\{K_{\ell_1}+K_{u_1}\}$	K_{ℓ_2}	K_{u_2}	$\frac{10^6}{EI_2}\{K_{\ell_2}+K_{u_2}\}$	$K_{\ell\theta}$	$K_{u\theta}$	$\frac{10^6}{GJ}\{K_{\ell\theta}+K_{u\theta}\}$
11	05504	60 121	3 297	$33\ 379\times 10^{-3}$	0	15 863	$3\ 785\times 10^{-3}$	3668	4 3700	$17\ 873\times 10^{-}$
12	09159	0	-6 443	$-3\ 391\times 10^{-3}$	0	9 795	$2\ 337\times 10^{-}$	-1567	24 708	92 659·
13	-005355	14408	4 101	$2\ 917\times 10^{-3}$	0	6 293	$1\ 502\times 10^{-3}$	0	-2 590	-9 773·
22	5591	0	26 755	$14\ 082\times 10^{-3}$	60 121	13 684	$17\ 610\times 10^{-3}$	.0669	139 697	527 37'
23	-07267	0	-35 125	$-18\ 487\times 10^{-3}$	0	4 497	$1\ 073\times 10^{-3}$	0	-14 645	-55 260·
33	03655	04604	56 992	$30\ 021\times 10^{-3}$	0	3 101	$.740\times 10^{-3}$	0	1 532	5 782·

Deflections

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Q_1 $\bar{F} = (-20, 49.9, 2.4)$ $\bar{\delta} = (3.457, 25.893, -3.431)$ $|\delta| = 26.35$

Q_2 $\bar{F} = (20, 49.9, 2.4)$ $\bar{\delta} = (-5.658, -29.556, 3.646)$ $|\delta| = 30.313$

Compliances and Their Contributors

Configuration: Vertical Ladder, Position IV Material: Aluminum

$$C_{ij} = \frac{10^6}{EI_1}\left\{K_{l1ij} + K_{u1ij}\right\} + \frac{10^6}{EI_2}\left\{K_{l2ij} + K_{u2ij}\right\} + \frac{10^6}{GJ}\left\{K_{lQij} + K_{uQij}\right\}$$

$l_l = 715$ $e_1 = 9.027$ $\frac{10^6}{EI_1} = 5263\times10^{-3}$ $\frac{10^6}{EI_2} = 2386\times10^{-3}$ $\frac{10^6}{GJ} = 3\ 7733\times10^{-3}$

$l_u = 552$ $e_3 = 10.884$

ij	C_{ij}	K_{l1}	K_{u1}	$\frac{10^6}{EI_1}\{K_{l1}+K_{u1}\}$	K_{l2}	K_{u2}	$\frac{10^6}{EI_2}\{K_{l2}+K_{u2}\}$	K_{lQ}	K_{uQ}	$\frac{10^6}{GJ}\{K_{lQ}+K_{uQ}$
11	07124	121 84	5 202	$66\ 866\times10^{-3}$	0	12 737	$3\ 039\times10^{-3}$	4642	0921	$1\ 337\times10$
12	$-\ 02118$	0	13 554	$7\ 134\times10^{-3}$	0	36 433	8.693×10^{-3}	-1933	-4.684	$-37\ 004\times10$
13	$-\ 00417$	2 3074	$-11\ 624$	-4.904×10^{-3}	0	$-3\ 249$	$-\ 775\times10^{-3}$	0	4129	$1\ 262\times10$
22	9692	0	40 416	$21\ 272\times10^{-3}$	121 84	104 640	54.035×10^{-3}	0847	236 801	[illegible] 841
23	$-\ 08698$	0	$-43\ 493$	$-22\ 892\times10^{-3}$	0	$-9\ 327$	$-1\ 302\times10^{-3}$	0	$-20\ 871$	$-62\ 788$
33	03259	0583	60 165	$31\ 6[illegible]\times10^{-3}$	0	334	198×10^{-3}	0	1329	$694\times10^{-}$

Deflections

Q_1 $\bar{F} = (-20, 50, 0)$ $\bar{\delta} = (-2\ 484, 48\ 884, -4\ 261)$ $|\delta| = 49\ 13$

Q_2 $\bar{F} = (20, 50, 0)$ $\bar{\delta} = (.364, 48\ 036, -4\ 437)$ $|\delta| = 48\ 24$

Compliances and Their Contributors

Configuration: Vertical Ladder, Position I Material: CRES

$$C_{ij} = \frac{10^6}{EI_1}\left\{K_{\ell 1ij} + K_{u1ij}\right\} + \frac{10^6}{EI_2}\left\{K_{\ell 2ij} + K_{u2ij}\right\} + \frac{10^6}{GJ}\left\{K_{\ell Qij} + K_{uQij}\right\}$$

$\ell_\ell = 584.33$ $e_1 = 6\ 352$ $\frac{10^6}{EI_1} = 51677\times10^{-3}$ $\frac{10^6}{EI_2} = 16483\times10^{-3}$ $\frac{10^6}{GJ} = 2\ 6094\times10^{-3}$

$\ell_u = 552$ $e_3 = 10\ 875$

ij	C_{ij}	$K_{\ell 1}$	K_{u1}	$\frac{10^6}{EI_1}\{K_\ell + K_{u1}\}$	$K_{\ell 2}$	K_{u2}	$\frac{10^6}{EI_2}\{K_\ell + K_{u2}\}$	$K_{\ell c}$	K_{uc}	$\frac{10^6}{GJ}\{K_{\ell c} + K_{uc}\}$
[illegible]	2211	66 505	35 269	$52\ 594\times10^{-3}$	0	1506	02482×10^{-3}	3794	64 200	$168\ 51\times10^{-3}$
12	− 01309	0	1.500	7752×10^{-3}	0	2.8337	$.4671\times10^{-3}$	−1619	−5 3318	−14.335×
13	004593	1 0844	− 6707	2138×10^{-3}	0	−2.0027	-3301×10^{-3}	0	1 5049	4 709
22	02564	0	1288	0666×10^{-3}	66 505	80 512	$24\ 233\times10^{-3}$	06911	4428	1 336×
23	− 006104	0	− 1350	-0698×10^{-3}	0	−34 238	$-5\ 643\times10^{-3}$	0	− 1499	− 39115×
33	004702	02356	1870	1088×10^{-3}	0	27 062	$4\ 461\times10^{-3}$	0	05074	1324×

Deflections

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Q_1 $\overline{F} = (-20, 494, 77)$ $\overline{\delta} = (-5\ 033, 1\ 481, -357)$ $|\delta| = 5\ 259$

Q_2 $\overline{F} = (20, 491, 77)$ $\overline{\delta} = (3\ 811, -958, -173)$ $|\delta| = 3\ 933$

Compliances and Their Contributors

Configuration: Vertical Ladder, Position I   Material: Carbon/Epoxy

$$C_{ij} = \frac{10^6}{EI_1}\left\{K_{\ell 1ij} + K_{u1ij}\right\} + \frac{10^6}{EI_2}\left\{K_{\ell 2ij} + K_{u2ij}\right\} + \frac{10^6}{GJ}\left\{K_{\ell Qij} + K_{uQij}\right\}$$

$\ell_\ell = 584\ 33$ $e_1 = 6.352$ $\frac{10^6}{EI_1} = 52589 \times 10^{-3}$ $\frac{10^6}{EI_2} = 16774 \times 10^{-3}$ $\frac{10^6}{GJ} = 2\ 6531 \times 10^{-3}$

$\ell_u = 552$ $e_3 = 10\ 884$

ij	C_{ij}	$K_{\ell 1}$	K_{u1}	$\frac{10^6}{EI_1}\{K_\ell + K_{u1}\}$	$K_{\ell 2}$	K_{u2}	$\frac{10^6}{EI_2}\{K_\ell + K_{u2}\}$	$K_{\ell c}$	K_{uc}	$\frac{10^6}{GJ}\{K_{\ell Q} + K_{uQ}\}$
11	2249	66.505	35.268	$53\ 521 \times 10^{-3}$	0	15062	0253×10^{-3}	3794	64 200	171 336 ×
12	− 01331	0	1.4999	7888×10^{-3}	0	2 8337	4753×10^{-2}	−16205	−5 3303	−14 572 ×
13	004672	1 0844	− 67068	2176×10^{-3}	0	−2 0027	-3359×10^{-3}	0	1 8057	4 791 ×
22	02609	0	12882	06775×10^{-2}	66.505	80 512	24.661×10^{-3}	06922	44256	1 3578 ×
23	− 006212	0	− 12498	-07098×10^{-3}	0	−34.238	$-5\ 743 \times 10^{-3}$	0	− 14992	− 3978 ×
33	004735	.02358	18705	11077×10^{-2}	0	27 063	$4\ 540 \times 10^{-3}$	0	05079	1346 ×

Deflections

Q_1 $\overline{F} = (-20, 494, 77)$ $\overline{\delta} = (-5\ 120, 1\ 507, -363)$ $|\delta| = 5\ 349$

Q_2 $\overline{F} = (20, 494, 77)$ $\overline{\delta} = (3\ 876, 975, -177)$ $|\delta| = 4\ 001$

Compliances and Their Contributors

Configuration: Vertical Ladder, Position IV Material: Carbon/Epoxy

$$C_{ij} = \frac{10^6}{EI_1}\left\{K_{\ell 1ij} + K_{u1ij}\right\} + \frac{10^6}{EI_2}\left\{K_{\ell 2ij} + K_{u2ij}\right\} + \frac{10^6}{GJ}\left\{K_{\ell Qij} + K_{uQij}\right\}$$

$\ell_\ell = 7.15$ $e_1 = 6\,352$ $\frac{10^6}{EI_1} = 52589\times10^{-3}$ $\frac{10^6}{EI_2} = 16774\times10^{-3}$ $\frac{10^6}{GJ} = 2\,6531\times10^{-6}$

$\ell_u = 552$ $e_3 = 10\,884$

ij	C_{ij}	$K_{\ell 1}$	K_{u1}	$\frac{10^6}{EI_1}\{K_{\ell}+K_{u1}\}$	$K_{\ell 2}$	K_{u2}	$\frac{10^6}{EI_2}\{K_{\ell}+K_{u2}\}$	$K_{\ell Q}$	K_{uQ}	$\frac{10^6}{GJ}\{K_{\ell Q}+K_{uQ}\}$
11	07055	121 242	5 447	$16\,940\times10^{-3}$	0	12 737	$2\,137\times10^{-3}$	4642	0927	$1\,478\times10^{-3}$
12	00450	0	13 862	$7\,290\times10^{-3}$	0	36 446	$6\,113\times10^{-3}$	-1983	-4 634	-12 953
13	-00493	1 [illegible]	-12 040	$-5\,473\times10^{-3}$	0	-3 249	-545×10^{-3}	0	4129	$1\,095\times$
22	6376	0	40 110	$21\,093\times10^{-3}$	121 842	104 640	$37\,990\times10^{-3}$	0847	236 301	628 481
23	-07977	0	-43 408	$-22\,828\times10^{-3}$	0	-9 327	$-1\,565\times10^{-3}$	0	-20 871	-55 73
33	03676	02885	60 338	$31\,746\times10^{-3}$	0	8314	1395×10^{-3}	0	1 839	$4\,879\times$

Deflections

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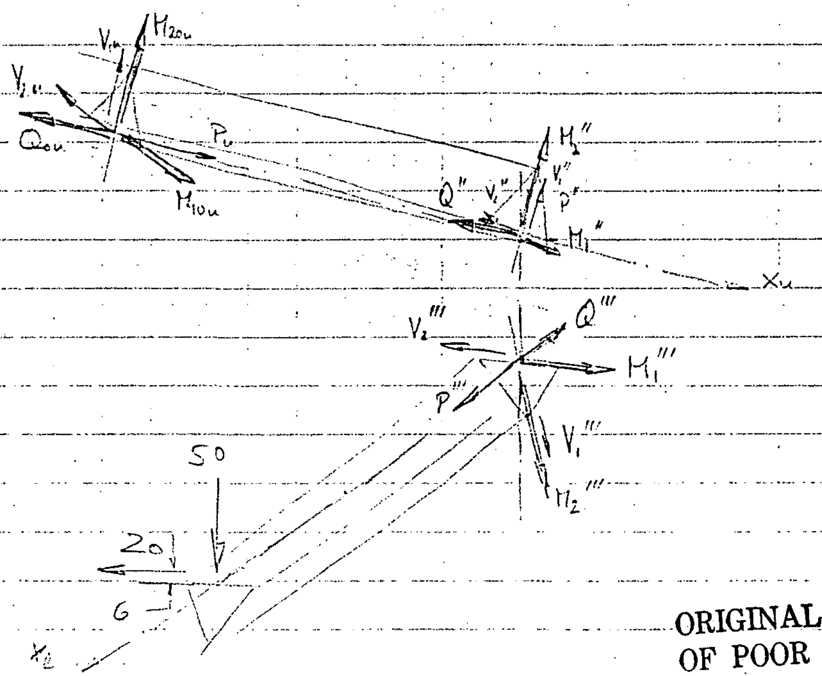
Q_1 $\overline{F} = (-20, 50, 0)$ $\overline{\delta} = (-1\,389, 34\,371, -3\,890)$ $|\delta| = 34\,62$

Q_2 $\overline{F} = (20, 50, 0)$ $\overline{\delta} = (1\,436, 34\,380, 1\,739)$ $|\delta| = 34\,46$

APPENDIX H

HORIZONTAL LADDER CONFIGURATION

Forces in horizontal-ladder truss



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∴ lower arm $V_1''' = 49.4$

$V_2''' = 20$

$P''' = 7.7$

$x_l = 0$ $M_1''' = 49.4 \times 584.33 = 28\,866$

$M_2''' = 20 \times 584.33 = 11,687$

$Q''' = -120$

note transformation matrix can be left same as for vertical ladder but force vector is changed to

$$\begin{array}{cccccccc} Q_2 & Q_1 & & & & & Q_1 & Q_2 \\ \begin{bmatrix} +9.95 \\ -45.37 \\ 20.32 \\ -10\,325 \\ 26998 \\ 5683 \end{bmatrix} & \begin{bmatrix} -19.95 \\ -46.72 \\ 17.33 \\ 8811 \\ 23593 \\ -8999 \end{bmatrix} & \begin{bmatrix} -V_2'' \\ +V_1'' \\ +P'' \\ -M_2'' \\ M_1'' \\ +Q'' \end{bmatrix} & = & \begin{bmatrix} T \end{bmatrix} & \begin{bmatrix} -V_2''' \\ V_1''' \\ P \\ -M_{20}''' \\ M_{10}''' \\ Q''' \end{bmatrix} & \begin{bmatrix} -20 \\ 49.4 \\ 7.7 \\ -11\,687 \\ 28866 \\ -120 \end{bmatrix} & \begin{bmatrix} +20 \\ 49.4 \\ 7.7 \\ +11\,687 \\ 28866 \\ +120 \end{bmatrix} \end{array}$$

62)
6-1-73

then real $V_1'' = -46.72$
$V_2'' = 19.95$
$P'' = 17.85$
$M_1' = 28,595$
$M_2'' = -8,847$
$Q'' = -8,999$

at $x_u = l_u - 37.5$ $V_1 = -46.72$
$V_2 = 19.95$
$P = 17.85$
$M_1 = 28\,595 - 46.72 \times 37.5 = 26\,843$
$M_2 = -8,811 + 19.95 \times 37.5 = -8,063$
$Q = -8,999$

at $x_l = 37.5$ $V_1 = 49.4$
$V_2 = 20$
$P = 7.7$
$M_1 = 28\,866 - 49.4 \times 37.5 = 27,014$
$M_2 = 11,687 - 20 \times 37.5 = 10,937$
$Q = -120$

	lower arm	Peak (see p 66)	upper arm
P_1	739.1	4	-119.6
P_2	163.3		1026.7
P_3	-894.7		-849.0
P_4	36.1	4863	-477.3
P_5	47.7	4863	398.1
P_6	-35.2	4863	-467.0
P_7	34.0	10,488	313.0

	panel 1	panel 2		panel 14	panel 15
$P_{1,1}$	790.7	764.9	9067	1026.0	684.0
$P_{1,2}$	137.5	163.3	9067	684.7	1026.7
$P_{1,3}$	-965.7	-823.7	2681	-831.0	-947.0

M.S = .075

$\boxed{62a}$

6-8-76

reverse V_2 to -20

at x_1 = 37.5   at $x_u = l_u - 37.5$

V_1	49.4	-45.67
V_2	-20	-19.95
P	7.7	20.38
M_1	27,014	27,285
M_2	-10,937	9,575
Q	[illegible] 120	5,683

$\hat{f}_1$	795.7	960.5
$\hat{f}_2$	111.7	-36.5
$\hat{f}$	-894.7	-903.7
f_p	47.7	237.7
$\hat{f}_3$	36.1	-315.1
$\hat{f}_d$	35.2	505.0
$\hat{f}_l$	-24.0	-208.2

	panel 1	panel 2	panel 16	panel 15
$\hat{f}_{11}$	[illegible]	739.1	512.7	736.1
$\hat{f}_{12}$	137.5	111.7	127.4	-36.5
$\hat{f}_{13}$	-956.1	-823.5	$\boxed{-847.0}$	-960.4

M.S. = .055

6-1-78

back longeron gets 831 lb compression

P_{crit} must be at least 3 × 831 lb

then $$\frac{\pi^2 E \frac{\pi}{64}(1.25^4 - ID^4)}{4 \times 37.5^2} > 2493$$

aluminium $ID^4 < -453$ not even full rod ok!

carbon epoxy $ID < .9985$ for strength

for stiffness thicker wall may be required

Other members (Carbon/Epoxy only)

longerons 1, 2 $$\frac{\pi^3 \times 20 \times 10^6 (1.25^4 - ID^4)}{64 \times 37.5^2} > 3 \times 1026.7$$

$ID < 1.188$ ($\frac{1}{32}$ wall)

a-members (477.3 lb comp.) → $ID < 1.197$

d-members less load

note stiffness may require thicker walls.

Weight 15 + 20 panels

$$\overset{280.91}{(2 \times 51.204 + 51.205 + 1.5 \times 34.866 + 2 \times 37.5)} .39396$$

$+ 37.5 \times .4882$

$+ 34.866 \times .2332$

$37.5 \times .07069$

110.67

18.31

8.13

2.6

139.75

@ .057 lb/in³ → 7.966 lb/panel

159.3 lower / 119.5 upper

for deformations matrix is slightly different

because $Q'' - Q_0 l = -V_2''' \times 6$ $\quad H_1''' = V_1''' l_e \quad H_2''' = V_2''' l_e$

$$\begin{bmatrix} -V_{2u} \\ +V_{1u} \\ P_u \\ -H_{2u} \\ H'_{1u} \\ Q_u \end{bmatrix} = \begin{pmatrix} \bar{T}_{11} & \bar{T}_{12} & \bar{T}_{13} \\ \bar{T}_{21} & \bar{T}_{22} & \bar{T}_{23} \\ \bar{T}_{31} & \bar{T}_{32} & \bar{T}_{33} \\ [\bar{T}_{41} + \bar{T}_{44} l_e - \bar{T}_{46} \cdot 6] & \bar{T}_{42} + \bar{T}_{45} l_e & \bar{T}_{43} \\ [\bar{T}_{51} - \bar{T}_{54} l_1 - \bar{T}_{56} \cdot 6] & \bar{T}_{52} + \bar{T}_{55} l_e & \bar{T}_{53} \\ [\bar{T}_{61} + \bar{T}_{64} l_e - \bar{T}_{66} \cdot 6] & \bar{T}_{62} + \bar{T}_{65} l_e & \bar{T}_{63} \end{pmatrix} \begin{bmatrix} -V_2''' \\ V_1''' \\ P' \end{bmatrix}$$

different

or

$$\begin{matrix} V_{1u} \\ V_{2u} \\ P_u \\ H'_{1u} \\ H_{2u} \\ Q_u \end{matrix} = \begin{pmatrix} \bar{T}_{22} & -\bar{T}_{21} & \bar{T}_{23} \\ -\bar{T}_{12} & \bar{T}_{11} & -\bar{T}_{13} \\ \bar{T}_{32} & -\bar{T}_{31} & \bar{T}_{23} \\ \bar{T}'_{52} & -\bar{T}'_{51} & \bar{T}_{53} \\ -\bar{T}'_{42} & \bar{T}'_{41} & -\bar{T}_{43} \\ \bar{T}'_{62} & -\bar{T}'_{61} & \bar{T}_{63} \end{pmatrix} \begin{matrix} V_1'' \\ V_2' \\ ? \end{matrix}$$

[illegible]

new matrix for upper arm terms

$$[T_\Delta] = \begin{array}{ccc} -.85429 & -.026228 & -.51913 \\ .010525 & .99765 & -.067725 \\ .51968 & -.0633205 & -.85201 \\ \begin{array}{c} 592\,608 \\ \cancel{560.35} \end{array} & -9\,7212 & -62\,106 \\ \begin{array}{c} 15\,580 \\ \cancel{14\,732} \end{array} & -478\,39 & -1\,6323 \\ \begin{array}{c} -37\,613 \\ -\cancel{35\,566} \end{array} & -358\,55 & 3\,9416 \end{array}$$

note - lower arm $G(x) = -V_2 \cdot 6 - V_2 e_3 = -V_2(e_3 + 6)$

$$\text{then} \quad \frac{\partial U_\ell}{\partial V_1} = V_1 \frac{1}{EI_1} \frac{\ell_\ell^3}{3} + P \frac{e_1 \ell_\ell^2}{2EI_1}$$

$$\frac{\partial U_\ell}{\partial V_2} = V_2 \left(\frac{\ell_2^3}{3EI_2} + \frac{(c_3 + 6)^2 \ell_\ell}{GJ} \right)$$

$$\frac{\partial U_\ell}{\partial P} = V_1 \frac{e \ell_\ell^2}{2EI_1} + P \frac{e_1^2 \ell_\ell}{EI_1}$$

for numerical values assume wall thicknesses that produce approximately a 275 lb truss

all main members same ID except knee braces ID = 1.1 and back scissor (.5 OD / .4 ID)

then length of tubing per panel

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$2a + 3c + d + 1.5b = 318.4$ in

total number of panels 35 → 11145 in (929 ft)

length of knee braces and [illegible] 1220 A = .2332 [illegible]
[illegible] scissor 1313 A = .0750 [illegible]
[illegible] scissor [illegible] A = .4822 [illegible]

275 lb → allows total volume of 4,825 in^3
for structural tubing 4447 in or 399 in^2 [illegible]

I.D. = 1.027

note scissor needs thicker wall

$ID_s = .97$ → $ID_{a,b,d,i} = 1.034$ use 1.03

then
$$\frac{1}{EI_1} = \frac{1}{30\cdot 10^6}\ [illegible]\ \frac{4}{\pi}\left(\frac{1}{2(1.25^4-1.03^4)} + \frac{1}{(1.25^4-.97^4)}\right) = 18195\cdot 10^{-9}$$

$$\frac{1}{EI_2} = \frac{4}{3\ [illegible]}\ \frac{1}{20\ [illegible]}\ \frac{4}{\pi(1.25^4-1.03^4)} = 20880\times 10^{-9}$$

$$e_1 = 30.194\ \frac{1}{1+2\frac{(1.25^4-1.03^4)}{(1.25^4-.97^4)}} = 11.531$$

$$e_3 = 30.194\ \frac{\left(1-\frac{[illegible]}{375} + \frac{3}{[illegible]}\left(\frac{34.8[illegible]}{375}\right)^3\right)}{(1+[illegible])} = 12.556$$

$$\frac{1}{GJ} = \frac{375}{[illegible]}\ [illegible] = 14280\times 10^{-9}$$

Compliances and Their Contributors

Configuration: Horizontal Ladder, Position I Material: Carbon/Epoxy

$$C_{ij} = \frac{10^6}{EI_1}\left\{K_{l1ij} + K_{u1ij}\right\} + \frac{10^6}{EI_2}\left\{K_{l2ij} + K_{u2ij}\right\} + \frac{10^6}{GJ}\left\{K_{l\theta ij} + K_{u\theta ij}\right\}$$

$l_l = 524.33$ $e_1 = 11.551$ $\frac{10^6}{EI_1} = 18195 \times 10^{-3}$ $\frac{10^6}{EI_2} = 20880 \times 10^{-3}$ $\frac{10^6}{GJ} = 14280 \times 10^{-3}$

$l_u = 562.5$ $e_3 = 13.556$

ij	C_{ij}	K_{l1}	K_{u1}	$\frac{10^6}{EI_1}\{K_{l1}+K_{u1}\}$	K_{l2}	K_{u2}	$\frac{10^6}{EI_2}\{K_{l2}+K_{u2}\}$	$K_{l\theta}$	$K_{u\theta}$	$\frac{10^6}{GJ}\{K_{l\theta}+K_{u\theta}$
11	02845	66 505	83 054	$27\ 212 \times 10^{-3}$	0	19500	0407×10^{-3}	0	80184	1 193
12	01077	0	-3 2615	-5934×10^{-3}	0	-1 9071	-3982×10^{-3}	0	7 9020	11 758
13	-006639	1 9721	-37 355	$-6\ 4379 \times 10^{-3}$	0	-22625	04724×10^{-3}	0	-10321	-15358
22	13780	0	18901	03439×10^{-3}	66 505	36770	$21\ 564 \times 10^{-3}$	22347	77 872	116 206
23	-000808	0	2 3878	43446×10^{-3}	0	1 2989	27121×10^{-3}	0	-1 0172	-1 5136
33	005688	07797	30 718	$5\ 6033 \times 10^{-3}$	0	30860	$06444 \times 10^{-}$	0	01323	01976

Deflections

Q_1 $\overline{F} = (494, 20, 77)$ $\bar{\delta} = (1\ 570, 3\ 282, -300)$ $|\delta| = 3\ 650$

Q_2 $\overline{F} = (491, -20, 77)$ $\bar{\delta} = (1\ 139, -2\ 230, 388)$ $|\delta| = 2\ 534$

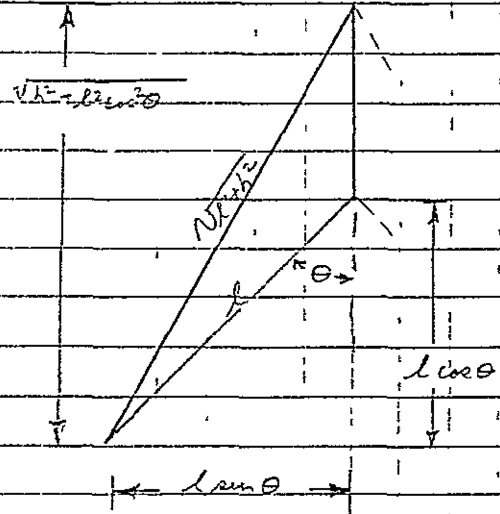
APPENDIX I

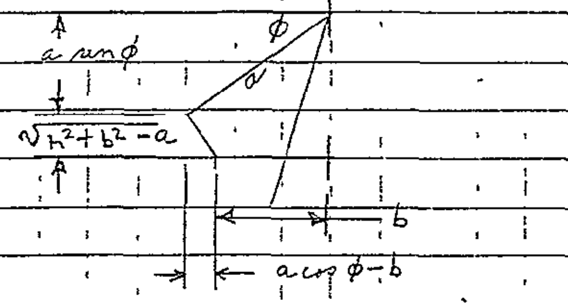
KNEE BRACE SYNCHRONIZER ANALYSIS

Prepared	Name JC STAMMREICH	Date 7-11-78	**ASTRO** RESEARCH CORPORATION	Page 1 of 11
Checked			TITLE KNEE BRACE SYNCHRONIZER ANALYSIS	Program EVA1A
Approved				Job No 11792.

This analysis is a rewritten copy of Dr John Hedgepeth's analysis of 5-8-75

Problem – develop a relationship between the knee brace angle, ϕ, and the free panel angle, θ, of the EVATA truss structure.





side view of knee brace

h = deployed depth of truss

top view of knee brace

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setting top view equal to side view:

$$\sqrt{h^2+l^2\cos^2\theta}-l\cos\theta=a\sin\phi+\sqrt{h^2+a^2\sin^2\phi+2ab\cos\phi-2a\sqrt{h^2-b^2}}$$

divide by l and solve for $\cos\theta$

$$\cos\theta=\frac{c^2-K^2}{2K}$$

where $c = h/l$

$$K=\frac{a}{l}\sin\phi+\sqrt{\left(\frac{h}{l}\right)^2+\left(\frac{a}{l}\right)^2\sin^2\phi+2\frac{a}{l}\frac{b}{l}\cos\phi-2\frac{a}{l}\sqrt{\left(\frac{h}{l}\right)^2-\left(\frac{b}{l}\right)^2}}$$

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Preliminary EVATA dimensions:

$l = 33''$

$b = 18''$

$h = 22''$

$a = 23''$

programming equations c, k into TI 52

ϕ	θ
10	12.18
11	26 64
10.5	20,88
12	34 94
13	41.08
15	50.14
20	64.21
25	72.89
30	78 79
35	83 44
40	86.73
45	88 94
46	89 26
47	89 52
48	89.74
49	89 89
50	89.98
50 71	90,00
9.80	5.4
9 76	2,24
9.752	1.95
9,755	1,41
9,152	0 42
9 7518	0,23

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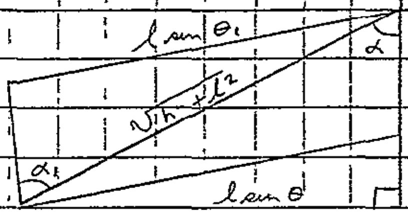
Effect of inaccuracies



for a given ϕ, θ is known and

$$\alpha = \sin^{-1} \frac{\sin\theta}{\sqrt{1+\left(\frac{h}{l}\right)^2}}$$

if the sensor angle θ_1 is wrong then a difference Δ will occur between α_1 and α

$$\alpha_1 = \alpha + \Delta$$

$$\theta_1 = \sin^{-1}\left(\sqrt{1+\left(\frac{h}{l}\right)^2} \sin\alpha_1\right)$$

θ	$\Delta = 1$	$\Delta = -1$
5	6.2	3.8
10	11.2	8.8
20	21.2	18.8
30	31.3	28.7
40	41.3	38.7
50	51.5	48.6
60	61.7	58.4
70	72.3	67.9
75	78	72.4
80	85.3	76.6
85	–	79.8
87	–	80.7
90	–	81.2

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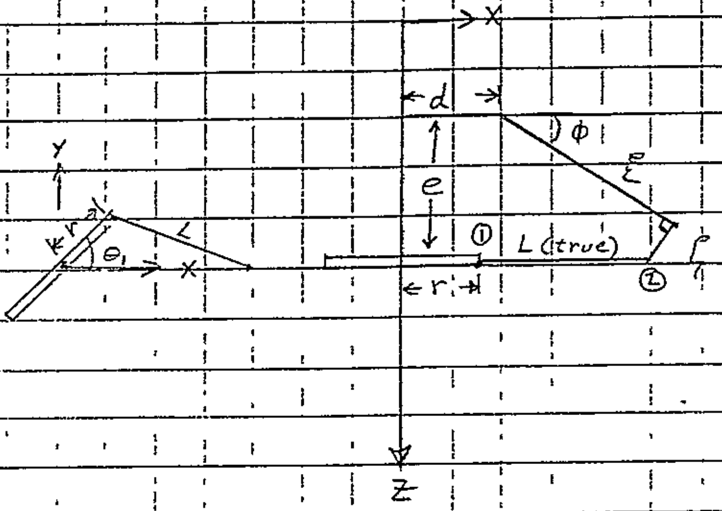
Synchronizer Geometry



for point ①

$x = r \cos \theta_1$

$y = r \sin \theta_1$

$z = e$

point ②

$x = d + \xi \cos \phi - \zeta \sin \phi$

$y = 0$

$z = \xi \sin \phi + \zeta \cos \phi$

$$L^2 = (d + \xi \cos\phi - \zeta \sin\phi - r\cos\theta_1)^2 + r^2 \sin^2\theta_1 + (\xi \sin\phi + \zeta\cos\phi - e)^2$$

$$= e^2 + d^2 + \xi^2 + \zeta^2 + 2d(\xi\cos\phi - \zeta\sin\phi) - 2e(\xi\sin\phi + \zeta\cos\phi)$$

$$+ r^2 - 2r\cos\theta_1 (d + \xi\cos\phi - \zeta\sin\phi)$$

$$2r(d + \xi\cos\phi - \zeta\sin\phi)\cos\theta_1 = r^2 - L^2 + (d + \xi\cos\phi - \zeta\sin\phi)^2$$

$$+ (\xi\sin\phi + \zeta\cos\phi - e)$$

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When $\phi = 50.71°$, $\theta = 90°$ so:

$$L^2 = r^2 + (d + \xi\cos\phi_5 - \zeta\sin\phi_1)^2 + (\xi\sin\phi_5 + \zeta\cos\phi_5 - e)^2$$

so.

$$2r\cos\phi_1 = (d + \xi\cos\phi - \zeta\sin\phi) + \frac{(\xi\sin\phi + \zeta\cos\phi - e)^2 - (\xi\sin\phi_5 + \zeta\cos\phi_5 - e)^2 - (d + \xi\cos\phi_5 - \zeta\sin\phi_5)^2}{d + \xi\sin\phi - \zeta\sin\phi}$$

When $\phi = 9.75°$, $\theta = 0$ so

$$2r = (d + \xi\cos\phi_1 - \zeta\sin\phi_1) + \frac{(\xi\sin\phi_1 + \zeta\cos\phi_1 - e)^2 - (\quad)^2 - (\quad)^2}{d + \xi\cos\phi_1 - \zeta\sin\phi_1}$$

let $x = \xi\cos\phi - \zeta\sin\phi$

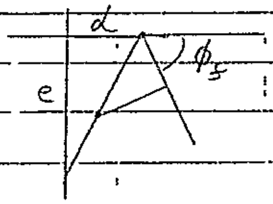
$z = \xi\sin\phi + \zeta\cos\phi$

then $L^2 - r^2 = (d + x_5)^2 + (e - z_5)^2$

$$r = d - x_1 + \frac{(e - z_1)^2 - [L^2 - r^2]}{2(d + x_1)}.$$

$$\cos\theta_1 = \frac{\dfrac{(d+x)^2 + (e-z)^2 - [L^2 - r^2]}{d + x}}{2r}$$

we have "lockup" at $\phi = \phi_5$ when



$$e = \frac{-z_5}{x_5} d$$

for $d = 1$, $\phi_f = 50.719$, $\phi_1 = 7.75°$

ε	.5	.5	0	.5	.3	.2	.2	.01	.15
ζ	1.0	1.0	1.0	1.5	.9	.6	.4	.02	1.000
e	4483	2.2308	.8182	1.583	1.583	1583	2.2308	2.2308	1.000
ϕ	14	21.4	19.1	18.7	21 3	22.7	23.7	25.4	19
10	2.6	38.8							34.8
15	34.9	50.9	46.6	45	50.9	53.5	55.3	58.3	46
20		60.8							55.6
25	50 8	69.4	65.4	62.9	69.5	72	73 5	75 9	64.0
30		76.9			77	79.1	80.2	81.9	72.1
35	67.4	83.1	80.9	78.4	83.3	84.6	85.2	86.2	79.3
40	77	87.8	86.9	85 3	87.8	88.3	88 6	88.9	85.5
45	88.2	89 96	89.94	89 88	89.96	89.98	89.98	89.98	87.9
50									
r	5886	5387		.614	4051	2843	.2562	.01456	.4244
L	.9839	1.432		.680	1.009	1.272	2.0139	2 4224	5895

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Partial Rotation

assume that $\theta_1(\phi_1) = 0$

$$\theta_1(\phi_5) = \theta_0 + 90^\circ$$

since $$2r\cos\theta_1 = \frac{(d+x)^2 + (e-z)^2 - [L^2 - r^2]}{d+x_1}$$

$$2r = \frac{(d+x_1)^2 + (e-z_1)^2 - [L^2 - r^2]}{d+x_1}$$

$$2r\cos\theta_0 = \frac{(d+x_5)^2 + (e-z_5)^2 - [L^2 - r^2]}{d+x_5}$$

$$L^2 - r^2 = \frac{(d+x_1)[(d+x_5)^2 + (e-z_5)^2] - (d+x_5)\cos\theta_0[(d+x_1)^2 + (e-z_1)^2]}{d+x_1 - (d+x_5)\cos\theta_0}$$

$$r = \frac{(d+x_1)^2 + (e-z_1)^2 - (d+x_5)^2 - (e-z_5)^2}{2[d+y_1 - (d+x_5)\cos\theta_0]}$$

$$r = \frac{d(x_1 - x_5) + e(z_5 - z_1)}{d+x_1 - (d+x_5)\cos\theta_0}$$

if $\frac{d\theta_1}{d\phi} = 0$ at $\phi = \phi_5$

$$\frac{d}{d\phi}\left[\frac{(d+x)^2 + (e-z) - [L^2 - r^2]}{d+x}\right]_{\phi=\phi_5}$$

if $\frac{dx}{d\phi} = -z$, $\frac{dz}{d\phi} = x$

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$$\frac{(d+x_f)^2+(e-z_f)^2-\left[L^2-r^2\right]}{d+x_f} = 2\,\frac{d z_f + e x_f}{z_f}$$

substituting,

$$\frac{d z_f + e x_f}{z_f} = \frac{(d+x_1)^2+(e-z_1)^2-(d+x_f)^2-(e-z_f)^2}{2\left[d+x_1-(d+x_f)\cos\theta_0\right]}\cos\theta_0$$

$$= \frac{d(x_1-x_f)-e(z_1-z_f)}{d+x_1-(d+x_f)\cos\theta_0}\cos\theta_0$$

solving for e,

$$e = \frac{d z_f\left[d+x_1-(d+x_f)\cos\theta_0\right]-z_f d(x_1-x_f)\cos\theta_0}{-x_f\left[d+x_1-(d+x_f)\cos\theta_0\right]+z_f(z_f-z_1)\cos\theta_0}$$

$$e = \frac{d z_f(d+x_1)(1-\cos\theta_0)}{x_f(d+x_1)-\left[x_f(d+x_f)-z_f(z_f-z_1)\right]\cos\theta_0}$$

$$= -\frac{d z_f}{x_f}\,\frac{1-\cos\theta_0}{1-\left(\frac{d+x_f}{d+x_1}\right)\cos\theta_0-\left(\frac{z_f}{x_f}\right)\left(\frac{z_f-z_1}{d+x_1}\right)}$$

and

$$r = \frac{1-\frac{d+x_f}{d+x_1}+e\,\frac{z_f-z_1}{d+x_1}}{1-\frac{d+x_f}{d+x_1}\cos\theta_0}$$

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for $d = 1$, $\phi_\xi = 50.71$

ε	ξ	ϕ_1	θ_0	e	$L^2 - r^2$	r	L
1	0	9.75	75	-2.183			
1	1	9.75	65	2.637	1.44126	1.11651	1.63947
1	2	9.75	65	1.5286			
2	2	9.75	65	2.0621	.31343	1.28842	1.3683
2	2	13	50	1.4410	1.32336	1.1681	1.63946
1	1	13	50	1.90772	-0.19847	1.07486	97819

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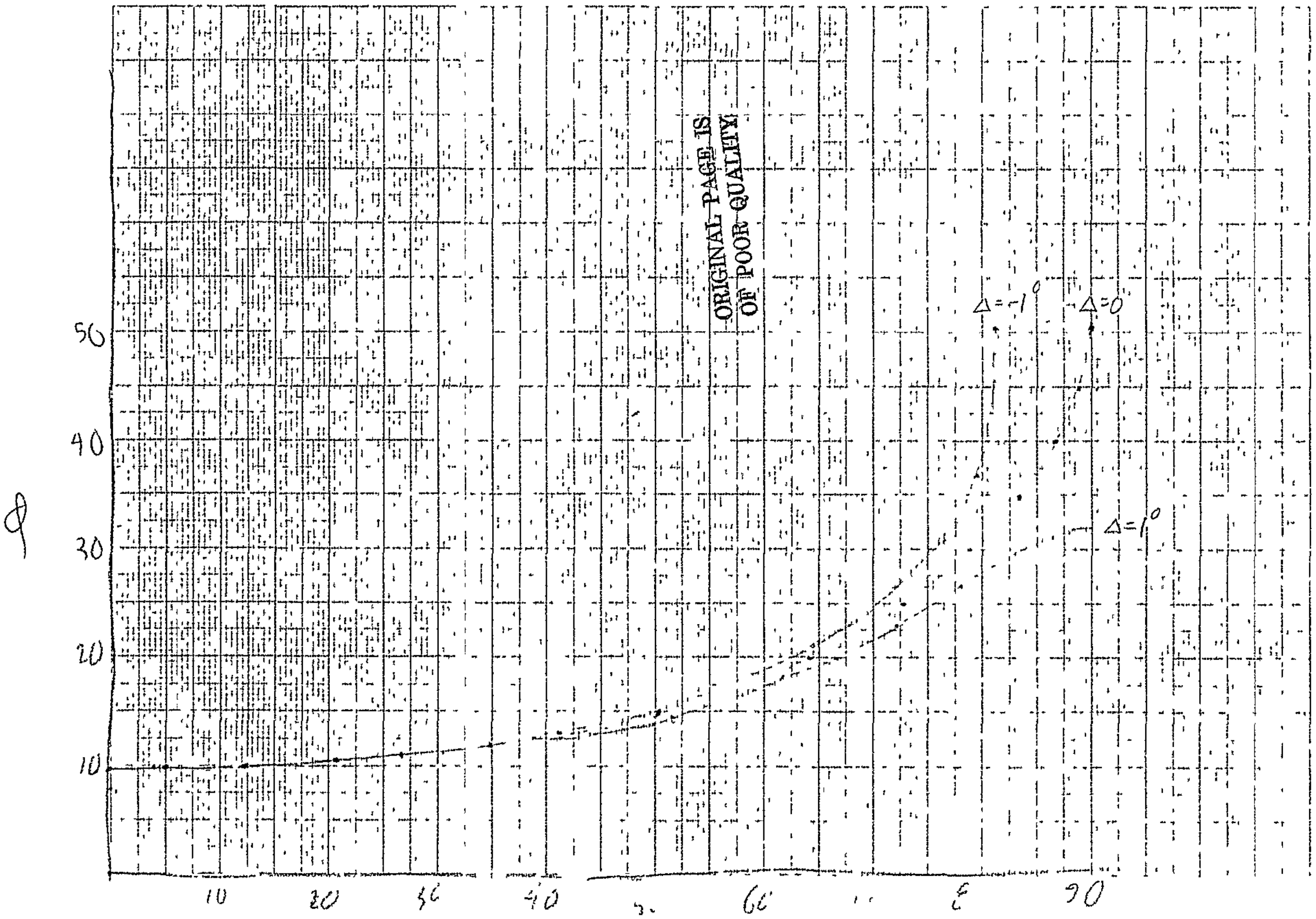
Table of ϕ vs θ_1

	ϕ_1	9.75°	9.75°	13°	13°
	θ_0	65°	65°	50°	50°
	ε	2	1	2	1
	ζ	2	1	2	1
ϕ'	e	2.0621	2.637	1.4410	1.90772
9.75		25.69°	26.6°		
10		29.99	30.6°		
11		35.83°			
12		39.16°	41.1°		
15		47.33°	49.5°	51.7°	52.8°
20		56.32°	59.3°	62°	63.8°
25		63.39°	66.8°	68.8°	71°
30		69.47°	73°	74.3°	76.6°
35		74.96°	78.4°	79°	81.2°
40		80.06°	83.0°	83.1°	84.9°
45		84.86°	86.8°	86.7°	87.87°
50		89.42°	89.72°	89.65°	89.82°



EVATA
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