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ABSTRACT

An estimation has been made of the principal long period spherical harmonic parameters in the representation for the M_2 ocean tide from the orbital histories of three satellites — 1967-92A (TRANSIT), Starlette, and GEOS-3. The data used were primarily the evolution of the orbital inclinations of the satellites, with the addition of the longitude of the ascending node from GEOS-3. The results are:

$$C^+_{22} = 3.42 \pm 0.24 \text{ cm.} \qquad \epsilon^+_{22} = 325^\circ\!.5 \pm 3^\circ\!.9$$

$$C^+_{42} = 0.97 \pm 0.12 \text{ cm.} \qquad \epsilon^+_{42} = 124^\circ\!.0 \pm 6^\circ\!.9$$

These values agree quite well with recent numerical models and another recent determination from satellite data.

Dissipational tidal friction in the oceans is known to provide the largest contribution to the observed deceleration in the lunar mean longitude. Further, only the second degree components of the ocean tide contribute significantly to this secular decay (Lambeck, 1975). The M_2 parameters obtained here infer an $\dot{N}$ of -25 ± 3 arc seconds/century^2, in good agreement with other investigators. The range of current determinations of $\dot{N}$ is from -24.6 to -27.2 arc second/century^2. Considerably different techniques have been used to derive the estimates: the study of ancient eclipses, transits of Mercury, Lunar Laser Ranging, and another satellite solution.

M_2 OCEAN TIDE PARAMETERS AND THE DECELERATION OF THE MOON'S MEAN LONGITUDE FROM SATELLITE ORBIT DATA

INTRODUCTION

In the last decade and a half, there have been a number of investigations concerning the effects of solid and fluid earth tides on satellite orbits. With improved tracking techniques, more accurate modelling of the Earth's gravity field, and more data from satellites in different orbits, it has been possible to extract increasingly better information from the satellite orbit data about tidal effects and the resulting implications regarding lunar orbit evolution.

Ocean tidal effects are most easily observed in the inclination histories of satellites, it being the best behaved of the orbital elements for tidal studies. In some instances, the perturbations can be seen satisfactorily in the longitude of the ascending node. Tidal constituents such as S_2, K_1, K_2, and P_1 typically cause variations up to several tenths of an arc second in amplitude, with periods of tens or hundreds of days. The most intriguing ocean tide to study is the M_2 because it has the greatest effect on the Earth's oceans. However, its influence on satellite orbital elements is small, usually no more than a few hundredths of an arc second in amplitude with periods averaging about two weeks. As a consequence, very precise data must be derived in order to obtain a significant recovery of parameters for the M_2 ocean tide. Table 1 presents a comparison of the expected and observed M_2 effects on the orbits of the three satellites used in this investigation.

Lambeck et al. (1974) presented analytical expressions in terms of spherical harmonics for the ocean tide effects on satellite orbits. They indicate that the observable perturbations are long period in nature (on the order of days) and are produced by only the first few low degree and order terms in the development of each tidal constituent.

This formulation of Lambeck et al. (1974) has been adopted by a number of recent investigations of satellite derived ocean tidal effects. Felsentreger et al. (1976) and

Felsentreger et al. (1978) estimated ocean tide parameters from GEOS-1, GEOS-2, and 1967-92A observational data. These were single satellite solutions making use of inclination data only, so separation between the second and fourth degree harmonics could not be obtained. Cazenave et al. (1977) studied the orbits of GEOS-1 and 1967-92A and presented estimates of the (2,2) and (4,2) parameters for the S_2 and M_2 ocean tides. However, their results for S_2 are probably corrupted by questionable solar radiation pressure modelling and their M_2 values are dominated by the 1967-92A data. Goad and Douglas (1977a) successfully derived values of the (2,2) and (4,2) parameters for the M_2 ocean tide from 1967-92A and GEOS-3 data. They also used these results to evaluate the deceleration of the Moon's mean longitude, according to the procedure described by Lambeck (1975).

In this present work, we have used considerably more data for 1967-92A and GEOS-3 than either our earlier work or that used by Goad and Douglas (1977a).* Furthermore, to strengthen the solution we have incorporated data from Starlette, which has a significantly different inclination than the other two satellites.

ANALYSIS PROCEDURE AND ANALYTIC FORMULATION

Our analysis procedure begins with obtaining a definitive set of satellite osculating orbital elements from the tracking data. These elements are then converted to "mean" Keplerian orbital elements. The "meaning" procedure consists of removing the first order perturbations from the osculating elements, and then passing these preliminary mean elements through a low-pass filter to numerically remove residual high frequency effects. The final mean elements are the result of filtering over two day spans. Note that prior to the filtering, the direct lunar perturbations are removed in order to eliminate errors in the lunar ocean tide parameter determination caused by damping the direct effect due to the

*There has been a considerable amount of cooperation between Goad and Douglas and the authors for the purpose of generating the precision mean element data. Thus, there is a substantial data overlap.

averaging (Goad and Douglas, 1977a). After filtering, the effects of the direct lunar perturbations are reinserted.

The mean elements are then processed through an orbit determination program which computes a theoretical orbit based upon long period, resonant and secular perturbations. This theoretical orbit is differenced with the mean elements to produce a set of "residuals" from which the tidal perturbations can be isolated.

In order to separate the ocean and solid Earth tidal effects, a solid Earth tide model with $k_2 = 0.3$ and lag angle $\delta_2 = 0°$ is used in the orbit determination procedure. This k_2 value has been well established to ±.01 from seismic data, Earth tide measurements, and Earth rotation observations. This modelling is necessary because the same frequencies are present in both tides; otherwise, meaningful ocean tide parameters could not be separated. In addition, Goad and Douglas (1977b) have shown that the only satellite derived M_2 ocean tide parameters consistent with both the recent numerical ocean tides and values of the deceleration of the lunar mean longitude are those obtained with $k_2 = 0.3$ and $\delta_2 = 0°$.

The formulation we use for the ocean tide potential is expressed in terms of the equatorial Keplerian orbital elements of the satellite (Lambeck et al., 1974; Felsentreger et al., 1976)

$$\Delta U = g\frac{\rho}{\bar{\rho}}\sum_{j=0}^{\infty}\sum_{\ell=0}^{j}\sum_{p=0}^{j}\sum_{q=-\infty}^{\infty}(1+k_j')\left(\frac{3}{2j+1}\right)\left(\frac{R}{a}\right)^{j+1} F_{j\ell p}(I)G_{jpq}(e)C_{j\ell}^{\pm}\cdot$$

$$\cdot\begin{bmatrix} \pm\sin\gamma_{j\ell pq}^{\pm} \\ \mp\cos\gamma_{j\ell pq}^{\pm}\end{bmatrix}\begin{matrix} j-\ell \text{ even} \\ j-\ell \text{ odd}\end{matrix} \tag{1}$$

where

$$\gamma_{j\ell pq}^{\pm} = (j-2p)\omega + (j-2p+q)M + \ell(\Omega-\theta) \pm 2\pi f T \pm \epsilon_{j\ell}^{\pm}$$

g = mean acceleration of gravity at surface of Earth

ρ = mean density of sea water

$\bar{\rho}$ = average density of the Earth

k_j' = load deformation coefficient of degree j

R = mean radius of the Earth

a = semi-major axis

I = inclination

e = eccentricity

ω = argument of perigee

M = mean anomaly

Ω = longitude of ascending node

θ = Greenwich mean sidereal time

$F_{j\ell p}(I), G_{jpq}(e)$ = inclination and eccentricity functions (Kaula, 1966)

$2\pi fT$ = argument for tidal constituent

$C_{j\ell}^{\pm}$ = spherical harmonic coefficients

$\epsilon_{j\ell}^{\pm}$ = phase angles

Long period terms occur only with $C_{j\ell}^{+}$ and when $j - 2p + q = 0$. Perturbations in the elements of each satellite can be found by substituting (1) into the Lagrange Planetary Equations and performing the integration under the assumption that only the arguments are time varying. The resulting perturbation equations for the principal second and fourth degree harmonics of the global M_2 ocean tide are as follows:

1967-92A

$$\Delta I = \frac{0.01003 \text{ arc sec}}{\text{cm}} C_{22}^{+} \sin(2\Omega - 2\lambda' + \epsilon_{22}^{+})$$

$$+ \frac{0.009606 \text{ arc sec}}{\text{cm}} C_{42}^{+} \sin(2\Omega - 2\lambda' + \epsilon_{42}^{+}) \qquad (2)$$

STARLETTE

$$\Delta I = \frac{0.006268 \text{ arc sec}}{\text{cm}} C^+_{22} \sin(2\Omega - 2\lambda' + \epsilon^+_{22})$$

$$- \frac{0.01190 \text{ arc sec}}{\text{cm}} C^+_{42} \sin(2\Omega - 2\lambda' + \epsilon^+_{42}) \tag{3}$$

GEOS-3

$$\Delta I = \frac{0.01288 \text{ arc sec}}{\text{cm}} C^+_{22} \sin(2\Omega - 2\lambda' + \epsilon^+_{22})$$

$$- \frac{0.003267 \text{ arc sec}}{\text{cm}} C^+_{42} \sin(2\Omega - 2\lambda' + \epsilon^+_{42}) \tag{4}$$

$$\Delta\Omega = \frac{-0.002394 \text{ arc sec}}{\text{cm}} C^+_{22} \cos(2\Omega - 2\lambda' + \epsilon^+_{22})$$

$$- \frac{0.03464 \text{ arc sec}}{\text{cm}} C^+_{42} \cos(2\Omega - 2\lambda' + \epsilon^+_{42}) \tag{5}$$

where

λ' = mean longitude of the Moon (referred to ecliptic)

At this point, we correct for the two-day averaging. When the averaging was performed, both the solid Earth and ocean tidal effects were present in the data. Therefore, in order to recover the uncorrupted ocean tide signals, the solid Earth tide model used in the orbit determination step must be reconstructed and a correction applied. The solid Earth tide potential outside the Earth is (Lambeck, 1975)

$$\Delta U_E = \frac{Gm^*}{a^*} \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \sum_{p=0}^{\ell} \sum_{q=-\infty}^{\infty} \sum_{j=0}^{\ell} \sum_{h=-\infty}^{\infty} k_\ell \left(\frac{R}{a^*}\right)^{\ell} \left(\frac{R}{a}\right)^{\ell+1} \frac{(\ell - m)!}{(\ell + m)!} \eta_m F_{\ell m p}(I^*) G_{\ell p q}(e^*) \cdot$$

$$\cdot F_{\ell m j}(I) G_{\ell j h}(e) \cos(\nu^*_{\ell m p q} - \nu_{\ell m j h} + \epsilon_{\ell m p q}) \tag{6}$$

where

$$\nu^*_{\ell mpq} = (\ell - 2p)\omega^* + (\ell - 2p + q)M^* + m\Omega^*$$

$$\nu_{\ell mjh} = (\ell - 2j)\omega + (\ell - 2j + h)M + m\Omega$$

Here, $(a^*, e^*, I^*, M^*, \omega^*, \Omega^*)$ are the equatorial Keplerian elements of the Moon, m^* is the mass of the Moon, and k_ℓ is the Love number of degree ℓ.

Also,

$$\eta_m = \begin{cases} 1, & m=0 \\ 2, & m\neq 0. \end{cases}$$

Again, the long period part of the potential is extracted and substituted into the Lagrange Planetary Equations which are integrated as before. The perturbation equations which are generated are the following (for the M_2 constituent):

1967-92A

$$\Delta I_E = (1.04713 \text{ arc sec})\, k_2 \cos(\nu^*_{2200} - 2\Omega + \epsilon_{2200}) \tag{7}$$

STARLETTE

$$\Delta I_E = (0.66813 \text{ arc sec})\, k_2 \cos(\nu^*_{2200} - 2\Omega + \epsilon_{2200}) \tag{8}$$

GEOS-3

$$\Delta I_E = (1.36861 \text{ arc sec})\, k_2 \cos(\nu^*_{2200} - 2\Omega + \epsilon_{2200}) \tag{9}$$

$$\Delta \Omega_E = (-0.63760 \text{ arc sec})\, k_2 \sin(\nu^*_{2200} - 2\Omega + \epsilon_{2200}) \tag{10}$$

OCEAN TIDAL PARAMETER ESTIMATION

Data utilized for the 1967-92A, Starlette, and GEOS-3 satellites spanned time periods of 312, 252, and 382 days, respectively. Frequency analyses were performed on the residuals to ascertain the periodicities to be included in a series of least squares fits to the residuals. These least squares analyses produced amplitudes and phases for the observed harmonic variations in the residuals. Figures 1, 2, 3, and 4 display the residuals and the fits, along with the observed amplitudes for the M_2 effects. The oscillations caused by the M_2 tide can be clearly seen in each instance. The rms of fits for the four figures are .036, .051, .030, and .034 arc seconds, respectively.

The expressions in Equations 2, 3, 4, 5, 7, 8, 9, 10, along with the amplitudes and phases for the M_2 tide obtained in the breakdown of the residuals, were then used in a subsequent least squares solution for the $(2,2)^+$ and $(4,2)^+$ M_2 ocean tide parameters. In the process, the correction for the averaging was made (see Jeffreys and Jeffreys, 1972) and the values $k_2 = 0.3$ and $\delta_2 = 0°$ were used for the solid Earth tide parameters. The results are shown in Table 2, along with several other recent determinations. The uncertainties are the formal standard deviations of the solution -- the realistic errors are probably somewhat larger.

DECELERATION OF MOON'S MEAN LONGITUDE

The tidal potentials (1) and (6) can be used to study the effects of the tides on the lunar orbit evolution. Lambeck (1975) pointed out that the tidal perturbation expressions obtainable from these potentials are valid for any satellite of the Earth, and thus are applicable to the Moon itself. He also demonstrated that the deceleration of the lunar mean longitude was caused principally by the second degree components of the ocean tide and possibly the second degree solid tide (if the phase angles are assumed to be different from zero). A more recent paper by Lambeck (1977) gives an excellent historical and technical overview on the subject of the dissipation of tidal energy in the oceans.

The mean motion N of the Moon can be expressed as:

$$N = \sqrt{\frac{G(M + m^*)}{a^3}} \tag{11}$$

where M is the mass of the Earth. Taking the time derivative of (11) results in

$$\dot{N} = -\frac{3}{2}\left(\frac{N}{a}\right)\dot{a}, \tag{12}$$

which gives the secular rate of change of N (Lambeck, 1975; 1977). The purely secular terms in (6) are those for which $\nu^*_{\ell mpq} - \nu_{\ell mjh} = 0$, or p = j and q = h. These terms are substituted into the Lagrange Planetary Equation for a, resulting in an expression for $\dot{a}$ which is inserted in (12). Then, the contribution to $\dot{N}$ from the solid Earth tide is

$$\dot{N}_E = \frac{-3\,Gm^*}{(a^*)^3} \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \sum_{p=0}^{\ell} \sum_{q=-\infty}^{\infty} k_\ell \left(\frac{R}{a^*}\right)^{2\ell+1} \frac{(\ell-m)!}{(\ell+m)!}\,\eta_m$$

$$[F_{\ell mp}(I^*)\,G_{\ell pq}(e^*)]^2 \cdot (\ell - 2p + q) \sin \epsilon_{\ell mpq} \tag{13}$$

Table 3 summarizes the principal second degree solid tide contributions to $\dot{N}$.

The main ocean tide variations in $\dot{N}$ are provided by the second degree terms for the M_2, N_2, and O_1 constituents, and only for $\gamma^+_{j\ell pq}$ (Lambeck, 1975). In the ocean tide potential (1), the (nearly) secular M_2 term arises when j = 2, ℓ = 2, p = 0, q = 0; for N_2, j = 2, ℓ = 2, p = 0, q = 1; for O_1, j = 2, ℓ = 1, p = 0, q = 0. Table 4 presents these contributions.

In order to compute a value for $\dot{N}$, we use the fact that the solid Earth tide phase angles $\epsilon_{\ell mpq}$ are approximately equal to $m\delta_2$, where δ_2 is the solid Earth tide lag angle. In this work, the assumption has been made that $\delta_2 = 0°$. Goad and Douglas (1977b) have shown that a change in the assumed values for k_2 and δ_2 will result in a corresponding alteration in the ocean tide parameters derived from satellite data. Therefore, the solid Earth tide contributions to $\dot{N}$ given in Table 3 must be taken to be zero.

This leaves the ocean tidal effects listed in Table 4. We use our satellite derived M_2 parameters presented in Table 2 and the N_2 and O_1 models of R. Estes of Business and Technological Systems, Inc., Seabrook, Maryland:

N_2		O_1	
C^+_{22}	ϵ^+_{22}	C^+_{21}	ϵ^+_{21}
.58 cm	358°	2.70 cm	159°

The result is a value for $\dot{N}$ of -25 ± 3 arc seconds/century^2. Table 5 presents a comparison of this estimate with several other recent determinations.

DISCUSSION

The satellite derived $(2,2)^+$ parameters for the M_2 ocean tide are in accord with the numerical models of Estes and Schwiderski and the satellite solution of Goad and Douglas (1977a), if one assumes the realistic uncertainties to be at least 50% larger than the formal standard deviations. The $(4,2)^+$ values are in good agreement, with the noticeable exceptions of the C_{42}^+ of Schwiderski and the ϵ_{42}^+ of Estes. The data used by Goad and Douglas (1977a) form a subset of the data used in the present investigation, so the consistency between the two models is not unexpected. However, the agreement with the numerical models is quite satisfying.

The credibility of our M_2 ocean tide solution is further enhanced by the close accord between our computed value for the deceleration of the lunar mean longitude and the other recently reported estimates. Under the assumption that $k_2 = 0.3$ and $\delta_2 = 0°$ for the solid Earth tide, the major contribution to the $\dot{N}$ comes from the $(2,2)^+$ parameters for the M_2 ocean tide.

It is evident from the results of this investigation that studies of close Earth satellite orbits are able to provide important information about the tidal forces acting on the Earth. We anticipate that the new laser data for Starlette and GEOS-3, in addition to data for the SEASAT Satellite, will be even more precise for studies of this sort.

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TABLE 1. M_2 OCEAN TIDAL PERTURBATION AMPLITUDES

SATELLITE	ΔI (ARC SEC)		ΔΩ (ARC SEC)		PERIOD (DAYS)
	EXPECTED*	OBSERVED	EXPECTED*	OBSERVED	
1967-92A ($I = 89^\circ.3$)	.024 → .044	.040	.005 → .009		13.6
Starlette ($I = 49^\circ.8$)	.032 → .046	.042	.011 → .023		10.5
GEOS-3 ($I = 115^\circ.0$)	.046 → .069	.039	.025 → .050	.029	17.2

*From existing numerical models.

TABLE 2. M_2 OCEAN TIDE PARAMETERS–RECENT SOLUTIONS

SOURCE	C^{+}_{22} (cm)	ϵ^{+}_{22} (deg)	C^{+}_{42} (cm)	ϵ^{+}_{42} (deg)
FELSENTREGER et al.[1]	3.42 ± 0.24	325.5 ± 3.9	.97 ± 0.12	124.0 ± 6.9
GOAD & DOUGLAS (1977a)[1]	3.23	331	.87	113
ESTES (BTS)[2]	3.31	333	.96	145
SCHWIDERSKI (NSWC)[3]	3.59	326	1.34	126

[1] Satellite Solutions.

[2] Business and Technological Systems, Inc., Seabrook, Md. – Numerical Solution

[3] Naval Surface Weapons Center, Dahlgren, Va. – Numerical Solution.

Obtained from C. Goad, private communication.

TABLE 3. SUMMARY OF PRINCIPAL SOLID EARTH TIDAL EFFECTS ON THE DECELERATION OF THE MOON'S MEAN LONGITUDE

TIDAL CONSTITUENT	$\Delta\dot{N}$ (ARC SECONDS/CENTURY^2)
M_2	$-870.8\ k_2 \sin \epsilon_{2200}$
N_2	$-48.3\ k_2 \sin \epsilon_{2201}$
O_1	$-112.1\ k_2 \sin \epsilon_{2100}$
L_2	$-0.3\ k_2 \sin \epsilon_{220(-1)}$
J_1	$0.4\ k_2 \sin \epsilon_{211(-1)}$
Q_1	$-6.2\ k_2 \sin \epsilon_{2101}$

TABLE 4. SUMMARY OF PRINCIPAL OCEAN TIDAL EFFECTS ON THE DECELERATION OF THE MOON'S MEAN LONGITUDE

TIDAL CONSTITUENT	$\Delta\dot{N}$ (ARC SECONDS/CENTURY^2)
M_2	$C_{22}^{+}(-8.04\cos\epsilon_{22}^{+} + 0.22\sin\epsilon_{22}^{+})$
N_2*	$C_{22}^{+}(-2.32\cos\epsilon_{22}^{+} + 0.06\sin\epsilon_{22}^{+})$
O_1*	$C_{21}^{+}(-0.32\cos\epsilon_{21}^{+} - 1.41\sin\epsilon_{21}^{+})$

*Estes (BTS) numerical models (obtained from C. Goad, private communication).

TABLE 5. RECENT VALUES FOR THE DECELERATION OF THE MOON'S MEAN LONGITUDE

SOURCE	$\dot{N}$ (ARC SECONDS/CENTURY^2)
FELSENTREGER et al.[1]	-25 ± 3
GOAD AND DOUGLAS (1977b)[2]	-26 ± 3
MORRISON AND WARD (1975)[3]	-26 ± 2
MULLER (1976)[4]	-27.2 ± 1.7
CALAME AND MULHOLLAND (1978)[5]	-24.6 ± 1.6

[1] Satellite Solution.

[2] Satellite Solution.

[3] From Transits of Mercury.

[4] From Ancient Eclipses and Transits of Mercury.

[5] From Lunar Laser Ranging.

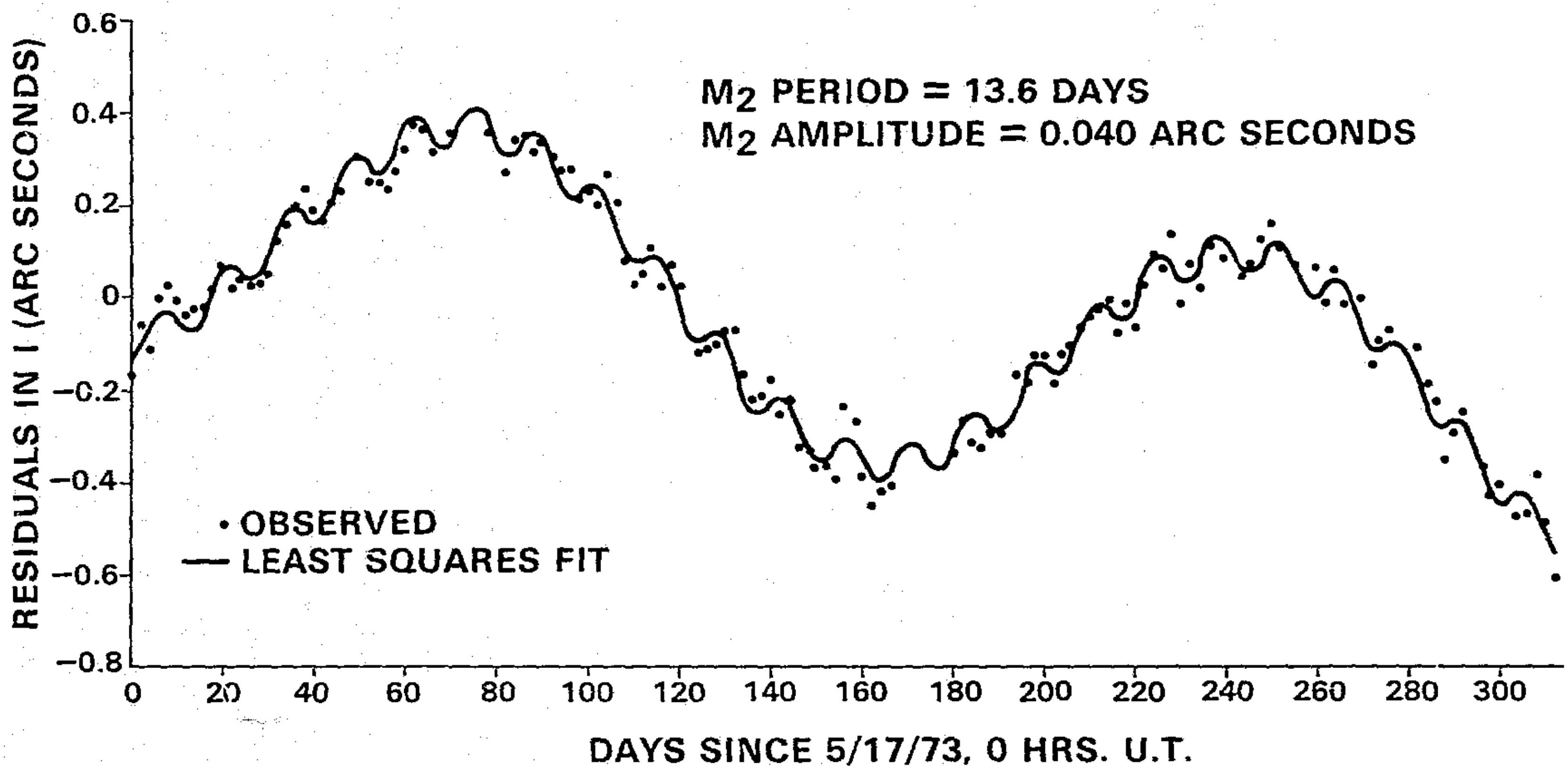

Figure 1. Inclination Residuals for 1967-92A (Transit)

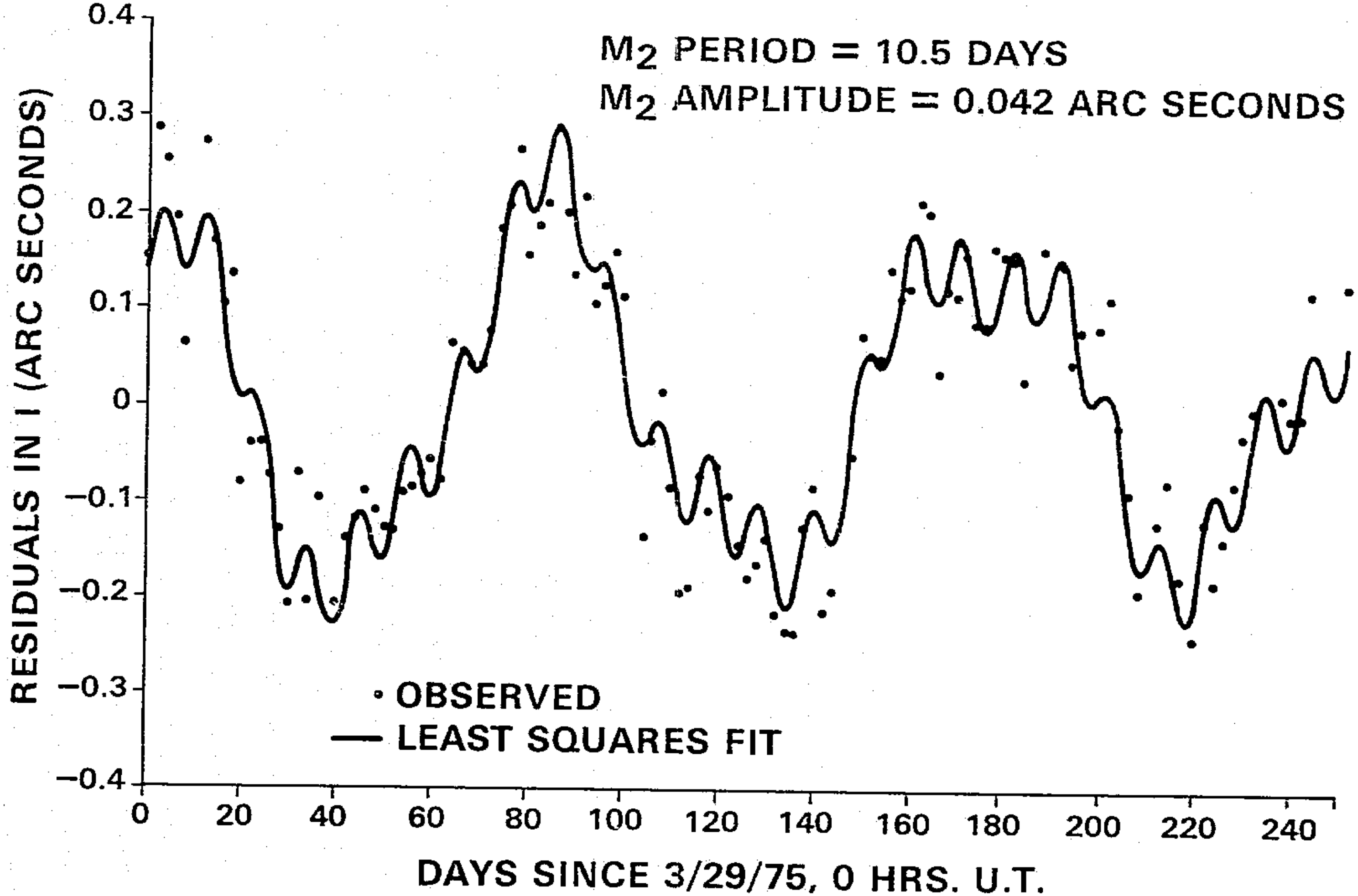

Figure 2. Residuals in Inclination for Starlette

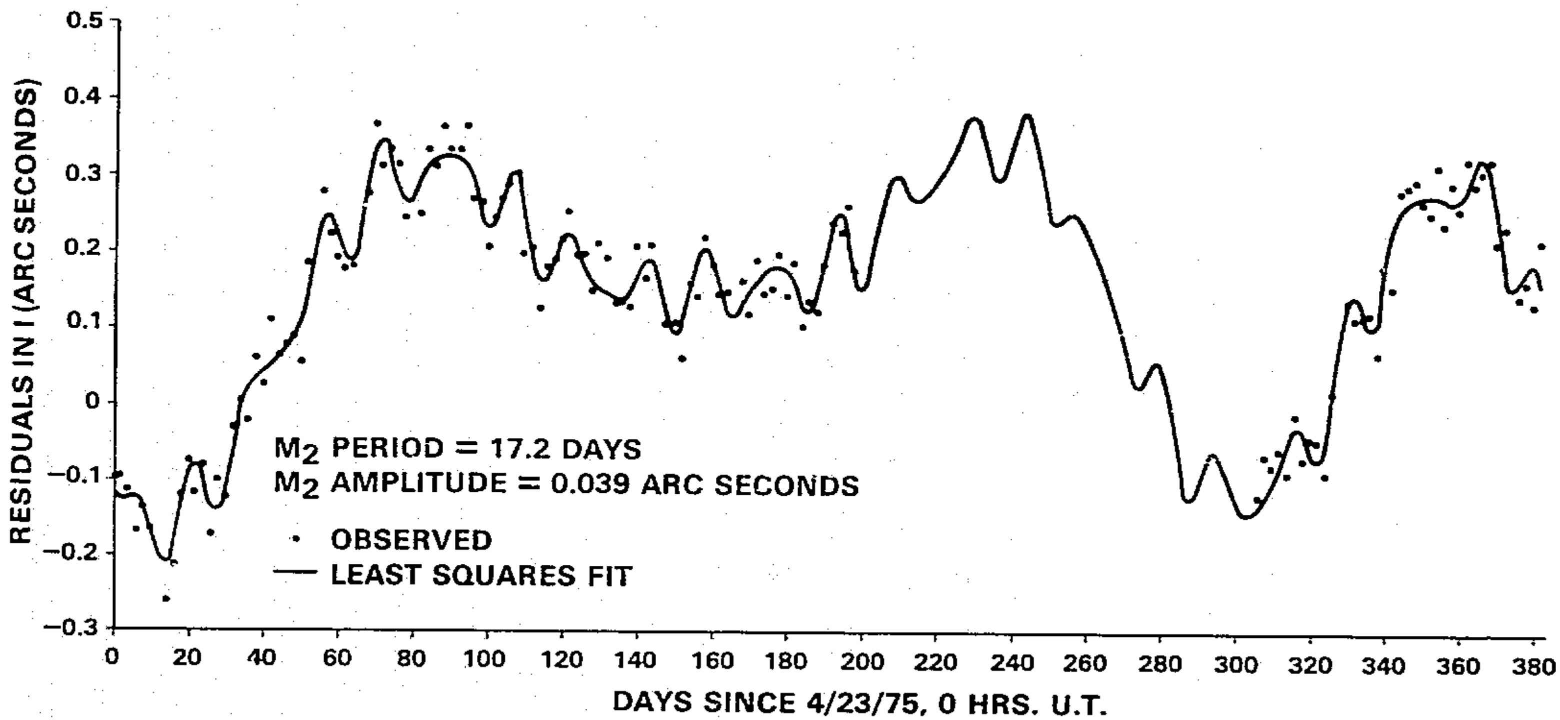

Figure 3. Inclination Residuals for GEOS-3

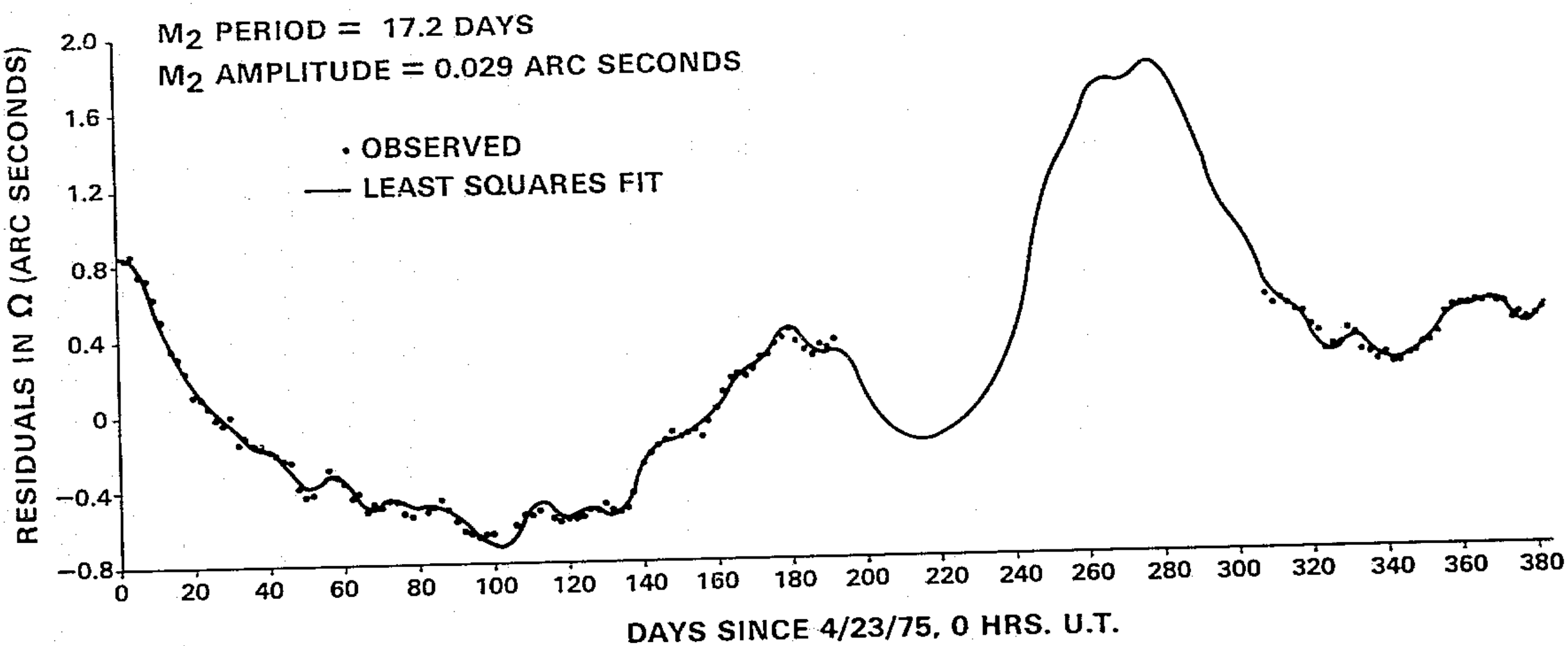

Figure 4. Residuals in Longitude of Ascending Node for GEOS-3