NASA TECHNICAL MEMORANDUM NASA TM 75544

COMPTON INTERACTION OF FREE ELECTRONS WITH INTENSE LOW FREQUENCY RADIATION

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Translation of "Komptonovskoye Vzaimodeystviye Svobodnykh Elektronov s Intensivnym Nizkochastotnym Izlucheniyem," Academy of Sciences USSR, Institute of Space Research, Moscow, Report Pr-283, 1976, pp. 1-30.

(NASA-TM-75544) COMPTON INTERACTION OF FREE ELECTRONS WITH INTENSE LOW FREQUENCY RADIATION (National Aeronautics and Space Administration) 32 p HC A03/MF A01 CSCL 20H
N78-32838
Unclas
G3/72 31634

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 October 1978

STANDARD TITLE PAGE

1. Report No. NASA TM 75544	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle COMPTON INTERACTION OF FREE ELECTRONS WITH INTENSE LOW FREQUENCY RADIATION		5. Report Date October 1978
		6. Performing Organization Code
7. Author(s) A.F. Illarionov and D.A. Kompaneyets Institute of Space Research, Academy of Sciences USSR, Moscow		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		11. Contract or Grant No. NASw-3199
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Komptonovskoye Vzaimodeystviye Svobodnykh Elektronov s Intensivnym Nizkochastotnym Izlucheniyem," Academy of Sciences USSR, Institute of Space Research, Moscow, Report Pr-283, 1976, pp. 1-30.

16. Abstract Electron behavior in an intense low frequency radiation field, with induced Compton scattering as the primary mechanism of interaction, is investigated. Evolution of the electron energy spectrum is studied, and the equilibrium spectrum of relativistic electrons in a radiation field with high brightness temperature is found. The induced radiation pressure and heating rate of an electron gas are calculated. The direction of the induced pressure depends on the radiation spectrum. The form of spectrum, under the induced force can accelerate electrons to superrelativistic energies is found.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified		

ANNOTATION

The behavior of an electron moving with arbitrary velocity in a given strong low frequency radiation field is considered, when induced Compton scattering is the primary mechanism of interaction of electrons with radiation. The evolution of the energy spectrum of the electrons is investigated in a diffuse approximation, and the equilibrium spectrum of relativistic electrons is found in a radiation field with a high brightness temperature. The induced radiation pressure, which acts on a moving electron and the induced heating rate of the electron gas in an isotropic radiation field are calculated. It is shown that the direction of the induced force, in contrast to the well known spontaneous braking force, depends on the radiation spectrum. The condition of the form of the spectrum, under which the induced force can accelerate electrons up to superrelativistic energies, is found.

COMPTON INTERACTION OF FREE ELECTRONS WITH INTENSE LOW FREQUENCY RADIATION

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The induced Compton scattering of electromagnetic radiation by free electrons (see survey [1]) can play an important part in astrophysics, in the interaction of powerful radio emission pulsars, quasars and other objects surrounded by a rarefied plasma [2-4], as well as under laboratory conditions, in examination of plasma heating by laser, maser and ultrahigh frequency unit emissions [5-7]. /3*

It is known that this nonlinear process [8] leads to electron heating [5,9], the development of induced radiation pressure [10,11], change in the radiation spectrum [12] and in particular, the development of narrow spectral details and solitons in the continuous radiation spectrum [13], to divergence or convergence of the radiation beam [14], etc.

The question of the behavior of relativistic electrons in a given radiation field is considered in the present work, i.e., we will be interested in:

1. The energy distribution of electrons, in situations when the plasma is sufficiently rarefied, and scattering processes, which result in electron diffusion in impulse space, play the basic part in establishment of the energy distribution of the electrons;

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*Numbers in the margins indicate pagination in the foreign text.

2. Induced radiation pressure which acts on free electrons;

3. Relativistic electron heating by induced light scattering.

Precise relativistic formulas were obtained for all quantities which characterize electron behavior in an isotropic radiation field, which are valid for arbitrary electron energies and arbitrary radiation spectra. The resulting formulas are fairly simple and convenient for the calculation of /4
induced effects, and they permit the study of both nonrelativistic and superrelativistic asymptotics.

A large number of studies has been devoted to the question of the induced interaction of radiation with a relativistic plasma [15-17]. The relationship of heating rate of monoenergetic superrelativistic electrons to their energies, obtained in these works, are valid only for radiation spectra of a given type. However, for precisely these spectra, the plasma heating pattern differs qualitatively from monoenergetic electron heating. This is connected with pulsed electron diffusion, as a consequence of induced scattering [10], which washes out the monoenergetic distribution in these spectra very much faster than it heats them [18]. At the same time, in a broad class of spectra, the electron heating pattern differs from both the results of works [16-17] and [18]. The induced effects are analyzed in our work, for arbitrary radiation spectra.

In the case of a high brightness temperature of the radiation field $kT_b \gg mc^2$, allowance for induced electron heating and cooling in spontaneous scattering results in establishment of the equilibrium energy distribution of relativistic electrons $dNe/d\varepsilon \sim \varepsilon^2 \exp(-A\varepsilon^n)$, $4 \leq n \leq 5$, as a function of the radiation spectrum. The average electron energy in this distribution is $<\varepsilon> \approx mc^2 \left(\frac{kT_b}{mc^2}\right)^{1/n}$.

Besides a systematic increase in electron energy, a systematic change in its impulse occurs in induced radiation scattering, i.e., induced radiation pressure acts on the electrons. In an isotropic radiation field, this pressure can be directed both opposite and in the same direction as the electron velocity, /5 depending on the type of radiation spectrum. In the electron rest system, this pressure was calculated in work [11], and it was shown in survey [1] that, in this system, the pressure always slows down the electrons. However, the presence of induced heating in the electron rest system results in the induced pressure in a laboratory system differing significantly from the pressure found in work [11], and precisely the heating results in a change in direction of the induced pressure, in certain situations.

The effects being considered, which develop upon induced scattering of light by electrons, are classical, i.e., they ulimately do not include the Planck constant h. However, the calculation of these effects in quantum language, in which electromagnetic radiation is considered as a photon gas, is significantly more convenient than the classical [19].

We will consider the electromagnetic field to be random, and we will characterize it by the quantity $N(\nu,\vec{n})$ the photon filling number with frequency ν and direction of propagation $\vec{n}$ of phase space. Then, in this state, the probability of photon scattering from any other state is proportional to $1+N(\nu,\vec{n})$. One corresponds to spontaneous scattering and

N, to induced. The change in frequency of the photon upon scattering by an electron moving with random velocity v, equals [20]

$$\boxed{\frac{\nu'}{\nu} = \frac{1-\frac{\vec{v}\vec{n}}{c}}{1-\frac{\vec{v}\vec{n}'}{c}+\frac{h\nu}{mc^2\gamma}\left(1-\vec{n}\vec{n}'\right)}}, \qquad (1)$$

where ν, ν' and $\vec{n}$, $\vec{n}'$ are the frequency and direction of propagation of the photon before and after scattering, $\gamma=(1-v^2/c^2)^{-1/2}$, c is the velocity of light, m is the electron mass. When the Thomson approximation is valid, the last term in the denominator of (1), connected with output, is a small correction to the Doppler effect compared with h. Thus, in the scattering of low frequency photons by electrons, the change in frequency is primary connected with the Doppler effect. However, the induced scattering effects owe their existence to this small correction connected with output. /16

We proceed to the electron rest system, and we show that, without accounting for output in induced scattering, there is no exchange of either energy or impulse between an electron and the radiation field. This is easily seen from the following example. We place an electron in crossed radiation beams. The induced scattering of photons from the first beam is possible only in the direction of the second beam, and the number of photons from the first beam scattered from the second per unit time is proportion to N_1N_2. Photons from the second beam with the same velocity (proportional to N_1N_2) are scattered in the first, in which, by disregarding output, the frequency in scattering remains as before. Thus, with induced scattering, the radiation field does not change and the electrons acquire neither impulse nor energy, i.e.,

both induced heating and induced radiation pressure are absent. In the reverse transition in a laboratory system (where the electrons move at velocity $\vec{v}$), the induced heating and induced pressure rates are, in accordance with the Lorentz tranform $Q=W+\vec{\theta}\vec{L}$, $\vec{f}=\vec{L}+\frac{\vec{\theta}}{c^2}W$, where Q and $\vec{f}$ are the rates of change of energy and impulse in the laboratory system, and W and $\vec{L}$ are the same in the rest system. It is evident that, if heating and pressure equalled zero in the rest system, they equal zero in any other inertial coordinate system. Thus, the pure Doppler effect makes no contribution to induced electron /8
heating, and only allowance for the output effects results in the effects indicated.

We note that the conditions of applicability of the Thomson approximation are different in the cases of induced and spontaneous scattering. This is connected with the fact that induced radiation scattering by a relativistic electron occurs mainly at small angles $1-\vec{n}\vec{n}'\sim 1/\gamma^2$. In this case, the Thomson approximation is valid at $h\nu \ll mc^2\gamma$.

1. Electron Diffusion in Impulse Space

We consider a system of electrons in a given radiation field with high brightness temperature, when induced scattering effects are important. Upon scattering radiation, an electron changes its impulse both systematically and randomly. While the impulse of an electron changes by a relatively small amount as a result of each scattering, to find the electron distribution function in impulse space ϕ, the Fokker-Planck equation can be used, according to which

$$\frac{\partial \varphi}{\partial t} + v_i \frac{\partial \varphi}{\partial x_i} = \frac{\partial}{\partial p_i}\left(\frac{\partial}{\partial p_k} D_{ik}\varphi - f_i \varphi \right), \qquad (2)$$

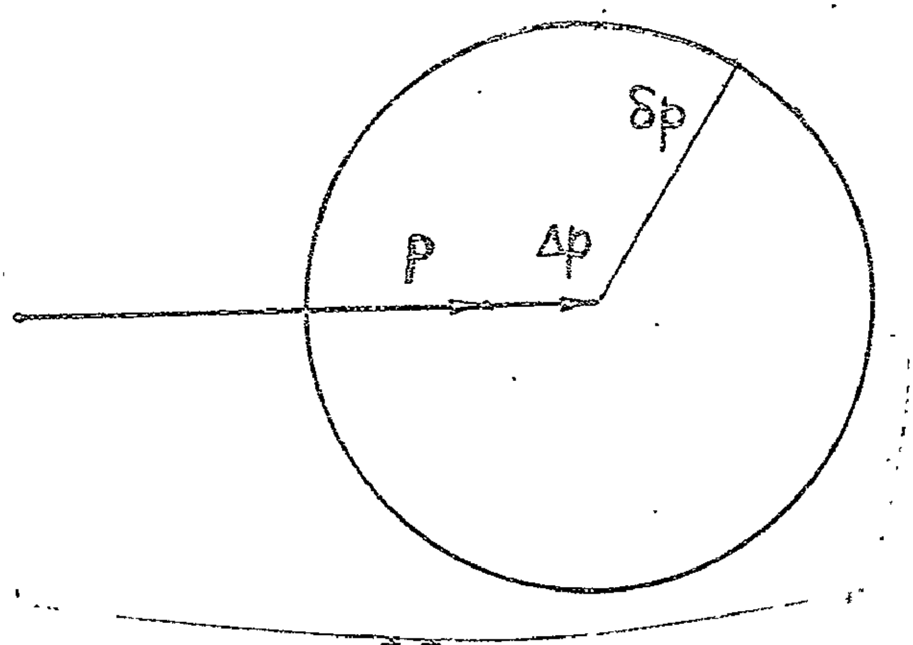

Fig. 1. Systematic $\vec{\Delta p}=\vec{f}\Delta t$ and random δp= change of initial impulse $\vec{p}$ of group of electrons in small time Δt, connected with pressure and diffusion, respectively; induced pressure results in relatively small value of Δp compared with random δp; the figure represents the situation in the case of accelerating induced pressure and nonrelativistic electrons; in this case, Δp is proportional to p, and δp does not depend on p.

where f_i, the average force on an electron from the direction of the radiation field, and D_{ik}, the electron impulse diffusion tensor in this radiation field, are determined by the formulas

$$\begin{pmatrix} f_i \\ \mathcal{D}_{ik} \end{pmatrix} = \int \begin{pmatrix} \Delta p_i \\ \frac{\Delta p_i \Delta p_k}{2} \end{pmatrix} N(\nu,\vec{n})\left[1+N(\nu',\vec{n}')\right] c\sigma \frac{2\nu^2}{c^3}\, d\nu\, d\vec{n}\, d\vec{n}', \tag{3}$$

here, /17

$$\Delta p_i = \frac{h}{c}\left(\nu n_i - \nu' n_i'\right), \tag{4}$$

is the transmission of an impulse to an electron as a result of a single scattering, and the Thomson scattering cross section

$$\sigma = \left(\frac{e^2}{mc^2}\right)^2 \frac{1}{2\gamma^2} \frac{1-\vec{\beta}\vec{n}}{(1-\vec{\beta}\vec{n}')^2} \left[1 + \left(1 - \frac{1-\vec{n}\vec{n}'}{\gamma^2(1-\vec{\beta}\vec{n})(1-\vec{\beta}\vec{n}')}\right)^2\right]. \qquad (5)$$

In formula (3), one in the square brackets corresponds to spontaneous scattering, and $N(\nu', \vec{n}')$ to induced. The quantity $\Delta p_i \Delta p_k \sim h^2$. Therefore, spontaneous diffusion is a quantum quantity and induced diffusion, classical [10]. This is evident, if the diffusion coefficient is written as the classical quantity, spectral intensity

$$F(\nu, \vec{n}) = \frac{2h\nu^3}{c^2} N(\nu, \vec{n}); \qquad [F] = \frac{\text{эрг}}{\text{см}^2\ \text{сек стер гц}} \qquad (6)$$

Written this way, constant h disappears from the expression for induced D_{ik}. The radiation pressure on an electron, both spontaneous and induced, is a classical quantity. Calculation of $\vec{f}_{ind}$ by formula (3) must be carried out, with allowance for the first correction of $h\nu/mc^2$ in the expression for frequency ν' in Δp_i and $N(\nu', \vec{n}')$. In this case, allowance for the quantum correction to the scattering cross section gives a zero contribution. In calculation of the diffusion coefficient, quantum corrections need not be taken into account at all, in substituting $\nu' = \nu \frac{1-\vec{\beta}\vec{n}}{1-\vec{\beta}\vec{n}'}$, in formula (3). With this substitution taken into account and with the use of formulas (4) and (5), it can be shown, by direct differentiation of formula (3), that the induced pressure $f_{i\ ind} = \frac{dD_{ik}}{dp_k}$, and equation (2) coincides with the equation, which is known in quasilinear plasma theory [21]. In the case of uniform electron and photon distributions, equation (2) has the form /10

$$\frac{\partial \varphi}{\partial t} = \frac{\partial}{\partial p_i}\left(D_{ik}\frac{\partial \varphi}{\partial p_k} - f_{i\,sp}\varphi\right). \tag{7}$$

In the case of an isotropic radiation field, coefficient D_{ik} can be presented in the form

$$D_{ik} = D_\ell(p)\frac{p_i p_k}{p^2} + D_t(p)\left(\delta_{ik} - \frac{p_i p_k}{p^2}\right), \tag{8}$$

where D_ℓ and D_t are the longitudinal and transverse diffusion coefficients, respectively.

In this case, equation (7) is written in the following manner

$$\frac{\partial \varphi}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}p^2\left(D_\ell \frac{\partial \varphi}{\partial p} - f_{sp}\varphi\right) + \frac{D_t}{p^2}\Delta_{\varphi,\theta}\varphi, \tag{9}$$

where $\Delta_{\phi,\theta}$ is the angular part of the Laplacian in spherical coordinates. We multiply both parts of equation (7) by $\varepsilon=\gamma mc^2$, and we integrate it over phase space. After substituting $\phi=\frac{1}{4\pi p^2}\delta(q-p)$, we obtain the heating rate of monoenergetic electrons. In the calculation on one electron

$$\frac{d\varepsilon}{dt} = Q + \vec{f}_{sp}\vec{v} = \frac{1}{p^2}\frac{d}{dp}\left(p^2 v D_\ell\right) + \vec{f}_{sp}\cdot\vec{v}. \tag{10}$$

In a similar manner, by multiplying (7) by $\vec{p}$ and substituting

$\phi=\delta(\vec{q}-\vec{p})$, we obtain an expression for the radiation pressure on a moving electron

$$\vec{f}=\vec{f}_{sp}+\vec{f}_{ind}=\vec{f}_{sp}+\frac{\vec{p}}{p}\left(\mathcal{D}_e'+\frac{2}{p}(\mathcal{D}_e-\mathcal{D}_t)\right). \quad (11)$$

The known expression for the spontaneous force [22] can be obtained from formula (3) /11

$$\vec{f}_{sp}=-\frac{4}{3}\sigma_T\frac{\vec{v}}{c}\mathcal{E}_r\gamma^2,$$

where $\sigma_T=\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$ is the Tompson cross section, and $E_r=\frac{4\pi}{c}\int F d\nu$ is the radiation energy density.

2. Induced Pressure Acting on Electron in Radiation Field

We find the expression for the induced pressure acting on an electron, moving at velocity $\vec{v}=c\vec{\beta}$ in a given radiation field, for simplicity of calculation, through quantities written in the electron rest system. In accordance with the Lorentz transform

$$\vec{f}_{ind}=\int\left(\Delta\vec{p}+\frac{\vec{v}}{c^2}\Delta\varepsilon\right)N(\nu,\vec{n})\,N(\nu',\vec{n}')\,c\sigma\frac{2\nu^2}{c^3}\,d\nu\,d\vec{n}\,d\vec{n}', \quad (12)$$

where $\Delta\vec{p}=\frac{h}{c}(\nu\vec{n}-\nu'\vec{n}')$ and $\Delta\varepsilon=h(\nu-\nu')$ are the change in impulse and energy of the electrons in the rest system in single scattering, the Thomson scattering cross section $\sigma=r_e\frac{1+\cos^2\alpha}{2}$, $r_e=\frac{e^2}{mc^2}$ is the electron radius, $\cos\alpha=\vec{n}\vec{n}'$, α is the scattering angle, frequency of photon with $h\nu\ll mc^2$ after scattering by a rest electron $\nu'=\nu-\frac{h\nu^2}{mc^2}(1-\cos\alpha)$. By expanding formula (12) /12
$N(\nu',\vec{n}')$ in a $\frac{\nu'-\nu}{\nu}$ power series and by substituting the highest (second of h) order terms, known beforehand not to disappear

after integration by angle, we obtain an expression for the induced pressure

$$\vec{f}_{ind} = \frac{r_e^2 h^2}{mc^5} \int (1-\cos\alpha)(1+\cos^2\alpha)\, N(\nu,\vec{n}) \cdot \left[N(\nu,\vec{n}')(\vec{n}+\vec{\beta}) + (\vec{n}'-\vec{n})\,\nu \frac{dN(\nu,\vec{n}')}{d\nu} \right] \nu^4 d\nu\, d\vec{n}\, d\vec{n}'. \tag{13}$$

Let an electron move in an isotropic radiation field with a given spectrum N(ν). Since filling number N is an invariant quantity, the radiation spectrum in the electron rest system (already anisotropic, of course) has the form

$$N(\nu,\vec{n}) = N(\nu\gamma(1+\beta\cos\theta)). \tag{14}$$

Here and below, $\beta\cos\theta = \frac{\vec{v}\vec{n}}{c}$, and the plus sign in the parenthesis corresponds to the fact that $\vec{n}$ is the direction of the photon wave vector, which differs by angle π from the direct from which we receive the photon.

Initially, we find the induced pressure in the non-relativistic limit. Expression (14), in the first approximation of β<<1 has the form

$$N(\nu,\vec{n}) = N(\nu) + \vec{\beta}\vec{n}\ \nu \frac{dN(\nu)}{d\nu}.$$

By substituting this formula in (13) and by calculating the integrals, with the expression for the scattering angle $\cos\alpha = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi')$ taken into account, by recording the answer through the spectral intensity (6), we obtain

$$\vec{f}_{ind} = \frac{2\pi}{15} \frac{\sigma_T}{mc} \frac{\vec{v}}{c} \left[11 \int \frac{F^2}{\nu^2} d\nu - 14 \int \left(\frac{dF}{d\nu}\right)^2 d\nu \right]. \tag{15}$$

Here and everywhere further on, in calculation of effects connected with induced scattering, we assume that $F(\nu)$ decreases more rapidly than $\nu^{1/2}$, as $\nu \to 0$. /13

It was pointed out in survey [1] that the pressure in the electron rest system always is directed against the velocity. In distinction from this pressure (15), i.e., the pressure calculated in the laboratory system can, depending on the radiation spectrum, be directed both against and with the electron rate of movement. For example, for spectra of the form $F \sim \nu^{\alpha} e^{-\nu a}$, with $1/2 < \alpha < 11/7$, the pressure accelerates the electron and, with $\alpha > 11/7$, slows it down. The critical value of index α depends on the rate of decay of the radiation spectrum at high frequencies.

As an example, we compare the spontaneous and induced pressures acting on a nonrelativistic electron, moving in an isotropic quasi-Planck $N(\nu) = A[\exp(\frac{h\nu}{kT}) - 1]^{-1}$ radiation field. The induced pressure in this spectrum is braking, and introduction of factor $A \gg 1$ permits description of a situation with a high radiation brightness temperature $T_b = \frac{h\nu}{k} N(\nu) = AT$, in the low frequency $h\nu \ll kT$ region of the spectrum, where $N \gg 1$. For the ratio of the pressures, we obtain

$$\boxed{\frac{f_{ind}}{f_{sp}} \simeq 0{,}03 \frac{kT_e}{mc^2}}$$

Normally, the induced pressure constitutes a small correction to the spontaneous (on the surface of the sun, it equals only $2 \cdot 10^{-7}$). However, in an astrophysical situation, for example, near that of pulsars, the radiation is essentially nonplanckian and the brightness temperature of the radio emission is tremendous, /14 $T_b \approx 10^{30} K \approx 10^{20} \frac{mc^2}{k}$. Under these conditions, the induced pressure plays a major role in the interaction of the radiation with the surrounding plasma. It must be taken into account, in the interaction of powerful radiation beams with a rarefied laboratory plasma.

For the case of relativistic electrons moving in an isotropic radiation field, the induced pressure is determined by expression (13), with formula (14) taken into account. The result of exact calculations of the multiple integrals included in expression (13) for the pressure, in the case of arbitrary radiation spectra, is presented in the appendix. The induced pressure, just as in the nonrelativistic case, can, depending on the form of the radiation spectrum, both slow down an electron and accelerate it. The sign of the effect depends only on the behavior of the spectrum at low frequencies. Thus, for spectra

$$F(\nu) \sim \nu^{\alpha}, \qquad \nu \to 0 \tag{16}$$

asymptotic at $\gamma \gg 1$, the induced pressure at $\alpha > 1$ slows down an electron, and it has the form

$$\vec{f}_{ind} = -\frac{12\pi\sigma_T h^2}{mc^5}\,\frac{\vec{\beta}}{\gamma^3}\int_0^1 \Phi(y)\, y^2\left(1-\frac{y}{3}\right) dy, \tag{17}$$

and it behaves as γ^{-3} [18]. Here,

$$\Phi(y) = \int_0^{\infty} N(\nu)\, N(\nu y)\, \nu^4\, d\nu \tag{18}$$

is a universal function, which, as we shall see subsequently, determines the dependence of all the induced effects on the radiation spectrum.

In the case of spectrum (16), with $1/2<\alpha<1$, the pressure /15
accelerates an electron, and its asymptote coincides with the asymptotic heating rate (see the following section) in such spectra, and it equals

$$\vec{f}_{ind} = \frac{\vec{v}}{c^2} Q \sim \gamma^{-1-2\alpha}.$$

3. Induced Electron Heating in Isotropic Radiation Field

An electron in a radiation field changes its energy due to both induced and spontaneous scattering. Since the heating of electrons due to spontaneous scattering is a quantum effect proportional to h, it will not be considered. In the case of a nonrelativistic electron and isotropic radiation, the induced heating rate follows [5,9] from the equation of A.S. Kompaneyets [8]

$$Q = \frac{8\pi\sigma_T h^2}{mc^4}\int N^2\nu^4 d\nu = \frac{2\pi\sigma_T}{m}\int \frac{F^2}{\nu^2} d\nu. \qquad (19)$$

Due to spontaneous heating, an electron can acquire an energy on the order of the average energy of a photon in the radiation. Radiation with a high brightness temperature T_b[1] at low frequencies is capable of more. It heats electrons to energies much greater than the average photon energy, due to induced scattering. Thus, radiation with a Rayleigh-Jeans spectrum $F=\frac{2\nu^2}{c^2}kT_b$, at
$0<\nu<\nu_0$ and $F=0$ and $\nu>\nu_0$, $h\nu_0<<kT_b$, is capable of heating electrons /16
to energies $<\varepsilon>=\frac{3}{2}kT_{eq}=\frac{3}{8}kT_\ell$ (for the relativistic case, see formula (24)). In the case $kT_b>>mc^2$, which is not a rarity under astrophysical conditions, electrons are capable of being

[1]Consideration of induced scattering is equivalent to a change to high brightness temperatures.

heated to relativistic energies. In this section, we find expressions which determine relativistic electron heating.

The expression for induced heating of an electron moving at velocity $\vec{v}=\vec{\beta}c$ is similar to that of the induced pressure and, with the same designations, it has the form

$$Q=\int(\Delta\varepsilon+\vec{v}\,\Delta\vec{p})\,N(\nu,\vec{n})\,N(\nu',\vec{n}')\,c\sigma\,\frac{2\nu^2}{c^3}\,d\nu\,d\vec{n}\,d\vec{n}'. \qquad (20)$$

In the nonrelativistic case $\beta \ll 1$, this formula gives result (19). After integration (see Appendix), formula (20) leads to the expression

$$Q=\frac{12\pi\sigma_T h^2}{mc^4}\int_0^{\beta}\phi(y')\,G_Q(\beta,\beta')\,d\beta', \qquad (21)$$

where
$$G_Q(\beta,\beta')=\frac{\beta'^2}{\gamma^5(1+\beta')^5\beta^8}\Big[(30-24\beta^2+2\beta^4)\ln\frac{y'}{y}-60\beta+28\beta^3+$$
$$+\beta'(5-\beta^2)(3+\beta'^2)\left(\frac{\gamma'}{\gamma}\right)^2+5\beta'(3-\beta^2)^2\Big],$$

$$y=\frac{1-\beta}{1+\beta},\qquad y'=\frac{1-\beta'}{1+\beta'},$$

and function ϕ is determined by formula (18). In the limit $\beta,\beta' \ll 1$, the function

$$G_Q(\beta,\beta')=\frac{4\beta'^3}{\beta^8}\left[\beta^4-2\beta^2\beta'^2+2\beta'^4\right]. \qquad (21')$$

We note that the expression for Q does not change upon substitution of β for $-\beta$. The behavior of Q, as a function of quantity γ, is determined by the form of $\phi(y)$ at small y, i.e., by the radiation spectrum at low frequencies. In the case of spectrum (16), at $\alpha>2$, $Q\,\frac{\ln\gamma}{\gamma^5}$. In the case $1/2<\alpha<2$, we have $Q\sim\gamma^{-1-2\alpha}$. At $\alpha=2$, $Q\sim\ln^2\gamma/\gamma^5$. /17

4. Electron Impulse Diffusion Coefficients. Equilibrium State.

In an isotropic radiation field, the electron impulse distribution functions obey equation (7), where $D_\ell=D_{ik}\frac{p_ip_k}{p}$, and $D_t=\frac{1}{2}(D_{ii}-D_\ell)$. Quantity D_ℓ is calculated from general formulas (3), with (4) taken into account, similar to the case of quantities f_{ind} and Q (see Appendix), and it equals

$$\mathcal{D}_\ell = \frac{12\pi\sigma_T h^2}{c^4}\int_0^\beta \Phi(y')\,G_{\mathcal{D}_\ell}(\beta,\beta')\,d\beta', \qquad (22)$$

where $$G_{\mathcal{D}_\ell}(\beta,\beta')=\frac{2\beta'^2}{\gamma^4\beta^8(1+\beta')^5}\Big[(3-\beta^2)\ln\frac{y}{y'}+2\beta-2\beta'\Big(\frac{\gamma'}{\gamma}\Big)^2+2(\gamma^2+\gamma^{-2})(\beta-\beta')\Big].$$

In the limit $\beta,\beta'\ll1$, in accordance with the result of [6],

$$G_{\mathcal{D}_\ell}(\beta,\beta')=\frac{4\beta'^2}{15\beta^8}\left(11\beta^5-15\beta'\beta^4+10\beta'^3\beta^2-6\beta'^5\right). \qquad (22')$$

To find the form of D_t, we use the connection between D_ℓ and D_t, which follows from equations (10) and (11)

$$Q-\vec{v}\vec{f}_{ind}\equiv\frac{W}{v^2}=\frac{1}{m}\left(\mathcal{D}_\ell\gamma^{-3}+2\mathcal{D}_t\gamma^{-1}\right), \qquad (23)$$

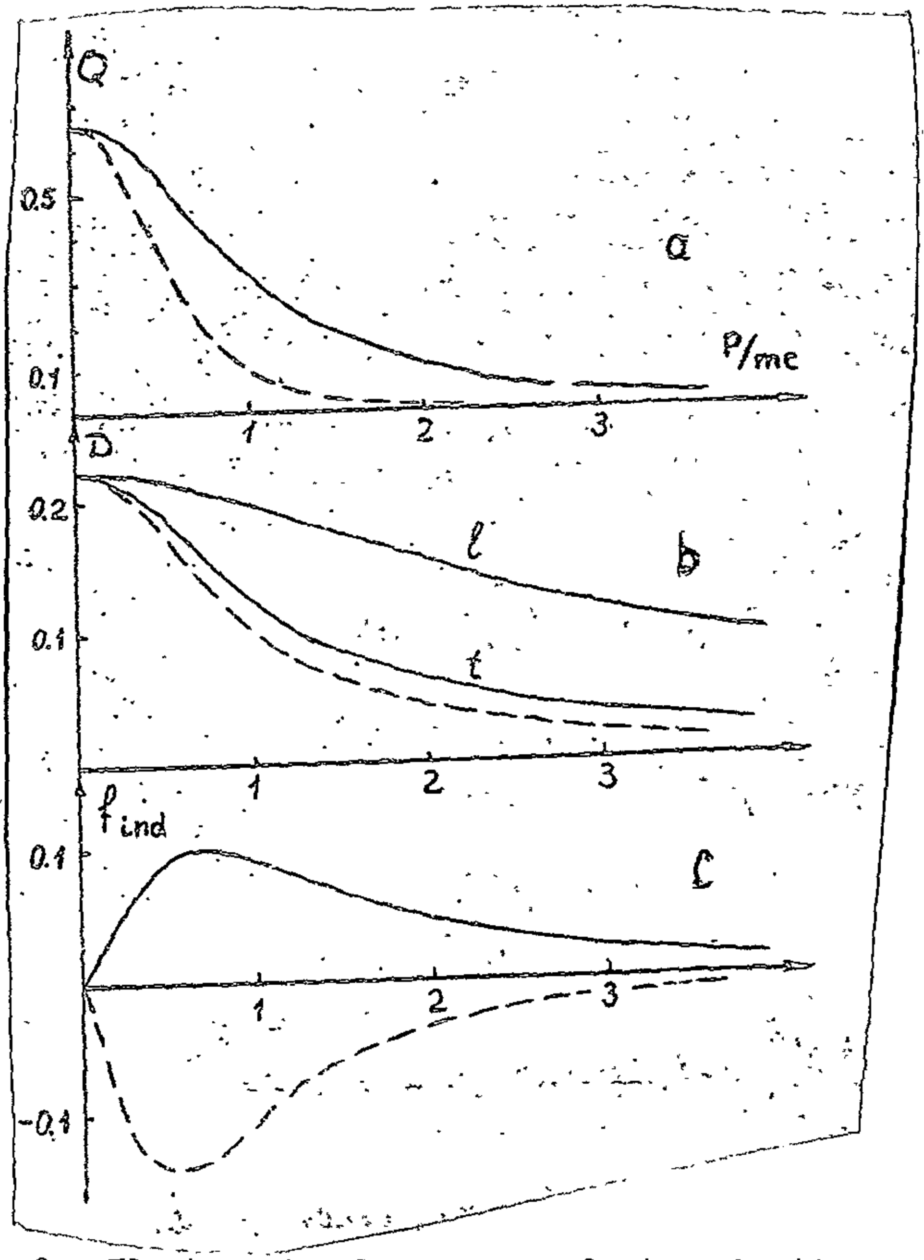

Fig. 2. Electron impulse vs. a. electron heating rate Q, b. diffusion coefficients D_ℓ and D_t and c. induced pressure f_{ind} for radiation spectra of the type $F\sim\nu^\alpha e^{-a\nu}$; the dashed line corresponds to a Wien spectrum with $\alpha=3$ and the solid line, to a spectrum with $\alpha=3/4$; in the case $\alpha=3$, D_ℓ practically coincides with D_t; in case c, a positive f_{ind} corresponds to an accelerating pressure; the values are reduced to units of $Q_0=\frac{12\pi\sigma_t h^2}{mc^4}\int N^2\nu^4 d\nu$, $D_0=mQ_0$, $f_0=Q_0c^{-1}$.

where W has the meaning of the rate of increase of energy by an electron in its rest system.

In the nonrelativistic limit, $D_\ell = D_t = \frac{m}{3}Q$.

In the relativistic case, with $\gamma \gg 1$, the behavior of quantity D_ℓ (as in the cases of Q and f_{ind}) depends on the /19
behavior of the spectrum at low frequencies. In the case of a spectrum of the type of (16), $D_\ell \sim \gamma^{-2}$ at $\alpha > 1$, and $D_\ell \sim \gamma^{-2\alpha}$ at $1/2 < \alpha < 1$. As it is easy to show, the asymptotic value of W is proportional to γ^{-1}, and the formula for $W/\gamma^2 c$ coincides with the formula for pressure (17), and it is valid for any radiation spectra. It follows from formula (23) and the asymptotes D_ℓ and W that, regardless of the radiation spectrum, $D_t = mW\gamma^{-1} \sim \gamma^{-2}$.

We find the equilibrium electron impulse distribution function. It follows from equation (9) that $\phi_{eq}(p) \sim \exp\left[-\int_0^p \frac{f_{sp}}{D_\ell}\, dp'\right]$. In the nonrelativistic case, the problem of the electron energy distribution in their Compton interactions with radiation with a broad spectrum was solved by Zel'dovich and Levich [9]. The steady state function proves to be Maxwellian, with temperature

$$T_{eq} = \frac{h}{4k} \frac{\int N^2 \nu^4 d\nu}{\int N \nu^3 d\nu} \qquad (24)$$

We consider the case of relativistic electrons, which corresponds to the condition $kT_{eq} \gg mc^2$. Depending on the form of radiation spectrum (16) at low frequencies, the electron equilibrium distribution function has the form

$$\varphi_{eq}(p) \sim \exp(-ap^n), \qquad (25)$$

where n=5 at $\alpha>1$ and $n=3+2\alpha$ at $1/2<\alpha<1$, and parameter $a \approx \frac{mc^2}{kT_{eq}}(mc)^{-n}$. In this case, the average energy $\langle\varepsilon\rangle = c\langle p\rangle \simeq ca^{-1/n}$, and ϕ_{eq} (25) corresponds to a flat electron spectrum, cut off at high energies by the spontaneous braking pressure.

5. Evolution of Electron Spectra of a Result of Induced Scattering and Electron Gas Heating.

Analytical solution of equation (9) in the isotropic case, /20
with diffusion coefficient D_ℓ (see (22)), which is dependent on impulse in a complicated manner, does not appear possible. However, asymptotic solution of the diffusion equation at large $p>mc$, where $D_\ell(p)$ is a certain power function of impulse $D=D_0 p^{-k}$, is easy to find

$$\varphi(p,t) = \frac{\varkappa}{4\pi\left(\varkappa^2 D_0 t\right)^{3/\varkappa}\Gamma\left(\frac{3}{\varkappa}\right)}\exp\left(-\frac{p^{\varkappa}}{\varkappa^2 D_0 t}\right), \quad (26)$$

where $\varkappa=k+2$. This solution is valid in the region of impulses, where the sponanteous braking pressure can be disregarded, i.e., at $p \ll \langle p\rangle$ and in times $t < \frac{\langle p\rangle^{\varkappa}}{D_0}$, when equilibrium distribution (25) still has not been reached.

Based on the resulting solution, the electron gas heating rate with induced radiation scattering can be found

$$\frac{d\bar{\varepsilon}}{dt} = \int Q(p)\,\varphi(p,t)\,4\pi p^2\,dp. \quad (27)$$

By substituting Q from asymptote (21), we find that, depending on the radiation spectrum at low frequencies (16),

$$\frac{d\bar{\varepsilon}}{dt} \sim \begin{cases} t^{3/4}, & \alpha>1 \\ t^{\frac{2\alpha+1}{2\alpha+2}}, & \frac{1}{2}<\alpha<1. \end{cases}$$

The dependence of the relativistic electron heating rate on the average electron energies with a spectrum in form (26) corresponds to this:

$$\frac{d\bar{\varepsilon}}{dt} \sim \begin{cases} \bar{\varepsilon}^{-3}, & \alpha > 1 \\ \bar{\varepsilon}^{-1-2\alpha}, & \frac{1}{2} < \alpha < 1 \end{cases} \qquad (28)$$

In the case of equilibrium distribution of electrons (25), their induced heating is compensated by spontaneous cooling. However, cooling of the electron gas is determined by the high energy end of the electron spectrum, and cooling (at $\alpha \gtrsim 1$), by the presence of semirelativistic particles with $p \simeq mc$, since integral (27) is accumulated precisely in region $p \simeq mc$. In the case $1/2<\alpha<1$, heating is determined by energetic electrons, which results in the same dependence of heating rate on the average energy of electrons with distribution (26), as for monoenergetic electrons. /21

Together with heating of the electron gas, we consider the problem of the time evolution of the average electron impulse, as a result of induced scattering. Interest in this problem is due to the fact that the induced radiation pressure, in principle, can accelerate the motion of an electron up to superrelativistic energies. It follows from equation (9) that some induced processes (without allowance for the spontaneous braking pressure) cannot in themselves lead to an equilibrium distribution of electron energies. The effect of the spontaneous pressure is equivalent to the presence of an effective "reflecting" wall, at energies on the order of $\lesssim\varepsilon>$. Since the equilibrium impulse distribution is symmetrical, regardless of the initial conditions in the direction of the induced pressure, the final electron distribution has zero impulse. However, the direction of the induced pressure significantly affects the pattern of evolution of the total impulse distribution. Thus, in the case of a radiation spectrum which results in an induced pressure accelerating in the relativistic limit ($\alpha<1$), the inequality $D_\ell \gg D_t$ is valid, and equation (9) (the spontaneous braking pressure can be disregarded for the present) can be written in the form /22

$$\boxed{\frac{\partial\varphi(\vec{p})}{\partial t}=\frac{1}{p^2}\frac{\partial}{\partial p}p^2\mathcal{D}_\ell\frac{\partial\varphi(\vec{p})}{\partial p}}$$

This form of equation corresponds to strong diffusion in direction $\vec{p}$ and an increase in total electron impulse up to $\langle p\rangle$ in the given direction.

6. Spectrally Narrow Radiation Lines

The formulas presented above for the quantities which describe the induced effects (electron heating rate, diffusion coefficients, etc.) permit fairly simple calculation of them for any specific radiation spectrum. In particular, it is easy to carry out this calculation for existing characteristic features of the case of spectrally narrow radiation, consisting of one or more lines. This situation occurs in cosmic masers, or it can be brought into being in a laser experiment. The case of one narrow line was considered in [6]. The basic results of [6] (diffusion coefficients and electron heating) follow from formulas (21) and (22) presented above, with the substitution of nonrelativistic expressions for functions $a(\beta,\beta')$ (see (21') and (22')). In the case of one narrow line $\delta\nu \ll \nu_0$, overlap function $\phi(\frac{1-\beta'}{1+\beta'})$ (see formula (18)) decreases rapidly at $\beta' > \frac{\delta\nu}{\nu_0}$. For example, in the case of a line with a

Gaussian profile $N \sim \exp[-(\nu-\nu_0)^2/2(\delta\nu)^2]$, with dispersion $(\delta\nu)^2 \ll \nu_0^2$, function $\phi \sim \exp[-\nu_0^2\beta'^2/(\delta\nu)^2]$. Therefore, the values of D_ℓ and Q decrease rapidly with increase in electron velocity. Thus, for a Gaussian line which is greater at /23
$\beta \ll \frac{\delta\nu}{\nu_0}$, ~~the values of~~ $Q = \frac{\sqrt{\pi}\, F_0^2 \sigma_T}{m \nu_0^2 \delta\nu}$ and $D_\ell = D_t = \frac{mQ}{3}$ decrease at $\beta > \frac{\delta\nu}{\nu_0}$ with increase in β, as

$$Q \simeq \frac{3\sqrt{\pi}\,\sigma_T F_0^2}{m \nu_0^3}\left(\frac{\delta\nu}{\nu_0}\right)^3 \frac{1}{\beta^4} \qquad (29)$$

$$\mathcal{D}_\ell = \frac{11\pi\sigma_T F_0^2}{10\,\nu_0^3}\left(\frac{\delta\nu}{\nu_0}\right)^2 \frac{1}{\beta^3}$$

where $F_0 = \int F d\nu = 2hc^{-2}\nu_0^3 \int N d\nu$ is the total radiation output in the line. At the same time, a decrease in value of D_t with increase in β is slower:

$$\mathcal{D}_t = \frac{23\pi\sigma_T F_0^2}{70\,\nu_0^3}\,\frac{1}{\beta}\,. \qquad (29')$$

An interesting situation arises with several narrow lines in the radiation spectrum. We consider the case of two lines displaced with respect to each other by frequency $\Delta\nu \ll \delta\nu_1, \delta\nu_2$, in which we assume that $\Delta\nu \ll \nu_1 \simeq \nu_2$ when the nonrelativistic case is of interest.

At small $\beta < \frac{\Delta\nu}{2\nu_1}$, the quantities which describe the induced effects of both lines add up arithmetically. Quantity F_0^2 in formulas (29) and (29') is replaced by $F_1^2 + F_2^2$, where F_1 and F_2 are the total radiation output in the first and second lines. However, at $\beta \geq \frac{\Delta\nu}{\nu_1+\nu_2} = \frac{\Delta\nu}{2\nu_c}$, an induced photon transition from one line to another by scattering becomes possible. Function ϕ has a sharp maximum (with dispersion

$[(\delta\nu_1)^2+(\delta\nu_2)^2]/4\nu_c^2 \ll 1)$, at $\beta' = \frac{\Delta\nu}{2\nu_c}$. This results in a considerable increase in the induced effects at $\beta \gtrsim \frac{\Delta\nu}{2\nu_c}$:

$$\boxed{Q = \frac{3\pi\sigma_T}{2m}\frac{F_1 F_2}{\nu_c^3} G_Q\left(\beta, \frac{\Delta\nu}{2\nu_c}\right) \simeq \frac{3\pi\sigma_T F_1 F_2}{4m\,\nu_c^3}\left(\frac{\Delta\nu}{\nu_c}\right)^3 \frac{1}{\beta^4},} \tag{30}$$

$$\boxed{\mathcal{D}_\ell = \frac{3\pi F_1 F_2 \sigma_T}{2\nu_c^3} G_{\mathcal{D}_\ell}\left(\beta, \frac{\Delta\nu}{2\nu_c}\right) \approx \frac{11\pi\sigma_T F_1 F_2}{10\cdot\nu_c^3}\left(\frac{\Delta\nu}{\nu_c}\right)^2 \frac{1}{\beta^3}.}$$

At the same time, overlap of the lines at $\beta > \frac{\Delta\nu}{2\nu_c}$ shows up slightly in the quantity

$$\boxed{\mathcal{D}_t \simeq \frac{23\pi\sigma_T}{70}\frac{(F_1+F_2)^2}{\nu_c^3}\frac{1}{\beta}.} \tag{30'}$$

It is evident from this that, at $\beta > \frac{\Delta\nu}{2\nu_c}$, the difference in the line frequencies $\Delta\nu$ (but not $\delta\nu_1$ and $\delta\nu_2$) plays the part of effective width of the radiation spectrum. Thus, for example, the heating rate of Maxwellian electrons with temperature $kT_e \gtrsim mc^2(\frac{\Delta\nu}{\nu_c})^2$ considerably exceeds the heating rate of a colder plasma with $kT_e < mc^2(\frac{\Delta\nu}{\nu_c})^2$.

In conclusion, we thank Ya.B. Zel'dovich and R.A. Syunyayev for interest in the work and valuable comments.

APPENDIX

As an example, we present the calculation of the value /25 of W, for an electron moving in an isotropic radiation field at velocity $v=c\beta$. Quantity W, which has the meaning of the rate of accumulation of energy by an electron in its rest system, is determined by formula (23), and it equals

$$W = \int \Delta\varepsilon \, N(\nu, \vec{n}) \, N(\nu', \vec{n}') \, c\sigma \, \frac{2\nu^2}{c^3} \, d\nu \, d\vec{n} \, d\vec{n}' .$$

After substituting $\Delta\varepsilon = h(\nu - \nu')$ and integrating over azimuth angles ϕ and ϕ', we obtain

$$W = \frac{3\pi\sigma_T h^2}{4mc^4} \int P_W(\mu, \mu') \, N(\nu\gamma(1+\beta\mu)) \, N(\nu\gamma(1+\beta\mu')) \, \nu^4 \, d\nu \, d\mu \, d\mu' \tag{A.1}$$

where

$$\mu = \cos\theta, \qquad \mu' = \cos\theta';$$

$$P_W(\mu, \mu') = 3 - \mu^2 - \mu'^2 - 5\mu\mu' + 3\mu^2\mu'^2 + 3\mu\mu'^3 + 3\mu^3\mu' - 5\mu^3\mu'^3 .$$

We present the scheme of calculation of angular integrals in (A.1). We substitute (A.1) in the form

$$W = \int d\nu \, \nu^4 \sum_{i,k} a_{ik} I_i I_k , \tag{A.2}$$

where $0 \le i, k \le 4$, $\quad I_k = (\nu\gamma\beta)^{-1-k} \displaystyle\int_{\nu\gamma(1-\beta)}^{\nu\gamma(1+\beta)} N(x)\, x^k \, dx$,

and coefficients a_{ik} are functions of parameter β. By differentiating I_k, we obtain

$$\nu \frac{dI_k}{d\nu} = -(k+1) I_k + \left(\frac{1+\beta}{\beta}\right)^{k+1} N(\nu\gamma(1+\beta)) - \left(\frac{1-\beta}{\beta}\right)^{k+1} N(\nu\gamma(1-\beta)) . \tag{A.3}$$

By integrating expression $\int I_i \frac{dI_k}{d\nu} \nu^5 d\nu$ by parts and by using (A.3) for $\nu\frac{dI_k}{d\nu}$ and for $\nu\frac{dI_i}{d\nu}$, we obtain the formula

$$(k+i-3)\int I_i I_k \nu^4 d\nu = \tag{A.4}$$

$$= \left(\frac{1+\beta}{\beta}\right)^{i+1} \int I_k N(\nu\gamma(1+\beta))\nu^4 d\nu - \left(\frac{1-\beta}{\beta}\right)^{i+1} \int I_k N(\nu\gamma(1-\beta))\nu^4 d\nu +$$

plus symmetric expression relative to the indexes. /26

By substituting the variables, we obtain

$$\int I_k N(\nu\gamma(1\pm\beta))\nu^4 d\nu = \frac{(1\pm\beta)^{k-4}}{\gamma^5\beta^{k+1}} \int_{y_0}^{1} \Phi(y)\, y^{\frac{3}{2}\pm\left(k-\frac{3}{2}\right)} dy\,; \tag{A.5}$$

where $\phi(y)$ is the overlap function introduced by formula (18), $y_0=\frac{1-\beta}{1+\beta}$. At $i+k\neq3$, formula (A.4) is written in the form

$$\int I_i I_k \nu^4 d\nu = \frac{(1+\beta)^{i+k-3}}{(i+k-3)\gamma^5\beta^{i+k+2}} \int_{y_0}^{1} \Phi(y)(y^i+y^k)\left[1-\left(\frac{y_0}{y}\right)^{i+k-3}\right]dy. \tag{A.6}$$

At $i+k=3$, this formula, after the limiting transition $i+k\to3$, has the form

$$\int I_i I_k \nu^4 d\nu = \frac{1}{\gamma^5\beta^5} \int_{y_0}^{1} \Phi(y)(y^i+y^k)\ln\frac{y}{y_0}\, dy\,. \tag{A.7}$$

And, finally, by substituting the resulting formulas (A.5-A.7) in (A.2), after the substitutions $y'=\frac{1-\beta'}{1+\beta'}$, and $y=y_0=\frac{1-\beta}{1+\beta}$, we obtain

$$W = \frac{12\pi\sigma_T h^2}{mc^4} \int_0^{\beta} \phi(y')\, G_W(\beta,\beta')\, d\beta' , \tag{A.8}$$

where
$$G_W(\beta,\beta') = \frac{1}{\gamma(1+\beta')^5\beta^8} \left\{ \frac{25-9\beta^2+3\beta'^2(\beta^2-5)}{\gamma^4}\left[\ln\frac{y'}{y} - 2(\beta-\beta')\right] + \right. \tag{A.9}$$

$$\left. + 2\beta^3(5-8\beta^2+\beta^4)(\beta'^2-\beta^2) + \frac{2}{3}\left[\frac{25-9\beta^2}{\gamma^4} + 2\beta^4(3\beta^2-1)\right](\beta'^3-\beta^3) \right. . \tag{A.9}$$

In the nonrelativistic limit, with $\beta,\beta' \ll 1$ /27

$$G_W(\beta,\beta') = \left(\frac{92}{105}\beta^7 - \frac{4}{3}\beta'^3\beta^4 + \frac{8}{5}\beta'^5\beta^2 - \frac{8}{7}\beta'^7\right)\cdot\beta^{-8}$$

The superrelativistic asymptote gives

$$G_W(1,\beta') = \frac{4(1-\beta')^2(1+2\beta')}{3\gamma(1+\beta')^5} = \frac{y'^2}{\gamma}\left(1-\frac{y'}{3}\right)\left|\frac{dy'}{d\beta'}\right| . \tag{A.10}$$

$D_\ell = D_{ik}\frac{p_i p_k}{p^2}$, where D_{ik} is determined by formula (3), is calculated, (like find and Q), by changing to the electron rest system and by substituting Δp_i in accordance with (4). After integration over ϕ,ϕ'

$$D_\ell = \frac{3\pi\sigma_T h^2}{8c^4}\gamma \int P_D(\mu,\mu')\, N(\nu\gamma(1+\beta\mu))\, N(\nu\gamma(1+\beta\mu'))\, \nu^4 d\nu\, d\mu\, d\mu',$$

where

$$P_D(\mu,\mu') = (\mu-\mu')^2(3-\mu^2-\mu'^2+3\mu^2\mu'^2) .$$

By calculating this integral according to the scheme represented above, we obtain expression (23).

The induced heating is found by the formula (see (10))

$$Q = \frac{1}{p^2}\frac{d}{dp}\left(\mathcal{D}_e p^2 \vartheta\right).$$

The result is represented by formula (21).

In a similar manner, based on formula (23), we find the quantity

$$\mathcal{D}_t = \frac{12\pi\sigma_T h^2}{c^4}\int^{\beta} \Phi(y')\, G_{\mathcal{D}_t}(\beta,\beta')\, d\beta' \tag{A.11}$$

where

$$G_{\mathcal{D}_t}(\beta,\beta') = \frac{1}{2}\left(\frac{G_W}{\gamma} - \frac{G_{\mathcal{D}_e}}{\gamma^2}\right).$$

At $\beta, \beta' \ll 1$

$$G_{\mathcal{D}_t}(\beta,\beta') = \left(\frac{46}{105}\beta^7 - \frac{22}{15}\beta'^2\beta^5 + \frac{4}{3}\beta'^3\beta^4 - \frac{8}{15}\beta'^5\beta^2 + \frac{8}{35}\beta'^7\right)\beta^{-8}. \tag{A.12}$$

In the case $\gamma \gg 1$

$$\boxed{G_{\mathcal{D}_t}(1,\beta') = \frac{1}{2\gamma}\, G_W(1,\beta')}. \tag{A.13}$$

We find the induced pressure acting on an electron moving in an isotropic radiation field from the relationship (see (23))

$$\vec{f}_{ind} = \frac{\vec{\vartheta}}{\vartheta^2}\left(Q - \frac{W}{\gamma^2}\right).$$

From formulas (21) and (A.8), we obtain

$$\vec{f}_{ind} = \frac{12\pi\sigma_T h^2}{c^5}\,\frac{\vec{\beta}}{\beta}\int_0^{\beta} \Phi(y')\, G_f(\beta,\beta')\, d\beta', \tag{A.14}$$

where $\boxed{G_f = \frac{G_Q - G_W \gamma^{-2}}{\beta}}$

At $\beta, \beta' \ll 1$ the expansion of a_f is combersome. In the case of a wide radiation spectrum, we expand ϕ in a β' power series, to the second order in β'. We note that, at $\beta'=0$, the relationship $\frac{d\phi}{d\beta'}=5\phi$ is satisfied. After in integrating expression $\int\phi a_f d\beta'$ and the necessary algebraic transformations, we reach formula (15). For the case of a spectrally narrow radiation ($\frac{\delta\nu}{\nu}\ll 1$, see section 6), at $\beta>\frac{\delta\nu}{\nu}$, the first term of the expansion $a_f=-\frac{92}{105}\frac{1}{\beta^2}$, which corresponds to the relationship $\vec{f}_{ind}\simeq-\frac{\vec{\beta}}{\beta}\frac{2Dt}{p}$. In the superrelativistic asymptote $a_f=-a_W\gamma^{-2}$, for spectra which fall to zero more rapidly than the first power of the frequency and $a_f=a_Q$, with a slower decrease in spectral intensity. /29

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