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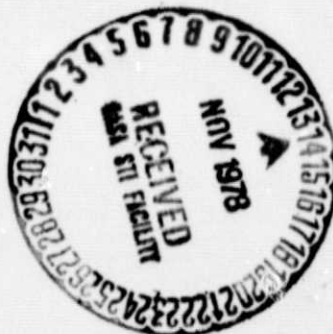
Design of Transonic Airfoil Sections Using a Similarity Theory

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DESIGN OF TRANSONIC AIRFOIL SECTIONS USING A SIMILARITY THEORY

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SUMMARY

A study of the available methods for transonic airfoil and wing design indicates that the most powerful technique is the numerical optimization procedure. However, the computer time for this method is relatively large because of the amount of computation required in the searches during optimization. The optimization method requires that "base" and "calibration" solutions be computed to determine a "minimum drag" direction. The design space is then computationally searched in this direction; it is these searches that dominate the computation time. A recent similarity theory allows certain transonic flows to be calculated rapidly from the base and calibration solutions. In this report the application of the similarity theory to design problems is examined with the object of at least partially eliminating the costly searches of the design optimization method. An example of an airfoil design is presented.

INTRODUCTION

New interest in reducing the fuel consumption of existing and future aircraft has been prompted by the world-wide fuel shortage. A logical place to start the search for lower fuel consumption is to reduce drag, especially at the transonic speeds at which most aircraft cruise. The most common description of a transonic flow is when there is a supersonic "bubble" totally embedded in a subsonic flow. The supersonic bubble may be terminated by a shock wave producing wave drag or, in certain circumstances, may return to subsonic conditions through an isentropic compression with no wave drag. This wave drag is associated with the entropy change across the shock. Soon after a shock wave appears in the flow, the drag will increase rapidly with increasing free-stream Mach number, leading to the definition of a "drag rise Mach number," which is defined as the free-stream Mach number at which this rapid drag rise begins. One of the main objects of designing a wing for transonic speeds is to obtain as high a "drag rise Mach number" as possible, subject to certain constraints. The obvious way to reduce the wave drag, at least in a two-dimensional study, is to use a supercritical shock-free airfoil section where there is no shock wave and, consequently, no wave drag. However, these shock-free airfoils may have undesirable off-design characteristics, such as strong shock waves suddenly appearing when the Mach number is perturbed slightly from its design value. Thus, an important constraint in the design

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of airfoils is that there should be good off-design behavior. In wing design (i.e., three-dimensional flows) the design can be altered by spanwise changes (e.g., sweep or twist); and this can complicate the design procedure.

Methods for calculating transonic flow characteristics around given airfoils were first derived by Murman and Cole (ref. 1), with a later correction by Murman (ref. 2), using the concept of type-dependent finite differences applied to the transonic small disturbance equation. Later work resulted in methods using the full potential equation such as that by Jameson (ref. 3). Three-dimensional extensions of the small disturbance methods were made by Ballhaus, Bailey, and Frick (ref. 4), and extensions of the full potential equation methods were made by Jameson and Caughey (ref. 5). All these methods are for isentropic flow, which, strictly speaking, cannot produce a wave drag that (as noted above) would be a consequence of an entropy change. However, isentropic flow does produce a momentum deficit across the shock wave that can be thought of as a wave drag (ref. 6), an assumption that seems to be corroborated by experiment. These calculation methods must be differenced (ref. 2) in conservative form otherwise the shock location and strength may be incorrect.

The earliest attempt to design transonic airfoil sections used the hodograph transformation, in which the velocity components are the independent variables rather than the usual geometric variables (x,y). The hodograph transformation leads to a linear equation for transonic flow rather than the usual nonlinear equation in the physical coordinate system. Best known of these methods are those of Nieuwland (ref. 7) and Bauer, Garabedian, and Korn (ref. 8). These methods require complex mappings and transformations and can be difficult to use. In addition, neither the design pressure distribution or the design Mach number is known in advance. Only shock-free designs are possible, thus precluding the inclusion of off-design criteria into the design process. Also, it is difficult to apply constraints, such as maintaining a specified minimum lift coefficient.

A second type of design method is the reversal of the direct finite difference procedures, with the pressure distribution specified at a given Mach number and the corresponding airfoil shape obtained. Examples of this type of procedure are given by Steger and Klineberg (ref. 9) and by Carlson (ref. 10). The main problems associated with this line of attack are:

1. The specified pressure may not produce a realistic airfoil (e.g., nonclosure of the trailing edge).

2. Constraints on the lift coefficient or section thickness are difficult to implement.

3. Off-design criteria cannot be incorporated into the design process.

A third method of transonic wing design is the numerical optimization procedure developed by Hicks (refs. 11 and 12) and his coworkers. The numerical optimization method seeks to minimize some specific parameter (e.g., the drag coefficient, C_D) for a set of design variables describing the airfoil

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geometry, while satisfying a number of specified constraints. The constraints may be aerodynamic (e.g., lift-coefficient or off-design criteria) or geometric (e.g., airfoil thickness or volume). The design method uses an aerodynamic analysis program coupled with a numerical optimization program; the design process is briefly outlined below.

A base airfoil profile is chosen and is perturbed by the use of shape functions which control the final profile. The coefficients of these shape functions are the design variables. The optimization scheme perturbs each of these coefficients in turn, returning to the aerodynamic analysis program for the evaluation of the drag coefficient after each perturbation. After the change of C_D for each change of design variable has been noted, the optimization program computes the gradient of C_D (∇C_D) with respect to each design variable. The optimization program then increments the coefficients one to four times in the $-\nabla C_D$ direction, searching for a minimum value of C_D that satisfies the constraints. At each increment, the aerodynamic analysis program is used to compute the drag and aerodynamic characteristics and any off-design constraints. If a constraint is reached or the drag increases (due to nonlinearity), then a new gradient is computed and the process repeated. The computing time of this optimization procedure is fairly large. The bulk of the computing time, however, is used in searching for the minimum drag, when the analysis program has to be used at each incremental step. The inclusion of off-design criteria in the constraints require an additional calculation of the aerodynamic characteristics at the specified off-design Mach number. An outcome of this design effort (refs. 11 and 12) is the determination of useful airfoil shape functions that, when added to the base airfoil, permit a large class of airfoil contours; these shape functions always give closure of the airfoil. Extensions of the numerical optimization procedure to finite wings has been reported by Hicks and Henne (ref. 13). In this three-dimensional procedure, five shape functions are used at two span-wise stations, together with an angle of attack variation, giving a total of eleven design parameters. While angle of attack and wing twist are computationally efficient to use in the design code, other design criteria (such as sweep, aspect ratio, and taper) require a great expenditure of computing power. This is because these design variables affect the location of the wing relative to the finite difference mesh (which is usually sheared so that the grid lines coincide with the leading and trailing edges), and modifications to the mesh are desirable for each change. Another application of this design procedure is by Haney, Waggoner, and Ballhaus (ref. 14).

It would seem from the above discussion that the most powerful of the available design methods is the numerical optimization technique, since aerodynamic constraints such as lift coefficient, off-design criteria, and geometric constraints (such as wing thickness) can be incorporated into the design process. However, the computing requirements can be considerable, especially for three-dimensional designs. The bulk of the computing time is used in the searches where the aerodynamic analysis program must be used at each step. If some way of reducing the time for the searches could be found, then the computer requirements would be less formidable.

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Recently, a similarity theory has been derived by Nixon (refs. 15 and 16) by which a range of transonic flow solutions can be found by simple algebra, provided two solutions, a base solution and a calibration solution, are known. The analysis is based on a perturbation expansion of the transonic equations, which leads to a linear perturbation equation. The nonlinear phenomena of shock movement due to a perturbation is treated by using a strained coordinate system in which the shock location is invariant. Essentially, this device treats the nonlinear flow changes due to a perturbation in some flow characteristic, such as Mach number, by a nonlinear combination of two linear problems that can be solved in sequence. Since these equations are linear, the principle of superposition applies; and hence, the effect of several perturbations can be considered at once. The nonlinearity appears only in the last step of the procedure, which is the nonlinear combination of the linear problems. Generally, if N parameters are perturbed, then the procedure requires N + 1 solutions composed of one base solution and N calibration solutions. Once these solutions are known, any related "nearby" solution can be obtained rapidly. Using a CDC-7600 computer for a three-dimensional, two-parameter example, eight cases can be computed in 0.38 s of CPU time. The theory has been applied to three-dimensional wings (ref. 16) with multiple shock waves. The main restriction of the theory is that shock waves cannot be generated or destroyed in the perturbation; although in principle a shock of zero strength can be treated, that is, a shock-free supercritical airfoil.

Since the computation time required to calculate the "nearby" flow characteristics is so small, it seems that this similarity method would be extremely advantageous in the numerical optimization design procedure. The costly searches previously carried out by reversion to the aerodynamic analysis program can now be done using the inexpensive similarity theory. Also, since the similarity theory gives the "nearby" solutions as analytic rather than numerical functions of the parameters, it is possible that the optimization procedure itself could be improved by a study of the analytic dependence of the design criteria on these parameters. Furthermore, off-design constraints in the transonic regime, e.g., drag at a slightly lower Mach number, can be easily computed since a change in the Mach number is just another perturbation parameter. This avoids the need to calculate off-design characteristics at each point in the design loop using the aerodynamic program. Another example is the effect of wing sweep angle in three-dimensional flows; which, again, is just another perturbation parameter, the effect of which need only be computed once.

This report begins to investigate the applications of the similarity theory to design problems. Only two-dimensional flows are considered, and a total of five parameters are used to characterize the shape functions. At this stage an optimization procedure is not used, since the basic aim is to establish the validity of the design applications of the theory. The main objective in the present design study is to reduce the wave drag coefficient of an airfoil and to deduce the necessary ground rules for this design objective. Applications of these rules to the design of an airfoil section are presented, and the design pressure distribution agrees satisfactorily with a direct calculation.

BASIC PRINCIPLES

If an airfoil section is specified by several shape functions with unknown amplitudes, the problem under consideration is to choose these amplitudes such that some design criteria is met. Generally, in the course of choosing these design parameters, any shock waves in the flow will change location, thus invalidating the usual type of perturbation theory. This is because the pressure changes in the region traversed by the shock wave are not small, even for a small change in shock locations. A means of treating this kind of perturbation is given by Nixon (ref. 15), using a strained coordinate system in which the shock location is invariant with changes in the perturbation variable. Using this technique, the nonlinear transonic problem is split into two linear problems which can be solved in sequence. Because of this linearity, complex solutions can be constructed from simpler solutions by superposition. In a design problem the effect of each design parameter can therefore be calculated separately and the principle of superposition used to construct a solution giving the effect of any combination of the design parameters.

The analysis given by Nixon (ref. 15) concerns only the transonic small disturbance equation and since design methods are most accurate if the full potential equation is used, the strained coordinate technique must be developed for such an equation.

If (x,y) is a Cartesian coordinate system, nondimensionalized with respect to the airfoil chord and with x aligned with the airfoil chord, then the full potential equation is

$$(a^2 - u^2)\phi_{xx} - 2uv\phi_{xy} + (a^2 - v^2)\phi_{yy} = 0 \tag{1}$$

where u and v are the velocity components in the x and y directions, respectively, nondimensionalized with respect to the free-stream velocity q_∞, and are given by

$$u = \frac{\partial\phi}{\partial x}, \qquad v = \frac{\partial\phi}{\partial y} \tag{2}$$

where ϕ is the velocity potential; a is the speed of sound, again nondimensionalized with respect to q_∞ and given by

$$a^2 = \frac{1}{M_\infty^2} + \left(\frac{\gamma - 1}{2}\right)(1 - u^2 - v^2) \tag{3}$$

M_∞ is the free-stream Mach number. The boundary condition on the airfoil surface is given by

$$\frac{v(x,y_s)}{u(x,y_s)} = y_{s_x}(x) \tag{4}$$

where $y = y_s(x)$ defines the surface of the airfoil. An infinite distance upstream of the airfoil

$$u = q_\infty \cos\alpha$$

$$v = q_\infty \sin\alpha \tag{5}$$

where α is the angle of attack.

The pressure velocity relation is given by

$$C_p(x,y) = \frac{2}{\gamma} M_\infty^2 \left[1 + (\frac{\gamma - 1}{2})M_\infty^2 (1 - u^2 - v^2) \right]^{\gamma/\gamma-1} \tag{6}$$

It is now proposed to change the airfoil boundary by some small amount characterized by ϵ. Thus, the new airfoil is defined by

$$y = \bar{y}_s(x) = y_s(x) + \epsilon y_s^{(1)}(x) \tag{7}$$

and the corresponding boundary condition is

$$\frac{v(x,\bar{y}_s)}{u(x,\bar{y}_s)} = y_{s_x}(x) + \epsilon y_{s_x}^{(1)}(x) \tag{8}$$

The question is, given the solution of the problem defined by equations (1) and (4), find the solution of the problem defined by equations (1) and (8). As in reference 15, it is assumed that any shock waves in the flow are normal to the x-axis and that there is only one shock wave on each airfoil surface. The strained coordinate x' is then introduced where

$$x = x' + \epsilon\delta x_s x_1(x') \tag{9}$$

where $\epsilon\delta x_s$ is the shock movement and $x_1(x')$ is given in reference 15 by

$$\left.\begin{array}{ll} x_1(x') = \dfrac{x'(1 - x')}{x'_s(1 - x'_s)} & 0 \leqslant x' \leqslant 1 \\[2ex] x_1(x') = 0 & x' < 0 \\[2ex] & x' > 1 \end{array}\right\} \tag{10}$$

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where x'_s is the original shock location and $\epsilon \delta x'_s$ is the shock movement. The variables ϕ, u, etc., are then expanded as the series

$$\phi(x,y) = \phi_o(x',y) + \epsilon \phi_1(x',y) + \ldots \tag{11}$$

and

$$u(x,y) = u_o(x',y)\left[1 - \epsilon \delta x_s x_{1_x}(x')\right] + \epsilon u_1(x',y) \ldots \tag{12}$$

$$v(x,y) = v_o(x',y) + \epsilon v_1(x',y) \ldots$$

$$a(x,y) = a_o(x',y) + \epsilon a_1(x',y) \ldots \tag{13}$$

The expression for $a(x,y)$ can be found in terms of u, v, and δx_s, using equations (3) and (9).

Substituting equations (9), (11), (12), and (13) into equations (1) and (8) and equating coefficients of ϵ gives, to first order in ϵ, the following two equations with associated boundary conditions

$$(a_o^2 - u_o^2)\phi_{o_{x'x'}} - 2u_o v_o \phi_{o_{x'y}} + (a_o^2 - v_o^2)\phi_{o_{yy}} = 0 \tag{14}$$

$$\frac{v_o(x',y_s)}{u_o(x',y_s)} = y_{s_{x'}}(x') \tag{15}$$

and

$$(a_o^2 - u_o^2)\phi_{1x'x'} - 2u_o v_o \phi_{1x'y} + (a_o^2 - v_o^2)\phi_{1yy} + 2(a_o a_1 - u_o u_1)\phi_{o_{x'x'}}$$

$$- 2(u_o v_1 + v_o u_1)\phi_{o_{x'y}} + 2(a_o a_1 - v_o v_1)\phi_{oyy}$$

$$= -2\phi_{o_{x'x'}} u_o^2 \delta x_s x_{,x'} - 2\phi_{o_{x'x'}} \delta x_s x_{1_x}(a_o^2 - u_o^2)$$

$$- (a_o^2 - u_o^2)\delta x_s x_{1_{x'x}}\phi_{o_{x1}} + 2u_o v_o \delta x_s x_{1_{x'}}\phi_{o_{x'y}} \tag{16}$$

$$v_1(x',\overline{y}_s) = y_{s_{x'}}^{(1)}(x')u_o(x',y_s) + \left(u_1 - \delta x_s u_o x_{1_{x'}}\right)y_{s_{x'}}(x')$$

$$+ x_1(x')y_{s_{x'x'}}(x') \tag{17}$$

Provided the perturbation is not a function of M_∞, then equations (16) and (17) do not contain the parameter ϵ. Hence, if the variables ϕ_1, δx_s, etc., are known for one value of ϵ, then the value of the variables for any other value of ϵ can be found by simple proportion. Thus,

$$\epsilon u_1(x',y) = \frac{\epsilon}{\epsilon_o}\left[\epsilon_o u_1(x',y)\right]$$

$$\epsilon \delta x_s = \frac{\epsilon}{\epsilon_o}\left[\epsilon_o \delta x_s\right] \tag{18}$$

The omission of the Mach number variation from the analysis is because the term a_1 contains M_∞; hence equation (16) is not independent of ϵ. However, Mach number variations can be treated if it is assumed that for such changes the transonic small disturbance equation is a valid approximation. In this case, the analysis of reference 16 can be used.

While the linear perturbation equation, equation (16), can be solved for u_1, δx_s, etc., it is much more convenient to use the same technique to solve both the base and calibration solutions. Equation (16) multiplied by ϵ_o represents the first order of magnitude in ϵ_o the difference of two solutions to equation (1). Hence, expressions for u_1 and δx_s in equation (13) can be found by a suitable combination of two known nonlinear results.

If, by some method, the solution to the base problem defined by equations (14) and (15) is known and if the solution to some perturbed problem, characterized by some parameter ϵ_o, is also known (the calibration solution), then the terms $(\epsilon_o u_1)$ and $(\epsilon_o \delta x_s)$ can be found as follows:

1. The change in the shock location $\epsilon_o \delta x_s$ between the base and calibration solutions is easily found by inspection.

2. If $u^{(1)}(\hat{x},y)$ is the solution of the perturbed problem and if $u^{(0)}(x',y)$ is the solution of the base problem, then

$$\epsilon_o u_1(x',y) = u^{(1)}(\hat{x},y) - u^{(0)}(x',y)\left[1 - \epsilon_o \delta x_s x_{1_{x'}}(x')\right] \tag{19}$$

where

$$\hat{x} = x' + \epsilon_o \delta x_s x_1(x') \tag{20}$$

and $x_1(x')$ is given by equation (10).

Having obtained δx_s and $u_1(x',y)$ one can then obtain the final solutions

$$u(x,y) = u^{(0)}(x',y)\left[1 - \epsilon \delta x_s x_{1_{x'}}(x')\right] + \epsilon u_1(x',y) \tag{21}$$

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and

$$x = x' + \varepsilon \delta x_s x_1(x') \tag{22}$$

The other velocity component is simply given by

$$v_1(x,y) = \frac{1}{\varepsilon_0}[v^{(1)}(x,y) - v^{(0)}(x,y)] \tag{23}$$

where $v^{(0)}(x,y)$ and $v^{(1)}(x,y)$ are the solutions of the base and calibration problems, respectively. The total velocity in the y direction is then given by

$$v(x,y) = v^{(0)}(x,y) + \varepsilon v_1(x,y) \tag{24}$$

The pressure coefficient can then be found from equations (6), (21), (22), and (24).

Since the basic equation derived in the preceding sections are linear, the effect of more than one parameter can be obtained by superposition. Thus, for N parameters, equations (21), (22), and (24) can be generalized to give

$$u(x,y) = u^{(0)}(x',y)\left[1 - \sum_{i=1}^{N} \varepsilon_i \delta x_{s_i} x_{1_{x'}}(x')\right] + \sum_{i=1}^{N} \varepsilon_i u_{1_i}(x',y) \tag{25}$$

where

$$x = x' + \sum_{i=1}^{N} \varepsilon_i \delta x_{s_i} x_1(x') \tag{26}$$

$$v(x,y) = v^{(0)}(x,y) + \sum_{i=1}^{N} \varepsilon_i v_{1_i}(x,y) \tag{27}$$

The N parameters are denoted by ε_i $(i=1,N)$, the change in shock location due to the ith parameter change is $\varepsilon_i \delta x_{s_i}$.

DESIGN APPLICATIONS

In the direct calculations, the parameters ε are specified and $u(x,y)$ and $v(x,y)$ on the airfoil surface are obtained. In a design application, the velocity $u(x,y)$ on the airfoil surface can be specified at $N - 1$ stations and the ε_0 found; the Nth equation for the ε_i is found by specifying the total shock movement $\sum_{i=1}^{N} \varepsilon_i \delta x_{s_i}$, thus giving the relationship

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between the coordinate systems x and x'. Once $u(x,y)$ on the airfoil surface and the ϵ_i are known, the $v(x,y)$ can be obtained from the tangency boundary conditions. An alternative to specifying $u(x,y)$ is to use some form of optimization procedure. The parameters ϵ_i may be changes in angle of attack, geometric shape functions, or, if the small disturbance equation is used, changes in Mach number.

As shown in reference 16, the adequate calculation of the drag coefficient involves a flow field calculation around the shock waves and is a cubic equation in the parameter ϵ. This would lead to a complicated analysis in the design application. Therefore, for optimization purposes, it is proposed that the possibility of minimizing the function $|C_p^+ - C_p^*|$ rather than C_D is investigated, where C_p^+ is the surface pressure just ahead of the shock wave and C_p^* is the critical pressure. The applicability of this assumption rests on C_D being a monotonic function of $|C_p^+ - C_p^*|$. In figure 1, a plot of C_D against $|C_p^+ - C_p^*|$ for 14 different direct calculations is shown, and it may be seen that within the limits of numerical accuracy C_D is indeed a monotonic function of $|C_p^+ - C_p^*|$.

An obvious design objective is to reduce the drag coefficient by reducing the shock strength to zero or nearly zero. In theory, this should give something close to a shock-free airfoil. In order to establish some ground rules for such a design objective, a test case using the transonic small disturbance equation with linear boundary conditions is considered. The base airfoil is a 10% biconvex section at zero angle of attack and the object is to find the free-stream Mach number at which the shock strength is nearly zero. The base Mach number is 0.828 and the calibration Mach number is 0.838. A shock fitting small disturbance code was used to compute these solutions. The design Mach number is 0.7905 and the result obtained by the similarity solutions is compared to a direct calculation in figure 2. The difference in shock locations and pressure distributions ahead of the shock are probably due to the magnitude of the perturbation in Mach number being too large. However, the main discrepancy between the similarity and direct results is in the large supersonic expansion in the former behind the shock wave. This expansion is due to the postshock expansion behavior exhibited by the base and calibration solutions, which apparently does not scale when a shock-free limit is approached. This suggests that in order to adequately compute shock-free or nearly shock-free solutions, both base and calibration solutions should fairly closely model the essential flow features of the final design. For example, the rapid postshock expansion should not be large in the base and calibration solutions. In figure 3, the pressure distribution close to the shock at $M_\infty = 0.798$ for the same biconvex airfoil is shown, but with a base Mach number of 0.818 and a calibration Mach number of 0.808. It can be seen that the rapid postshock expansion has been eliminated. These results lead to the following design criteria for shock-free or nearly shock-free designs.

1. The base and calibration solutions should represent all of the essential features of the final design.

2. The flow must not accelerate supersonically behind a shock wave (including a zero strength shock) and must be supersonic just ahead of the shock (i.e., the shock must be compressive).

3. The final design should be realistic, that is, there should be no crossover of the upper and lower surfaces of the airfoil.

4. For a shock-free design, it is desirable that any discontinuities in the pressure gradient at the "shock" be at a minimum in order to have a smooth recompression.

A final criterion is that the perturbations should not be too large. Generally, this is indicated when the strained coordinate overshoots the airfoil chord, that is, gives values of x that lie outside the airfoil. Consequently, a fifth condition is as follows:

5. For values of the strained coordinate x' on the airfoil chord, the coordinate x must always lie on the chord line.

These then are the criteria used in the preliminary tests of design applications of the similarity theory.

EXAMPLE

In order to test the above ideas, an airfoil is designed using the ideas of Hicks (refs. 11 and 12). The base airfoil is a laminar flow design.

Perturbations of the form

$$y = a_i(\sin \pi x b_i)^3 \qquad i = 1,5 \tag{28}$$

are used to modify the airfoil geometry. The values of b_i and a_i used in computing the calibration solutions are shown in table 1. Both base and calibration solutions were computed using a full potential equation code. The object is to reduce the shock strength and hence, the wave drag by choosing $a_i(i = 1,5)$. The magnitude of the perturbations is limited to 1-1/2 times that used in the base and calibration solutions, although the sign can change. This is effectively a constraint on section thickness. The free-stream Mach number is 0.74 and the angle of attack is zero. The optimization is simply done by a search of the relevant range of parameters with 12 steps in each range. This scheme is not by any means the best, but is easy to program and is used here only to validate the theory. The magnitude of the parameters found by this procedure is given in table 1. The resulting pressure distribution is shown in figure 4 and compared to a direct calculation. It can be seen that the predicted and direct calculations agree fairly well. The total computing time is 3.7 s on a CDC-7600 computer, provided the base and calibration solutions are known.

In this paper, only one design iteration is considered; that is, only one set of base and calibration solutions is used. This serves to test the ideas;

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and, in any case, one such computation may be sufficient. If the base and calibration solutions do need to be recalculated, then the whole procedure can easily be repeated.

CONCLUDING REMARKS

The similarity theory developed by Nixon (refs. 15 and 16) for direct calculations is extended to design optimization problems. Some ground rules for the design of shock-free or nearly shock-free airfoils are deduced. A simple example is computed. The advantages of the scheme of incorporating the similarity theory into the numerical optimization design procedure are as follows:

1. In the hitherto expensive searches, the computing time can be reduced considerably using the similarity theory.

2. Off-design criteria can be easily and inexpensively incorporated into the design process.

3. In three-dimensional applications, the difficult design parameters, such as wing sweep or taper, can be much more easily taken into account since only one calibration solution is required rather than the multiple finite difference calculations required in the existing numerical optimization scheme.

4. The analytic nature of the similarity theory may lead to improvements in the optimization scheme itself.

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TABLE 1. DESIGN PARAMETERS

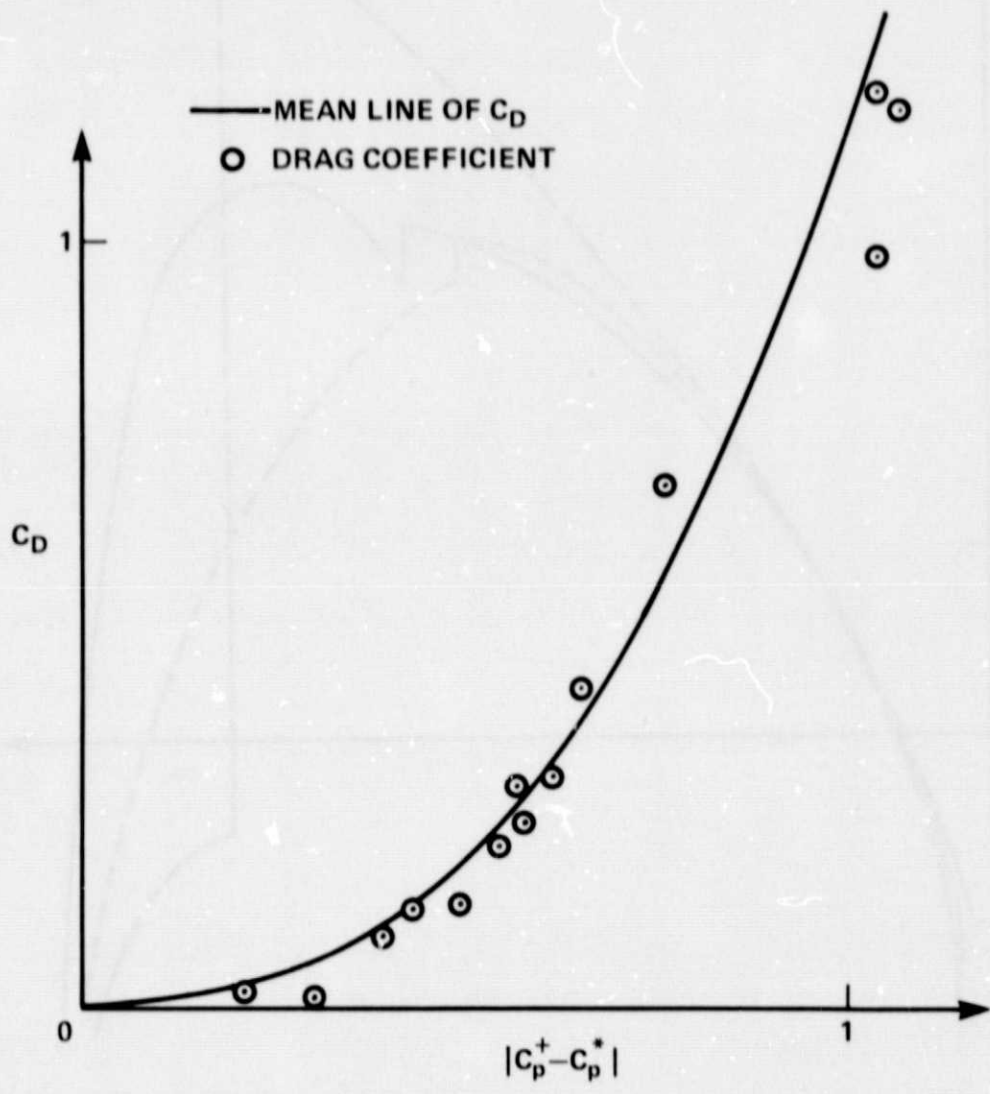| i	b_i	a_i (calibration)	a_i (design)
1	0.2314	-0.003	0.0034
2	0.3654	0.0005	0.0008
3	0.6603	0.002	-0.0013
4	0.8681	0.001	0.0001
5	2.4094	-0.002	-0.0022

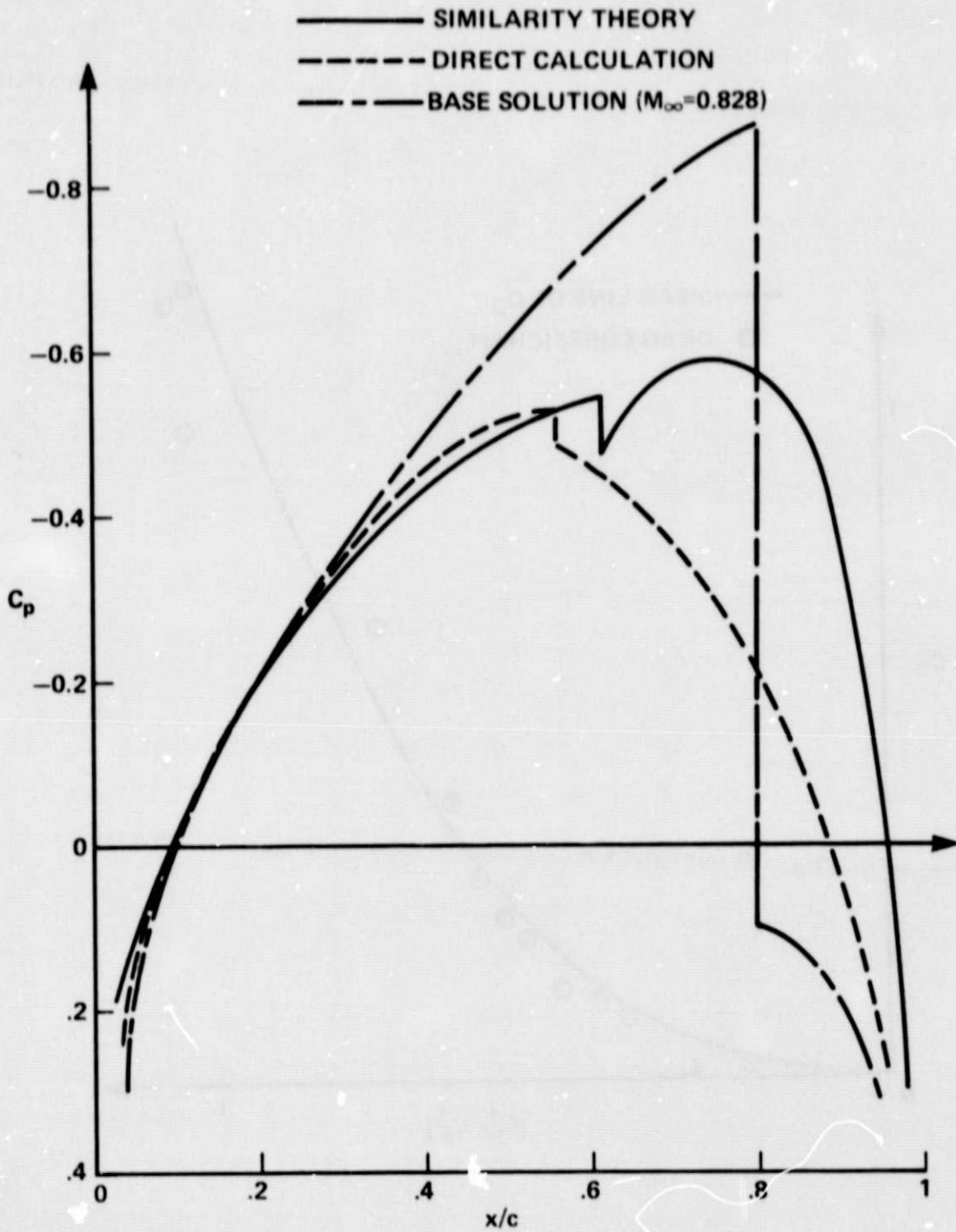Figure 1.- Variation of drag coefficient with $\left|c_p^+ - c_p^*\right|$.

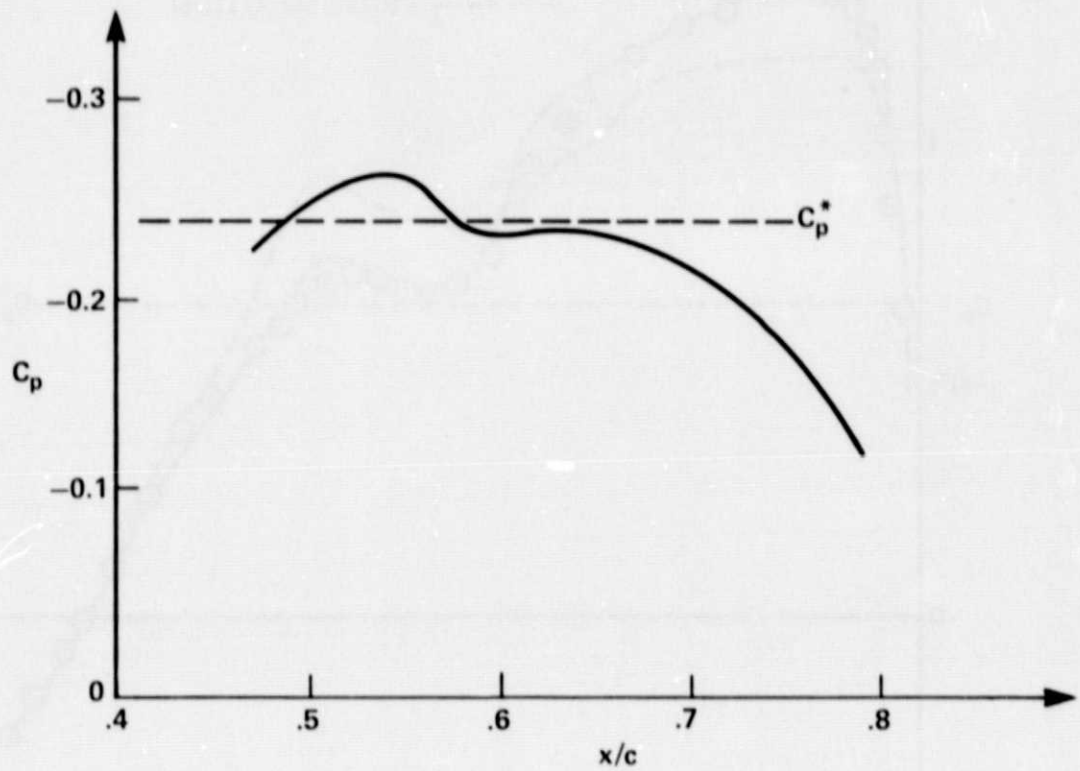Figure 2.- Pressure distribution around a 10% biconvex airfoil; $M_\infty = 0.7905$.

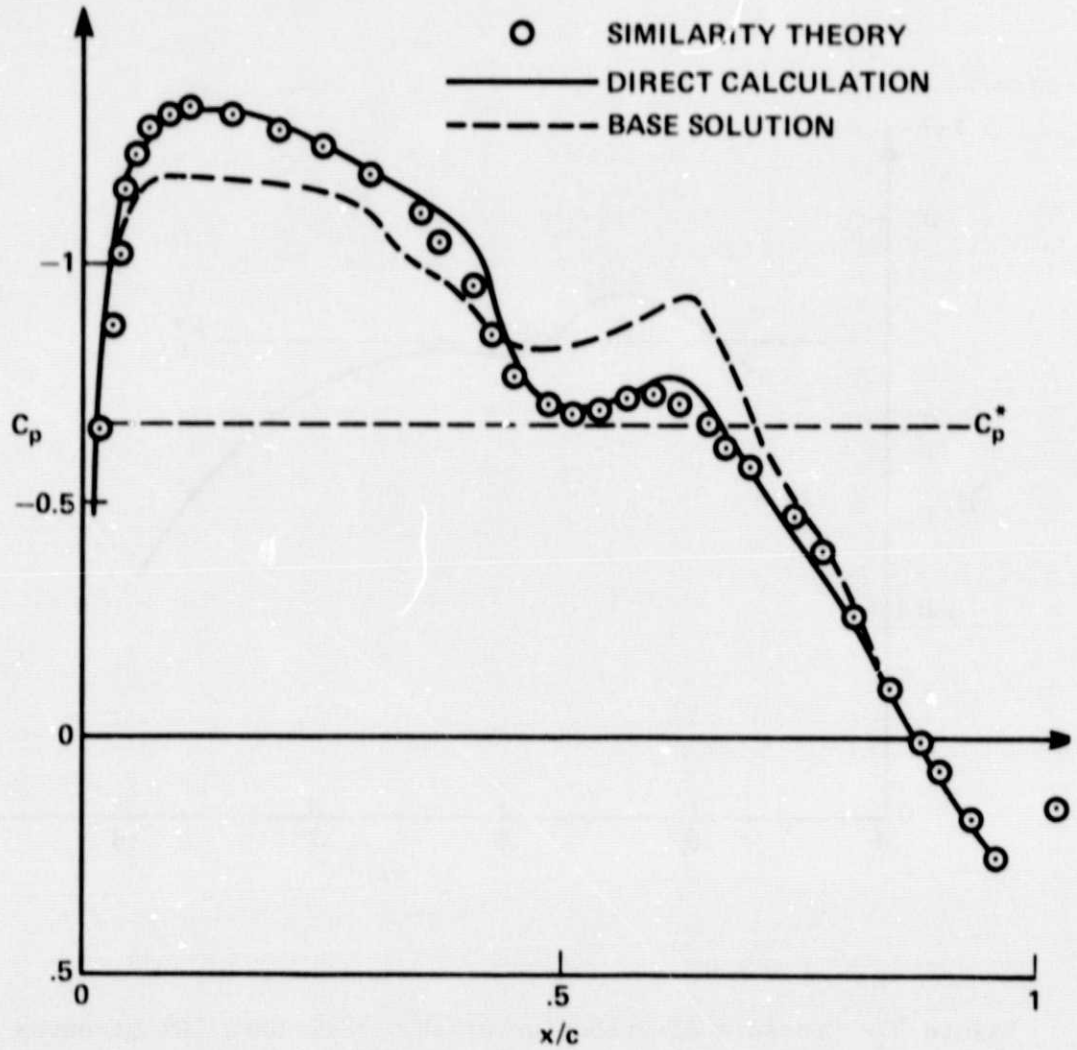Figure 3.- Pressure distribution at the shock wave 10% biconvex airfoil; $M_\infty = 0.798$.

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Figure 4.- Airfoil design, $\alpha = 0°$, $M_{\infty} = 0.74$ (upper surface only).

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