BOUNDS ON THICKNESS AND LOADING NOISE OF ROTATING BLADES AND THE FAVORABLE EFFECT OF BLADE SWEEP ON NOISE REDUCTION

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SUMMARY

In this paper the maxima of amplitudes of thickness and loading noise harmonics are established when the radial distribution of blade chord, thickness ratio, and lift coefficient is specified. It is first shown that only airfoils with thickness distribution and chordwise loading distributions which are symmetric with respect to midchord need be considered for finding the absolute maxima of thickness and loading noise. The resulting chordwise thickness and load distributions for these maximum noise conditions require infinite slope at some points along the chord but otherwise are uniform. It is shown that sweeping the blades reduces the thickness and loading noise, but there is no optimum sweep which generates the lowest noise.

INTRODUCTION

In the design of a high tip-speed rotating blade such as a helicopter rotor or a propeller, one important acoustic question is: given radial (spanwise) load distribution, thickness ratio, and chord distribution of the blade, can the maximum of the level of each of the sound harmonics be established? These maxima, of course, correspond to the worst possible acoustic design. If these maximum levels are kept within acceptable limits, then neither the chordwise load distribution nor the airfoil shape would be of concern in the acoustic design. Another question which comes to mind next is whether sweeping the blade tips appropriately can result in the lowest possible noise. In this paper both of the above questions are studied and answered.

The starting point of our analysis is the following equation. Let S be the mean surface of the blade, h and p be the local thickness and load distribution on this surface, respectively. Then, the acoustic pressure $p'(\vec{x},t)$ in the far field is given by the equation

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$$4\pi r_o p'(\vec{x},t) = \rho_o \frac{\partial^2 \Psi}{\partial t^2} + \frac{\hat{r}_i}{c} \frac{\partial L_i}{\partial t} \tag{1-a}$$

$$\Psi(\vec{x},t) = \int_S \left[\frac{h}{|1-M_r|}\right]_{ret} dS \tag{1-b}$$

$$L_i(\vec{x},t) = - \int_S \left[\frac{pn_i}{|1-M_r|}\right]_{ret} dS \tag{1-c}$$

The first term in eq. (1-a) is called the thickness noise. This formulation of thickness noise was derived by Hanson (ref. 1) and by Farassat (ref. 2), using a different approach. The second term in eq. (1-a) is the loading noise.

It is assumed that the blade system, lying in y_1y_2-plane, is not in motion as a whole. That is, only hovering rotors and static propellers are considered here. It is also assumed that unsteady loading noise is negligible. For high-speed rotating blades, this assumption is justified for observer positions where the sources on the blades appear noncompact. Under this assumption, the acoustic pressure will be periodic with fundamental frequency based on blade passage frequency. For simplicity one blade is considered in the analysis.

The nth Fourier component of the noise, $p'_n(\vec{x})$, is found from eq. (1-a) to be

$$4\pi r_o p'_n(\vec{x}) = - \rho_o n^2 \omega^2 \Psi_n(\vec{x})$$

$$- ikn \hat{r}_i L_{in}(\vec{x}) \tag{2}$$

In the following analysis, the surface integrals with respect to S, used in evaluation of $\Psi_n(\vec{x})$ and $L_{in}(\vec{x})$, are written in an unconventional manner in chordwise direction. Written in this form, the effect of sweep can be introduced easily. The bounds are obtained in two stages as follows. First, it is shown that if the airfoil shape is deformed in such a way that the chordwise distance between the points of equal thickness on the airfoil surface is not changed, then the thickness noise is maximum if the airfoil is made symmetric with respect to a radius of the blade disc at each radial position. A similar result holds for the chordwise loading distribution. Therefore to obtain the absolute maxima of thickness and loading noise, only airfoil shapes and chordwise loading with midchord symmetry should be considered. These and the related result concerning sweep do not apply at high frequencies due to the mathematical limitations of some of the inequalities used in their derivation. The range of applicability of these results is, however, wide, particularly in the case of helicopter rotors and conventional propellers.

In all the examples in this paper a rotor blade of 5-m radius and uniform chord of 0.4 m is used. The number of blades is two and the tip Mach number is 0.95.

SYMBOLS

A	function of η (used in eq. (12))
B	function of η (used in eq. (14)), number of blades
b	chordwise variable (see fig. 1), m
b'	mean chord in blade tip region, m
b_o	blade chord (function of η), m
b_p	value of $b \leq b_o$ where $\sin(nb/2\eta)$ achieves its peak, m
C	coefficient used in eq. (5)
c	speed of sound, m/sec
c_ℓ	section lift coefficient (a function of η)
$g(\vec{y}),g(\vec{y}')$	arbitrary positive functions
h	blade section thickness variable (see fig. 1), m
h_m	maximum thickness of blade section (function of η), m
I,I_γ	integrals defined in eq. (7)
J_n	Bessel function of first kind on nth order
k	wave number, ω/c
L_i,L_{in}	surface integral used in calculation of loading noise, and its amplitude of the nth Fourier component of L_i (see eq. (1-c) and (2)), i=1,2,3
M_r	Mach number along radiation direction
M_t	tip Mach number of the blade
n	harmonic number
n_i	unit normal to surface S, direction from pressure to suction side of the blade. i=1,2,3
p	load distribution of the blade, Pa
p_m	peak section load (function of η), Pa
p'	acoustic pressure, Pa
p'_n	amplitude of the nth Fourier component of p', Pa

$Q_n(\vec{x})$	function defined in eq. (10-a)
r	$\lvert\vec{x}-\vec{y}\rvert$, m
r_o	observer distance from center of rotation, m
$\hat{r}_i$	$(x_i-y_i)/r$, radiation vector
R, R_i	blade outer and inner radius, respectively, m
S	mean surface of the blade
t	observer time, sec
$\tilde{t}$	thickness ratio of the blade (a function of η)
$T_n(\vec{x})$	function defined in eq. (10-b)
T	period of the sound, sec
$\vec{x}$	observer position vector, origin at rotation center
$\vec{y}$	source position vector, origin at rotation center
α	geometric angle of attack (function of η), deg
β	azimuthal angle, rad
β_c	$(\beta_t+\beta_\ell)/2$ azimuthal angle of point C midway points A and B in fig. 1, rad
β_t, β_ℓ	functions of η and h or p indicating azimuthal angles of points A and B in fig. 1, rad
$\gamma(\vec{y})$	arbitrary function (see eq. 8)
$\delta(b-b_p)$	Dirac delta function
ε	angle between axis of rotation and $\vec{x}$, rad
η	radial position variable, m
μ	variable defining the degree of blade sweep
ρ_o	density of the undisturbed medium, kg/m^3
τ	source time, sec
Ψ	surface integral used in calculation of thickness noise (see eq. (1-b))
Ψ_n	amplitude of the nth Fourier component of Ψ
ω	angular velocity of the blade, rad/sec

DERIVATION OF THE BOUNDS

In this section, our attention will be focused on Ψ_n. The manipulations for L_{in} are identical to those of Ψ_n. If T is the period of the sound, then

$$\Psi_n(\vec{x}) = \frac{1}{T}\int_0^T \Psi(\vec{x},t)e^{in\omega t}\,dt \tag{3}$$

For our purpose, the volume $\Psi(\vec{x},t)$ will be written as

$$\Psi(\vec{x},t) = \int_{R_i}^{R} \eta d\eta \int_0^{h_m} dh \int_{\beta_t}^{\beta_\ell} [|1-M_r|]^{-1}_{ret}\, d\beta \tag{4}$$

where η is the radial position and h_m is the maximum thickness of the airfoil. The azimuthal angle is denoted by β. The angles β_t and β_ℓ are the azimuthal angles of points A and B, respectively, in fig. (1). Note that $h_m=h_m(\eta)$ $\beta_t=\beta_t(\eta,h)$, and $\beta_\ell=\beta_\ell(\eta,h)$. The only dependence on $\vec{x}$ and t in eq. (4) comes through the integrand so that the time integral in eq. (3) commutes with all the integrals in eq. (4).

We now introduce the source time τ in a manner used by Hawkings and Lowson (ref. 3). Since $dt=[|1-M_r|]_{ret}\,d\tau$, the Doppler singularity in eq. (4) is cancelled. Writing $t=\tau+r/c$ and using the well-known integration with respect to τ which results in a Bessel function of first kind (ref. 3), we get

$$\Psi_n(\vec{x}) = C\int_{R_i}^{R} \eta d\eta \int_0^{h_m} dh \int_{\beta_t}^{\beta_\ell} e^{-in\beta} J_n(nk\eta\sin\epsilon)d\beta \tag{5}$$

where $C=(i)^n e^{iknr_o}$ is a constant. Let $\beta_c=(\beta_t+\beta_\ell)/2$ and use $\beta_\ell-\beta_t\approx b/\eta$ where $b=b(\eta,h)$ is the chordwise distance between points A and B in fig. (1). Then integrating eq. (5) with respect to β results in

$$\Psi_n(\vec{x})=-2iC\int_{R_i}^{R}\int_0^{h_m} \eta J_n(nk\eta\sin\epsilon)\, e^{-in\beta_c}\sin(\frac{nb}{2\eta})\, dhd\eta \tag{6}$$

We note that the angle $\beta_c=\beta_c(\eta,h)$. We will show that the maximum of Ψ_n when only β_c varies corresponds to β_c=constant, that is at each radial station and for all $0\leq h\leq h_m$, points A and B should be located symmetrically with respect to one and the same radius of the blade disk. We need the following result. If $g(\vec{y})\geq 0$ and $\gamma(\vec{y})$ is an arbitrary function, then in any region D

$$I_\gamma = \left|\int_D e^{i\gamma}\, g\, d\vec{y}\right| \leq \int_D g\, d\vec{y} = I \tag{7}$$

To prove this, we note that

$$I^2 - |I_\gamma|^2 = \int_D \int_D 2\sin^2 \frac{1}{2}\left[\gamma(\vec{y})-\gamma(\vec{y}\,')\right] g(\vec{y})g(\vec{y}\,')d\vec{y}d\vec{y}\,' \geq 0 \tag{8}$$

In fact, strict inequality $I_\gamma < I$ holds in most cases since the integrand in eq. (8) has to be non-zero only in a small region in DxD. Note that if $\gamma(\vec{y})=$ constant, $|I_\gamma| = I$.

In eq. (6), $J_n(nk\eta\sin\varepsilon)\geq 0$ even for transonic and low supersonic speeds, since $nk\eta\sin\varepsilon\leq nM_t$ where M_t is the tip Mach number. If $\sin(\frac{nb}{2\eta})\geq 0$, then we can apply the result proved above. This would require $nb/2\eta\leq\pi$ or $n\leq 2\pi\eta/b$. Since for high-speed blades, the tip region is responsible for the generation of the noise, a reasonable value for n is $n\leq 2\pi\vec{x}\cdot 7R/b'=4.4R/b'$ where b' is the mean chord in the blade tip region. For blades with B blades; we must have $nB\leq 4.4R/b'$. For two-bladed helicopter rotor blades the following result typically holds up to twenty fifth harmonics of the blade passage frequency. Applying eq. (7) to eq. (6), we get

$$|\Psi_n(\vec{x})| \leq 2\int_{R_i}^{R}\int_0^{h_m} \eta J_n(nk\eta\sin\varepsilon)\sin(\frac{nb}{2\eta})\, dh d\eta \tag{9}$$

In exactly similar fashion, we can show that

$$|Q_n(\vec{x})| \equiv |\hat{r}_1 L_{1n}+\hat{r}_2 L_{2n}| \leq \frac{2}{k}\int_{R_i}^{R}\int_0^{p_m} J_n(nk\eta\sin\varepsilon)\sin(\frac{nb}{2\eta})\sin\alpha(\eta)dp d\eta\ , \tag{10-a}$$

and

$$|T_n(\vec{x})| \equiv |\hat{r}_3 L_{3n}| \leq 2|\cos\varepsilon| \int_{R_i}^{R}\int_0^{p_m} \eta J_n(nk\eta\sin\varepsilon)\sin(\frac{nb}{2\eta})\cos\alpha(\eta)dp d\eta \tag{10-b}$$

where $p_m=p_m(\eta)$ is the peak chordwise loading and $\alpha(\eta)$ is the geometric angle of attack. Equations (10) and (11) describe bounds on torque and lift or thrust noise, respectively, when they are used in eq. (2).

We have shown above that, if we are only allowed to deform the airfoil shape or the chordwise loading in such a way that the chordwise distance between points of equal thickness or equal loading is kept fixed, the maximum thickness and loading noise correspond to symmetrical positioning of such points with respect to the same radius. Incidentally, in this case the thickness noise and loading noise are 90 degrees out of phase.

To find the absolute maxima of thickness and loading noise, eqs. (9) and (10) will be used. It is assumed that the airfoil thickness and load distribution functions are monotonic with respect to variable b. This assumption is satisfied in most cases of interest. To be specific, thickness noise will be considered first. The right side of eq. (9) can be written as

$$\int_{R_i}^{R} \int_{0}^{h_m} \eta J_n(nk\eta\sin\varepsilon)\sin\left(\frac{nb}{2\eta}\right) dh d\eta$$

$$= - \int_{R_i}^{R} \int_{0}^{b_o} \eta J_n(nk\eta\sin\varepsilon)\sin\left(\frac{nb}{2\eta}\right)\frac{dh}{db}\, db d\eta \tag{11}$$

where $b_o=b_o(\eta)$ is the blade chord. To maximize the last integral, take

$$\frac{dh}{db} = - A\delta(b-b_p) \tag{12}$$

where A is a function determined by the maximum thickness of the airfoil and $b_p=b_p(\eta)$ is the value of $b\leq b_o$ where $\sin(nb/2\eta)$ achieves its peak. If $\tilde{t}(\eta)$ is the thickness ratio of the airfoil, then $A(\eta)=b_o(\eta)\tilde{t}(\eta)$. Using eqs. (9), (11) and (12), we find that

$$|\Psi_n(\vec{x})|_{max} = 2 \int_{R_i}^{R} \eta b_o \tilde{t} J_n(nk\eta\sin\varepsilon)\sin\left(\frac{nb_p}{2\eta}\right) d\eta \tag{13}$$

Similarly, to maximize the integrals in eqs. (10-a) and (10-b), take

$$\frac{dp}{db} = - B\delta(b-b_p) \tag{14}$$

where B is a function determined by spanwise loading of the blade. If $c_\ell(\eta)$ is section lift coefficient of the blade, then $B=\rho_o b_o \eta^2\omega^2 c_\ell/2b_p$. Equations (10-a) and (10-b) then give

$$|Q_n(\vec{x})|_{max} = \frac{\rho_o \omega^2}{k} \int_{R_i}^{R} \frac{b_o \eta^2 c_\ell}{b_p} J_n(nk\eta \sin\varepsilon) \sin(\frac{nb_p}{2\eta}) \sin\alpha(\eta) d\eta \quad (14\text{-a})$$

$$|T_n(\vec{x})|_{max} = \rho_o \omega^2 |\cos\varepsilon| \int_{R_i}^{R'} \frac{b_o \eta^3 c_\ell}{b_p} J_n(nk\eta \sin\varepsilon) \sin(\frac{nb_p}{2\eta}) \cos\alpha(\eta)\ d\eta \quad (14\text{-b})$$

We have shown that for all blades with a given thickness ratio $\tilde{t}(\eta)$ at each radial station, the rms amplitude of the nth harmonic of thickness noise has the following bound

$$4\pi r_o |p_n'(\vec{x})| \le \sqrt{2}\ \ \rho_o n^2 \omega^2 |\Psi_n(\vec{x})|_{max} \quad (15)$$

Similarly for all blades with a given section lift coefficient $c_\ell(\eta)$, the rms amplitude of the nth harmonic of torque and thrust (lift) noise have the bounds

$$4\pi r_o |p_n'(\vec{x})| \le \sqrt{2}\ nk\ |Q_n(\vec{x})|_{max} \quad (16\text{-a})$$

$$4\pi r_o |p_n'(\vec{x})| \le \sqrt{2}\ nk |T_n(\vec{x})|_{max} \quad (16\text{-b})$$

respectively.

Equations (12) and (14) show that the thickness function and chordwise load distribution function which generate maximum noise have infinite slope at the same two points which are symmetrically located with respect to the mid-chord. The corresponding thickness and chordwise load distributions are rectangular. Note also that $|p_n'|$ is maximized by different thickness and load distributions for different n. In general, therefore the results of equations (15) and (16) are expected to be too pessimistic.

Figure 2 shows some spanwise aerodynamic data for a two-bladed helicopter rotor. These performance data were calculated by a strip theory-momentum analysis described in reference 4. The blade thickness ratio is 8 percent, the blade radius is 5 m and the chord is 0.4 m. The blade planform is rectangular. The rotor rpm is 626. Figure 3 shows the calculated thickness and loading noise with the theoretical bounds obtained above. The chordwise load distribution at each radial position was obtained from the Garabedian-Korn program (ref. 5). The airfoil section used is NACA 0008. The observer position is 50 m from the rotor center and 30° below the rotor (ε=120°). The bound for loading noise is obtained by adding the right sides of eqs. (16-a) and (16-b). It is seen that both bounds are very coarse although the bound on loading noise is not as pessimistic as that of thickness noise. For the first harmonic level of loading noise, the reason for the theoretical bound being lower than the calculated level is not known. It may be due to the fact that the drag force (skin friction and wave drag) obtained from the Garabedian-Korn program has a component normal to the chord which was used in the acoustic calcualtions.

For B blades, substitute nB in all the equations derived above.

THE EFFECT OF BLADE SWEEP

We have shown that $|\Psi_n(\vec{x})|$ is maximum when $\beta_c(\eta,h)$=constant. One way of reducing the level of the thickness and loading noise is blade sweep. This can be seen from eq. (8). The question arises whether a blade sweep can be selected which generates the least noise. We will show that among the blades with gradually increasing sweep towards the tip, there is no optimal sweep.

To be specific, we take $\beta_c=-\mu\eta^2$ where $\mu>0$. The same argument holds as long as $\partial\beta_c/\partial\eta < 0$. From eq. (6), we have

$$\Psi_n(\vec{x}) = \int_{R_i}^{R} \eta\, J_n(nk\eta\sin\varepsilon)\, e^{in\mu\eta^2/2}\, d\eta \int_0^{h_m} \sin(\frac{nb}{2\eta})\, dh$$

$$= \int_{R_i}^{R} g(\eta)\, e^{in\mu\eta^2/2}\, d\eta \qquad (17)$$

where $g(\eta)>0$ is defined as

$$g(\eta) = \eta\, J_n(nk\eta\sin\varepsilon) \int_0^{h_m} \sin(\frac{nb}{2\eta})\, dh \qquad (18)$$

It is assumed, as before, that $nb/2\eta\leq\pi$. We note that as μ increases, so does the blade sweep. We have

$$\frac{d|\Psi_n(\vec{x})|^2}{d\mu} = \frac{d}{d\mu}\int_{R_i}^{R}\int_{R_i}^{R} g(\eta)g(\eta')\cos[\frac{n\mu}{2}(\eta^2-\eta'^2)]d\eta d\eta'$$

$$= -n\mu \int_{R_i}^{R}\int_{R_i}^{\eta'} (\eta^2-\eta'^2)g(\eta)g(\eta')\sin[\frac{n\mu}{2}(\eta^2-\eta'^2)]d\eta d\eta' \qquad (19)$$

If it is assumed that $\sin[\frac{n\mu}{2}(R^2-R_i^2)]>0$, that is $n\leq 2\pi/\mu(R^2-R_i^2)$, then

$$\frac{d|\Psi_n(\vec{x})|^2}{d\mu} < 0 \tag{20}$$

For all practical angles of sweep, the above restriction on n is less strict than previously obtained restriction $n\leq 2\pi\eta/b$. The above result indicates that the levels of harmonics of the thickness noise decreases as the blade sweep increases. This result is also valid for loading noise.

To test the validity of the above result, figure 4 shows the thickness noise spectra of three blades with increasing sweep. The tip Mach number is 0.95 and the thickness ratios of all the blades are 8 percent. The observer is in the rotor plane and 50 m from the rotor center. It is seen that the above conclusion is indeed correct and should hold up to the 22nd harmonic. In fact it holds for much higher harmonics. The airfoil section used in the calculations is NACA 0008.

CONCLUSIONS

In this paper, bounds are established on thickness and loading noise of rotating blades. Only steady loading noise is considered which restricts the results to high tip speeds. It is shown that only chordwise thickness and load distributions with midchord symmetry need be considered to establish these bounds. The resulting thickness and load distributions have infinite slopes at two points symmetrically located with respect to the midchord. Due to the fact that the amplitude of each harmonic of the spectrum is maximized, the resulting bounds are too coarse. A more appropriate approach may be to search for chordwise thickness and load distributions which maximize overall acoustic level.

It is also shown that sweeping the blade tips is beneficial in reducing the radiated noise. Also for blades with sweeps that increase towards the tip, there is no optimal sweep for minimum noise level.

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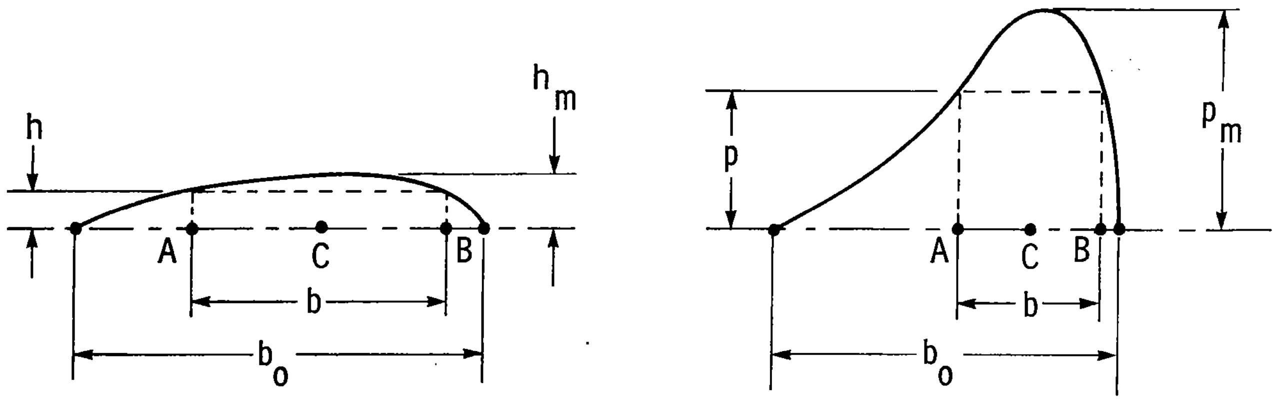


(a) Thickness. (b) Load.

Figure 1.- Chordwise distributions.

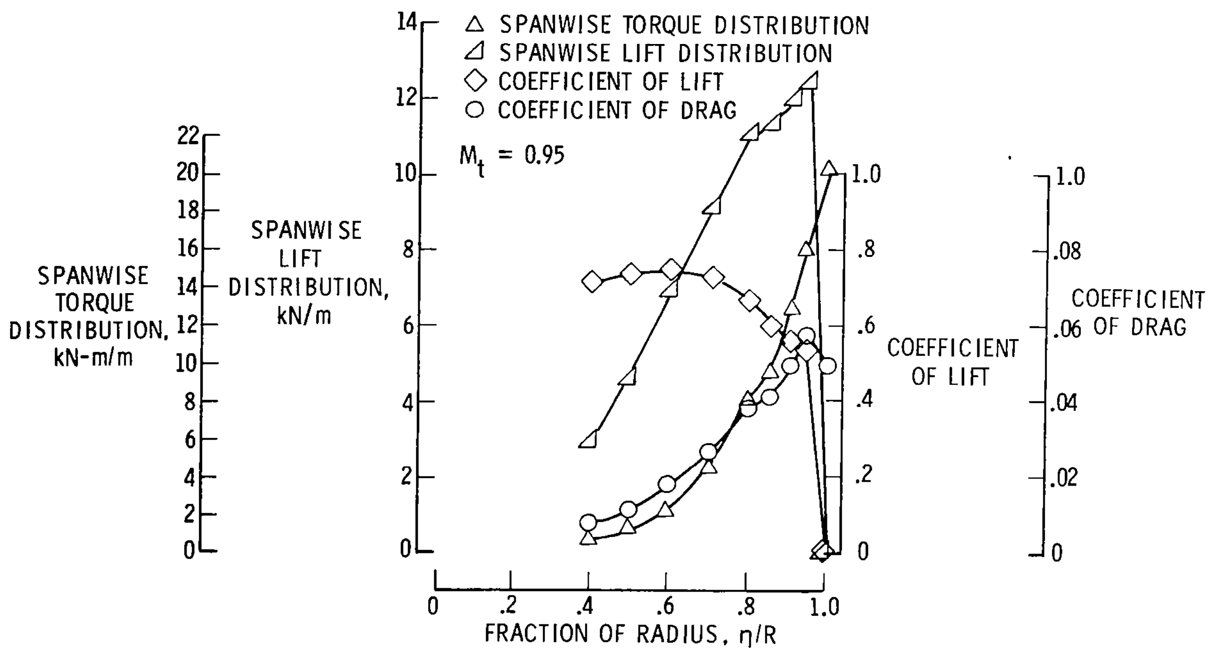
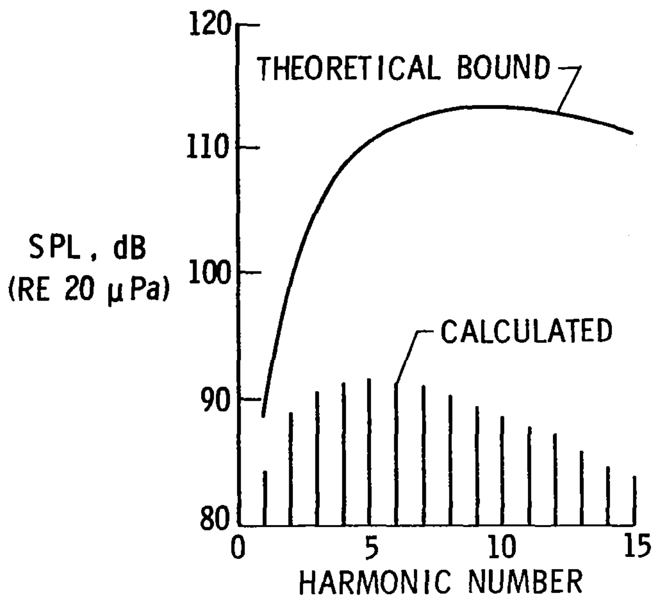
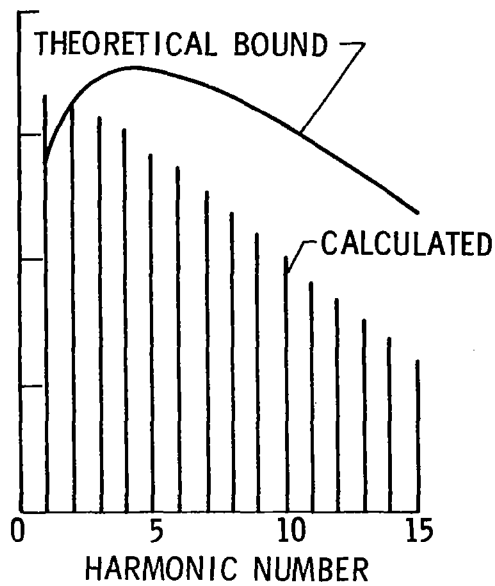


Figure 2.- Performance curves for two-bladed helicopter.



(a) Thickness noise.



(b) Loading noise.

Figure 3.- Comparison of theoretical bound with calculated thickness and loading noise spectra.

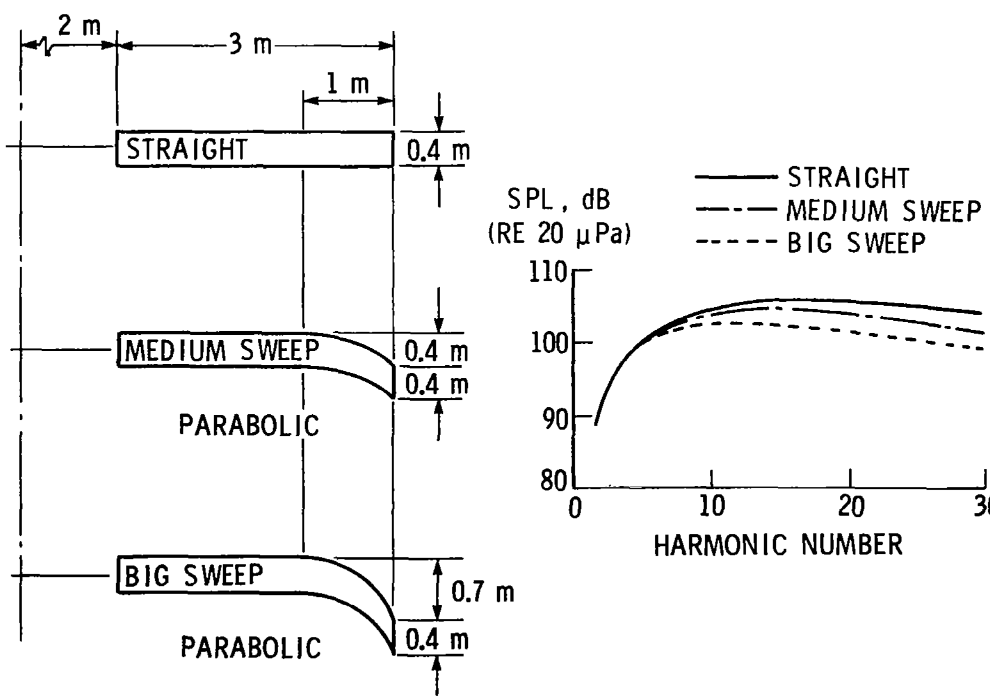


Figure 4.- Effect of blade sweep on thickness noise.