## General Disclaimer

## One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.
$\square$




# EVALUATION OF WAVELENGTH GROUPS FOR <br> DISCRIMINATION OF AGRICULTURAL COVER TYPES 

R. Kumar<br>Instituto de Pesquisas Espaciais - INPE Conselho Nacional de Desenvolvimento Cientifico e Tecnolögico - CNPq 12.200 - Sāo Josē dos Campos, SP, Brazil


#### Abstract

Multispectral scanner data in twelve spectral channels, in the wavelength range 0.46 to $11.7 \mathrm{\mu m}$, acquired in July 1971 for three flightlines, were analysed by applying automatic pattern recognition techniques. These twelve spectral channels were divided into four wavelength groups (W1, W2, W3 and W4), each consisting of three wavelength groups - - with respect to their estimated probability of correct classification ( $P$ ) -- in discriminati:.g agricultural cover types The same analysis was also done for the data acquired in August, to investigate the effect of time on these results. The effect of deletion of each of the - wavelength groups on $P_{C}$, in the subsets of one to nine channeis, is given. Values of $P_{c}$ for all possible combinations of wavelength groups, in the subsets of one to eleven channels are also given.


## 1. INTRODUCTION

Multispectral scanner (MSS) data were analysed in subsets, . one to twelve spectral channels, in the wavelength range 0.46 to $11.7 \mu \mathrm{~m}$, for selected flightlines of the 1971 Corn Blight Watch Experiment. These tw ave spectral channels were divided into four wavelength groups ( $W 1$, W2, W3 ai $\ddagger$ W4), each consisting of three spectral channels (Table I). The purpose of his study was to determine the statistical separability of multispectral measurements from agricultural cover types for evaluation of these wavelength groups. The agricultural cover types selected were: corn, soybeans, green forage (hay 8 pasture), and forest. In particular, the objectives of the study were: (1) To study the effect of deletion of all possible combinations of the four wavelength groups -- W1, W2, W3 and W4, on the statistical separability and corresponding estimated probability of correct classification ( $P$ ) of the agricultural cover types. (2) To develop a criterion for evaluatfon of a combination of wavelength groups, based on the estimation of the probability of correct classification obtained by using this combination of wavelengin groups in discrimination of agricultural cover types. (3) To investigate the effect of time on these results.

The literature review of the statistical separability of agricultural cover types was done by Kumar and Silva (1977) ${ }^{2}$. In addition, they analysed the multispectral scanner data in wavelength range 0.46 to 11.7 um for three flightines. They found that in the subsets of one to six spectral channels, the combination of wavelength regions (where $V, N, M$ and $T$ denote the visibie, near infrared, middle infrared and thermal infrared wavelength regions, respectively): $V$, $V M$, VNM, VNMT, VVNMT, VVNFMT, respectively, wore found to be the best choices for getting good overall statistical separahility of the agricultural cover types for the data acquired on July 16 as well as August 12.

An effort was made to explain these results on the basis of spectral properties of agricultural cover types. The overall statistical separability of the agricultural cover types was found to be greater for the data of August 12 than the data of July 16. Kunar ${ }^{3}$ (1977) did a further analysis of similar nature, to evaluate explicitly each spectral channel, each wavelength region and all possible combinations of wavelength regions for statistical separability, in terms of estimated probability of correct classi:ication for agricultural cover types. Deletion of the channel $7(0.61$ to $0.70 \mu \mathrm{~m})$ reduced $P_{c}$ by about two percent. The deletion of each of the other channels caused no reduction, or less reduction, in the values of $P_{f}$, as compared to this ciannel. The deletion of the spectral channels constituting the visible wavelength region caused more reduction in $P_{c}$, as compared to the spectral channels constituting any of the other wavelength regions. The deletion of the spectral channels of near infrared wavelength region caused relatively small changes in the values of $P_{C}$. In the subsets of one to six spectral channels, the combination of wavelength regions $V$, VM, VMT, VNMT, VVNMT, VVNMMT; and $T, N T$, VNT or VST, VNMT, VVNMT or VNMMT, VVIMMT were found to be the best choices for the data of middle of July and middle of August respectively.

Since the spectral channels were divided into four wavelength regions, the results obtained could be interpreted on a physical basis; on the other hand, each wavelength region ( $V, N, M, T$ ) is not represented ecually, in the sense that each wavelength region does not have the same number of spectral channels. The author does not have data of a multispectral scanner where each wavelength region has equal number of channels. However, the author feli a definate need for doing an analysis of similar nature by dividing the available twelve
 each wavelength group consisting of three channels, so that each wavelength group is equally represented in the sense that it has an efual number of channels. In addition, this analysis is more complete in tie following aspects: (1) The effect of deletion of all possible combinations of the four wavelength groups on the estimated value of $\mathrm{P}_{\mathrm{C}}$ is given. (2) In addition to the average values of ${ }^{C}$, maximum as well as minimum values of ${ }^{\circ} \mathrm{c}$ for ali possible combinations of wavelengtin groups in the subsets of one to twelve spectral channels are give.

## $\because$. METHOD OF ANALYSIS

Multispectra: scanner data in twelve spectral channels in the wavelength range 0.46 to $11 . \mu \mathrm{m}$, collectes with an optical-mechanica: seanner at altitudes of 914 ; 2133 meters 33000 to 7000 feet) over iestern Indiana were analysed by apply. 'g automatic $!$ ttern recognition rechnicues. The waveiength bands of these twe, ve spectral tannels are Eiven in Tabiez. The data of three selected flightlines, acquired ir. July of 1971, were anaivsej. Each of these three flightines had fair or good amount of each of the four agricultural cover types: corn, soybeans, green forage and forest. These three finghilines were selected carefully so that these combined couid be consicered to be representative of the four agricultural cover types in the festern Indiana.

Black and white photography and gray scale printouts of the spectral channels of the flightlines were used to aid in locating the boundaries of the fields on the Digital Display. Sufficient number of fields of each agricultural cover type were selected carefully so that they could be assumed to be representative of the flightline.

Using the same three flightlines and tweive spectral channels, an identical analysis was perforned on the deta acquired in august of 1971, to study the effect of time on the statistical separability of agricultural cover types. The multispectral scanner data was acquired on both dates (July and August) between 10.30 a.m. and noon time (local solar time). In addition, these data were of good quality and free from problems like lack of stificient eround observations, excessive cloud cover, etc. The analysis was done for the data acquired in July and August, because corn and soybeans have reached their maximum vegetative growth by these times, and one month of time is sufticient for significant =hanges to occur in the spectral properties of agricultural coser types. The author wanted to avoid the analysis of data from late

September afterwards, because soybeans are harvested in September-October. The - ${ }^{-u t h o r ~ t r i e d ~ t o ~ k e e p ~ a l l ~ t h e ~ v a r i a b l e s ~ o t h e r ~ t h a n ~ t i m e ~ u n i f o r m ~ i n ~ t h e ~ t w o ~}$ - (July and August) sets of data as far as possible. For example, an effort was rade to select about the same field boundaries for the two sets of data. A total of more than 550 fields taken from three flightlines were analysed.

Each field was treated as an independent unit and the fields of the same agricultural cover type were put in the same class. The statistics algorithm was used to compute the mean vector and covariance matrix (mean and standard deviation) of the classes. Histograms of the agricultural classes defined above were used to check unimodality of the statistical distributions in individual channels. The classes were redefined to eliminate distinct muitiple modes. Divergence is defined for any two density functions. In the case of normal variables with unequal covariance matrices, divergence in a spectral channels $C_{1}, C_{2}, \ldots, C_{n}$ is given in terms of mean vectors and co:ariance matrices of the classes ${ }^{4}, 5$.

A modified form of the divergence DT, referred to as "transformed divergence", has a behavior ${ }^{4}$, more like the probability of currect classification than the divergence, D.
$D_{T}=2\{1-\exp (-D / 8)\}$
Iransformed divergence has been used throughout this study.


#### Abstract

Although divergence only provides a measure of the distance between two class densities, its use has been extended to the multiclass case by taicing the average over all pairs?. Let DTij denote. the divergence between classes i and $j$ of a certain flightline, then the average-divergence over eil class pairs of four classes (each agricultural cover was treated as a separaie class) is given by


$D_{\text {IAVG }}=\frac{1}{6}\left[D_{T 12}+D_{T 13}+D_{T 14}+D_{T 23}+D_{T 24}+D_{T 34}\right]$
Let $D_{I M I N}=-$ minimir of $\left\{\begin{array}{c}D_{I 12}, D_{I 13}, D_{I 14}, D_{I 23}, D_{I 24}, D_{T 34}\end{array}\right\}$

Let superscri ts 1 and 2 w: th the symbol " $D_{T}$ " denote the values of transformed diverg. ace for the cata acquired in middle of July and midटle of
 -eq. (3)) in first, second and third IIIghtline. respectiveiy, for the daca sequired in middle of July.

Let $\bar{D}_{\text {TAVG }}^{i}=\frac{1}{3}\left[D_{\text {TAVG1 }}^{i}+D_{\text {TAVG2 }}^{i}+D_{\text {IAVG3 }}^{i}\right] \quad$, $i=1,2$

Let $\bar{D}_{\text {TMIN }}^{i}=$ minimum of $\left[D_{\text {TMIN1 }}^{i}, D_{\text {IMIN2 }}^{i}, D_{\text {IMIN }}^{i}\right] \quad, i=1,2$

Assuming each agricultural class has a fultivariate gaussian distribution, the feature selection processor was used to find $\overline{\mathrm{D}}$ TAVG and $\overline{\mathrm{D}}$ TMIN in all possible combinations of one to twelve spectral channels out of the available twelve spectral channels.

Let $D_{\text {TMAX }}^{i}\{$ subset of $r$ spectral channels $\}=\max \left\{\mathrm{D}_{\text {TAVG }}^{i}\right\}, i=1,2$
maximized over all possible subsets of $r$ spectral channels out of the available twelve spectral channels. From the values of the average transformed divergence, ciassification accuracy can be reasonably predicted from the results of Swain et al. (1973) .

Table I gives the wavelength interval and the corresponding wavelength group of each of the twelve spectral channels. Tables II and III give the effect of deletion of each of the four wavelength groups and all possible combinations of the four wavelength groups respectively on $\bar{D} \frac{1}{T M A X}$ and $\bar{D}$ ? ${ }^{2}$ ax , in terms of the corresponding estimated probability of correct classification ( $P_{c}$ ).

To fulfill one of the main objectives of the study -- evaluation of all possible combinations of wavelength groups -- the following criterion is proposed.

Each of 12 available channels of the multispectral scanner can be placed in one of the four wavelength groups W1, W2, W3 and W4 (Table I). Thus, any combination of spectral channels can be called as the corresponding combination of the wavelength groups. For example, channel combination 14710 is called "combination of wavelength groups W1 W2 W3 W4". For a given combination of wavelength groups, for example, W1 W2 W3 W4, DTAVG and the corresponding value of $P_{C}$ were calculated using the curve of Swain and King ${ }^{6}$, for ali possible combinations of four spectral channels that constitute the combination WI W2 W3 W4. The minimum, maximum and the mean of these values of $P_{c}$ were calculated for all possible combinations of wavelength groups, in the subsets of one to twelve spectral channels, and is shown in Table. iv, for the data of middle of July as well as middle of August.

## 3. RESULTS AND DISCUSSION

Table II shows that deletion of each of the wavelength groups causes the following maximum reductions in $P_{C}$ for the data of July: W1 (0.5, subset of five channels), W2 ( 0.4 , subset of four channels), W3 (1.0, subset of one channel), W4 ( 2.3, subset of $t h$ ) channels). The corresponding values for reductions of $I$ for the data $\begin{gathered}\text { 'quire in August are: } W \text { ( } 0.2 \text {, subs it of nine }\end{gathered}$ channels) W2 ( $0 . E$ subset of n ne channels) W3 ( 0.5 , subset af three channels), W ( 1.5 , subset of 0 :e channel).

Each wavelength region (visible, near infrared, middle infrared and thermal infrared) contains inge indent valuable information. Deletion of W4 causes more reduction in $P_{c}$ the.. deletion of any of the other wavelengin groups because it is the only group tin .t contains all the spectral channels of two wavelength regions -- middle in: cared and thermal infrared. Deletion of W3 causes more reduction in $P_{C}$ thar $W 1$ or W2 because it has ore spectral channel in the visible and two in the near infrared wavelength region, whereas $W 1$ or $W 2$ has the spectral channels only in the visible wavelength region.

Similarly Table III shows that, among the combinations of two wavelength groups, the deletion of $W 3$ and $W 4$ causes most reduction in $P_{c}$, because it has all the spectral channels of the near infrared, middle infrared and thermal infrared wavelength regions and one spectral channel of the visible wavelength region. Similarly, among the combinations of three wavelength groups, deletion of $W 2$, $W 3$ and $W 4$ causes the most reduction.

Table IV is quite useful, since it evaluates all possible combinations of wavelength groups in the subsets of one to eleven spectral channels, in terms of $P_{c}$. It shows that in the subset of one channel, wi has the highest value of $P_{C}$. This is mainly because channel 7 (red) is an excellent channel for discriminating agricultural cover types. Besides other seasons pointed out by Kumar and Silva<super> (1977), the separability of agricultural cover types in this channel was relatively high due to their low variance in this channel. In the subset of two to four spectral channels, the combination of wavelengii groups W2 W4, W2 W3 W4 and W1 W3 ( 2 W 4 ), respectively, for the data of July; and W3 W4, W2 W3 W4 and W2 W3 (2W4), respectively, for the data of August, are found to be the best choices. This work has much application to

## OXCINAL RACE OB ROOR ginnty

feature selection (i.e. selecting best subset of mepectral channels, out of N available channels, of the existing multispectral scanners), and deciding wavelength bands of future satellites.

The author gratefully acknowledges: the Laboratory for Applications of Remote Sensing, Purdue University, for their permission to use the multispectral scanner data, obtained under the NASA Grant No. 15-005-112; Dr. Nelson de Jesus Parada, the Director of the Instituto de Pesquisas Espaciais (INPE), for his permission to publish this work; Dr. Celso de Renna Souza and Dr. Derli Chaves Machado da Silva, for their continuous encouragement and assistance; Mr. Nandamudi L. Vijay Kumar, for his help in preparation of the tables.

## REFERENCES

1: Bauer, M.E., in M.E. Bauer (ed.), "1971 Corn Blight Watch Experiment Einal Report", National Aeronautics and Space Administration, Lyndon B. Johnson Space Center, Houston, Texas, 1974.
2. Kumar, R. and L. Silva, "Separability of Agricultural Cover Types by Remote Sensing in the Visible and Infrared Wavelength Regions", IEEE Transactions on Geoscience Electronics, Jan. 1977.
3. Kumar, R., "Evaluation of Spectral Channels and Wavelength Regions for Separability of Agricultural Cover Types", Proceedings of the Eleventh International Symposium on Remote Sensing of Environment, Environmental Research Institute of Michigan, Ann Arbor, Michigan, 1977:
4. Swain, P.H., "Pattern Recognition: A Basis for Remote Sensing Data Analysis", LARS Information Note 111572, Laboratory for Applications of Remote Sensing, Purdue University, W. Lafayette, Indiana, 40 p., 1972.
5. Marill, T. and D.M. Green, "On the Effectiveness of Receptors in Recognition Systems", IEEE Trans. on Information Theory, Vol. IT-9, PP. 11-17, 1963.
6. Swain, P.H. anr R.C. King, "Two Effective Eeature" Selection Criteria for Multispectra. Remote Sensing", in International Joint Conference on Pattern Reco 2 ition (LARS Information Note 042673, Purciue University). Washington, I.C., 1973.
7. Fu, K.S. and P... Min, "On Feature Selection in Multiclass Pattern Recognition", Technical Report TR-EE 68-17, School of Electrical Engineering, urdue University, W. Lafayette, Indiana, $1 \neq 68$.

TABI $\because$ I. WAVELENGTH BANDS OF THE SPECTRAL CHANHIELS
Channel No. Wavelength Band
Wavelength Region Navelength Group

| 1 | $0.46=0.49$ |
| ---: | ---: |
| 2 | $0.48=0.51$ |
| 3 | $0.50=0.54$ |
| 4 | $0.52=0.57$ |
| 5 | $0.54=0.60$ |
| 6 | $0.58=0.65$ |
| 7 | $0.61=0.70$ |
| 8 | $0.72=0.92$ |
| 9 | $1.00=1.40$ |
| 10 | $1.50=1.80$ |
| 11 | $2.00=2.60$ |
| 12 | $9.30=11.70$ |


| visible | 1 |
| :---: | ---: |
| visible | 2 |
| visible | 1 |
| visible | 2 |
| visible | 2 |
| visible | 2 |
| visible | 3 |
| near infrared | 3 |
| near infrared | 3 |
| middle infrared | 4 |
| middle infrared | 4 |
| thermal infrared | 4 |

table il. effect of deletion of each wavelength group on the percentage of correct classification

## Number of <br> Channels in Values $0:$ Probsility of Correct Classification Estimated from $\mathrm{D}_{\text {TMAX }}$ the subset

|  | $\mathrm{A}_{0}$ | $A_{1}$ | $A_{2}$ | $\mathrm{A}_{3}$ | $\mathrm{A}_{4}$ | $\mathrm{B}_{0}$ | $B_{1}$ | $B_{2}$ | $E_{3}$ | $\mathrm{B}_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 84.3 | 84.3 | 84.3 | 83.3 | 84.3 | 35.4 | 85.4 | 85.4 | 83.9 | 85.4 |
| 2 | 89.2 | 89.2 | 89.2 | 88.3 | 86.9 | 91.1 | 91.1 | 91.1 | 90.6 | 90.4 |
| 3 | 90.9 | 90.9 | 90.9 | 90.2 | 90.0 | 94.3 | 94.2 | 94.3 | 93.8 | 93.5 |
| 4 | 92.6 | 92.6 | 92.3 | 92.2 | 90.6 | 96.2 | 96.2 | 96.1 | 25.c | 95.0 |
| 5 | 93.9 | . 93.7 | 93.7 | 93.8 | 91.8 | 96.8 | 96.8 | 96.7 | ¢ 6.7 | 95.9 |
| 6 | 94.6 | 94.2 | 94.2 | 94.4 | 92.4 | 97.3 | 97.3 | 97.1 | 97.1 | 96.2 |
| 7 | 95.0 | 94.6 | 94.7 | 94.7 | 93.0 | 97.5 | 97.5 | 97.3 | ¢7.2 | 96.4 |
| 8 | 95.4 | 95.0 | 95.1 | 94.9 | 93.5 | 97.7 | 97.6 | 97.5 | 97.3 | 96.5 |
| 9 | 95.7 | 95.2 | 95.4 | 95.0 | 93.7 | 97.8 | 97. C | 97.5 | 07.3 | 96.5 |
| 10 | 95.9 | np | np | np | np | 97.8 | n\% | np | np | np |
| 11 | 96.0 | np | np | np | np | 97.8 | np | np | np | np |
| 12 | 96.1 | np | np | $n \mathrm{p}$ | np | 97.9 | nf | np | np | np |

Note: This able gives tae va:ues of perctatage pr bability of orrect classification ( $P_{C}$ ) estimated from the valuev of $D_{T M E X}$ (scseq. (6)), Swain and King (1973) ${ }^{6}$. $A_{0}, A_{1}, A_{2}, A_{3}$ and $A$. denote $=$ ie $\because$ ilues of $P_{C}$ when using all available channels, deleting sectral chan is in the wavelength groups $1,2,3$ and 4 respectively for the data acquired in the middle of July. $B_{0}, B_{1}, B_{2}, B_{3}$ and $B_{4}$ ce ote corresp...ing quantities as $A_{0}, A_{1}, A_{2}, A_{3}$ and $A_{4}$ respectively for th data accuir, 1 in the middle of August. "np" denotes that this channel co bination was not possible.

## ONOL WrGEE IS OF POOR QUALITY

TABLE III. EFEECT OF DELETION OF COMBINATION OF WAVELENGTH GROUPS OA TEE PERCENTAGE OF CORRECT CLASSIFICATION

## DATA OF JULY

| (A) |  | Values of Probability of Correct Classification ( $P_{C}$ ) Estimated from $\mathrm{B}_{\text {TMAX }}$ (see eq. (6))After Deletion of Combination of Wavelength Regions |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | W1W2 | W1W3 | W2W4 | W2W3 | W2W4 | W3W4 | W1W2W3 | W1W2W4 | W1W3W4 | W2W3W4 |
| 1 | 84.3 | 84.3 | 83.30 | 34.3 | 82.68 | 84.3 | 83.30 | 82.68 | 84.3 | 83.30 | 81.59 |
| 2 | 89.2 | 89.2 | 88.33 | 96, 38 | 86.85 | 85.79 | 34.74 | 86.12 | 86.88 | 86.06 | 83.70 |
| 3 | 90.9 | 90.9 | 90.18 | 86.96 | 90.84 | 89.62 | 36.40 | 88.56 | 89.51 | 86.45 | 83.94 |
| 4 | 92.6 | 96.26 | 52.25 | 90.50 | 91.21 | 90.87 | 88.86 | np | np | np | np |
| 5 | 93.9 | 92.89 | 92.90 | 91.35 | 91.93 | 91.64 | 91.64 | np | np | np | np |
| 6 | 94.6 | 93.89 | 93.62 | 92.29 | 92.91 | 92.68 | 92.67 | np | np | np | np |

## DATA OF AUGUST



| 85.4 | 85.4 | 83.88 | 85.4 | 33.88 | 84.05 | 93.51 | 83.62 | 83.88 | 83.51 | 83.51 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 9.1 .1 | 91.1 | 90.95 | 90.57 | 90.01 | 90.26 | 88.14 | 90.88 | 90.71 | 98.93 | 84.20 |
| 94.3 | 91.64 | 93.95 | 93.55 | 93.62 | 92.33 | 90.53 | 92.17 | 92.10 | 89.36 | 85.47 |
| 96.2 | 95.77 | 95.96 | 95.01 | 95.16 | 94.50 | 90.71 | $n p$ | $n p$ | $n p$ | $n p$ |
| 96.8 | 96.44 | 96.69 | 96.26 | 96.05 | 95.88 | 32.05 | $n p$ | $n p$ | $n p$ | $n p$ |
| 97.3 | 96.77 | 96.77 | 96.38 | 96.18 | 96.01 | 33.89 | $n p$ | $n p$ | $n p$ | $n p$ |

[^0]
## ORIGINAL PAGE E OF POOR QUALIM


\%


# ORIGNal pace is ol moor purcica 












 u




＝$\stackrel{\text { ®n }}{\sim}$


 がいの きッ～ッ～
 a
$a^{2} \geqslant$





 $\mathcal{Z a}_{a} \cup \frac{5}{2}$



## ـ




「ロッツm $\dot{0} 000^{\circ}$
 ＝NNi andrn
 FNMN OMOMn まN～～N さNNが





 がペーベッ









 $\because$ ットツ゚ロのー












$$
a^{0}{ }_{3}^{\infty}
$$


 $20 \times$ mmmosm：


$$
\mathcal{S a}^{u \cdot \frac{C}{x}}
$$


5.1
5.2
.37
4.7
5.0
5.2
5.2
5.4
5.2
5.4
5.5




WGC
（3W1）W2（3W3）（2W4）
$(3 W 1) W 2(2 W 3)(3 W 4)$
$(3 W 1)(3 W 3)(3 W 4)$
$(2 W 1)(3 W \mathrm{C})(3 W 3) W 4$
$(2 W 1)(2 W 2)(2 W 3)(2 W 4) 9$
$(2 W 1)(3 W 2)(3 W 4)$
$(2 W 1)(2 W 3)(3 W 3)(2 W 4) 95$
$(2 W 1)(2 W 2)(2 W 3)(3 W 4) 9$
$(2 W 1) W 2(3 W 3)(3 W 4)$
$W 1(3 W 2)(3 W 3)(2 W 4)$
$W 1(3 W 2)(2 W 3)(3 W 4)$
$W 1(2 W 2)(3 W 3)(3 W 4)$
$(3 W 2)(3 W S)(3 W 4)$ $N=10$

$$
\begin{aligned}
& \text { のaのaのののaかo } \\
& \begin{array}{l}
(3 W 1)(3 W 2)(3 W 3) W 4 \\
(3 W 1)(3 W 2)(2 W 3)(2 W 4) 95.2 \\
(3 W 4.72 \\
94
\end{array} \\
& \text { (3W1)(3W2)(2W3)(2W4)95.2 94.72 94.52 97. } \\
& \text { (3W1) (2W2)(3W3) (2W4)95.4 44.95 95.7197.5 } \\
& \left.\begin{array}{l}
(3 W 1)(2 W 2)(3 W 3)(2 W 4) 95.4 \\
(3 W 1)(2 W 2)(2 W 3)(3 W 4) 95.95 \\
95
\end{array}\right) 95.7197 .97 \\
& \begin{array}{lll}
(3 W 1)(2 W 2)(2 W 3)(3 W 4) 95.5 & 95.14 & 95.69 \\
(3 W 1) & 972(3 W 3)(3 W 4) & 95.66 \\
95.64 & 95.67 & 97 .
\end{array} \\
& \begin{array}{l}
(3 W 1) W 2(3 W 3)(3 W 4) \\
(2 W 1)(3 W 2)(3 W 3)(2 W 4) 95.5 \\
\hline
\end{array} \\
& (2 W 1)(S W 2)(2 W 3)(3 W 4) 95.6 \text { y } 5.4595 .8297 .7
\end{aligned}
$$ por example： 14 dnong yłsuptenem sozouop $=9: \%$

combination in $N$
-a



[^0]:    liote: "np" denotes that deletion of combination of wavelength regions was not possible. W1, W2, W3 and W4 denote wavelength groups $1,2,3$ and 4 respectively (see Table I).
    $N=$ Number of channels in the subset $P_{c}^{*}=P_{c}$ Without Deletion

