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THE PRACTICAL IMPACT OF ELASTOHYDRODYNAMIC LUBRICATION

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Elastohydrodynamic lubrication has had its most significant impact on, among all the types of concentrated contact mechanisms, rolling element bearings. EHL technology, through its inclusion in computer codes, now provides us with more effective methods for optimizing bearing design and for predicting bearing life, power loss, temperature and dynamic behavior. Bearing life prediction has advanced to a much more sophisticated level as compared to the calculation of fatigue life based on Lundberg-Palmgren theory. Application of elastohydrodynamics to gearing has, more or less, been limited to the calculation of pitch point film thicknesses. Techniques for calculating film thicknesses over the entire range of tooth meshes for arbitrarily shaped gear teeth (noninvolute, spur, helical, etc.) need to be developed. Elastomer seals with both unidirectional and reciprocating motion offer a fruitful application for the elastohydrodynamics of low modulus materials.

1. INTRODUCTION

Our present knowledge and understanding of elastohydrodynamics appears in an extensive array of analytical and experimental research papers in the literature. These are summarized in several excellent review papers such as those by Dowson (1), McGrew, et al. (2), and Cheng (3). It is not the purpose of this paper to rereview the state of the art of elastohydrodynamics, but rather to examine where and how it has been applied, and, if possible, to assess its impact. Papers dealing with EHL research, such as those summarized in (1-3), are relatively easy to uncover in a straightforward literature search since their dominant theme is elastohydrodynamics. Applications papers, however, are in general not so easily searched out because their theme is usually a machine element or system.

The object of this paper is to briefly trace the application and impact of elastohydrodynamics on rolling element bearing, gear and seal technologies. As will be shown, elastohydrodynamics has had its most significant impact on rolling element bearings with lesser impacts, as of this writing, on gears and seals. Elastohydrodynamic research has, firstly, lead to a better understanding of how bearings, gears and seals as well as traction drive elements, metal forming processes and human joints function. Secondly, it has impacted the design of these mechanisms. Its influence on design will be traced. Finally, additional areas of research with applications potential will be briefly discussed.

2. APPLICATIONS OF ELASTOHYDRODYNAMIC RESEARCH

2.1 Rolling Element Bearings - Quasi Static Analyses

2.1.1 Ball Bearings

The evolution of analytic techniques for predicting ball bearing performance and life presents an interesting story because it has been significantly influenced by elastohydrodynamic research. The first widely used ball bearing analyses were conceived by Jones (4 and 5) before there was a general awareness of elastohydrodynamic lubrication. Jones made the assumption that coulomb friction existed at the ball race contacts and developed his equations for a quasi-static analysis on that assumption. This leads to the commonly known "race-control" theory which assumes that pure rolling (except for Heathcote interfacial slip) can only occur at one of the ball-race contacts. All of the spinning required for dynamic equilibrium of the balls would then take place at the other or "noncontrolling" race contact. Jones' analysis proved to be quite useful and accurate for predicting the effects of speed and load on fatigue life, but it was not useful in predicting the onset and magnitude of skidding (ball and cage slip) which generally occurs under high-speed light-load conditions. Harris (6) extended Jones' analysis, retaining the assumption of coulomb friction but allowing frictional resistance to gyroscopic moments at the noncontrolling as well as the controlling raceway contacts. Harris' analysis (6) is probably quite adequate for predicting bearing performance under conditions of dry film lubrication when there is a complete absence of any EHL film.

Harris (7) first incorporated EHL relationships into a ball bearing analysis. He used the Archard and Cowking (8) point contact film thickness equation, assumed the lubricant to be Newtonian and that its viscosity was a function of pressure and temperature. As shown in figure 1, the analysis agreed better with skidding data than did the race control theory.

A revised version of Harris' computer program called SHABERTH was developed incorporating actual traction data from a disk machine. This program was then revised by Coe, et al. (9) substituting subroutines for traction force based on Allen, et al. (10) and film thickness versus load based on Loewenthal, et al. (11). These two computer programs were then used in an attempt to correlate predicted and measured bearing operating temperatures and power loss. Typical data are shown in figure 2 (9). In general the revised SHABERTH program showed better agreement with 120-mm bore ball bearing experimental data. The principal difference between the two programs is the film thickness load relationship assumed. It should be noted that there is controversy regarding the film thickness load relationship at high values of contact stress. Gentle's, et al. data (12) for point contact is seen to be in conflict with the data used by Loewenthal, et al. (11) which was obtained using an X-ray technique and elliptical contacts.

In the early development of EHL analyses it was assumed that the inlet region of the contact was flooded with lubricant, and much of the early experiments were carried out with systems having a copious supply of lubricant. Orcutt, et al. (13) first pointed out the possibility of starved EHL contacts due to restricted lubricant supply. A number of investigations, both analytical and experimental, have since been conducted which confirm the possibility of starved EHL contacts occurring in machine elements, and which now allow calculation of the effects of the degree of starvation on film thickness. (See Hamrock, et al. (14) for example). Starvation is most likely to occur in sparsely lubricated bearings such as instrument bearings (13), and possibly in high speed bearings, as well, even under conditions of copious lubricant supply. What needs to be done to be able to apply starvation theory is to develop the means, based on system design and operating variables, to calculate the degree of starvation in operating bearings.

The key elements in more accurately predicting bearing performance are better predictions of film thickness and tractive force. Elastohydrodynamics has certainly stimulated research on lubricant rheology which is leading to the development of better models for calculating tractive forces.

2.1.2 Cylindrical Roller Bearings

The development of analytic tools for predicting performance and life of cylindrical roller bearings parallels that for ball bearings. As with ball bearings, the assumption of Coulomb friction at the roller-race contacts resulted in poor correlation with skidding data. Roller bearing skidding at high speeds and light loads is a more severe problem than it is in ball bearings.

Harris (15) first introduced elastohydrodynamics into a roller bearing analysis. He assumed that the lubricant viscosity was an exponential function of pressure and temperature and used Dowson and Higginson's method for determining forces (16). Poplawski (17) extended Harris' analysis to include fluid churning loss, cage pilot surface friction and cage unbalance effects. Finally, Rumbarger, et al. (18) calculated film thickness based on Dowson and Higginson (16) and Cheng's thermal effects (19), and introduced a traction force subroutine fitted to disk machine data. The correlation of these three analyses with experimental data, taken from (18), is shown in figure 3. The increasing degree of sophistication of the film thickness and tractive force calculation methods is evident in the improved correlation with experimental data.

An attempt was made by Ford, et al. (20) to explain instabilities in roller bearing cage slip as being caused by lubricant traction curves which exhibit a pronounced peak. Cage slip instabilities, in which the magnitude of cage slip cycles back and forth between two distinctly different levels, has been observed by a number of researchers. Traction curves exhibiting a peak would presumably allow the equilibrium of forces to be satisfied at two distinctly different cage speeds for a given shaft speed. An understanding of cage instabilities and other transient behavior modes in rolling element bearings is necessary to further progress in bearing technology.

A detailed study of the sources of power loss in cylindrical roller bearings was carried out by Astridge, et al. (21). Bearing power loss in aircraft turbine engines is critical because of the limited heat sink available to which the frictional heat generated can be rejected. Surprisingly, the most significant single source of heat generation was found to be the EHL films between the rollers and races. The Dowson and Higginson (16) expression for rolling friction was used. Two cases were examined - one a relatively low flow rate lubrication system, and one a high flow rate system. The heat generation in the EHL films was 62 and 60 percent, respectively, of the total bearing heat generation. Studies such as (21) provide valuable insight and guidance for designers faced with problems of optimizing design for minimum heat generation or other bearing parameters.

2.1.3 Tapered Roller Bearings

Wren, et al. (22) used Dowson and Higginson (16) and Archard and Cowking (8) to calculate, with the appropriately developed kinematics, the roller-cone, roller-cup, and roller-rib film thicknesses. Investigations of this type and those of Karna (23) can be used to optimize the contour of the roller end-rib geometries. This contact, illustrated in figure 4, is critical to the successful operation of tapered roller bearings at high speeds. High speed tapered roller bearings for application in advanced transmissions and engines are under development.

2.2 Rolling Element Bearings - Dynamic Analyses

2.2.1 Ball and Cylindrical Roller Bearings

The quasi-static analyses previously described are suitable only for describing steady-state bearing operation since they tacitly assume that an equilibrium of forces exists at all times. Evidences of transient behavior and instabilities have been observed, for example, in gyro bearings by Horsch (24). These have provided the stimulus for full scale dynamic analyses which will, hopefully, improve our understanding of cage instabilities and other transient phenomena. Because of the even greater complexity of dynamic analyses as compared to the quasi-static type, accurate relationships for film thickness and traction are even more critical.

Walters (25) made the first attempt at an analysis to explain cage dynamics in gyro spin axis ball bearings. Although he simplified the analysis by placing certain constraints on ball motion, he obtained some useful results. The EHL traction analysis developed by Walowit and Kannel was used by Walters. Gupta (26) solved the generalized differential equations of motion of the ball in an angular contact ball bearing. He used Dowson and Higginson (16) with Cheng's (19) thermal and side leakage effects for film thickness calculation, and three traction models. Two traction models were developed experimentally and the third model assumed was a constant traction coefficient. Solutions for ball motion and skid were obtained. Gupta continued his dynamic analyses for both cylindrical roller bearings (27 and 28) and ball bearings (29 and 30). Again, Dowson and Higginson with Cheng's thermal effects are used for film thickness calculation together with traction models developed for two lubricants by Smith, Walowit, and McGrew. Some of the results obtained from Gupta's analyses are still being interpreted, but there is no doubt that they represent an advance in our ability to understand and predict bearing dynamic behavior.

2.3 Rolling Element Bearings - Life Prediction

Considerable advancements have been made over the past decade in the analysis and understanding of rolling contact failure mechanisms and life prediction. Elastohydrodynamics has, without a doubt, provided the principal stimulus for these advances. It brought about an awareness of, and consequently related research on, the many failure modes that can occur in rolling element bearings. These include classical subsurface fatigue, surface initiated fatigue, and wear, depending on the elastohydrodynamic conditions present. Other bearing failure modes are not germane to this discussion.

Tallian (31) first investigated the concept of competing failure modes, the engineering parameters influential in causing contact failure by any failure mode, and related failure mechanisms to the engineering parameters. Of particular interest here is the appearance in (31), shown in figure 5, of what is now known as the lambda curve - the relationship of bearing life to the ratio of minimum film thickness to composite surface roughness (now commonly designated as lambda). Tallian recognized the dependence of the failure mode, and thus of bearing life, on the value of lambda. He reported observing surface distress at values of lambda below 1.6, but noted that the severity of the surface distress was not only dependent on the value of lambda but also on the boundary lubricating abilities of the lubricant and the nature of the rolling surfaces (i.e., solid lubricant coatings).

Skurka conducted fatigue life tests of cylindrical roller bearings operating under various lambda values (32), and developed an empirical equation to predict the effect of varying lubricant and surface finish conditions on fatigue life. Similarly, Danner (33) conducted fatigue tests with tapered roller bearings operating under various lambda values. In contrast with previous data on ball and cylindrical roller bearings, Danner's tapered roller bearing data showed a less severe decrease in bearing life at lambda values less than 1. The bearings used to accumulate the data reported by Danner were carburized and generally had considerably rougher surface finishes as contrasted with the through hardened, more finely finished bearings reported on by Tallian and Skurka. One observation that can be made after surveying the life-lambda curves is that there is considerable scatter in life among bearing lots tested at conditions resulting in low values of lambda. Accordingly, the authors of (34) recommend an average curve based on Tallian's and Skurka's data (fig. 6). The bearing life calculated from AFBMA is corrected by multiplying by the Lubrication-Life Correction Factor. Reference 34, as a design manual for the prediction of ball and roller bearing life taking into account elastohydrodynamic as well as other factors, is a testimony to the impact of EHL on the increasingly sophisticated bearing life prediction techniques which have evolved. Anreason, et al. (35) developed an analytical expression for the life adjustment factor which takes account of the lubrication influence on bearing life. Andreason's expression is more convenient to use than the graph presented in reference 34.

It is easy to visualize why there is more variation in life among bearing lots run at low lambdas than at high lambdas. At lambda values above 3 the EHL film is thick and continuous with no significant asperity contact. The stress distribution is close to ideal, failures are classical subsurface fatigue and, for a given lubricant, only the basic material properties constitute a determining factor. At low lambdas the situation is infinitely more complex. With frequent or even continuous asperity contact, microgeometry and starvation effects as well as lubricant-material chemistry become critically important. Because of its complexity the low lambda or "mixed EHL" regime is less well understood. It is also here where the greatest potential gains can be made through further research simply because the machine elements operating here are the victims of early mortality. It is not surprising, from these observations, that much of EHL research is now concerned with the low lambda region. Liu, et al. (36) determined that, at lambda values below 0.5, the variation in lambda explains only a minor portion of the scatter of fatigue lives. Bearing type, material type, surface roughness range, and pre-existing defects all apparently contribute to the life variation at low lambdas. Moyer, et al. (31) discusses the life results obtained with tapered roller bearings in terms of a time transit-hydraulic crack propagation hypothesis. A regression analysis conducted on the data obtained with 28 lots of bearings indicated that the time transit is a factor that cannot be neglected.

2.4 Applications of Elastohydrodynamics to Bearing Testing and Field Problems

There have, undoubtedly, been innumerable examples of the application of elastohydrodynamics to predict bearing EHL film conditions and bearing life before finalization of design. These examples generally do not appear in publications because they are merely part of routine design procedures in many engineering departments. Two examples of how elastohydrodynamics assisted in defining and solving bearing problems will be described.

Bamberger, et al. (38) conducted fatigue tests with 120-mm bore angular contact ball bearings at temperatures from 205° to 315° C (400° to 600° F) in an inert environment to prevent oxidation of the test lubricant. Because of the very low viscosity of the test lubricant at the highest test temperature it was necessary to closely examine the EHL film conditions. Preliminary tests were conducted in a rolling disk machine to determine the film thicknesses at the three test temperatures and expected bearing contact stresses. It was determined that the bearing raceways would have to be honed rather than ground and polished to obtain a surface finish fine enough to insure satisfactory values of lambda at the ball-race contacts. Table I shows the film thickness and lambda values obtained from the disk tests and the ratios of bearing life to AFBMA calculated life. The bearing L_{10} life at 315° C (600° F) was approximately 15 times the expected AFBMA life. Only very slight surface distress was noted in the raceway tracks of the bearings tested at 315° C (600° F). No surface distress was present in the bearings tested at 205° C (400° F) and 260° C (500° F). Avoidance of excessive surface distress and degraded life would not have been possible at the highest test temperature without honed raceways, a nonstandard procedure at the time.

Russell, et al. (39) recount a story of how a group of 58 aircraft turbine engine mainshaft ball bearings with seriously discrepant outer raceway surface finishes were inadvertently installed in production engines, and the steps that were necessary to take, first, to assess the problem, and then to correct it. The bearings in this lot had outer race track surface finishes ranging from 0.152 μm (6 μin.) to 0.559 μm (22 μin.) with the majority in the range of 0.355 μm (14 μin.). An analysis was made on the basis of EHL theory to determine the effect of the rough surface finishes on bearing life and rig tests were conducted to verify the analysis. The fatigue life determined during component tests was found to be significantly less than the life of similar bearings with proper finishes and considerably less than predicted by available analytical methods. The results of the analysis and component tests are shown in figure 7. The discrepancy between analytical life prediction and the assumed life based on the test points is believed due to the wide range of surface finishes. Based on these test results, a prediction was made of the number of failures expected in actual engines and engines having suspect bearings were successfully retrofitted to preclude the possibility of premature bearing failures.

2.5 Gears

A gear tooth meshing cycle is more difficult to analyze than is a ball or roller-race contact. In the case of bearings a steady state situation prevails; in the case of a gear tooth meshing cycle a highly unsteady state condition of changing kinematics (slide-roll ratio), contact load, contact geometry, and surface temperature prevails from the first point of tooth tip contact to the end of contact at the tooth dendendum. To further complicate the situation, the EHL conditions that exist throughout most of the tooth meshing cycle are anything but ideal. In all but the most lightly loaded gears, tooth contacts are, at best, in the mixed EHL if not outright boundary lubrication regime. This is the least well understood region. In the case of rolling bearings operating at low lambdas we have seen that factors which are not yet well understood cause variations of, and relative unpredictability of lives. Such will probably be the case with gears until we develop a better understanding of microgeometry, thermal, and lubricant-material chemistry effects.

Applications of elastohydrodynamics to gearing have, for the most part, been limited to predictions of contact conditions at the pitch point, or to a simplification of the transient conditions occurring throughout the meshing cycle to a quasi-steady state situation. Dowson, et al. (40) assumed a steady state situation and developed methods for calculating film thickness. The utility of this approach was shown by Dudley (41) who observed some correlation between gear tooth wear and film thickness calculated on the basis of the quasi-steady state assumption.

Gu (42) developed a criterion, based on a comparison of two physical time scales of the gear system, for applying steady-state EHL theories to the analysis of involute gear contacts. Gu's technique can be used to calculate film thickness, contact pressure and surface temperature. He recommends using the pitch point values for film thickness and contact pressure, and the tooth tip contact for surface temperature as representative of the most severe values for the gear system.

Akin (43) extended the work of Dowson and Higginson (40) deriving film thickness expressions for external and internal spur and helical gears and for straight and spiral bevel gears. Akin's relationships apply only at the tooth pitch lines and for bevel gears at mid-face only.

Finkin, et al. (44) examined the validity of using EHL criteria to predict gear scoring. He used data from three published experimental investigations of gear scoring. He concluded that gear scoring occurs under fully boundary-lubricated conditions at lambda values considerably below the onset of the mixed EHL regime. He also concluded that isothermal EHL film thicknesses are in error both in magnitude and in trend with velocity, as is the assumption that pitch point film thickness is a reliable indicator of minimum film thickness. Wellauer, et al. (45) found that the lambda parameter could not be used independently of pitch line velocity as a means to determine the probability of tooth surface distress. Figure 8 shows Wellauer's relationship among lambda, pitch line velocity and the probability of tooth surface distress. Wellauer's observations were as follows:

(1) When lambda exceeds 2, no tooth distress occurs.

(2) When lambda is between 0.7 and 2, distress can be pitting, wear or scoring, depending on the precise conditions of film surface texture and lubricant.

(3) When lambda is less than 0.7 boundary lubrication prevails and lubricant surface physical and chemical interactions, loads and temperatures importantly control distress modes and rates.

Bowen (46) found that a critical change in gear tooth pitting behavior seems to occur at a lambda of about 0.3. Below this value gear life is insensitive to load. At lambda values of 0.4 and above there is a strong dependence of pitting life on load. There appears to be at least qualitative agreement among the findings of Finkin, Wellauer, and Bowen.

Townsend (47) presents a useful overall view of gear operation, and the role of elastohydrodynamics in predicting gear performance. He reviews methods for calculating film thickness and emphasizes the importance of knowing what the properties of the lubricant and its additives are when it is determined that the gear system is operating in the mixed EHL or boundary lubrication regime.

2.6 Reciprocating Seals

Attempts to advance the state of the art of reciprocating seals through the application of elastohydrodynamics have, so far, been concerned with gaining a better understanding of separating film development between elastomer O-rings and pistons, the performance of pumping ring seals, and some studies of the effects of O-ring form on film development. Both elastohydrodynamic and inverse hydrodynamic analytic methods and photoelastic and optical interferometric experimental methods have been used.

Dowson, et al. (48 and 49) conducted experiments which showed good agreement with an isoviscous theoretical curve developed by Herrebrugh. The complexity of a true elastohydrodynamic analysis limits its feasibility to a study under more or less uniform velocity conditions, however, so Hirano, et al. (50) used inverse hydrodynamic theory considering both the wedge and squeeze film terms simultaneously. Hirano, et al. (51) treated the case of sinusoidally varying velocity and found considerable differences in seal profiles in the pumping and reverse strokes. The ratio of contact width to length of stroke was found to be important. Hirano, et al. (51) in a photoelastic experimental study found upper and lower values of the lambda parameter between which elastohydrodynamic films existed. Low ratios of the length of stroke to contact width inhibit EHL film formation, and in the absence of EHL films the effects of tangential surface traction predominate, causing asymmetry of profiles and pressures.

Field, et al. (52) conducted optical interferometry experiments with a rectangular cross section elastomer and found that the shape of the leading edge was important in the development of an EHL film for this particular geometry. Blok, et al. (53) had previously shown that the pressure distribution is little affected by the film profile because of the low modulus of the elastomer.

Gibson, et al. (54) studied the breakaway friction of O-ring seals. They found that starting friction can be predicted with reasonable quantitative accuracy from EHL theory, and that starting friction depends on the dwell period, the acceleration, and the terminal velocity of the previous stroke. They found that starting friction effects continue until the seal has moved two contact widths at constant velocity. Their results are summarized nicely in figure 9, showing the strong effects of dwell time and terminal velocity.

Pumping rings constitute another type of low capacity sealing device. Zull, et al. (55) conducted an EHL analysis of the pumping ring model shown in figure 10. Although the pumping ring itself is not a low modulus elastomer it is mounted on a secondary O-ring seal whose behavior affects the overall pressure pattern acting on the pumping ring. The pumping ring was treated as a flexible body and its deformations computed by a finite element analysis. Performance calculations were made for a range of geometric variables (ring length, thickness, taper, and modulus; rod radius, stroke and frequency; O-ring location and fluid dynamic viscosity and outlet pressure).

3. CONCLUDING REMARKS

It has been shown that the use of elastohydrodynamics in the analysis of rolling element bearings is well established. Relationships for minimum film thickness and tractive force have been incorporated into computer codes used for bearing performance prediction. The lambda parameter (ratio of film thickness to composite surface roughness) has been shown to be important in predicting bearing life and failure mode. At values of lambda below 3, failure modes other than the classic subsurface initiated fatigue can occur. The variation in life becomes appreciable at low lambdas and analysis indicates that in this region of mixed EHL or boundary lubrication a number of failure causing factors, not yet well understood, come into play. These include bearing type, material type, surface roughness and texture, pre-existing defects, lubricant supply and lubricant-material chemistry. It is here that further studies are needed. This region is the least well understood, and bearings operating in the mixed EHL regime suffer short lives so the potential for gain is the greatest.

The application of elastohydrodynamics to gears is complicated by two principal factors: many gears operate in the low lambda region, and the rapidly changing conditions in a tooth meshing cycle make an analysis based on steady-state EHL theory approximate at best. Because many gears operate at low lambdas, failures occur by scuffing and scoring as well as tooth pitting and breakage. In a tooth meshing cycle slide-roll ratios, surface temperatures, contact pressures and contact geometry all change making it necessary to weigh carefully the value of applying steady state theory. Despite these complexities steady state isothermal EHL theory has demonstrated a reasonable degree of correlation with gear performance. If further gains are to be made in predicting gear distress and failure modes and rates, additional work to develop a better understanding of microgeometry, thermal and lubricant-material chemistry effects must be done.

Much useful work has been done to gain a better understanding of film formation in reciprocating elastomer seals. Here again, the physical situation of time varying velocities makes the steady state EHL theory of value only in predicting what is happening at a given time instant and then only approximately. Further experiments may be the most fruitful avenue here. Wedge and squeeze effects are both critical to overall seal leakage, friction, and wear. There is an acute need for long-life reciprocating seals; it will probably be the key element in determining, for example, the feasibility of the Stirling engine in many applications. The importance of seal geometry and operating variables to the establishment of EHL films should be the subject of further studies.

APPENDIX

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TABLE I. - EHL FILM THICKNESSES AND TEST BEARING LIFE RESULTS (AFTER BAMBERGER (38))

Temperature, °C (F)	EHL film thickness,[a] μm (μin.)	Film parameter, lambda	Ratio, bearing L_{10} life / AFBMA life
205 (400)	0.253 (10)	3.6	17
260 (500)	.172 (6.8)	2.5	24
315 (600)	.127 (5)	1.8	15

[a]Obtained from rolling disk machine.

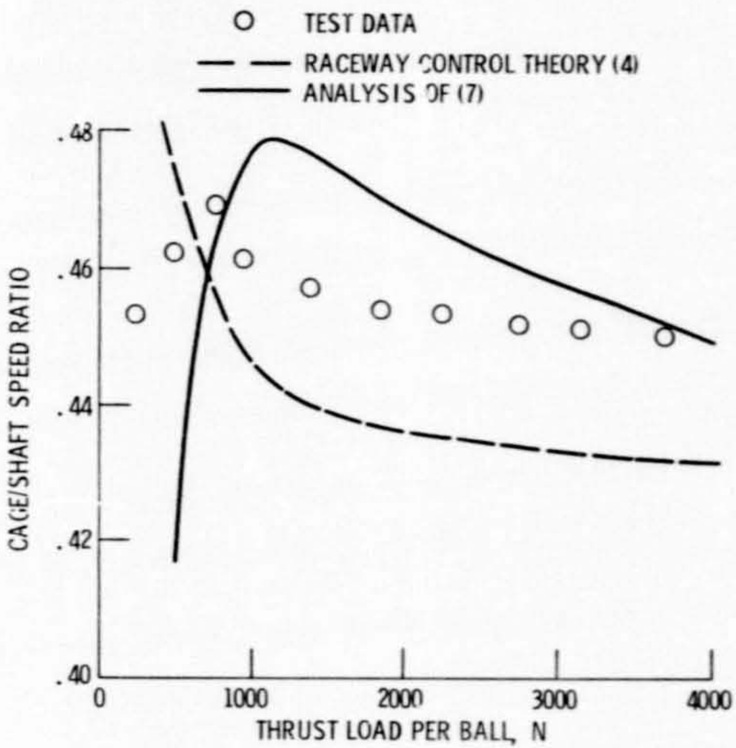

Figure 1. - Cage/shaft speed ratio versus thrust load per ball. (After Harris (7).)

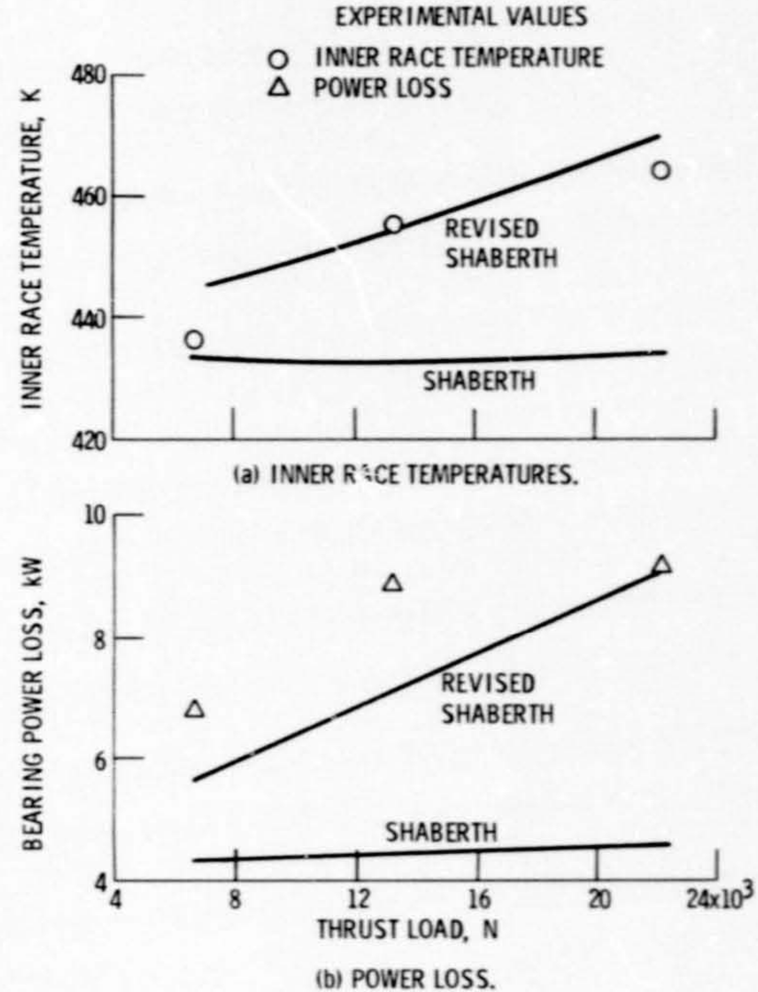

Figure 2. - Comparison of measured and calculated values of bearing operating characteristics as functions of thrust load using two versions of SHABERTH. Shaft speed, 16 700 rpm; lubricant flow rate, 8.3×10^{-3} cubic meter per minute (2.2 gal/min); volume of lubricant, 2 percent. (After Coe (9).)

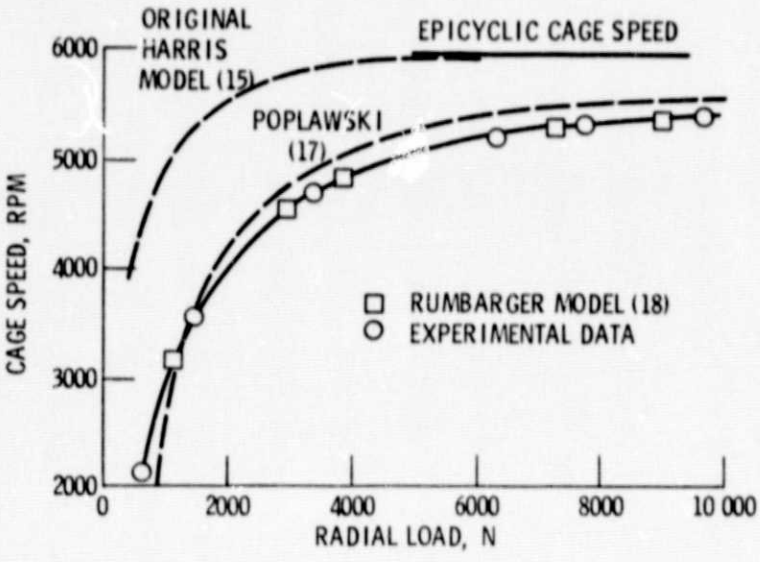

Figure 3. - Roller bearing cage speed correlation. (After Rumbarger (18).)

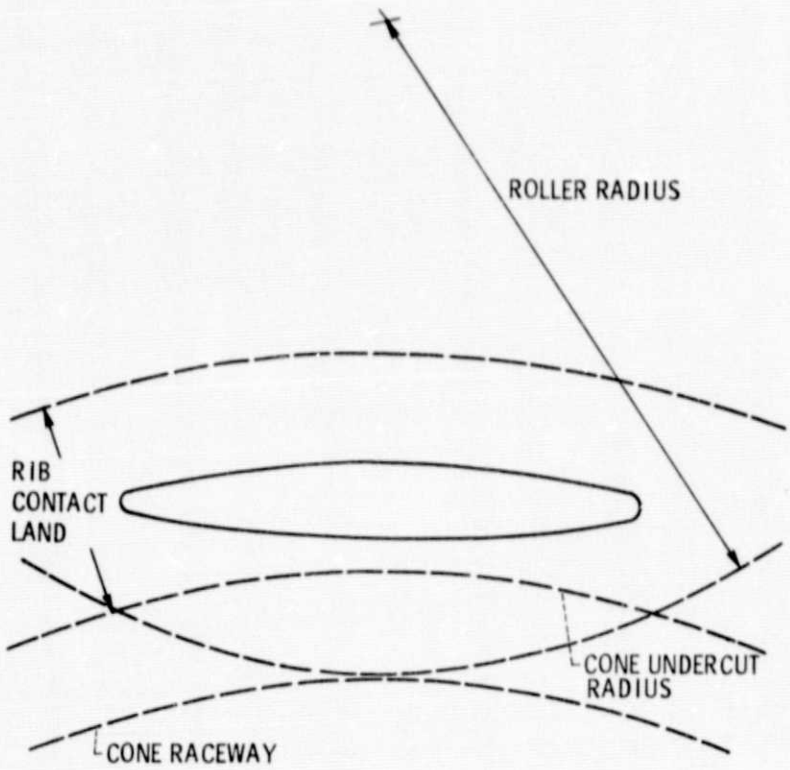

Figure 4. - Projected roller-rib contact in a tapered roller bearing. (After Wren (22).)

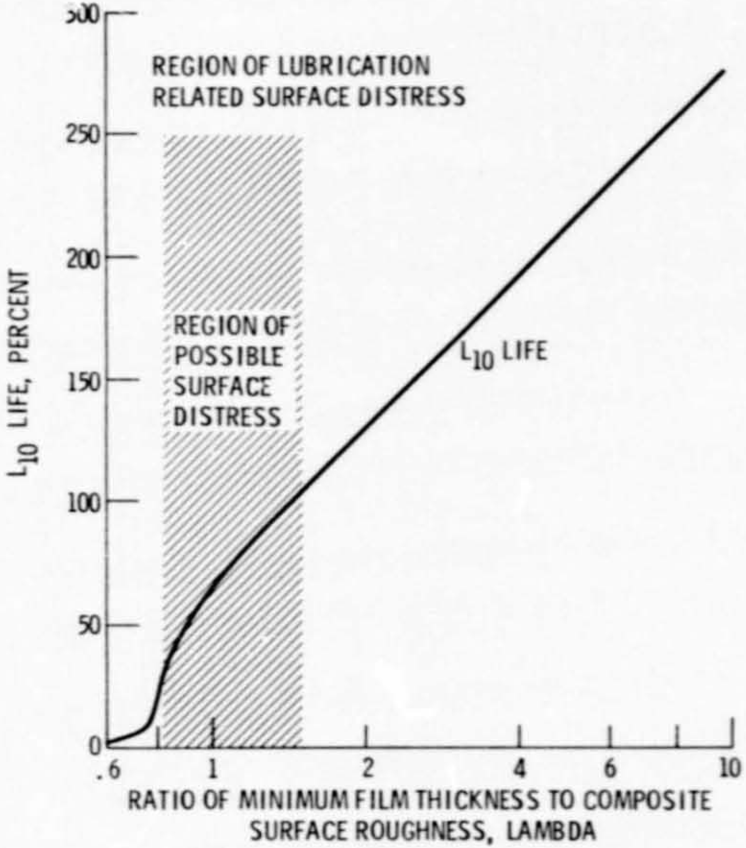

Figure 5. - Group fatigue life L_{10} as a function of lambda. (After Tallian (31).)

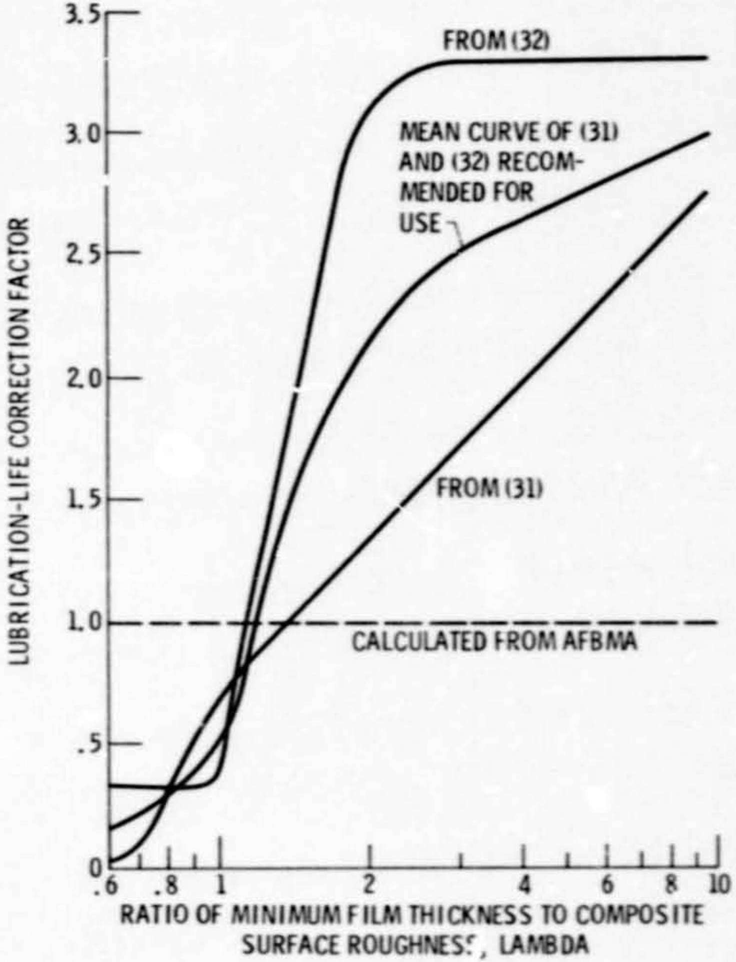

Figure 6 - Lubrication-life correction factor as a function of lambda. (From (34).)

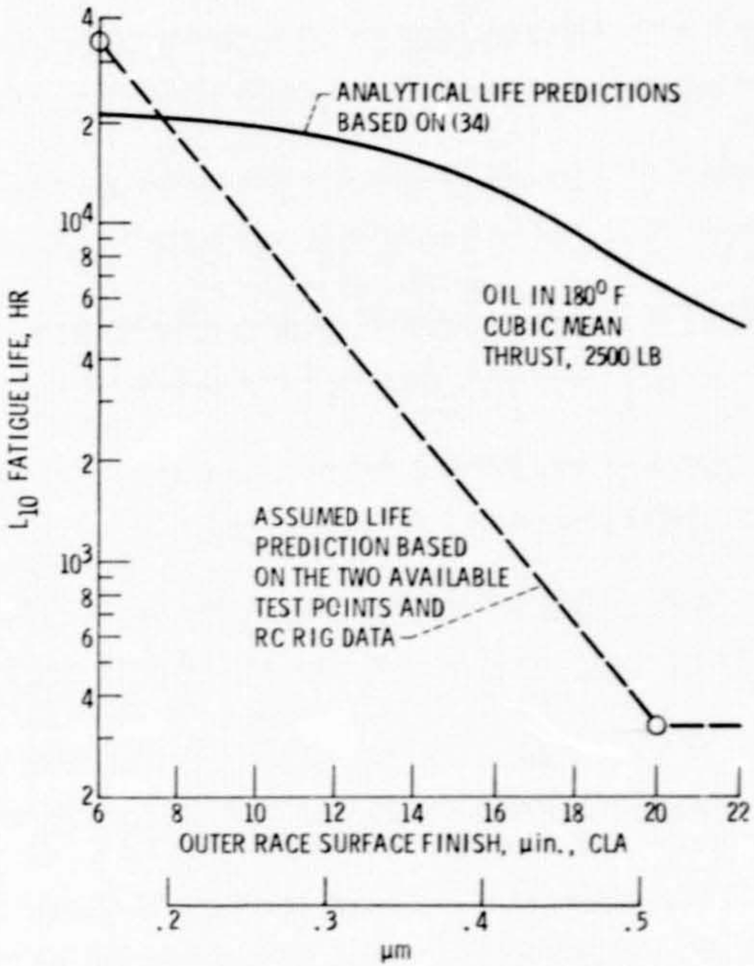

Figure 7. - Bearing life at field operating conditions as function of surface finish. (After Russell (39).)

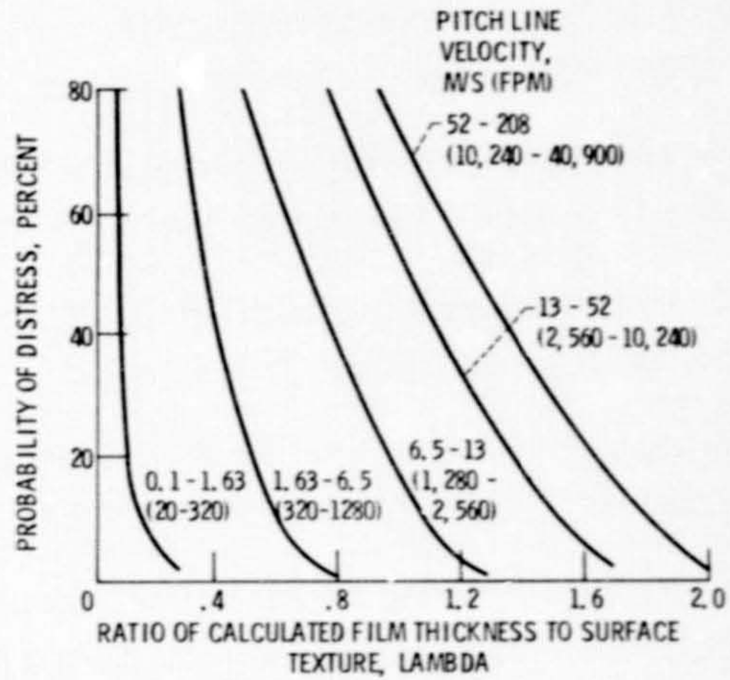

Figure 8. - Probability of tooth surface distress. (After Wellauer (45).)

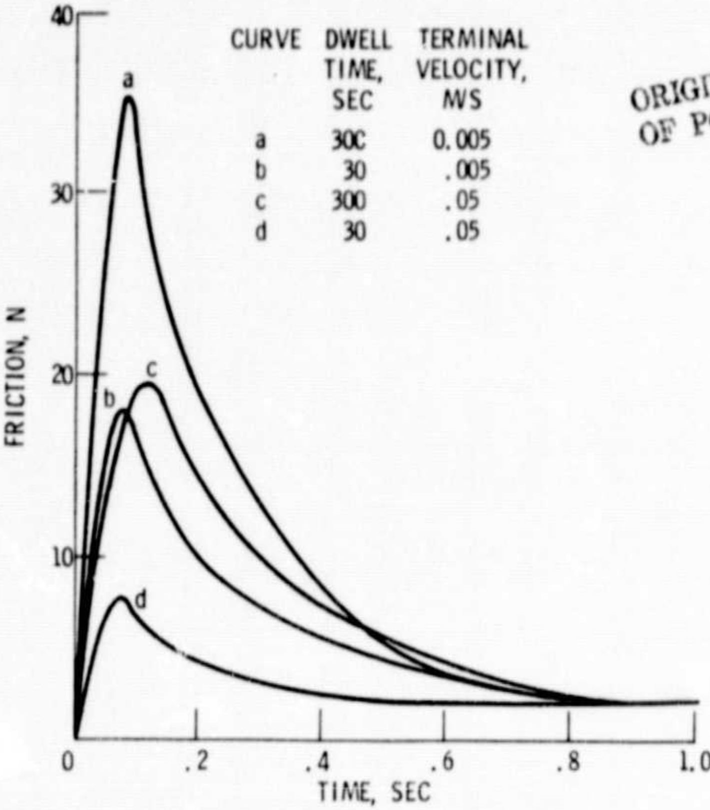

Figure 9. - Typical experimental traces of starting friction for varying terminal velocity and dwell time. Sealed pressure, 11 bar (after Gibson (54).)

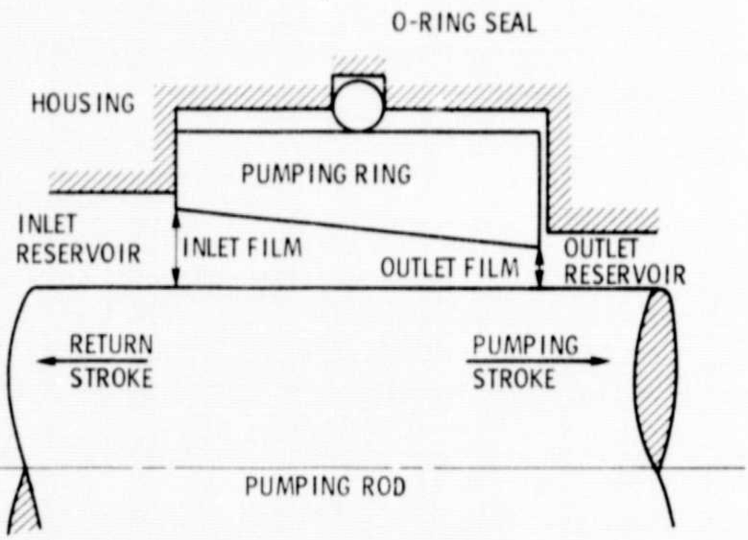

Figure 10. - Pumping ring model. (After Zull (55).)