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647  
NASA CR-135243  
Technion TME-312

R-0 9-19-77

11-15-77

2/78

(NASA-CR-135243) RADIAL FORCES IN A  
MISALIGNED RADIAL FACE SEAL Final Report  
(Technion - Israel Inst. of Tech.) 27 p HC  
A03/MF A01 CSCL 11A

N78-32430 -78

Unclas -78

G3/37 32836

## RADIAL FORCES IN A MISALIGNED RADIAL FACE SEAL

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Prepared for  
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August 1977

Grant NSG-7317

1. Report No. <b>NASA CR-135243</b>	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle <b>RADIAL FORCES IN A MISALIGNED RADIAL FACE SEAL</b>		5. Report Date <b>August 1977</b>	6. Performing Organization Code
		8. Performing Organization Report No. <b>TME-312</b>	
7. Author(s) <b>I. Etsion</b>		10. Work Unit No.	
9. Performing Organization Name and Address <b>Technion - Israel Institute of Technology Haifa, Israel</b>		11. Contract or Grant No. <b>NSG-7317</b>	
		13. Type of Report and Period Covered <b>Contractor Report</b>	
12. Sponsoring Agency Name and Address <b>National Aeronautics and Space Administration Washington, D.C. 20546</b>		14. Sponsoring Agency Code	
		15. Supplementary Notes <b>Final report. Project Manager, Lawrence P. Ludwig, Fluid System Components Division, NASA Lewis Research Center, Cleveland, Ohio 44135</b>	
16. Abstract  <p>Radial forces on the primary seal ring of a flat misaligned seal are analyzed, taking into account the radial variation in seal clearance. An analytical solution for both hydrostatic and hydrodynamic effects is presented that covers the whole range from zero to full angular misalignment. The net radial force on the primary seal ring is always directed so as to produce a radial eccentricity which generates inward pumping. Although the radial force is usually very small, in some cases it may be one of the reasons for excessive leakage through both the primary and secondary seals of a radial face seal.</p>			
17. Key Words (Suggested by Author(s)) <b>Face seal; Radial face seal; Mechanical seal; Shaft seal</b>		18. Distribution Statement <b>Unclassified - unlimited STAR Category 37</b>	
19. Security Classif. (of this report) <b>Unclassified</b>	20. Security Classif. (of this page) <b>Unclassified</b>	21. No. of Pages <b>26</b>	22. Price*

## SYMBOLS

- C - seal clearance along centerline
- $F_d$  - radial hydrodynamic friction force
- $\bar{F}_d$  - nondimensional force,  $F_d/6\mu\omega(\frac{r_o}{C})r_o^2$
- $F_s$  - radial hydrostatic friction force
- $\bar{F}_s$  - nondimensional force,  $F_s/\pi r_o C(p_i - p_o)$
- h - film thickness
- $I_1, I_2$  - integrals defined in eqs. (36) and (37)
- J - given by eq. (39)
- p - pressure
- R - nondimensional radius,  $r/r_o$
- r - radial coordinate
- u - tangential velocity
- v - radial velocity
- x, y, z - cartesian coordinates (Fig. 2)
- 
- $\gamma$  - angle of tilt
- $\epsilon$  - tilt parameter,  $\gamma r_o/C$
- $\bar{\epsilon}$  -  $\epsilon R_m$
- $\theta$  - angular coordinate
- $\mu$  - viscosity
- $\tau_r$  - radial shear stress
- $\tau_\theta$  - tangential shear stress
- $\omega$  - rotational angular velocity

## Subscripts

- |                            |                           |
|----------------------------|---------------------------|
| d - hydrodynamic component | o - outer radius          |
| i - inner radius           | s - hydrostatic component |
| m - mean radius            |                           |

## INTRODUCTION

Much attention has been paid in the last two decades to the problem of radial face seals operating mechanisms. In particular, axial forces, which separate the primary seal faces, and tilting moments were studied. However, radial forces which may cause a radial displacement of the primary seal ring have been totally overlooked.

The secondary seal in a radial face seal (Fig. 1) is usually an elastic member such as an O ring or a piston ring. Its main task is to prevent leakage via the radial clearance between the primary seal ring and the seal housing. Due to its flexibility, however, it also permits radial displacement of the primary seal ring. It was previously shown [1], [2] that radial eccentricity combined with angular misalignment of the primary seal ring affect the leakage through the primary seal. Radial forces may well be the origin of radial eccentricity and hence directly influence primary seal leakage. The radial forces also press the flexible element of the secondary seal against one side of the seal housing while relieving the pressure on the opposite side. As a result, the flexible element may lose contact with the rigid surface and the secondary seal starts leaking.

In addition to their influence on leakage, radial forces may also affect the dynamic stability of the seal. In two recent papers [3], [4] the dynamic behavior of a radial face seal was treated and the axial friction force in the secondary seal was considered. This friction provides damping of axial and angular vibrations of the primary seal ring and is dependent on the radial pressure exerted by the primary seal ring and seal housing on the flexible element of the secondary seal.

Hence, radial forces which affect the distribution of this radial pressure have a direct influence on seal stability.

Both hydrodynamic and hydrostatic effects can produce radial forces in a misaligned radial face seal. The hydrodynamic component is due to nonaxisymmetric tangential and radial shear, and the hydrostatic component is mainly due to nonaxisymmetric hydrostatic pressure distribution [5].

In order to establish a better understanding of radial face seal mechanism of operation the problem of radial forces in a misaligned seal is being pursued in this paper. It is believed that such an analysis, which so far has been missing, will prove helpful in further studies of a realistic seal model.

ANALYSIS

Applying the usual assumptions of the theory of lubrication [6], the momentum equations for laminar flow are, in polar coordinates, given by

$$\frac{\partial^2 v}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial r} \quad (1)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{r} \frac{\partial p}{\partial \theta} \quad (2)$$

Integrating twice, and using the boundary conditions (see Fig. 2),

$$v = 0 \text{ at } y = 0 \text{ and } y = h$$

$$u = 0 \text{ at } y = 0$$

and

$$u = \omega r \text{ at } y = h.$$

We have, for the radial and tangential velocities,

$$v = \frac{1}{2\mu} \frac{\partial p}{\partial r} y(y-h) \quad (3)$$

and

$$u = \frac{1}{2\mu} \frac{1}{r} \frac{\partial p}{\partial \theta} y(y-h) + \omega r \frac{y}{h} \quad (4)$$

The radial and tangential shear stress components are

$$\tau_r = \mu \frac{\partial v}{\partial y} = \frac{1}{2} \frac{\partial p}{\partial r} (2y-h) \quad (5)$$

and

$$\tau_\theta = \mu \frac{\partial u}{\partial y} = \frac{1}{2r} \frac{\partial p}{\partial \theta} (2y-h) + \mu \frac{\omega r}{h} \quad (6)$$

Seals usually have radius ratios,  $r_i/r_o$ , close to unity, in which case it is common to use the narrow seal approximation and neglect the circumferential pressure gradient. Hence, for narrow seals, the shear stress on the primary seal ring, at

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$y = 0$ , is

$$\tau_r = -\frac{h}{2} \frac{\partial p}{\partial r} \quad (7)$$

$$\tau_\theta = \mu \frac{\omega r}{h} \quad (8)$$

The pressure  $p$  is obtained from the Reynolds equation which, for a narrow seal and incompressible fluid, has the form

$$\frac{\partial}{\partial r}(rh^3 \frac{\partial p}{\partial r}) = 6\mu\omega r \frac{\partial h}{\partial \theta} \quad (9)$$

Eq. (9) is linear in  $p$  and hence can be solved separately for the hydrodynamic and hydrostatic pressure distributions. Each pressure distribution can then be separately integrated to obtain the various performance characteristics of the seal, including the friction force. In [5] and [7] solutions of eq. (9) are presented for the hydrostatic and hydrodynamic pressure components, respectively:

$$p_s = (p_i - p_o) \frac{h_i^2}{h_o^2 - h_i^2} \left( \frac{h_o^2}{h^2} - 1 \right) + p_o \quad (10)$$

and

$$p_d = 3\mu\omega CR_m \frac{(r_o - r)(r - r_i)}{h_m h^2} \epsilon \sin\theta \quad (11)$$

where  $h$ , the film thickness for a misaligned seal (Fig. 2), is given by:

$$h = C + y r \cos\theta \quad (12)$$

### Hydrostatic Component

From eq. (10) it is clear that the hydrostatic pressure,  $p_s$ , is symmetric about the line BB (Fig. 3) which connects the points of maximum and minimum film thickness. Hence, the radial shear stress,  $\tau_r$ , due to the hydrostatic pressure, is also symmetric about that line. As a result, the hydrostatic shear force has a component only along the z axis. This component is in the positive z direction:

$$(F_s)_z = -2 \int_0^{r_o} \int_0^{\pi} \tau_r \cos\theta r dr d\theta \quad (13)$$

From eqs. (10) and (12) we find

$$\frac{\partial p_s}{\partial r} = -2(p_i - p_o) \frac{h_1^2 h_o^2}{h_o^2 - h_1^2} \frac{\gamma \cos\theta}{h^3} \quad (14)$$

Hence, from eq. (7) the radial hydrostatic shear stress is

$$\tau_r = (p_i - p_o) \frac{h_1^2 h_o^2}{h_o^2 - h_1^2} \frac{\gamma \cos\theta}{h^2} \quad (15)$$

Substituting eq. (15) into eq. (13) and neglecting curvature effects

$$(F_s)_z = -2r_m(p_i - p_o) \int_0^{r_o} \int_0^{\pi} \frac{h_1^2 h_o^2}{h_o^2 - h_1^2} \frac{\gamma \cos^2\theta}{h^2} dr d\theta \quad (16)$$

where  $r_m$  is the mean radius  $(r_i + r_o)/2$ . Integrating first over the radius we have

$$\int_{r_i}^{r_o} \frac{dr}{h^2} = -\frac{1}{\gamma \cos\theta} \left( \frac{1}{h_o} - \frac{1}{h_i} \right)$$

and eq. (16) becomes

$$(F_s)_z = -2r_m(p_i - p_o) \int_0^\pi \frac{h_i h_o}{h_o + h_i} \cos\theta d\theta \quad (17)$$

The film thickness,  $h$ , in eq. (12) can be written in the form

$$h = C(1 + \epsilon R \cos\theta) \quad (18)$$

where  $\epsilon$  is a tilt parameter given by  $\epsilon = \gamma r_o / C$ , and  $R$  is a dimensionless radius in the form  $R = r / r_o$ . Also  $h_i + h_o = 2h_m$ , where  $h_m$  is the film thickness at the mean radius  $r_m$ . Hence,

$$\frac{h_i h_o}{h_i + h_o} \cos\theta = \frac{C}{2} \frac{\cos\theta + \epsilon(1+R_1)\cos^2\theta + \epsilon^2 R_1 \cos^3\theta}{1 + \bar{\epsilon} \cos\theta} \quad (19)$$

where

$$\bar{\epsilon} = \epsilon R_m \quad (20)$$

From the Journal Bearing Integrals [8]:

$$\int_0^\pi \frac{\cos\theta}{1 + \bar{\epsilon} \cos\theta} d\theta = -\frac{\pi}{\bar{\epsilon}} [(1 - \bar{\epsilon}^2)^{-1/2} - 1] \quad (21)$$

$$\int_0^\pi \frac{\cos^2\theta}{1 + \bar{\epsilon} \cos\theta} d\theta = \frac{\pi}{\bar{\epsilon}^2} [(1 - \bar{\epsilon}^2)^{-1/2} - 1] \quad (22)$$

$$\int_0^\pi \frac{\cos^3\theta}{1 + \bar{\epsilon} \cos\theta} d\theta = \frac{\pi}{\bar{\epsilon}^3} [(1 - \bar{\epsilon}^2)^{-1/2} + \frac{\bar{\epsilon}^2}{2}] \quad (23)$$

Substituting eq. (19) into eq. (17) and using eqs. (21) to (23) yields

$$(F_s)_z = - (p_i - p_o) r_m C \frac{\pi}{\epsilon R_m} \left\{ [(1 - \bar{\epsilon}^2)^{-1/2} - 1] (1 - \frac{R_1}{R_m^2}) + R_1 \frac{\bar{\epsilon}^2}{2} \right\} \quad (24)$$

Noting that

$$1 - \frac{R_i}{R_m} = \frac{1}{R_m} \left[ \left( \frac{1+R_i}{2} \right)^2 - R_i \right] = \left( \frac{1-R_i}{1+R_i} \right)^2 \quad (25)$$

the radial hydrostatic force in the z direction becomes

$$(F_s)_z = \pi r_o C (p_o - p_i) \bar{F}_s \quad (26)$$

where  $\bar{F}_s$  is a dimensionless force given by

$$\bar{F}_s = \frac{1}{\epsilon} \left[ (1 - \epsilon^2 R_m^2)^{-1/2} - 1 \right] \left( \frac{1-R_i}{1+R_i} \right)^2 + \frac{\epsilon}{2} R_i \quad (27)$$

A simpler expression can be obtained for small values of  $\epsilon$  by the approximation

$$(1 - \epsilon^2 R_m^2)^{-1/2} = 1 + \frac{1}{2} \epsilon^2 R_m^2 \quad (28)$$

Substituting eq. (28) into eq. (27) yields for very small tilts

$$\bar{F}_s = \frac{\epsilon}{2} R_m^2 \quad (29)$$

From eqs. (27) and (29) it can be seen that  $\bar{F}_s$  is always positive. Hence, the direction of  $(F_s)_z$  as given by (26) depends on  $(p_o - p_i)$ . When the high pressure is on the inside periphery of the seal  $(p_o - p_i) < 0$ , and hence, by (26),  $(F_s)_z < 0$ , which means that the radial hydrostatic force acting on the primary seal ring is directed towards the point where  $\theta=0$  (see Fig. 2). When the high pressure is on the outside,  $(p_o - p_i) > 0$ , and  $(F_s)_z$  is directed towards the point  $\theta=\pi$ .

## Hydrodynamic Component

Assuming a full fluid film condition, the hydrodynamic pressure,  $p_d$ , given by eq. (11) is antisymmetric about the line BB (Fig. 3). Hence, the radial hydrodynamic shear stress,  $\tau_r$ , which is also antisymmetric about that line, produces a net force only along the x axis. The film thickness,  $h$ , is symmetric about the line BB and therefore, by eq. (8),  $\tau_\theta$  is also symmetric about that line resulting again in a net force along the x axis. The total hydrodynamic friction force in the positive x direction is (see Fig. 3)

$$(F_d)_x = -2 \int_0^\pi \int_{r_1}^{r_0} (\tau_r \sin\theta + \tau_\theta \cos\theta) r dr d\theta . \quad (30)$$

From eqs. (11) and (12)

$$\frac{\partial p_d}{\partial r} = 6\mu\omega r_m \frac{h_o h_i - h_m h}{h_m h^3} \frac{\sin\theta}{\cos\theta} . \quad (31)$$

Hence, from eq. (7), the radial hydrodynamic shear stress is

$$\tau_r = -3\mu\omega r_m \frac{h_o h_i - h_m h}{h_m h^2} \frac{\sin\theta}{\cos\theta} . \quad (32)$$

From eq. (8) the tangential hydrodynamic shear stress, when curvature effects are neglected, is

$$\tau_\theta = \frac{\mu\omega r_m}{h} . \quad (33)$$

Substituting eqs. (32) and (33) into eq. (30) and neglecting curvature effects, we have

$$(F_d)_x = 6\mu\omega r_m^2 \int_0^\pi \int_{r_1}^{r_0} \left( \frac{h_o h_i - h_m h}{h_m h^2} \frac{\sin^2\theta}{\cos\theta} - \frac{\cos\theta}{3h} \right) r dr d\theta . \quad (34)$$

Integrating first over the radius, noting that  $\gamma = \epsilon C/r_0$ , yields

$$(F_d)_x = 6\mu\omega r_0^2 \left(\frac{r_0}{C}\right) R_m^2 \left\{ \int_0^\pi [(1-R_i) \frac{\sin^2\theta}{\cos\theta(1+\epsilon R_m \cos\theta)} - \frac{\sin^2\theta}{\epsilon \cos^2\theta} \ln \frac{1+\epsilon \cos\theta}{1+\epsilon R_i \cos\theta}] d\theta - \int_0^\pi \frac{1}{3\epsilon} \ln \frac{1+\epsilon \cos\theta}{1+\epsilon R_i \cos\theta} d\theta \right\} \quad (35)$$

The first integral of eq. (35) represents the contribution of the radial hydrodynamic shear stress to  $(F_d)_x$ . The second integral is the contribution of the tangential shear due to the angular velocity.

In order to integrate eq. (35), we define the integrals

$$I_1(R, \theta) = \int \frac{\sin\theta}{\cos\theta(1+\epsilon R \cos\theta)} \ln(1+\epsilon R \cos\theta) d\theta = \frac{1}{\epsilon \cos\theta} \ln(1+\epsilon R \cos\theta) + R \ln \frac{1+\epsilon R \cos\theta}{\epsilon \cos\theta} \quad (36)$$

and

$$I_2(\theta) = \int \frac{\sin\theta d\theta}{\cos\theta(1+\epsilon R_m \cos\theta)} = \ln \frac{1+\epsilon R_m \cos\theta}{\epsilon \cos\theta} \quad (37)$$

We also define the sum

$$J(\theta) = I_1(1, \theta) - I_1(R_i, \theta) - (1-R_i)I_2(\theta) \quad (38)$$

where  $I_1(1, \theta)$  and  $I_1(R_i, \theta)$  are obtained from eq. (36) by substituting  $R=1$  and  $R=R_i$ , respectively. Thus,  $J(\theta)$  becomes

$$J(\theta) = \frac{1}{\epsilon \cos\theta} \ln \frac{1+\epsilon \cos\theta}{1+\epsilon R_i \cos\theta} + \ln \frac{1+\epsilon \cos\theta}{1+\epsilon R_m \cos\theta} - R_i \ln \frac{1+\epsilon R_i \cos\theta}{1+\epsilon R_m \cos\theta} \quad (39)$$

Integrating by parts the two terms within the first integral of eq. (35) and using eqs. (36) and (37), we have

$$\int \frac{\sin^2\theta d\theta}{\cos\theta(1+\epsilon R_m \cos\theta)} = I_2(\theta) \sin\theta - \int I_2(\theta) \cos\theta d\theta \quad (40)$$

and

$$\int \frac{\sin^2 \theta}{\epsilon \cos^2 \theta} \ln(1 + \epsilon R \cos \theta) d\theta = r_1(R, \theta) \sin \theta - \int I_1(R, \theta) \cos \theta d\theta \quad (41)$$

Hence, from eqs. (40), (41) and (38), eq. (35) can be written in the form

$$(F_d)_x = 6\mu\omega \left(\frac{r_0}{C}\right) r_0^2 \bar{F}_d \quad (42)$$

where  $\bar{F}_d$  is a dimensionless force given by

$$\bar{F}_d = R_m \left( -J(\theta) \sin \theta \Big|_0^\pi + \int_0^\pi J(\theta) \cos \theta d\theta - \frac{1}{3\epsilon} \int_0^\pi \ln \frac{1 + \epsilon \cos \theta}{1 + \epsilon R_1 \cos \theta} d\theta \right) \quad (43)$$

The first term in eq. (43) vanishes at both limits of integration. Thus, substituting eq. (39) into eq. (43), we have

$$\begin{aligned} \bar{F}_d = R_m^2 \left[ \int_0^\pi \left( \frac{1}{\epsilon} \ln \frac{1 + \epsilon \cos \theta}{1 + \epsilon R_1 \cos \theta} + \cos \theta \ln \frac{1 + \epsilon \cos \theta}{1 + \epsilon R_m \cos \theta} \right. \right. \\ \left. \left. - R_1 \cos \theta \ln \frac{1 + \epsilon R_1 \cos \theta}{1 + \epsilon R_m \cos \theta} \right) d\theta - \frac{1}{3\epsilon} \int_0^\pi \ln \frac{1 + \epsilon \cos \theta}{1 + \epsilon R_1 \cos \theta} d\theta \right] \end{aligned} \quad (44)$$

The first and last terms in eq. (44) can be readily integrated using the result [9]

$$\int_0^\pi \ln(1 + \epsilon R \cos \theta) d\theta = \pi \ln \frac{1 + (1 - \epsilon^2 R^2)^{1/2}}{2} \quad (45)$$

For the other two terms in (44) we integrate by parts, which gives

$$\int_0^\pi \cos \theta \ln(1 + \epsilon R \cos \theta) d\theta = \epsilon R \int_0^\pi \frac{\sin^2 \theta}{1 + \epsilon R \cos \theta} d\theta \quad (46)$$

From [8], we find

$$\int_0^{\pi} \frac{\sin^2 \theta}{1 + \epsilon R \cos \theta} d\theta = \frac{\pi}{\epsilon^2 R^2} [1 - (1 - \epsilon^2 R^2)^{1/2}] . \quad (47)$$

Combining eqs. (45), (46) and (47) with eq. (44), the final result for the nondimensional hydrodynamic force is obtained:

$$\begin{aligned} \bar{F}_d = \frac{\pi R_m^2}{\epsilon} \left\{ \frac{2}{3} \ln \frac{1 + (1 - \epsilon^2)^{1/2}}{1 + (1 - \epsilon^2 R_i^2)^{1/2}} - (1 - \epsilon^2)^{1/2} + (1 - \epsilon^2 R_i^2)^{1/2} \right. \\ \left. - \frac{1 - R_i}{R_m} [1 - (1 - \epsilon^2 R_m^2)^{1/2}] \right\} \end{aligned} \quad (48)$$

A simpler expression for this radial hydrodynamic friction force,  $\bar{F}_d$ , can be found for small tilts by using the approximations

$$(1 - \epsilon^2 R^2)^{1/2} = 1 - \frac{1}{2} \epsilon^2 R^2 \quad \text{and} \quad \ln(1 - \frac{1}{4} \epsilon^2 R^2) = - \frac{1}{4} \epsilon^2 R^2$$

Eq. (48) thus becomes for very small tilts

$$\bar{F}_d = \frac{\pi \epsilon}{12} R_m^2 (1 - R_i^2) \quad (49)$$

where a positive value of  $\bar{F}_d$  means that the force,  $(F_d)_x$ , acting on the primary seal ring is directed towards the point

$\theta = 3\pi/2$  (Fig. 3).

It should be noted that if the radial shear stress,  $\tau_r$ , is omitted in eq. (30), the nondimensional hydrodynamic force  $\bar{F}_d$  becomes

$$\bar{F}_d = - \frac{\pi R_m^2}{3\epsilon} \ln \frac{1 + (1 - \epsilon^2)^{1/2}}{1 + (1 - \epsilon^2 R_i^2)^{1/2}} \quad (50)$$

which, for very small tilts, is approximated by the same expression as in eq. (49). This indicates that the radial hydrodynamic friction force is mainly due to the angular velocity  $\omega$ .



## RESULTS AND DISCUSSION

The results for the nondimensional radial force components  $\bar{F}_s$  and  $\bar{F}_d$  calculated from eqs. (27) and (48) are presented in Table I and Figs. 4 and 5. These results cover the whole range of angular misalignment from  $\epsilon=0$  to  $\epsilon=1$  and radius ratio,  $r_i/r_o$ , from 0.8 to 0.99.

The nondimensional hydrostatic force,  $\bar{F}_s$ , is nearly independent of the radius ratio and varies almost linearly with  $\epsilon$  (Fig. 4). The linear behavior becomes more pronounced as the radius ratio increases. An interesting result is that the expression for very small tilts (eq. (29))  $\bar{F}_s = \epsilon R_m^2 / 2$  is also a good approximation at high tilts, even up to  $\epsilon=1$ . For example, at  $r_i/r_o=0.8$  and  $\epsilon=1$ , the difference between the small tilt expression for  $\bar{F}_s$  and the complete one given by eq. (27) is only 2.5 percent; this deviation decreases further as  $r_i/r_o$  increases.

The nondimensional hydrodynamic component,  $\bar{F}_d$ , is more strongly influenced by both the radius ratio and the angular misalignment. As seen in Fig. 5,  $\bar{F}_d$  increases both with increasing tilt parameter and with a smaller radius ratio.

The hydrostatic and hydrodynamic components of the radial friction force acting on the primary seal ring are perpendicular to each other. The hydrostatic component,  $F_s$ , acts along the line BB which connects the highest and lowest points of the primary seal ring. When the high pressure is on the inside periphery of the seal,  $F_s$  is directed towards the point of maximum film thickness; when the high pressure is on the outside periphery,  $F_s$  acts in the direction of the minimum film thickness.

The hydrodynamic component,  $F_d$ , is always directed towards the point  $\theta = 3\pi/2$ . Hence, the resultant radial force is always

directed towards the region  $\pi \leq \theta \leq 2\pi$ . As far as leakage is concerned, a radial displacement of the primary seal ring in the region  $\pi \leq \theta \leq 2\pi$  will produce inward pumping [1], [2], provided that  $\theta$  is measured in the direction of  $\omega$  beginning from the point of maximum film thickness (see Fig. 2). As stated above, the hydrodynamic component,  $F_d$ , is always directed towards the point  $\theta = 3\pi/2$ . This hydrodynamic force can cause inward pumping. Indeed, inward pumping in radial face seals has been experimentally observed by several investigators [1], [10]. The intensity of the inward pumping depends on both the magnitude and direction of the radial eccentricity vector of the primary seal ring [2]. This radial eccentricity vector depends on the hydrostatic and hydrodynamic forces.

For a numerical example, consider a seal having the following dimensions and operating conditions:

outer radius, $r_o$ , cm	5
radius ratio, $r_i/r_o$	0.9
seal clearance, $C$ , cm	0.0025
angular velocity, $\omega$ , rad/sec	100
fluid viscosity, $\mu$ , N-sec/m <sup>2</sup>	$1.27 \times 10^{-3}$
pressure differential, $(p_i - p_o)$ , N/m <sup>2</sup>	$10^6$

For this seal, the maximum forces which occur at  $\epsilon=1$  are  $F_s = 1.79\text{N}$  and  $F_d = 1.74\text{N}$ . Hence, the total radial force is 2.5N at an angle  $\theta = -44$  deg. A radial force of this magnitude is a very small one and would be even smaller for values of  $\epsilon$  less than unity. Thus, depending on the flexibility of the secondary seal member, the effects of the radial forces

may be negligible. This has to be examined individually for each case. If, for example, the outer radius is doubled, the resultant radial force at  $\epsilon = 1$  now becomes 14.4N at an angle  $\theta = - 75$  deg. The force would become even higher for a smaller seal clearance, C, and higher angular velocities.

Radial force effects appear to be more pronounced when the high pressure is on the outside periphery of the seal. This is because such an arrangement is inherently unstable [5] due to nonrestoring hydrostatic moments which tend to increase any angular misalignment and thereby raise the magnitude of the radial forces.

## CONCLUDING REMARKS

Radial forces which result from both hydrostatic and hydrodynamic effects are analyzed for a misaligned radial face seal. These forces are usually small and in many cases can be considered to be negligible. When the radial forces are large enough to cause a radial displacement of the primary seal ring, they will generate inward pumping. Such a radial displacement may initiate leakage via the secondary seal and also affect the dynamic stability of the primary seal ring.

It is postulated that radial forces effects are most pronounced when the high pressure is on the outside periphery of the seal. In this case the inward pumping adds to the hydrostatic leakage and may cause seal failure due to an excessive leakage.

## REFERENCES

1. Findlay, J.A.: Inward Pumping in Mechanical Face Seals. J. Lub. Tech., Trans. ASME, Ser. F, Vol. 91, No. 3, July 1969, pp. 417-426.
2. Sneek, H.J.: The Misaligned, Eccentric Face Seal. J. Lub. Tech., Trans. ASME, Ser. F, Vol. 91, No. 4, Oct. 1969, pp. 695-703.
3. Shapiro, W. and Colsher, R.: Steady State and Dynamic Analysis of a Jet Engine, Gas Lubricated Shaft Seal. ASLE Trans., Vol. 17, No. 3, July 1974, pp. 190-200.
4. Ludwig, L.P. and Allen, G.P.: Face Seal Lubrication II - Theory of Response to Angular Misalignment. NASA TN D-8102, March 1976.
5. Etsion, I.: Nonaxisymmetric Incompressible Hydrostatic Pressure Effects in Radial Face Seals. NASA TN D-8343, Nov. 1976.
6. Pinkus, O. and Sternlicht, B.: Theory of Hydrodynamic Lubrication. McGraw-Hill Book Co., 1961.
7. Etsion, I.: Hydrodynamic Effects in a Misaligned Radial Face Seal. TME Report No. 310, March 1977, Department of Mechanical Engineering, Technion, Haifa, Israel.
8. Booker, J.F.: A Table of the Journal Bearing Integral. J. Basic Eng., Trans. ASME, Ser. D, Vol. 87, June 1965, pp. 533-535.

9. Dwight, H.B.: Tables of Integrals and Other Mathematical Data. The Macmillan Co., 1957.
10. Denny, D.F.: Some Measurements of Fluid Pressures between Plane Parallel Thrust Surfaces with Special Reference to Radial-Face Seals. Wear, Vol. 4, 1961, pp. 64-83.

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$r_i/r_0$	$\epsilon$	$F_s$	$F_d$	$F_d(\tau_r=0)$
0.80	C.1	0.041	0.008	0.008
	C.2	0.081	0.016	0.016
	C.3	0.122	0.024	0.024
	C.4	0.162	0.034	0.034
	C.5	0.203	0.045	0.045
	C.6	0.244	0.060	0.059
	C.7	0.285	0.079	0.078
	C.8	0.327	0.107	0.106
	C.9	0.370	0.158	0.156
	1.0	0.416	0.241	0.399
0.85	C.1	0.043	0.006	0.006
	C.2	0.086	0.013	0.013
	C.3	0.128	0.020	0.020
	C.4	0.171	0.028	0.028
	C.5	0.214	0.037	0.037
	C.6	0.257	0.049	0.049
	C.7	0.300	0.065	0.065
	C.8	0.344	0.090	0.090
	C.9	0.388	0.136	0.135
	1.0	0.436	0.387	0.379
0.90	C.1	0.045	0.005	0.005
	C.2	0.090	0.009	0.009
	C.3	0.135	0.014	0.014
	C.4	0.181	0.020	0.020
	C.5	0.226	0.027	0.027
	C.6	0.271	0.036	0.036
	C.7	0.316	0.048	0.048
	C.8	0.362	0.068	0.067
	C.9	0.408	0.105	0.105
	1.0	0.456	0.347	0.342
0.92	C.1	0.046	0.004	0.004
	C.2	0.092	0.008	0.008
	C.3	0.138	0.012	0.012
	C.4	0.184	0.017	0.017
	C.5	0.230	0.023	0.023
	C.6	0.277	0.030	0.030
	C.7	0.323	0.040	0.040
	C.8	0.369	0.057	0.057
	C.9	0.416	0.090	0.089
	1.0	0.464	0.323	0.319

Table I. - Nondimensional Radial Force Components

$r_i/r_0$	$\epsilon$	$F_s$	$F_d$	$F_d(\tau_r=0)$
0.94	C.1	0.047	0.003	0.003
	C.2	0.094	0.006	0.006
	C.3	0.141	0.009	0.009
	C.4	0.188	0.013	0.013
	C.5	0.235	0.017	0.017
	C.6	0.282	0.023	0.023
	C.7	0.329	0.032	0.032
	C.8	0.377	0.045	0.045
	C.9	0.424	0.072	0.072
	1.0	0.473	0.292	0.282
0.96	C.1	0.048	0.002	0.002
	C.2	0.096	0.004	0.004
	C.3	0.144	0.006	0.006
	C.4	0.192	0.009	0.009
	C.5	0.240	0.012	0.012
	C.6	0.288	0.016	0.016
	C.7	0.336	0.022	0.022
	C.8	0.384	0.031	0.031
	C.9	0.433	0.051	0.051
	1.0	0.482	0.250	0.248
0.98	C.1	0.049	0.001	0.001
	C.2	0.098	0.002	0.002
	C.3	0.147	0.003	0.003
	C.4	0.196	0.005	0.005
	C.5	0.245	0.006	0.006
	C.6	0.294	0.008	0.008
	C.7	0.343	0.011	0.011
	C.8	0.392	0.017	0.017
	C.9	0.441	0.028	0.028
	1.0	0.491	0.187	0.186
0.99	C.1	0.050	0.001	0.001
	C.2	0.099	0.001	0.001
	C.3	0.149	0.002	0.002
	C.4	0.198	0.002	0.002
	C.5	0.248	0.003	0.003
	C.6	0.297	0.004	0.004
	C.7	0.347	0.006	0.006
	C.8	0.396	0.008	0.008
	C.9	0.446	0.014	0.014
	1.0	0.495	0.137	0.137

Table I. - Concluded



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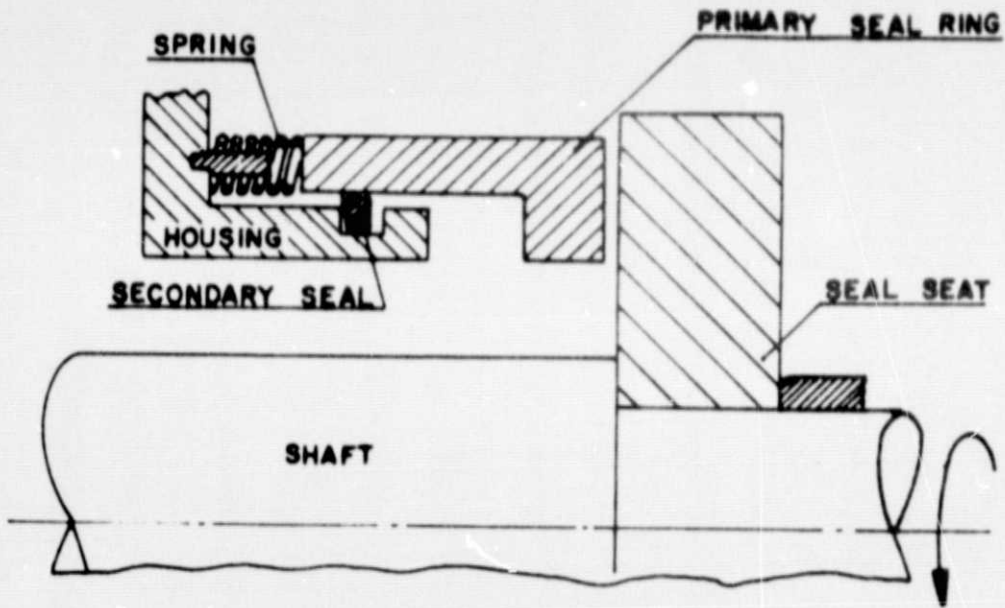


Figure 1 - Radial face seal, identifying nomenclature.

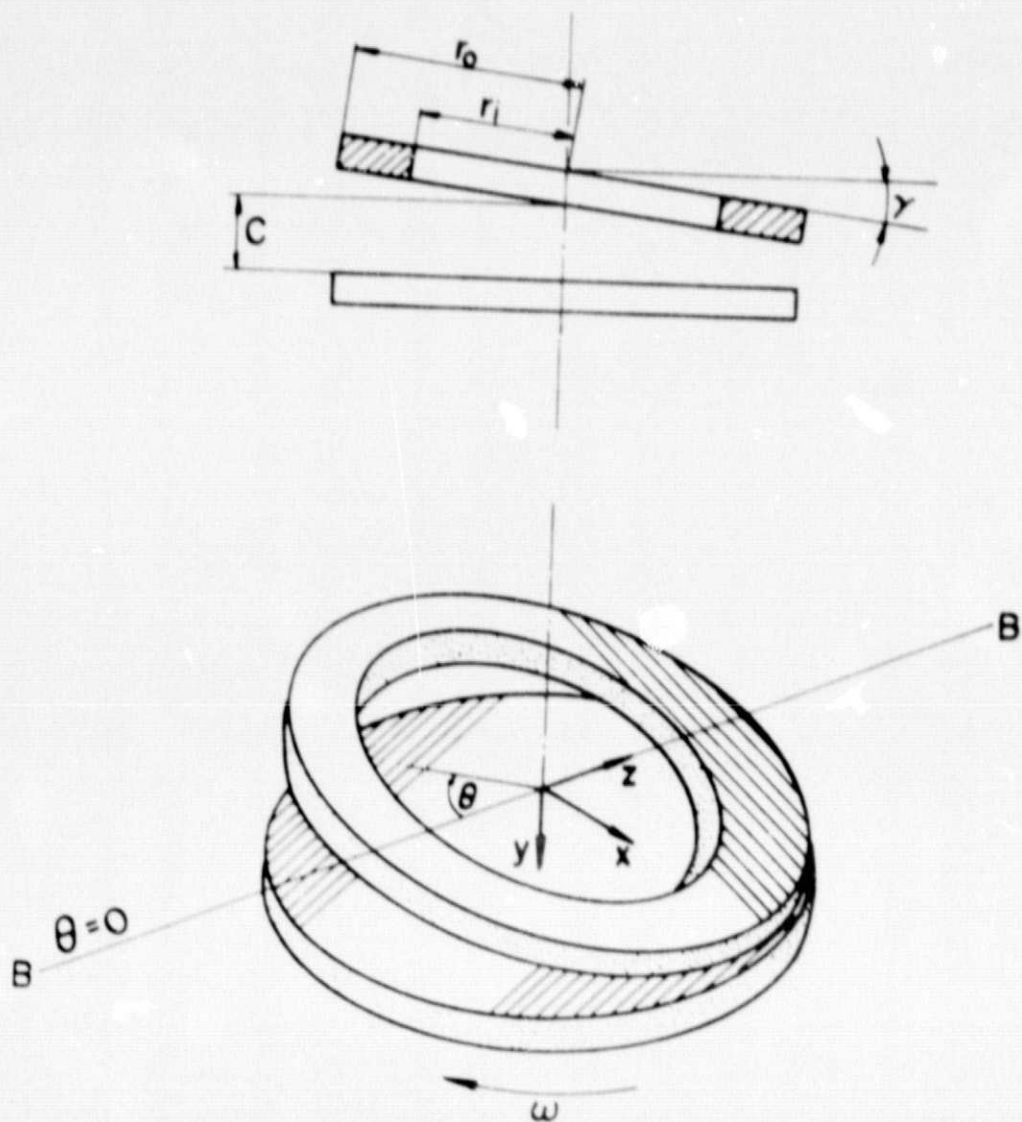


Figure 2 - Face seal with angular misalignment.

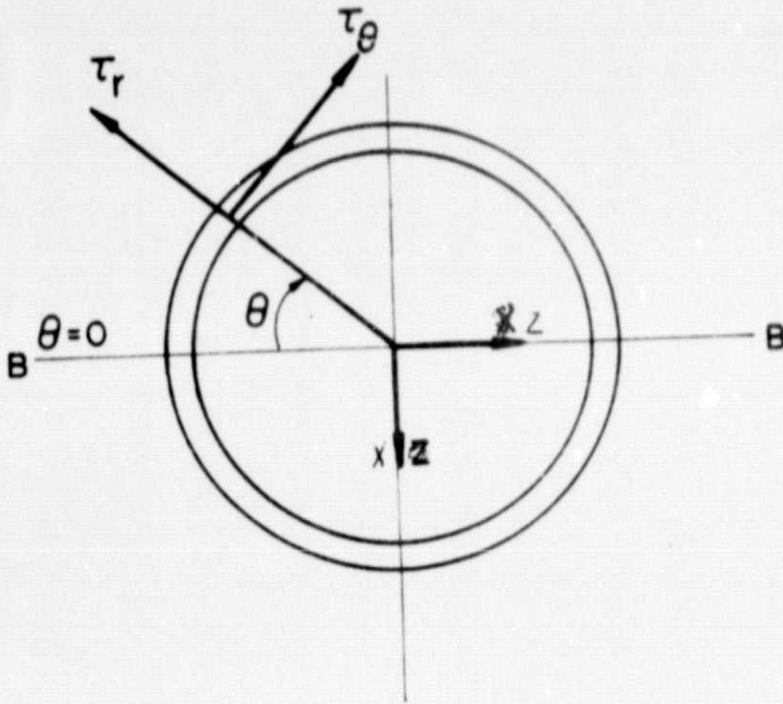


Figure 3 - Primary seal ring and shear stress components.

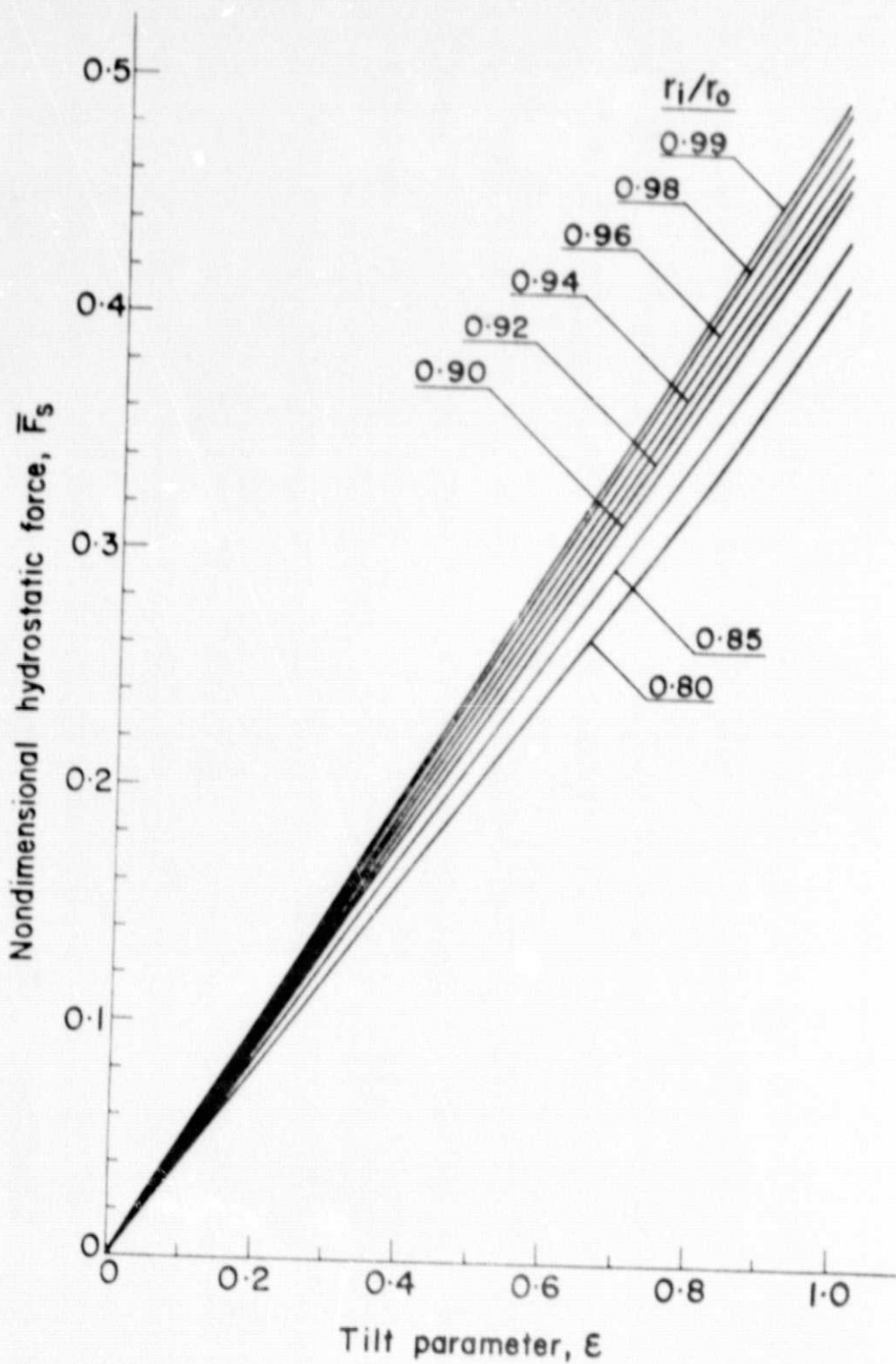


Figure 4 - Nondimensional hydrostatic radial force as a function of tilt parameter for various radius ratios.

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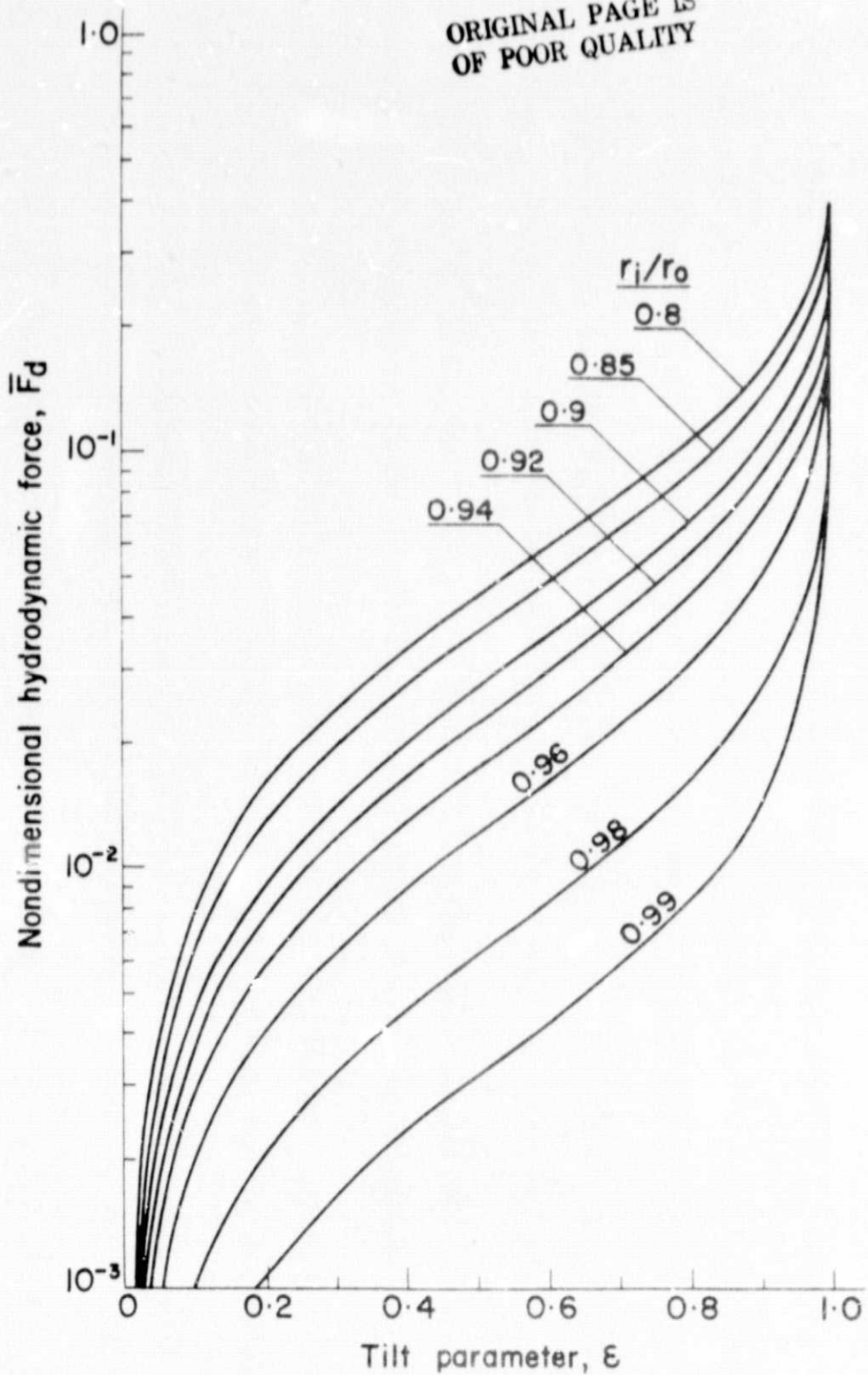


Figure 5 - Nondimensional hydrodynamic radial force as a function of tilt parameter for various radius ratios.