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A SIMPLE ELEMENT FOR MULTILAYER BEAMS IN NASTRAN THERMAL STRESS ANALYSIS

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I. INTRODUCTION

In the study of structural integrity for electronic packages under thermal cycles, an important consideration is the distortion due to the differential thermal expansions of multilayer dissimilar materials. A simple antecedent for such cases is the classical bimetallic beam analysis by S. Timoshenko. In the application of NASTRAN, such structural members are usually represented by bar elements with multi-point constraint cards to enforce the interface conditions. While this is a very powerful method in principle, one finds that in practice the process for specification of constraints becomes tedious and error prone, unless the geometry is simple and the number of grid points low. An alternative approach has been found within the framework of the NASTRAN program. This approach makes use of the idea that a thermal distortion in a multilayer beam may be similar to a homogenous beam with a thermal gradient across the cross section. This paper contains the exact mathematical derivation for the equivalent beam, and all the necessary formulae for the equivalent parameters in NASTRAN analysis. Some numerical examples illustrate the simplicity and ease of this approach for finite element analysis such as NASTRAN.

II. ANALYSIS

Consider an n-layer composite beam of dissimilar materials at constant temperature T, having an external loading N_x as the axial force, and M_y as the bending moment. The cross section is shown in Figure 1.

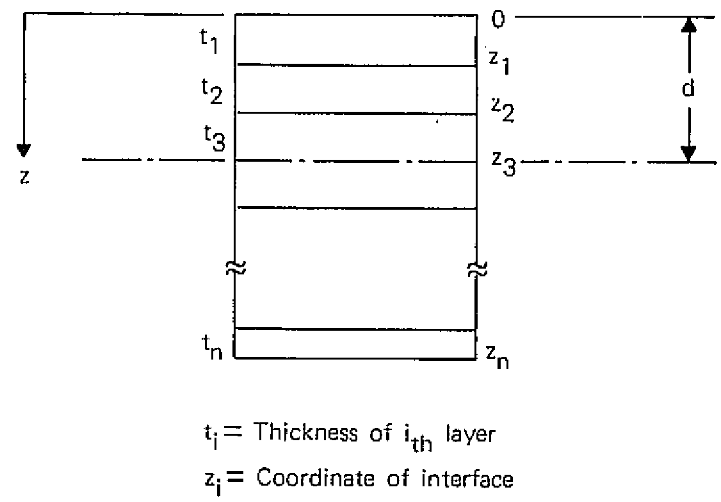

Fig. 1 - Cross Section of n layer Composite Beam

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t_1, t_2..t_n are the thicknesses of various layers and z_1..z_n are the coordinates at the interface. With the coordinates as specified,

$$zi = \sum_{j=1}^{i} t_j$$

Strain at any point can be written as

$$e_{xx} = e + \frac{z-d}{r} - \alpha T \tag{1}$$

where e_{xx} = elastic strain

e = average axial strain of composite
z = coordinate of the point
d = composites neutral axis
r = radius of curvature
αT = thermal strain
α = thermal coefficient of expansion of the layer

$$\sigma_{xx} = E\, e_{xx}$$

The axial force N_x and bending moment M_y are given by

$$N_x = \int \sigma_{xx}\, dA$$

$$M_y = \int (z-d)\, \sigma_{xx}\, dA \tag{2}$$

Let E_i, α_i, t_i be the Young's modulus, thermal coefficient of expansion and thickness of ith layer with uniform width b.

Define $z_o = 0$ and,

$$\beta = \int EdA = b \int_{o}^{zn} Edz$$

$$= b \sum_{i=1}^{n} E_i \; (z_i - z_{i-1})$$

$$= b \sum_{i=1}^{n} E_i \; t_i \tag{3}$$

$$\gamma = \int EzdA = b \int_{o}^{zn} Ezdz$$

$$= \frac{b}{2} \sum_{i=1}^{n} E_i \; (z_i^2 - z_{i-1}^2 \tag{4}$$

$$\delta = \int Ez^2 dA = b \int_{o}^{zn} Ez^2 dz$$

$$= \frac{b}{3} \sum_{i=1}^{n} E_i \; (z_i^3 - z_{i-1}^3 \tag{5}$$

$$N_T = \int E \alpha T \cdot dA = bT \int_{o}^{zn} E \alpha \, dz$$

$$= bT \sum_{i=1}^{n} E_i \; t_i \; \alpha_i \tag{6}$$

$$M_T = \int E \alpha Tz \cdot dA = bT \int_{o}^{zn} E \alpha \, zdz$$

$$= \frac{bT}{2} \sum_{i=1}^{n} E_i \; \alpha_i \; (z_i^2 - z_{i-1}^2 \tag{7}$$

then

$$N_x = \int \sigma_{xx} dA$$

$$= b \int_o^{zn} \left(Ee + E \frac{(z-d)}{r} - \alpha T \right) dz$$

$$= \beta e + \frac{\gamma - \beta d}{r} \div N_T \qquad (8)$$

Similarly M_y can be shown as

$$M_y = \gamma e + \frac{\delta - \gamma d}{r} - M_T - d\left(\beta e + \frac{\gamma - \beta d}{r} - N_T \right) \qquad (9)$$

From Equation (3), since N_x should be independent of r,

therefore

$$\beta d - \gamma = o$$

$$d = \frac{\gamma}{\beta} = \text{distance of neutral axis},$$

substituting this in Equations (8) and (9) we get

$$N_x = \beta e - Nt \qquad (10)$$

$$M_y = \left(\delta - \frac{\gamma^2}{\beta} \right) \frac{1}{r} - \left(M_T - \frac{\gamma}{\beta} N_T \right) \qquad (11)$$

Thus the external force and moment have been derived for n layer structure. Expression $\delta - \frac{\gamma^2}{\beta}$ is simply area stiffness [in which E is included] moment of inertia about centroid, and can be proved as follows.

δ = Area Stiffness M.I. about coordinate axis[as defined by Equation 5] .

MI about centroid [using parallel axis theorem] =

$$= .\delta - \text{Area Stiffness} \times d^2$$

$$= \delta - \beta \cdot \frac{\gamma^2}{\beta^2}$$

$$= \delta - \frac{\gamma^2}{\beta}$$

Consider now an equivalent homogenous beam of area A, centroid moment of inertia I, Young's modulus E, thermal coefficient of expansion having thermal gradient $\frac{dT}{dz}$.

Then

$$e_{xx} = e + \left(\frac{z}{r} - z\,\frac{dT}{dz}\,.\alpha\right) - \alpha T$$

where the coordinate axes are the neutral axes of the beam. By using the previous formulation it can be derived that

$$N_x = AEe - AE\alpha T \qquad (12)$$

$$M_y = \frac{I}{r} - I\alpha \frac{dT}{dz} \qquad (13)$$

Comparing Equation 10 to 12 and 11 to 13, the following parametric relationships can be written for equivalency.

$$AE = \beta \qquad (14)$$

$$AE\alpha T = N_T \qquad (15)$$

$$I = \delta - \frac{\gamma^2}{\beta} \qquad (16)$$

$$I\alpha \frac{dT}{dz} = P_T - \frac{\gamma}{\beta} N_T \qquad (17)$$

Thus assuming E for the equivalent beam, Area A, α, I, $\frac{dT}{dz}$ can be calculated from Equations 14 through 17.

Thus a nonhomogenous beam [specified as a finite element in NASTRAN] at constant temperature can be solved by a mathematical equivalent homogenous beam with temperature gradient at the cross section.

III. TIMOSHENKO'S BIMETALLIC BEAM

It can further be proved that for two layer structure, in the absence of external forces, Equation 9 degenerates to Timoshenko bimetallic beam.

Consider now a bimetallic beam of thickness $t_1 = t_2 = t$, and therefore

$z_1 = t$, $z_2 = 2t$.

From Equations 3, 4, 5, 6 and 7

$$\beta = \frac{bt}{2}\ (E_1 + E_2)$$

$$\gamma = \frac{bt^2}{8}\ \left[E_1 + 3E_2\right]$$

$$\delta = \frac{b}{24}\ t^3\ \left[E_1 + 7E_2\right]$$

$$N_T = bT\ \frac{t}{2}\ (E_1\alpha_1 + E_2\alpha_2)$$

$$M_T = \frac{bt}{8}\ t^2\ (E_1\alpha_1 + 3E_2\alpha_2)$$

$$M_T - \frac{\gamma}{\beta} N_T$$

$$= \frac{bt^2}{8}\ T\left[E_1\alpha_1 + 3E_2\alpha_2\right] - \frac{bt^2}{8}\ T\ \left[E_1\alpha_1 + E_2\alpha_2\right]\ \left[\frac{E_1+3E_2}{E_1+E_2}\right]$$

$$= \frac{bt^2}{4}\ T\left[\alpha_2 - \alpha_1\right]\ \frac{E_1 E_2}{E_1+E_2}$$

$$\delta - \frac{\gamma^2}{\beta}$$

$$= \frac{b}{24} t^3\ \left[E_1 + 7E_2\right] - \frac{bt^3}{32}\ \frac{(E_1+3E_2)^2}{E_1+E_2}$$

$$= \frac{bt^3}{96}\ (E_1+E_2) + \frac{bt^3}{8}\ \left(\frac{E_1 E_2}{E_1 + E_2}\right)$$

Substituting in Equation 11, we get

$$M_y = \frac{1}{r}\ \left\{\frac{bt^3}{96}\ (E_1+E_2) + \frac{bt^3}{8}\left(\frac{E_1E_2}{E_1+E_2}\right)\right\} - \frac{bt^2}{4}\ T\ (\alpha_2 - \alpha_1)\ \frac{E_1E_2}{E_1+E_2}$$

$$= \frac{bt^2}{4}\ \frac{E_1E_2}{E_1+E_2}\ \left[\frac{\frac{4}{bt^2}\ 1\ (E_1+E_2)\ \left(\frac{1}{E_1} + \frac{1}{E_2}\right) - \frac{t}{2}}{r} - (\alpha_2 - \alpha_1)\ T\right]$$

In the absence of external force, the expression in parentheses is zero, which is identical to Timoshenko's bimetallic beam.

IV. EXAMPLE

For the purpose of illustration, the example in Strength of Materials Part I by Timoshenko [1] will be discussed. To model a bimetallic beam section, one defines four grid points and two bar elements. For each pair of grid points, continuity of the interface displacements must be specified. Multipoint constraints or MPC cards must be used to relate the displacements and rotation of one bar element to the other of the following form

$$u_1 + h_1 \theta_1 = u_2 + h_2 \theta_2$$

It should be noted that this could become quite complicated when many elements are used to represent a curved beam in space.

An equivalent beam with the same mechanical response will now be constructed. The bending curvature of the bimetallic beam will now be induced by the TEMPRB Bulk Data card. The bending stress in the original bimetallic beam due to differential expansion will now be replaced by the thermal moment thermal gradient defined by the TEMPRB card for each bar element. In the example in reference [1], page 219, we shall use

$$b = h = 1,\ E_1 = 1 \times 10^7,\ E_2 = 1.15 \times 10^7,\ \alpha_1 = 1 \times 10^{-4},\ \alpha_2 = 2 \times 10^{-4}$$

Then it turns out that the equivalent beam should be

$$A = 1.8695,\ I = .62091,\ E = 1.15 \times 107,\ \alpha = 1.5348 \times 10^{-4};$$

with 100 degree rise in temperature, the gradient should be 48.8.

V. DISCUSSION

The method proposed in this paper has been found to be very useful in analysis dealing with deformation associated with multi-layered curved beam structures undergoing thermal loads. The main advantages are (1) elimination of the time-consuming task in specifying multipoint constraints, and (2) reduction in number of grid points.

VI. REFERENCES

1. S. Timoshenko, Strength of Materials Part I, D. Van Nostrand, Inc., 1940