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SUMMARY

An existing computer program, used for predicting the natural frequencies and mode shapes of helicopter rotor blades, has been refined to improve program accuracy and versatility. The program is based on the Holzer-Myklestad approach adapted for rotating beams. Coupled vertical (out-of-plane), horizontal (in-plane), and torsional mode characteristics can be determined for a variety of hub and blade configurations of practical interest. The resulting program is documented by presenting the recursion equations and techniques for determining natural frequencies and mode shapes, input data requirements and descriptions of various program outputs. The accuracy of the program is demonstrated by comparing computed results with exact solutions to classical problems and experimental data.

INTRODUCTION

The calculation of undamped natural frequencies and mode shapes of rotor blades is an important first step in analyzing the dynamic behavior of helicopter rotor systems. An existing analysis and computer program for determining blade modes and frequencies, which has been used by both government and industry for a number of years, is described in reference 1. This program also provides data on punched cards for input to the flight simulation program described in references 2 and 3. It is based on the Holzer-Myklestad approach adapted for rotating beams and represents the helicopter blade by a lumped-mass system which includes effects of rotary inertia, blade geometric pitch, and inertial and elastic coupling among the five degrees of freedom at each mass station (radial motion is not included). Shear deformation, aerodynamic effects, and the built-in coning (precone) are not represented. Discrete spanwise variations of blade properties and various hub configurations are modeled in the program. A unique feature of the program of reference 1 is the ability to include the effects of support system impedance on the blade natural frequencies and mode shapes. The capability to represent both a variety of hub configurations and support system effects are features not generally found in other computer programs of this type.

Because of the program's uniqueness, the present authors have made considerable use of it. However, experience has identified a number of errors and limitations in the computer program. In anticipation of future need for this modal analysis program and possible further expansion of its capabilities, the analysis has been improved and all known errors in the program have been corrected. Major changes to the computational procedure involved a different iteration scheme to determine natural frequencies and a different technique for computing mode shapes. Other changes to the program were made to reduce the size of the source program and core requirements needed for execution, increase run time efficiency, and expand program capabilities by increasing the allowed numbers of mass stations and rotor rotational speeds. The recursion equations, relating the state vectors at adjacent stations, were changed in the new program version and are substantiated by the work described in reference 4.

The resulting analysis is extensively different from the original version and has significant improvements in program size, efficiency, and versatility. The purpose of this report is to discuss the equations and techniques used, describe the final program, and present a program source listing. The accuracy of the program is demonstrated by correlating predicted results with exact solutions for classical problems and with experimental data.

SYMBOLS

Values in text are given in U.S. Customary Units and SI units. The calculations were made in U.S. Customary Units. The computer program values and related material are given in U.S. Customary Units. Conversion factors for these units in the computer output are given in appendix A.

a,b,c	distances along A-, B-, and C-axes
B_1	blade section constant, in^6 (cm^6)
D	horizontal force perpendicular to Z-axis, lb (N)
E	Young's modulus of elasticity, lb/in^2 (N/m^2)
EI	blade structural stiffness in bending, $lb\text{-}in^2$ ($N\text{-}m^2$)
F	centrifugal force, lb (N)
F_H	in-plane force due to centrifugal force acting through a center of gravity horizontally offset from pitch axis, lb (N)
F_X	in-plane moment due to centrifugal force acting through a center of gravity horizontally offset from pitch axis, in-lb (N-m)
F_Y	out-of-plane moment due to centrifugal force acting through a center of gravity vertically offset from pitch axis, in-lb (N-m)
GJ	blade structural stiffness in torsion, $lb\text{-}in^2$ ($N\text{-}m^2$)
g	acceleration due to gravity, in/sec^2 (m/sec^2)
I	segment second mass moment of inertia relative to pitch axis with rotary inertia effects included, $in\text{-}lb\text{-}sec^2$ ($m\text{-}N\text{-}sec^2$)
I'	cross-sectional second mass moment of inertia relative to center of gravity, $lb\text{-}sec^2$ ($N\text{-}sec^2$)
JHUB	number of nonfeathering hub segments
K_c	blade pitch control system stiffness, in-lb/rad (N-m/rad)

K_{ip}	hub in-plane translational stiffness per blade, lb/in (N/m)
K_{op}	hub out-of-plane translational stiffness per blade, lb/in (N/m)
K_T	rotor shaft torsional stiffness, in-lb/rad (N-m/rad)
K_β	blade flapping or rotor teetering spring rate, in-lb/rad (N-m/rad)
K_ψ	blade lagging spring rate, in-lb/rad (N-m/rad)
k_a	segment mass radius of gyration, in. (m)
L	vertical force perpendicular to Z-axis, lb (N)
M	out-of-plane bending moment, in-lb (N-m)
M_{ip}	hub in-plane translational inertia per blade, lb-sec^2/in. (N-sec^2/m)
M_{op}	hub out-of-plane translational inertia per blade, lb-sec^2/in. (N-sec^2/m)
m	segment mass, lb-sec^2/in. (N-sec^2/m)
P	number of zero frequency modes
Q	in-plane bending moment, in-lb (N-m)
R	rotor radius, in. (m)
r	center-of-gravity offset from blade pitch axis, in. (m)
S	shear center offset from blade pitch axis, in. (m)
T	torsional moment, in-lb (N-m)
$T_{\psi\psi}$	torsional inertia term representing centrifugal stiffening, in-lb-sec^2 (N-m-sec^2)
W	blade tip weight, lb (N)
w	blade segment weight per unit length, lb/in. (N/m)
X,Y,Z,A,B,C	coordinate axes (see fig. 1)
x,y,z	distances along X-, Y-, and Z-axes
$\bar{z}$	blade segment length, in. (m)
z	radical distance to end of blade segment, in. (m)
β	elastic bending rotation in vertical (YZ) plane, rad
Δ	determinant of boundary condition coefficient matrix

δ_x	elastic deflection in x-direction (see fig. 1), in. (m)
δ_y	elastic deflection in y-direction (see fig. 1), in. (m)
θ	geometric pitch angle between blade principal structural axis and horizontal (XZ) plane at a segment end, deg
$\tilde{\theta}$	geometric pitch angle between blade principal structural axis and horizontal (XZ) plane at a segment mid-point, deg
$\tilde{\theta}'$	rate of change of geometric pitch angle with span at a segment mid-point, rad/in. (rad/m)
θ_c	blade root collective pitch angle, deg
θ_t	blade linear twist, deg
λ	auxiliary function associated with Δ
ϕ	elastic torsional rotation in vertical (XY) plane, rad
ψ	elastic bending rotation in horizontal (XZ) plane, rad
Ω	rotor angular velocity, rad/sec
ω	blade natural frequency, rad/sec (cpm in tables)

Subscripts:

a,b,c	rotating coordinate system fixed to local structural axes at each station after deformation
i,j,l,N or N + 1	blade station number
x,y,z	rotating coordinate (reference) system fixed to hub

METHOD OF ANALYSIS

The method of analysis considers the rotor blade as a discretized system of finite elements with each element representing a segment of the blade. The user-specified input data pertains to discretized structural properties which are average values over each segment. The data are referenced to the local segment axis system and to either the segment center of gravity (inertial properties) or pitch axis (elastic properties). Since the pitch axis is the reference axis in the analysis, the inertial properties are transferred to the pitch axis. All properties are then rotated into a hub-fixed axis system oriented so that the vertical axis is parallel with the axis of rotation. Details of the data conversion are presented in appendix B. Figure 1 illustrates the local segment-fixed axis system (A,B,C) and the hub-fixed axis system (X,Y,Z) with sign conventions. The jth segment is shown in figure 1 having a width ℓ_j and an inboard station z_j as measured from the center of rotation. Figure 2 illustrates the shear center and center-of-gravity offsets from the pitch axis in the (A,B,C) and (X,Y,Z) systems and the sign conventions. The positive directions for r_b, r_c, and r_x are opposite to the B-, C-, and X-axes, respectively.

The blade segments are converted to a finite-element representation with all the inertia lumped at the inboard end and a stiffness element extending over the segment length. By using the sign conventions for the displacements, forces, and moments shown in figure 3, using the free-body diagrams of figure 4, assuming harmonic motion, and requiring force and moment equilibrium and continuity of displacements across the lumped masses, the following recursion equations result:

$$\phi_j = \frac{\bar{z}_j}{(GJ)_j + F_j k^2_{a,j}}\Big[(F_{x,j} + F_j S_{x,j})\beta_{j+1} + (F_{y,j} - F_j S_{y,j})\psi_{j+1} - (S_{x,j})L_{j+1}$$

$$+ (S_{y,j})D_{j+1} + (F_{H,j} S_{x,j})\phi_{j+1} - T_{j+1}\Big] + \phi_{j+1} \qquad (1)$$

$$\beta_j = \left[1 + \frac{\bar{z}_j^{\,2}F_j}{2(EI)_{yy,j}}\right]\beta_{j+1} + \left[\frac{\bar{z}_j^{\,2}F_j}{2(EI)_{xy,j}}\right]\psi_{j+1} - \left[\frac{\bar{z}_j}{(EI)_{yy,j}}\right]M_{j+1} - \left[\frac{\bar{z}_j}{(EI)_{xy,j}}\right]Q_{j+1}$$

$$- \left[\frac{\bar{z}_j^{\,2}}{2(EI)_{yy,j}}\right]L_{j+1} - \left[\frac{\bar{z}_j^{\,2}}{2(EI)_{xy,j}}\right]D_{j+1} - \left[\frac{F_{y,j}\bar{z}_j}{(EI)_{xy,j}} + \frac{F_{x,j}\bar{z}_j}{(EI)_{yy,j}} - \frac{F_{H,j}\bar{z}_j^{\,2}}{2(EI)_{yy,j}}\right]\phi_{j+1}$$

$$(2)$$

$$\psi_j = \left[\frac{\bar{z}_j^{\,2}F_j}{2(EI)_{xy,j}}\right]\beta_{j+1} + \left[1 + \frac{\bar{z}_j^{\,2}F_j}{2(EI)_{xx,j}}\right]\psi_{j+1} - \left[\frac{\bar{z}_j}{(EI)_{xy,j}}\right]M_{j+1} - \left[\frac{\bar{z}_j}{(EI)_{xx,j}}\right]Q_{j+1}$$

$$- \left[\frac{\bar{z}_j^{\,2}}{2(EI)_{xy,j}}\right]L_{j+1} - \left[\frac{\bar{z}_j^{\,2}}{2(EI)_{xx,j}}\right]D_{j+1} - \left[\frac{F_{y,j}\bar{z}_j}{(EI)_{xx,j}} + \frac{F_{x,j}\bar{z}_j}{(EI)_{xy,j}} - \frac{F_{H,j}\bar{z}_j^{\,2}}{2(EI)_{xy,j}}\right]\phi_{j+1}$$

$$(3)$$

$$\begin{aligned}\delta_{y,j} = &-\left[\frac{F_j\bar{z}_j^{\,3}}{6(EI)_{yy,j}} + \bar{z}_j\right]\beta_{j+1} - \left[\frac{F_j\bar{z}_j^{\,3}}{6(EI)_{xy,j}}\right]\psi_{j+1} + (S_{x,j})(\phi_j - \phi_{j+1})\\ &+ \left[\frac{\bar{z}_j^{\,2}}{2(EI)_{yy,j}}\right]M_{j+1} + \left[\frac{\bar{z}_j^{\,2}}{2(EI)_{xy,j}}\right]Q_{j+1} + \left[\frac{\bar{z}_j^{\,3}}{6(EI)_{yy,j}}\right]L_{j+1} + \left[\frac{\bar{z}_j^{\,3}}{6(EI)_{xy,j}}\right]D_{j+1}\\ &+ \left[\frac{F_{y,j}\bar{z}_j^{\,2}}{2(EI)_{xy,j}} + \frac{F_{x,j}\bar{z}_j^{\,2}}{2(EI)_{yy,j}} - \frac{F_{H,j}\bar{z}_j^{\,3}}{6(EI)_{yy,j}}\right]\phi_{j+1} + \delta_{y,j+1} \qquad (4)\end{aligned}$$

$$\begin{aligned}\delta_{x,j} = &-\left[\frac{F_j\bar{z}_j^{\,3}}{6(EI)_{xy,j}}\right]\beta_{j+1} - \left[\frac{F_j\bar{z}_j^{\,3}}{6(EI)_{xx,j}} + \bar{z}_j\right]\psi_{j+1} - (S_{y,j})(\phi_j - \phi_{j+1})\\ &+ \left[\frac{\bar{z}_j^{\,2}}{2(EI)_{xy,j}}\right]M_{j+1} + \left[\frac{\bar{z}_j^{\,2}}{2(EI)_{xx,j}}\right]Q_{j+1} + \left[\frac{\bar{z}_j^{\,3}}{6(EI)_{xy,j}}\right]L_{j+1} + \left[\frac{\bar{z}_j^{\,3}}{6(EI)_{xx,j}}\right]D_{j+1}\\ &+ \left[\frac{F_{y,j}\bar{z}_j^{\,2}}{2(EI)_{xx,j}} + \frac{F_{x,j}\bar{z}_j^{\,2}}{2(EI)_{xy,j}} - \frac{F_{H,j}\bar{z}_j^{\,3}}{6(EI)_{xy,j}}\right]\phi_{j+1} + \delta_{x,j+1} \qquad (5)\end{aligned}$$

$$L_j = L_{j+1} + \omega^2\left[(mr_x)_j\phi_j + m_j\delta_{y,j}\right] \qquad (6)$$

$$D_j = D_{j+1} + (\omega^2 + \Omega^2)\left[(mr_y)_j\phi_j + m_j\delta_{x,j}\right] \qquad (7)$$

$$\begin{aligned}M_j = M_{j+1} &+ (F_j)(\delta_{y,j} - \delta_{y,j+1}) - \left[\Omega^2 z_j (mr_x)_j\right]\phi_j + (\bar{z}_j)L_{j+1}\\ &+ (\omega^2 + \Omega^2)(I_{yy,j}\beta_j + I_{xy,j}\psi_j) \qquad (8)\end{aligned}$$

$$\begin{aligned}Q_j = Q_{j+1} &+ (F_j)(\delta_{x,j} - \delta_{x,j+1}) - \left[\Omega^2 z_j (mr_y)_j\right]\phi_j + (\bar{z}_j)D_{j+1}\\ &+ \omega^2(I_{xy,j}\beta_j + I_{xx,j}\psi_j) \qquad (9)\end{aligned}$$

$$T_j = T_{j+1} + (F_{H,j})(\delta_{y,j} - \delta_{y,j+1}) + (\omega^2 I_{zz,j} + \Omega^2 T_{\phi\phi,j})\phi_j + \left[(\omega^2 + \Omega^2)(mr_y)_j\right]\delta_{x,j} + \left[\omega^2 (mr_x)_j\right]\delta_{y,j} \tag{10}$$

where the coefficients in these equations are defined in appendix B.

Equations (1) to (10) are in general agreement with the equations of reference (4), which presents a detailed development of these recursion formulas. There are two significant differences, however, between the equations in this report and the final equations of reference 4. The equations given in this report are based on the assumption that cross-sectional principal structural stiffness and mass axes are parallel. Therefore, the terms in the equations of reference 4 that arise by assuming they are not parallel are omitted in equations (1) to (10). Secondly, the term $F_j k^2_{a,j}$ in equation (1) is not retained in reference 4.

Equations (1) to (10) are used to calculate deflections, slopes, forces, and moments at the inboard end of each segment because of the conditions at the outboard end of that segment. The calculation process begins by assuming a unity value for one of the tip displacements and zero for all others. The corresponding tip forces and moments are not all zero because a lumped mass is allowed at that station (tip mass). Thus, for a unity value of the tip out-of-plane slope, the forces and moments are calculated from equations (6) to (10) and yield

$$\beta_{N+1} = 1 \tag{11a}$$

$$M_{N+1} = (\Omega^2 + \omega^2) I_{yy,N+1} \tag{11b}$$

$$Q_{N+1} = \omega^2 I_{xv,N+1} \tag{11c}$$

where β, M, and Q represent the out-of-plane slope and moment and in-plane moment, respectively, with sign conventions shown in figure 3. All other forces and moments at the tip are zero. By successively applying equations (1) to (5) to determine deflections and slopes at each mass station and equations (6) to (10) to calculate the forces and moments acting at the outboard end of the next inboard elastic element, the conditions all along the blade are determined for specified values of collective pitch angle and blade twist, rotor rotational speed, and the assumed frequency of vibration. This procedure is repeated for the same vibratory frequency by individually assuming unity values for the other tip displacements (ψ_{N+1}, $\delta_{x,N+1}$, ϕ_{N+1}, and $\delta_{y,N+1}$).

To analyze a particular rotor blade, it is necessary to specify the geometric and structural properties relating the manner in which the blade is mounted to the hub. The hub configuration and support system impedance characteristics are expressed as five boundary condition equations in terms of the displacement variables, forces, and moments at a particular station. To satisfy the prescribed boundary condition equations with the conditions calculated along the blade by equations (1) to (10), the unknown ω must take on only select values. Thus, the eigenvalue problem is formed where the vibratory frequency is systematically varied, and the conditions along the blade are calculated for each unity tip displacement. These data are then used to see whether the boundary conditions are satisfied.

In the analysis three basic sets of boundary condition equations are used and the resulting modal characteristics are designated as pertaining to collective, cyclic, or scissors type modes, depending upon which set of boundary conditions is imposed. The form of the boundary conditions for each mode type is summarized in table I and the different spring and mass terms are shown in figure 5. Reference 5 presents a discussion of these mode types and how they are combined to describe rotor mode characteristics.

The collective mode is characterized by symmetric vertical or out-of-plane and antisymmetric horizontal or in-plane deflections of opposing pairs of blades on the rotor. The appropriate boundary conditions for the in-plane direction are elements 2 and 4 of the first column of table I and pertain to a spring-restrained K_T pinned joint at the center of rotation. The first and third elements give the conditions for a clamped joint attached to a movable hub, where the hub impedance is represented by a single-degree-of-freedom mass-spring system. These characteristics describe the boundary conditions for the out-of-plane direction. The first four conditions of the first column are applied at the center-line station. The torsional equation relates the twist and torque at the pitch horn radial attachment point. The term K_c represents the effective spring rate of the control system.

The cyclic modes have symmetric in-plane and antisymmetric out-of-plane deflection shapes about the center of rotation. The boundary conditions for the cyclic modes are given in the second column of table I. The first and third elements of that column describe the flapping-spring-restrained K_β pinned conditions in the out-of-plane direction. Elements 2 and 4 are for the in-plane direction where the representation is a clamped joint fixed to a flexible hub having lumped stiffness and mass characteristics described by K_{ip} and M_{ip}, respectively. The torsional boundary condition is the same for the cyclic and collective modes.

For the scissors modes, the in-plane and out-of-plane boundary conditions at the center line represent clamping of the blades to an immovable hub ($\delta_{y,1} = \delta_{x,1} = \beta_1 = \psi_1 = 0$). For a rotor having three or more blades, the torsional condition is the same as for the collective and cyclic modes. For two-bladed rotors, the torsional deflection is zero (clamped) at the pitch horn attachment radial station. These conditions are shown in the third column of table I. For the in-plane and out-of-plane directions, an alternate form of the boundary conditions is used whenever it is desirable to represent offset flapping and lagging hinges as in the case of an articulated rotor. (See the fourth column of table I.) If the offset of the flapping hinge is not zero, the zero slope condition is replaced by an equation relating the out-of-plane moment and slope at the hinge station using a flapping spring term K_β. Similarly, for an offset lagging hinge, the slope condition is changed to a model of a spring-restrained K_ψ pinned joint at the lagging hinge radial station.

The values of deflection, slope, moment, and shear calculated at the stations for which the boundary conditions apply and due to a value of unity for one coordinate at the outboard tip can be substituted into the left-hand side of the boundary condition equations for a particular mode type. These equations are then used to form one column of the boundary condition coefficient matrix. By repeating the substitution of conditions associated with unity values of the other tip coordinates, the complete coefficient matrix is generated. The matrix terms may be thought of as partial derivatives of each boundary condition equation with respect to an individual tip displacement. As an example, for a blade having a pitch horn offset the coefficient matrix [c] for the scissors mode would be (see table I):

$$[c] = \begin{bmatrix} \frac{\partial \delta_{y,1}}{\partial \beta_{tip}} & \frac{\partial \delta_{y,1}}{\partial \psi_{tip}} & \frac{\partial \delta_{y,1}}{\partial \delta_{x,tip}} & \frac{\partial \delta_{y,1}}{\partial \Phi_{tip}} & \frac{\partial \delta_{y,1}}{\partial \delta_{y,tip}} \\ \frac{\partial \delta_{x,1}}{\partial \beta_{tip}} & \frac{\partial \delta_{x,1}}{\partial \psi_{tip}} & \frac{\partial \delta_{x,1}}{\partial \delta_{x,tip}} & \frac{\partial \delta_{x,1}}{\partial \Phi_{tip}} & \frac{\partial \delta_{x,1}}{\partial \delta_{y,tip}} \\ \frac{\partial \beta_{1}}{\partial \beta_{tip}} & \frac{\partial \beta_{1}}{\partial \psi_{tip}} & \frac{\partial \beta_{1}}{\partial \delta_{x,tip}} & \frac{\partial \beta_{1}}{\partial \Phi_{tip}} & \frac{\partial \beta_{1}}{\partial \delta_{y,tip}} \\ \frac{\partial \psi_{1}}{\partial \beta_{tip}} & \frac{\partial \psi_{1}}{\partial \psi_{tip}} & \frac{\partial \psi_{1}}{\partial \delta_{x,tip}} & \frac{\partial \psi_{1}}{\partial \Phi_{tip}} & \frac{\partial \psi_{1}}{\partial \delta_{y,tip}} \\ \frac{\partial (T - K_c\Phi)}{\partial \beta_{tip}} & \frac{\partial (T - K_c\Phi)}{\partial \psi_{tip}} & \frac{\partial (T - K_c\Phi)}{\partial \delta_{x,tip}} & \frac{\partial (T - K_c\Phi)}{\partial \Phi_{tip}} & \frac{\partial (T - K_c\Phi)}{\partial \delta_{y,tip}} \end{bmatrix} \qquad (12)$$

where the subscript 1 refers to the station at the center of rotation. The form of the boundary condition coefficient matrix is dependent on the mode type to be calculated. Because the boundary condition equations are homogeneous, the following matrix equation can be formed to determine the tip deflections for each mode:

$$[C]\begin{Bmatrix}\beta_{tip}\\ \psi_{tip}\\ \delta_{x,tip}\\ \phi_{tip}\\ \delta_{y,tip}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\\0\end{Bmatrix} \tag{13}$$

For nontrivial values of tip deflection, the determinant of the coefficient matrix must vanish and yield a polynomial in terms of the squares of all natural frequencies. In general, the variation of the determinant of the matrix with frequency is such that only iteration techniques, where the frequency is incremented to find a crossover point, are reliable. Such frequency stepping techniques, however, are inefficient. As shown in reference 6, an auxiliary function can be generated which obviates the need for frequency stepping. The auxiliary function has the desirable feature of monotonically decreasing with increasing frequency for frequencies less than the natural frequency being sought. (See fig. 6.) The behavior of the auxiliary function is due to the removal of all roots of the determinant below that of interest and allows use of extrapolation techniques for convergence to each natural frequency. The auxiliary function is defined by the relation

$$\lambda_i(\omega)=\frac{\Delta(\omega)}{(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)\ \ldots\ (\omega^2-\omega_{i-1}^2)} \tag{14}$$

where the ith natural frequency is desired. At the ith natural frequency, the values of λ and Δ are both zero. In the computer program the auxiliary function is used to extrapolate to the natural frequencies. Once the natural frequency is determined, that root is also removed from the determinant before proceeding with the calculations for the frequency of the next mode. In practice, the P zero frequency roots, which correspond to the rigid-body modes, must also be removed before this technique will work properly (a fact not mentioned in ref. 6). This removal may be accomplished by noting the slope of the auxiliary function for values of frequency near zero. If the magnitude of λ increases for increasing frequency, then λ is divided by successively higher powers of ω^2 at the trial point until convergence is assured. The ω^{2P} factor is maintained in the denominator of the auxiliary function (eq. (14)) for all successive mode calculations.

Substitution of the calculated natural frequency into the boundary condition coefficient matrix leads to five homogeneous equations in terms of the five unknown displacements at the outboard tip (eq. (13)). To solve for the relative magnitudes of the tip deflections, an inverse iteration technique (ref. 7) is used. The inverse of the coefficient matrix is used to premultiply an initial trial vector. The resulting vector is used as the new trial vector and is again premultiplied by the inverse matrix. Four iterations are performed, each resulting vector being normalized by the largest element. The final vector corresponds to the relative tip deflections for that mode. By using these values and the spanwise distributions of deflections, slopes, shears, and moments for each unit tip deflection, the mode shapes and associated shear and moment distributions are calculated. The displacements are left in the X,Y,Z-axis system. The shears and moments are resolved back into the local segment axis systems, however, by using the twist and collective pitch angles.

DESCRIPTION OF COMPUTER PROGRAM

General Description of Program

The analytical methods have been implemented as a digital computer program coded in FORTRAN IV language. The program is run on the Control Data Corp. CYBER 175 computer system at the Langley Research Center, using an FTN compiler and NOS 1.2 operating system. By comparing the new program with the one described in reference 1, the lengths of the source decks are 1222 and 2398 cards, compilation times are 3.6 and 6.6 CPU (Central processing unit) seconds, and computer core requirements in octal are 54000 and 112000, respectively. A sample case with a 20-segment representation was run on both programs. The new version required 25 CPU seconds for execution and the old version required 27 CPU seconds. The new program can calculate natural frequencies and mode shapes for up to 30 blade segments (user specified in the new program), and up to 10 rotor speeds. All errors in the original program known to the authors have been corrected in this program.

The computer program consists of a main program, called BLDANL, and eight subroutines (AMPLTD, CARDS, COEF, INPT, PLOUT, START, SUMMY, and MODSHAP). A listing of the computer program is given in appendix C. In addition, a library subroutine, MATINV, is also required and is documented in appendix D. The program also uses a CalComp plotting package. A flow chart of the program is presented in figure 7. The various parts of the program are described in subsequent paragraphs.

Program BLDANL

The main program controls the computation of the blade natural frequencies for all combinations of collective pitch, rotor speed, and mode type. BLDANL initially calls subroutine INPT to read the input data for each case and then convert the input into the desired lumped-mass representation. The elastic and inertial properties for each station are rotated from the local A,B,C-axis system to the horizontal-vertical X,Y,Z-axis system. The program then computes the forces due to rotation at each station for each value of rotor speed.

The calculated characteristics in the X,Y,Z-axis system are input to subroutine COEF which then returns the determinant of the boundary coefficient matrix (eq. (12)) for each desired estimate of the natural frequency. In practice, the values of the determinant are used to extrapolate toward the point where the determinant has a zero value. Once the program has converged on the first natural frequency, the process is continued for successively higher modes until the specified upper frequency is exceeded. This procedure is followed for each appropriate mode type: collective, cyclic, or scissors.

After all desired natural frequencies have been determined, the associated mode shapes may be calculated. A call to subroutine AMPLTD causes the mode shapes to be calculated, printed out, plotted, or punched depending on the input options selected. Subroutine SUMRY is then called to print out a summary of all calculated natural frequencies. Also, if requested, subroutine PLOUT is called to plot the natural frequencies as a function of rotor rotational speed for each collective pitch angle and mode type. The program then proceeds to the next input case.

Subroutine AMPLTD

Subroutine AMPLTD is called by BLDANL to compute the mode shapes for the previously determined natural frequencies. Subroutine COEF is first called to calculate the matrix of the root-boundary conditions for individual unit tip displacements at the calculated natural frequency. The matrix is inverted by using the library subroutine MATINV and an inverse matrix iteration scheme is used to calculate the values of the deflections and slopes at the tip. From the transformation terms computed in subroutine COEF, the deflections, slopes, moments, and shears at each station are calculated and listed. The part of the generalized mass associated with each deflection or rotation component is calculated and used to determine the principal deflection direction (that having the largest generalized mass): vertical, horizontal, or torsion. The number of node points of the mode shape in the principal direction is then determined.

If requested, the mode shapes may be plotted or punched out through calls to MODSHAP or CARDS. The generalized mass components are also printed. Finally, a check of convergence for both natural frequency and mode shape is made by matrix multiplication of the boundary condition matrix and the matrix of tip deflections and slopes. The resulting matrix, which should be a null matrix, is printed out for each listed mode shape.

Subroutine CARDS

The subroutine CARDS is the only subroutine which produces punched output for use as input data in the flight simulation program of references 2 and 3. It is divided into three parts - the first punches the input inertia data, the second punches the components of the mode shape, and the third punches the cyclic detuning cards which are used to specify variations of natural frequency with collective pitch and rotor speed. The first part, called by subroutine START, takes blade mass and inertia data which are divided into an arbitrary number of segments and recasts it into 20 equal length segments. The second part, called by the subroutine AMPLTD, takes the mode shape data and recasts it into 20 equal segments as well. The first six natural frequencies for each combination of rotor collective pitch, rotor rotational speed, and mode type are stored. These frequencies are used in the third part to compute information for the cyclic detuning cards.

Subroutine COEF

The subroutine COEF is used to form the root boundary condition coefficient matrix for a specified frequency. The five generalized coordinates at the outboard tip - vertical slope, horizontal slope, horizontal deflection, torsional rotation, and vertical deflection - are individually set to a value of unity whereas the remaining tip deflections and slopes are set to zero. By using the elastic and inertial properties of the X,Y,Z-coordinate system and the specified frequency, the deflection, rotation, shear, and moment at each station along the blade are calculated for each unit tip displacement. For the particular mode type and rotor configuration the combination of five root conditions to be zeroed are selected. The previously calculated values of the conditions on the blade at the boundary stations are inserted into the appropriate row and column of the coefficient matrix. When the boundary coefficient matrix is completed, subroutine MATINV is called to compute the determinant of the matrix or its inverse.

Subroutine INPT

The subroutine INPT is called by BLDANL to read the input data from punched cards. INPT first reads the program option card and then either a full data deck or changes to the previous case. For multiple-case runs, a namelist format may be used for all cases except the first. If desired, subroutine CARDS may be called to output the blade inertia properties on punched cards. Subroutine START is also called from INPT.

Subroutine PLOUT

The subroutine PLOUT produces plots of the variation of natural frequencies with rotor speed. One figure is produced for each type of mode. For each figure, data for all combinations of collective pitch angle and rotor rotational speed are plotted. The maximum inertial plane associated with each natural frequency is distinguished by different plotting symbols. Subroutine PLOUT is called by program BLDANL.

Subroutine START

This subroutine is called by INPT. The locations of the pitch horn, flapping, and lagging hinge offsets are determined in terms of segment number and location within the appropriate segment. The centrifugal force acting at each station is calculated along with blade mass and second mass moment of inertia about the flapping hinge or center line. For linear twist distributions, the twist at each station is also calculated in subroutine START. The input inertia data which pertain to individual segments are averaged to generate inertia properties at each station and products and cross-products of inertia are calculated for each station as well. The input data are printed out for identification purposes.

Subroutine SUMMY

Subroutine SUMMY is called by the main program BLDANL to list a summary of the natural frequencies for each combination of rotor collective pitch, rotational speed, and mode type. Listed along with the natural frequencies are the maximum inertial (principal deflection) plane pertaining to each frequency and the number of node points of the component of the mode shape identified by the inertia plane.

Subroutine MODSHAP

Subroutine MODSHAP is called by subroutine AMPLTD to plot out the mode shape. The horizontal and vertical deflection and the torsional rotation are plotted as a function of the blade radial station nondimensionalized by the radius. Each component is identified by a distinct symbol. For convenience, the associated natural frequency is also given in the plot.

Description of Program Input Requirements

The program reads the input from file 5. The format for the input data deck is specified in table II and a sample deck listing is provided in table III. The program input and output data are expressed in U.S. Customary Units. Conversion factors for obtaining SI Units are given in appendix A. In table II the program names are given with the symbols used in this report shown parenthetically, where appropriate. A data deck consists of card types 1 to 18. Multiple cases can be run by placing the data decks one behind the other with no separators. For multiple-case runs, the namelist format option can be used for all cases except the first. The various inputs are described in the following paragraphs.

In table II, the first card specifies the program options via descriptive names which may be input in any order. The allowed names are DECK, NAMElist, PUNCh, SHAPe, MODEs, ALLModes, PLOT, TORSion, and NLTWist. Only the first four characters of each name are required. DECK instructs the program to read a new case input deck. The NAMElist option allows the user to read in only the changes to the previous case by using a namelist format. PUNCh is used to generate punched cards suitable for input into the computer program of references 2 and 3. The SHAPe option causes the program to generate CalComp plots of the blade mode shapes for the case with the reference values of rotor speed and collective pitch angle. The MODEs input results in printouts of the mode shape data provided by SHAPe. ALLModes is used when mode shape printouts for all combinations of rotor speed and collective pitch angle are desired. The PLOT option generates CalComp plots of the variation of blade natural frequencies as a function of rotor speed for each collective pitch angle. TORSion causes the torsional degree of freedom to be included in the analysis. The NLTWist option instructs the program to read in a nonlinear twist distribution using card 11 (card 11 is omitted if NLTWist is not used).

Card type 3 is used to specify the restraint conditions imposed on the blade motions by the pylon and collective pitch control systems. The pylon restraint is provided by lumped masses and springs in the out-of-plane and in-plane directions (MOP, MIP, KOP, and KIP) and a mast torsional wind-up spring KT. The control system stiffness is represented by KC. The term KBETA may be used to provide a rotor teetering spring or blade flapping spring in the case of an articulated rotor. The lagging spring rate is input by KPSI.

The blade may be divided up to a maximum of 30 segments, the inboard JHUB segments being nonfeathering. If all segments are of equal length, a nonzero value of ABBAR is input and card type 7 is omitted. Nonuniform segment lengths are input by setting ABBAR to zero and specifying the radial station of the outboard end of each segment on card 7. The rotor geometry is specified by the blade TWIST or TWD, number of BLADES, and the radial locations of the lagging (CHOFF) and flapping (FHOFF) hinges, and pitch horn (PHOFF). To delete specific hinges, a zero input value is used. The BLADES term is used only to suppress the calculation of collective and cyclic mode data when the input value is greater than 2.0.

The program determines natural frequencies below the specified upper limit, FLAST, for all combinations of rotor collective pitch angle (RCOLL) and rotor speed (RRPM). Up to 10 rotor speeds (IRPM) and 3 collective pitch angles (IRCOL) may be investigated for any one case. However, if the PUNCh option is used, three collective pitch angles and three rotor speeds must be input, as required by the program of references 2 and 3, the second values of each parameter being the average of the first and third values. The SHAPe and MODEs options provide output data pertaining to the next to the highest values of both rotor speed and collective pitch angle if three or more values are specified. For one or two values of rotor speed and/or collective pitch, the reference values are the first or second, respectively.

Each segment is assumed to have constant properties over its length. Card types 8 to 10 and 12 to 18 are used to input average segment characteristics, eight values per card. On card 10 the tip weight is input after the N values of segment weight per unit length (WTPL). The mass moments of inertia (KYEB and KYEC) are resolved about the center of gravity. The locations of the shear center and center of gravity relative to the pitch axis are input based on the sign conventions of figure 2. Note that the positive directions for RB and RC (center-of-gravity offsets in b- and c-directions) are opposite to the B- and C-axis convention. The N + 1 value of RB and RC pertain to the eccentricity of the tip weight relative to the pitch axis.

Description of Program Output Data

The information output by the program includes printer listings and, optionally, punched cards or Calcomp plots depending on the program control options selected by the user on card type 1. (See discussion on input data requirements.) The program generates a listing of the identification, geometric, and structural data that were input and a summary table of all calculated natural frequencies. Samples of these outputs, obtained from the input data listed in table III, are presented in tables IV and V.

The input data list includes geometric and structural parameters in the same units as originally read in, as well as segment lengths and values of blade twist and centrifugal force at each station. The total blade mass and flapping inertia are calculated by considering only the blade mass outboard of any flapping hinge and are listed. The hub and control system impedance parameters are also reproduced on the listing. The calculated natural frequencies are summarized for each appropriate mode type (that is, collective, cyclic, and/or scissors) and at each combination of root collective pitch angle and rotor speed. The values of natural frequency are nondimensionalized by rotor speed prior to being output or they are presented in units of cycles per minute for nonrotating cases. For each frequency listed, the plane containing the largest contribution to the total generalized mass of that mode is identified and the number of node points in the mode shape component associated with that plane are listed.

If either the MODEs or ALLModes options is selected, spanwise distributions of the mode shapes, moments, and shear forces at each station are printed out as shown in table VI. When the ALLModes option is exercised, these data are listed for all combinations of appropriate mode type, root collective pitch angle, and rotor speed. For the MODEs option, data are printed for each mode type, but for only one reference combination of collective pitch and rotor speed. The reference collective pitch angle is the last value input if only one or two input values are specified. If three or more collective pitch angles are requested, the reference value is the next to last value input. The reference rotor speed is determined in a similar fashion. The values in the mode shape columns correspond to deflections in the vertical and horizontal planes. However, the associated forces and moments are calculated in the local beam and chord axis system and reflect blade built-in twist and collective pitch angles. When TORSion is input on card type 1, the torsional rotations and moments for each mode are also listed. The displacements, moments, and shears are normalized so that the maximum deflection along the blade in either the vertical or horizontal plane is 1 in. (2.54 cm) or the maximum torsional rotation is 1 radian. After the spanwise mode shape listings, the parts of the total generalized mass attributed to each deflection plane, vertical, horizontal, or torsion, are printed. Finally, a check is made to see whether the boundary conditions are adequately satisfied and the calculated boundary condition values are printed. (See previous discussion concerned with imposing boundary conditions.) Both the generalized masses and boundary condition values reflect the normalization process applied to the mode shapes.

Use of the SHAPe option causes CalComp plots of the spanwise distribution of the mode shapes to be generated. A sample plot is shown in figure 8. One plot is made for each calculated frequency for all the mode types but only for the one combination of reference collective pitch and rotor speed. The different displacements, vertical, horizontal, or torsion, are plotted individually and reflect the normalization process applied to the listed data (maximum deflections of 1 in. (2.54 cm) or 1 radian). The abscissa represents the spanwise stations for the deflection values normalized by the rotor radius. For identification purposes, the mode type and natural frequency for each plot are given above the figure.

The PLOT option produces separate plots of blade natural frequency as a function of rotor speed for collective, cyclic, and scissors mode types. For each plot, a sample of which is shown in figure 9, the natural frequencies for all combinations of collective pitch angle and rotor speed are shown. The plotted data distinguishes between modes having maximum generalized masses in the three component directions (vertical, horizontal, or torsion). For identification purposes, the case number, title, and other information are given in each plot.

When the PUNCh option is used, punched cards are generated by the program compatible with the input requirements of the rotorcraft simulation computer program documented in references 2 and 3. The format for the punched deck is given in table VII and a sample output deck is listed in table VIII. The punched output consists of four parts: a problem identification card, cards containing the blade inertia characteristics, the mode shapes for the six lowest frequencies associated with each mode type and cyclic detuning information. When the PUNCh option is used, three values each of collective pitch and rotor speed must be input, the second values being the average of the low and high input quantities.

Although the program has the capability of treating up to 30 segments of unequal length, the simulation program of references 2 and 3 requires data for only 20 segments of equal length. Therefore, the input rotor inertia characteristics are converted by using linear interpolation. The moments of inertia in both beam and chord directions are also changed to reflect a transfer from the center of gravity to the feathering axis prior to being punched out. Each set of inertia properties is punched out in successive fields on three cards. The calculated mode shapes for the reference values of rotor speed and collective pitch have also been interpolated to yield values for a system with 20 segments of equal length. The units of the mode shape components are changed to feet for the vertical and horizontal deflections and degrees for the torsional amplitudes. For each of the first collective, cyclic, and scissor modes (up to six each), 11 cards are output. The first 10 cards define the three mode shape components (δ_x, δ_y, and ϕ) for adjacent station pairs starting at the center of rotation. The tip displacements are punched in the first three fields of the 11th card with the next three fields containing the natural frequency, normalized by the rotor speed, mode type designation, and an assumed modal damping ratio of 0.02. Identification data are punched in the remaining fields of all mode shapes cards for ease of handling. The mode type designations are 1 (collective), -1 (cyclic), and 0 (scissors).

One cyclic detuning card is generated for each punched mode shape. The purpose of the cyclic detuning card is to specify the variation of mode frequency with rotor speed and collective pitch angle. The first four fields of each cyclic detuning card list the normalized frequency of a particular mode for conditions of lowest values of rotor speed and pitch angle, lowest rotor speed and highest pitch angle, highest rotor speed and low pitch angle and, finally, the highest values of pitch angle and rotor speed. The reference values of rotor speed and pitch angle are also punched on each cyclic detuning card, as well as the case name, mode number, and type indicator.

PROGRAM CORRELATION RESULTS

The accuracy of the analysis is verified through correlation of computed results for selected problems, having either closed-form solutions or accurate solutions available in the literature. The correlation is based on comparisons of natural frequencies and mode shape characteristics. The first case considered is that of a nonrotating, uncoupled, uniform beam with pure bending in one plane and torsion allowed. Free-free and clamped-free conditions are examined and the lowest five modes in each direction and for each set of boundary conditions are determined. The second case expands on the first by adding a tip mass in a manner which affects the bending vibrations only. The third case is for a simply supported, nonrotating beam having uncoupled torsion and one bending degree of freedom. The fourth case adds elastic coupling between the bending and torsion vibrations for the simply supported beam. The fifth correlation case deals with a rotating propeller for which experimental data were available. Unfortunately, suitable data pertaining to a helicopter rotor could not be found by the authors. The analytical representations for each of these cases are summarized in tables IX and X.

Case 1 - Nonrotating Uncoupled Uniform Beam

The program input data for the nonrotating, uncoupled, uniform beam in bending and torsion are presented as case number 1 in table IX. Although the exact solution does not include the effects of rotary inertia, the computer program does. Therefore, negative values of the sectional mass moment of inertia (determined by eq. (B1)) are input to the program so that the net segment rotary inertia effects in the program are eliminated. Comparisons of computed and exact results (refs. 8 and 9) for the first case are tabulated in tables XI to XIV. The differences between the computed and exact natural frequencies for both sets of boundary conditions are always less than 0.8 and 1.7 percent for bending and torsion, respectively. The discrepancies increase with mode number and the computed frequencies are always smaller. The trends of bending mode shape correlation are similar to those for natural frequency. The computed and exact torsion mode shapes match for all modes and both sets of boundary conditions.

Case 2 - Nonrotating Uncoupled Uniform Beam With Tip Mass

Table XV illustrates the effect of adding a tip mass on the clamped-free bending frequencies. The analytical model is shown in table IX as case number two. The torsion frequencies are unchanged from the values indicated in table XIV. The calculated bending frequencies vary by up to 1.5 percent relative to the values obtained from reference 10 (for the first vertical bending mode). The trend of increasing discrepancy with mode order observed for the previous examples is not present in this case. In fact, the smallest difference occurs for the third bending mode (0.3 percent). For the range of data in table XV, the correlation is still good.

Cases 3 and 4 - Uniform Simply Supported Beam

Coupled bending-torsion vibrations are analyzed in reference 11, where equations are given for calculating exact natural frequencies and mode shapes. The example problem is that of a uniform, simply supported (pinned-pinned vertical bending and clamped-clamped torsion) beam with a horizontal offset between the center of gravity and shear center axes to induce coupling. These boundary conditions cannot be achieved at the outboard tip by the analysis documented in this report without slight modifications. The required changes to the computer program include replacement of the code generating the unity values of vertical and torsional deflections and all terms pertaining to shears and moments at the outboard tip. Code representing unity values of the vertical shear and torsional moment are then substituted into the program. These changes were made and the analytical representation, given as cases 3 and 4 in table IX, used for correlation purposes. Tables XVI and XVII present comparisons for calculated and exact results with and without the coupling added. Without the coupling the largest discrepancies in frequency are less than 0.02 and 1.7 percent for the highest bending and torsion modes, respectively, thus verifying the representation. For the coupled problem (table XVII) the differences between computed and exact frequencies are always less than 0.7 percent with the difference generally increasing with mode order. The relative bending-torsion amplitudes for the mode shapes show good agreement between computed and exact results.

Case 5 - Model Propeller

Experimental vibration data, pertaining to model propeller blades, are given in reference 12 and were used in reference 13 for correlation with the analysis presented in that report. The structural characteristics presented in these two references were used to obtain the representation shown in table X by interpolating to intermediate stations. The effective torsional stiffness which was used included both the nominal value of GJ and the contribution of another term dependent on the square of the twist rate as shown in table X. The second term is included in the torsional equation of motion in reference 13 but not in the equations given in this report (necessitating its inclusion with the GJ values). The effect of this term is to increase all frequencies alike without regard to the values of collective pitch or rotor speed. Its omission in the present equations is based on its insignificance for the twist rates used on conventional rotor designs.

Experimental and calculated variations of natural frequency with collective pitch angle and rotor rotational speed are illustrated in figure 10 for the first and second bending and the first torsion natural frequencies. Excellent agreement between experimental and calculated results are achieved for the two bending frequencies. (See figs. 10(a) and 10(b).) A discrepancy exists between calculated and measured variations of torsional frequency with collective pitch, as shown in figure 10(c). As discussed in reference 13, the angle between local structural axes on the blade and the plane of rotation is a function of built-in twist, collective pitch angle, and steady-state twist deflections caused by rotation. The last item is not included in the linear analysis documented in this report. However, this term is not significant for conventional helicopter blade designs.

CONCLUDING REMARKS

An existing computer program, used for predicting the natural frequencies and mode shapes of helicopter rotor blades, has been modified to improve program efficiency, accuracy, and versatility. The equations and techniques used in the original version were extensively changed. This program is based on the Holzer-Myklestad approach adapted for rotating beams and represents the rotor blade as a series of lumped masses connected by stiffness elements. Elastic and inertia coupling between vertical, horizontal, and torsional deflections are included in the analysis. Spanwise variations in structural stiffness, mass, blade twist, rotary inertia, and center of gravity and shear center offsets from the pitch axis may be input. Provisions are also included for representing various hub configurations and the effects of pylon impedance on blade modes.

The resulting program is documented by presenting in this report the equations and techniques for determining natural frequencies and mode shapes, input data requirements, and descriptions of the various program outputs. A source listing of the program is also presented. The accuracy of the program is indicated by comparing computed results with exact solutions and experimental data. For uniform, nonrotating beam problems, natural frequency predictions are within 2 percent of exact solutions for the first five bending and torsion modes and for a variety of boundary conditions. The predicted mode shapes closely match exact answers. Experimental data measured on a rotating, highly twisted propeller were also used for correlation. The calculated bending frequencies showed excellent agreement with measured data.

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APPENDIX A

CONVERSION OF U.S. CUSTOMARY UNITS TO SI UNITS

The input and output data associated with the computer program are expressed in U.S. Customary Units. Conversion factors for obtaining data in SI Units are given in the following table based on reference 14:

Physical quantity	Units		Conversion factor[a]
	U.S. Customary	SI	
Force, Weight	lb	N	4.4482
Frequency	cpm	Hz	0.01667
Length	in.	m	0.02540
Linear spring	lb/in.	N/m	175.13
Mass	lb-sec^2/in.	N-sec^2/m	175.13
Moment	in-lb	N-m	0.11298
Rotational speed	rpm	rad/sec	0.10472
Rotational spring	in-lb/rad	N-m/rad	0.11298
Second mass moment of inertia . . .	lb-sec^2	N-sec^2	4.4482
Structural stiffness	lb-in^2	N-m^2	0.28698×10^{-2}

[a] Multiply values given in U.S. Customary Unit by conversion factor to obtain equivalent value in SI Unit.

APPENDIX B

DEVELOPMENT OF TERMS APPEARING IN THE RECURSION EQUATIONS

To determine the natural frequencies and mode shapes associated with a rotor blade, the blade is segmented, and average values of the structural properties of each of the N segments are determined. These properties are associated with the midpoint of each segment by the program and the cross-sectional second mass moments of inertia are related to the section mass centroid. Figure 1 illustrates the local A,B,C segment-fixed axis system and overall X,Y,Z-axis system with sign conventions. The reference axis is the undeformed Z pitch axis of the blade. The jth segment is shown in figure 1 having a width $\bar{z}_j$ and an inboard station z_j as measured from the center of rotation.

The mass moments of inertia of the segment cross section for the beam and chord directions I'_{bb} and I'_{cc} are transferred to the reference (blade pitch) axis and the rotary inertia for the segment is added. The polar moment of inertia is also determined by the program. The relationships are:

$$I_{bb,i} = I'_{bb,i}\bar{z}_i + \frac{w_i\bar{z}_i}{g}\left(\frac{\bar{z}_i^2}{12} + r_{b,i}^2\right) \qquad (i = 1, 2, \ldots, N) \qquad \text{(B1)}$$

$$I_{cc,i} = I'_{cc,i}\bar{z}_i + \frac{w_i\bar{z}_i}{g}\left(\frac{\bar{z}_i^2}{12} + r_{c,i}^2\right) \qquad (i = 1, 2, \ldots, N) \qquad \text{(B2)}$$

$$I_{aa,i} = (I'_{bb,i} + I'_{cc,i})\bar{z}_i + \frac{w_i\bar{z}_i}{g}\left(r_{b,i}^2 + r_{c,i}^2\right) \qquad (i = 1, 2, \ldots, N) \qquad \text{(B3)}$$

$$k_{a,i}^2 = \frac{I_{aa,i}g}{\bar{z}_i w_i} \qquad (i = 1, 2, \ldots, N) \qquad \text{(B4)}$$

where w_i is the weight per unit length for the ith segment. The terms $r_{b,i}$ and $r_{c,i}$ are the center-of-gravity offsets from the reference axis and the sign convention for these terms is given in figure 2(a).

Because the analysis represents each segment as a weightless stiffness element with the inertial properties lumped at its inboard end, the inertias for adjacent segment midpoints are averaged to determine the values at each jth station. Thus,

$$m_j = \frac{w_{j-1}\bar{z}_{j-1} + w_j\bar{z}_j}{2g} \qquad (j = 2, 3, \ldots, N) \qquad \text{(B5)}$$

$$(mr_b)_j = \frac{w_{j-1}\bar{z}_{j-1}r_{b,j-1} + w_j\bar{z}_jr_{b,j}}{2g} \qquad (j = 2, 3, \ldots, N) \qquad \text{(B6)}$$

$$(mr_c)_j = \frac{w_{j-1}\bar{z}_{j-1}r_{c,j-1} + w_j\bar{z}_jr_{c,j}}{2g} \qquad (j = 2, 3, \ldots, N) \qquad \text{(B7)}$$

$$(mr_br_c)_j = \frac{w_{j-1}\bar{z}_{j-1}r_{b,j-1}r_{c,j-1} + w_j\bar{z}_jr_{b,j}r_{c,j}}{2g} \qquad (j = 2, 3, \ldots, N) \qquad \text{(B8)}$$

$$(I_{bb,j}) = \frac{I_{bb,j-1} + I_{bb,j}}{2} \qquad (j = 2, 3, \ldots, N) \qquad \text{(B9)}$$

$$(I_{cc,j}) = \frac{I_{cc,j-1} + I_{cc,j}}{2} \qquad (j = 2, 3, \ldots, N) \qquad \text{(B10)}$$

$$(I_{aa,j}) = \frac{I_{aa,j-1} + I_{aa,j}}{2} \qquad (j = 2, 3, \ldots, N) \qquad \text{(B11)}$$

Equation (B5) represents the lumped mass m_j at the jth station. For the end stations including the tip weight W

$$m_1 = \frac{w_1\bar{z}_1}{2g} \qquad \text{(B12a)}$$

$$m_{N+1} = \frac{w_n \bar{z}_n}{2g} + \frac{W}{g} \tag{B12b}$$

$$(mr_b)_1 = \frac{w_1 \bar{z}_1 r_{b,1}}{2g} \tag{B13a}$$

$$(mr_b)_{N+1} = \frac{w_N \bar{z}_N r_{b,N}}{2g} + \frac{W r_{b,N+1}}{g} \tag{B13b}$$

$$(mr_c)_1 = \frac{w_1 \bar{z}_1 r_{c,1}}{2g} \tag{B14a}$$

$$(mr_c)_{N+1} = \frac{w_N \bar{z}_N r_{c,N}}{2g} + \frac{W r_{c,N+1}}{g} \tag{B14b}$$

$$(mr_b r_c)_1 = \frac{w_1 \bar{z}_1 r_{b,1} r_{c,1}}{2g} \tag{B15a}$$

$$(mr_b r_c)_{N+1} = \frac{w_N \bar{z}_N r_{b,N} r_{c,N}}{2g} + \frac{W r_{b,N+1} r_{c,N+1}}{g} \tag{B15b}$$

$$I_{bb,1} = \frac{I'_{bb,1} \bar{z}_1}{2} + \frac{w_1 \bar{z}_1}{2g}\left(\frac{\bar{z}_1^{\,2}}{12} + r_{b,1}^2\right) \tag{B16a}$$

$$I_{bb,N+1} = \frac{I'_{bb,N} \bar{z}_N}{2} + \frac{w_N \bar{z}_N}{2g}\left(\frac{\bar{z}_N^{\,2}}{12} + r_{b,N}^2\right) + \frac{W r_{b,N+1}^2}{g} \tag{B16b}$$

$$I_{cc,1} = \frac{I'_{cc,1}\bar{z}_1}{2} + \frac{w_1\bar{z}_1}{2g}\left(\frac{\bar{z}_1^2}{12} + r_{c,1}^2\right) \tag{B17a}$$

$$I_{cc,N+1} = \frac{I'_{cc,N}\bar{z}_N}{2} + \frac{w_N\bar{z}_N}{2g}\left(\frac{\bar{z}_N^2}{12} + r_{c,N}^2\right) + \frac{W r_{c,N+1}^2}{g} \tag{B17b}$$

$$I_{aa,1} = \frac{\left(I'_{bb,1} + I'_{cc,1}\right)\bar{z}_1}{2} + \frac{w_1\bar{z}_1}{2g}\left(r_{b,1}^2 + r_{c,1}^2\right) \tag{B18a}$$

$$I_{aa,N+1} = \frac{\left(I'_{bb,N} + I'_{cc,N}\right)\bar{z}_N}{2} + \frac{w_N\bar{z}_N}{2g}\left(r_{b,N}^2 + r_{c,N}^2\right) + \frac{W}{g}\left(r_{b,N+1}^2 + r_{c,N+1}^2\right) \tag{B18b}$$

Equations (B5) to (B18b) apply to the local axis system at each station. Because the equations of motion are written for the horizontal-vertical or X,Y-axes, the blade inertial properties must be rotated into the latter system using the pitch angle at each station defined from the twist θ_t and collective pitch angle θ_c.

$$\theta_j = 0 \qquad (j = 1, 2, \ldots, \text{JHUB}) \tag{B19a}$$

$$\theta_{\text{JHUB}+1} = \theta_c \tag{B19b}$$

$$\theta_{j+1} = \theta_j + \frac{\bar{z}_j}{R}\theta_t \qquad (j = \text{JHUB}+1, \ldots, N) \tag{B19c}$$

where JHUB is an input quantity representing the number of nonfeathering hub segments. For general twist distributions, input values of twist are used, and the collective pitch angle is added to those values. The inertia representation in the X,Y,Z-axis system is given by the following relationships:

$$(mr_x)_j = (mr_c)_j \cos\theta_j + (mr_b)_j \sin\theta_j \qquad (j = 1, 2, \ldots, N+1) \tag{B20}$$

$$(mr_y)_j = (mr_c)_j \sin\theta_j - (mr_b)_j \cos\theta_j \qquad (j = 1, 2, \ldots, N+1) \tag{B21}$$

$$I_{yy,j} = I_{bb,j} \cos^2 \theta_j + I_{cc,y} \sin^2 \theta_j - 2(mr_b r_c)_j \sin \theta_j \cos \theta_j$$

$$(j = 1, 2, \ldots, N+1) \qquad \text{(B22)}$$

$$I_{xx,j} = I_{bb,j} \sin^2 \theta_j + I_{cc,j} \cos^2 \theta_j + 2(mr_b r_c)_j \sin \theta_j \cos \theta_j$$

$$(j = 1, 2, \ldots, N+1) \qquad \text{(B23)}$$

$$I_{xy,j} = (I_{bb,j} - I_{cc,j}) \sin \theta_j \cos \theta_j + (mr_b r_c)_j (\cos^2 \theta_j - \sin^2 \theta_j)$$

$$(j = 1, 2, \ldots, N+1) \qquad \text{(B24)}$$

$$I_{zz,j} = I_{aa,j} \qquad (j = 1, 2, \ldots, N+1) \qquad \text{(B25)}$$

$$I_{\phi\phi,j} = (I_{bb,j} - I_{cc,j})(\cos^2 \theta_j - \sin^2 \theta_j) - 4(mr_b r_c)_j \sin \theta_j \cos \theta_j$$

$$(j = 1, 2, \ldots, N+1) \qquad \text{(B26)}$$

where the last term pertains to the so-called "tennis racquet" effect in the equation for torsional moment.

The elastic properties are rotated into the X,Y,Z-axis system, using the total pitch angle at the midpoint of each segment obtained from averaging the values of pitch at the segment endpoints. The shear center (elastic axis) offsets from the reference axis (see fig. 2(b)) are found by

$$S_{x,i} = S_{c,i} \cos \bar{\theta}_i + S_{b,i} \sin \bar{\theta}_i \qquad (i = 1, 2, \ldots, N) \qquad \text{(B27)}$$

$$S_{y,i} = S_{b,i} \cos \bar{\theta}_i - S_{c,i} \sin \bar{\theta}_i \qquad (i = 1, 2, \ldots, N) \qquad \text{(B28)}$$

The structural stiffness representation in the X,Y,Z-axis system is

$$\frac{1}{(EI)_{yy,i}} = \frac{\cos^2 \bar{\theta}_i}{(EI)_{b,i}} + \frac{\sin^2 \bar{\theta}_i}{(EI)_{c,i}} \qquad (i = 1, 2, \ldots, N) \qquad \text{(B29)}$$

$$\frac{1}{(EI)_{xx,i}} = \frac{\sin^2 \bar{\theta}_i}{(EI)_{b,i}} + \frac{\cos^2 \bar{\theta}_i}{(EI)_{c,i}} \qquad (i = 1, 2, \ldots, N) \qquad \text{(B30)}$$

$$\frac{1}{(EI)_{xy,i}} = \left[\frac{1}{(EI)_{b,i}} - \frac{1}{(EI)_{c,i}}\right] \sin \bar{\theta}_i \cos \bar{\theta}_i \qquad (i = 1, 2, \ldots, N) \qquad \text{(B31)}$$

The torsional stiffnesss $(GJ)_i$ is unchanged.

At each station the centrifugal force and associated moments created by the offset of the center of gravity from the reference axis are calculated by using the formulas

$$F_j = \Omega^2 \sum_{k=j+1}^{N+1} m_k z_k \qquad (j = 1, 2, \ldots, N) \qquad \text{(B32)}$$

$$F_{H,j} = \Omega^2 \sum_{k=j+1}^{N+1} (mr_x)_k \qquad (j = 1, 2, \ldots, N) \qquad \text{(B33)}$$

$$F_{x,j} = \Omega^2 \sum_{k=j+1}^{N+1} (mr_x)_k z_k \qquad (j = 1, 2, \ldots, N) \qquad \text{(B34)}$$

$$F_{y,j} = \Omega^2 \sum_{k=j+1}^{N+1} (mr_y)_k z_k \qquad (j = 1, 2, \ldots, N) \qquad \text{(B35)}$$

where Ω is the rotor rotational speed and z_k is the radial distance to each station (fig. 1)

$$z_1 = 0.0 \qquad \text{(B36a)}$$

$$z_{j+1} = \sum_{k=1}^{j} \bar{z}_k \qquad \text{(B36b)}$$

APPENDIX C

COMPUTER PROGRAM LISTING

The computer program listing is given in this appendix.

```
      PROGRAM BLDANL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7)
C
C       THIS PROGRAM COMPUTES THE NATURAL FREQUENCIES AND MODE SHAPES
C            FOR A HELICOPTER ROTOR BLADE. TEETERING, ARTICULATED, AND
C            HINGELESS BLADES CAN BE TREATED.  FURTHER INFORMATION
C            CONCERNING THIS PROGRAM MAY BE FOUND IN NASA TM 78670
C
      COMMON /COMA/ JHUB,N1,LOT,POUT,ITLE(10),NAME,DAY,NPG
     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES
      COMMON/ COMC/ WTPL(31),EIB(30),EIC(30).GJ(30),THD(31)
      COMMON/COMB/IRCOL,XRCOL(3),IRPM,CRPM(10),ZBAR(30),EYEB(30),SB(30),
     1 EYEC(30),SC(30),SM'31),RB(31),RC(31),TH(31),WT(30)
      COMMON/COMD/CMAT(5,5,,SOMMAT(100,3),IPLN(100,3),INODE(100,3),
     1  MM3,MM5,CT(31),ST(31),IB,IST,IBS(3,10,3),IBE(3,10,3)
      COMMON/ H/ VMX(30),VQX(30),VMY(30),F(30),FTX(30),FTY(30),SX(30),
     1 SY(30),OMEGA2,FTH(30)
      COMMON/COMT/ EYB(31),EYC(31),EYR(31),EMRB(31),EMRC(31),EMRR(31),
     1 EMRX(31),EMRY(31),EMBBW(31),EMPPW(31),THHO(31),EMBPW(31)
      COMMON /COME/ ZB(155),ZX(155),ZQ(155),ZL(155),ZS(155),ZY(155),
     1 ZM(155),ZD(155),ZH(155),ZT(155)
      LOGICAL LOT,TRSN
      DIMENSION PP(2),IN(3),XQSOM(2),SMZ(31),SMZRX(31),SMZRY(31)
     1 ,SMRX(31)
      DATA CVRPS/0.1047198/
      CALL PSEUDO
      CALL LEROY
      CALL DATE(DAY)
   10 CALL INPT(PLAST)
      MM3=3
      MM5=5
      IF(  TRSN  ) GO TO 20
      MM3=2
      MM5=4
 20   NOB=1
      DO 30 I=1,3
      IN(I)=0
      DO 30 J=1,10
      DO 30 K=1,3
      IBS(I,J,K)=0
      IBE(I,J,K)=-1
   30 CONTINUE
      ENDXQ=(PLAST*CVRPS)**2
      DO 25 I=1,125,31


      ZB(I)=0.0
      ZS(I)=0.0
      ZY(I)=0.0
      ZX(I)=0.0
      ZH(I)=0.0
      ZM(I)=0.0
      ZQ(I)=0.0
      ZT(I)=0.0
      ZL(I)=0.0
   25 ZD(I)=0.0
      ZB(1)=1.0
      ZS(32)=1.0
      ZY(125)=1.0
      ZH(94)=1.0
      ZX(63)=1.0
C*************************
C COLLECTIVE ANGLE SWEEP *------------------------------------------------
C*************************
      DO 700 IST=1,IRCOL
```

APPENDIX C

```
C*************************************************************
C CALCULATE COEFFICIENTS DEPENDENT ON COLLECTIVE ANGLE *
C*************************************************************

      DO 80 I=NCS,N1
      ZTH=TH(I)
      IF(I.GT.JHUB) ZTH=ZTH+XRCOL(IST)
      ST(I)=SIN(ZTH)
      CT(I)=COS(ZTH)
      SSTH=ST(I)**2
      CCTH=1.-SSTH
      SCTH=ST(I)*CT(I)
      EMRX(I)=EMRC(I)*CT(I)+EMRB(I)*ST(I)
      EMRY(I)=EMRC(I)*ST(I)-EMRB(I)*CT(I)
      EMBBW(I)=EYB(I)*CCTH+EYC(I)*SSTH-2.0*EMRR(I)*SCTH
      EMPPW(I)=EYB(I)*SSTH+EYC(I)*CCTH+2.0*EMRR(I)*SCTH
      EMBPW(I)=(EYB(I)-EYC(I))*SCTH+EMRR(I)*(CCTH-SSTH)
      THMD(I)=(EYB(I)-EYC(I))*(CCTH-SSTH)-4.0*EMRR(I)*SCTH
      IF(I.EQ.N1) GO TO 80
      ZTH=0.5*(TH(I)+TH(I+1))
      IF(I.GT.JHUB) ZTH=ZTH+XRCOL(IST)
      STH=SIN(ZTH)
      CTH=COS(ZTH)
      SSTH=STH**2
      CCTH=1.-SSTH
      SCTH=STH*CTH
      SX(I)=SC(I)*CTH+SB(I)*STH
      SY(I)=SB(I)*CTH-SC(I)*STH
      VMX(I)=ZBAR(I)*((1.0/EIB(I))-(1.0/EIC(I)))*SCTH
      VOX(I)=ZBAR(I)*((SSTH/EIB(I))+(CCTH/EIC(I)))
      VMY(I)=ZBAR(I)*((CCTH/EIB(I))+(SSTH/EIC(I)))
 80   CONTINUE
      SMRX(N1)=0.0
      SMZ(N1)=0.0
      SMZRX(N1)=0.
      SMZRY(N1)=0.
      J=N1
      DO 85 I=1,N
      K=J
      J=J-1
      SMRX(J)=SMRX(K)+EMRX(K)
      SMZ(J)=SMZ(K)+SM(K)*Z(K)
      SMZRX(J)=SMZRX(K)+EMRX(K)*Z(K)
 85   SMZRY(J)=SMZRY(K)+EMRY(K)*Z(K)
      NOB=JHUB+1
C*****************
C ROTOR RPM SWEEP *-----------------------------------------------------------
C*****************


      DO 710 IB= 1,IRPM
C*********************************************************
C CALCULATE COEFFICIENTS DEPENDENT ON ROTOR RPM *
C*********************************************************
      OMEGA2=CRPM(IB)**2
      DO 110 I=1,N
      FTH(I)=OMEGA2*SMRX(I)
      F(I)=OMEGA2*SMZ(I)
      FTX(I)=OMEGA2*SMZRX(I)
      FTY(I)=OMEGA2*SMZRY(I)
      KAPSQ=((EYEB(I)+EYEC(I))*386.4/WTPL(I))+RB(I)*RB(I)+RC(I)*RC(I)
      WT(I)=0.0
      IF(TRSN) WT(I)=ZBAR(I)/(GJ(I)+F(I)*KAPSQ)
  110 CONTINUE
C*************** **************************** *****************
C  CALCULATE THE FIRST NATURAL FREQUENCY FOR EACH MODE TYPE  *
C**************************************************************
      J=1
C      BYPASS COLLECTIVE AND CYCLIC MODES FOR MORE THAN 2 BLADES
      IF(BLADES.NE.2.0) J=3
      DO 200 I=J,3
      IF(IN(I).EQ.100) GO TO 200
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```
      XQSOM(1)=(20.0*CVRPS)**2
      XQSOM(2)=(30.0*CVRPS)**2
      CALL COEF(I,.FALSE.,2,XQSOM,PP)
      IEXP=0
  120 RATIO=ABS(PP(2)/PP(1))
      IF(RATIO.LT.1.0) GO TO 129
      IEXP=IEXP+1
      PP(1)=PP(1)/XQSOM(1)
      PP(2)=PP(2)/XQSOM(2)
      GO TO 120
  129 EXTRAP=PP(2)*(XQSOM(2)-XQSOM(1))/(PP(1)-PP(2))+XQSOM(2)
      TEST=ABS((EXTRAP-XQSOM(2))/XQSOM(2))
      IF(TEST.LT.0.001) GO TO 130
  127 XQSOM(1)=XQSOM(2)
      PP(1)=PP(2)
      XQSOM(2)=EXTRAP
  125 CALL COEF(I,.FALSE.,1,XQSOM(2),PP(2))
      PP(2)=PP(2)/XQSOM(2)**IEXP
      GO TO 129
  130 CONTINUE
      IN(I)=IN(I)+1
      IBS(IST,18,I)=IN(I)
      NMODE=IN(I)
      SOMNAT(NMODE,I)=EXTRAP
      XREF=SOMNAT(NMODE,I)/2.0
      MODE=NMODE
      IF(MODE.EQ.100) GO TO 189
C*************************************************************
C  CALCULATE HIGHER NATURAL FREQUENCIES FOR EACH MODE TYPE   *
C*************************************************************
  135 XQSOM(1)=SOMNAT(MODE,I)*1.01
      XQSOM(2)=SOMNAT(MODE,I)*1.02
      CALL COEF(I,.FALSE.,1,XQSOM(1),PP(1))
  137 DENOML=XQSOM(1)**IEXP
      DENOMR=XQSOM(2)**IEXP
      DO 140 K=NMODE,MODE
      DENOML=DENOML*(XQSOM(1)-SOMNAT(K,I)*1.001)
  140 DENOMR= DENOMR*(XQSOM(2)-SOMNAT(K,I)*1.001)
      CALL COEF(I,.FALSE.,1,XQSOM(2),PP(2))
      FUNCL=PP(1)/DENOML
      FUNCR=PP(2)/DENOMR
      EXTRAP=(XQSOM(2)**0.5+FUNCR*(((XQSOM(2)-XREF)**0.50-(XQSOM(1)-
     1 XREF)**0.50)/(FUNCL-FUNCR)))**2
      IF(EXTRAP.GT.ENDXQ) GO TO 190


      TEST=ABS((EXTRAP-XQSOM(2))/XQSOM(2))
      IF(TEST.LT.0.001) GO TO 160
      PP(1)=PP(2)
      XQSOM(1)=XQSOM(2)
      XQSOM(2)=EXTRAP
      GO TO 137
  160 MODE=MODE+1
      SOMNAT(MODE,I)=EXTRAP
      XREF=SOMNAT(MODE,I)/2.0
      IF(MODE.EQ.100) GO TO 189
      GO TO 135
  189 CONTINUE
      WRITE(6,188) I
  188 FORMAT(//* ITERATION STOPPED FOR TYPE *,I2,* MODES - 100 NATURAL F
     1REQUENCIES MAX.*)
  190 CONTINUE
      IN(I)=MODE
      IBE(IST,18,I)=MODE
  200 CONTINUE
C CALCULATE AND PRINT OUT MODE SHAPES *
      CALL AMPLTD
  710 CONTINUE
```

```
  700 CONTINUE                                                          BLDA 198
C        PRINT OUT SUMMARY OF NATURAL FREQUENCIES                       BLDA 199
      CALL SUMFY                                                        BLDA 200
C PLOT NATURAL FREQ. VS ROTOR RPM *                                     BLDA 201
      IF(LOT) CALL PLOUT(PLAST)                                         BLDA 202
      GO TO 10                                                          BLDA 203
      END                                                               BLDA 204
```

```
      SUBROUTINE AMPLTD                                                 AMPL   1
C***********************************************************            AMPL   2
C THIS SUBROUTINE CALCULATES AND PRINTS OUT MODE SHAPES *               AMPL   3
C***********************************************************            AMPL   4
      COMMON /COMA/ JHUB,M1,LOT,POUT,ITLE(10),NAME,DAY,NPG              AMPL   5
     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES        AMPL   6
      COMMON/COMB/IRCOL,XRCOL(3),IRPM,CRPM(10),ZBAR(30),EYEB(30),SB(30),AMPL   7
     1 EYEC(30),SC(30),SM(31),RB(31),RC(31),TH(31),WT(30)               AMPL   8
      COMMON/COMD/CMAT(5,5),SOMNAT(100,3),IPLN(100,3),INODE(100,3),     AMPL   9
     1  MM3,MM5,CT(31),ST(31),IB,IST,IBS(3,10,3),IBE(3,10,3)            AMPL  10
      COMMON /COME/ ZB(155), ZX(155), ZQ(155), ZL(155), ZS(155), ZY(155)AMPL  11
     *,ZM(155), ZD(155), ZH(155), ZT(155)                               AMPL  12
      COMMON /HINGES/ LCH,LCHP1,LFH,LFHP1 ,CHOFF,FHOFF,FCH,FFH          AMPL  13
     1 ,RPMPUN,COLPUN,LPH,LPHP1,PHOFF,FPH,             LOTS             AMPL  14
      COMMON/COMT/ EYB(31),EYC(31),EYR(31),EMRB(31),EMRC(31),EMRR(31),  AMPL  15
     1 EMRX(31),EMRY(31),EMBBW(31),EMPPW(31),THHO(31),EMBPW(31)         AMPL  16
      LOGICAL POUT,TRSN,LOTS,MODPLOT,INPUN                              AMPL  17
      DIMENSION VEC(5),A(5),B(3,31),DMAT(5,5),IPIVOT(5),IWK(10)         AMPL  18
      DATA CVCPM/9.549297/                                              AMPL  19
C**************************************                                 AMPL  20
C MODE LOOP  M=1 FOR COLLECTIVE MODE *                                  AMPL  21
C            M=2 FOR CYCLIC MODES   *                                   AMPL  22
C            M=3 FOR SCISSORS MODES  *                                  AMPL  23
C**************************************                                 AMPL  24
      DO 227 M=1,3                                                      AMPL  25
C     BYPASS COLLECTIVE AND CYCLIC MODES FOR A MULTIBLADED ROTOR        AMPL  26
      IF(M.NE.3.AND.BLADES.NE.2.0)  GO TO 227                           AMPL  27
      MODENO=0                                                          AMPL  28
      IF(IBS(IST,IB,M).EQ.0) GO TO 227                                  AMPL  29
C*********************************************                          AMPL  30
C SWEEP NATURAL FREQUENCIES STORED IN SOMNAT *                          AMPL  31
C*********************************************                          AMPL  32


      NPT = 1+ IBE(IST,IB,M) -IBS(IST,IB,M)                             AMPL  33
      DO 223 NP=1,NPT                                                   AMPL  34
      NPS = NP +IBS(IST,IB,M) -1                                        AMPL  35
      CALL COEF(M,.TRUE.,1,SOMNAT(NPS,M),DUMYY)                         AMPL  36
C                                                                       AMPL  37
C       ITERATE  TRIAL VECTOR FOR TIP DEFLECTIONS                       AMPL  38
C                                                                       AMPL  39
      DO 4 I=1,MM5                                                      AMPL  40
      DO 1 J=1,MM5                                                      AMPL  41
    1 DMAT(I,J)=CMAT(I,J)                                               AMPL  42
    4 CONTINUE                                                          AMPL  43
      SOMNAT(NPS,M)=CVCPM*SQRT(SOMNAT(NPS,M))                           AMPL  44
      FNAT=SOMNAT(NPS,M)                                                AMPL  45
      CALL MATINV(5,MM5,CMAT,0,BBB,1,DETERM,ISCALE,IPIVOT,IWK)          AMPL  46
      VEC(1)=0.0                                                        AMPL  47
      VEC(2)=0.0                                                        AMPL  48
      DO 5 I=3,MM5                                                      AMPL  49
    5 VEC(I)=1.0/1.E50                                                  AMPL  50
      ITRY=0                                                            AMPL  51
   15 CONTINUE                                                          AMPL  52
      ITRY=ITRY+1                                                       AMPL  53
      AMAX=0.0                                                          AMPL  54
      DO 25 I=1,MM5                                                     AMPL  55
```

```
      A(I)=0.0
      DO 20 J=1,MM5
   20 A(I)=A(I)+VEC(J)*CMAT(I,J)
      IF(ABS(A(I)).GT.ABS( AMAX)) AMAX=A(I)
   25 CONTINUE
      DO 30 I=1,MM5
      A(I)=A(I)/AMAX
   30 CONTINUE
      IF(ITRY.EQ.4) GO TO 100
      DO 40 I=1,MM5
   40 VEC(I)=A(I)/1.E50
      GO TO 15
C
C      CALCULATE MODE SHAPES AND GENERALIZED INERTIA
C
  100 CONTINUE
      DO 60 I=1,MM5
      VEC(I)=0.0
      DO 50 J=1,MM5
      VEC(I)=VEC(I)+DMAT(I,J)*A(J)
   50 CONTINUE
   60 CONTINUE
      DO 70 I=1,MM5
      A(I)=A(I)/A(MM5)
   70 CONTINUE
      L=N1+1
      GIB=0.0
      GIP=0.0
      DO 115 I=1,N1
      L=L-1
      GIBB=ZB(I)*A(1)+ZB(31+I)*A(2)+ZB(124+I)+ZB(62+I)*A(3)
      GIPP=ZS(I)*A(1)+ZS(31+I)*A(2)+ZS(124+I)+ZS(62+I)*A(3)
      B(3,L)=0.0
      B(1,L)=ZY(I)*A(1)+ZY(31+I)*A(2)+ZY(124+I)+ZY(62+I)*A(3)
      B(2,L)=ZX(I)*A(1)+ZX(31+I)*A(2)+ZX(124+I)+ZX(62+I)*A(3)
      IF(.NOT.TRSN) GO TO 113
      GIBB=GIBB+ZB(93+I)*A(4)
      GIPP=GIPP+ZS(93+I)*A(4)
      B(1,L)=B(1,L)+ZY(93+I)*A(4)
      B(2,L)=B(2,L)+ZX(93+I)*A(4)
      B(3,L)=ZH(I)*A(1)+ZH(31+I)*A(2)+ZH(124+I)+ZH(62+I)*A(3)+ZH(93+I)*
     1  A(4)
  113 CONTINUE
      GIB=GIB+EMBBW(L)*GIBB**2

      GIP=GIP+EMPPW(L)*GIPP**2
  115 CONTINUE
      ABSCL=0.
      DO 120 I=1,MM3
      DO 120 J=1,N1
      ABSB=ABS(B(I,J))
      IF(ABSCL.GT.ABSB) GO TO 120
      SCALE=B(I,J)
      ABSCL=ABSB
  120 CONTINUE
      SCALE=1./SCALE
      GIB=GIB*SCALE**2
      GIP=GIP*SCALE**2
      DO 150 I=1,MM3
      DO 150 J=1,N1
  150 B(I,J)=B(I,J)*SCALE
      GIX=0.0
      GIY=0.0
      GIT=0.0
      DO 317 J=1,N1
      GIY=GIY+SM(J)*B(1,J)**2
      GIX=GIX+SM(J)*B(2,J)**2
```

```
      GIT=GIT+EYR(J)* B(3,J)**2                                         AMPL 122
  317 CONTINUE                                                          AMPL 123
      GIV=GIB+GIY                                                       AMPL 124
      GIM=GIP+GIX                                                       AMPL 125
      IPLN(NPS,M)=1                                                     AMPL 126
      IF(GIM.GT.GIV .AND. GIM.GT.GIT) IPLN(NPS,M)=2                     AMPL 127
      IF(GIT.GT.GIV .AND. GIT.GT.GIM) IPLN(NPS,M)=3                     AMPL 128
      FREQPR=FNAT                                                       AMPL 129
      IF(RRPM(IB).NE.0.0) FREQPR=FNAT/RRPM(IB)                          AMPL 130
C       COMPUTE NUMBER OF NODES                                         AMPL 131
      K=IPLN(NPS,M)                                                     AMPL 132
      INODE(NPS,M)=0                                                    AMPL 133
      KK=JHUB1                                                          AMPL 134
      IF(JHUB.EQ.0) KK=2                                                AMPL 135
      DO 160 I=KK,N                                                     AMPL 136
      PROD=B(K,I)*B(K,I+1)                                              AMPL 137
      IF(PROD.GE.0.0) GO TO 160                                         AMPL 138
      INODE(NPS,M)=INODE(NPS,M)+1                                       AMPL 139
  160 CONTINUE                                                          AMPL 140
      IF(LOTS) GO TO 190                                                AMPL 141
      IF(.NOT.POUT) GO TO 225                                           AMPL 142
      IF(RCOLL(IST).NE.COLPUN.OR.RRPM(IB).NE.RPMPUN) GO TO 225          AMPL 143
C     PLOT MODF SHAPE IF REQUESTED                                      AMPL 144
      IF(MODPLOT) CALL MODSHAP(B,RCOLL(IST),RRPM(IB),FREQPR,M,NP,NPT)   AMPL 145
  190 CONTINUE                                                          AMPL 146
C       PRINT OUT MODE SHAPE                                            AMPL 147
      NPG=NPG+1                                                         AMPL 148
      WRITE(6,901) NPG,DAY,NAME,(ITLE(J),J=1,10)                        AMPL 149
  901 FORMAT(1H1,18X,4HPAGE,I3,25X,34HMYKLESTAD ANALYSIS  NASA TM 78670 AMPL 150
     1 ,25X,A10,//10X,5HCASE ,I6,23X,10A6/)                             AMPL 151
      WRITE(6,904) RCOLL(IST),RRPM(IB)                                  AMPL 152
  904 FORMAT(37X,F6.2,2X,23HDEGREES ROOT COLLECTIVE,10X,F7.1,2X,16HROTORAMPL 153
     1 SPEED, RPM)                                                      AMPL 154
      IF(         M.EQ.1) WRITE(6,902)    FNAT                          AMPL 155
  902 FORMAT (36X,27HCOLLECTIVE MODE OF BLADE AT ,F9.2,4H CPM )         AMPL 156
      IF(         M.EQ.2) WRITE(6,903) FNAT                             AMPL 157
  903 FORMAT (38X,23HCYCLIC MODE OF BLADE AT ,F9.2,4H CPM )             AMPL 158
      IF(M.EQ.3)  WRITE(6,916)   FNAT                                   AMPL 159
  916 FORMAT (38X,25HSCISSORS MODE OF BLADE AT , F9.2,4H CPM )          AMPL 160
      IF(RRPM(IB).NE.0.0) WRITE(6,  920) FREQPR                         AMPL 161
  920 FORMAT(1H+,76X,3HOR ,F7.3,8H PER REV)                             AMPL 162
      WRITE(6,909)                                                      AMPL 163
  909 FORMAT(/10X,9HBLADE STA,8X,11HDEFLECTIONS ,19X,7HMOMENTS ,20X,     AMPL 164
     1  12HSHEAR FORCES ,12X,5HTWIST ,6X,6HTORQUE /13X,2HIN,16X,2HIN    ,AMPL 165

     2 25X,5HIN-LB,26X,5HLB    ,15X,3HRAD,7X,5HIN-LB,/26X,              AMPL 166
     3  4HVERT,4X,5HHORIZ,13X,4HBEAM,8X,5HCHORD,13X,4HBEAM,6X,5HCHORD / AMPL 167
     4 2X,16(8H******** ))                                              AMPL 168
      L=N1+1                                                            AMPL 169
      DO 220 J=1,N1                                                     AMPL 170
      L=L-1                                                             AMPL 171
      BM=(ZM(L)*A(1)+ZM(31+L)*A(2)+ZM(124+L)+ZM(62+L)*A(3))*SCALE       AMPL 172
      Q=(ZQ(L)*A(1)+ZQ(31+L)*A(2)+ZQ(124+L)+ZQ(62+L)*A(3))*SCALE        AMPL 173
      EL=(ZL(L)*A(1)+ZL(31+L)*A(2)+ZL(124+L)+ZL(62+L)*A(3))*SCALE       AMPL 174
      DE=(ZD(L)*A(1)+ZD(31+L)*A(2)+ZD(124+L)+ZD(62+L)*A(3))*SCALE       AMPL 175
      IF(.NOT.TRSN) GO TO 235                                           AMPL 176
      T =(ZT(L)*A(1)+ZT(31+L)*A(2)+ZT(124+L)+ZT(62+L)*A(3)+ZT( 93+L)*A(4AMPL 177
     1 ))*SCALE                                                         AMPL 178
      BM=BM+ZM(93+L)*A(4)*SCALE                                         AMPL 179
      Q =Q +ZQ(93+L)*A(4)*SCALE                                         AMPL 180
      EL=EL+ZL(93+L)*A(4)*SCALE                                         AMPL 181
      DE=DE+ZD(93+L)*A(4)*SCALE                                         AMPL 182
  235 CONTINUE                                                          AMPL 183
      IF(PHOFF.NE.0.0 .AND. J.LE.LPH  ) T=0.0                           AMPL 184
      QBM=BM*CT(J)+Q*ST(J)                                              AMPL 185
      QCD=Q*CT(J)-BM*ST(J)                                              AMPL 186
      FBM=EL*CT(J)+DE*ST(J)                                             AMPL 187
```

```
      FCD=DE*CT(J)-EL*ST(J)                                             AMPL 188
      WRITE(6,911) J, Z(J), B(1,J), B(2,J), QBM, QCD, FBM, FCD          AMPL 189
 911  FORMAT(6X,I3,F8.2,F14.3,F8.3,F19.0,F12.0,F16.0,F11.0)             AMPL 190
      IF(  TRSN  ) WRITE(6,912) B(3,J), T                               AMPL 191
 912  FORMAT(1H+,97X,F15.3,F13.0)                                       AMPL 192
  220 CONTINUE                                                          AMPL 193
      WRITE(6,914)                                                      AMPL 194
  914 FORMAT(16(8H********))                                            AMPL 195
      WRITF(6,407) GIV,GIH,GIT,(VEC(I),I=1,MM5)                         AMPL 196
  407 FORMAT(/10X,35HGENERALIZED INERTIA (IN-LBF-SEC**2),  5X,4HVERT,F8.AMPL 197
     1 5,5X,5HHORIZ,F8.5,5X,5HTWIST,F8.5,/,10X,24HBOUNDARY CONDITION CHEAMPL 198
     2CK,5X,5E12.5)                                                     AMPL 199
  225 CONTINUE                                                          AMPL 200
      IF(.NOT.INPUN) GO TO 223                                          AMPL 201
C           PUNCH OUT MODE SHAPE                                        AMPL 202
      IF(MODEND.LT.6) CALL CARDS(M,MODEND,NPS,B,1)                      AMPL 203
 223  CONTINUE                                                          AMPL 204
 227  CONTINUE                                                          AMPL 205
      RETURN                                                            AMPL 206
      END                                                               AMPL 207
```

```
      SUBROUTINE CARDS(M,MODEND,NPS,B,IFIRST)                           CARD   1
C*************************************************************          CARD   2
C THIS SUBROUTINE  PUNCHES OUT MODE SHAPES    *                          CARD   3
C*************************************************************          CARD   4
      COMMON /COMA/ JHUB,N1,LOT,POUT,ITLE(10),NAME,DAY,NPG              CARD   5
     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES        CARD   6
      COMMON/ COMC/ WTPL(31),EIB(30),EIC(30),GJ(30),THD(31)             CARD   7
      COMMON/COMB/IRCOL,XRCOL(3),IRPM,CRPM(10),ZBAR(30),EYEB(30),SB(30),CARD   8
     1 EYEC(30),SC(30),SM(31),RB(31),RC(31),TH(31),WT(30)               CARD   9
      COMMON/COMD/CMAT(5,5),SDMNAT(100,3),IPLN(100,3),INODE(100,3),     CARD  10
     1  MM3,MM5,CT(31),ST(31),IB,IST,IBS(3,10,3),IBE(3,10,3)            CARD  11
      COMMON /HINGES/ LCH,LCHP1,LFH,LFHP1 ,CHOFF,FHOFF,FCH,FFH          CARD  12
     1 ,RPMPUN,COLPUN,LPH,LPHP1,PHOFF,FPH,             LOTS             CARD  13
      DIMENSION W(21),RHOIXX(21),RHOIYY(21),B(3,31),PUN(22,3),D(6,3,6)  CARD  14
      DATA W,RHOIXX,RHOIYY/63*0.0/                                      CARD  15
      IF(IFIRST.GT.0) GO TO 35                                          CARD  16
C                                                                       CARD  17
C         RECAST BLADE RUNNING WEIGHT AND INERTIA DATA INTO 20 EQUAL    CARD  18
C                    SEGMENTS                                           CARD  19


      WRITE(7,1) NAME,(ITLE(I),I=1,10)                                  CARD  20
    1 FORMAT(I6,4X,10A6)                                                CARD  21
      W(21)=WTPL(N1)                                                    CARD  22
      RHOIXX(21)=WTPL(N1)*RB(N1)**2                                     CARD  23
      RHOIYY(21)=WTPL(N1)*RC(N1)**2                                     CARD  24
      DELR=Z(N1)/20.0                                                   CARD  25
      J=1                                                               CARD  26
      W(1)=0.0                                                          CARD  27
      RHOIXX(1)=0.0                                                     CARD  28
      RHOIYY(1)=0.0                                                     CARD  29
      DO 15 I=1,20                                                      CARD  30
      STATO=I*DELR                                                      CARD  31
    5 CONTINUE                                                          CARD  32
      IF(Z(J+1).GT.STATO) GO TO 10                                      CARD  33
      W(I)=W(I)+WTPL(J)*ZBAR(J)                                         CARD  34
      RHOIXX(I)=RHOIXX(I)+(EYEB(J)+WTPL(J)*RB(J)**2/386.4)*ZBAR(J)      CARD  35
      RHOIYY(I)=RHOIYY(I)+(EYEC(J)+WTPL(J)*RC(J)**2/386.4)*ZBAR(J)      CARD  36
      J=J+1                                                             CARD  37
      IF(Z(J).EQ.STATO) GO TO 15                                        CARD  38
      GO TO 5                                                           CARD  39
   10 FRAC=STATO-Z(J)                                                   CARD  40
      W(I)=W(I)+FRAC*WTPL(J)                                            CARD  41
      RHOIXX(I)=RHOIXX(I)+FRAC*(EYEB(J)+WTPL(J)*RB(J)**2/386.4)         CARD  42
```

APPENDIX C

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      RHOIYY(I)=RHOIYY(I)+FRAC*(EYEC(J)+WTPL(J)*RC(J)**2/386.4)         CARD  43
      FRAC=ZBAR(J)-FRAC                                                 CARD  44
      W(I+1)=FRAC*WTPL(J)                                               CARD  45
      RHOIXX(I+1)=FRAC*(EYEB(J)+WTPL(J)*RB(J)**2/386.4)                 CARD  46
      RHOIYY(I+1)=FRAC*(EYEC(J)+WTPL(J)*RC(J)**2/386.4)                 CARD  47
      J=J+1                                                             CARD  48
   15 CONTINUE                                                          CARD  49
      DO 20 I=1,20                                                      CARD  50
      W(I)=W(I)/DELR                                                    CARD  51
      RHOIXX(I)=RHOIXX(I)/DELR                                          CARD  52
   20 RHOIYY(I)=RHOIYY(I)/DELR                                          CARD  53
      WRITE(7,30) (W(I),I=1,21)                                         CARD  54
      WRITE(7,30) (RHOIXX(I),I=1,21)                                    CARD  55
      WRITE(7,30) (RHOIYY(I),I=1,21)                                    CARD  56
   30 FORMAT(7F10.5)                                                    CARD  57
      RETURN                                                            CARD  58
   35 CONTINUE                                                          CARD  59
C                                                                       CARD  60
C          RECAST MODE SHAPE INTO 20 EQUAL SEGMENTS                     CARD  61
C                                                                       CARD  62
      MODENO=MODENO+1                                                   CARD  63
      IF(RCOLL(IST).NE.COLPUN.OR.RRPM(IB).NE.RPMPUN) GO TO 702          CARD  64
      DO 40 K=1,3                                                       CARD  65
      PUN(1,K)=B(K,1)                                                   CARD  66
   40 PUN(21,K)=B(K,N1)                                                 CARD  67
      DELR=Z(N1)/20.0                                                   CARD  68
      J=1                                                               CARD  69
      DO 55 I=2,20                                                      CARD  70
      STATO=(I-1)*DELR                                                  CARD  71
   45 CONTINUE                                                          CARD  72
      IF(Z(J+1).GT.STATO) GO TO 50                                      CARD  73
      J=J+1                                                             CARD  74
      IF(Z(J).LT.STATO) GO TO 45                                        CARD  75
      DO 47 K=1,3                                                       CARD  76
   47 PUN(I,K)=B(K,J)                                                   CARD  77
      GO TO 55                                                          CARD  78
   50 FRAC=(STATO-Z(J))/ZBAR(J)                                         CARD  79
      DO 52 K=1,3                                                       CARD  80
   52 PUN(I,K)=B(K,J)+FRAC*(B(K,J+1)-B(K,J))                            CARD  81
   55 CONTINUE                                                          CARD  82
      DO 60 I=1,21                                                      CARD  83
      PUN(I,1)=PUN(I,1)/12.0                                            CARD  84
      PUN(I,2)=PUN(I,2)/12.0                                            CARD  85
   60 PUN(I,3)=PUN(I,3)*57.2958                                         CARD  86


      FNAT=SGMNAT(NPS,M)                                                CARD  87
      IF(RRPM(IB).NE.0.0) FNAT=FNAT/RRPM(IB)                            CARD  88
      PUN(22,1)=FNAT                                                    CARD  89
      IF(M.EQ. 1)PUN(22,2)= 1.                                          CARD  90
      IF(M.EQ. 2)PUN(22,2)=-1.                                          CARD  91
      IF(M.EQ. 3)PUN(22,2)=0.                                           CARD  92
      PUN(22,3)=.02                                                     CARD  93
      IF(ABS(PUN(22,1)-1.0).LE.0.05.AND.M.EQ.2) GO TO 200               CARD  94
      GO TO 460                                                         CARD  95
  200 DO 450 KN=1,22                                                    CARD  96
      PUN(KN,3)=0.0                                                     CARD  97
  450 CONTINUE                                                          CARD  98
  460 CONTINUE                                                          CARD  99
      ISW=PUN(22,2)                                                     CARD 100
      DO  600  KKK=1,21,2                                               CARD 101
      KKKP1=KKK+1                                                       CARD 102
      WRITE(7,27) (PUN(KKK,I),I=1,3),(PUN(KKKP1,I),I=1,3),NAME,         CARD 103
     1 MODENO,ISW                                                       CARD 104
  600 CONTINUE                                                          CARD 105
      RETURN                                                            CARD 106
  702 CONTINUE                                                          CARD 107
C          PUNCH CYCLIC DETUNING CARD FOR EACH MODE SHAPE               CARD 108
```

```
      IF(IB.EQ.1.AND.IST.EQ.1)D(1,M,MODENO)=SOMNAT(NPS,M)               CARD 109
      IF(IB.EQ.1.AND.IST.EQ.3)D(2,M,MODENO)=SOMNAT(NPS,M)               CARD 110
      IF(IB.EQ.3.AND.IST.EQ.1)D(3,M,MODENO)=SOMNAT(NPS,M)               CARD 111
      IF(IB.EQ.3.AND.IST.EQ.3)D(4,M,MODENO)=SOMNAT(NPS,M)               CARD 112
   27 FORMAT(6F10.6,10X,I6,2I2)                                         CARD 113
    7 CONTINUE                                                          CARD 114
      L=1                                                               CARD 115
      IF(BLADES.GT.2.0) L=3                                             CARD 116
      DO 8 IM=L,3                                                       CARD 117
      DO 8 IJ=1,6                                                       CARD 118
      D(5,IM,IJ)=(RCOLL(3)-RCOLL(1))*.5                                 CARD 119
      D(6,IM,IJ)=(RRPM(3)-RRPM(1))*.5                                   CARD 120
      IF(IM.EQ.1) ISW=1                                                 CARD 121
      IF(IM.EQ.2) ISW=-1                                                CARD 122
      IF(IM.EQ.3) ISW =0                                                CARD 123
      IF(IB.EQ.3.AND.IST.EQ.3.AND.MODENO.EQ.6.AND.M.EQ.3)               CARD 124
     1WRITE(7,28) (D(I,IM,IJ),I=1,6),RCOLL(2),RRPM(2),NAME,IJ,ISW       CARD 125
   28 FORMAT(6F10.2,2F5.0,I6,2I2)                                       CARD 126
    8 CONTINUE                                                          CARD 127
      RETURN                                                            CARD 128
      END                                                               CARD 129
```

```
      SUBROUTINE COEF( IA  ,    DET, IMAX, SQSOM, PP)                   COEF   1
C****************************************************************       COEF   2
C THIS SUBROUTINE CALCULATES THE DEFLECTION OF EACH STATION     *       COEF   3
C AS A FUNCTION OF DEFLECTIONS AT THE ROTOR BLADE TIP FOR       *       COEF   4
C                                                               *       COEF   5
C                          IA=1 FOR COLLFCTIVE MODES            *       COEF   6
C                          IA=2 FOR CYCLIC MODES                *       COEF   7
C                          IA=3 FOR SCISSORS MODES              *       COEF   8
C                                                               *       COEF   9
C       IMAX=NO OF FREQUENCIES TO BE CALCULATED                 *       COEF  10
C       SQSOM(1 TO IMAX) CONTAINES SQUARES OF FREQUENCIES       *       COEF  11
C                                                               *       COEF  12
C       DET=.TRUE. USED TO FIND MODE SHAPE FOR KNOWN NATURAL FREQ. *    COEF  13
C                                                               *       COEF  14
C       DET=.FALSE. USED TO FIND THE DETERMINANTS OF THE BOUNDARY *     COEF  15
C       CONDITION MATRICIES                                     *       COEF  16
C****************************************************************       COEF  17
      COMMON /COMA/ JHUB,N1,LOT,POUT,ITLE(10),NAME,DAY,NPG              COEF  18

     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES        COEF  19
      COMMON/COMB/IRCOL,XRCOL(3),IRPM,CRPM(10),ZBAR(30),EYEB(30),SB(30),COEF  20
     1 EYEC(30),SC(30),SM(31),RB(31),RC(31),TH(31),WT(30)               COEF  21
      COMMON/COMD/CMAT(5,5),SOMNAT(100,3),IPLN(100,3),INODE(100,3),     COEF  22
     1  MM3,MM5,CT(31),ST(31),IB,IST,IBS(3,10,3),IBE(3,10,3)            COFF  23
      COMMON/ H/ VMX(30),VQX(30),VMY(30),F(30),FTX(30),FTY(30),SX(30),  COEF  24
     1 SY(30),OMEGA2,FTH(30)                                            COEF  25
      COMMON/ COMJ/ KIP,MIP,KOP,MOP,KBETA,KPSI,KT,KC                    COEF  26
      COMMON /HINGES/ LCH,LCHP1,LFH,LFHP1 ,CHOFF,FHOFF,FCH,FFH          COEF  27
     1 ,RPMPUN,COLPUN,LPH,LPHP1,PHOFF,FPH,         LOTS                 COEF  28
      COMMON/COMT/ EYB(31),EYC(31),EYR(31),EMRB(31),EMRC(31),EMRR(31),  COEF  29
     1 EMRX(31),EMRY(31),EMBBW(31),EMPPW(31),THHO(31),EMBPW(31)         COEF  30
      COMMON /COME/ ZB(155), ZX(155), ZQ(155), ZL(155), ZS(155), ZY(155)COEF  31
     *,ZM(155), ZD(155), ZH(155), ZT(155)                               COEF  32
      REAL KIP,MIP,KOP,MOP,KBETA,KPSI,KT,KC                             COEF  33
      LOGICAL DET, TRSN                                                 COEF  34
      DIMENSION SQSOM(1),PP(2),IPIVOT(5),IWK(10)                        COEF  35
C CALCULATE DEFLECTION COEFFICIENTS *                                   COEF  36
      DO 220 II=1,IMAX                                                  COEF  37
      SOMS=SQSOM(II)                                                    COEF  38
      SOMSY=SOMS+OMEGA2                                                 COEF  39
      ZM(1)=SOMSY*EMBBW(N1)                                             COEF  40
      ZQ(1)=SOMS*EMBPW(N1)                                              COEF  41
```

```
      ZM(32)=SOMSY*EMBPW(N1)
      ZQ(32)=SOMS*EMPPW(N1)
      ZD(63)=SOMSY*SM(N1)
      ZT(63)=SOMSY*EMRY(N1)
      ZM(94)=-OMEGA2*Z(N1)*EMRX(N1)
      ZQ(94)=-OMEGA2*Z(N1)*EMRY(N1)
      ZL( 94)=SOMS*EMRY(N1)
      ZD( 94)=SOMSY*EMRY(N1)
      ZT( 94)=SOMS*EYR(N1)+OMEGA2*THHO(N1)
      ZL(125)=SM(N1)*SOMS
      ZT(125)=SOMS*EMRX(N1)
      DO 135 J=1,5
      IF(.NOT.TRSN .AND. J.EQ.4) GO TO 135
      M=N1
      L1=J*31-30
      DO 130 I=2,N1
      L2=L1
      L1=L1+1
      M=M-1
      ZOVTO=ZBAR(M)/2.0
      ZSQSX=ZBAR(M)**2/6.0
      VSHEAR=ZL(L2)-F(M)*ZB(L2)
      HSHEAR=ZD(L2)-F(M)*ZS(L2)
      DH=WT(M)*(HSHEAR*SY(M) - VSHEAR*SX(M) + FTY(M)*ZS(L2) + FTX(M)*
     1 ZB(L2) + ZH(L2)*SX(M)*FTH(M) - ZT(L2))
      DB=-ZOVTO*(VMY(M)*VSHEAR+VMX(M)*HSHEAR) - VMY(M)*ZM(L2) - VMX(M)*
     1 ZQ(L2) - ZH(L2)*(FTY(M)*VMX(M)+FTX(M)*VMY(M)-ZOVTO*FTH(M)*VMY(M))
      DS=-ZOVTO*(VMX(M)*VSHEAR+VQX(M)*HSHEAR) - VMX(M)*ZM(L2) - VQX(M)*
     1 ZQ(L2) - ZH(L2)*(FTY(M)*VQX(M)+FTX(M)*VMX(M)-ZOVTO*FTH(M)*VMX(M))
      DY=ZSQSX*(VMY(M)*VSHEAR+VMX(M)*HSHEAR) - ZBAR(M)*ZB(L2) + VMY(M)*
     1 ZOVTO*ZM(L2) + VMX(M)*ZOVTO*ZQ(L2) + ZH(L2)*(FTY(M)*ZOVTO*VMX(M)
     2 +FTX(M)*ZOVTO*VMY(M)-FTH(M)*ZSQSX*VMY(M)) + SX(M)*DH
      DX=ZSQSX*(VQX(M)*HSHEAR+VMX(M)*VSHEAR) - ZBAR(M)*ZS(L2) + VMX(M)*
     1 ZOVTO*ZM(L2) + VQX(M)*ZOVTO*ZQ(L2) + ZH(L2)*(FTY(M)*ZOVTO*VQX(M)
     2 +FTX(M)*ZOVTO*VMX(M)-FTH(M)*ZSQSX*VMX(M)) - SY(M)*DH
      IF(BL[illegible]ES.EQ.2.0) GO TO 120
      IF(FHOFF.EQ.0.0 .AND.CHOFF.EQ.0.0) GO TO 120
      IF(Z(M).LT.FHOFF) DB=-ZB(L2)
      IF(Z(M).LT.FHOFF) DY=-ZY(L2)
      IF(Z(M).LT.CHOFF) DS=-ZS(L2)
      IF(Z(M).LT.CHOFF) DX=-ZX(L2)
  120 ZB(L1)=ZB(L2)+DB
      ZS(L1)=ZS(L2)+DS
      ZY(L1)=ZY(L2)+DY


      ZX(L1)=ZX(L2)+DX
      ZH(L1)=ZH(L2)+DH
      ZL(L1)=ZL(L2)+SOMS*(EMRX(M)*ZH(L1)+SM(M)*ZY(L1))
      ZD(L1)=ZD(L2)+SOMSY*(EMRY(M)*ZH(L1)+SM(M)*ZX(L1))
      ZM(L1)=ZM(L2)+F(M)*DY+ZBAR(M)*ZL(L2)+SOMSY*(EMBPW(M)*ZS(L1)+EMBBW(
     1 M)*ZB(L1))-OMEGA2*Z(M)*EMRX(M)*ZH(L1)
      ZQ(L1)=ZQ(L2)+F(M)*DX+ZBAR(M)*ZD(L2)+SOMS*(EMPPW(M)*ZS(L1)+EMBPW(M
     2)*ZB(L1))-OMEGA2*Z(M)*EMRY(M)*ZH(L1)
      ZT(L1)=ZT(L2)+FTH(M)*DY+SOMSY*EMRY(M)*ZX(L1)+SOMS*EMRX(M)*ZY(L1)
     2 +(SOMS*EYR(M)+OMEGA2*THHO(M))*ZH(L1)
  130 CONTINUE
  135 CONTINUE
C CALCULATE ROOT CONDITIONS PER INDIVIDUAL TIP DEFLECTIONS
      L=N1-31
      DO 200 J=1,MM5
      L=L+31
      IF(.NOT.TRSN) GO TO 193
C  TORSION COMPONENT BOUNDARY CONDITION
      CMAT(5,J)=ZT(L-LPH)-FPH*(ZT(L-LPH)-ZT(L-LPHP1))-KC*(ZH(L-LPH)-FPH*
     1(ZH(L-LPH)-ZH(L-LPHP1)))
      GO TO 197
  193 IF(J.EQ.4) L=L+31
```

APPENDIX C

```
  197 CONTINUE
      IF(IA-2) 400,500,600
  400 CONTINUE
C  BOUNDARY CONDITIONS FOR COLLECTIVE MODES
      CMAT(1,J)=ZL(L)-(KOP-SOMS*MOP)*ZY(L)
      CMAT(2,J) = ZX(L)
      CMAT(3,J) = ZB(L)
      CMAT(4,J)=ZQ(L)-KT*ZS(L)
      GO TO     200
  500 CONTINUE
C  BOUNDARY CONDITIONS FOR CYCLIC MODES
      CMAT(1,J) = ZY(L)
      CMAT(2,J)=ZD(L)-(KIP-SOMS*MIP)*ZX(L)
      CMAT(3,J)=ZM(L)-KBETA*ZB(L)
      CMAT(4,J) = ZS(L)
      GO TO     200
  600 CONTINUE
C     BOUNDARY CONDITIONS FOR SCISSORS MODE
      CMAT(1,J)=ZY(L-LFH)-FFH*(ZY(L-LFH)-ZY(L-LFHP1))
      CMAT(2,J)=ZX(L-LCH)-FCH*(ZX(L-LCH)-ZX(L-LCHP1))
      CMAT(3,J) = ZB(L)
      CMAT(4,J) = ZS(L)
      IF(TRSN .AND. BLADES.EQ. 2.0)
     1              CMAT(5,J)=ZH(L-LPH)-FPH*(ZH(L-LPH)-ZH(L-LPHP1))
      IF(FHOFF.GT.0.0) CMAT(3,J)=ZM(L-LFH)-FFH*(ZM(L-LFH)-ZM(L-LFHP1))
     1 -KBETA*(ZB(L-LFH)-FFH*(ZB(L-LFH)-ZB(L-LFHP1)))
      IF(CHOFF.GT.0.0) CMAT(4,J)=ZQ(L-LCH)-FCH*(ZQ(L-LCH)-ZQ(L-LCHP1))
     1 -KPSI*(ZS(L-LCH)-FCH*(ZS(L-LCH)-ZS(L-LCHP1)))
 200  CONTINUE
      IF(DET)  RETURN
      CALL MATINV(5,MM5,CMAT,0,BBB,0,DETERM,ISCALE,IPIVOT,IWK)
      PP(II)=DETERM*10.0**(100*ISCALE)
  220 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INPT(FLIST)
C*************************************************************
C         THIS SUBROUTINE READS INPUT DATA
C*************************************************************


      COMMON /COMA/ JHUB,N1,LOT,POUT,ITLE(10),NAME,DAY,NPG
     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES
      COMMON/COMB/IRCOL,XRCOL(3),IRPM,CRPM(10),ZBAR(30),EYEB(30),SB(30),
     1 EYEC(30),SC(30),SM(31),RB(31),RC(31),TH(31),WT(30)
      COMMON/COMC/WTPL(31),EIB(30),EIC(30),GJ(30),THO(31)
      COMMON /HINGES/ LCH,LCHP1,LFH,LFHP1 ,CHOFF,FHOFF,FCH,FFH
     1 ,RPMPUN,COLPUN,LPH,LPHP1,PHOFF,FPH,            LOTS
      COMMON/ COMJ/ KIP,MIP,KOP,MOP,KBETA,KPSI,KT,KC
      REAL KIP,MIP,KOP,MOP,KBETA,KPSI,KT,KC
      LOGICAL LTWS,      LOT,POUT,TRSN,LOTS,MODPLOT,INPUN
      DIMENSION M(6)
      NAMELIST /INPUT/ NAME,KT,MOP,MIP,KIP,KOP,KBETA,KPSI,
     1,C,N,JHUB,AZBAR,TWIST,BLADES,CHORD,CHOFF,FHOFF,PHOFF,PLAST,IRPM,
     2RRPM,IRCOL,RCOLL,Z,EIB,EIC,WTPL,THO,EYEB,EYEC,GJ,SB,SC,RB,RC
      DATA CVR/0.0174533/
C
C         READ PROGRAM OPTIONS CARD
C
  10  READ (5,901) M
  901 FORMAT(A4,6X,A4,6X,A4,6X,A4,6X,A4,6X,A4)
      IF(EOF(5)) 320,30
  30  CONTINUE
      K = 0
```

APPENDIX C

```
      MODPLOT=.FALSE.
      LOTS=.FALSE.
      LOT = .FALSE.
      POUT = .FALSE.
      INPUN=.FALSE.
      TRSN = .FALSE.
      LTWS = .FALSE.
      DO 965 I=1,6
      IF(M(I).EQ.4HDECK) K=1
      IF(M(I).EQ.4HNAME) K=-1
      IF(M(I).EQ.4HMODE) POUT=.TRUE.
      IF(M(I).EQ.4HALLM) LOTS=.TRUE.
      IF(M(I).EQ.4HALLM) POUT=.TRUE.
      IF(M(I).EQ.4HPLOT) LOT=.TRUE.
      IF(M(I).EQ.4HPUNC) INPUN=.TRUE.
      IF(M(I).EQ.4HTORS) TRSN=.TRUE.
      IF(M(I).EQ.4HNLTW) LTWS=.TRUE.
      IF(M(I).EQ.4HSHAP) MODPLOT=.TRUE.
  965 CONTINUE
      IF(K.EQ.0) GO TO 319
C
C       IF K = 0      NO INPUT SELECTED
C          K = 1    READ IN DECK
C          K = -1    READ NAMELIST
C
      NPG=0
      IF(K.EQ.1) GO TO 20
C READ CHANGES TO PREVIOUS CASE BY NAMELIST INPUT
      READ(5,INPUT)
      N1=N+1
      GO TO 55
C         READ INPUT DATA DECK
   20 CONTINUE
      READ(5,902) NAME,(ITLE(I),I=1,10)
  902 FORMAT(I6,4X,10A6)
      READ(5,904) KT,MOP,MIP,KOP,KIP,KBETA,KPSI,KC
  904 FORMAT(8F10.0)
      READ(5,905) N,JHUB,AZBAR,TWIST,BLADES,CHOFF,FHOFF,PHOFF,PLAST
  905 FORMAT(2I5,6F5.0,F10.0)
      READ(5,2) IRPM,(RRPM(I),I=1,IRPM)
      READ(5,2) IRCOL,(RCOLL(I),I=1,IRCOL)
    2 FORMAT(I5,10F5.0)
      N1=N+1
      IF(AZBAR.EQ.0.0) READ(5,904) (Z(I),I=2,N1)


      READ(5,904) (EIB(I),I=1,N)
      READ(5,904) (EI (I),I=1,N)
      READ(5,904) (WTPL(I),I=1,N1)
      IF(LTWS) READ(5,904) (THD(I),I=1,N1)
      READ(5,904) (EYEB(I),I=1,N)
      READ(5,904) (EYEC(I),I=1,N)
      READ(5,904) (   GJ(I),I=1,N)
      READ(5,904) (SB(I), I=1,N)
      READ(5,904) (SC(I), I=1,N)
      READ(5,904)(RB(I),I=1,N1)
      READ(5,904)(KC(I),I=1,N1)
 55   JHUB1=JHUB+1
      Z(1)=0.0
      IF(AZBAR.EQ.0.0) GO TO 140
      DO 144 I=1,N
      Z(I+1)=Z(I)+AZBAR
      ZBAR(I)=AZBAR
  144 CONTINUE
      GO TO 145
  140 CONTINUE
      DO 147 I=1,N
      ZBAR(I)=Z(I+1)-Z(I)
```

```
147 CONTINUE
145 CONTINUE
    IF(LTWST) GO TO 160
    DO 150 I=1,JHUB1
150 THD(I)=0.0
    TWRATE=TWIST/Z(N1)
    DO 151 I=JHUB1,N
151 THD(I+1)=THD(I)+ZBAR(I)*TWRATE
160 CONTINUE
    DO 165 I=1,N1
165 TH(I)=THD(I)*CVR
    IF(INPUN) CALL CARDS(N,MODENO,NPS,8,0)
    CALL START
    RETURN
319 CONTINUE
    WRITE(6,321)
321 FORMAT(4X,47HNEITHER DECK OR NAMELIST INPUT OPTIONS SELECTED  )
320 CONTINUE
    CALL CALPLT(0.0,0.0,999)
    STOP
    END
```

```
      SUBROUTINE PLOUT(PLAST)
C*************************************
C THIS SUBROUTINE PRODUCES FAN PLOTS *
C*************************************
      COMMON /COMA/ JHUB,N1,LOT,POUT,ITLE(10),NAME,DAY,NPG
     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES
      COMMON/COMB/IRCOL,XRCOL(3),IRPM,CRPM(10),ZBAR(30),EYEB(30),SB(30),
     1 EYEC(30),SC(30),SM(31),RB(31),RC(31),TH(31),WT(30)
      COMMON/COMD/CMAT(5,5),SOMMAT(100,3),IPLN(100,3),INODE(100,3),
     1  MM3,MM5,CT(31),ST(31),IB,IST,IBS(3,10,3),IBE(3,10,3)
      LOGICAL TRSN
      DIMENSION XM(4),YM(4)
      ISIZE=1
      CALL CALPLT(0.5,0.,-3)
      XM(1)=0.0
      XM(2)=RRPM(IRPM)+100.
      XMAX = 4.
      XMIN=.5+IFIX((8.1-XMAX)*.5)


      XM(3)=0.0
      IRPMC=RRPM(IRPM)
      DO 20 IY=1,10
      IF((IY*200-IRPMC).GT.0) GO TO 30
   20 CONTINUE
   30 CONTINUE
      IF(IY.EQ.3 .OR. IY.EQ.6 .OR. IY.EQ.9) IY=IY+1
      IF(IY.EQ.7) IY=IY+1
      XM(4)=IY*50.0
      YM(1)=1.0
      YM(2)=PLAST
      MAXY=MIN0(8,IFIX(YM(2)*.01+0.9)-IFIX(YM(1)*.01+0.9))
      YMAX=MAXY
      YM(3)=0.0
      YM(4)=4.*XM(4)
    3 DO 435 I=1,3
      IF(I.NE.3.AND.BLADES.NE.2.0) GO TO 435
      NBLD=BLADES
      CALL AXES(XMIN,            1.0,90.0,YMAX,YM(3),YM(4),1.0,0.0,
     121HNATURAL FREQUENCY,CPM,0.15,21)
      CALL CALPLT(0.0,1.25,-3)
      IF(I.EQ.1) CALL NOTATE(3.7,0.10+YMAX,.125,15HCOLLECTIVE MODE,0.,15
     1)
```

```
      IF(I.EQ.2) CALL NOTATE(3.9,0.10+YMAX,.125,11HCYCLIC MODE,0.,11)
      IF(I.EQ.3) CALL NOTATE(3.8,0.10+YMAX,.125,13HSCISSORS MODE,0.,13)
      CET=3.56-.258*FLOAT(IRCOL)
      CALL NOTATE(CET,8.5,.1,17HROOT COLLECTIVE =,0.,17)
      CET=CET+1.657
      DO 5 J=1,IRCOL
      CALL NUMBER(CET, 8.5, .1, RCOLL(J), 0., 1)
      CET=CET+.429
      IF(J.GE.IRCOL)  GO TO 5
      CALL NOTATE(CET,8.5,.1,1H,,0.,1)
      CET=CET+.086
 5    CONTINUE
      CALL NOTATE(CET,8.5,.1,4HDEG.,0.,5)
      IF(.NOT.TRSN) GO TO 4
      CALL PNTPLT(2.36,8.85,3,ISIZE)
      CALL NOTATE(2.93,8.80,0.1,7HTORSION,0.0,7)
    4 CONTINUE
      CALL PNTPLT(2.36,9.05,2,ISIZE)
      CALL NOTATE(2.68,9.0,.1,11HHORIZ PLANE,0.,11)
      CALL PNTPLT(2.36,9.25,1,ISIZE)
      CALL NOTATE(2.76,9.2,.1,10HVERT PLANE,0.,10)
      CALL NOTATE(2.25,9.4,.1,17HSYM MAX AMPLITUDE,0.,18)
      CALL CALPLT(2.25,9.375,3)
      CALL CALPLT(2.48,9.375,2)
      CALL CALPLT(2.68,9.375,3)
      CALL CALPLT(3.77,9.375,2)
      DO 6 KK=1,10
      XX=2.0+(KK-1)*0.58
      CALL NOTATE(XX,9.8,0.1,ITLE(KK),0.0,6)
    6 CONTINUE
      CALL NOTATE(2.38,10.00,.15,33HCOUPLED BLADE NATURAL FREQUENCIES,0.
     1,33)
      XNAME=NAME
      CALL NOTATE(0.5,10.0,0.1,4HCASE,0.0,4)
      CALL NUMBER(1.0,10.0,0.1,XNAME,0.0,0)
      CALL NOTATE(7.43,10.00,.1,10H(          ),0.,10)
      CALL NOTATE(7.456,10.00,.1,DAY   ,0.,9)
      CALL CALPLT(XMIN,-0.25,-3)
      CALL AXES(.0,.0,.0,XMAX,XM(3),XM(4),1.,.0,15HROTOR SPEED,RPM,.15,
     1-15)
      DO 200 IFF=1,8
      YSPOT=IFF
      IF(I.NE.1) GO TO 210
C    THIS PATH FOR COLLECTIVE MODES


      IF(MOD(IFF,NBLD).NE.0) GO TO 210
      GO TO 236
  210 IF(I.NE.2) GO TO 220
C   THIS PATH FOR CYCLIC MODES
      IF(MOD(IFF,NBLD).EQ.0) GO TO 220
      GO TO 236
  220 IF(I.NE.3) GO TO 200
C   THIS PATH FOR SCISSOR MODES
  236 CONTINUE
C
C        PLOT 1, 2, 3 PER REV LINES ON PLOT
C
      XPOS=XMAX
      YPOS=YSPOT
      IF(YSPOT.LE.YMAX) GO TO 235
      YPOS=YMAX
      XPOS=YPOS/(0.25*YSPOT)
  235 CONTINUE
      CALL DASHLN(0.0,0.0,XPOS,YPOS ,0.2)
      CALL NUMBER(XPOS,YPOS-0.05,0.1,YSPOT,0.0,-1)
      CALL NOTATE(XPOS+0.0857,YPOS-0.05,0.1,4H/REV,0.0,4)
      CALL CALPLT(0,0,3)
```

```
  200 CONTINUE                                                          PLOU 108
C         PLOT ROTOR NATURAL FREQUENCY                                  PLOU 109
      DO 410 IB=1,IRPM                                                  PLOU 110
      DO 410 IST=1,IRCOL                                                PLOU 111
      J1=IBS(IST,IB,I)                                                  PLOU 112
      J2=IBE(IST,IB,I)                                                  PLOU 113
      IF(J2.LT.J1) GO TO 410                                            PLOU 114
      DO 400 J=J1,J2                                                    PLOU 115
      K=IPLN(J,I)                                                       PLOU 116
      X=RRPM(IB)/XM(4)-XM(3)                                            PLOU 117
      Y=SOMNAT(J,I)/YM(4)-YM(3)                                         PLOU 118
      CALL PNTPLT(X,Y,K,ISIZE)                                          PLOU 119
 400  CONTINUE                                                          PLOU 120
 410  CONTINUE                                                          PLOU 121
      CALL CALPLT(9.0-XMIN,-1.0,-3)                                     PLOU 122
  435 CONTINUE                                                          PLOU 123
      CALL NFRAME                                                       PLOU 124
      RETURN                                                            PLOU 125
      END                                                               PLOU 126
```

```
      SUBROUTINE START                                                  STAR   1
C****************************************************************       STAR   2
C         THIS SUBROUTINE PRINTS OUT INPUT DATA AND RECASTS DATA TO     STAR   3
C                 LUMPED MASS AT STATION BOUNDARY REPRESENTATION        STAR   4
C****************************************************************       STAR   5
      COMMON /COMA/ JHM,B,NI,LOT,POUT,ITLE(10),NAME,DAY,NPG             STAR   6
     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES        STAR   7
      COMMON/COMB/IRCOL,XRCOL(3),IRPM,CRPM(10),ZBAR(30),EYEB(30),SB(30),STAR   8
     1 EYEC(30),SC(30),SP(31),RB(31),RC(31),TH(31),WT(30)               STAR   9
      COMMON/ COMC/ WTPL(31),EIB(30),EIC(30),GJ(30),THD(31)             STAR  10
      COMMON /HINGES/ LCH,LCHP1,LFH,LFHP1 ,CHOFF,FHOFF,FCH,FFH          STAR  11
     1 ,RPMPUN,COLPUN,LPH,LPHP1,PHOFF,FPH,           LOTS               STAR  12
      COMMON/ COMJ/ KIP,MIP,KOP,MOP,KBETA,KPSI,KT,KC                    STAR  13
      COMMON/COMT/ EYS(31),EYC(31),EYR(31),EMRA(31),EMRC(31),EMRR(31),  STAR  14
     1 EMRX(31),EMRY(31),EMBBW(31),EMPPW(31),THHD(31),EMBPW(31)         STAR  15
      REAL KIP,MIP,KOP,MOP,KBETA,KPSI,KT,KC                             STAR  16
      LOGICAL TRSN                                                      STAR  17
      DIMENSION CF(30)                                                  STAR  18
      DATA CVR,CVRPS/ 0.0174533,0.1047198/                              STAR  19
            R=Z(N1)                                                     STAR  20


      RPMPUN=RRPM(IRPM)                                                 STAR  21
      IF(IRPM.GT.2) RPMPUN=RRPM(IRPM-1)                                 STAR  22
      COLPUN=RCOLL(IRCOL)                                               STAR  23
      IF(IRCOL.GT.2) COLPUN=RCOLL(IRCOL-1)                              STAR  24
C      COMPUTE BLADE ANGLE AT END OF EACH SEGMENT    RCOLL IN DEGREES   STAR  25
C          XCOL IN RADIANS                                              STAR  26
      DO 710 I=1,IRCOL                                                  STAR  27
 710  XRCOL(I)=RCOLL(I)*CVR                                             STAR  28
      DO 750 I=1,IRPM                                                   STAR  29
 750  CRPM(I)=RRPM(I)*CVRPS                                             STAR  30
C         COMPUTE BLADE STATION FOR FLAPPING HINGE, CHORDWISE HINGE, AND STAR  31
C             PITCH HORN OFFSET                                         STAR  32
      SUMZ=0.0                                                          STAR  33
      DO 874 I=1,N                                                      STAR  34
      K=I                                                               STAR  35
      SUMZ=SUMZ+ZBAR(I)                                                 STAR  36
      IF(SUMZ.GT.PHOFF) GO TO 875                                       STAR  37
  874 CONTINUE                                                          STAR  38
  875 CONTINUE                                                          STAR  39
      LPHP1=K                                                           STAR  40
      LPH=K-1                                                           STAR  41
      FPH=(PHOFF-SUMZ+ZBAR(LPHP1))/ZBAR(LPHP1)                          STAR  42
      SUMZ=0.0                                                          STAR  43
```

```
      DO 876 I=1,N
      K=I
      SUMZ=SUMZ+ZBAR(I)
      IF(SUMZ.GT.FHOFF) GO TO 877
  876 CONTINUE
  877 CONTINUE
      LFHP1=K
      LFH=K-1
      FFH=(FHOFF-SUMZ+ZBAR(LFHP1))/ZBAR(LFHP1)
      SUMZ=0.0
      DO 878 I=1,N
      K=I
      SUMZ=SUMZ+ZBAR(I)
      IF(SUMZ.GT.CHOFF) GO TJ 879
  878 CONTINUE
  879 CONTINUE
      LCHP1=K
      LCH=K-1
      FCH=(CHOFF-SUMZ+ZBAR(LCHP1))/ZBAR(LCHP1)
C      COMPUTE CENTRIFUGAL FORCE,BLADE MASS,AND BLADE FLAPPING INERTIA
      FLPINT=WTPL(N1)*(R-FHOFF)**2
      TOTMAS=WTPL(N1)
      DO 85 I=1,N
      CF(I)=0.0
      DO 87 IJ=I,N
      CF(I)=CF(I)+WTPL(IJ)*ZBAR(IJ)*(Z(IJ)+Z(IJ+1))*.5
   87 CONTINUE
      CF(I)=(CF(I)+WTPL(N1)*R)*CVRPS*CVRPS/386.4
      IF(I.LE.LFH) GO TO 85
      TOTMAS=TOTMAS+WTPL(I)*ZBAR(I)
      FLPINT=FLPINT+(WTPL(I)*ZBAR(I)*(Z(I)+Z(I+1)-2.*FHOFF)**2)/4.0
   85 CONTINUE
      FLPINT=FLPINT/(32.2*144.)
C PRINT OUT INPUT *
      NPG=NPG+1
      WRITE(6,905) NPG,DAY,NAME,(ITLE(J),J=1,10)
  905 FORMAT(1H1,18X,4HPAGE,I3,25X,34HMYKLESTAD ANALYSIS   NASA TM 78670
     1 ,25X,A10,//10X,5HCASE ,I6,23X,10A6/)
      WRITE(6,906)
      DO 904 I=1,N
      WRITE(6,910) I,ZBAR(I),EIB(I),EIC(I),WTPL(I),THD(I),CF(I),EYEB(I),
     1EYEC(I),RB(I),RC(I)
  910 FORMAT(I4,F8.2,2E11.4,F8.4,F8.2,E11.4,1X,2E13.4,29X,2F8.3)
      IF(.NOT.TRSN) GO TO 904


      WRITE(6,908) GJ(I),SB(I),SC(I)
  908 FORMAT(1H+,87X, E13.4,2F8.3)
  904 CONTINUE
      WRITE(6,909) WTPL(N1),THD(N1),RB(N1),RC(N1)
  909 FORMAT(3X,3HTIP,27X,F9.4,F8.2,67X,2F8.3/)
  906 FORMAT(6X,6HSEGMNT,7X,8H   EI     ,9X,5HWT/IN,4X,5HTWIST,5X,4HC.F.,
     1 10X,3HIBB,10X,3HICC,8X,8H   GJ     ,3X,12HSHEAR OFFSET,6X,11HC.G. O
     2FFSET/7X,4HLNTH,6X,4HBEAM,5X,5HCHORD,13X,6H(INBD),4X,6H(INBD),44X
     3,4HBEAM,4X,5HCHORD,3X,4HBEAM,3X,5HCHORD/7X,4H  IN,6X,14H-- LB-IN**
     42 --,5X,5HLB/IN,5X,3HDEG,3X,9HLB/RPM**2,9X,15HIN-LB-SEC**2/IN, 8X,
     58HLB-IN**2,5X,2HIN,6X,2HIN,6X,2HIN,6X,2HIN)
      WRITE(6,882) BLADES,R,TOTMAS,JHUB,FLPINT
  882 FORMAT(10X,F8.0,7H BLADES,18X,6HRADIUS,F8.2,3H IN,25X,12HBLADE WEI
     1GHT,F6.2,3H LB/10X,I8,13H HUB SEGMENTS,54X,18HBLADE FLAP INERTIA ,
     2 F8.2,11H SLUG-FT**2/)
      WRITE(6,883) FHOFF,KBETA,MOP,KOP,CHOFF,KPSI,MIP,KIP,PHOFF,KC,KT
  883 FORMAT(1X,13HFLAP HNG OFST,F8.2,3H IN,5X,11HFLAP SPRING,E9.3,10H I
     1N-LB/RAD,5X,16HHUB O.P. INERTIA,F6.3,13H LB-SEC**2/IN,5X,
     2 14HHUB O.P. STIFF,E9.3,6H LB/IN/1X,13HLAG HNG OFST ,F8.2,3H IN,
     35X,11HLAG SPRING ,E9.3,10H IN-LB/RAD,5X,16HHUB I.P. INERTIA,F6.3,1
     43H LB-SEC**2/IN,5X,14HHUB I.P. STIFF,E9.3,6H LB/IN/1X,15HPITCH HOR
     5N OFST,F6.2,3H IN,5X,11HCNTRL STIFF,E9.3,10H IN-LB/RAD/25X,
```

```
     6 17HMAST TORSION SPR ,E9.3,10H IN-LB/RAD/)
C CALCULATE COEFFICIENTS INDEPENDENT OF COLLECTIVE ANGLE AND ROTOR RPM
      DO 305 I=2,N
      SEGIN=0.5*WTPL(I-1)*ZBAR(I-1)/386.4
      SEGOUT=0.5*WTPL(I)*ZBAR(I)/386.4
      SM(I)=SEGIN+SEGOUT
      EMRB(I)=SEGIN*RB(I-1)+SEGOUT*RB(I)
      EMRC(I)=SEGIN*RC(I-1)+SEGOUT*RC(I)
      EMRR(I)=SEGIN*RB(I-1)*RC(I-1)+SEGOUT*RB(I)*RC(I)
      EYB(I)=0.5*(EYEB(I-1)*ZBAR(I-1)+EYEB(I)*ZBAR(I))+SEGIN*(ZBAR(I-1)
     1**2/12.0+RB(I-1)**2)+SEGOUT*(ZBAR(I)**2/12.0+RB(I)**2)
      EYC(I)=0.5*(EYEC(I-1)*ZBAR(I-1)+EYEC(I)*ZBAR(I))+SEGIN*(ZBAR(I-1)
     1**2/12.0+RC(I-1)**2)+SEGOUT*(ZBAR(I)**2/12.0+RC(I)**2)
  305 EYR(I)=EYB(I)+EYC(I)-(SEGIN*ZBAR(I-1)**2+SEGOUT*ZBAR(I)**2)/6.0
      SM(1)=0.5*WTPL(1)*ZBAR(1)/386.4
      SM(N1)=0.5*WTPL(N)*ZBAR(N)/386.4
      EMRB(1)=SM(1)*RB(1)
      EMRB(N1)=SM(N1)*RB(N)+WTPL(N1)*RB(N1)/386.4
      EMRC(1)=SM(1)*RC(1)
      EMRC(N1)=SM(N1)*RC(N)+WTPL(N1)*RC(N1)/386.4
      EMRR(1)=SM(1)*RC(1)*RB(1)
      EMRR(N1)=SM(N1)*RC(N)*RB(N)+WTPL(N1)*RC(N1)*RB(N1)/386.4
      EYB(1)=0.5*(EYEB(1)*ZBAR(1))+SM(1)*(ZBAR(1)**2/12.0+RB(1)**2)
      EYB(N1)=0.5*(EYEB(N)*ZBAR(N))+SM(N1)*(RB(N)**2+ZBAR(N)**2/12.0)
     1 +WTPL(N1)*RB(N1)**2/386.4
      EYC(1)=0.5*(EYEC(1)*ZBAR(1))+SM(1)*(ZBAR(1)**2/12.0+RC(1)**2)
      EYC(N1)=0.5*(EYEC(N)*ZBAR(N))+SM(N1)*(RC(N)**2+ZBAR(N)**2/12.0)+WT
     1PL(N1)*RC(N1)**2/386.4
      EYR(1)=EYB(1)+EYC(1)-SM(1)*ZBAR(1)**2/6.0
      EYR(N1)=EYB(N1)+EYC(N1)-SM(N1)*ZBAR(N)**2/6.0
      SM(N1)=SM(N1)+WTPL(N1)/386.4
      RETURN
      END
```

```
      SUBROUTINE SUMMY
C*****************************************************************
C THIS SUBROUTINE PRINTS OUT A SUMMARY OF NATURAL FREQUENCIES *
C*****************************************************************
      COMMON /COMA/ JHUB,N1,LOT,POUT,ITLE(10),NAME,DAY,NPG
     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES


      COMMON/COMB/IRCOL,XRCOL(3),IRPM,CRPM(10),ZBAR(30),EYEB(30),SB(30),
     1 EYEC(30),SC(30),SM(31),RB(31),RC(31),TH(31),WT(30)
      COMMON/COMD/CMAT(5,5),SOMNAT(100,3),IPLN(100,3),INODE(100,3),
     1 MM3,MM5,CT(31),ST(31),IB,IST,IBS(3,10,3),IBE(3,10,3)
      COMMON /HINGES/ LCH,LCHP1,LFH,LFHP1 ,CHOFF,FHOFF,FCH,FFH
     1 ,RPMPUN,COLPUN,LPH,LPHP1,PHOFF,FPH,          LOTS
      DIMENSION PLNE(4)
      DATA PLNE/6HVERT  ,6HHORIZ ,6HTORSN ,6H      /
      NPG=NPG+1
      WRITE(6,901) NPG,DAY,NAME,(ITLE(J),J=1,10)
  901 FORMAT(1H1,18X,4HPAGE,I3,25X,34HMYKLESTAD ANALYSIS  NASA TM 78670
     1,25X,A10/15X,5HCASE ,I6,18X,10A6/)
      IF(BLADES.GT.2.0) GO TO 10
      WRITE(6,904)
  904 FORMAT(13X,19HC O L L E C T I V E, 27X, 15HS C I S S O R S , 34X,
     1  11HC Y C L I C ,//
     1        6X,3(40HNATURAL    ROOT  ROTOR   MAX  NUMBER    ,3X),/7X,3(
     135HFREQ      COLL  SPEED INERTIA  OF  ,8X),/7X,3(37H/REV      DEG
     2   RPM   PLANE  NODES      ,6X) )
      GO TO 15
   10 CONTINUE
      WRITE(6,905)
  905 FORMAT(59X, 15HS C I S S O R S ,/   /
```

APPENDIX C

```
     1       49X, 40HNATURAL    ROOT  ROTOR   MAX  NUMBER       / 50X, SUMM  30
     135HFREQ      COLL  SPEED INERTIA  OF         /50X, 37H/REV      DEGSUMM  31
     2  RPM   PLANE  NODES          /)                                 SUMM  32
  15 CONTINUE                                                          SUMM  33
     LINES=0                                                           SUMM  34
     IN1=1                                                             SUMM  35
     IN2=1                                                             SUMM  36
     IN3=1                                                             SUMM  37
     DO 60  IST=1,IRCOL                                                SUMM  38
     LINES=LINES+1                                                     SUMM  39
     WRITE(6,902)                                                      SUMM  40
 902 FORMAT (1H  )                                                     SUMM  41
     DO 50  IB=1,IRPM                                                  SUMM  42
     MD=IBE(IST,IB,3)-IBS(IST,IB,3) +1                                 SUMM  43
     MA=MD                                                             SUMM  44
     IF(BLADES.GT.2.0) GO TO 20                                        SUMM  45
     MB=IBE(IST,IB,1)-IBS(IST,IB,1)+1                                  SUMM  46
     MC=IBE(IST,IB,2)-IBS(IST,IB,2)+1                                  SUMM  47
     MA=MAX0(MB,MC,MD)                                                 SUMM  48
  20 CONTINUE                                                          SUMM  49
     IF(MA.EQ.0) GO TO 50                                              SUMM  50
     IF(LINES+MA.LT.40) GO TO 23                                       SUMM  51
     NPG=NPG+1                                                         SUMM  52
     WRITE(6,901) NPG,DAY,NAME,ITLE                                    SUMM  53
     IF(BLADES.GT.2.0) GO TO 21                                        SUMM  54
     WRITE(6,904)                                                      SUMM  55
     GO TO 22                                                          SUMM  56
  21 CONTINUE                                                          SUMM  57
     WRITE(6,905)                                                      SUMM  56
  22 CONTINUE                                                          SUMM  59
     LINES=0                                                           SUMM  60
  23 CONTINUE                                                          SUMM  61
     WRITE(6,902)                                                      SUMM  62
     LINES=LINES+1                                                     SUMM  63
     MA=MAX0(MB,MC,MD)                                                 SUMM  64
     LINES=LINES+MA                                                    SUMM  65
     DO 40  I=1,MA                                                     SUMM  66
     WRITE(6,902)                                                      SUMM  67
     IF(I.GT.MD) GO TO 25                                              SUMM  68
     FNAT=SOMNAT(IN3,3)                                                SUMM  69
     IF(RRPM(IB).NE.0.0) FNAT=FNAT/RRPM(IB)                            SUMM  70
     IP=IPLN(IN3,3)                                                    SUMM  71
     WRITE(6,906) FNAT,RCOLL(IST),RRPM(IB),PLNE(IP),INODE(IN3,3)       SUMM  72
 906 FORMAT(1H+,45X,F10.3,2F7.1,3X,A6,I3)                              SUMM  73


     IN3=IN3+1                                                         SUMM  74
  25 IF(BLADES.GT.2.0) GO TO 40                                        SUMM  75
     IF( I.GT.MB) GO TO 30                                             SUMM  76
     FNAT=SOMNAT(IN1,1)                                                SUMM  77
     IF(RRPM(IB).NE.0.0) FNAT=SOMNAT(IN1,1)/RRPM(IB)                   SUMM  78
     IP=IPLN(IN1,1)                                                    SUMM  79
     WRITE(6,907) FNAT,RCOLL(IST),RRPM(IB),PLNE(IP),INODE(IN1,1)       SUMM  80
 907 FORMAT(1H+, [illegible]X,F10.3,2F7.1,3X,A6,I3)                    SUMM  81
     IN1=IN1+1                                                         SUMM  82
30   IF(I.GT.MC) GO TO 40                                              SUMM  83
     FNAT=SOMNAT(IN2,2)                                                SUMM  84
     IF(RRPM(IB).NE.0.0) FNAT=SOMNAT(IN2,2)/RRPM(IB)                   SUMM  85
     IP=IPLN(IN2,2)                                                    SUMM  86
     WRITE(6,908) FNAT,RCOLL(IST),RRPM(IB),PLNE(IP),INODE(IN2,2)       SUMM  87
 908 FORMAT(1H+,88X,F10.3,2F7.1,3X,A6,I3)                              SUMM  88
     IN2=IN2+1                                                         SUMM  89
40   CONTINUE                                                          SUMM  90
50   CONTINUE                                                          SUMM  91
60   CONTINUE                                                          SUMM  92
     RETURN                                                            SUMM  93
     END                                                               SUMM  94
```

APPENDIX C

```
      SUBROUTINE MODSHAP(B,COLL,RPM,FREQPR,M,K,KEND)
C
C          THIS SUBROUTINE PLOTS THE MODE SHAPE
      COMMON /COMA/ JHUB,N1,LOT,POUT,ITLE(10),NAME,DAY,NPG
     1,JHUB1,RRPM(10),RCOLL(3),Z(31),INPUN,MODPLOT,N,TRSN,BLADES
      LOGICAL TRSN
      DIMENSION B(3,31),R(33),BEAM(33),CHORD(33),TORSION(33),MODE(3)
      DATA MODE/10HCOLLECTIVE,10HCYCLIC     ,10HSCISSORS  /
      XORIG=0.0
      XSCALE=0.2
      XLENTH=1.0/XSCALE
      YORIG=-1.0
      YSCALE=0.50
      DO 10 I=1,N1
      R(I)=Z(I)/Z(N1)
      BEAM(I)=B(1,I)
      CHORD(I)=B(2,I)
      TORSION(I)=B(3,I)
   10 CONTINUE
      R(N1+1)=XORIG
      R(N1+2)=XSCALE
      BEAM(N1+1)=YORIG
      BEAM(N1+2)=YSCALE
      CHORD(N1+1)=YORIG
      CHORD(N1+2)=YSCALE
      TORSION(N1+1)=YORIG
      TORSION(N1+2)=YSCALE
      I=MOD(K,2)
      IF(I.GT.0) CALL CALPLT(1.0,6.5,-3)
      CALL LINPLT(R,BEAM,N1,1,1,1,1,0)
      CALL LINPLT(R,CHORD,N1,1,1,2,1,0)
      H=0.15
      IF(RPM.GT.0.0) GO TO 15
      CALL NUMBER(0.8,4.5,H,FREQPR,0.0,0)
      CALL NOTATE(1.7,4.5,H,4H CPM,0.0,4)
      GO TO 16
   15 CONTINUE
      CALL NUMBER(0.8, 4.5,H,FREQPR,0.0,3)
      CALL NOTATE(1.5,4.5,H,6H / REV,0.0,6)
   16 CONTINUE
      CALL NOTATE(2.5,4.5,H,MODE(M),0.0,10)
      XSCALE=XSCALE*(-1.0)
      CALL NUMBER(0.8,4.3,H,RPM,0.0,0)
      CALL NOTATE(1.6,4.3,H,3HRPM,0.0,3)


      CALL NUMBER(0.8,4.1,H,COLL,0.0,1)
      CALL NOTATE(1.6,4.1,H,4HCOLL,0.0,4)
      CALL AXES(XLENTH,0.0,180.0,XLENTH,1.0,XSCALE,-1.0,0.0,6HRADIUS,H,6
     1)
      CALL AXES(0.0,0.0,90.0,4.0,-1.00,YSCALE,-1.0,0.0,10HDEFLECTION,H,
     110)
      CALL PNTPLT(4.2,4.6,1,1)
      CALL NOTATE(4.4,4.5,H,4HVERT,0.0,4)
      CALL PNTPLT(4.2,4.4,2,1)
      CALL NOTATE(4.4,4.3,H,5HHORIZ,0.0,5)
      IF(.NOT.TRSN) GO TO 20
      YSCALE=YSCALE*(-1.0)
      CALL LINPLT(R,TORSION,N1,1,1,3,1,0)
      CALL PNTPLT(4.2,4.2,3,1)
      CALL NOTATE(4.4,4.1,H,7HTORSION,0.0,7)
   20 CONTINUE
      IF(I.GT.0) CALL CALPLT(0.0,-5.5,-3)
      IF(K.EQ.KEND .OR. I.EQ.0) CALL NFRAME
      RETURN
      END
```

APPENDIX D

LIBRARY SUBROUTINE MATINV LISTING

The listing for the library subroutine MATINV follows:

```
      SUBROUTINE MATINV(MAX,N,A,M,B,IOP,DETERM,ISCALE,IPIVOT,IWK)
C*************************************************************-*****************
C
C     PURPOSE - MATINV INVERTS A REAL SQUARE MATRIX A.
C               IN ADDITION THE ROUTINE SOLVES THE MATRIX
C               EQUATION AX=B,WHERE B IS A MATRIX OF CONSTANT
C               VECTORS. THERE IS ALSO AN OPTION TO HAVE THE
C               DETERMINANT EVALUATED. IF THE INVERSE IS NOT
C               NEEDED, USE GELIM TO SOLVE A SYSTEM OF SIMULTANEOUS
C               EQUATIONS AND DETFAC TO EVALUATE A DETERMINANT
C               FOR SAVING TIME AND STORAGE.
C
C     USE       - CALL MATINV(MAX,N,A,M,B,IOP,DETERM,ISCALE,IPIVOT,IWK)
C
C                 MAX - THE MAXIMUM ORDER OF A AS STATED IN THE
C                       DIMENSION STATEMENT OF THE CALLING PROGRAM.
C
C                 N   - THE ORDER OF A, 1.LE.N.LE.MAX.
C
C                 A   - A TWO-DIMENSIONAL ARRAY OF THE COEFFICIENTS.
C                       ON RETURN TO THE CALLING PROGRAM, A INVERSE
C                       IS STORED IN A.
C                       A MUST BE DIMENSIONED IN THE CALLING PROGRAM
C                       WITH FIRST DIMENSION MAX AND SECOND DIMENSION
C                       AT LEAST N.
C
C                 M   - THE NUMBER OF COLUMN VECTORS IN B.
C                       M=0 SIGNALS THAT THE SUBROUTINE IS
C                       USED SOLELY FOR INVERSION,HOWEVER,
C                       IN THE CALL STATEMENT AN ENTRY CORRE-
C                       SPONDING TO B MUST BE PRESENT.
C
C                 B   - A TWO-DIMENSIONAL ARRAY OF THE CONSTANT
C                       VECTOR B. ON RETURN TO CALLING PROGRAM,
C                       X IS STORED IN B. B SHOULD HAVE ITS FIRST
C                       DIMENSION MAX AND ITS SECOND AT LEAST M.
C
C                 IOP   COMPUTE DETERMINANT OPTION.
C                        IOP=0 COMPUTES THE MATRIX INVERSE AND
C                              DETERMINANT.
C                        IOP=1 COMPUTES THE MATRIX INVERSE ONLY.
C
C               DETERM- FOR IOP=0-IN CONJUNCTION WITH ISCALE
C                       REPRESENTS THE VALUE OF THE DETERMINANT
C                       OF A, DET(A),AS FOLLOWS.
C                        DET(A) =(DETERM)(10**100(ISCALE))
C                       THE COMPUTATION DET(A) SHOULD NOT BE
C                       ATTEMPTED IN THE USER PROGRAM SINCE IF
C                       THE ORDER OF A IS LARGER AND/OR THE
C                       MAGNITUDE OF ITS ELEMENTS ARE LARGE(SMALL),
C                       THE DET(A) CALCULATION MAY CAUSE OVERFLOW
C                     (UNDERFLOW). DETERM SET TO ZERO FOR
C                     SINGULAR MATRIX CONDITION, FOR EITHER
C                     IOP=1OR 0. SHOULD BE CHECKED BY PROGRAMER
C                     ON RETURN TO MAIN PROGRAM.
C
```

```
C              ISCALE  - A SCALE FACTOR COMPUTED BY THE
C                        SUBROUTINE TO AVOID OVERFLOW OR
C                        UNDERFLOW IN THE COMPUTATION OF
C                        THE QUANTITY.DETERM.
C
C              IPIVOT  - A ONE DIMENSIONAL INTEGER ARRAY
C                        USED BY THE SUBPROGRAM TO STORE
C                        PIVOTOL INFORMATION. IT SHOULD BE
C                        DIMENSIONED AT LEAST N. IN GENERAL
C                        THE USER DOES NOT NEED TO MAKE USE
C                        OF THIS ARRAY.
C
C                 IWK  - A TWO-DIMENSIONAL INTEGER ARRAY OF
C                        TEMPORARY STORAGE USED BY THE ROUTINE.
C                        IWK SHOULD HAVE ITS FIRST DIMENSION
C                        MAX. AND ITS SECOND 2.
C
C     REQUIRED ROUTINES-
C
C     REFERENCE          -FOX.L. AN INTRODUCTION TO NUMERICAL
C                                LINEAR ALGEBRA
C
C     STORAGE            - 542 OCTAL LOCATIONS
C
C     LANGUAGE           -FORTRAN
C
C     FORTRAN
C     LIBRARY FUNCTIONS -ABS
C
C     LATEST REVISION - JULY 1973 -CMPB
C
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
C
      DIMENSION IPIVOT(N).A(MAX.N).B(MAX.N).IWK(MAX.2)
      EQUIVALENCE (IROW.JROW). (ICOLUM.JCOLUM). (AMAX. T. SWAP)
C
C     INITIALIZATION
C
      ISCALE=0
      R1=10.0**100
      R2=1.0/R1
      DETERM=1.0
      DO 20 J=1.N
   20 IPIVOT(J)=0
      DO 550 I=1.N
C
C     SEARCH FOR PIVOT ELEMENT
C
      AMAX=0.0
      DO 105 J=1.N
      IF (IPIVOT(J)-1) 60. 105. 60
   60 DO 100 K=1.N
      IF (IPIVOT(K)-1) 80. 100. 740
```

```
   80 IF (ABS(AMAX)-ABS(A(J,K)))85,100,100
   85 IROW=J
      ICOLUM=K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IF (AMAX) 110,106,110
  106 DETERM=0.0
      ISCALE=0
      GO TO 740
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
      IF (IROW-ICOLUM) 140, 260, 140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      IF(M) 260, 260, 210
  210 DO 250 L=1, M
      SWAP=B(IROW,L)
      B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260 IWK(I,1)=IROW
      IWK(I,2)=ICOLUM
      PIVOT=A(ICOLUM,ICOLUM)
      IF(IOP.EQ.1) GO TO 321
      IF(PIVOT)1000,106,1000
C
C     SCALE THE DETERMINANT
C
 1000 PIVOTI=PIVOT
      IF(ABS(DETERM)-R1)1030,1010,1010
 1010 DETERM=DETERM/R1
      ISCALE=ISCALE+1
      IF(ABS(DETERM)-R1)1060,1020,1020
 1020 DETERM=DETERM/R1
      ISCALE=ISCALE+1
      GO TO 1060
 1030 IF(ABS(DETERM)-R2)1040,1040,1060
 1040 DETERM=DETERM*R1
      ISCALE=ISCALE-1
      IF(ABS(DETERM)-R2)1050,1050,1060
 1050 DETERM=DETERM*R1
      ISCALE=ISCALE-1
 1060 IF(ABS(PIVOTI)-R1)1090,1070,1070
 1070 PIVOTI=PIVOTI/R1
      ISCALE=ISCALE+1
      IF(ABS(PIVOTI)-R1)320,1080,1080
 1080 PIVOTI=PIVOTI/R1
      ISCALE=ISCALE+1
      GO TO 320
 1090 IF(ABS(PIVOTI)-R2)2000,2000,320
```

```
 2000 PIVOTI=PIVOTI*R1
      ISCALE=ISCALE-1
      IF(ABS(PIVOTI)-R2)2010,2010,320
 2010 PIVOTI=PIVOTI*R1
      ISCALE=ISCALE-1
  320 DETERM=DETERM*PIVOTI
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  321 IF(PIVOT)330,106,330
  330 A(ICOLUM,ICOLUM)=1.0
      DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
      IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
      IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
      A(L1,ICOLUM)=0.0
      DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
      IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
      DO 710 I=1,N
      L=N+1-I
      IF (IWK(L,1)-IWK(L,2))630,710,630
  630 JROW=IWK(L,1)
      JCOLUM=IWK(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

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TABLE I.- SUMMARY OF BOUNDARY CONDITION EQUATIONS[a]

Collective mode	Cyclic mode	Scissors mode (no hinge offsets)	Scissors mode (hinge offsets)
$L - (K_{op} - M_{op}\omega^2)\delta_y = 0$	$\delta_y = 0$	$\delta_y = 0$	[b] $(\delta_y = 0)$
$\delta_x = 0$	$D - (K_{ip} - M_{ip}\omega^2)\delta_x = 0$	$\delta_x = 0$	[c] $(\delta_x = 0)$
$\beta = 0$	$M - K_\beta\beta = 0$	$\beta = 0$	[b] $(M - K_\beta\beta = 0)$
$Q - K_T\psi = 0$	$\psi = 0$	$\psi = 0$	[c] $(Q - K_\psi\psi = 0)$
[d] $(T - K_c\phi = 0)$	[d] $(T - K_c\phi = 0)$	[d] $(T - K_c\phi = 0)$	[d] $(T - K_c\phi = 0)$
		[e] $(\phi = 0)$	

[a] Boundary condition applied at center of rotation (station 1) unless otherwise noted.

[b] Boundary condition applied at flapping hinge offset.

[c] Boundary condition applied at lagging hinge offset.

[d] Boundary condition applied at pitch horn offset.

[e] For two-bladed rotors only and applied at pitch horn offset.

TABLE II.- DESCRIPTION OF PROGRAM INPUT REQUIREMENTS

Card type	Format	Variable	Units	Description
1	6(A4,6X)	----------	----------	Program options: DECK, NAME, PUNC, SHAP, MODE, ALLM, PLOT, TORS, and NLTW
2	I6,4X,10A6	NAME	----------	Problem number
		ITLE	----------	Problem title
3	8F10.0	KT (K_T)	in-lb/rad	Mast torsional stiffness per blade
		MOP (M_{op})	lb-sec^2/in.	Hub out-of-plane inertia per blade
		MIP (M_{ip})	lb-sec^2/in.	Hub in-plane inertia per blade
		KOP (K_{op})	lb/in.	Hub out-of-plane stiffness per blade
		KIP (K_{ip})	lb/in.	Hub in-plane stiffness per blade
		KBETA (K_β)	in-lb/rad	Blade flapping spring
		KPSI ($K\psi$)	in-lb/rad	Blade lagging spring
		KC (K_c)	in-lb/rad	Control system stiffness
4	2I5,6F5.0,F10.0	N	----------	Number of segments
		JHUB	----------	Number of nonfeathering segments
		AZBAR ($\bar{z}$)	in.	Segment length for uniform distribution
		TWIST (θ_t)	deg	Linear twist from center of rotation to tip
		BLADES	----------	Number of blades
		CHOFF	in.	Lagging hinge offset from center line
		FHOFF	in.	Flapping hinge offset from center line
		PHOFF	in.	Pitch horn offset from center line

TABLE II.- Concluded

Card type	Format	Variable	Units	Description
		PLAST	cpm	Upper limit of frequency calculations
5	I5,10F5.0	IRPM		Number of rotor speeds
		RRPM	rpm	Rotor speeds
6	I5,10F5.0	IRCOL		Number of rotor collective pitch angles
		RCOLL (θ_c)	deg	Rotor collective pitch angles
7	8F10.0	^{a}X	in.	Outboard end stations of segments
8	8F10.0	^{a}EIB (EI_b)	$lb\text{-}in^2$	Beamwise bending stiffness
9	8F10.0	^{a}EIC (EI_c)	$lb\text{-}in^2$	Chordwise bending stiffness
10	8F10.0	^{b}WTPL (w,W)	lb/in, lb	Weight per unit length and tip weight
11	8F10.0	^{c}THD (θ)	deg	Nonlinear twist distribution at station
12	8F10.0	^{a}EYEB (I'_{bb})	$lb\text{-}sec^2$	Beamwise second mass moment of inertia
13	8F10.0	^{a}EYEC (I'_{cc})	$lb\text{-}sec^2$	Chordwise second mass moment of inertia
14	8F10.0	^{a}GJ	$lb\text{-}in^2$	Torsional stiffness
15	8F10.0	^{a}SB (S_b)	in.	Beamwise shear center offset
16	8F10.0	^{a}SC (S_c)	in.	Chordwise shear center offset
17	8F10.0	^{d}RB (r_b)	in.	Beamwise offset of center of gravity
18	8F10.0	^{d}RC (r_c)	in.	Chordwise offset of center of gravity

[a] N values are input, one for each segment.

[b] N values are input and the tip weight is listed in field N + 1.

[c] N + 1 values are input, one for each station.

[d] N values are input and the offsets pertaining to the tip weight are input in field N + 1.

TABLE III.- SAMPLE INPUT DATA DECK

```
                                        Card column
Card type 00000000011111111112222222222333333333344444444445555555555666666666677777777778
          12345678901234567890123456789012345678901234567890123456789012345678901234567890
    1     DECK      TORSION   PLOT      MODE      SHAPE     PUNCH
    2     100000              COMPUTER PROGRAM SAMPLE CASE
    3     166000.   .99       .259      1150.     1150.             0.       0.   15500.
    4        30    2  .0   .0    4.   3.   3.   3. 15000.
    5         3 1100 1200 1300
    6         3    0    6   12
                 1.5       3.0     4.935      6.87       9.1    11.235     13.37     14.87
    7           17.1      18.8      20.5      22.5    25.179    27.857    30.536    33.214
              35.893    38.571     41.25    43.929    46.607    49.286    51.964    54.643
              57.321       60.       61.     61.25      61.5     62.05
          100000000.100000000.1500000.  1500000.  500000.   182900.   182900.   1198000.
    8     500000.   135000.   135000.   79400.    62900.    62900.    62900.    62900.
          62900.    62900.    62900.    62900.    62900.    62900.    62900.    62900.
          62900.    62900.    100000.   100000.   10000.    10000.
          100000000.100000000.1500000.  1500000.  1250000.  911400.   911400.   1198000.
    9     1250000.  972000.   972000.   820000.   560000.   560000.   560000.   560000.
          560000.   560000.   560000.   560000.   560000.   560000.   560000.   560000.
          560000.   560000.   1000000.  1000000.  100000.   100000.
              1.7083    1.7083     .4185     .4185     .1806     .0705     .0705    .12268
   10          .1652    .02777    .02777    .02779    .02766    .02766    .02766    .02766
              .02766    .02766    .02766    .02766    .02766    .02766    .02766    .02766
              .02766    .02766    .04315    .08273    .03744    .01101       0.0
                  0.        0.        0.        0.        0.        0.        0.        0.
   12             0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.
                .001      .001    .00057    .00057   .000143  .0000285  .0000285   .000062
   13        .000114   .000023   .000023   .000064   .000063   .000063   .000063   .000063
             .000063   .000063   .000063   .000063   .000063   .000063   .000063   .000063
             .000063   .000063   .000131   .000074   .000068   .000023
              .1E+10    .1E+101000000.  1000000.  213500.   202000.   202000.   910500.
   14     213500.   109400.   91390.    75900.    75900.    75900.    75900.    75900.
          75900.    75900.    75900.    75900.    75900.    75900.    75900.    75900.
          75900.    75900.    114000.   114000.   11400.    11400.
                  0.        0.        0.        0.        0.        0.        0.        0.
   15             0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.        0.        0.
   16             0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.        0.        0.
   17             0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.       0.0
                  0.        0.        0.        0.        0.        0.        0.        0.
   18             0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.        0.        0.
                  0.        0.        0.        0.        0.        0.        0.
```

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TABLE IV.- ILLUSTRATION OF PROGRAM STANDARD OUTPUT (INPUT SUMMARY)

```
              PAGE   1                       MYKLESTAD ANALYSIS  NASA TM 78670                              78/07/12.

        CASE 100000                                COMPUTER PROGRAM SAMPLE CASE

      SEGMNT        EI                WT/IN     TWIST      C.F.          IBB           ICC           GJ       SHEAR OFFSET        C.G. OFFSET
       LNTH     BEAM       CHORD               (INBD)     (INBD)                                               BEAM     CHORD     BEAM     CHORD
        IN      -- LB-IN**2 --        LB/IN      DEG     LB/RPM**2         IN-LB-SEC**2/IN           LB-IN**2      IN        IN        IN        IN
   1   1.50   .1000E+09  .1000E+09   1.7083     0.00   .2309E-02     0.               .1000E-02   .1000E+10   0.000     0.000     0.000     0.000
   2   1.50   .1000E+09  .1000E+09   1.7083     0.00   .2254E-02     0.               .1000E-02   .1000E+10   0.000     0.000     0.000     0.000
   3   1.94   .1500E+07  .1500E+07    .4185     0.00   .2090E-02     0.               .5700E-03   .1000E+07   0.000     0.000     0.000     0.000
   4   1.94   .1500E+07  .1500E+07    .4185     0.00   .1999E-02     0.               .5700E-03   .1000E+07   0.000     0.000     0.000     0.000
   5   2.23   .5000E+06  .1250E+07    .1806     0.00   .1864E-02     0.               .1430E-03   .2135E+06   0.000     0.000     0.000     0.000
   6   2.14   .1829E+06  .9114E+06    .0765     0.00   .1772E-02     0.               .2850E-04   .2020E+06   0.000     0.000     0.000     0.000
   7   2.14   .1829E+06  .9114E+06    .0765     0.00   .1729E-02     0.               .2850E-04   .2020E+06   0.000     0.000     0.000     0.000
   8   1.50   .1198E+07  .1198E+07    .1227     0.00   .1676E-02     0.               .6200E-04   .9105E+06   0.000     0.000     0.000     0.000
   9   2.23   .5000E+06  .1250E+07    .1652     0.00   .1603E-02     0.               .1140E-03   .7135E+06   0.000     0.000     0.000     0.000
  10   1.70   .1350E+06  .9720E+06    .0278     0.00   .1435E-02     0.               .2300E-04   .1094E+06   0.000     0.000     0.000     0.000
  11   1.70   .1350E+06  .9720E+06    .0278     0.00   .1411E-02     0.               .2300E-04   .9139E+05   0.000     0.000     0.000     0.000
  12   2.00   .7940E+05  .8200E+06    .0278     0.00   .1389E-02     0.               .6400E-04   .7590E+05   0.000     0.000     0.000     0.000
  13   2.68   .6290E+05  .5600E+06    .0277     0.00   .1351E-02     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  14   2.68   .6290E+05  .5600E+06    .0277     0.00   .1301E-02     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  15   2.68   .6290E+05  .5600E+06    .0277     0.00   .1245E-02     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  16   2.68   .6290E+05  .5600E+06    .0277     0.00   .1184E-02     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  17   2.68   .6290E+05  .5600E+06    .0277     0.00   .1117E-02     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  18   2.68   .6290E+05  .5600E+06    .0277     0.00   .1044E-02     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  19   2.68   .6290E+05  .5600E+06    .0277     0.00   .9660E-03     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  20   2.68   .6290E+05  .5600E+06    .0277     0.00   .8525E-03     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  21   2.68   .6290E+05  .5600E+06    .0277     0.00   .7925E-03     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  22   2.68   .6290E+05  .5600E+06    .0277     0.00   .6973E-03     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  23   2.68   .6290E+05  .5600E+06    .0277     0.00   .5965E-03     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  24   2.68   .6290E+05  .5600E+06    .0277     0.00   .4900E-03     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  25   2.68   .6290E+05  .5600E+06    .0277     0.00   .3779E-03     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  26   2.68   .6290E+05  .5600E+06    .0277     0.00   .2603E-03     0.               .6300E-04   .7590E+05   0.000     0.000     0.000     0.000
  27   1.00   .1000E+06  .1000E+07    .0432     0.00   .1369E-03     0.               .1310E-03   .1140E+06   0.000     0.000     0.000     0.000
  28    .25   .1000E+06  .1000E+07    .0627     0.00   .6280E-04     0.               .7400E-04   .1140E+06   0.000     0.000     0.000     0.000
  29    .25   .1000E+05  .1000E+06    .0374     0.00   .2692E-04     0.               .6600E-04   .1140E+05   0.000     0.000     0.000     0.000
  30    .55   .1000E+05  .1000E+06    .0110     0.00   .1062E-04     0.               .2300E-04   .1140E+05   0.000     0.000     0.000     0.000
   TIP                               0.0000     0.00                                                                               0.000     0.000

                4. BLADES                            RADIUS   62.65 IN                                BLADE WEIGHT  4.14 LB
                 2 HUB SEGMENTS                                                                       BLADE FLAP INERTIA     .45 SLUG-FT**2

FLAP HNG OFST     3.00 IN      FLAP SPRING0.          IN-LB/RAD       HUB O.P. INERTIA  .990 LB-SEC**2/IN       HUB O.P. STIFF .115E+04 LB/IN
LAG HNG OFST      3.00 IN      LAG SPRING 0.          IN-LB/RAD       HUB I.P. INERTIA  .259 LB-SEC**2/IN       HUB I.P. STIFF .115E+04 LB/IN
PITCH HORN OFST   3.00 IN      CNTRL STIFF  .155E+05 IN-LB/RAD
                             MAST TORSION SPR   .166E+06 IN-LB/RAD
```

TABLE V.- ILLUSTRATION OF PROGRAM STANDARD OUTPUT (NATURAL FREQUENCY SUMMARY)

```
    PAGE 10                 MYKLESTAD ANALYSIS  NASA TM 78670                     78/07/12.
 CASE 100000                 COMPUTER PROGRAM SAMPLE CASE

                                  S C I S S O R S

                        NATURAL     ROOT  ROTOR   MAX  NUMBER
                         FREQ       COLL  SPEED INERTIA  OF
                         /REV       DEG   RPM   PLANE  NODES


                          .297      0.0 1100.0   HORIZ   0
                         1.042      0.0 1100.0   VERT    0
                         2.970      0.0 1100.0   VERT    1
                         3.690      0.0 1100.0   HORIZ   2
                         5.029      0.0 1100.0   VERT    2
                         8.675      0.0 1100.0   TORSN   0
                         9.333      0.0 1100.0   VERT    3
                         9.887      0.0 1100.0   HORIZ   3

                          .297      0.0 1200.0   HORIZ   0
                         1.042      0.0 1200.0   VERT    0
                         2.932      0.0 1200.0   VERT    2
                         3.603      0.0 1200.0   HORIZ   1
                         4.819      0.0 1200.0   VERT    2
                         7.962      0.0 1200.0   TORSN   0
                         8.795      0.0 1200.0   VERT    3
                         9.157      0.0 1200.0   HORIZ   3

                          .297      0.0 1300.0   HORIZ   0
                         1.042      0.0 1300.0   VERT    0
                         2.902      0.0 1300.0   VERT    2
                         3.314      0.0 1300.0   HORIZ   2
                         4.669      0.0 1300.0   VERT    2
                         7.360      0.0 1300.0   TORSN   0
                         8.319      0.0 1300.0   VERT    3
                         8.946      0.0 1300.0   HORIZ   3


                          .297      6.0 1100.0   HORIZ   0
                         1.042      6.0 1100.0   VERT    0
                         2.968      6.0 1100.0   VERT    1
                         3.691      6.0 1100.0   HORIZ   2
                         5.021      6.0 1100.0   VERT    2
                         8.674      6.0 1100.0   HORIZ   2
                         9.333      6.0 1100.0   VERT    3
                         9.887      6.0 1100.0   HORIZ   3
```

TABLE VI.- ILLUSTRATION OF PROGRAM OPTIONAL OUTPUT (MODE SHAPE LISTING)

```
        PAGE  9                   MYKLESTAD ANALYSIS  NASA TM 78670                        78/07/12.

  CASE 100000                          COMPUTER PROGRAM SAMPLE CASE

                               6.00  DEGREES ROOT COLLECTIVE          1200.0  ROTOR SPEED, RPM
                              SCISSORS MODE OF BLADE AT 10988.81 CPM OR   9.157 PER REV

   BLADE STA        DEFLECTIONS                 MOMENTS                      SHEAR FORCES              TWIST      TORQUE
      IN                IN                       IN-LB                          LB                     RAD        IN-LB
                    VERT    HORIZ           BEAM        CHORD              BEAM       CHORD
********************************************************************************************************************
   1    0.00       0.000   0.000            -483.       4523.              -161.      1907.            -.000         0.
   2    1.50       0.000   0.000            -241.       2262.              -161.      1507.            -.000         0.
   3    3.00        .000   -.000              -0.          0.                -2.      1916.            -.000         0.
   4    4.94       -.012    .108               4.      -2763.                -2.      1517.            -.000         0.
   5    6.87       -.023    .209               7.      -4972.                -2.      1212.            -.000         0.
   6    9.10       -.033    .308               8.      -6512.                -1.       771.             .000         0.
   7   11.24       -.041    .375               9.      -7378.                -0.       474.             .000         0.
   8   13.37       -.044    .406               9.      -7900.                 0.       277.             .000         0.
   9   14.87       -.044    .406               8.      -7960.                 0.        40.             .000         0.
  10   17.10       -.041    .381               6.      -7231.                 1.      -391.             .000         0.
  11   18.80       -.036    .340               5.      -6240.                 1.      -627.             .000         0.
  12   20.50       -.030    .281               3.      -5195.                 1.      -699.             .000         0.
  13   22.50       -.020    .191               1.      -3901.                 1.      -734.             .000         0.
  14   25.18       -.003    .034              -1.      -2165.                 1.      -777.             .000         0.
  15   27.86        .017   -.148              -2.       -314.                 1.      -786.             .000         0.
  16   30.54        .037   -.335              -3.       1386.                 1.      -747.             .000         0.
  17   33.21        .055   -.505              -3.       2897.                 0.      -641.             .000         0.
  18   35.89        .069   -.638              -3.       4130.                -0.      -530.             .000         0.
  19   38.57        .077   -.720              -2.       5005.                -1.      -369.             .000         0.
  20   41.25        .079   -.739              -1.       5466.                -1.      -179.             .000         0.
  21   43.93        .073   -.590               1.       5492.                -1.        12.             .000         0.
  22   46.61        .060   -.572               2.       5099.                -1.       198.             .000         0.
  23   49.29        .040   -.389               3.       4344.                -0.       338.             .000         0.
  24   51.96        .015   -.153               3.       3327.                -0.       438.             .000         0.
  25   54.64       -.014    .126               2.       2186.                 0.       478.             .000         0.
  26   57.32       -.046    .432               1.       1694.                 0.       445.             .000         0.
  27   60.00       -.079    .752               0.        248.                 0.       334.             .000         0.
  28   61.00       -.092    .873               0.         51.                 0.       189.             .000         0.
  29   61.25       -.095    .903               0.         20.                 0.        83.             .000         0.
  30   61.50       -.098    .933               0.          4.                 0.        36.             .000         0.
  31   62.05       -.105   1.000              -0.          1.                 0.        11.             .000         0.
********************************************************************************************************************

  GENERALIZED INERTIA (IN-LBF-SEC**2)      VERT  .00002      HORIZ  .00134      TWIST  .00006
  BOUNDARY CONDITION CHECK          .13467E-04 -.13009E-03 -.10753E-02  .89407E-07  .11765F-03
```

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TABLE VII.- DESCRIPTION OF PUNCHED OUTPUT DATA

Card	Description	Format
1	Identification card containing case number and title	I6, 4X, 10 A6
2-4	Distribution of weight per unit length and tip weight, lb/in. and lb	7F10.5
5-7	Beamwise second mass moment of inertia, lb-sec^2	7F10.5
8-10	Chordwise second mass moment of inertia, lb-sec^2	7F10.5
11-76	Collective mode deflection shapes, case number, mode number, mode type	6F10.6, 10x, I6, 2I2
77-142	Cyclic mode deflection shapes, case number, mode number, and mode type	6F10.6, 10x, I6, 2I2
143-208	Scissors mode deflection shapes, case number, mode number, and mode type	6F10.6, 10x, I6, 2I2
209-214	Collective mode cyclic detuning information, reference values of collective pitch and rotor speed, case number, mode number, and mode type	6F10.2, 2F5.0, I6, 2I2
215-220	Cyclic mode cyclic detuning information, reference values of collective pitch and rotor speed, case number, mode number, and mode type	6F10.2, 2F5.0, I6, 2I2
221-226	Scissors mode cyclic detuning information, reference values of collective pitch and rotor speed, case number, mode number, and mode type	6F10.2, 2F5.0, I6, 2I2

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TABLE VIII.- ILLUSTRATION OF PROGRAM OPTIONAL OUTPUT (PUNCHED CARD DECK)

```
          Card column
Card no.  00000000011111111112222222222333333333344444444445555555555666666666677777777778
          12345678901234567890123456789012345678901234567890123456789012345678901234567890
   1      100000              COMPUTER PROGRAM SAMPLE CASE
   2         1.66569    .41850    .22423    .07050    .11534    .09809    .02778
   3          .02769    .02766    .02766    .02766    .02766    .02766    .02766
   4          .02766    .02766    .02766    .02766    .02766    .03493    .00000
   5         0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   6         0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   7         0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   8          .00099    .00057    .00023    .00003    .00006    .00007    .00004
   9          .00006    .00006    .00006    .00006    .00006    .00006    .00006
  10          .00006    .00006    .00006    .00006    .00006    .00008    .00000
 143        0.000000  0.000000  -.000000  -.000000   .000142  -.000000         100000 1  0
 144        -.000008   .004437  -.000000  -.000016   .008737  -.000000         100000 1  0
 145        -.000021   .013048  -.000000  -.000026   .017368  -.000000         100000 1  0
 146        -.000029   .021698  -.000000  -.000031   .026040  -.000000         100000 1  0
 147        -.000031   .030394  -.000000  -.000028   .034761  -.000000         100000 1  0
 148        -.000025   .039143  -.000000  -.000021   .043536  -.000000         100000 1  0
 149        -.000016   .047938  -.000000  -.000011   .052347  -.000000         100000 1  0
 150        -.000006   .056764  -.000000   .000000   .061186  -.000000         100000 1  0
 151         .000006   .065612  -.000000   .000012   .070040  -.000000         100000 1  0
 152         .000019   .074471  -.000000   .000025   .078902  -.000000         100000 1  0
 153         .000031   .083333  -.000000   .296742  0.000000   .020000         100000 1  0
 154        0.000000  0.000000  -.000001   .000139  -.000000  -.000001         100000 2  0
 155         .004340  -.000008  -.000001   .008547  -.000016  -.000001         100000 2  0
 156         .012777  -.000022  -.000001   .017032  -.000027  -.000001         100000 2  0
 157         .021302  -.000031  -.000001   .025602  -.000032  -.000001         100000 2  0
 158         .029937  -.000032  -.000001   .034303  -.000029  -.000001         100000 2  0
 159         .038697  -.000024  -.000001   .043110  -.000018  -.000001         100000 2  0
 160         .047539  -.000012  -.000001   .051982  -.000005  -.000001         100000 2  0
 161         .056437   .000003  -.000001   .060903   .000012  -.000001         100000 2  0
 162         .065379   .000021  -.000001   .069862   .000031  -.000001         100000 2  0
 163         .074350   .000041  -.000001   .078841   .000051  -.000001         100000 2  0
 164         .083333   .000062  -.000001  1.042099  0.000000   .020000         100000 2  0
 165        0.000000  0.000000  -.000000  -.000299  -.000037  -.000000         100000 3  0
 166        -.009328  -.001156  -.000000  -.018191  -.002251  -.000000         100000 3  0
 167        -.026208  -.003240  -.000000  -.033260  -.004107  -.000000         100000 3  0
 168        -.039590  -.004877  -.000000  -.044140  -.005430  -.000000         100000 3  0
 169        -.046418  -.005709  -.000000  -.046243  -.005686  -.000000         100000 3  0
 170        -.043545  -.005349  -.000000  -.038672  -.004740  -.000000         100000 3  0
 171        -.031681  -.003870  -.000000  -.022634  -.002747  -.000000         100000 3  0
 172        -.011621  -.001385  -.000000   .001216   .000196  -.000000         100000 3  0
 173         .015803   .001984  -.000000   .031686   .003924  -.000000         100000 3  0
 174         .048479   .005570  -.000000   .065806   .008077  -.000000         100000 3  0
 175         .083333   .010207  -.000000  2.529757  0.000000   .020000         100000 3  0
 176        0.000000  0.000000  -.000000   .000040  -.000332  -.000000         100000 4  0
 177         .001247  -.010239  -.000000   .002409  -.019753  -.000000         100000 4  0
```

TABLE VIII.- Concluded

Card no.	Card column								
178	.003451	-.028273	-.000000	.004367	-.035556	-.000000	100000	4	0
179	.005098	-.041587	-.000000	.005639	-.045915	-.000000	100000	4	0
180	.005925	-.048201	-.000000	.005896	-.047912	-.000000	100000	4	0
181	.005521	-.044773	-.000000	.004848	-.039179	-.000000	100000	4	0
182	.003893	-.031295	-.000000	.002680	-.021327	-.000000	100000	4	0
183	.001235	-.009524	-.000000	-.000411	.003834	-.000000	100000	4	0
184	-.002234	.018535	-.000000	-.004178	.034127	-.000000	100000	4	0
185	-.006201	.050288	-.000000	-.008268	.066756	-.000000	100000	4	0
186	-.010352	.083333	-.000000	3.485074	0.000000	.020000	100000	4	0
187	0.000000	0.000000	-.000001	.000357	.000038	-.000001	100000	5	0
188	.011066	.001181	-.000001	.021045	.002245	-.000001	100000	5	0
189	.027529	.002934	-.000001	.030200	.003214	-.000001	100000	5	0
190	.030277	.003214	-.000001	.024751	.002618	-.000001	100000	5	0
191	.013430	.001407	-.000001	-.002264	-.000269	-.000001	100000	5	0
192	-.019715	-.002129	-.000001	-.036044	-.003867	-.000001	100000	5	0
193	-.048997	-.005244	-.000001	-.056601	-.006048	-.000001	100000	5	0
194	-.057310	-.006115	-.000001	-.050169	-.005346	-.000001	100000	5	0
195	-.034221	-.003638	-.000001	-.010980	-.001154	-.000001	100000	5	0
196	.017761	.001915	-.000001	.049801	.005344	-.000001	100000	5	0
197	.083333	.008912	-.000001	4.814033	0.000000	.020000	100000	5	0
198	0.000000	0.000000	10.247496	.002421	-.010704	10.262930	100000	6	0
199	.002766	-.012937	10.700472	.002878	-.014182	12.241085	100000	6	0
200	.002332	-.013490	14.147736	.001209	-.010882	15.298205	100000	6	0
201	-.000213	-.006750	17.819172	-.001868	-.001214	22.042276	100000	6	0
202	-.003446	.005244	26.650284	-.004599	.012056	31.055474	100000	6	0
203	-.004996	.018255	35.196218	-.004623	.023033	39.063002	100000	6	0
204	-.003573	.025756	42.627106	-.002076	.025972	45.861841	100000	6	0
205	-.000446	.023447	48.743058	.006977	.018175	51.249152	100000	6	0
206	.001841	.010165	53.319931	.002036	.000113	54.969884	100000	6	0
207	.001618	-.011357	56.194762	.000776	-.023645	56.985361	100000	6	0
208	-.000241	-.036239	57.295800	7.960946	0.000000	.020000	100000	6	0
221	326.48	326.46	385.70	385.67	6.00	100.00	6.1200.100000	1	0
222	1146.34	1146.29	1354.72	1354.66	6.00	100.00	6.1200.100000	2	0
223	2827.16	2816.80	3252.45	3239.18	6.00	100.00	6.1200.100000	3	0
224	4058.79	4065.72	4308.07	4317.59	6.00	100.00	6.1200.100000	4	0
225	5526.68	5521.96	6038.11	6032.12	6.00	100.00	6.1200.100000	5	0
226	9542.68	9537.19	9567.74	9560.10	6.00	100.00	6.1200.100000	6	0

TABLE IX.- UNIFORM BEAM ANALYTICAL REPRESENTATIONS USED FOR CORRELATION STUDIES

Property	Case 1	Case 2	Case 3[a]	Case 4[a]
$(EI)_b$, lb-in^2	8000	8000	8000	8000
$(EI)_c$, lb-in^2	32×10^9	32×10^9	32×10^9	32×10^9
GJ, lb-in^2	8717	8717	8717	8717
w, lb/in.	0.024	0.024	0.024	0.024
W, lb	0	0.6	0	0
I'_{bb}, lb-sec^2	-0.5176×10^{-5}	-0.5176×10^{-5}	-0.5176×10^{-5}	-0.5176×10^{-5}
I'_{cc}, lb-sec^2	0.1087×10^{-3}	0.1087×10^{-3}	0.1087×10^{-3}	0.1087×10^{-3}
N	25	25	25	25
$\bar{z}$, in.	1.0	1.0	1.0	1.0
S_b, in.	0	0	0	0
S_c, in.	0	0	0	-0.7454
r_b, in.	0	0	0	0
r_c, in.	0	0	0	0
Ω, rpm	0	0	0	0
θ_c, deg	0	0	0	0
θ_t, deg	0	0	0	0
K_ζ, in-lb/rad	0	0	∞	∞
K_{ip}, lb/in.	0	0	∞	∞
K_{op}, lb/in.	0	0	0	0
K_T, in-lb/rad	0	0	∞	∞
K_β, in-lb/rad	0	0	0	0
K_ψ, in-lb/rad	0	0	0	0
M_{ip}, lb-sec^2/in. . . .	0	0	0	0
M_{op}, lb-sec^2/in. . . .	0	0	0	0

[a] Program was modified to achieve proper boundary conditions at the outboard tip.

TABLE X.- ROTATING PROPELLER ANALYTICAL REPRESENTATION USED FOR CORRELATION STUDIES (CASE 5)

z, in.	$(EI)_b$, lb-in^2	$(EI)_c$, lb-in^2	$GJ + EB_1\bar{\theta}'^2$, lb-in^2	w, lb/in.	θ_t, deg	I'_{bb}, lb-sec^2	I'_{cc}, lb-sec^2
4.0	∞	∞	∞	------	0	-----------	--------
5.0	10 000 000	300 000 000	1 000 000	0.5000	-2.55	0.000010000	0.002000
6.0	800 000	100 000 000	700 000	.3993	-5.10	.000007137	.001692
7.0	180 000	51 000 000	575 000	.3100	-7.65	.000005069	.001400
8.0	140 000	47 900 000	480 000	.2761	-10.20	.000003001	.001265
9.0	110 000	46 800 000	420 000	2600	-12.75	.000002507	.001225
10.0	92 000	45 800 000	382 000	.2437	-15.50	.000002013	.001184
11.0	72 000	45 100 000	348 000	.2408	-18.25	.000001755	.001204
12.0	63 000	44 400 000	320 000	.2379	-21.00	.000001496	.001223
13.0	53 000	44 100 000	295 000	.2294	-23.55	.000001270	.001228
14.0	42 000	43 800 000	275 000	.2209	-26.05	.000001043	.001233
15.0	37 000	43 800 000	261 000	.2166	-28.45	.000001042	.001245
16.0	32 000	43 800 000	251 000	.2123	-31.05	.000001042	.001256
17.0	29 500	44 100 000	241 000	.2081	-33.45	.000000882	.001263
18.0	27 000	44 400 000	230 000	.2039	-35.55	.000000722	.001270
19.0	26 500	45 800 000	214 000	.2018	-37.60	.000000691	.001289
20.0	26 000	47 200 000	196 000	.1997	-39.55	.000000659	.001307
21.0	25 500	49 300 000	173 000	.1955	-41.50	.000000638	.001311
22.0	25 000	51 400 000	153 000	.1912	-43.00	.000000616	.001314
23.0	24 500	53 700 000	125 000	.1912	-44.30	.000000627	.001340
23.5	24 000	54 900 000	100 000	.1912	-44.90	.000000632	.001353
24.0	24 000	56 000 000	78 000	.1912	-45.55	.000000638	.001366

TABLE XI.- COMPARISONS OF COMPUTED FREE-FREE BENDING NATURAL FREQUENCIES AND MODE SHAPES WITH EXACT SOLUTIONS FOR A NONROTATING, UNIFORM BEAM (CASE 1)

Natural frequency, ω, cpm

First mode		Second mode		Third mode		Fourth mode		Fifth mode	
Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact
968.68	969.88	2667.9	2673.5	5225.8	5241.2	8631.3	8663.9	12883	12942

Mode shape

z_j/R	First mode		Second mode		Third mode		Fourth mode		Fifth mode	
	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact
0	-0.608	-0.608	0	0	0.714	0.711	0	0	-0.714	-0.707
.04	-.605	-.604	-.108	-.108	.697	.694	.199	.198	-.672	-.665
.08	-.594	-.594	-.212	-.212	.647	.644	.381	.379	-.551	-.545
.12	-.576	-.576	-.312	-.311	.566	.564	.534	.531	-.365	-.360
.16	-.552	-.551	-.403	-.403	.459	.457	.645	.641	-.136	-.132
.20	-.521	-.520	-.484	-.483	.330	.328	.705	.700	.109	.111
.24	-.483	-.482	-.551	-.551	.186	.184	.709	.704	.342	.341
.28	-.439	-.438	-.605	-.604	.034	.032	.657	.652	.534	.531
.32	-.389	-.389	-.642	-.641	-.119	-.120	.555	.549	.664	.659
.36	-.333	-.333	-.662	-.661	-.265	-.266	.408	.403	.716	.709
.40	-.272	-.272	-.663	-.662	-.397	-.397	.231	.226	.684	.675
.44	-.207	-.206	-.646	-.645	-.508	-.507	.036	.032	.572	.563
.48	-.136	-.136	-.611	-.609	-.592	-.591	-.160	-.163	.394	.384
.52	-.062	-.062	-.557	-.556	-.645	-.643	-.341	-.343	.171	.162
.56	.016	.016	-.486	-.485	-.663	-.661	-.493	-.493	-.070	-.077
.60	.097	.098	-.399	-.397	-.646	-.643	-.602	-.600	-.301	-.305
.64	.181	.182	-.297	-.295	-.592	-.588	-.658	-.655	-.492	-.493
.68	.268	.268	-.182	-.180	-.503	-.499	-.657	-.652	-.620	-.617
.72	.356	.356	-.055	-.053	-.381	-.377	-.595	-.588	-.667	-.661
.76	.446	.446	.082	.084	-.231	-.226	-.474	-.467	-.625	-.616
.80	.537	.537	.226	.227	-.056	-.052	-.302	-.294	-.494	-.483
.84	.629	.629	.375	.377	.137	.142	-.086	-.078	-.284	-.272
.88	.721	.721	.529	.530	.344	.347	.163	.170	-.010	.001
.92	.814	.814	.685	.686	.559	.562	.433	.438	.308	.317
.96	.907	.907	.842	.843	.779	.786	.715	.718	.650	.655
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XII.- COMPARISONS OF COMPUTED CLAMPED-FREE BENDING NATURAL FREQUENCIES AND MODE SHAPES WITH EXACT SOLUTIONS FOR A NONROTATING, UNIFORM BEAM (CASE 1)

Natural frequency, ω, cpm

First mode		Second mode		Third mode		Fourth mode		Fifth mode	
Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact
609.23	690.68	3811.1	3820.8	10654	10698	20842	20964	34394	34656

Mode shape

x_j/R	First mode		Second mode		Third mode		Fourth mode		Fifth mode	
	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact
0	0	0	0	0	0	0	0	0	0	0
.04	.003	.003	-.017	-.017	.044	.044	-.084	-.083	.133	.130
.08	.011	.011	-.062	-.062	.157	.156	-.277	-.274	.410	.401
.12	.024	.024	-.128	-.128	.307	.306	-.500	-.494	.667	.650
.16	.042	.042	-.211	-.210	.466	.464	-.681	-.671	.778	.756
.20	.064	.064	-.301	-.301	.608	.605	-.766	-.754	.583	.660
.24	.090	.090	-.395	-.394	.712	.707	-.729	-.715	.396	.377
.28	.120	.120	-.485	-.484	.761	.755	-.567	-.553	-.002	-.013
.32	.154	.154	-.566	-.565	.748	.741	-.307	-.295	-.393	-.392
.36	.190	.190	-.634	-.633	.670	.663	.005	.013	-.659	-.646
.40	.230	.230	-.685	-.684	.534	.526	.313	.316	-.719	-.697
.44	.272	.272	-.715	-.713	.350	.343	.559	.556	-.556	-.529
.48	.316	.317	-.722	-.720	.137	.131	.698	.689	-.220	-.196
.52	.363	.363	-.704	-.701	-.085	-.091	.705	.691	.185	.198
.56	.411	.411	-.661	-.658	-.296	-.299	.579	.562	.533	.532
.60	.461	.461	-.592	-.589	-.473	-.474	.344	.326	.717	.700
.64	.512	.512	-.500	-.497	-.598	-.597	.047	.031	.681	.652
.68	.564	.564	-.385	-.382	-.659	-.655	-.256	-.266	.436	.403
.72	.617	.617	-.251	-.247	-.647	-.641	-.505	-.508	.060	.032
.76	.671	.671	-.099	-.096	-.561	-.553	-.649	-.643	-.328	-.343
.80	.725	.725	.067	.070	-.404	-.395	-.656	-.643	-.603	-.600
.84	.780	.780	.244	.246	-.188	-.178	-.518	-.499	-.673	-.652
.88	.835	.835	.428	.430	.076	.085	-.247	-.226	-.501	-.467
.92	.890	.890	.617	.618	.371	.378	.123	.141	-.113	-.078
.96	.945	.945	.808	.809	.682	.686	.550	.562	.415	.438
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XIII.- COMPARISONS OF COMPUTED FREE-FREE TORSION NATURAL FREQUENCIES AND MODE SHAPES WITH EXACT SOLUTIONS FOR A NONROTATING, UNIFORM BEAM (CASE 1)

Natural frequency, ω', cpm

First mode		Second mode		Third mode		Fourth mode		Fifth mode	
Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact
11004	11012	21965	22023	32840	33035	43584	44047	54157	55058

Mode shape

z_j/R	First mode		Second mode		Third mode		Fourth mode		Fifth mode	
	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact
0	-1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000	-1.000
.04	-.992	-.992	.969	.969	-.930	-.930	.876	.876	-.809	-.809
.08	-.969	-.969	.876	.876	-.729	-.729	.536	.536	-.309	-.309
.12	-.930	-.930	.729	.729	-.426	-.426	.063	.063	.309	.309
.16	-.876	-.876	.536	.536	-.063	-.063	-.426	-.426	.809	.809
.20	-.809	-.809	.309	.309	.309	.309	-.809	-.809	1.000	1.000
.24	-.729	-.729	.063	.063	.637	.637	-.992	-.992	.809	.809
.28	-.637	-.637	-.187	-.187	.876	.876	-.930	-.930	.309	.309
.32	-.536	-.536	-.426	-.426	.992	.992	-.637	-.637	-.309	-.309
.36	-.426	-.426	-.637	-.637	.969	.969	-.187	-.187	-.809	-.809
.40	-.309	-.309	-.809	-.809	.809	.809	.309	.309	-1.000	-1.000
.44	-.187	-.187	-.930	-.930	.536	.536	.729	.729	-.809	-.809
.48	-.063	-.063	-.992	-.992	.187	.187	.969	.969	-.309	-.309
.52	.063	.063	-.992	-.992	-.187	-.187	.969	.969	.309	.309
.56	.187	.187	-.930	-.930	-.536	-.536	.729	.729	.809	.809
.60	.309	.309	-.809	-.809	-.809	-.809	.309	.309	1.000	1.000
.64	.426	.426	-.637	-.637	-.969	-.969	-.187	-.187	.809	.809
.68	.536	.536	-.426	-.426	-.992	-.992	-.637	-.637	.309	.309
.72	.637	.637	-.187	-.187	-.876	-.876	-.930	-.930	-.309	-.309
.76	.729	.729	.063	.063	-.637	-.637	-.992	-.992	-.809	-.809
.80	.809	.809	.309	.309	-.309	-.309	-.809	-.809	-1.000	-1.000
.84	.876	.876	.536	.536	.063	.063	-.426	-.426	-.809	-.809
.88	.930	.930	.729	.729	.426	.426	.063	.063	-.309	-.309
.92	.969	.969	.876	.876	.729	.729	.536	.536	.309	.309
.96	.992	.992	.969	.969	.930	.930	.876	.876	.809	.809
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XIV.- COMPARISONS OF COMPUTED CLAMPED-FREE TORSION NATURAL FREQUENCIES AND MODE SHAPES WITH EXACT SOLUTIONS FOR A NONROTATING, UNIFORM BEAM (CASE 1)

Natural frequency, ω, cpm

First mode		Second mode		Third mode		Fourth mode		Fifth mode	
Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact
5504.9	5505.8	16493	16517	27416	27529	38231	38541	48895	49552

Mode shape

z_j/R	First mode		Second mode		Third mode		Fourth mode		Fifth mode	
	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact	Computed	Exact
0	0	0	0	0	0	0	0	0	0	0
.04	.063	.063	-.187	-.187	.309	.309	-.426	-.426	.536	.536
.08	.125	.125	-.368	-.368	.588	.588	-.771	-.771	.905	.905
.12	.187	.187	-.536	-.536	.809	.809	-.969	-.969	.992	.992
.16	.249	.249	-.685	-.685	.951	.951	-.982	-.982	.771	.771
.20	.309	.309	-.809	-.809	1.000	1.000	-.809	-.809	.309	.309
.24	.368	.368	-.905	-.905	.951	.951	-.482	-.482	-.249	-.249
.28	.426	.426	-.969	-.969	.809	.809	-.063	-.062	-.729	-.729
.32	.482	.482	-.998	-.998	.588	.588	.368	.368	-.982	-.982
.36	.536	.536	-.992	-.992	.309	.309	.729	.729	-.930	-.930
.40	.588	.588	-.951	-.951	.000	.000	.951	.951	-.588	-.588
.44	.637	.637	-.876	-.876	-.309	-.309	.992	.992	-.063	-.063
.48	.685	.685	-.771	-.771	-.588	-.588	.844	.844	.482	.482
.52	.729	.729	-.637	-.637	-.809	-.809	.536	.536	.876	.876
.56	.771	.771	-.482	-.482	-.951	-.951	.125	.125	.998	.998
.60	.809	.809	-.309	-.309	-1.000	-1.000	-.309	-.309	.809	.809
.64	.844	.844	-.125	-.125	-.951	-.951	-.685	-.685	.368	.368
.68	.876	.876	.063	.063	-.809	-.809	-.930	-.930	-.187	-.187
.72	.905	.905	.249	.249	-.588	-.588	-.998	-.998	-.685	-.685
.76	.930	.930	.426	.426	-.309	-.309	-.876	-.876	-.969	-.969
.80	.951	.951	.588	.588	.000	.000	-.588	-.588	-.951	-.951
.84	.969	.969	.729	.729	.309	.309	-.187	-.187	-.637	-.637
.88	.982	.982	.844	.844	.588	.588	.249	.249	-.125	-.125
.92	.992	.992	.930	.930	.809	.809	.637	.637	.426	.426
.96	.998	.998	.982	.982	.951	.951	.905	.905	.844	.844
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE XV.- COMPARISONS OF COMPUTED UNCOUPLED BENDING AND TORSION NATURAL FREQUENCIES WITH REFERENCE SOLUTIONS FOR A CLAMPED, NONROTATING, UNIFORM BEAM WITH A TIP WEIGHT[a] (CASE 2)

Mode	Reference frequency,[b] cpm	Computed frequency, cpm
Vertical	266	270
Vertical	2 837	2 817
Torsion	5 506	5 505
Vertical	8 847	8 823
Torsion	16 517	16 493

[a] The tip weight, as represented, affects only the bending natural frequencies.

[b] See reference 10.

TABLE XVI.- COMPARISONS OF COMPUTED UNCOUPLED BENDING AND TORSION NATURAL FREQUENCIES WITH EXACT SOLUTIONS FOR A NONROTATING, SIMPLY SUPPORTED, UNIFORM BEAM (CASE 3)

Mode	Computed frequency, cpm	Exact frequency, cpm
Vertical	1 711	1 711
Vertical	6 846	6 846
Torsion	11 004	11 012
Vertical	15 402	15 403
Torsion	21 966	22 023
Vertical	27 380	27 382
Torsion	32 841	33 035
Vertical	42 779	42 785
Torsion	43 582	44 047
Torsion	54 158	55 058

TABLE XVII.- COMPARISONS OF COMPUTED COUPLED BENDING AND TORSION MODAL CHARACTERISTICS WITH EXACT SOLUTIONS FOR A NONROTATING, SIMPLY SUPPORTED, UNIFORM BEAM (CASE 4)

Mode	Computed		Exact solution	
	ω, cpm	ϕ/δ_y, in^{-1} (a)	ω, cpm	ϕ/δ_y, in^{-1} (a)
1	1 704	-0.011	1 704	-0.011
2	6 726	-.046	6 727	-.046
3	11 050	54.629	11 057	53.817
4	14 740	-.113	14 750	-.111
5	22 354	12.953	22 411	12.985
6	25 032	-.220	25 094	-.215
7	34 315	5.322	34 497	5.384
8	36 417	-.369	36 665	-.356

[a] The quantity δ_y is measured at the center of gravity.

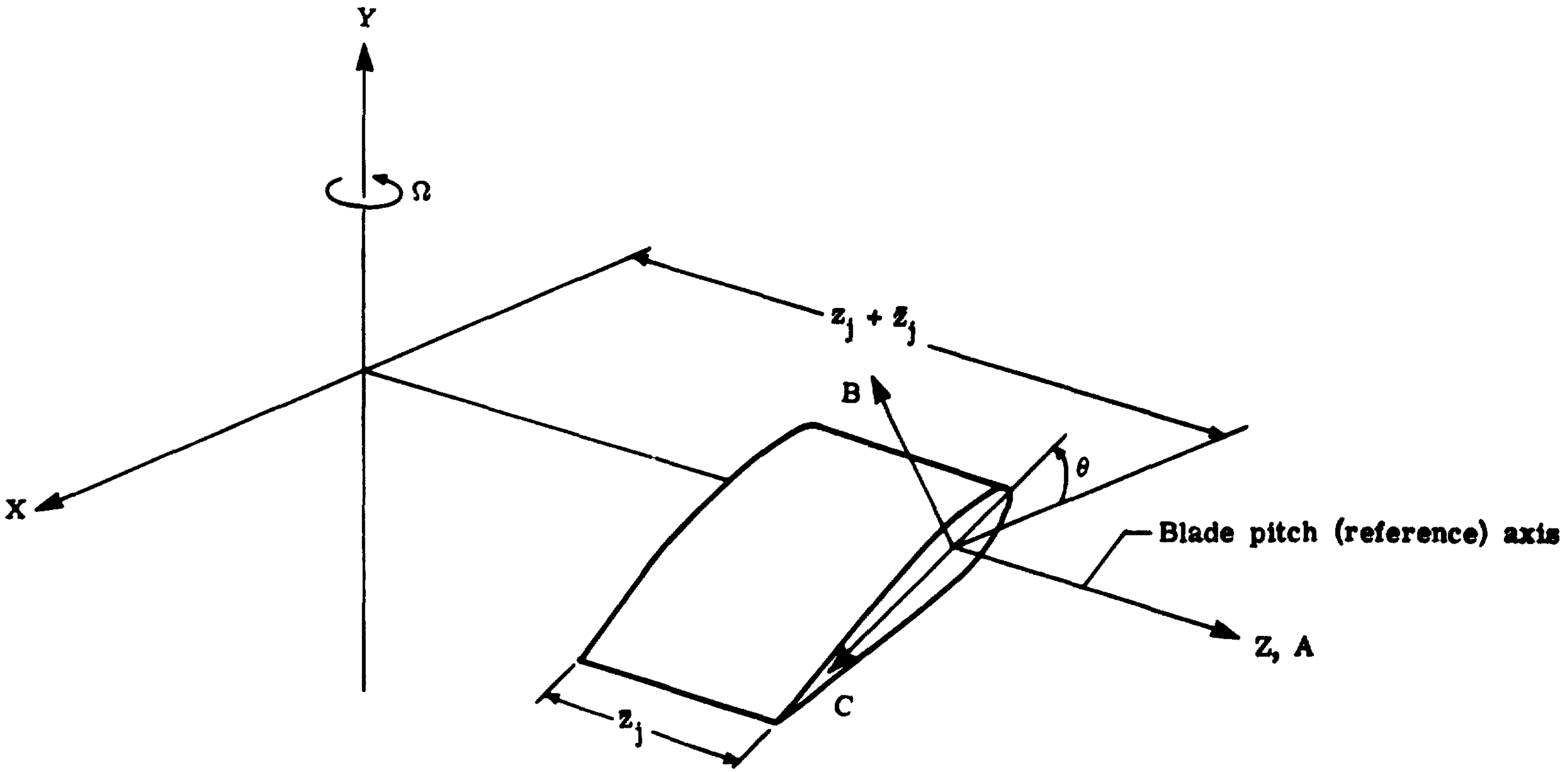
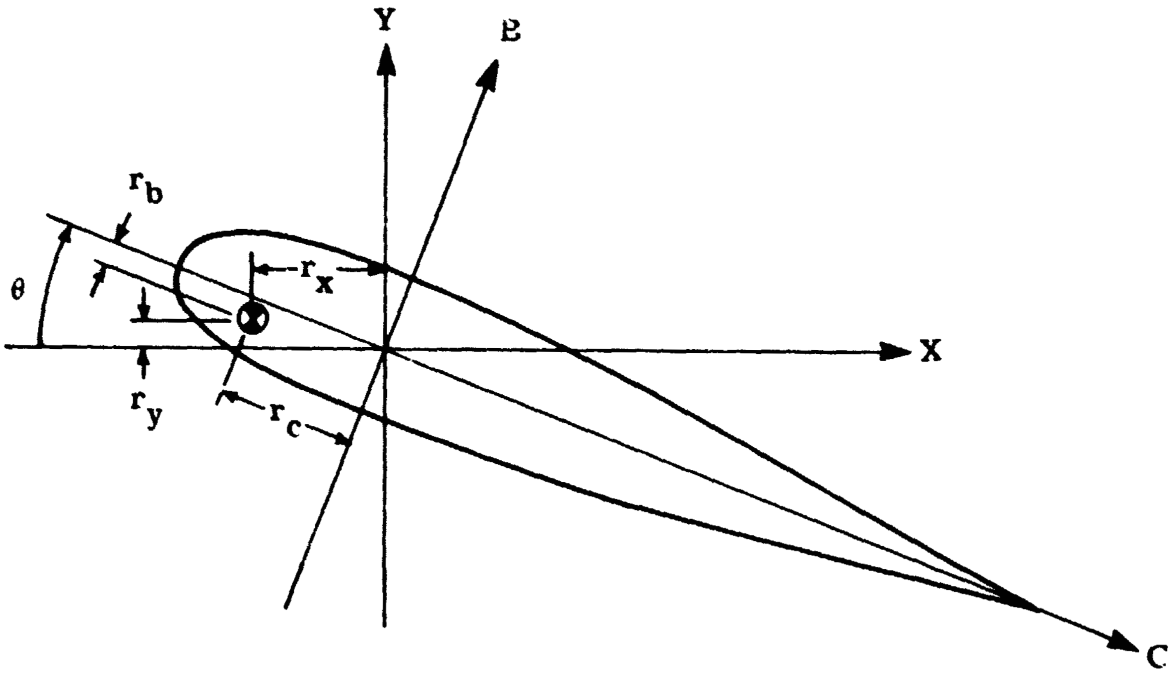
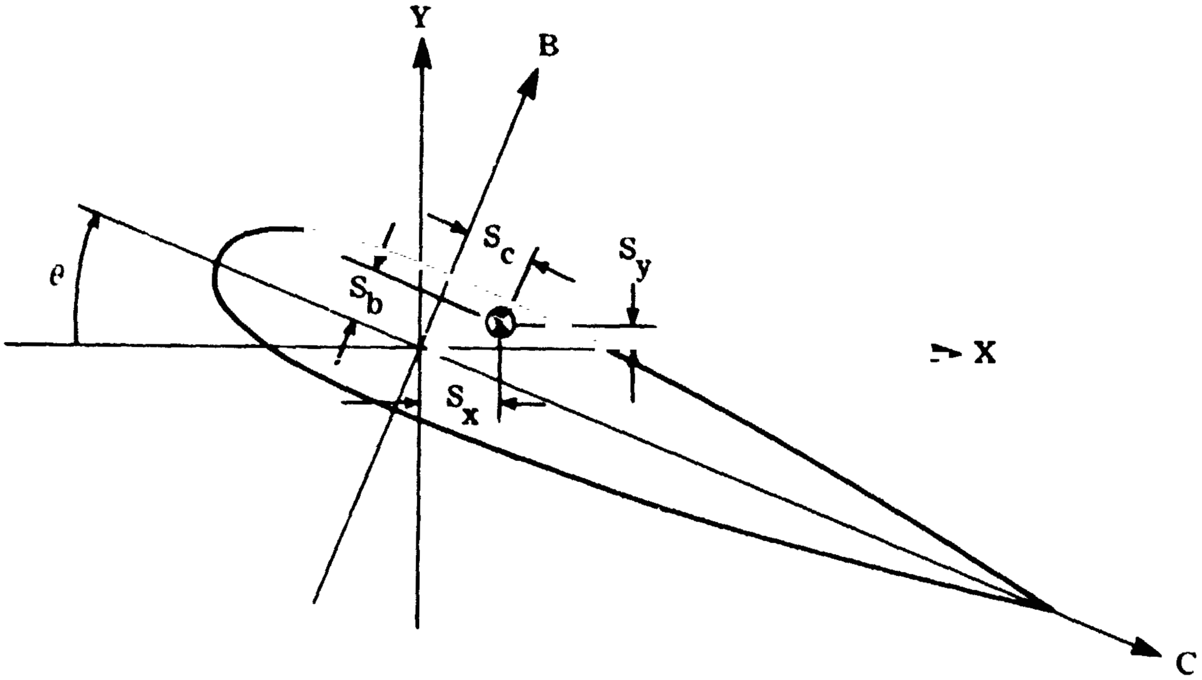


Figure 1.- Undeformed blade coordinate systems.

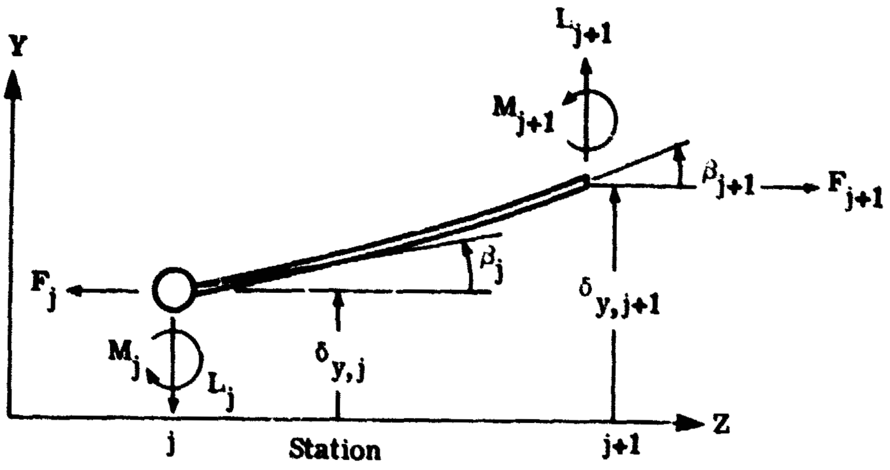


(a) Center-of-gravity offset.

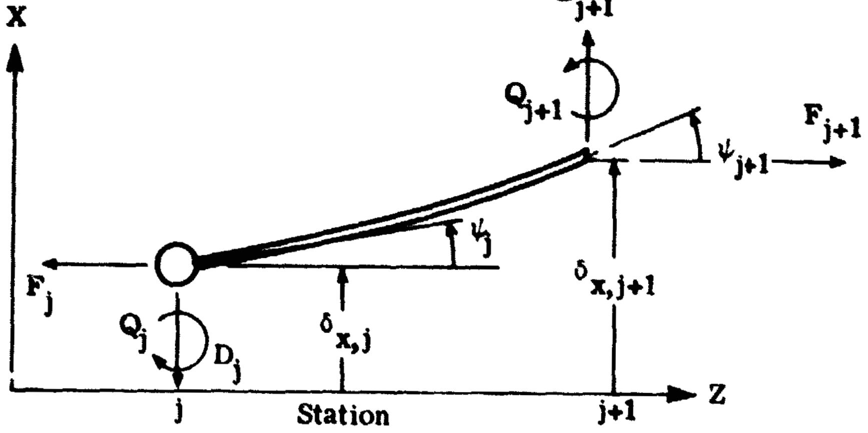


(b) Shear center offset.

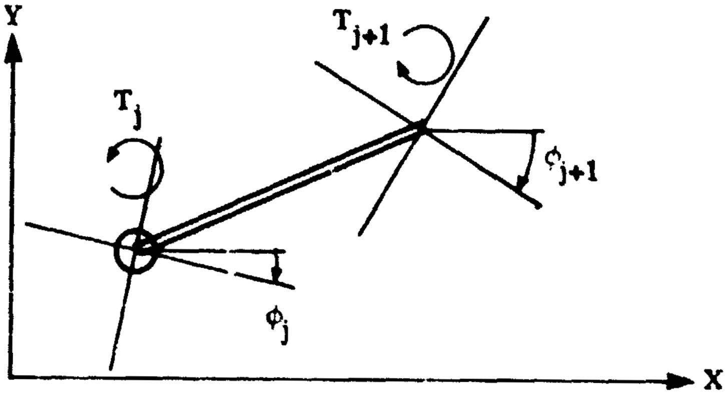
Figure 2.- Sign conventions for center-of-gravity and shear center offsets from pitch axis.



(a) Vertical plane.

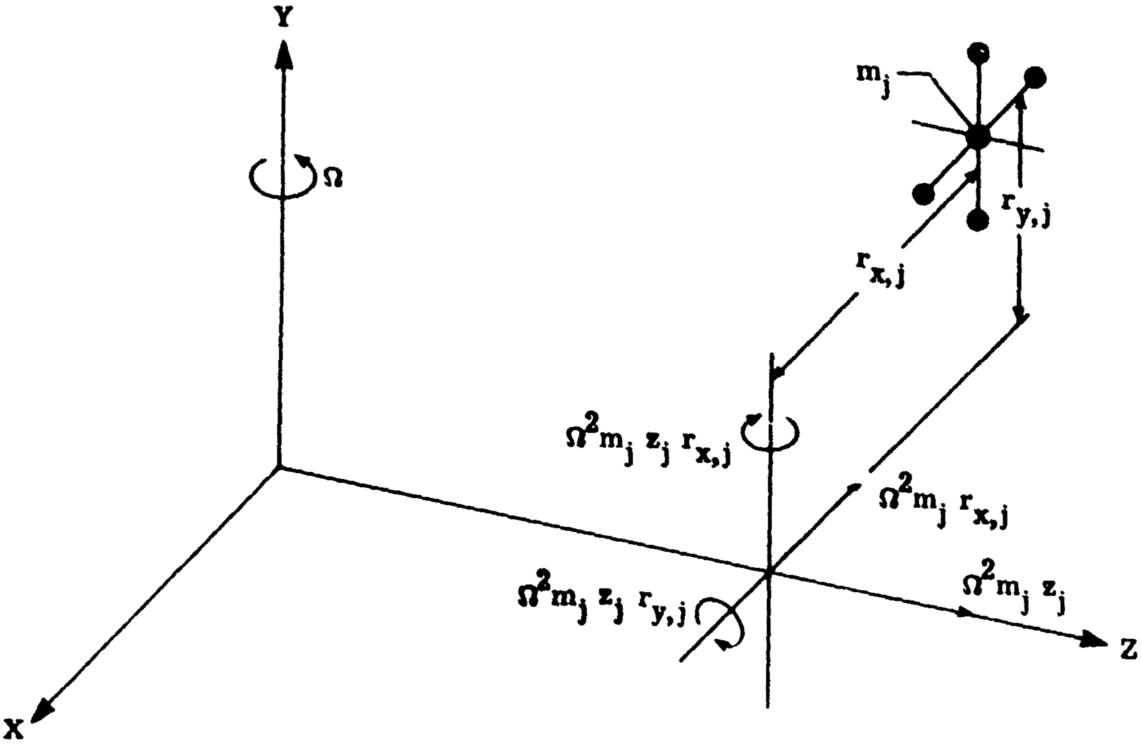


(b) Horizontal plane.



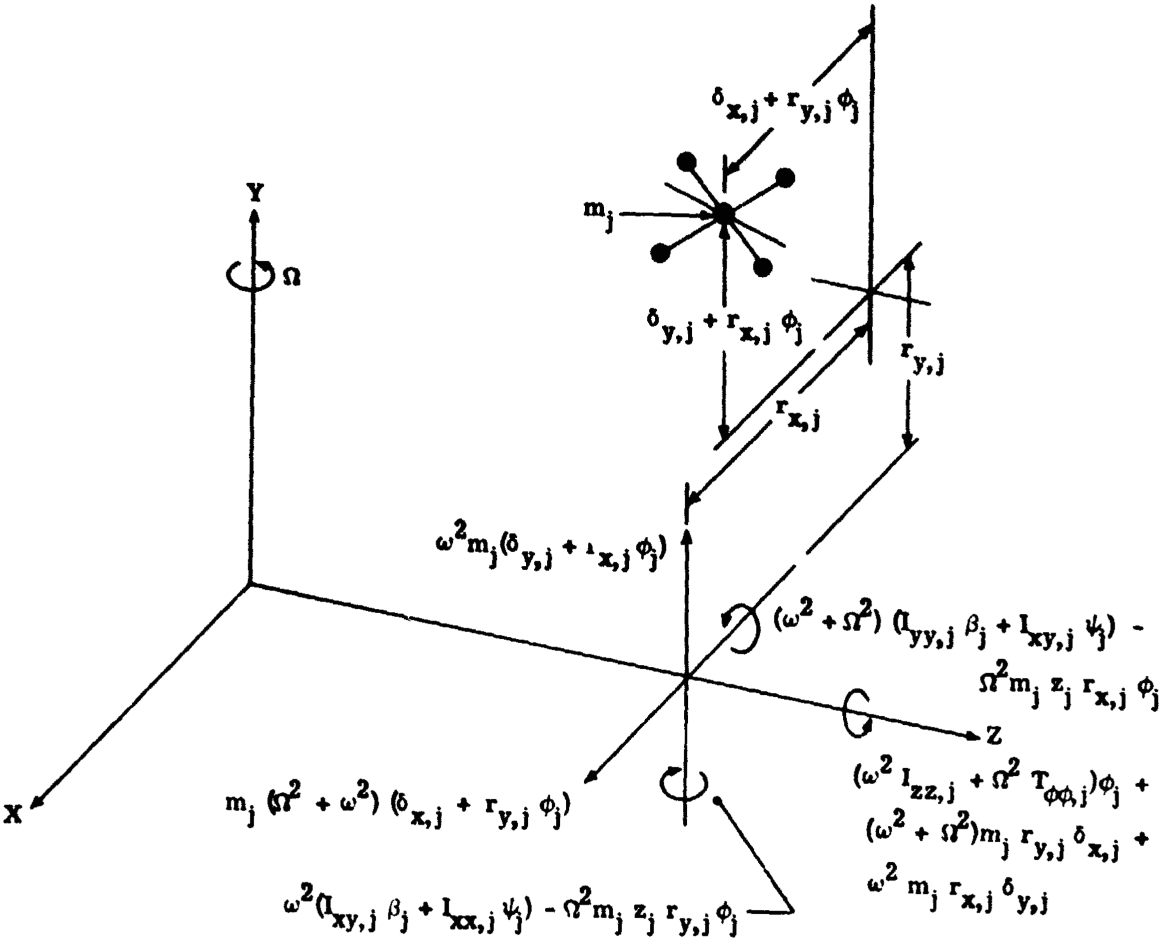
(c) Torsion plane.

Figure 3.- Sign conventions for deflections, slopes, shears, and moments associated with a blade segment. Arrows indicate positive directions.



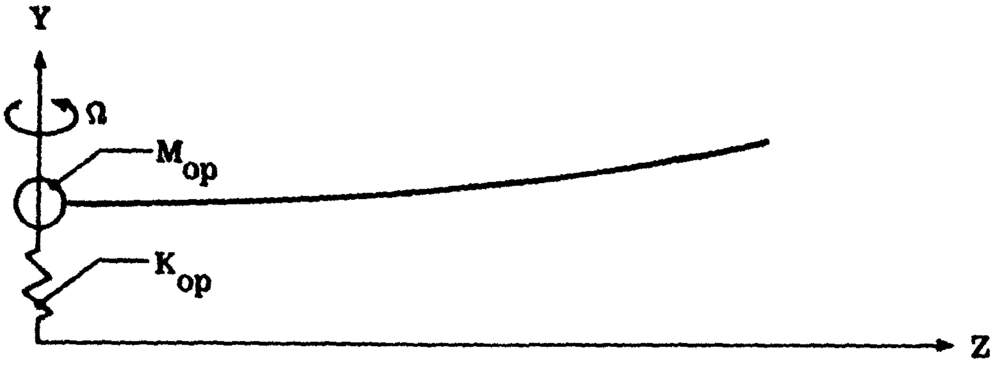
(a) Forces and moments independent of deformation.

Figure 4.- System of forces and moments at a single mass station.

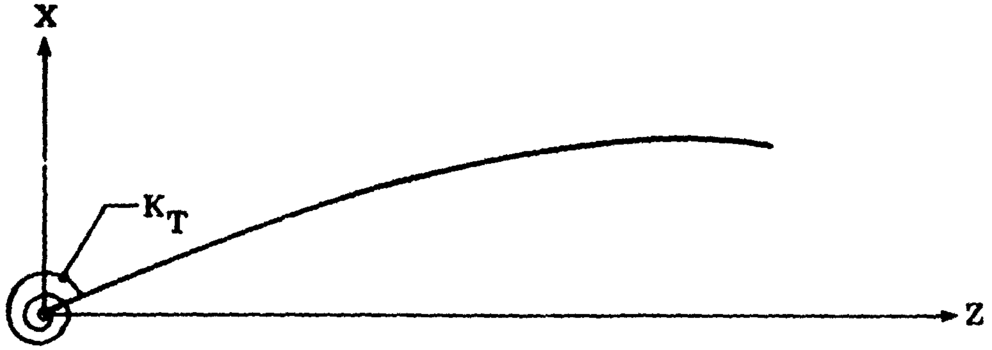


(b) Forces and moments dependent on deformation.

Figure 4.- Concluded.

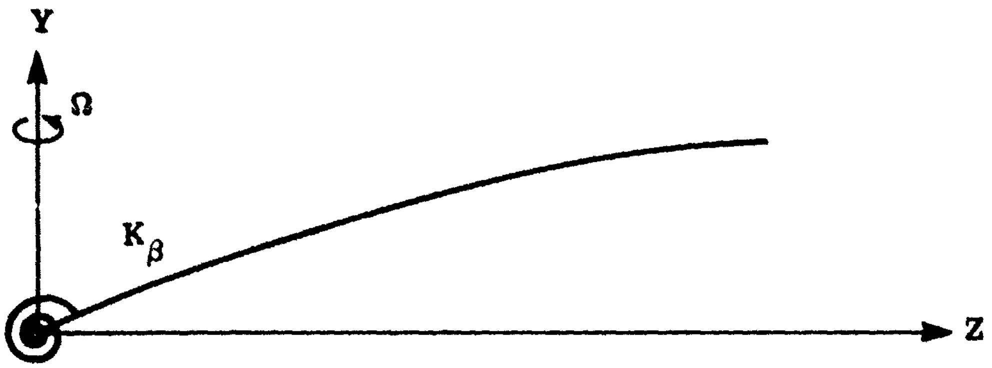


(a) Collective out-of-plane condition.

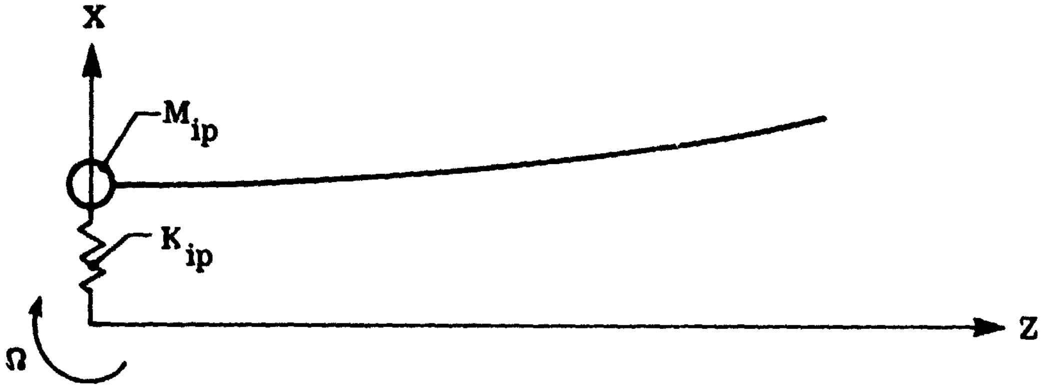


(b) Collective in-plane condition.

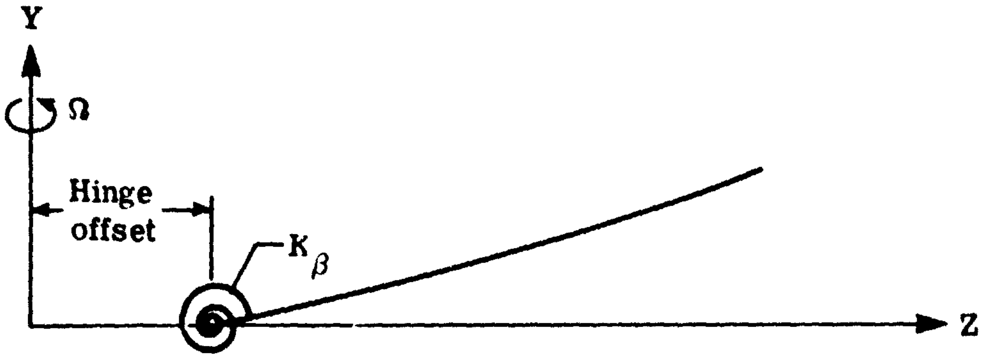
Figure 5.- Hub boundary condition representations.



(c) Cyclic out-of-plane condition.

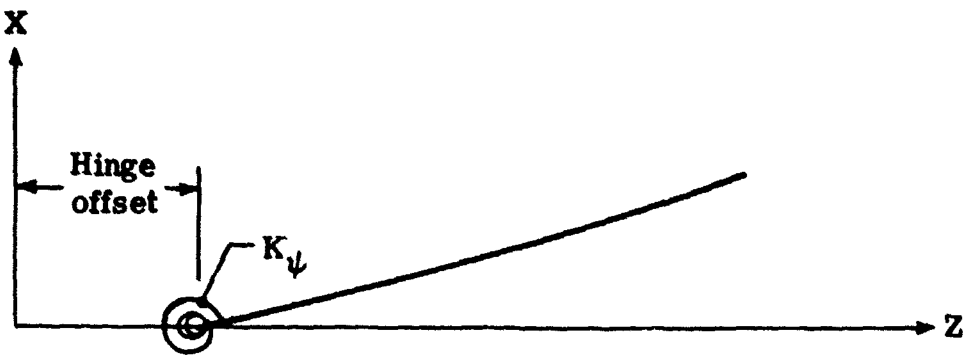


(d) Cyclic in-plane condition.

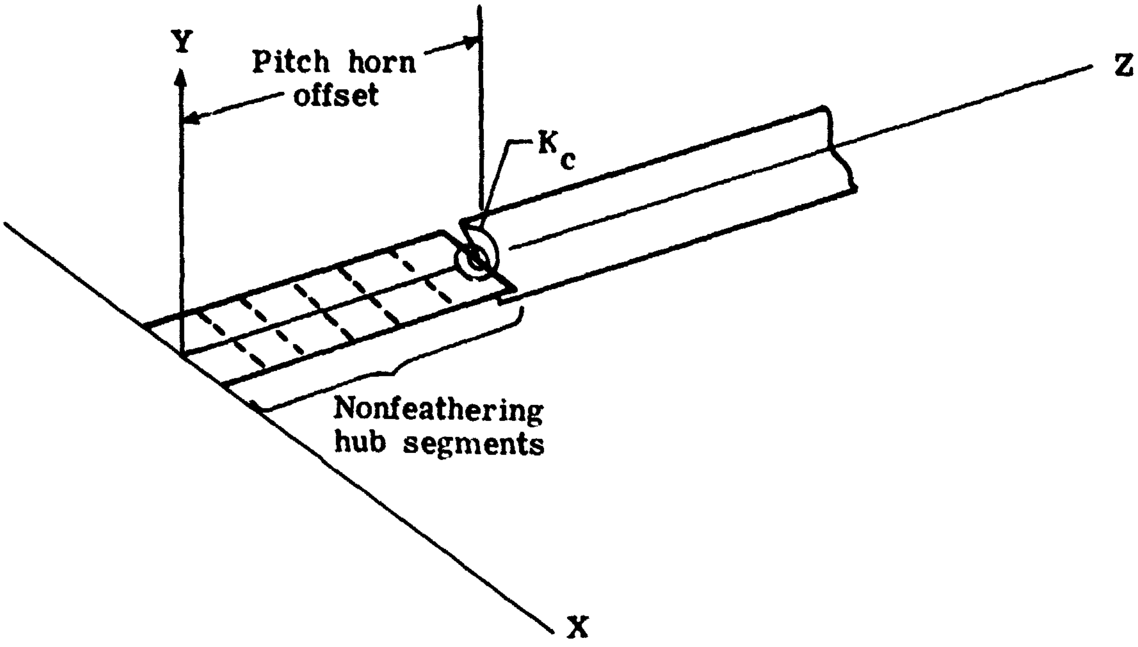


(e) Scissors out-of-plane condition.

Figure 5.- Continued.

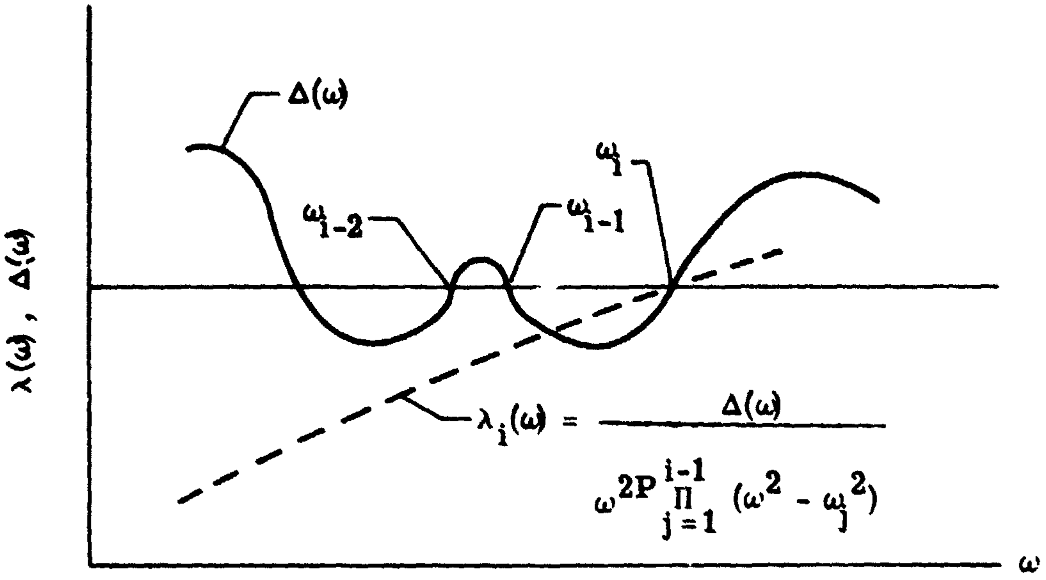


(f) Scissors in-plane condition.

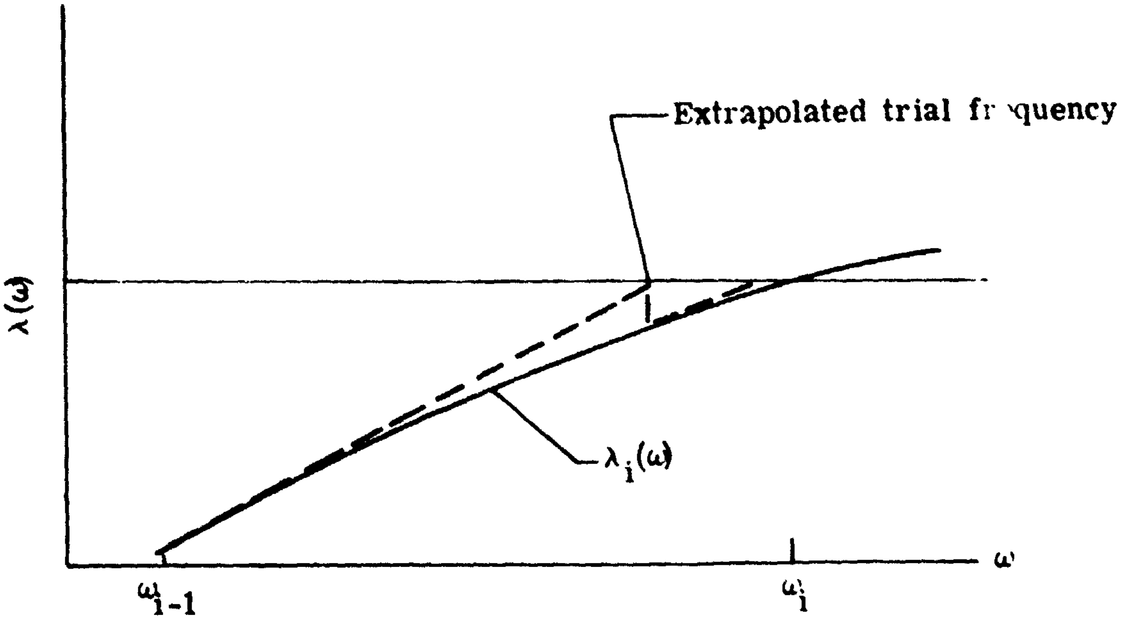


(g) Torsion condition.

Figure 5.- Concluded.



(a) Schematic of typical determinant and auxiliary functions.



(b) Illustration of natural frequency iteration technique.

Figure 6.- Determination of natural frequencies using the auxiliary function.

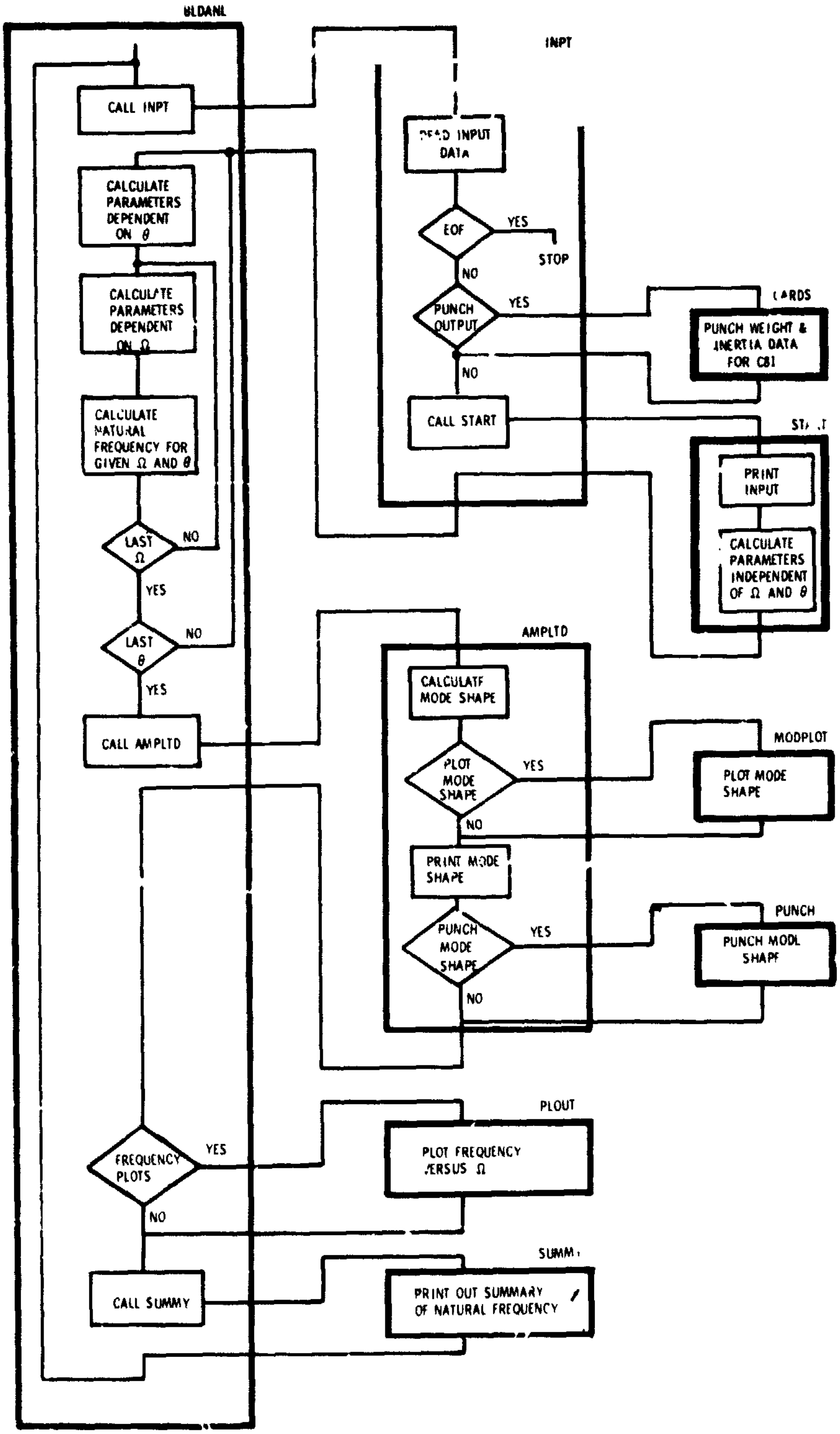


Figure 7.- Computer program flow chart and logic.

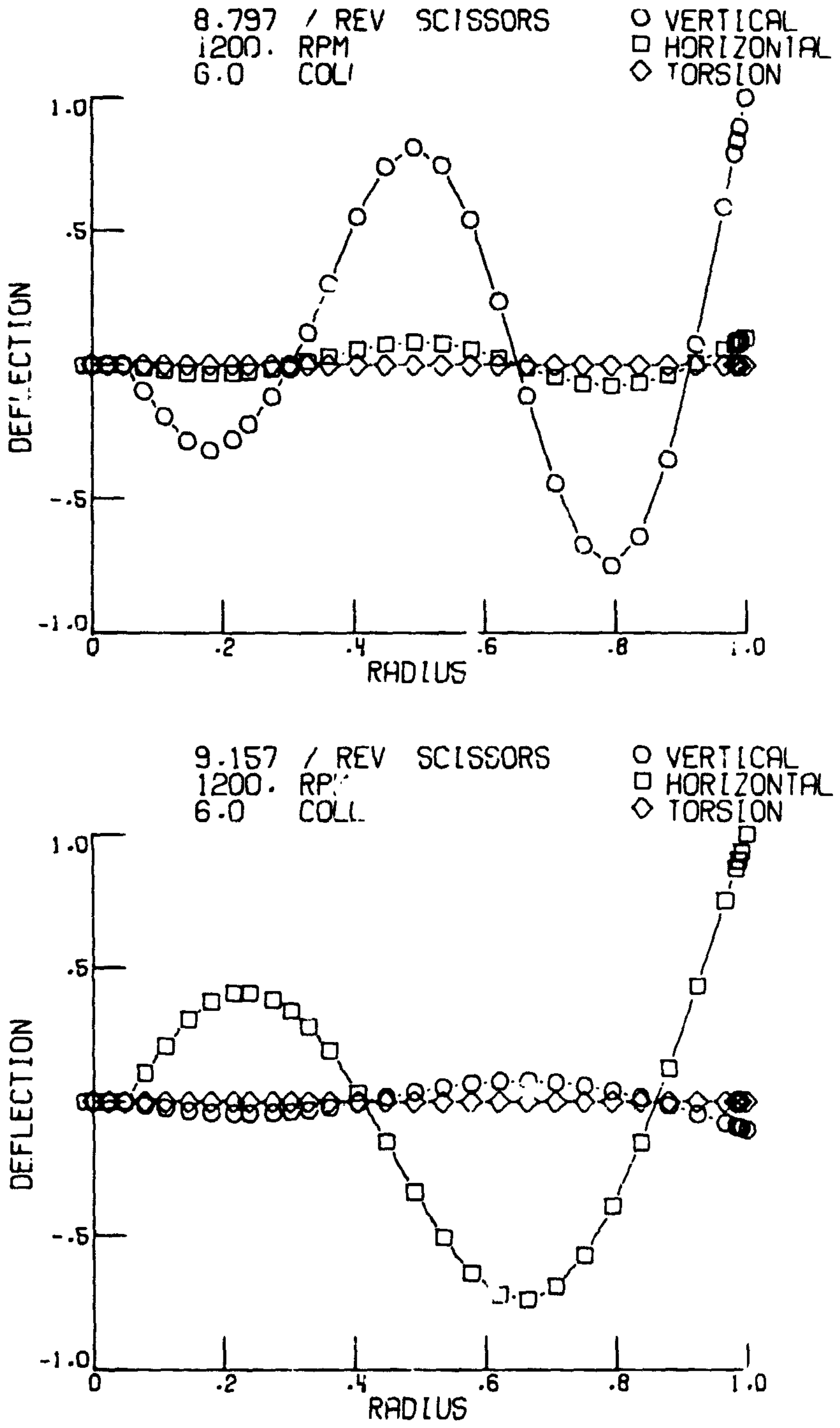


Figure 8.- Illustration of program optional output (mode shape plots).

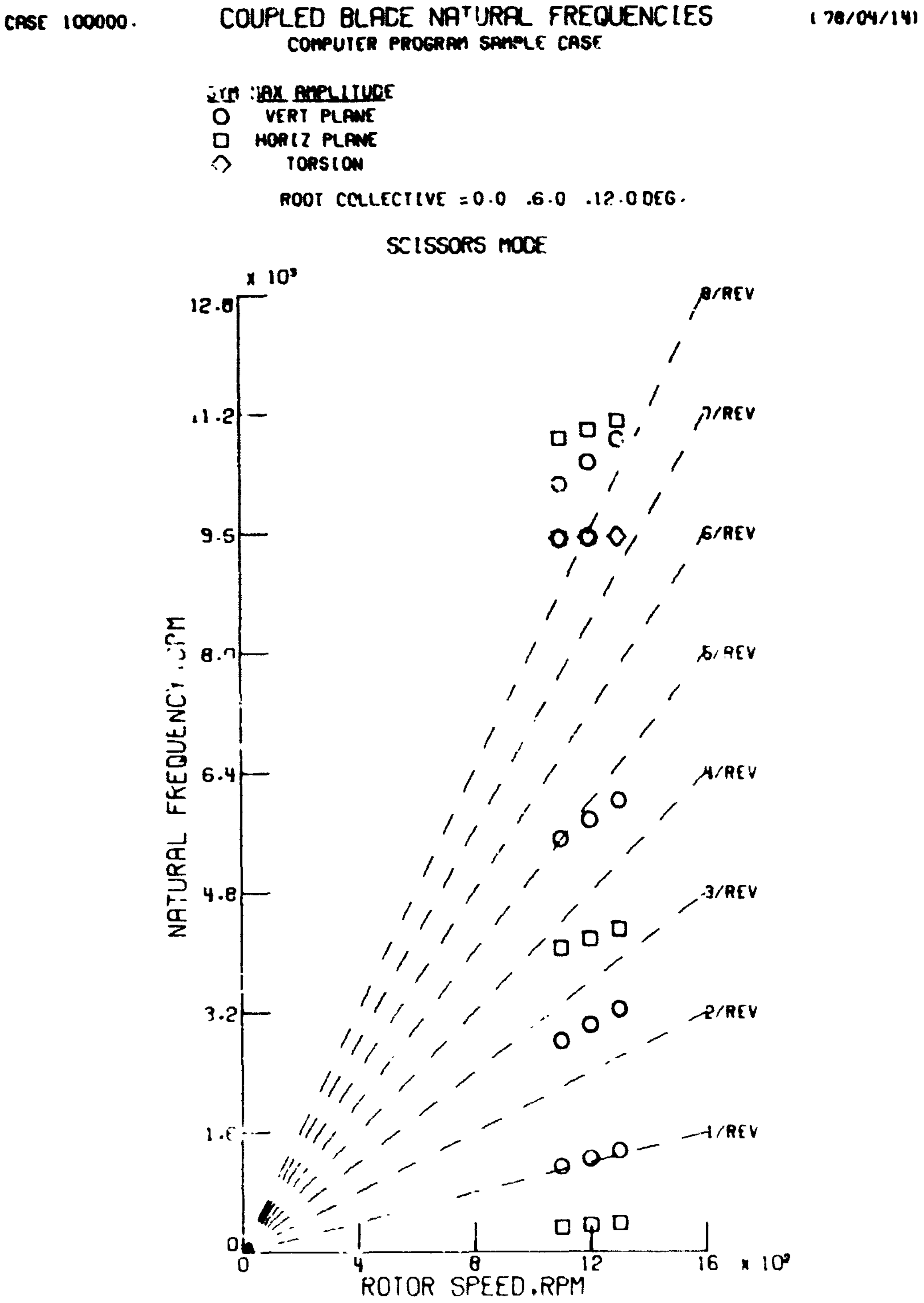
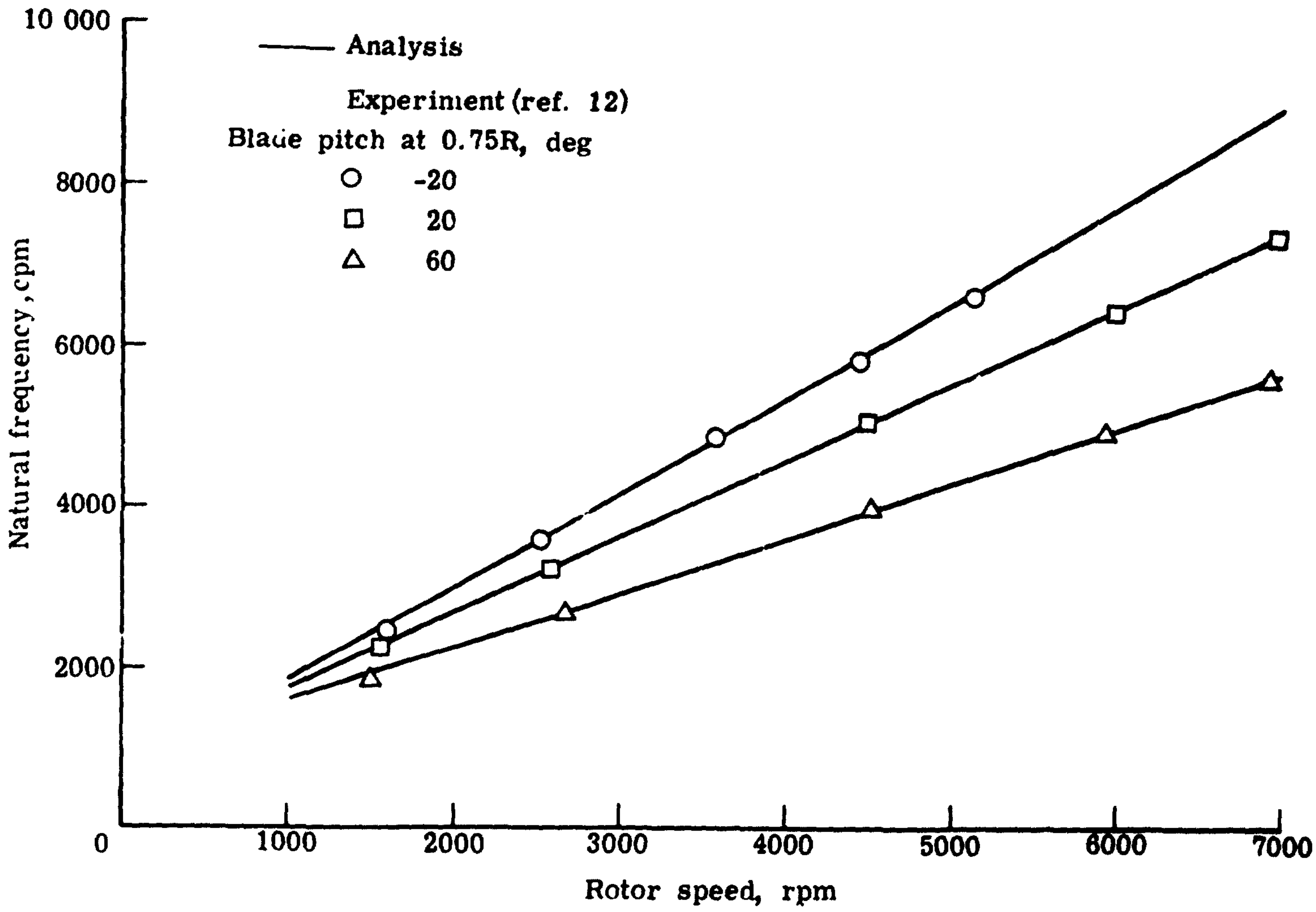
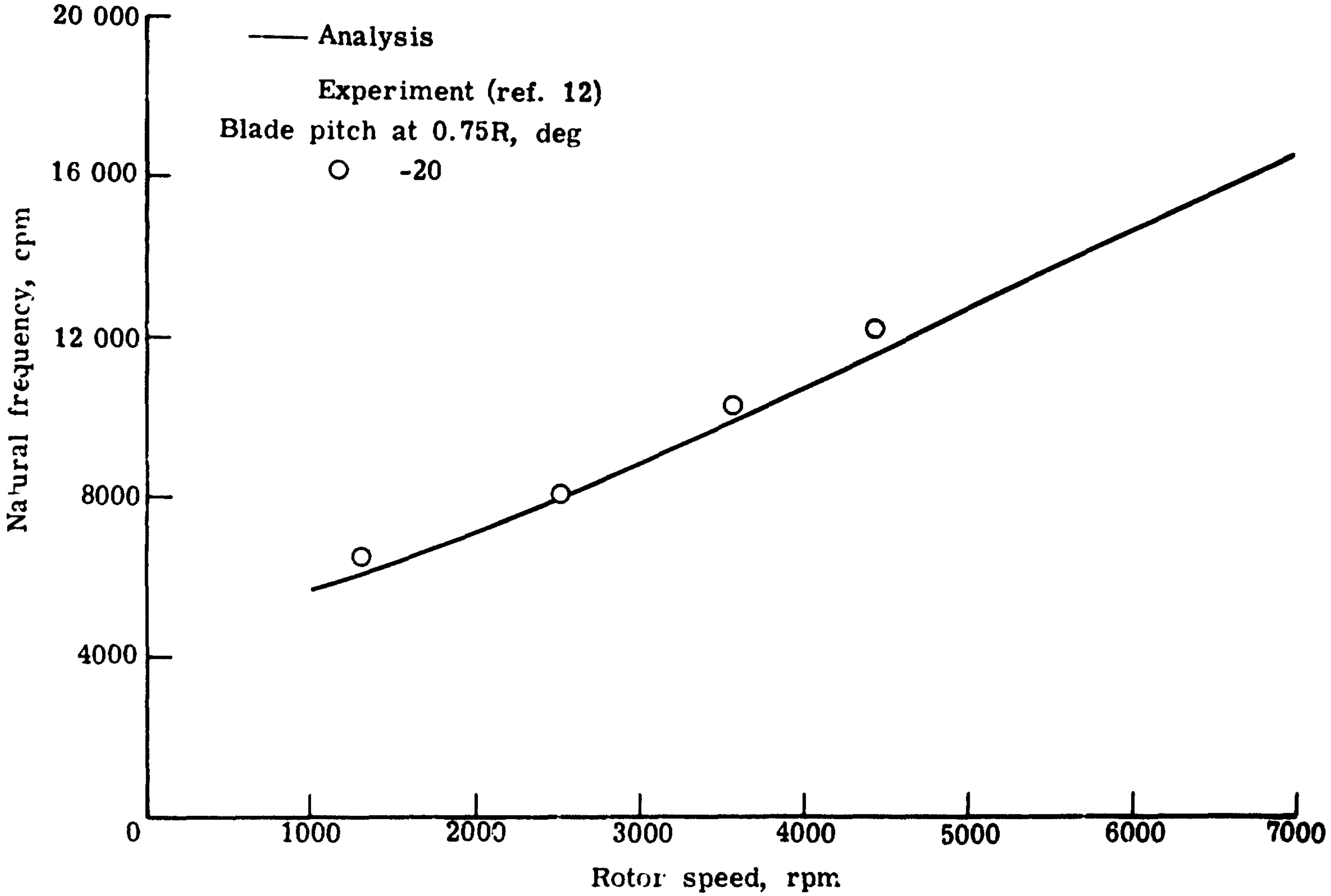


Figure 9.- Illustration of program optional output (plot of variation of natural frequency with rotor speed).



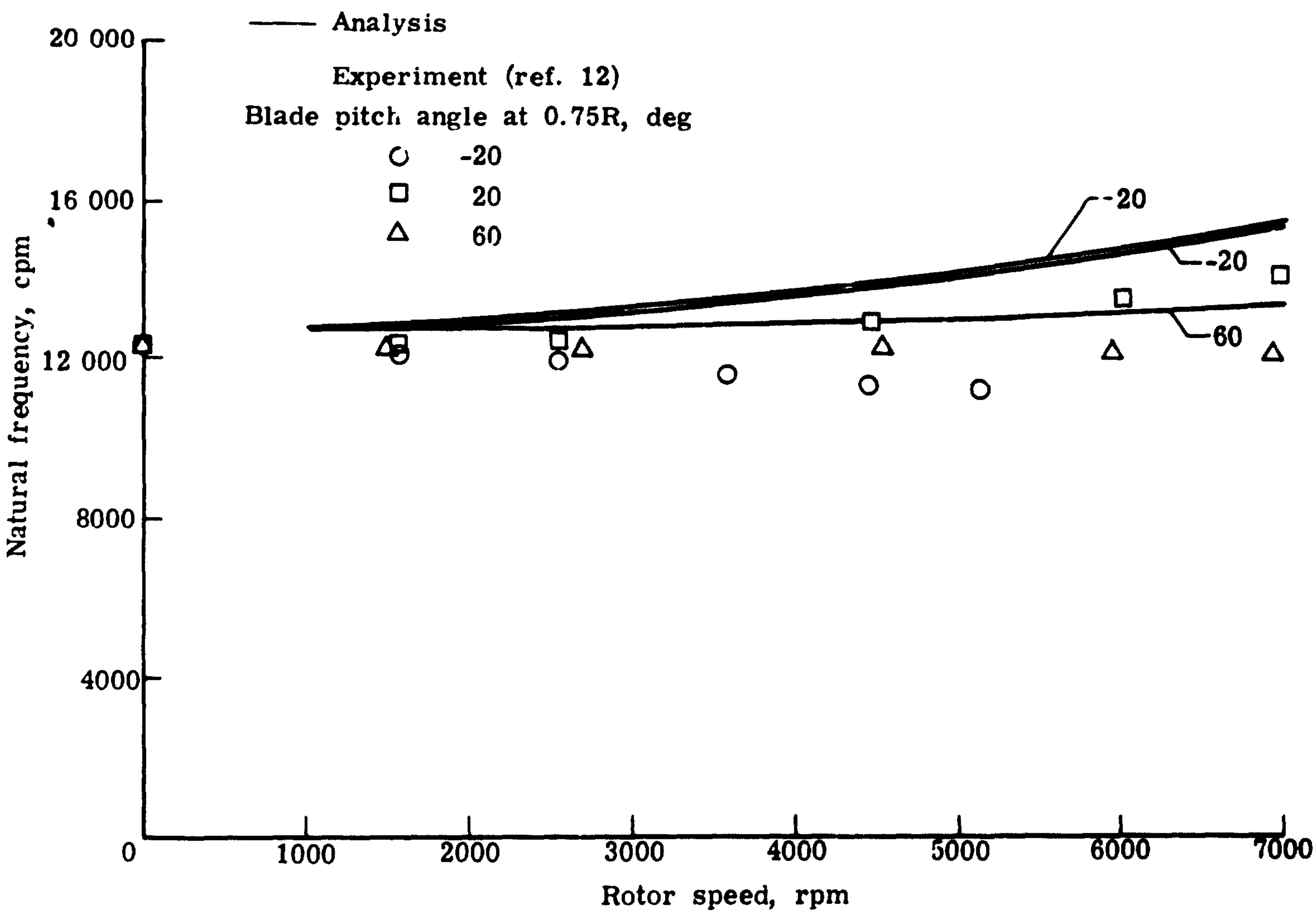
(a) First bending natural frequency.

Figure 10.- Comparison of calculated and measured natural frequencies for a rotating, nonuniform, twisted propeller blade.



(b) Second bending natural frequency.

Figure 10.- Continued.



(c) First torsion natural frequency.

Figure 10.- Concluded.

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15. Supplementary Notes

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16. Abstract

An existing computer program, used for predicting the natural frequencies and mode shapes of helicopter rotor blades, has been refined to improve program accuracy and versatility. The program is based on the Holzer-Myklestad approach adapted for rotating beams. Coupled vertical (out-of-plane), horizontal (in-plane), and torsional mode characteristics can be determined for a variety of hub and blade configurations of practical interest. The resulting program is documented by presenting the recursion equations and techniques for determining natural frequencies and mode shapes, input data requirements, and descriptions of various program outputs. The accuracy of the program is demonstrated by comparing computed results with exact solutions to classical problems and experimental data.

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