N 8 9 . 1 5 8 1

MEANS TO ACHIEVE WIDE SWATH WIDTHS
IN
SYNTHETIC APERTURE SATELLITE BORNE RADARS

LOUIS J. CUTRONA
DEPARTMENT OF APPLIED PHYSICS AND INFORMATION SCIENCES
UNIVERSITY OF CALIF., SAN DIEGO
LA JOLLA, CALIFORNIA

SUMMARY

In deriving the characteristics of a synthetic aperture radar which is carried in a high speed vehicle there are a number of constraints as well as a number of degrees of freedom among the parameter values which may be selected for the radar system. It is the purpose of this paper to show how these constraints and the available degrees of freedom affect the swath width, resolution, area converage rate, average power, system complexity, and system parameters of the radar.

The organization of this paper is as follows: The radar range equation including processing gains for pulse compression and synthetic aperture generation is the starting point. System geometry considerations are introduced. For simplicity flat earth geometry is used, although it is realized this is not a good model for satellite borne radars. Next the constraints are introduced. These include those needed to avoid ambiguities in both range and azimuth, those needed to acheive the desired resolution, and those needed to achieve the desired swath width.

It is found, if only a single channel radar system is used, that the number of degrees of freedom needed are not available. There are a variety of ways in which these added channels can be introduced. They may be multiple along track beams, or a combination of along track and along range beams.

The multiple along track channel case is analyzed in Section 1.0. It is referred to as Case I. The multiple along range case is analyzed in Section 2.0. It is referred to as Case II.

1.0 TECHNICAL DISCUSSION OF MULTIPLE ALONG-TRACK CHANNELS

Section 1.0 considers the case of a synthetic aperture radar in which multiple channels in the along track direction are introduced.

The corresponding analysis for multiple beams in the along track range direction is given in Section 2.0.

Much of the analysis of Sections 1.1 to 1.7 is analogous to that used by the author in two papers analyzing the properties of synthetic aperture sonars.[1,2]

1.1 PRELIMINARY SIGNAL TO NOISE RATIO COMBINATIONS

The signal to noise ratio at the output of a single channel radar using both pulse compressio and synthetic aperture generation is given as

$$\left(\frac{\hat{S}}{N}\right)_{out} = \left[\frac{\hat{P}_T G_T \sigma A_{rec}}{(4\pi)^2 R^4 k T_N B}\right]\left[\frac{\tau_i}{\tau_o}\right]\left[\frac{prf\ R\lambda}{2\delta_a V}\right] \tag{1}$$

In equation (1), the first factor gives the usual radar range signal to noise ratio expressio . the second factor gives the improvement in signal to noise ratio due to pulse compression, while the third factor gives the improvement in signal to noise ratio due to synthetic aperture generation.

It is useful to introduce the following relations into equation (1)

$$\left.\begin{aligned} G_T &= \frac{4\pi H D}{\lambda^2} \\ A_{rec} &= H D \\ P_{ave} &= \hat{P}_T \tau_i\ prf \\ \theta_E &= \frac{\lambda}{H}\ ;\ \theta_B = \lambda/D \\ \beta\tau_o &\simeq 1 \end{aligned}\right\} \tag{2}$$

Combination of equations (1) and (2) gives

$$\frac{\hat{S}}{N} = \frac{P_{ave} D^2 \sigma \lambda}{8\pi R^3 kT_N \theta_E^2 \delta_a V} \qquad (3)$$

At this point parameter values in equation (3) are unconstrained. The nature of the constraints is introduced in subsequent sections. When the constraints have been determined, they will be introduced into equation 3.

The definition of quantities used in all equations is given in a glossary of terms. Equations 1,2, and 3 have been previously derived by the author in reference 3.

1.2 SYSTEM GEOMETRY

The geometry used is shown in Figure 1 for the flat earth case. This is done for simplicity only.

In Figure 1, the following relations apply. The quantities h, θ, and W are assumed to be the independent variables.

$$\begin{aligned} d &= h \cot\theta \\ R_{min} &= \sqrt{h^2 + d^2} \\ R_{max} &= \sqrt{h^2 + (d + W)^2} \\ \frac{\sin\theta_E}{W} &= \frac{\sin\theta}{R_{max}} = \frac{\sin E}{R_{min}} \\ \theta &= \theta_E + E \\ W &= (R_{max} - R_{min}) \frac{\cos\theta_E/2}{\cos(\theta - \theta_E/2)} \end{aligned} \qquad (4)$$

Figure 2 gives a plot of $W/(R_{max} - R_{min})$.

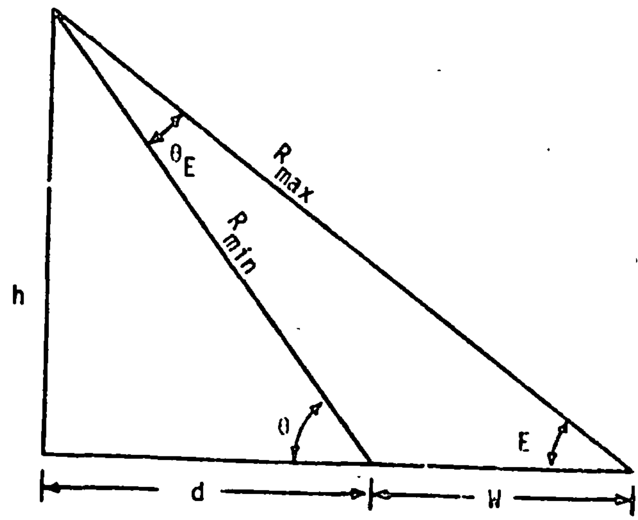

Figure 1. Geometry (flat earth approximation)

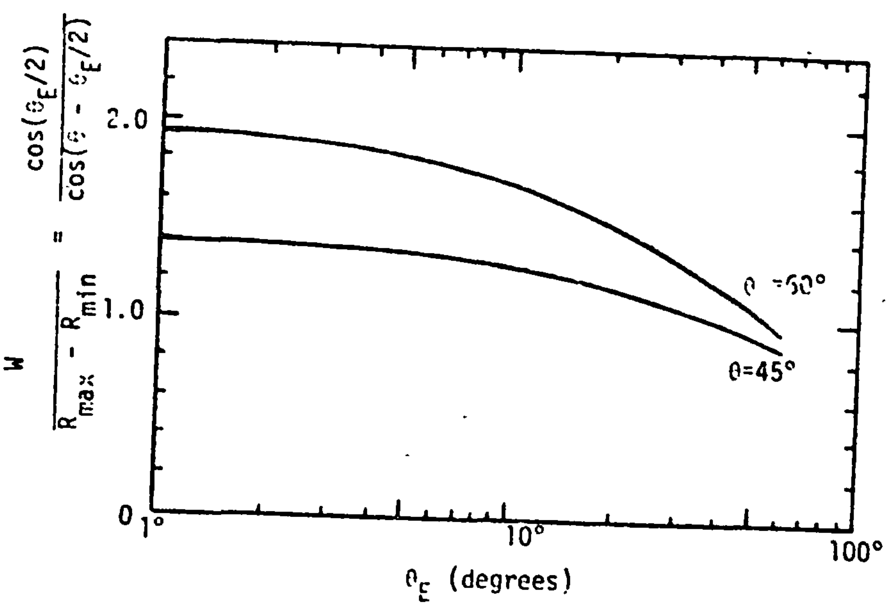

Figure 2. Plot of $W/(R_{max} - R_{min})$ vs θ_E

1.3 RANGE SWATH SELECTION, RANGE AMBIGUITY AVOIDANCE, AND RANGE RESOLUTION

For the geometry of Figure 1, one first selects W, the desired swath width on the ground. Together with h, and θ, this determines all the geometric parameters in Figure 1 using equations 4.

From the last of equations 4, the quantity W is related to $(R_{max} - R_{min})$.

The unambiguous range, R_u, may be chosen to have any value greater than $R_{max} - R_{min}$, i.e.,

$$\left.\begin{array}{l} \beta R_u = (R_{max} - R_{min}) \\ \beta \leq 1 \end{array}\right\} \qquad (5)$$

To prevent eclipsing one needs also

$$\left.\begin{array}{ll} \frac{2}{c} R_{max} \leq M T & (6) \\ \frac{2}{c} R_{min} \geq (M - 1) T + \tau_i & \end{array}\right\}$$

where M is an integer.

These relations are illustrated in Figure 3.

The quantities R_u, T, and prf are related by the expressions

$$R_u = \frac{cT}{2} = \frac{c}{2\ prf} \qquad (7)$$

One additional requirement is necessary to avoid range ambiguities; namely: potentially ambiguous ranges must receive limited illumination by tailoring the elevation pattern of the beam so that θ_E is given by the third of equations 4 and, hence, H, the vertical antenna aperture, is given by

$$H = \lambda/\theta_E \qquad (8)$$

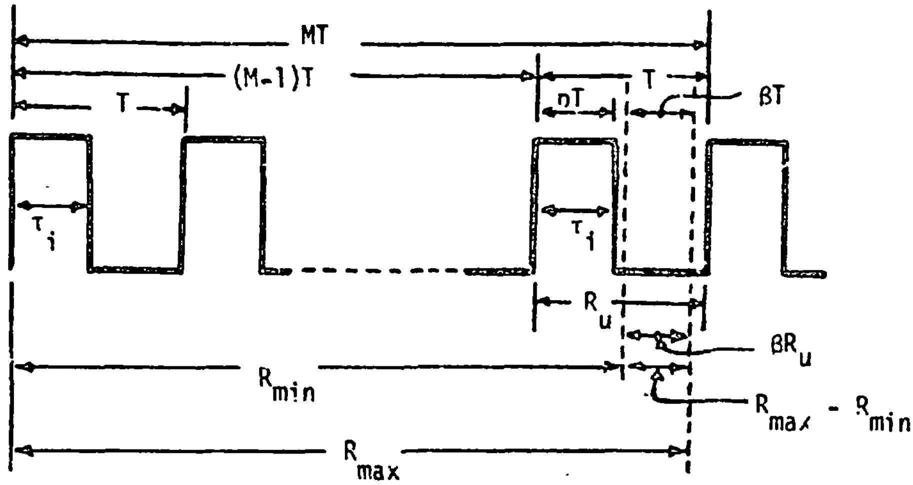

Figure 3. Relations among R_{max}, R_{min}, T, prf, and τ_i.

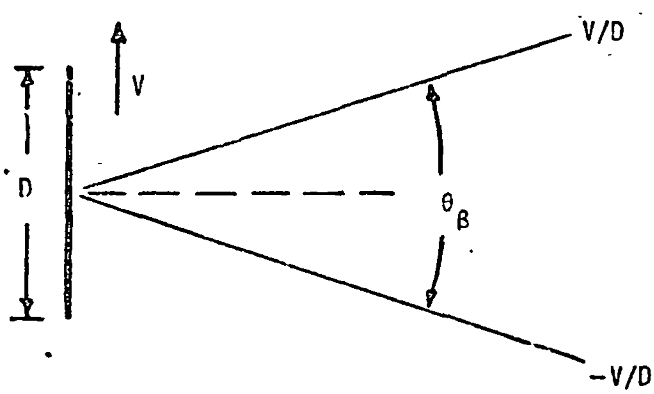

Figure 4. Doppler frequency shift for a moving antenna.

Having selected the values for W, R_u and λ, the quantities R_u, T, prf and H are now determined.

Range resolution, δ_r, of course depends on the radar bandwidth B in accordance with the relations

$$\delta_r = \frac{c}{2B} = \frac{c\,\tau_o}{2} \quad (9)$$

1.4 <u>AZIMUTH AMBIGUITY AVOIDANCE</u>

It can be shown easily that given an antenna with horizontal aperture D, moving with velocity V in a direction parallel to D generates a Doppler frequency shift, f_d, given by

$$f_d = \pm V/D \quad (10)$$

at its three db beam width points. This is illustrated in Figure 4.

Since the radar systems of concern are sampled data systems, one needs to sample the signals in the antenna beam at a rate at least twice the value given by equation (10). Hence

$$\text{prf} = 2\gamma \frac{V}{D}$$
$$\gamma \gtrsim 1 \quad (11)$$

Recall however, that prf has been set by choice of R_u. Also V is a parameter whose value is set by primarily non-radar consideration. Thus equation (11) is really a constraint on the value of D - namely:

$$D = \frac{2\gamma V}{\text{prf}} = \frac{4\gamma V}{c} R_u \gtrsim \frac{4V}{c} R_u \quad (12)$$

In writing the second form of equation (12), use was made of equation (7).

A useful relation is obtained by solving equation (12) for $R_u V$. This quantity is the area rate mapped in the slant range plane. If $\dot{A}$ represents this quantity, one has

$$\dot{A} = R_u V = \frac{c}{4\gamma} D \leq \frac{c}{4} D \qquad (13)$$

Thus the area which can be mapped per second is determined only by D and γ.

Choice of the value of D according to equation (12) prevents azimuth ambiguities if the sidelobe levels of the antenna are properly kept below some acceptable level.

1.5 ACHIEVEMENT OF ALONG TRACK RESOLUTION

From equations 12 or 13 one notes that the value of D is set by the selection of swath width, or by the area rate desired. In some cases the value of D resulting from these considerations may be greater than $2\delta_a$, where δ_a is desired along track resolution.

In such cases, the use of a single beam cannot achieve the desired resolution, because the width of the segment illuminated by the radar is too short.[4] However, there is no reason why one cannot use multiple beams. What one needs to do is illuminate a segment whose length is at least that needed to achieve the synthetic aperture length one needs.

The length of synthetic aperture needed (non-squint case) is given by

$$L_{SAR} = \frac{R\lambda}{2\delta_a} \qquad (14)$$

Given an antenna aperture of horizontal aperture, D, one can form n beams using a technique such as that used in a Butler Matrix so that the same aperture is used to form the n beams. In this case the segment illuminated at range R is

$$L_I = \frac{n\lambda R}{D} \qquad (15)$$

One needs to choose n so that

$$\left.\begin{array}{c} L_{SAR} = \alpha\ L_I \\ \alpha \lessapprox 1 \end{array}\right\} \qquad (16)$$

The multiple beam configuration is illustrated in Figure 5.
Combination of equations 14, 15, and 16 gives

$$n = \left[\frac{D}{2\alpha\delta_a}\right]_{GE} \qquad (17)$$

Equation (17) gives the number of beams necessary. In equation 17, the symbol $[x]_{GE}$ is to be interpreted as the smallest integer greater than or equal to x.

The first of equations (6) with the equal sign chosen can be written as

$$R_{max} = M\frac{cT}{2} = M\ R_u \qquad (18)$$

Thus M may be interpreted as the number of pulses simultaneously in transit during radar opertion.

Use of equations 12 and 18 in equation 17 gives

$$n\ M = \frac{1}{\alpha}\left(\frac{V2R_{max}}{c\ \delta_a}\right) \qquad (19)$$

Thus the product of the number of beams by number of pulses under way simultaneously is given by the right hand side of equation 19.

The quantity of $2R_{max}/c$ is the time required for a radar signal to traverse the path from radar to target and then back to the target. Multiplication of this time by V gives the distance traversed by the radar during this round-trip interval. This distance divided by δ_a gives the product n M needed except for the factors γ and α which relate to an oversampling factor and over illumination factor respectively.

1.6 SIGNAL TO NOISE RATIO WITH CONSTRAINTS

In writing the equations leading to equation (3) a single radiated beam was assumed so that P_{ave} in this equation is the average power in a single radiated beam. Since n beams are required, with n given by equation (17), the total average power, P_{Total}, is given by solving equation 3 for P_{ave}, and then multiplying this quantity by n. One has

$$P_{ave} = \left(\frac{\hat{S}}{N}\right) \left[\frac{8\pi R^3 k T_N \theta_E^2 \delta_a V}{D^2 \sigma \lambda}\right] \qquad (20)$$

and

$$\left.\begin{aligned} P_{Total} &= \left(\frac{\hat{S}}{N}\right) \left[\frac{8\pi R^3 k T_N \theta_E^2 \delta_a V}{D^2 \sigma \lambda}\right] \left[\frac{D}{2\alpha \delta_a}\right] \\ &= \left(\frac{\hat{S}}{N}\right) \left[\frac{4\pi R^3 k T_N \theta_E^2}{\alpha \sigma \lambda}\right] \left[\frac{V}{D}\right] \end{aligned}\right\} \qquad (21)$$

From equations 13 and 18 several equivalent expressions for V/D may be obtained: namely

$$\frac{V}{D} = \frac{c}{4\gamma R_u} = \frac{c M}{4\gamma R_{max}} \qquad (22)$$

Combination of equations 21 and 22 gives

$$P_{Total} = \left(\frac{\hat{S}}{N}\right) \left[\frac{4\pi R^3 k T_N \theta_E^2}{\alpha \sigma \gamma}\right] \left[\frac{c M}{4\gamma R_{max}}\right] \qquad (23)$$

It will be noted that equation (23), evaluated at maximum range, predicts that the average power required varies as the square of maximum range.

1.7 ANTENNA CONSIDERATIONS

It has been shown in Section 1.2 that

$$\sin \theta_E = \frac{W \sin\theta}{R_{max}} \tag{24}$$

and that

$$W = (R_{max} - R_{min}) \quad \frac{\cos \theta_{E/2}}{\cos(\theta - \theta_{E/2})} \tag{25}$$

If it is assumed that θ_E is sufficiently small so that

$$\sin \theta_E \approx \theta_E \tag{26}$$

then combination of equations 4,8, and 26 gives

$$H = \frac{\lambda R_{max}}{W \sin \theta} \tag{27}$$

One may rewrite equation 25 as follows:

$$\frac{W}{R_u} = \frac{\beta W}{(R_{max} - R_{min})} = \frac{\beta \cos (\theta_{E/2})}{\cos (\theta - \theta_{E/2})} \tag{28}$$

Substitution of W from equation 28 into equation 27 gives

$$H = \left[\frac{\lambda R_{max}}{\sin \theta}\right] \left[\frac{\cos (\theta - \theta_{E/2})}{R_u \beta \cos (\theta_{E/2})}\right] \tag{29}$$

The area, A, of the antenna can be obtained by multiplying equation 20 by equation 12. The result is

$$A = H L = \left[\frac{\lambda R_{max}}{\sin \theta}\right] \left[\frac{\cos (\theta - \theta_{E/2})}{\cos (\theta_{E/2})}\right] \left[\frac{4\gamma V}{c}\right] \tag{30}$$

Since $\beta \leq 1$ and $\gamma \leq 1$, equation 30 shows that there is a minimum antenna area required.

One also recalls with respect to antenna requirements that D is given by equation 12 and that the number of beams required is given by equation 17.

2.0 THE MULTIPLE RANGE CHANNEL CASE

In section 3 the case in which the multiple channels are in the range coordinate is analyzed.

One starts by considering a single elevation channel with the value of θ_E' at first unspecified. As the analysis proceeds, one will need to provide multiple elevation channels.

Quantitites whose values differ in Case II from their values in Case I are designated by use of a prime on the appropriate symbol.

2.1 PRELIMINARY SIGNAL TO NOISE RATIO FOR CASE II

Equation (1) for the single beam case applies to the multiple range channel case. Equations (2) also apply. For this case one gets

$$\left(\frac{\hat{S}}{N}\right) = \left[\frac{P_{ave}\ c\ \lambda}{8\pi\ R^3\ k\ T_N\ \delta_a\ V}\right]\left[\frac{(H')^2\ (D')^2}{\lambda^2}\right] \qquad (31)$$

2.2 SYSTEM GEOMETRY

The system geometry in Figure 1 applies also for Case II as do also equations (4) except that the elevation angle, θ_E', will eventually be divided into a number of elevation channels which will span different range intervals. (See Figure 6).

One assumes for this case that h, and R_{max}, are independent variables. The third quantity to solve the geometry is specified later.

2.3 ACHEIVEMENT OF ALONG TRACK RESOLUTION

Given a desired resolution, δ_a, the required value for D is given[4] by

$$D' = 2\alpha\delta_a \qquad \alpha \leq 1 \tag{32}$$

2.4 AVOIDANCE OF ALONG TRACK AMBIGUITIES AND ACHIEVEMENT OF DESIRED SWATH WIDTH

Equation 11 applies to Case II. However, in this case, D has been specified by equation 32 so that equation 11 becomes a specification on prf. This, of course, also specifies T′ and R_u' - namely:

$$\left.\begin{aligned} T' &= \frac{1}{prf'} \\ R_u' &= \frac{c}{2\ prf'} = \frac{c}{4\gamma V}D = \frac{c2\delta_a}{4\gamma V} \end{aligned}\right\} \tag{33}$$

Comparison of equation 17 with equation 32 shows that

$$D = n D'$$

Hence (34)

$$R_u = n R_u'$$

and

$$W = n W' \tag{34}$$

Thus n beams in elevation are needed to span the desired swath width W.

Equation 27 is valid for Case II. Combination of this equation with equation 35 gives

$$H' = \frac{\lambda R_{max}}{W' \sin\theta'} = \frac{n\lambda R_{max}}{W\sin\theta'} \tag{36}$$

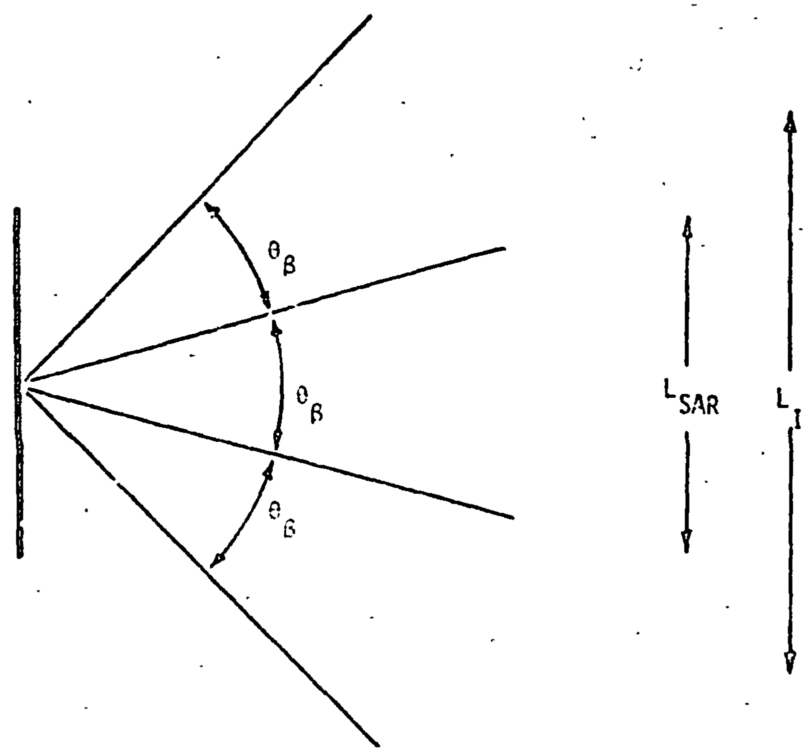

Figure 5. Use of multiple beams to illuminate the synthetic aperture length.

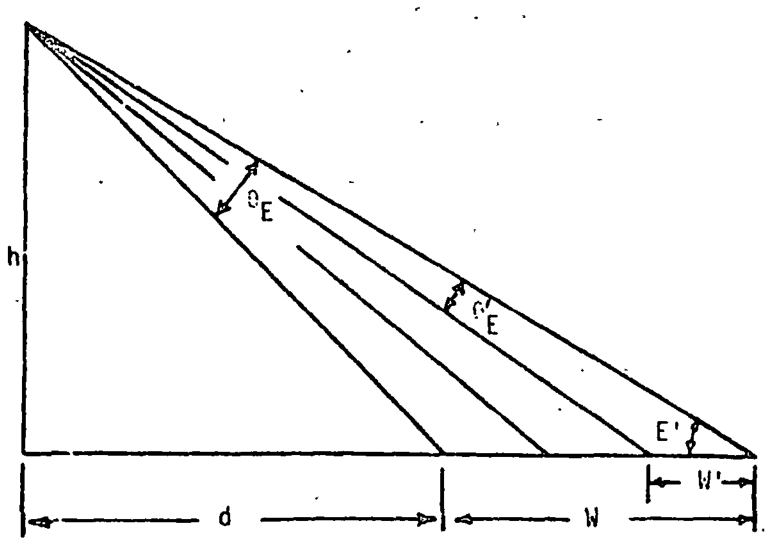

Figure 6. Partition of θ_E into range channels θ_E' .

As an approximation, let θ be considered as being approximately equal to θ'.

Comparison of (36) with equation 27 gives

$$H' = n\,H \qquad (37)$$

Choice of δ_a specifies D', and R_u'. Choice of θ_E (or W) specifies n. Choice of system geometry specifies H'.

It will be noted from equations 52, 53, and 54, that for Case II, the value of H' is n times the value of H for Case I, but that D' is 1/n the value of D such that the antenna areas remain constant.

The number of channels n for Case I and II are equal. In Case I, the channels are in the along track direction; in Case II, they are in the along range direction.

For both Cases I and II, there is a minimum antenna area required. This area is proportional to wavelength, λ, maximum range, R_{max}, and satellite speed V. It also depends on geometric factors as shown in equation 49.

The total average power required is the same for both Cases I and II.

From equation 47, one notes that the total average power, P_{Total}, required varies directly as the square of θ_E, and directly as M, directly with the noise temperature T_N, inversely as the target cross section σ.

The dependence of average power with range is as range cubed. However, at maximum range it varies as the square of range. This latter behavior is due to the fact that antenna area must be made proportional to R_{max}.

The analysis above has been based on a flat earth approximation only for the simplicity of a first analysis. It is realized that the actual geometry needs to be considered. This will be done at a later date.

2.5 SIGNAL TO NOISE RATIO WITH CONSTRAINTS FOR CASE II

Equation (31) gives the signal to noise for Case II in the absence of constraints. The important constraints for Case II are given by equations 34 and 37. If one forms the product $H'D'$, one gets using these equations

$$H'D' = (n\,H)\left(\frac{D}{n}\right) = H\,D \qquad (38)$$

Use of equation 38 in equation 31 shows that

$$\frac{\hat{S}}{N}_{\text{Case II}} = \frac{\hat{S}}{N}_{\text{Case I}} \qquad (39)$$

Thus the signal to noise ratio for Case II is identical to that for Case I.

For Case II, one needs the same average power as for Case I.

In Case I, n beams are used in the along track case and D is given by equation 12. For Case II n beams along the range direction are used.

In Case I, the value of D is n times greater than the value of D'.

In Case II, the value of H' is n times larger than H.

In both cases the area of the antenna has the same value, - namely: that given by equation 30.

3.0 SUMMARY OF RESULTS

The major results of these analyses are the constraints on parameter values and the effects of these constraints on determining the average power required.

For Case I, the primary resluts are those given by equations 7, 12, 13, 14, 17, 18, 19, 23, 29, and 30. These are repeated on the next page.

$$\text{prf} = \frac{c}{2R_u} \tag{40}$$

$$D = \frac{4\gamma V}{c} R_u \tag{41}$$

$$\dot{A} = R_u V = \frac{c}{4\gamma} D \tag{42}$$

$$L_{SAR} = \frac{R\lambda}{2\delta_a} \tag{43}$$

$$n = \left[\frac{D}{2\alpha\delta_a}\right]_{GE} = \left[\frac{2\gamma V R_u}{\alpha c \delta_a}\right] \tag{44}$$

$$R_{max} = MR_u \tag{45}$$

$$n\ M = \frac{\gamma}{\alpha}\left(\frac{2R_{max}}{c\ \delta}\ V\right) \tag{46}$$

$$P_{Total} = \left(\frac{\hat{S}}{N}\right)\left[\frac{4\pi R^3 k T_N \theta_E{}^2}{\alpha\sigma\lambda}\right]\left[\frac{cM}{4\gamma R_{max}}\right] \tag{47}$$

$$H = \frac{1}{\beta}\left(\frac{\cos(\theta-\theta_E/2)}{\cos(\theta_E/2)}\right)\left(\frac{\lambda R_{max}}{\sin\theta R_u}\right) \tag{48}$$

$$A = HD = 4\left(\frac{\gamma}{\beta}\right)\left(\frac{V}{c}\right)\left(\frac{\cos(\theta-\theta_E/2)}{\cos(\theta_E/2)}\right)\left(\frac{\lambda R_{max}}{\sin\theta}\right) \tag{49}$$

For Case II the primary results are given by equations 32, 33, 34, 37, 38, and 39. These are repeated below

$$D' = 2\,\alpha\,\delta_a \qquad (50)$$

$$\alpha \leq 1$$

$$R_u = \frac{c}{2prf'} = \frac{c}{4\gamma V} D' = \frac{c\,\delta_a}{2\gamma V} \qquad (51)$$

$$D' = D/n \qquad (52)$$

$$H' - n\,H \qquad (53)$$

$$H'D' = H\,D = A \qquad (54)$$

$$\left(\frac{\hat{S}}{N}\right)_{II} = \left(\frac{\hat{S}}{N}\right)_{I} \qquad (55)$$

For Case I, it will be noted that R_u and δ_a are chosen independently. For Case I choice of R_u leads to the specification of prf, D, and $\dot{A}$. Choice of δ_a then leads to a specification of L_{SAR} and n.

Choice of R_{max} and the system geometry leads to the specification of M, H, and A.

All of these constraints lead to the expression for total average power required.

For Case II, R_{max}, δ_a, and θ_E (or its equivalents such as W) are chosen independently.

GLOSSARY OF TERMS

A, A_{rec}	Area of Receiving Antenna
$\dot{A}$	Area mapped per second in slant range plane
B	Receiver Bandwidth
c	Speed of propagation of radar signals
D	Horizontal antenna aperture
d	See Figure 1
E	Elevation angle (see Figure 1)
f_d	Doppler frequency shift (see eq. 10)
G_T	Gain of Transmitting antenna
H	Vertical Antenna aperture
h	Altitude of radar above earth
k	Boltzmann's constant
L_{SAR}	Length of synthetic aperture
L_I	Length of illuminated segment
M	Number of pulses under way (see eq. 6 and 18)
N	Noise power in radar receiver
n	Number of radar channels
P_T	Peak transmitter power (one channel)
P_{ave}	Average transmitter power (one channel)
P_{Total}	Total average transmitter power (n channels)
R	Radar Range
R_{min}	Minimum Radar Range
R_{max}	Maximum Radar Range
R_u	Unambiguous Radar Range
S	Signal Power at Radar Output
T	Interpulse period
T_N	Receiver Noise temperature
V	Speed of translation of radar
W	Swath width

GLOSSARY OF TERMS (cont'd)

α	Constant (see eq. 16)
β	constant (see eq. 5)
γ	Constant (see eq. 11)
δ_a	Synthetic Aperture Resolution
δ_r	Range Resolution
λ	Radar wavelength
σ	Target cross-sectional area
τ_i	Duration of uncompressed pulse
τ_o	Duration of compressed pulse
θ	Elevation Angle (see Figure 1)
θ_E	Elevation Beamwidth (see Figure 1)

SPECIAL SYMBOL

$[x]_{GE}$ Signifies smallest integer greater than or equal to x.

4.0 REFERENCES

1. Cutrona, L. J., "Comparison of Sonar System Performance Achievable Using Synthetic Aperture Technique With Performance Achievable by More Conventional Means," J. Acoust. Soc. Am., 58, 336-348 (1975).

2. Cutrona, L. J., "Additional Characteristics of Synthetic Aperture Sonar Systems and a Further Comparison With Non Synthetic Aperture Sonar Systems," J. Acoust. Soc. Am., 61, 1213-1217 (1977).

3. Cutrona, L. J., "Synthetic Aperture Radar," in Radar Handbook, M. Skolnik Ed. (McGraw-Hill, New York, 1970) Chapter 23.

4. Cutrona, L. J., W. E. Vivian, E. N. Leith and G. O. Hall, "A High Resolution Radar Combat Surveillance System," I.R.E. Trans. Mil. Electron 5, 127-130 (April 1961).

CAPTIONS FOR FIGURES

Figure 1. Geometry (Flat Earth Approximation)

Figure 2. Plot of $W/(R_{max} - R_{min})$ vs θ_E

Figure 3. Relations among R_{max}, R_{min}, T, prf, and τ_i.

Figure 4. Doppler Frequency Shift For a Moving Antenna

Figure 5. Use of Multiple Beams to Illuminate the Synthetic Aperture Length.

Figure 6. Partition of θ_E, into Range Channels θ_E.