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THE SPEED OF LIGHT AS MEASURED BY

TWO TERRESTRIAL STABLE CLOCKS

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The Speed of Light as Measured by Two Terrestrial Stable Clocks*

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It is shown that despite the recent criticism within the special theory of relativity there exists an arrangement of stable clocks rotating with the earth which predicts diurnal variations of the one-way speed of light, as suggested in our previous paper.

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In our previous paper[1] we suggested the possibility that within the framework of special relativity the one-way speed of light signals measured by two stable clocks will vary due to the rotational motion of the earth. We made use of the statement: 'Since clocks A and B are assumed to move in an identical manner, they should continue to show simultaneity with respect to the (original) f-frame concept of time'. This assumption and the subsequent conclusions derived from it are criticized by Grøn[2] who claims they are false. Grøn has examined the rotational motion of the clocks A and B and has shown that if the clocks are separated by a distance $R\Delta\theta$ and are located on the equator in such a way that they are the same distance from the center, O, of the earth (i.e., OA = OB = R), then they do not move in an identical manner, from the f-frame's point of view. The f-frame was that in which the two clocks were initially synchronized. Thus, to the accuracy stated in our first paper the clocks A and B will yield, in particular, no diurnal variation in the one-way speed of light.

Grøn's result is correct. However, it must be stressed that his result hinges crucially on the set-up involved, namely that the two clocks A and B are situated at precisely the same distance from the axis of rotation.

The point of our paper[1] was to establish the diurnal variation of the one-way speed of light. Grøn has criticized the particular arrangement used to illustrate this variation. We agree with Grøn that two clocks fixed on the earth do not move in an identical manner with respect to an instantaneous rest frame. However, we disagree with the general statement that special relativity theory allows no diurnal variations, to first order in β, in the value of the one-way speed of light as measured on earth. We shall describe a different set-up in which two spatially separated clocks A and B are at rest in the same instantaneous rest frame to first order in β. One-half a day later these clocks read the simultaneity of the original f-frame while they are at rest (to first order in β) in a different frame f'. Therefore, one will measure a diurnal variation for the one-way speed of light signals, as suggested in our previous paper.[1] This may seem surprising in view of Grøn's result; however, as we stressed above, his result was derived for one particular configuration only.

In contrast to Grøn's treatment, we consider a more general case by allowing $OA \neq OB$ for the clocks A and B. We choose OA and OB to differ by an amount of the order of magnitude of the spatial separation of A and B

$$OA = OB + E, \qquad E \sim R\Delta\theta^{o} \ll R,$$

where R is the equatorial radius. We derive the exact expressions, rather than an approximate expression, for the accumulated time of each of clocks A and B from the f-frame's point of view. Our approximations in β and $\Delta\theta^{o} \sim E/R$ will be made at the end of the calculation. In the case of OA = OB and to first order in β, our result agrees with Grøn's approximate solution. If we go to higher orders, the result does not follow, as then the spatial separation between A and B is such that to the required order clocks A and B are not at rest in the same inertial frame. The result is true at higher orders if we choose the spatial separation ($\sim R\Delta\theta^{o}$) sufficiently small.

Because of the possible experimental test of the diurnal variation of the one-way speed of light in the future, we concentrate here upon the diurnal motion of the separated clocks. We choose the clock B to be on the equator, on the earth at a distance R (the equatorial radius) from the axis of rotation. The clock A, on the other hand, is behind B, in the sense of the rotation of the earth, on the equator, at a distance $OA = R_A > R$ from the axis of rotation. As in references 1 and 2, we choose A to be at the origin of the instantaneous f-frame at the initial time $t = 0$. The f^0 - frame is at rest relative to the axis of rotation of the earth and is coincident with the f-frame at $t^0 = t = 0$. The +y-axis is along the direction of motion of the clock A, and the +x-axis is in the radial direction. Thus, the f-frame moves with a velocity $v_1 = R_A\omega^0$ along the y^0-axis of the f^0 frame, where ω^0 is the rotational speed of the earth and is a constant in the f^0-frame. The Lorentz transformations between the inertial frames f and f^0 are thus

$$\begin{aligned} y &= \gamma_1 (y^0 - \beta_1 c t^0), \\ x &= x^0, \\ ct &= \gamma_1 (c t^0 - \beta_1 y^0), \end{aligned} \tag{1}$$

where, as usual, $\beta_1 = v_1/c$ and $\gamma_1 = (1 - \beta_1^2)^{-1/2}$.

The frame f' will be the instantaneous rest frame of clock A, after clock A has described one-half a revolution according to the f^0 frame. We choose this frame with that

at $y^0 = 0$, $t^0 = 0$, $y' = 0$, $t' = 0$

but $x' = x^0 + 2R_A$.

Then the Lorentz transformation from the f^0 frame to the f' - frame is

$$\begin{aligned} y' &= \gamma_1 (y^0 + \beta_1 c t^0), \\ ct' &= \gamma_1 (c t^0 + \beta_1 y^0). \end{aligned} \tag{2}$$

We note in passing that the transformation from the inertial frame f to the inertial frame f' is

$$y' = \hat{\gamma}\,(y + \hat{\beta}\,ct)$$
$$ct' = \hat{\gamma}\,(ct + \hat{\beta}\,y) \tag{3}$$
$$x' = x + 2R_A$$

where $\hat{\beta} = 2\beta_1/(1+\beta_1^2)$ and $\hat{\gamma} = (1-\hat{\beta}^2)^{-1/2}$.

In the f^0-frame, the rotational motion of the clock A is given by[2]

$$x^0 = -R_A(1 - \cos\theta_A^0), \tag{4}$$
$$y^0 = R_A \sin\theta_A^0,$$

where $\theta_A^0 = \omega^0 t^0$. Clearly the clock moves in the x^0-y^0 plane. Its velocity, at a later time $t^0 > 0$, has components

$$dy^0/dt^0 = V_1\cos\theta_A^0, \qquad dx^0/dt^0 = -V_1\sin\theta_A^0.$$

In the f-frame, the motion of A is also in the x-y plane and its components are, using (1) and (4),

$$V_{Ay}(t) = \frac{dy}{dt} = \frac{V_1(\cos\theta_A^0 - 1)}{1 - \beta_1^2\cos\theta_A^0}, \tag{5}$$

$$V_{Ax}(t) = \frac{dx}{dt} = \frac{-V_1\sin\theta_A^0}{\gamma_1(1-\beta_1^2\cos\theta_A^0)}.$$

Then, for the magnitude of this velocity we have

$$V_A^2(t) = V_1^2\left(2 - 2\cos\theta_A^0 - \beta_1^2\sin^2\theta_A^0\right)/\left(1-\beta_1^2\cos\theta_A^0\right)^2.$$

This in turn gives us

$$\left(1 - V_A^2(t)/c^2\right)^{1/2} = \gamma_1^{-2}\left(1 - \beta_1^2\cos\theta_A^0\right)^{-1}.$$

The clocks A and B are assumed to be initially synchronized from the f-frame's point of view, i.e., at t = 0, and so in the f^0 — frame since they are spatially separated they are not synchronized. If $\Delta\theta^\circ$ is the angle subtended at O by the radii OA and OB, then initially the clock B has coordinates in the f^0 — frame

$$
\begin{aligned}
y^\circ &= R_B \sin \Delta\theta^\circ \\
x^\circ &= R_B \cos \Delta\theta^\circ - R_A \\
t^\circ &= (\beta_1 R_B \sin \Delta\theta^\circ)/c \equiv t_i^\circ \quad ;
\end{aligned}
\tag{6}
$$

at later times, in the f^0 - frame, the motion of clock B is given by

$$
\begin{aligned}
x^\circ &= R_B \cos \theta_B^\circ - R_A , \\
y^\circ &= R_B \sin \theta_B^\circ ,
\end{aligned}
\tag{7}
$$

where $\theta_B^\circ = \omega^\circ(t^\circ - t_i^\circ) + \Delta\theta^\circ$. In the f-frame, the velocity of clock B will have components

$$
\begin{aligned}
V_{Bx}(t) &= \frac{dx}{dt} = - \frac{V_2 \sin \theta_B^\circ}{\gamma_1 (1 - \beta_1 \beta_2 \cos \theta_B^\circ)} , \\
V_{By}(t) &= \frac{dy}{dt} = \frac{V_2 \cos \theta_B^\circ - V_1}{1 - \beta_1 \beta_2 \cos \theta_B^\circ} ,
\end{aligned}
\tag{8}
$$

where $\beta_2 = V_2/c = R_B \omega^\circ/c$. Then we find for clock B

$$
(1 - V_B^2(t)/c^2)^{1/2} = (\gamma_1 \gamma_2)^{-1} (1 - \beta_1 \beta_2 \cos \theta_B^\circ)^{-1} . \tag{9}
$$

From equations (5) and (8) we see that, to first order in the small quantities $\Delta\theta^{0}$, β_1 and β_2, the clocks A and B are at rest in the same instantaneous inertial frame initially and one-half a day later. Explicitly we have initially

$$V_{Ax} = V_{Ay} = 0,$$

$$V_{Bx}/c = \beta_2 \Delta\theta^{0}(1 + \tfrac{1}{2}\beta_2^2),$$

$$V_{By}/c = -\tfrac{1}{2}\beta_2 \left[(\Delta\theta^{0})^2 + 2E/R_B \right](1 + \beta_2^2),$$

while at the latter time we have

$$V_{Ax} = 0,$$

$$V_{Ay}/c = -2\beta_1 (1 - \beta_1^2),$$

$$V_{Bx}/c = \beta_2 \Delta\theta^{0}(1 - \tfrac{3}{2}\beta_2^2),$$

$$V_{By}/c = -2\beta_2 (1 - (\Delta\theta^{0})^2/4 + E/2R_B)(1 - \beta_2^2).$$

We are now in a position to evaluate exactly the expressions for the accumulated time of each clock, as seen by an f-observer at a later f-time T, namely

$$\tau_j = \int_0^T dt \, (1 - V_j^2(t)/c^2)^{1/2}, \qquad j = A, B. \tag{10}$$

To evaluate this integral exactly, we should carry out the integration in the f^0-frame. It is straightforward to see that we have

$$\int_0^T dt = \int_0^{U_1} dt^{0} \, \gamma_1 (1 - \beta_1^2 \cos\theta_A^{0}) \tag{11}$$

for clock A, where the upper integration limit is

$$\mathcal{U}_1 \equiv t_A^{\circ}(T) = \gamma_1^{-1} T + \beta_1 y_A^{\circ}(T)/c , \tag{12}$$

while for clock B,

$$\int_0^T dt = \int_{L_2}^{\mathcal{U}_2} dt^{\circ} \gamma_1 (1 - \beta_1 \beta_2 \cos \theta_B^{\circ}) , \tag{13}$$

the upper and lower limits of integration being

$$\mathcal{U}_2 \equiv t_B^{\circ}(T) = \gamma_1^{-1} T + \beta_1 y_B^{\circ}(T)/c ,$$
$$L_2 \equiv t_B^{\circ}(0) = \beta_1 y_B^{\circ}(0)/c = t_i^{\circ} . \tag{14}$$

Then, we find at once the accumulated times

$$\tau_A = \gamma_1^{-1} [\gamma_1^{-1} T + \beta_1 y_A^{\circ}(T)/c] ,$$

$$\tau_B = \gamma_2^{-1} [\gamma_1^{-1} T + \beta_1 y_B^{\circ}(T)/c - t_i^{\circ}] \tag{15}$$

These expressions are exact in contrast to the expressions derived in ref. 2. However, we do not have exact explicit solutions for $y_A^{\circ}(T)$ and $y_B^{\circ}(T)$. One can easily see that these are determined by solving the following transcendental equations

$$y_A^{\circ}(T) = R_A \sin (\omega^{\circ} \gamma_1^{-1} T + \omega^{\circ} \beta_1 y_A^{\circ}(T)/c) ,$$

$$y_B^{\circ}(T) = R_B \sin (\omega^{\circ} \gamma_1^{-1} T + \omega^{\circ} \beta_1 y_B^{\circ}(T)/c - \omega^{\circ} t_i^{\circ} + \Delta\theta^{\circ}) . \tag{16}$$

We shall concentrate on the diurnal case, for which $y^{o}_{A}(\tau)$ is zero (i.e., in the f^{o} frame clock A describes one-half revolution). Thus, from (16) we see that this corresponds to

$$\gamma_1^{-1} T = \pi/\omega^{o} . \qquad (17)$$

The expression we are interested in evaluating is

$$\Delta\tau = \tau_B - \tau_A = (\gamma_2^{-1} - \gamma_1^{-1})\pi/\omega^{o} + \gamma_2^{-1}(\beta_1/c)\left[y^{o}_{B}(\tau) - R_B \sin\Delta\theta^{o}\right] . \qquad (18)$$

This expression for the difference in the accumulated times of clocks B and A , as seen by an f-observer, is to be compared with the difference between the proper f'-clock readings at the same events. We make use of the Lorentz transformations (2) and the solution to (16) for which $y^{o}_{A}(\tau) = 0$ to find

$$\Delta t' = t'_B - t'_A = 2\beta_1\gamma_1 y^{o}_{B}(\tau)/c , \qquad (19)$$

where t'_B corresponds to $t_B = T$ and t'_A corresponds to $t_A = T$.

We now compare (18) and (19) in different cases. First we consider the special case

$$R_A = R_B = R$$

in which case we also have

$$\beta_1 = \beta_2 = \beta , \quad \text{and} \quad \gamma_1 = \gamma_2 = \gamma .$$

Then

$$\Delta\tau = \gamma^{-1}\beta/c\left[y^{o}_{B}(\tau) - R_B \sin\Delta\theta^{o}\right]$$

while $\Delta t' = 2(\beta/c)\gamma\, y_B^0(\tau)$.

Now to lowest order in $\Delta\theta^0$ and to lowest orders in β, the solution of equation (16) gives us

$$y_B^0(\tau) = -R\Delta\theta^0$$

so that to this approximation we have $\Delta\tau = \Delta t'$. The validity of this approximation is for $\Delta\theta^0 \approx 10^{-3}$ or smaller. It is clear, however, that to higher orders of approximation, $\Delta\tau$ and $\Delta t'$ will be different, if we keep A and B fixed distance apart. However, if we choose $\Delta\theta^0 \approx 10^{-6}$ or smaller, then we find that $\Delta\tau = \Delta t'$ to third order in β and to first order in $\Delta\theta^0$ (higher orders do not contribute).

An interesting comment is that we can solve the equation

$$\Delta\tau = \Delta t'$$

for $y_B^0(\tau)$. It gives us

$$y_B^0(\tau) = \gamma^{-1} R \sin\Delta\theta^0 / (\gamma^{-1} - 2\gamma).$$

This cannot be a solution to the transcendental equation (16). However, if we agree to approximate to first order in $\Delta\theta^0$ only (that is, we retain linear terms in $\Delta\theta^0$ only), then this agrees with the corresponding approximate solution to (16).

We turn now to the more general case $R_A \neq R_B$ discussed earlier. We first note that the physical reason for the occurrence of a diurnal variation is that the stable clocks do not agree with the proper clocks of the instantaneous frame f'. The time difference (18) should be different from that of (19) for this to occur. Let us now concentrate on the quantity $\Delta\tau$ of (18). Since $\beta_1 > \beta_2$, the first term in (18) is positive, definite while the second term is negative due to the sign of $y_B^0(\tau)$. Furthermore, an examination of these shows that $\Delta\tau$ has a zero as a function of R_A. So, we can choose R_A such that $\Delta\tau = 0$. In particular, to lowest order we obtain the solution

$$R_A = R_B(1 + 2\Delta\theta^{\circ}/\pi). \tag{20}$$

Here $\Delta\theta^{\circ}$ is, as mentioned, assumed to be of the order of magnitude of 10^{-3} or smaller. This corresponds to $R_B\Delta\theta^{\circ}$ being 10^{4} meters or smaller. With this choice for R_A, the clocks A and B read the simultaneity with respect to the f-frame, when they are to first order in β instantaneously at rest in the f' frame. Thus, $\Delta\tau$ and $\Delta t'$ cannot be the same, except in the uninteresting case $\Delta\theta^{\circ} = 0$.

In the above we have compared the accumulated time difference

$$\Delta\tau = \tau_B - \tau_A$$

with the proper f'-time difference

$$\Delta t' = t'_B - t'_A$$

at an f-instant of time, i.e., $t_A = t_B = T$. We can also carry out the calculation at an f^{0}-instant,

i.e., $t^{0}_A = t^{0}_A = T^{0}$

or at an f'-instant

i.e., $t'_A = t'_B = T'$

In each case the calculation yields qualitatively the same answer, viz.

(I) for the special case $R_A = R_B = R$, we find to first order

$$\Delta\tau = \Delta t'$$

(II) for the general case $R_A \neq R_B$, $R_A = R_B + E$, we find

that $\Delta\tau \neq \Delta t'$.

Thus, the stable clocks A and B, rotating with the earth in the configuration discussed, do not read the simultaneity of the instantaneous proper-rest frame after one-half a revolution, when initially they were synchronized in their instantaneous rest frame. Direct measurement of the one-way speed of light using clocks A and B must yield diurnal variations in the speed of light.

In conclusion, we have shown that Grøn's criticism does not invalidate the suggestion of our earlier paper. In particular, we have demonstrated by means of an exact treatment of the rotational motion of clocks an arrangement of fixed stable clocks which realizes our previous suggestion. In this arrangement the clocks are at different distances from the axis of rotation. Direct measurement of the one-way speed of light using such arrangement will yield a diurnal variation to lowest order in β.

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