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CALCULATION OF ELECTROSTATIC FIELDS IN PERIODIC STRUCTURES OF COMPLEX SHAPE

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16. Abstract

A method is presented for developing a universal algorithm for calculating electrostatic fields in an infinite periodic structure consisting of electrodes of arbitrary shape which are located in mirror-symmetrical manner along the axis of electron-beam propagation. The method is based on the theory of R-functions, and the differential operators which are derived on the basis of the functions. Numerical results are presented and the accuracy of the results is examined. The algorithm can be applied to the investigation of such devices as electron-optic systems, microwave-oscillator components, mass spectrometers, and linear accelerators.

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ABSTRACT. A method is presented for developing
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A method for developing a universal algorithm for
calculating electrostatic fields in an infinite
periodic structure consisting of electrodes of
arbitrary shape which are randomly located in
mirror-symmetric manner along the electron-beam
propagation axis. The method is based on appli-
cation of the theory of R-functions. The accuracy
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The problem of the distribution of electrostatic field in struc-
tures with electrodes of complex form is of great practical value in
electron optics. In the majority of cases, the complex geometry of

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*Numbers in the margin indicate pagination in the foreign text.

electronic lenses does not permit the use of exact methods of solution; thus it becomes necessary to employ methods of approximation. Since the numerical realization of these methods is usually accomplished by means of an electronic computer, the choice of the method to be used should be made primarily in terms of its practical effectiveness.

In the present paper, a method is proposed for developing a universal algorithm for calculating electrostatic fields in an infinite periodic structure consisting of electrodes whose shape is arbitrary and which are located at random in mirror-symmetric fashion along the propagation axis of the electron beam.

The method is based on the use of special properties of R-functions and the introduction of the differential operators which are derived from them [1, 2]. R-functions are widely used in the /4 solution of a broad range of problems in electrostatics and electro-dyanmics [3 - 7]. The merits of the present method, which is based on the above principle, lies in the possibility of describing analytically the shape of any piecewise-smooth region and the broad applicability of the method to an entire class of similarly formulated problems. Moreover, the present method, as distinguished from the method of finite differences, requires a smaller expenditure of machine time and furnishes a solution in analytic form, which materially facilitates the analysis of the focusing properties of the structures under consideration.

Given the following problem: to find the potential distribution of the electrostatic field in a certain structure consisting of an infinite set of electrodes, piecewise smooth in shape, located along the z-axis so that $z_n = nl$ $(n = 0, \pm 1, \pm 2...)$ are the planes of symmetry for the entire structure as well as for those electrodes which are intersected by these planes (Figure 1a, b, c). If in this case the potentials of the electrodes have the same symmetric form, then the condition:

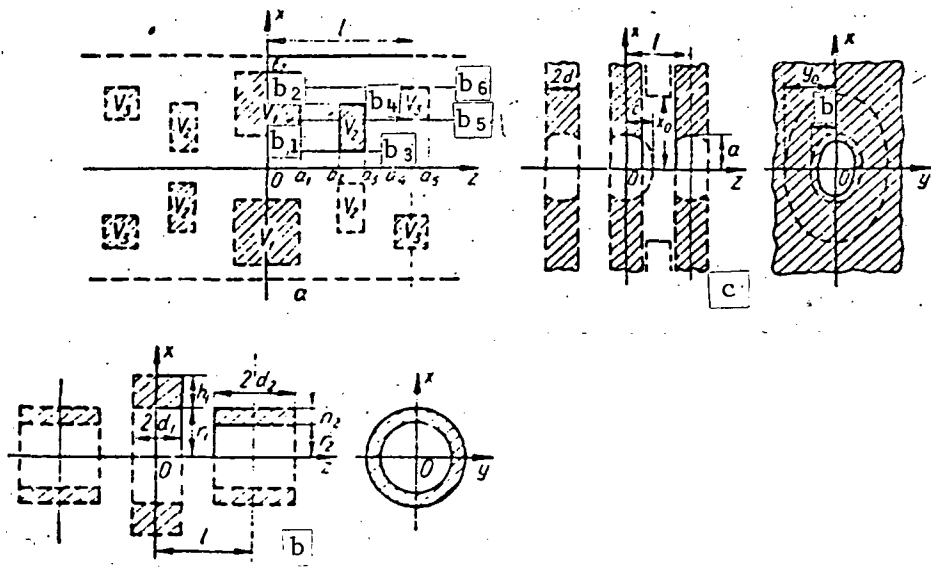$$\left. \frac{\partial U}{\partial \vec{v}} \right|_{z_n} = 0, \quad (n = 0, \pm 1, \pm 2...),$$ (1)

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Figure 1

must be satisfied in the planes, where U is the desired potential of the electrostatic field, and $\vec{v}$ is the normal to the corresponding planes.

The boundary condition (1) makes it possible to restrict the region to be considered along the z-axis to the planes z_n and z_{n+1}, and to reduce the boundary problem to be considered to the equivalent problem for one "cell" of this structure of length l along the z-axis.

Then the problem can be formulated as follows: in the region D, which is bounded with respect to z, the solution of the equation:

$$\Delta U = 0 \qquad (2)$$

subject to the boundary conditions

$$U\Big|_{\Gamma_i} = V_i; \quad \frac{\partial U}{\partial \vec{v}}\Big|_{\Gamma_{i+1}} = 0, \quad (i = 1, 2, 3 ..., s), \qquad (3)$$

where Γ_i are the surfaces or boundaries of the electrodes for which the corresponding values of the potentials V_i are given; Γ_{s+1} is the surface or the boundary formed by the planes (axes) of symmetry; $\vec{v}$ is the internal normal to Γ_{s+1} .

In those cases when the region D is bounded (Figure 1a), or can be bounded (Figure 1b) in the plane XOY, the solution for (2) with the conditions (3) can be represented in the following structural form:

$$U = \Phi_0 + \omega\Phi - \frac{\omega\omega_0}{\omega + \omega_0} D_1^{(0)} (\Phi_0 + \omega\Phi),$$ (4)

where

$$\Phi_0 = \frac{\sum\limits_{i=1}^{s} V_i \prod\limits_{j=1, j \neq i}^{s} \omega_j}{\sum\limits_{i=1}^{s} \prod\limits_{\substack{j=1 \\ j \neq i}}^{s} \omega_j + \prod\limits_{i=1}^{s} \omega_j};$$ (5)

$$\omega = \prod_{i=1}^{s} \omega_i; \quad D_1^{(0)} = \frac{\partial \omega_0}{\partial x} \frac{\partial}{\partial x} + \frac{\partial \omega_0}{\partial y} \frac{\partial}{\partial y} - \frac{\partial \omega_0}{\partial z} \frac{\partial}{\partial z},$$

where Φ is a function of class C^2 subject to determination; ω_j is the $\underline{/5}$ function which gives an analytic description of the surface Γ_i of the corresponding electrode; ω_0 is the function which gives the analytic description of the surface $\Gamma_{s\pm1}$ formed by the planes of symmetry bound:: ing the region D along the z axis.

It is not difficult to show that a solution in the form (4) subject to (5) satisfies the conditions (3), regardless of the form of the function Φ. For this purpose, it is sufficient to take account of the following properties of ω_j, ω_0, and the operator $D_1^{(0)}$ [1]:

$$\omega_j|_{\Gamma_j} = 0; \quad \omega_0|_{\Gamma_{s+1}} = 0; \quad \left.\frac{\partial \omega_0}{\partial \vec{v}}\right|_{\Gamma_{s+1}} = 1 \quad (j = 1, 2..., s),$$ (6)

$$\omega_j (M) > 0; \quad \omega_0 (M) > 0;$$

$$D_1^{(0)} (f)|_{\Gamma_{s+1}} = \left.\frac{\partial f}{\partial \vec{v}}\right|_{\Gamma_{s+1}}; \quad D_1^{(0)} (\omega_0 f)|_{\Gamma_{s+1}} = f|_{\Gamma_{s+1}},$$

where M is an arbitrary point of the region D, and f is any differentiable function.

In the case the region D is not bounded in the plane XOY and the dimensions of the electrodes in this plane are finite, it becomes necessary to take into account the behavior of the field at infinity.

In this case, a more general form of the solution must be selected:

$$U = \Phi_0 + W \Phi \left(\frac{x}{\Omega}; \frac{y}{\Omega}; z \right) - \frac{WW_0}{W+W_0} D_1^{(0)} \left(\Phi_0 + W \Phi \left(\frac{x}{\Omega}; \frac{y}{\Omega}; z \right) \right), \tag{7}$$

where

$$W = \frac{\omega}{\Omega^{m+1}}; \quad W_0 = \frac{\omega_0}{\Omega_0^{m+1}}; \quad \Omega = \sqrt[m]{1+\omega}; \quad \Omega_0 = \sqrt[m_0]{1+\omega_0};$$

and m and m_0 are the degrees of the functions ω and ω_0, respectively.

The solution in the form (7) not only satisfies the conditions (3), but also ensures the regularity of the function U at infinity for any choice of the function $\Phi\left(\frac{x}{\Omega}; \frac{y}{\Omega}; z\right)$ from the class C^2.

The arbitrariness which is permissible in the choice of the function ϕ makes it possible to seek it by any method of approximation such that the structure (4) or (7) provides the best solution to equation (2).

Let us represent Φ by means of the following approximation:

$$\Phi = \sum_{k+i+q=0}^{p} C_{kiq} \chi_{kiq},$$

where χ_{kiq} is a certain complete system of functions represented in the form:

$$\chi_{kiq} = P_k(x) P_i(y) P_q(z) \quad \text{for the structure (4);}$$

$$\chi_{kiq} = P_k\left(\frac{x}{\Omega}\right) P_i\left(\frac{y}{\Omega}\right) P_q(z) \quad \text{for the structure (7).}$$

On substituting (8) into (4) and (7), and by virtue of the linearity /6
of the operator $D_l^{(0)}$, we obtain:

$$U = \Psi_0 = \sum_{k+i+q=0}^{p} C_{kiq} \Psi_{kiq}, \tag{9}$$

where, for the structure (4),

$$\Psi_0 = \Phi_0 - \frac{\omega\omega_0}{\omega+\omega_0} D_1^{(0)}(\omega\chi_{kiq}), \tag{10}$$

$$\Psi_{kiq} = \omega\chi_{kiq} - \frac{\omega\omega_c}{\omega+\omega_0} D_1^{(0)}(\omega\chi_{kiq}).$$

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Analogously, we have for the structure (7):

$$\Psi_0 = \Phi_0 - \frac{WW_0}{W+W_0} D_1^{(0)} (W\chi_{klq}),$$

$$\Psi_{klq} = W\chi_{klq} - \frac{WW_0}{W+W_0} D_1^{(0)} (W\chi_{klq}).$$

(10a)

Next, we introduce the function:

$$V = \sum_{k+l+q=0}^{p} C_{klq} \Psi_{klq}.$$

(11)

The problem (2) with the mixed conditions (3) can be reduced relative to V to the following problem with uniform boundary conditions:

$$- \Delta V = \Delta \Psi_0; \quad V|_{\Gamma_l} = 0; \quad \frac{\partial V}{\partial \vec{v}}\bigg|_{\Gamma_{s+1}} = 0.$$

(12)

We shall seek the solution on the lineal of the functions V_n belonging to C^2 and satisfying the conditions (12). The given problem is equivalent to the problem of constructing an element of the energy space which realizes the minimum of the functional [8]:

$$I(V) = (- \Delta V, V) - 2 (V, \Delta \Psi_0) = - \int_D (\Delta VV + 2V\Delta \Psi_0) \, d\tau.$$

(13)

On applying Green's theorem to (13) we have:

$$I(V) = \int_D \left[\left(\frac{\partial V}{\partial x} \right)^2 + \left(\frac{\partial V}{\partial y} \right)^2 + \left(\frac{\partial V}{\partial z} \right)^2 + 2 \left(\frac{\partial V}{\partial x} \frac{\partial \Psi_0}{\partial x} + \frac{\partial V}{\partial y} \frac{\partial \Psi_0}{\partial y} + \right. \right.$$

$$\left. \left. + \frac{\partial V}{\partial z} \frac{\partial \Psi_0}{\partial z} \right) \right] d\tau - \int_s V \frac{\partial V}{\partial v} \, dS - 2 \int_s V \frac{\partial \Psi_0}{\partial v} \, dS.$$

(14)

Taking into account the conditions (6) and also for (7) the condition for the decrease of the potential at infinity, it is not difficult to verify that the surface integral in (14) equals zero.

Using the variational method of Rietz to minimize the functional <u>/7</u> (14), we obtain the following system of equations:

$$\sum_{k+l+q=0}^{p} C_{klq} \int_{D} \left(\frac{\partial \Psi_{klq}}{\partial x} \frac{\partial \Psi_{mnl}}{\partial x} + \frac{\partial \Psi_{klq}}{\partial y} \frac{\partial \Psi_{mnl}}{\partial y} + \frac{\partial \Psi_{klq}}{\partial z} \frac{\partial \Psi_{mnl}}{\partial z} \right) d\tau =$$

$$= \int_{\omega} \left(\frac{\partial \Psi_{0}}{\partial x} \frac{\partial \Psi_{mnl}}{\partial x} + \frac{\partial \Psi_{0}}{\partial y} \frac{\partial \Psi_{mnl}}{\partial y} + \frac{\partial \Psi_{0}}{\partial z} \frac{\partial \Psi_{mnl}}{\partial z} \right) d\tau. \tag{15}$$

Realization of the system (15) on an electronic computer permits the determination of the coefficient C_{klq} in the expansion (8), and therefore the solution of the problem posed.

The proposed method is illustrated below in an example of concrete structures which are used in electron optics.

Let us consider the structure (Figure 1a) consisting of two rows of bars with rectangular cross-section, infinitely long along the y-axis and distributed so that the planes $z_n = nl$, $(l=a_5)$ and the plane x = 0 are the symmetry planes of this structure. Moreover, above and below, the structure is bounded by the two conducting planes $x = \pm b_7$.

In particular, when the dimensions of the bars and the distance between them are the same $V_4 = 0$ and $V_1 = V_3 = -V_2$, we obtain the structure which is used for focusing the electron current in generators of diffraction radiation [9].

Taking into consideration the homogeneity of the structure along the y-axis and its boundedness with respect to the x-axis, we shall seek the solution in the form (4) for a two-dimensional region D bounded by the lines $z=0$, $z=a_5$, $x=0$, $x=b_7$ (the solid lines in Figure 1a).

For the given case:

$$\Phi_0 = \frac{V_1 \omega_2 \omega_3 \omega_4 + V_2 \omega_1 \omega_3 \omega_4 + V_3 \omega_1 \omega_2 \omega_4 + V_4 \omega_1 \omega_2 \omega_3}{\omega_2 \omega_3 \omega_4 + \omega_1 \omega_3 \omega_4 + \omega_1 \omega_2 \omega_4 + \omega_1 \omega_2 \omega_3 + \omega}; \tag{16}$$

$$\omega = \omega_1\omega_2\omega_3\omega_4; \quad \omega_1 = -[f_1\Lambda_0 f_2 f_3];$$

$$\omega_2 = -[f_4 f_5 \Lambda_0 f_6 f_7]; \quad \omega_3 = -[f_8\Lambda_0 f_9 f_{10}];$$

$$f_1 = a_1 - z \geq 0; \quad f_6 = x - b_3 \geq 0; \quad f_{11} = b_7 - x \geq 0;$$

$$f_2 = x - b_1 \geq 0; \quad f_7 = b_4 - x \geq 0; \quad f_{12} = z \geq 0;$$

$$f_3 = b_2 - x \geq 0; \quad f_8 = z - a_4 \geq 0; \quad f_{13} = a_5 - z \geq 0;$$

$$f_4 = z - a_2 \geq 0; \quad f_9 = x - b_5 \geq 0; \quad f_{14} = x \geq 0;$$

$$f_5 = a_3 - z \geq 0; \quad f_{10} = b_6 - x \geq 0;$$

$$\tag{17}$$

Λ_0 is the symbol for the operation of R-conjunction [1]

It can be verified by direct checking that (4) together with (16) and (17) satisfies (3) for any choice of Φ.

Representing Φ in the form (8) and using the method described above, we have for C_{kl} a system analogous to (15) where Ψ_0 and Ψ_{kl} have the form (10).

Let us consider a periodic structure (Figure 1b) consisting of infinite plates of thickness 2d, the potentials of which alternately equal V_1 and V_2. In the plates there are openings formed by the intersection of these plates with the surfaces of the form: /8

$$\frac{x^{2t}}{a^{2t}} + \frac{y^{2t}}{b^{2t}} + \frac{(z - z_n)^{2t}}{c^{2t}} = 1, \tag{18}$$

where $z_n = nl$ $(n = 0, \pm 1, \pm 2 \ldots)$; l is the distance between the center of two neighboring plates, t is the exponent $(t \geq 1)$; a, b, c, d are constants which determine the magnitude and shape of the openings, and $c > d$.

The appearance of an opening in (18) is selected in connection with the fact that, depending on the magnitude of the exponent and the relationships among the quantities a, b, c, and d, the possibility emerges of obtaining openings of the most varied shape. Thus, if $t \gg 1$, we obtain an opening of practically rectangular shape (slit diaphragm). Then t = 1 and a = b, we obtain various types of axially dymmetric openings from segmental (when c = a = b) to cylindrical (when $c \gg a, b$ and d).

Consequently, from (18) it is possible to obtain in particular cases the basic forms of diaphragms widely used in electrostatic focusing system.

We shall seek a solution in the region D bounded by the planes $z = 0$, $z=l$, $x=0$, $y=0$ (the solid lines in Figure lb), given on them conditions of the type (1). Note that, beginning with some $x_0 \gg a$, $y_0 \gg b$, the field between the plates may be considered as practically homogeneous with a linear distribution of the potential along the z-axis. This makes it possible to restrict the region D in the plane XOY to a cylindrical surface of the form:

$$\frac{x^{2l}}{x_0^{2l}} + \frac{y^{2l}}{y_0^{2l}} = 1,$$ (19)

along z, where $x_0 \gg a$, $y_0 \gg b$.

Here on the surface (19) in the interval $d \leqslant z \leqslant (l - d)$, it is necessary to assign a distribution of the potential analogous to that which exists between two infinite planes. Taking this into account, the boundary conditions for the region D can be written in the following form:

$$U|_{\Gamma_1} = V_1; \quad U|_{\Gamma_2} = V_2; \quad U|_{\Gamma_3} = \frac{V_2(z-d) + V_1(l-d-z)}{l-2d}; \quad \frac{\partial U}{\partial v}\Big|_{\Gamma_4} = 0,$$ (20)

where Γ_1, Γ_2 are the surfaces of the electrons; Γ_3 is the surface (19) bounding the region D in the plane XOY, Γ_4 is the surface formed by the planes of symmetry. We shall select the structure of the solution in the form (4), where for the given case:

(21)

$$\Phi_0 = \frac{V_1\omega_2\omega_3 + V_2\omega_1\omega_3 + \varphi\omega_1\omega_2}{\omega_2\omega_3 + \omega_1\omega_3 + \omega_1\omega_2 + \omega};$$

$$\omega = \omega_1\omega_2\omega_3; \quad \omega_1 = -[f_1\Lambda_0 f_2 f_3]; \quad \omega_2 = -[f_1\Lambda_0 f_4 f_5];$$

$$\omega_3 = -[f_2 f_4 \Lambda_0 f_6]; \quad \omega_0 = [f_7 f_8 \Lambda_0 f_9 \Lambda_0 f_{10}];$$

$$f_1 = \frac{x^{2l}}{a^{2l}} + \frac{y^{2l}}{b^{2l}} + \frac{z^{2l}}{c^{2l}} - 1 \geqslant 0; \quad f_2 = d - z \geqslant 0;$$

$$f_3 = d + z \geqslant 0; \quad f_4 = z - l + d \geqslant 0; \quad f_5 = l + d - z \geqslant 0;$$

$$f_6 = \frac{x^{2l}}{x_0^{2l}} + \frac{y^{2l}}{y_0^{2l}} - 1 \geqslant 0; \quad f_7 = z \geqslant 0;$$

$$f_8 = \frac{1}{l} \cdot (l - z) \geqslant 0; \quad f_9 = x \geqslant 0; \quad f_{10} = y \geqslant 0;$$

$$\varphi = \frac{V_1 f_9 + V_2 f_3}{2d - l}.$$

Representing Φ in the form (8) and performing the sequence of operations described above, we obtain the system (15) to determine C_{klq} . The limits of integration with respect to z are from 0 to 1; with respect to x, from 0 to x_0; with respect to y, from 0 to y_0.

It should be noted that the values x_0 and y_0 are selected in the process of numerical realization on an electronic computer, based on the condition that their subsequent increase must not influence (to a given degree of accuracy) the value of the potential of the field U at any point of the domain D under consideration.

Let us consider the periodic structure (Figure 1c) consisting of a system of cylindrical rings with internal radii r_1 and r_2, thickness h_1 and h_2, length $2d_1$ and $2d_2$, located along the z-axis at equal distances l between their centers. In the present instance, the region can be bounded only along the z-axis. Here the planes of symmetry have the following form in cylindrical coordinates:

$$z = 0; \quad z = l; \quad \rho \cos \varphi = 0; \quad \rho \sin \varphi = 0.$$

Since the region D is not closed along ρ, together with the boundary conditions (3) it is necessary to take into account also the behavior of the field at infinity. Therefore, the potential of the field U must be selected in the form of the structure (7) in the present case, thus ensuring the requisite behavior of the poten-tial and its partial derivatives when $\rho \rightarrow \infty$. The quantities occurring in (7) have the following form:

$$\Phi_0 = \frac{V_1\omega_2 + V_2\omega_1}{\omega_2 + \omega_1 + \omega}; \quad \omega = \omega_1\omega_2; \tag{22}$$

$$\omega_1 = -[f_1 f_2 \Lambda_0 f_3]; \quad \omega_2 = -[f_4 f_5 \Lambda_0 f_6];$$

$$\omega_0 = [f_7 \Lambda_0 f_8 \Lambda_0 f_9]; \quad f_1 = \rho - r_1 \geqslant 0;$$

$$f_2 = r_1 + h_1 - \rho \geqslant 0; \quad f_3 = d_1^2 - z^2 \geqslant 0;$$

$$f_4 = \rho - r_2 \geqslant 0; \quad f_5 = r_2 + h_2 - \rho \geqslant 0;$$

$$f_6 = (z - l + d_2)(l + d_2 - z) \geqslant 0;$$

$$f_7 = \frac{1}{l}(l - z) z \geqslant 0; \quad f_8 = \rho \cos\varphi \geqslant 0;$$

$$f_9 = \rho \sin\varphi \geqslant 0.$$

We shall represent Φ in the form of the following approximation:

$$\Phi = \sum_{k+l+q=0}^{p} C_{klq} P_k\left(\frac{\rho}{\Omega}\right) P_l(\varphi) P_q(z), \tag{23}$$

where P_k, P_l, P_q is some complete system of functions.

Substituting (23) into (7) and performing the operations described above, we obtain for finding C_{kl} a system of the type: /10

$$\sum_{k+l=0}^{p} C_{kl} \int_0^\infty \int_0^{\pi^2} \int_0^l \left(\frac{\partial\Psi_{klq}}{\partial\rho} \frac{\partial\Psi_{mnl}}{\partial\rho} + \frac{1}{\rho^2} \frac{\partial\Psi_{klq}}{\partial\varphi} \frac{\partial\Psi_{mnl}}{\partial\varphi} + \frac{\partial\Psi_{klq}}{\partial z} \frac{\partial\Psi_{mnl}}{\partial z}\right) \rho\,d\rho\,d\varphi\,dz =$$

$$= \int_0^\infty \int_0^{\pi^2} \int_0^l \left(\frac{\partial\Psi_0}{\partial\rho} \frac{\partial\Psi_{mnl}}{\partial\rho} + \frac{1}{\rho^2} \frac{\partial\Psi_0}{\partial\varphi} \frac{\partial\Psi_{mnl}}{\partial\varphi} + \frac{\partial\Psi_0}{\partial z} \frac{\partial\Psi_{mnl}}{\partial z}\right) \rho\,d\rho\,d\varphi\,dz, \tag{24}$$

where Ψ_0 and Ψ_{klq} have the form (10a), and in the present case:

$$\chi_{klq} = P_k\left(\frac{\rho}{\Omega}\right) P_l(\varphi) P_q(z). \tag{25}$$

The numerical realization of the system (24), as well as (15) also, was accomplished with the help of an electronic computer.

As an example of the numerical realization of the method described, the structure was selected which is represented in Figure la. The solution of the corresponding algorithm was accomplished on the electronic computer "Minsk-32" for various values of the dimensions of the bars and the distances between them. The system (15)

was solved and the values of the coefficients C_{kl} (in the two-dimensional case) obtained. Then the patterns of the distribution of the equipotential surfaces in the region D were constructed in accordance with formula (4), taking into account (8).
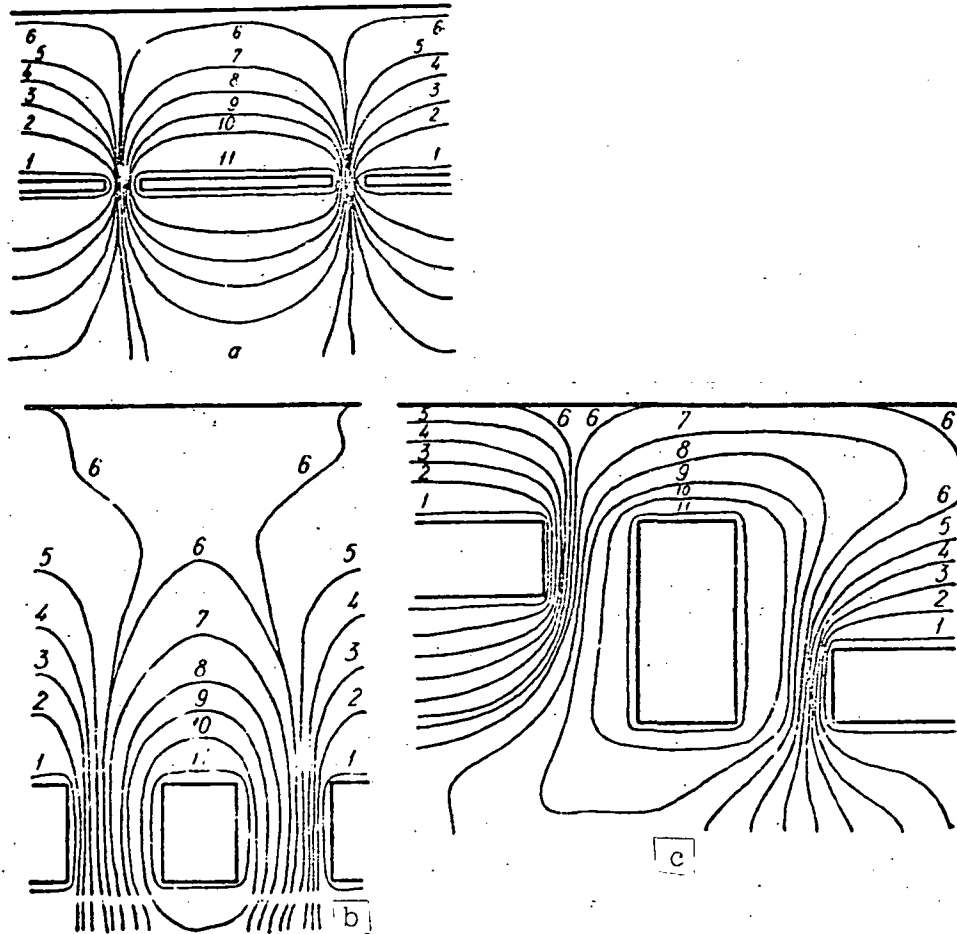

Figure 2

The polynomials of the form:

$$\chi_{kl} = x^k y^l, \quad (k+l=0; \ 1; \ 2; \ ..., p).$$

were chosen as a complete system of functions.

The set of coordinate functions Ψ_{kl} were chosen subject to the condition that their subsequent increase would provide a correction to the result not exceeding the prescribed precision of the numerical realization of the potential at any point of the domain D under consideration.

The results of the calculation in the form of patterns display-
ing the distribution of the equipotential surfaces are presented in
Figure 2a, b, c. The values of the potentials at the corresponding
boundaries are $V_1 = -100$, $V_2 = +100$, $V_3 = -100B$, $V_4 = 0$.

The equipotentials in Figure 2a, b, c are denoted by numerals,
representing voltages as follows:

$$1 - 100; \; 2 - 80; \; 3 - 60; \; 4 - 40; \; 5 - 20; \; 6 - 0; \; 7 - 20; \; 8 - 40; \; 9 - 60;$$
$$10 - 80; \; 11 - 100 \text{ V}$$

The relative sizes of the regions in Figure 2a:

$$a_1 = 0.225, \; a_2 = 0.275, \; a_3 = 0.725, a_4 = 0.775, \; a_5 = 1; \; b_1 = 0.5, \; b_2 = 0.501, \; b_3 = 0.5,$$
$$b_4 = 0.501, \; b_5 = 0.5, \; b_6 = 0.501, \; b_7 = 1.$$

The relative sizes of the regions in Figure 2b:

$$a_1 = 0.125, \; a_2 = 0.375, \; a_3 = 0.625, \; a_4 = 0.875, \; a_5 = 1; \; b_1 = 0.1, \; b_2 = 0.35, \; b_3 = 0.1,$$
$$b_4 = 0.35, \; b_5 = 0.1, \; b_6 = 0.35, \; b_7 = 1.$$

The relative sizes of the regions in Figure 2c:

$$a_1 = 0.25, \; a_2 = 0.4, \; a_3 = 0.6, \; a_4 = 0.75, \; a_5 = 1; \; b_1 = 0.55, \; b_2 = 0.75, \; b_3 = 0.25, \; b_4 =$$
$$= 0.750, \; b_5 = 0.25, \; b_6 = 0.45, \; b_7 = 1.$$

In conclusion it should be noted that the proposed method, and
the machine algorithm based on it make it possible to calculate the
field in structures with elements having arbitrary profiles, and it
can be used effectively during the development of electron optical
systems, the focusing devices of superhigh frequency generators
(in particular GDI*), mass spectrometers, and linear accelerators.

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*Translator's note.   Expansion unknown.

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