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Lunar Tidal Acceleration Obtained From Satellite-Derived Ocean Tide Parameters

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INTRODUCTION

Lambeck [1975] gives the ocean tide, ξ, as

$$\xi_\mu = \sum_{s=0}^{\infty} \sum_{t=0}^{s} \sum_{+}^{-} P_{st}(\sin\phi).$$

$$(C^{\pm}_{st})_\mu \quad \sin[\sigma(\tau) \pm t\lambda + (\varepsilon^{\pm}_{st})_\mu]$$

where the $(C^{\pm}_{st})_\mu$ and $(\varepsilon^{\pm}_{st})_\mu$ are functions of the spherical harmonic expansions of the μ tide component, and $\sigma(\tau)$ represents a linear combination of the solar and lunar angular quantities [Goad, 1977]. Lambeck [1975] also gives for the rate of change of the semi-major axis of the moon due to ocean tides

$$\dot{a} = 2\, k'_{stuv}\ (s-2u+v) \begin{bmatrix}\cos\\ \sin\end{bmatrix} (\varepsilon^{+}_{st})_\mu \,, \quad \begin{matrix}(s-t)\ \text{even}\\ (s-t)\ \text{odd}\end{matrix}$$

where k'_{stuv} is a function of the orbital semi-major axis, inclination, eccentricity, Earth and lunar masses, mean density of the Earth and oceans, the load deformation coefficient, and particularly the coefficient C^{+}_{st} as a linear factor. From elementary celestial mechanics, the secular change in the mean motion, n, of the moon is given by,

$$\dot{n} = -\frac{3}{2}\ \frac{n}{a}\ \dot{a}\ .$$

The critical factor in computing the tidal acceleration $\dot{n}$ of the moon is shown to be the quantity

$$C^{+}_{22}\cos\varepsilon^{+}_{22}$$

and Lambeck [1975] notes that for three different global M_2 ocean tide models this quantity varies by only 10% from the mean of the three. Taking this mean value for the dominant M_2 tide effect, the results by Dietrich for the O_1 tide, and estimating the N_2 tide as proportional to the corresponding tide raising M_2 and N_2 potentials, Lambeck [1975] obtains the estimate

$$\dot{n} = -35 \pm 4 \text{ arc-sec}/(100 \text{ yr})^2$$

for the lunar tidal acceleration if it is assumed that there is no contribution from a solid tide phase lag. He further notes that astronomically obtained values at the time of his publication ranged from -37 to -52 arc-sec/(100 yr)^2. Subsequently, the values estimated from astronomical data $\dot{n}_t = -27.2 \pm 1.7$ and $\dot{n}_t = -26 \pm 2$ have been obtained by Muller [1976] and Morrison and Ward [1976], respectively.

Artificial satellites can help resolve this problem because results obtained from them are independent of lunar or planetary data and do not require knowledge of any tidal mechanisms. However, until very recently there were no reliable estimates of M_2 ocean tide parameters from satellite orbit perturbations.

Previously, we published a constraint equation on the M_2 ocean tide coefficients that was in reasonable agreement with numerical ocean tide models [Goad and Douglas, 1977]. In the present paper we obtain additional equations from an analysis of the evolution of the orbit of GEOS-3 that enable an explicity solution for $C^+_{22} \cos\varepsilon^+_{22}$ for the M_2 tide and hence an estimate for the tidal acceleration of the moon. Our value of (-27.6 arc-sec/(100 yr)^2) is in agreement with the recent solution of Muller [1976] and Morrison and Ward [1975].

THE ORBIT OF GEOS-3

Table 1 gives the orbital specifications of GEOS-3. Osculating orbit elements for 100 successive two-day arcs were obtained from the Doppler tracking done in support of the GEOS-3 program by the Naval Surface Weapons Center (NSWC), Dahlgren, Virginia. The mean elements corresponding to the osculating elements are given in Table 2. All long periodic and secular perturbations are present in these elements. We have analyzed extensively only the mean inclination and node. The precision of the mean inclination is about 0."02. The mean node is less accurate (about 0."04) primarily due to the uncertainty of the prediction of UT1 time used in the data reductions.

Table 3 gives the perturbation on the inclination and node of GEOS-3 computed from our solution for the $(2,2)^+$ and $(4,2)^+$ coefficients. A detailed discussion of the tidal perturbations on satellite orbits is given by Lambeck et al., [1974]. We have used the results of that work for our own computations with the exception of eq. 6, p. 424 for the perturbation of the node. We are unable to verify this equation. Our application of the differential equation for the time rate of change of the node to the tidal potential developed by Lambeck et al. [1974] is

$$\Delta\Omega^{\pm}_{n\ell mpq} = \frac{4\pi(1+k'_\ell)}{2\ell+1} Ga_e\left(\frac{a_e}{a}\right)^{\ell+1} \frac{\rho_w}{Na^2(1-e^2)^{\frac{1}{2}}\sin i} C^{\pm}_{n\ell m} G_{\ell pq}(e)$$

$$\frac{1}{\dot{\gamma}^{\pm}}\left\{\frac{\partial F_{\ell mp}(i)}{\partial i} - \frac{3}{2}N\left(\frac{a_e}{a}\right)^2 \frac{C_{20}\sin i}{(1-e^2)^2} F_{\ell mp}(i)\frac{[(\ell-2p)\cos i - m]}{\dot{\gamma}^{\pm}}\right\}$$

$$\begin{bmatrix} \mp\cos \\ \\ \mp\sin \end{bmatrix}^{\ell-m \text{ even}}_{\ell-m \text{ odd}} \gamma^{\pm}_{n\ell mpq} \quad .$$

Table 1. GEOS-3 Orbital Characteristics

Epoch MJD 42525.0		April 23, 1975 0 hrs.
$\bar{a}$	7219 km	
$\bar{e}$	.0005	
$\bar{i}$	114.°99	
Perigee Altitude		837 km
M_2 Tidal Period		$17.^d2$

Table 2. Mean Keplerian Elements (GEOS-3)

TIME (MJD)	A (METERS)	E	I (DEGREES)	NODE (DEGREES)	PERIGEE (DEGREES)	M (DEGREES)
42525.0	7219574.686	0.0004882	114.99295425	274.48228311	48.61929980	65.50701482
42527.0	7219576.837	0.0004961	114.99299087	279.93371283	47.00311446	173.27369786
42529.0	7219572.559	0.0005015	114.99397525	285.38499338	45.67152893	280.76031555
42531.0	7219576.155	0.0005086	114.99463393	290.83616337	43.98767899	28.59823389
42533.0	7219572.073	0.0005164	114.99562204	296.28746873	42.78774795	135.95201999
42535.0	7219574.238	0.0005218	114.99572488	301.73890939	41.13261257	243.76504650
42537.0	7219572.882	0.0005309	114.99561991	307.19041251	39.92174652	351.13032729
42539.0	7219571.436	0.0005362	114.99518986	312.64196159	38.47385703	98.73922242
42541.0	7219573.570	0.0005459	114.99444588	318.09342861	37.17156580	206.19825897
42543.0	7219569.254	0.0005521	114.99424180	323.54482439	36.01641110	313.51555560
42545.0	7219573.324	0.0005609	114.99391104	328.99608398	34.56906609	61.12367634
42547.0	7219569.026	0.0005689	114.99455323	334.44735744	33.56860079	168.28514994
42549.0	7219571.735	0.0005757	114.99499185	339.89866681	32.09562306	275.92247787
42551.0	7219570.089	0.0005857	114.99546126	345.34998605	31.16074386	23.01769886
42553.0	7219569.198	0.0005918	114.99527682	350.80133848	29.89744636	130.44706980
42555.0	7219571.028	0.0006025	114.99428755	356.25255261	28.85059745	237.65536090
42557.0	7219566.985	0.0006091	114.99347473	1.70360447	27.81741672	344.85485147
42559.0	7219570.864	0.0006186	114.99243843	7.15455516	26.55940009	92.27767135
42561.0	7219566.453	0.0006269	114.99241018	12.60545494	25.74746318	199.25501034
42563.0	7219569.202	0.0006349	114.99232789	18.05637785	24.49012208	306.68052504
42565.0	7219567.228	0.0006449	114.99270061	23.50728037	23.74031043	53.59442345
42567.0	7219566.574	0.0006518	114.99290526	28.95820738	22.59592000	160.90844077
42569.0	7219568.256	0.0006626	114.99266438	34.40898950	21.71228138	267.95642636
42571.0	7219564.737	0.0006697	114.99233141	39.85965449	20.78038771	15.05729127
42573.0	7219568.924	0.0006800	114.99111435	45.31006141	19.73728709	122.26723834
42575.0	7219564.693	0.0006883	114.99029696	50.76041229	19.04384251	229.12854989
42577.0	7219567.736	0.0006972	114.98933636	56.21082932	17.92948451	336.41355882
42581.0	7219565.131	0.0007151	114.98950686	67.11202591	16.18396703	190.49801969
42583.0	7219566.517	0.0007259	114.98968300	72.56250532	15.45389532	297.39468011
42585.0	7219562.952	0.0007330	114.98984708	78.01280314	14.60623573	44.41364991
42587.0	7219566.869	0.0007433	114.98912932	83.46281261	13.72199271	151.46680914
42589.0	7219562.235	0.0007514	114.98865931	88.91273152	13.06162287	258.29730159
42591.0	7219565.454	0.0007605	114.98766943	94.36260605	12.03005925	5.50110623
42593.0	7219562.910	0.0007699	114.98719724	99.81257147	11.44164026	112.25827146
42595.0	7219563.112	0.0007776	114.98685618	105.26268114	10.46170420	219.41243317
42597.0	7219564.318	0.0007877	114.98677245	110.71287301	9.83150441	326.21181984
42599.0	7219561.383	0.0007950	114.98725815	116.16315081	9.01790348	73.19989689
42601.0	7219565.207	0.0008051	114.98739531	121.61327745	8.22776041	180.16154119
42603.0	7219560.770	0.0008132	114.98784634	127.06322070	7.58006815	286.98216955
42605.0	7219563.903	0.0008226	114.98740563	132.51298673	6.65241007	34.08449447
42607.0	7219561.330	0.0008317	114.98700401	137.96281066	6.12958575	140.77918178
42609.0	7219561.501	0.0008403	114.98643920	143.41290433	5.21237410	247.87388073
42611.0	7219562.539	0.0008504	114.98615356	148.86319629	4.65992834	354.59900950
42613.0	7219559.279	0.0008583	114.98650998	154.31360724	3.84854139	101.58887292
42615.0	7219562.991	0.0008685	114.98672715	159.76391339	3.14780111	208.46499581
42617.0	7219558.319	0.0008763	114.98758191	165.21410779	2.52067087	315.26937053
42619.0	7219561.978	0.0008858	114.98785328	170.66417906	1.70262960	62.26603000
42621.0	7219559.025	0.0008945	114.98834552	176.11419628	1.21418191	168.93067016
42623.0	7219560.054	0.0009032	114.98837432	181.56432407	0.35299262	275.97359388
42625.0	7219560.722	0.0009130	114.98821867	187.01458102	359.85344985	22.65015953
42627.0	7219558.176	0.0009206	114.98816581	192.46518683	359.04945093	129.63712208
42629.0	7219561.490	0.0009303	114.98780705	197.91601624	358.41607347	236.45048199
42631.0	7219556.846	0.0009377	114.98830480	203.36706358	357.78380139	343.26546799
42633.0	7219560.353	0.0009471	114.98873567	208.81800609	357.02878804	90.20469770
42635.0	7219557.094	0.0009553	114.98980396	214.26865890	356.53459240	196.88105583
42637.0	7219557.929	0.0009641	114.99038357	219.71923190	355.70262633	303.90058197
42639.0	7219558.150	0.0009732	114.99056390	225.16986256	355.19230573	50.59392884
42641.0	7219555.384	0.0009812	114.99053621	230.62079550	354.40690277	157.56847399
42643.0	7219558.751	0.0009909	114.98995538	236.07185924	353.83292007	264.32830313
42645.0	7219554.174	0.0009990	114.98992265	241.52302369	353.19614093	11.15390553
42647.0	7219558.299	0.0010088	114.98956967	246.97410239	352.49101388	118.04889695
42649.0	7219554.930	0.0010173	114.98995088	252.42502948	351.96192163	224.76626156
42651.0	7219556.446	0.0010265	114.99025898	257.87584338	351.15693013	331.76445972
42653.0	7219556.207	0.0010356	114.99078111	263.32649908	350.66055450	78.44924646
42655.0	7219553.725	0.0010439	114.99133581	268.77721182	349.89978055	185.40417145
42657.0	7219556.661	0.0010537	114.99115735	274.22795007	349.37340121	292.12061960
42659.0	7219552.157	0.0010617	114.99087588	279.67890073	348.73838121	38.94899783
42661.0	7219556.043	0.0010714	114.98968717	285.12992038	348.06106137	145.82048696
42663.0	7219552.319	0.0010793	114.98917904	290.58082283	347.51165009	252.56324225
42665.0	7219553.789	0.0010883	114.98893281	296.03148204	346.76354688	359.50949362
42667.0	7219553.204	0.0010972	114.98929690	301.48185294	346.30913482	106.15706275
42669.0	7219551.080	0.0011057	114.98985332	306.93219026	345.58549716	213.07952648
42671.0	7219554.080	0.0011155	114.98985824	312.38248331	345.07322279	319.78620356
42673.0	7219549.903	0.0011235	114.98993469	317.83283716	344.42349082	66.63381446
42675.0	7219553.957	0.0011334	114.98910507	323.28311257	343.80368210	173.45141768
42677.0	7219550.087	0.0011415	114.98852222	328.73329415	343.28049751	280.17131915
42679.0	7219551.873	0.0011510	114.98756363	334.18335273	342.59049522	27.06229016
42681.0	7219551.069	0.0011601	114.98700641	339.63322200	342.14420355	133.70484370
42683.0	7219549.171	0.0011687	114.98695552	345.08308744	341.43451214	240.61689648
42685.0	7219551.912	0.0011788	114.98713105	350.53290898	340.92741391	347.32228335
42687.0	7219547.539	0.0011869	114.98803872	355.98283404	340.29028296	94.16155739
42689.0	7219551.499	0.0011969	114.98819341	1.43276068	339.71341705	200.93979288
42691.0	7219547.190	0.0012047	114.98814855	6.88261769	339.21306735	307.64073960
42693.0	7219549.216	0.0012143	114.98720014	12.33238028	338.54803708	54.51068993
42695.0	7219547.822	0.0012233	114.98647234	17.78204173	338.09357953	161.16644274
42697.0	7219546.464	0.0012321	114.98620239	23.23177229	337.40865017	268.05924759
42699.0	7219548.895	0.0012418	114.98617641	28.68150700	336.94194945	14.72975061
42701.0	7219544.721	0.0012493	114.98692577	34.13141758	336.33790408	121.54223468
42703.0	7219548.457	0.0012595	114.98730920	39.58138736	335.79430251	228.29349975
42705.0	7219544.156	0.0012670	114.98810342	45.03142703	335.29408762	335.00125508
42707.0	7219546.349	0.0012769	114.98819128	50.48143750	334.65691241	81.84970786
42709.0	7219544.896	0.0012855	114.98813255	55.93135820	334.22881743	188.48537352
42711.0	7219543.679	0.0012941	114.98773320	61.38128983	333.58350429	295.34494745
42713.0	7219545.967	0.0013038	114.98705096	66.83125635	333.15570082	41.98310880
42715.0	7219541.606	0.0013113	114.98703143	72.28147489	332.58274493	148.77170182
42717.0	7219545.447	0.0013222	114.98704298	77.73198597	332.07016231	255.49896367
42719.0	7219540.818	0.0013298	114.98802941	83.18268858	331.58652204	2.19746060
42721.0	7219543.386	0.0013397	114.98855830	88.63332574	330.99237426	109.00990879
42723.0	7219541.471	0.0013473	114.98895142	94.08371801	330.58333570	215.63368886

Table 3. Amplitude and phase of inclination and node perturbations due to the M_2 ocean tide on GEOS-3 satellite.

Source	Inclination Amplitude	Inclination Phase	Node Amplitude	Node Phase
Observed	0".040	327°	0".027	291°
Pekeris and Accad*	0".051	339°	0".030	352°
Hendershott (Model 1)*	0.061	317°	0".030	286°
Hendershott (Model 2)*	0".065	276°	0".025	244°
Bogdanov and Magarik*	0".050	328°	0.049	290°

* As reported by Lambeck et al. [1974].

The difference appears in the indirect term arising from the interaction with the Earth's oblateness [Kaula, 1966, p. 49].

Note in Table 3 that the ocean tide effects on GEOS-3 are rather different on the inclination and node. The perturbation on the inclination comes almost entirely from the (2,2) term and that of the node from the (4,2) term. Using the methods employed in our previous paper [Goad and Douglas, 1977] to remove known perturbations, we obtain the observation equations for the M_2 ocean tides:

Inclination:

$$(3''.99 \pm 0.4) \times 10^{-2} \sin [\sigma(\tau) + 327° \pm 4°]$$

$$= \frac{1''.26}{\text{cm}} \times 10^{-2} \, C^{+}_{22} \sin [\sigma(\tau) + \varepsilon^{+}_{22}] - \frac{0''.32}{\text{cm}} \times 10^{-2} \, C^{+}_{42} \sin [\sigma(\tau) + \varepsilon^{+}_{42}] + \ldots$$

Node:

$$(2''.73 \pm 0.7) \times 10^{-2} \cos [\sigma(\tau) + 291° \pm 13°]$$

$$= - \frac{0''.24}{\text{cm}} \times 10^{-2} \, C^{+}_{22} \cos [\sigma(\tau) + \varepsilon^{+}_{22}] - \frac{3.38}{\text{cm}} \times 10^{-2} \, C^{+}_{42} \cos [\sigma(\tau) + \varepsilon^{+}_{42}] + \ldots$$

These equations can be used for a GEOS-3 solution alone for $(2,2)^+$ and $(4,2)^+$ quantities because the node and inclination on GEOS-3 have negligible correlation [Anderle, 1977, private communication]. However, a combined solution with 1967-92A yields more precise results. To demonstrate the precision of the GEOS-3 inclination data, we have prepared Figure 1 to show the observed and calculated values of the M_2 ocean tide perturbation of the inclination of GEOS-3.

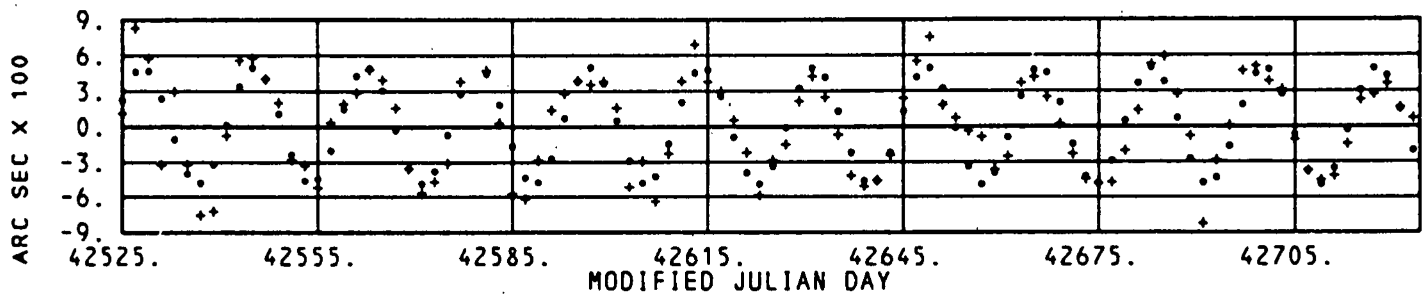

Figure 1. Observed and Calculated Inclination Values at M_2 Frequency for GEOS-3

. - calculated

+ - observed

COMPUTATION OF TIDAL PARAMETERS AND THE LUNAR TIDAL ACCELERATION

As shown by Lambeck et al. [1974], computation of ocean tide parameters from satellite orbit perturbations requires assumption of values for the solid tide Love number k_2 and the lag angle. Table 4 gives the results for ocean tidal parameters for various values of the solid tide phase lag with k_2 assumed to be 0.30, a value often quoted from seismic measurements.

Note in Table 4 that increasing the solid phase lag results in a large reduction in the value of C^{+}_{22}, until at 1° the value of either the GEOS-3 or the combined GEOS-3/1967-92A solution falls to less than 1/2 of the value obtained by Hendershott (quoted in Lambeck et al.[1974]). Thus, although solid and ocean tide effects on satellite orbits cannot be separated, assumption of a solid tide phase lag as large as 1° leads to unacceptable values for the ocean tide coefficients with $k_2 = 0.30$.

Using the combined GEOS-3/1967-92A results for the M_2 tide coefficients in the equations for $\dot{a}$, $\dot{n}$ and using the values given by Lambeck [1975] for the O_1 and N_2 values, we obtain the value $\dot{n} = -27.6 \pm 3$ arc sec/(100 yr)^2. Muller [1976] obtained -27.2 ± 1.7; and Morrison and Ward [1976] obtained -26 ± 2. All of these values are outside the range of -35 to -52 arc sec/(100 yr)^2 reported in Lambeck [1975].

Table 4. M_2 Tidal Estimates Obtained from GEOS-3 and Combined GEOS-3/1967-92A Orbit Perturbations for Various Solid Tide Phase Lags and k_2 = .30

Phase Lag (assumed)	C_{22}^{+}		ε_{22}^{+}		C_{42}^{+}		ε_{42}^{+}	
	Combined	GEOS-3	Combined	GEOS-3	Combined	GEOS-3	Combined	GEOS-3
0	3.23	2.86	331°	332°	0.87	1.05	113°	115°
0.5	2.76	2.39	325°	326°	0.87	1.05	113°	115°
1.0	2.32	1.95	318°	317°	0.87	1.05	113°	116°

DISCUSSION

The formal uncertainty of our solution for the M_2 tide parameters corresponds to ± 1.8 arc sec/(100 yr)^2 in $\dot{n}_t$. However, the O_1 and N_2 tides also contribute about -4.4 arc sec/(100 yr)^2 total lunar tidal acceleration. Because of the very approximate way in which parameters for these latter tidal components were estimated by Lambeck [1975], an uncertainty of perhaps 25% in the O_1 and N_2 contributions is conceivable, leading to an additional source of uncertainty of at least one arc sec/(100 yr)^2. Therefore, we estimate the total uncertainty of our value of the tidal $\dot{n}$ for the moon to be ± 3 arc sec/(100 yr)^2.

Another significant matter is concerned with the effect of a non-zero but small solid tide phase lag δ_2. Again assuming that the Love number k_2 has the value of 0.30, the total solid and fluid lunar tidal acceleration is given by

$$\dot{n}_{total} = -1040\ k_2 \sin(2\delta_2) - 8.32 C_{22}^{+} \cos\varepsilon_{22}^{+}$$

$$-4.4 (N_1 + O_2)$$

arc sec/(1-0 yr)^2 where the last term is that given by Lambeck [1975]. However, the affect of a non-zero value of δ_2 in a satellite solution for ocean tide parameters results in a compensating change in $C_{22}^{+}\cos\varepsilon_{22}^{+}$ that maintains a nearly constant value for the total $\dot{n}$. For example, assumption of $\delta_2 = 0.5°$ changes the value of the total solid/fluid $\dot{n}$ by only 1 arc sec/(100 yr)^2. Thus our value for the lunar tidal acceleration is insensitive to any small future adjustments of the value of δ_2.

Finally, some consideration is required of the method used by Lambeck to obtain the 2nd degree spherical harmonic coefficients from numerical ocean tide models. Lambeck [1975] notes that the three M_2 tide models he used to estimate the lunar $\dot{n}$ vary by 10% about their mean value for $C_{22}^{+}\cos\varepsilon_{22}^{+}$. However, his procedure for obtaining the coefficients can introduce considerable uncertainty into the process. For two of the models he obtained values of the tidal amplitude and phase at a 10° grid interval and then expanded the values in a series of harmonics. We decided to test the effect of grid-size on the coefficient estimates. E. Schwiderski (NSWC), provided us with the M_2 tidal phase and amplitude at each 1° of latitude and longitude based upon his latest numerical solution of the Laplace tidal equations. This model is being used in the analysis of satellite altimeter data at the NSWC (Schwiderski, 1977). Table 5 shows that the value of the critical $C^+_{22}\cos\varepsilon^+_{22}$ term does depend on the grid interval by (in this case) an amount corresponding to as much as 5.4 arc sec/(100 yr)^2. Thus it is critical in future analyses of numerical ocean tide models to compute spherical harmonic coefficients from data gridded at much finer than 10° intervals. It is also worth noting that the $\dot{n}_t$ obtained with zero solid phase lag and the Schwiderski ocean tide parameters is -28.9 arc sec/(100 yr)^2. No information is available to estimate the uncertainty of this result.

In spite of new results for the lunar tidal accelerations smaller in absolute value than $\dot{n}_t$ = -30 arc sec/(100 yr)^2, the source of this acceleration can be accounted for by the ocean tides alone. The $(2,2)^+$ term in the expansion of the ocean tides is equivalent to an equatorial bulge that leads the moon by 55° so that even though the amplitude is small compared to solid tide, the torque produced is large.

The results from perturbations of artificial satellites are especially satisfying because no knowledge of tidal mechanisms is required to obtain the principal component of the expansion of the ocean tides in spherical harmonics that is responsible for the lunar tidal acceleration. New, accurate numerical tide models for the O_1 and N_2 tides would be useful in further refining the results presented here.

Table 5. Effect of Grid Size on Estimates of Tidal Parameters

Grid Interval, deg.	C_{22}^{+}	ε_{22}^{+}	$C_{22}^{+} \cos\varepsilon_{22}^{+}$
10 x 10	3.26	325	2.67
8 x 8	3.81	331	3.33
5 x 5	3.38	325	2.76
3 x 3	3.65	325	2.99
1 x 1	3.59	326	2.98

REFERENCES

Goad, C. C., Application of digital filtering to satellite geodesy, NOAA Technical Report NOS71 NGS6, U. S. Dept. of Commerce, National Oceanic and Atmospheric Administration, National Ocean Survey, Rockville, MD, 1977.

Goad, C. C. and B. C. Douglas, Determination of M_2 ocean tide parameters from satellite orbit perturbations, J. of Geophys. Res., 82, 898-900, 1977.

Kaula, W. M., Introduction to Satellite Geodesy, Blaisdell, Waltham, Mass., 124 pp., 1966.

Lambeck, K., Effects of tidal dissipation in the oceans on the moon's orbit and the earth's rotation, J. Geophys. Res., 80, 2917-2925, 1975.

Lambeck, K., A. Cazenaye, and G. Balmino, Solid earth and ocean tides estimated from satellite orbit analyses, Reviews of Geophysics and Space Physics, 12, 421-434, 1974.

Morrison, L. V. and C. G. Ward, An analysis of the transits of Mercury 1677-1973, Monthly Notices of Royal Astronomical Society, 173, London, 183-206, 1975.

Muller, P. M., Determination of the cosmological rate of change of G and the tidal accelerations of earth and moon from ancient and modern astronomical data, JPL Special Publication 43-36, Jet Propulsion Laboratory, Pasadena, CA, 1976.

Schwiderski, E. W., Ocean tides and GEOS-3. Presented at the AGU Annual Spring Meeting, Washington, D.C., May 30-June 3, 1977.

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15. Supplementary Notes

16. Abstract Analysis of 100 sets of mean elements of GEOS-3 computed at 2-day intervals had yielded observation equations for the M_2 ocean tide from the long periodic variations of the inclination and node of the orbit. If the 2nd degree Love number is given the value $k_2 = 0.30$ and the solid tide phase angle is taken to be 0°, the values are

Inclination: $(3''.99 \pm 0.4) \times 10^{-2} \sin[\sigma(\tau) + 327° \pm 4°]$

$$= \frac{1''.26}{cm} \times 10^{-2} C^{+}_{22} \sin[\sigma(\tau) + \varepsilon^{+}_{22}] - \frac{0''.32}{cm} \times 10^{-2} C^{+}_{42} \sin[\sigma(\tau) + \varepsilon^{+}_{42}] + - - -$$

Node: $(2''.73 \pm 0.7) \times 10^{-2} \cos[\sigma(\tau) +]91° \pm 13°]$

$$= - \frac{0''.24}{cm} \times 10^{-2} C^{+}_{22} \cos[\sigma(\tau) + \varepsilon^{+}_{22}] - \frac{3.38}{cm} \times 10^{-2} C^{+}_{42} \cos[\sigma(\tau) + \varepsilon^{+}_{42}] + - - -$$

where $\sigma(\tau) + 2\Omega - 2M^* - 2\omega^* - 2\Omega^*$ and the starred quantities are lunar orbit elements. Combining these equations with the result obtained by Goad and Douglas [1977] for the satellite 1967-92A gives the M_2 ocean tide parameter values

$$C^{+}_{22} = 3.23 \pm .25 \text{ cm}, \quad \varepsilon^{+}_{22} = 331° \pm 6°$$

$$C^{+}_{42} = .87 \pm .19 \text{ cm}, \quad \varepsilon^{+}_{42} = 113° \pm 6°$$

Under the same assumption of zero solid tide phase lag, the lunar tidal acceleration is mostly (85%) due to the C^{+}_{22} term in the expansion of the M_2 tide with additional small contributions from the O_1 and N_2 tides. Using Lambeck's [1975] estimates for the latter we obtain for the tidal acceleration in lunar longitude the value $\dot{n}_t = -27.6 \pm 3$ arc sec/(100 yr)^2, in excellent agreement with the most recent determinations from ancient and modern astronomical data.

The mean elements of GEOS-3 are also presented in tabular form.

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