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# Theoretical Study of the Effect of Wind Velocity Gradients on 

 Longitudinal Stability and Control in Climbing and Level FlightWindsor L. Sherman

# Theoretical Study of the Effect of Wind Velocity Gradients on Longitudinal Stability and Control in Climbing and Level Flight 

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## SUMMARY

A change in the wind vector over a short distance along the flight path produces gradients in the wind pattern that have caused several severe airplane accidents during take-off and landing operations. Results presented in NASA TN D-8496, reference 2, showed that, in descending flight, a positive wind gradient (that is, a gradient that causes a decreasing head wind or changes a head wind into a tail wind) caused severe divergent motion in the control-fixed mode. A negative wind gradient (that is, a gradient that causes a decreasing tail wind or changes a tail wind into a head wind) caused oscillatory motions in the control-fixed mode which should not create a control problem. This paper reports the results obtained when the method of analysis used in reference 2 was applied to climbing and to straight and level flight. In straight and level flight, a variation of a head wind or a tail wind with distance along the flight path was found to cause effects similar to those found in descending flight. In climbing flight, it was found that a negative wind gradient caused a slightly divergent oscillation that presented no control problems. For a positive wind gradient the stability analysis indicated the usual divergence; however, the motion was an oscillation instead of the downward divergence found for descending flight. The results of motion studies indicated that adequate control of the airplane motions can be provided by automatic control systems.

## INTRODUCTION

A large change in the wind vector over a short distance along the flight path of an airplane can have disastrous effects on its subsequent flight, as shown in reference 1. In fact, wind gradients have caused several severe accidents during take-off and landing operations. A previous study reported in reference 2 assessed the effects of positive and negative gradients in the horizontal winds (with respect to altitude) on the longitudinal stability and control of an airplane flying descending flight paths. A positive gradient in the horizontal wind causes a decreasing head wind and can change a head wind into a tail wind, whereas a negative gradient causes a decreasing tail wind and can change a tail wind into a head wind. It was found that, in descending flight, a positive gradient in horizontal winds caused the long-period longitudinal mode to become unstable, with the time to double amplitude approaching 5 sec. However, there was little effect on the short-period mode. A negative gradient in horizontal winds produced negligible effects on both the short- and long-period modes.

In the study reported herein, the method of analysis and the equations of motion presented in appendix A of reference 2 were applied to determine the effect of positive and negative gradients in the horizontal wind component on the longitudinal stability and control of a large jet transport airplane in climbing or straight and level flight. Results presented are (1) spatial motions of the airplane as affected by the horizontal wind gradients for
control-fixed and automatic-control flight and (2) root locus plots displaying airplane stability information for the control-fixed mode.

SYMBOLS
$C_{D} \quad$ drag coefficient
$C_{D, O} \quad$ drag coefficient at $C_{L}=0$
$C_{D_{\alpha}}=\frac{\partial C_{D}}{\partial \alpha}, \mathrm{rad}^{-1}$
$C_{D_{\delta_{e}}}=\frac{\partial C_{D}}{\partial \delta_{e}}, \mathrm{rad}^{-1}$
$C_{L} \quad$ lift coefficient
$C_{L, 0} \quad$ lift coefficient at $\alpha=0$
$C_{L_{q}} \quad=\frac{\partial C_{L}}{\partial q}, \operatorname{rad}^{-1}-\sec$
$\mathrm{C}_{\mathrm{L}_{\alpha}}=\frac{\partial \mathrm{C}_{\mathrm{L}}}{\partial \alpha}, \mathrm{rad}^{-1}$
$C_{L \dot{\alpha}} \quad=\frac{\partial C_{L}}{\partial \dot{\alpha}}, \operatorname{rad}^{-1}-\mathrm{sec}$
$C_{\mathrm{L}_{\mathrm{e}}}=\frac{\partial \mathrm{C}_{\mathrm{L}}}{\partial \delta_{\mathrm{e}}}, \mathrm{rad}^{-1}$
$C_{m} \quad$ pitching-moment coefficient
$c_{m} \quad=\frac{\partial C_{m}}{\partial q}, \operatorname{rad}^{-1}-\sec$
$\mathrm{c}_{\mathrm{m}_{\alpha}}=\frac{\partial \mathrm{C}_{\mathrm{m}}}{\partial \alpha}, \mathrm{rad}^{-1}$
$c_{m_{\dot{\alpha}}} \quad=\frac{\partial c_{m}}{\partial \dot{\alpha}}, \mathrm{rad}^{-1}-\mathrm{sec}$
$c_{m \delta_{e}}=\frac{\partial c_{m}}{\partial \delta_{e}}, \mathrm{rad}^{-1}$
$\overline{\mathrm{c}} \quad$ mean aerodynamic chord, m
D horizontal distance along flight path, m
$\mathrm{F}_{\mathrm{T}} \quad$ thrust, N
$\mathrm{F}_{\mathrm{T}_{\mathrm{U}}} \quad=\frac{\partial \mathrm{F}_{\mathrm{T}}}{\partial \mathrm{U}}, \mathrm{N}-\mathrm{sec}-\mathrm{m}^{-1}$
$\mathrm{F}_{\mathrm{X}}, \mathrm{F}_{\mathrm{Y}}, \mathrm{F}_{\mathrm{Z}} \quad$ forces in $\mathrm{x}-, \mathrm{Y}$, and z -directions (stability-axis system), N $\mathrm{g} \quad$ acceleration of gravity, $9.80665 \mathrm{~m}-\mathrm{sec}^{-2}$
$I_{Y} \quad$ moment of inertia about $Y$ stability axis, $\mathrm{kg}-\mathrm{m}^{2}$
$\mathrm{K}, \mathrm{K}_{1}, \mathrm{~K}_{2}, \ldots, \mathrm{~K}_{7}$ gains in flight-path and airspeed control systems
$k_{1} \quad=\frac{\rho \mathrm{SU}_{0}}{m}, \sec ^{-1}$
$k_{2} \quad=\frac{\rho S \bar{c} U_{0}}{I_{Y}}, m^{-1}-\sec ^{-1}$
$k_{3}=\frac{\rho \mathrm{SU}_{0}{ }^{2}}{2 \mathrm{~m}}, \mathrm{~m}-\mathrm{sec}^{-2}$
$\mathrm{k}_{4}=\frac{\rho \mathrm{s} \mathrm{CU}_{0}{ }^{2}}{2 \mathrm{I}_{\mathrm{Y}}}, \sec ^{-2}$
M pitching moment, $N-m$
$M_{q} \quad=\frac{1}{I_{Y}}\left(\frac{\partial M}{\partial q}\right)=c_{m_{q}} k_{4}, \operatorname{rad}^{-1}-\sec ^{-1}$
$M_{u} \quad=\frac{1}{I_{Y}}\left(\frac{\partial M}{\partial u}\right)=C_{m^{k}}, m^{-1}-\sec ^{-1}$
$M_{\alpha} \quad=\frac{1}{I_{Y}}\left(\frac{\partial M}{\partial \alpha}\right)=C_{m_{\alpha}} k_{4}, \operatorname{rad}^{-1}-\sec ^{-2}$

$$
\begin{aligned}
& M_{\dot{\alpha}} \quad=\frac{1}{I_{Y}}\left(\frac{\partial M}{\partial \dot{\alpha}}\right)=C_{m_{\dot{\alpha}}} \dot{k}_{4}, \mathrm{rad}^{-1}-\mathrm{sec}^{-1} \\
& M_{\delta_{e}} \quad=\frac{1}{I_{Y}}\left(\frac{\partial M}{\partial \delta_{e}}\right)=c_{m_{\delta}}{ }^{k_{4}}, \operatorname{rad}^{-1} \sec ^{-2} \\
& \text { m mass, kg } \\
& \text { P period, sec } \\
& q \quad \text { pitch velocity, rad-sec }{ }^{-1} \\
& s \quad \text { wing area, } \mathrm{m}^{2} \\
& \text { t time, sec } \\
& t_{D} \quad \text { time to double amplitude, sec } \\
& \mathrm{t}_{1 / 2} \text { time to damp to half amplitude, sec } \\
& \mathrm{U}_{0} \quad \text { steady-state velocity, } \mathrm{m}-\mathrm{sec}^{-1} \\
& u \text { perturbation velocity, m-sec }{ }^{-1} \\
& \mathrm{v}_{\mathrm{C}} \quad \text { command velocity, m-sec }{ }^{-1} \\
& \mathrm{~V}_{\mathrm{R}} \quad \text { resultant velocity, } \mathrm{m}-\mathrm{sec}^{-1} \\
& \dot{\mathrm{~V}}_{1} \text { derivative with respect to time of component of } \mathrm{V}_{\mathrm{R}} \\
& \text { in } x \text {-direction (principal body-axis system), } m-\mathrm{sec}^{-1} \\
& \mathrm{v}_{\mathrm{w}, 1} \text { horizontal wind gradient, } \mathrm{m}-\mathrm{sec}^{-1}-\mathrm{m}^{-1} \\
& \mathrm{x}_{\mathrm{u}} \quad=-\frac{1}{\mathrm{~m}}\left(\frac{\partial \mathrm{~F}_{\mathrm{x}}}{\partial \mathrm{u}}\right)=-\mathrm{C}_{\mathrm{D}_{1}}-\frac{\mathrm{F}_{\mathrm{T}_{\mathrm{u}}}}{\mathrm{~m}}, \sec ^{-1} \\
& X_{\alpha} \quad=-\frac{1}{m}\left(\frac{\partial F_{X}}{\partial \alpha}\right)=-C_{D_{\alpha}} k_{3}, m-r a d^{-1}-\sec ^{-2} \\
& X_{\delta_{e}} \quad=-\frac{1}{m}\left(\frac{\partial F_{x}}{\partial \delta_{e}}\right)=-C_{D_{\delta_{e}}} k_{3 r}, m-\operatorname{rad}^{-1}-\sec ^{-2} \\
& \mathrm{z}_{\mathrm{q}} \quad=-\frac{1}{\mathrm{~m}}\left(\frac{\partial \mathrm{~F}_{\mathrm{Z}}}{\partial \mathrm{q}}\right)=-\mathrm{C}_{\mathrm{L}_{\mathrm{q}}} \mathrm{k}_{3}, \mathrm{~m}-\mathrm{rad}^{-1}-\mathrm{sec}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
& z_{u} \quad=-\frac{1}{m}\left(\frac{\partial F_{Z}}{\partial u}\right)=-C_{L_{1}}{ }^{k_{1}}, \sec ^{-1} \\
& \mathbf{z}_{\alpha} \quad=-\frac{1}{m}\left(\frac{\partial F_{z}}{\partial \alpha}\right)=-C_{L_{\alpha}} k_{3}, m-\operatorname{rad}^{-1}-\sec ^{-2} \\
& \mathrm{z}_{\dot{\alpha}} \quad=-\frac{1}{\mathrm{~m}}\left(\frac{\partial \mathrm{~F}_{\mathrm{Z}}}{\partial \dot{\alpha}}\right)=-\mathrm{C}_{\mathrm{L}_{\dot{\alpha}}} \mathrm{k}_{3}, \mathrm{~m}-\mathrm{rad}^{-1}-\mathrm{sec}^{-1} \\
& z_{\delta_{e}} \quad=-\frac{1}{m}\left(\frac{\partial F_{Z}}{\partial \delta_{e}}\right)=-C_{L_{\delta_{e}}} k_{3}, m-r a d^{-1}-\sec ^{-2}
\end{aligned}
$$

$\alpha$
$\dot{\alpha}$
$\Gamma_{c}$
$\Gamma_{0}$
$\gamma$
$\delta_{e}$
$\varepsilon_{1}$
$\varepsilon_{4}$
$\zeta$
$\theta$
$\rho$
$\sigma_{u}$
$\tau_{E}$
$\omega_{n}$
perturbation angle of attack, rad
$=\frac{d \alpha}{d t}, \mathrm{rad}-\mathrm{sec}^{-1}$
flight-path angle, rad
command flight-path angle, rad
steady-state flight-path angle, rad perturbation flight-path angle, rad
elevator deflection, rad
$=\Gamma_{C}-\Gamma, \operatorname{rad}$
$=V_{C}-V_{R}, m-\sec ^{-1}$
damping ratio
pitch angle, rad
air density, $\mathrm{kg}-\mathrm{m}^{-3}$
wind shear parameter, $\frac{U_{0} v_{w, l}}{g}$, dimensionless
engine time constant
undamped circular frequency, rad-sec ${ }^{-1}$

## ANALYSIS AND CONDITIONS OF STUDY

The linearized equations of longitudinal motion, equations (Al 8) of reference 2 , were used in this analysis. These equations are

$$
\left[\begin{array}{ccc}
\frac{d}{d t}-\frac{g}{2 U_{0}} \sigma_{u} \sin 2 \Gamma_{0}-x_{u} & -x_{\alpha} & g\left(\cos \Gamma_{0}-\sigma_{u} \cos 2 \Gamma_{0}\right)  \tag{1}\\
-z_{u}-\frac{g}{U_{0}}\left(\sigma_{u} \sin ^{2} \Gamma_{0}\right) & -\left(z_{\dot{\alpha}}+z_{q}\right) \frac{d}{d t}-z_{\alpha} & -\left(U_{0}+z_{q}\right) \frac{d}{d t}+g\left(\sin \Gamma_{0}-\sigma_{u} \sin 2 \Gamma_{0}\right) \\
-M_{u} & \frac{d^{2}}{d t^{2}}-\left(M_{\dot{\alpha}}+M_{q}\right) \frac{d}{d t}-M_{\alpha} & \frac{d^{2}}{d t^{2}}-M_{q} \frac{d}{d t}
\end{array}\right]=\left[\begin{array}{l}
u \\
z_{\delta_{e}} \\
x_{e}
\end{array}\right] \delta_{e}
$$

for horizontal wind gradients. The parameter $\sigma_{u}$ introduces the effect of horizontal wind gradients into the equations of motion. The horizontal wind shear parameter $\sigma_{u}$ is defined as

$$
\begin{equation*}
\sigma_{u}=\frac{\mathrm{U}_{0} \mathrm{v}_{\mathrm{w}, 1}}{\mathrm{~g}} \tag{2}
\end{equation*}
$$

where $U_{0}$ is the still-air speed and $v_{W, l}$ is the horizontal wind gradient. Head winds are considered negative. Positive $v_{w, 1}$ will

1. Decrease a head wind
2. Increase a tail wind
3. Change a head wind into a tail wind
and negative $\mathrm{v}_{\mathrm{w}, 1}$ will
4. Decrease a tail wind
5. Increase a head wind
6. Change a tail wind into a head wind

It should be noted from equation (2) that if the still-air speed is changed, a new value of $\sigma_{u}$ must be calculated in order to keep the wind gradient constant.

The wind gradients used in this study were obtained from the wind conditions recorded at the John $F$. Kennedy International Airport, Long Island, New York, on June 24 , 1975 (see ref. 1). This is the same wind condition used in reference 2.

The airplane used in this study is a large four-engine jet transport. The aerodynamic and physical characteristics of this airplane are given in the appendix. These data represent the airplane up to a Mach number of 0.3 with a flap setting of 0.4363 rad .

Before considering the effects of wind gradient, it is interesting to briefly consider the stability of the airplane in climbing fiight when there is no wind gradient. The stability was calculated by solving for the eigenvalues of the characteristics equation which is obtained from the determinant of the $3 \times 3$ matrix given on the left side of equations (1). Three flight-path angles were used: $\Gamma_{0}=0,0.08727$, and 0.1745 rad . The results of these calculations are summarized in table $I$. As can be seen from the data, the magnitude
table I.- effect of flight-path angle on longitudinal stability

$$
\left[\mathrm{U}_{0}=77.12 \mathrm{~m}-\mathrm{sec}^{-1} ; \quad \sigma_{\mathrm{u}}=0\right]
$$

| T0 |  | Short-period mode | Long-period mode |
| :---: | :---: | :---: | :---: |
| 0 | Roots of characteristic | $-0.7003289 \pm 0.8080260 \mathrm{i}$ | -0.0038872 $\pm 0.1355501 \mathrm{i}$ |
| . 08727 | equation . . . . . . . | $-.6986357 \pm .8114533 i$ | $-.0000726 \pm .7346378 i$ |
| . 1745 |  | $-.6968870 \pm .8144512 i$ | $.0037194 \pm .1331214 \mathrm{i}$ |
| 0 |  | 0.9895350 | 178.2777536 |
| . 08727 | $t_{1 / 2}$ or $t_{D}$, sec | . 9919332 | 9548.571641 |
| . 1745 |  | . 9944223 | -186.3185308 |
| 0 |  | 7.7742375 | 46.3532377 |
| . 08727 | P, sec. | 7.7431263 | 46.6673172 |
| . 1745 |  | 7.7146247 | 47.1989285 |
| 0 |  | 1.0694192 | 0.1356058 |
| . 08727 | $\omega_{n}$, rad-sec ${ }^{-1}$. . . . . | 1.0707700 | . 1346378 |
| . 1745 |  | 1.071770 | . 1331733 |
| 0 |  | 0.6548684 | 0.0286654 |
| . 08727 | ち • • • • • • • • • • | . 6524611 | . 0005390 |
| . 1745 |  | . 6501382 | -. 0279293 |

of the flight-path angle has little effect on the short-period mode, whereas the long-period mode changes from a lightly damped oscillation to a slightly divergent oscillation with a time to double amplitude of 186.32 sec . If $U_{0}$ is increased from $77.12 \mathrm{~m}-\mathrm{sec}^{-1}$ to $100 \mathrm{~m}-\mathrm{sec}^{-1}$, the divergent oscillation that occurred at $\Gamma_{0}=0.1745$ rad changes to a lightly damped oscillation that has a time to damp to half amplitude of 433 sec . These results indicate that in climbing flight, the stability of the long-period mode of the basic airplane is dependent on the flight-path angle and the airspeed. Combinations of these parameters that give unstable conditions should be avoided when studying the effect of wind gradients to insure that all changes in stability arise from the wind gradients. It should be noted that the unstable oscillations found for the basic airplane should cause no control problems because of the long times to double amplitude.

For straight and level flight, the flight condition was $\Gamma_{0}=0$ and $U_{0}=77.12 \mathrm{~m}-\mathrm{sec}^{-1}$. The long-period mode with $\sigma_{u}=0$ was a stable oscillation. The root for this mode was $-0.0038872 \pm 0.1355501 \mathrm{i}$ which indicates a lightly damped oscillation. The damping ratio was 0.0287 , the period was 46.35 sec , and the time to damp to half amplitude was 178.28 sec . In addition, these flight conditions provide for direct comparison with reference 2 since the parameter $\sigma_{u}$ retains the same value. The flight condition for climbing flight was $\Gamma_{0}=0.0524 \mathrm{rad}$ and $\mathrm{U}_{0}=77.12 \mathrm{~m}-\mathrm{sec}^{-1}$. The long-period-mode root for this flight condition with no wind gradient was $-0.00159960 \pm 1349260 i$ which indicates a stable, lightly damped oscillation with a period of 46.6 sec , a damping ratio of 0.0119 , and a time to damp to half amplitude of 433.22 sec . In this study, all variations fram these stability conditions can be attributed to the effects of wind gradient.

## RESULTS AND DISCUSSION

Equations (1) were used to determine the longitudinal stability of the selected airplane at the selected flight conditions, $\Gamma_{0}=0$ or 0.05236 rad and $U_{0}=77.12 \mathrm{~m}-\mathrm{sec}^{-1}$, for various values of the wind shear parameter $\sigma_{u}$. The maximum value of $\sigma_{u}$ was 2.0 , which for these conditions represents a $\mathrm{v}_{\mathrm{w}, 1}=0.25 \mathrm{~m}-\mathrm{sec}^{-1}-\mathrm{m}^{-1}$. The value of 2.0 for $\sigma_{\mathrm{u}}$ is a good representation of the wind gradients discussed in reference 1.

Straight and Level Flight ( $\left.\Gamma_{0}=0\right)$
The effect of positive and negative wind gradients on the long-period mode of the transport airplane is shown by the root locus plot presented in figure 1. In level flight, the gradient is considered to be a change in wind magnitude along the flight path. As $\sigma_{u}$ increased negatively from 0 , the only effect was a slight decrease in the stability of the long-period mode. However, as $\sigma_{u}$ increased positively from 0, the roots approached the real axis, and at $\sigma_{u}=1.0$, the oscillation broke down into an aperiodic mode with a time to damp to half amplitude of 59.68 sec . As $\sigma_{u}$ increased above 1.0, two aperiodic modes appeared, one of which was unstable. At $\sigma_{u}=2.0$, the unstable mode had time to double amplitude of 5.33 sec . Thus for values of $\sigma_{u}>1.0$, the longperiod mode was characterized by aperiodic, divergent motion. Wind gradient, positive or negative, had no significant effect on the short-period mode.

$$
\text { Climbing Flight } \Gamma_{0}=0.05236 \mathrm{rad}
$$

The root locus plot for the long-period longitudinal mode as affected by wind gradients during climbing flight is shown in figure 2. As $\sigma_{u}$ increased negatively from 0 , the airplane developed an unstable oscillation for $\sigma_{u}<-0.5$. This oscillation was very mild and the time to double amplitude was about 1407.68 sec and decreased as the magnitude of $\sigma_{u}$ increased until, at $\sigma_{u}=-2.0$, the time to double amplitude was 202.99 sec . This unstable condition occurred because the long-period mode was normally less stable in climbing flight than in level flight, and the slight decrease in stability caused by negative wind gradients was sufficient to introduce the unstable condition. Increasing
the flight-path angle caused a greater instability until, at $\Gamma_{0}=0.1745 \mathrm{rad}$ and $\sigma_{u}=-2.0$, the time to double amplitude had decreased to 47.13 sec . Increasing the flight speed from $77.12 \mathrm{~m}-\mathrm{sec}^{-1}$ to $100 \mathrm{~m}-\mathrm{sec}^{-1}$ caused an increase in the time to double amplitude. For $\Gamma_{0}=0.1745$ and $\sigma_{u}=-2.0$, a change from $U_{0}=77.12 \mathrm{~m}-\mathrm{sec}^{-1}$ to $100 \mathrm{~m}-\mathrm{sec}^{-1}$ changed the time to double amplitude from 47.13 sec to 71.27 sec . These calculations were made with $\sigma_{u}$ constant, which means that the gradient $v_{w, 1}$ decreased. If the gradient had been held constant, the value of $\sigma_{u}$ would have been -2.55 at $100 \mathrm{~m}-\mathrm{sec}^{-1}$.

For positive wind gradients, the roots approached the real axis as $\sigma_{u}$ increased from 0 until, for $\sigma_{u}$ slightly greater than 1.0, the oscillation broke down into two aperiodic modes, one of which was unstable. The time to double amplitude decreased as $\sigma_{u}$ increased until, at $\sigma_{u}=2.0$, it was only 5.39 sec . Increases in flight-path angle and speed produced no significant changes in the time to double amplitude for positive wind gradients.

The results of a stability analysis when wind gradients are included are difficult to interpret, since stability equations are perturbations from a steady-state condition and a wind gradient does not represent a steady state. The most that can be said is that the results represent the stability of the airplane when a wind described by $\sigma_{u}$ is encountered. For this reason, a greater understanding of the effect of wind gradients can be obtained by computing the spatial motions of the airplane.

## Spatial Motions and Automatic Controls

The equations used to calculate the spatial motions are the nonlinear longitudinal equations of motion with wind shear parameters added. The specific equations used in this study are equations (A4), (A6), and (A8) of appendix A of reference 2 .

In straight and level flight, positive and negative horizontal wind gradients produced the same type of motions as had been encountered in descending flight. Positive gradients produced a rapid downward divergence and negative gradients produced an oscillatory response. The flight-path and airspeed control systems used in reference 2 (fig. 3) provided adequate control in both cases. No motions are shown for straight and level flight.

Negative horizontal wind gradients in climbing flight produced no control problems and no motions are shown for this case. The airplane motions for positive horizontal wind gradients are shown in figure 4; the dashed line is for the control-fixed motions. When the airplane entered the gradient field, oscillations were set up in $\theta, \alpha$, and $\Gamma$, and the airplane climbed at a very low flight-path angle and finally exited the gradient field at $D \approx 5750 \mathrm{~m}$, which corresponds to 63 sec . This is about five times as long as it would have taken the airplane to climb 50 m in still air. As the airplane climbed into the positive wind gradient, the gradient caused the airplane to start a downward motion. As the airplane started to lose altitude, the sign of the gradient changed from positive to negative, the head wind increased, and the airplane started to climb. Because the climb changed the sign of the gradient back to positive, the downward motion began again and the cycle repeated. This cycle
continued and the airplane gained a small amount of altitude with each repetition and eventually cleared the gradient field.

The flight-path and airspeed control systems used in reference 2 (fig. 3) were added to the mathematical model. The controlled motion, shown by the solid curves (fig. 4(a)), resulted when the engine time constant $\tau E$ was equal to 2.5 and the speed error integration gain $K_{5}$ was equal to 0.5 . This time constant is representative of a modern jet fan engine (see ref. 2). The resulting motions, while not as oscillatory as the control-fixed motions showed large excursions in $\theta$ and $\alpha$ that were not reflected in the altitude track. The $\theta$ and $\alpha$ motions were large enough to be considered unsatisfactory. When the engine time constant was decreased to 0.1 , a time constant that corresponds to a thrust modulator (see ref. 2), the motions (short-long dashed lines in fig. $4(a))$ improved to the point where they were considered barely acceptable. No such poor response problem had been encountered with these control systems in the study of reference 2. A detailed study of the computer output indicated that the integrator gain $K_{5}$ in the speed control system was too high. It was reduced from 0.5 to 0.1 , and the motions shown in figure $4(\mathrm{~b})$ resulted. While the airplane was encountering the horizontal wind gradients, the motions were well controlled. These motions were well damped for both engine time constants, with a $\tau_{E}$ of 0.1 giving slightly better results than $\tau_{E}=2.5$.

These results are interesting in that the positive horizontal wind gradients in climbing flight did not produce the catastrophic motions that occurred in level and descending flight. The control-fixed motions shown in figure 4 are not motions that would present control problems to a competent pilot.

## CONCLUDING REMARKS

A study reported in NASA TN D-8496, reference 2, of the effect of horizontal wind gradients on the longitudinal stability and control of a jet transport has been extended to climbing and to straight and level flight. The stability analysis showed that in both climbing and level flight, positive gradients in the horizontal winds caused the long-period longitudinal mode to become aperiodic and divergent. In level flight, negative horizontal wind gradients had no important effect on airplane motions. However, in climbing flight, negative horizontal wind gradients caused the normally stable longperiod mode to change to a divergent oscillation. The time to double amplitude and the period of the unstable oscillation were sufficiently long so that the divergent oscillation did not pose a control problem.

For positive horizontal wind gradients in climbing flight, the divergence never fully developed because as the airplane started its downward motion, the gradient changed from positive and the airplane started to recover. An oscillation caused by the changing gradient developed and persisted until the airplane cleared the gradient layer. This result is interesting since it shows that positive horizontal wind gradients pose much less danger to a climbing airplane than to a descending airplane. The character of the motions for positive horizontal wind gradients in climbing flight should pose no control problem for a pilot. The flight-path and airspeed control systems used in
previous studies produced barely satisfactory control at an engine time constant of $\tau_{E}=0.1$. However, reduction of the integrator gain in the airspeed control system provided good control for the two values of the engine time constant used in this study.

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## APPENDIX

## AIRPLANE CHARACTERISTICS

The airplane used in this study is considered a typical narrow-body modern jet transport powered by four engines, each having approximately 67233 N of thrust.

The airplane's dimensional and mass characteristics are

$$
\begin{array}{ll}
\overline{\mathrm{c}}=7.01 \mathrm{~m} & \mathrm{I}_{\mathrm{Y}}=9933300 \mathrm{~kg}-\mathrm{m}^{2} \\
\mathrm{~S}=267.9 \mathrm{~m}^{2} & \text { Aspect ratio }=7.03 \\
\mathrm{~m}=90909.1 \mathrm{~kg} &
\end{array}
$$

The air density was $1.2929 \mathrm{~kg}-\mathrm{m}^{-3}$.

The aerodynamic data with respect to the stability axes, for the center of gravity at $0.25 \bar{c}$, flaps at 0.4363 rad , and a Mach number less than or equal to 0.3 are

$$
\begin{array}{ll}
C_{D_{\alpha}}=0.529 & C_{m_{\alpha}}=-0.241 \\
C_{L_{\alpha}}=4.87 & C_{m_{q}}=-0.707 \\
C_{L_{\alpha}}=0.0889267 & C_{D, \circ}\left(C_{L}=0\right)=0.038 \\
C_{L_{q}}=0.2831216 & C_{L, O}(\alpha=0)=0.705 \\
C_{m_{\alpha}}=-1.115 &
\end{array}
$$



Figure 1.- Root locus plot for long-period longitudinal mode of airplane affected by wind gradients. Straight and level flight; $\Gamma_{0}=0 ; U_{0}=77.12 \mathrm{~m}-\mathrm{sec}^{-1}$.


Figure 2.- Root locus plot for long-period longitudinal mode of airplane affected by wind gradients. Climbing flight; $\Gamma_{0}=0.05236 \mathrm{rad} ; \mathrm{U}_{0}=77.12 \mathrm{~m}-\mathrm{sec}^{-1}$.

Integrator and Gain


Gains and constants

| $\mathrm{K}^{2}$ | 0.8 |
| :---: | :---: |
| $\mathrm{~K}_{1}$ | 5.0 |
| $\mathrm{~K}_{2}$ | -1.0 |
| $\mathrm{~K}_{3}$ | 8.66 |
| $\mathrm{~T}_{\mathrm{E}}$ | 0.0357 |

(a) Flight-path control system.

Figure 3.- Automatic control systems.


Gains and constants
$\begin{array}{lll}\mathrm{K}_{4} & 1.0 \\ \mathrm{~K}_{5} & 0.5 \\ \mathrm{~K}_{6} & 345.67 \mathrm{C}_{\mathrm{D}} \mathrm{V}_{\mathrm{R}} \\ \mathrm{K}_{7} & 0.5 \\ \tau_{\mathrm{E}} & \text { Not fixed, values from } 0.1 \text { to } 4.0 \text { used }\end{array}$
(b) Air speed control system.

Figure 3.- Concluded.

(a) $K_{5}=0.5$.

Figure 4.- Control-fixed and automatic-control motions of an airplane affected by wind gradients in climbing flight. $\mathrm{U}_{0}=77.12 \mathrm{~m}-\mathrm{sec}^{-1} ; \quad \Gamma_{0}=0.05236 \mathrm{rad} ;$ $\sigma_{u}=2.0$.


Figure 4.- Concluded.

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15. Supplementary Notes
16. Abstract

A change in the wind vector over a short distance along the flight path (wind gradient) has caused several severe airplane accidents during take-off and landing. Results of a previous study (NASA TN D-8496) showed that, in descending flight, a positive wind gradient (decreasing head wind) caused severe divergent motion and a negative wind gradient (decreasing tail wind) caused oscillatory motion which should not create a control problem. This paper reports the results obtained when the same method of analysis was applied to climbing and to straight and level flight. In straight and level flight, a wind gradient was found to cause effects similar to those found in descending flight. In climbing flight, it was found that a negative wind gradient caused a slightly divergent oscillation that presented no control problems and a positive wind gradient caused oscillatory divergence. Results of motion studies indicated that adequate control of the airplane motions can be provided by automatic control systems.
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