View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by NASA Technical Reports Server

D6 (32)

N78-30457

EFFECT OF PIXEL DIMENSIONS ON SAR PICTURE QUALITY

JOHN R. PIERCE
VIJAYA N. KORWAR
DEPARTMENT OF ELECTRICAL ENGINEERING
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA 91125

SUMMARY

In an SAR mapping system, the pixel size for best picture quality is not always known. Here we investigate whether it is worthwhile looking at small pixels if it turns out after processing that such fine resolution was not necessary and that several adjacent pixels therefore need to be combined. The product of looks per pixel and number of pixels in the scene is kept constant. Assuming that the returns from all the resolution cells obey Rayleigh statistics, the expression for pixel SNR incorporating both speckle and additive white Gaussian noise is derived. We conclude that it is possible to use fine resolution and leave the large-area estimate slightly but not much worse than if the larger pixel size had been initially decided upon.

1.0 INTRODUCTION

Speckle reduction in SAR requires incoherent averaging over several independent looks at each pixel. [1], [2], [3]. If we can settle in advance on the best pixel size, we should take the maximum allowable looks at pixels of that size to get speckle reduction. But suppose we aren't sure what the best pixel size is, which might happen in mapping unknown surfaces. Might it not be better to design the system to take fewer looks at smaller pixels? (We ignore additional processing requirements which would arise for digital, though not for optical, processors, but do consider that the system power is fixed.) Suppose a large area G is divided into n = M/k resolvable sub-areas (n, M, k integers) and k looks are taken at each sub-area and the average of the k reflectivities is taken as the estimate of reflectivity for that sub-area. If it is later decided that an estimate of the reflectivity of area G is required, this can be obtained by averaging over the estimates for the n sub-areas. In the following, we define pixel SNR in the presence of speckle and additive white Gaussian noise as in [1] and derive the

ORIGINAL PAGE IS OF POOR QUALITY

expression for pixel SNR as a function of n. The return from each sub-area is assumed to obey Rayleigh statistics.

2.0 PIXEL SNR FOR k LOOKS AT n = M/k AREAS

Let $A_1, A_2, \cdots A_n$ be the mean power returns from the n sub-areas. Each component (in-phase (I) and quadrature (Q) of the return amplitude from the jth look at the ith sub-area is (after Butman and Lipes [1])):

$$r_{Iij} = a_{Iij} + n_{Iij} \tag{1}$$

$$r_{Qij} = a_{Qij} + n_{Qij} \tag{2}$$

where the a's are the signal components and the n's are noise components, i.e. the n_{Iij} and n_{Qij} are statistically independent white Gaussian noise with variance $N_o/2$ and the a_{Iij}, a_{Qij} are statistically independent Gaussian random variables with zero mean and variance $A_i/2$.

The power return is

$$p_{ij} = r^2_{Iij} + r^2_{Qij} \tag{3}$$

which is exponentially distributed with mean $(A_i + N_o)$ and variance $(A_i + N_o)^2$, for each j from 1 to k.

The estimate of power return from the ith sub-area, after averaging over k looks is

$$p_i = \frac{1}{k} \sum_{j=i}^{k} p_{ij} \tag{4}$$

The variance of p_i, for independent looks is

$$N_i = \frac{1}{k} \cdot \left[A_i + N_o\right]^2 \tag{5}$$

In the absence of noise, the mean signal power in p_i is $\frac{1}{k}$ times the sum of the mean signal powers for the p_{ij}, i.e.

$$\left.\overline{p_i}\right]_{N_o=0} \equiv S_i = \frac{k}{k} A_i = A_i \tag{6}$$

so that the pixel SNR which incorporates both noise and speckle is

$$\text{SNR} = \frac{\sqrt{k}\, A_i}{A_i + N_o} \tag{7}$$

This gives the pixel SNR that can be expected in the estimate of the power return or reflectivity of the ith sub-area (which has size G/n if all n

ORIGINAL PAGE IS
OF POOR QUALITY

sub-areas are equal). But now suppose that an estimate of the reflectivity of the large resolution cell of size G is required, after having processed the data to give a resolution size of G/n. This can be obtained by averaging over the estimates for the n sub-areas.

In this case the estimate of power from the whole area is

$$p = \frac{1}{n} \sum_{i=1}^{n} p_i \tag{8}$$

where p_i is exponentially distributed with mean $(A_i + N_o)$ and variance $(A_i + N_o)^2/k$.

Now the pixel SNR becomes

$$SNR\Big]_G = \frac{S}{\sqrt{N}} = \frac{\sqrt{k}\left[A_1 + \cdot\cdot + A_n\right]}{\sqrt{(A_1+N_o)^2 + \cdot\cdot + (A_n+N_o)^2}} \tag{9}$$

We assume now that there is available a total power MP which is incident on the n sub-areas in k looks. Thus, in one look, a total power $\frac{MP}{k}$ is incident on the whole area G and, if the n sub-areas are equal, $\frac{MP}{kn} = \frac{MP}{M} = P$ is incident on each sub-area G; let the mean return power in each look in this case be A. Thus, in the case where power P is incident on each of n > 1 sub-areas G/n, a total power nP is incident on the area G and the mean return must therefore be nA.

Therefore,
$$nA = A_1 + A_2 + \cdot\cdot + A_n \tag{10}$$
where $A_1, \cdot\cdot\ A_n$, as defined before, are the mean power returns from the n sub-areas.

Now consider two n-dimensional vectors

$$\underline{a} = ((A_1+N_o)(A_2+N_o)\ \cdots\ (A_n+N_o)) \quad \text{and} \tag{11}$$

$$\underline{b} = (\ 1 \quad 1 \quad \cdot\cdot \quad 1\ ). \tag{12}$$

Then Schwartz's inequality

$$a \cdot b \leq |a||b| \tag{13}$$

gives

$$n(A+N_o) \leq \sqrt{n}\sqrt{(A_1+N_o)^2 + \cdots + (A_n+N_o)^2} \tag{14}$$

Let

$$n(A+N_o) = r\sqrt{n}\sqrt{(A_1+N_o)^2 + \cdot\cdot\ (A_n+N_o)^2} \tag{15}$$

where $r \leq 1$.

Then Eq. (9) becomes

$$SNR\Big]_G = \sqrt{k} \cdot r \frac{\sqrt{n} \cdot A}{A + N_o} = \frac{\sqrt{kn}\ A}{A + N_o} = r \frac{\sqrt{M}\ A}{A + N_o} \tag{16}$$

where

$$r = \frac{\frac{1}{n} \sum_{i=1}^{n} (A_i + N_o)}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (A_i + N_o)^2}} \tag{17}$$

3.0 SPECIAL CASES

(1) When $n = 1$, we get $r = 1$, and

$$SNR\Big]_G = \frac{\sqrt{M}\ A}{A + N_o} \tag{18}$$

so that taking M looks at the large area G and averaging gives the best estimate of the reflectivity of the large area G.

(2) If the area G has uniform reflectivity so that the A_i's are all equal, then the equality sign holds in Eq. (14) and again $r = 1$, irrespective of what n is.

If the A_i's are not equal, then $r < 1$ so that we sacrifice something in pixel SNR by not having decided on the coarser resolution.

(3) When $N_o = 0$, i.e., no noise is present,

$$SNR\Big]_G = r\sqrt{M}; \qquad r = \frac{\sum_{i=1}^{n} A_i}{\sqrt{n} \sqrt{\sum_{i=1}^{n} A_i^2}} \tag{19}$$

which is unaltered if all A_i's are multiplied by a common factor, so that the variation of $SNR\Big]_G$ with n is independent of the power incident on area G.

4.0 CONSEQUENCES FOR VARIOUS DISTRIBUTIONS OF A_i FOR $N_o = 0$

(1) For the most extreme case where all but one of the A_i are zero:

$$r = \frac{1}{\sqrt{n}} \tag{20}$$

When $n = M$, this gives $r = \frac{1}{\sqrt{M}}$, so that $SNR\Big]_{G_{(n=M)}} = 1$ while if we had $n = 1$, it would have been $\mathrm{SNR}\big]_{G(n=1)} = \sqrt{M}$, which is expected since we are getting only one non-zero return in all the M looks and thus effectively taking only one look at the whole area. On the other hand, if all the return comes from only one of the M sub-areas, that in itself would be of considerable interest.

(2) Uniform distribution of the A_i's: If we assume that most scenes of interest can be classified as having A_i that are independent and uniformly distributed between 0 and A_m, then we can get the expected value of r as a function of n.

$$r^2(n) = \frac{\left(\frac{1}{n}\sum_{i=1}^{n} A_i\right)^2}{\frac{1}{n^2}\sum_{i=1}^{n} A_i^2} \tag{21}$$

The numerator is a random variable with mean equal to $(\frac{A_m}{2})^2$ and a variance that becomes very small as $n \to \infty$ while the denominator is a random variable with mean $\frac{A_m^2}{3}$ and negligible variance as $n \to \infty$, so that

$$\lim_{n\to\infty} \overline{r(n)} = \frac{\sqrt{3}}{2} = 0.866 \tag{22}$$

For finite n, $\overline{r^2(n)}$ is given by

$$\overline{r^2(n)} = \int \frac{(\alpha_1 + \cdot\cdot\ \alpha_n)^2}{\alpha_1^2 + \cdot + \alpha_n^2}\, p_{A_i}(\alpha_i) \cdot\cdot\ p_{A_n}(\alpha_n) d\alpha_i \cdot\cdot\ d\alpha_n \tag{23}$$

(where $p_{A_i}(\alpha_i)$ = probability density function of A_i) which is difficult to evaluate explicitly for general n. However, for n = 2, this can be evaluated for the uniform A_i distribution case, and gives

$$r(2) = \sqrt{\frac{1 + \ln 2}{2}} = 0.92 \tag{24}$$

(3) Exponential distribution of the A_i's: If the A_i's are independent and exponentially distributed with a mean value of A_m, then for $n \to \infty$, we have the numerator of Eq. (21) tending to A_m^2 and the denominator to $2A_m^2$ so that

$$\lim_{n\to\infty} r(n) = \frac{1}{\sqrt{2}} = 0.707 \tag{25}$$

Since all these values of r calculated in Eqs. (22), (24), (25) are not significantly different from 1, we conclude that, for a realistic distribution of the A_i's, the pixel SNR does not get much worse if n is doubled or quadrupled. It might be worthwhile doubling n for the sake of improved small-area resolution, (especially if k, the number of looks at each small area, is large enough to compensate for speckle) while leaving the estimate of the large area G if later desired, almost as good.

5.0 CONCLUSIONS

If we are not sure what the best pixel size is, it seems to be better to design the system to take fewer looks each at smaller pixels. If the chosen (larger) pixel size is indeed correct, we will have sacrificed something in SNR. But if a smaller pixel size is better, we can at least get some (noisier) information about the smaller pixels, and we still have almost as good information about the larger pixels as if we had looked at the whole large pixels only.

6.0 REFERENCES

1. Butman, S. A. and Lipes, R. G., "The effect of noise and diversity on SAR imagery," Deep Space Network Progress Report 42-29, Jet Propulsion Laboratory, Pasadena, California, p. 46, July and August, 1975.
2. Goodman, J. W., "Some fundamental properties of speckle," JOSA, 66, pp. 1145-1150, Nov. 1976.
3. Porcello, L. J., et al, "Speckle reduction in SARs," JOSA, 66, pp. 1305-1311, Nov. 1976.