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Department of Engineering
Hampton Institute
Hampton, Virginia 23668

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A SYSTEMS APPROACH TO THEORETICAL FLUID MECHANICS

-FUNDAMENTALS-

By

Joshua C. Anyiwo

Prepared for the:
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

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GENERAL INTRODUCTION

Theoretical fluid mechanics research essentially seeks to find complete quantitative and qualitative descriptions of the velocity, pressure, temperature and any other relevant property-fields of fluid flow systems of diverse boundary and initial conditions; and especially to understand how these property-fields determine, or are themselves influenced by, such special and often very important fluid dynamical phenomena as flow instability, transition and turbulence.

Early efforts in theoretical fluid mechanics focused on the so-called "potential flow" of ideal incompressible fluids. Quite a large class of flows could be described as potential flows and the analytical methods employed to describe such flows were almost perfect. However, viscosity was soon recognized as a real fluid dynamical property critical to an understanding, and to a complete description, of any important flow phenomenon. This knowledge led to the creation of the mathematical model of viscous fluids governed by the basic Navier-Stokes (N-S) equations. Virtually every effort in theoretical fluid mechanics since the formulation of the N-S equations has become one of finding solutions to complete or simplified versions of the N-S equations for prescribed boundary and initial conditions.

But there is now a rapidly growing belief among fluid mechanics researchers that either the N-S equations may not be a completely correct mathematical model of the general real fluid flow system, or they may not have been adequately understood by their users. It appears that some corrections need to be made regarding especially the expressions or conception of the fluid stress-strain relation employed in the N-S equations, the boundary conditions employed for the variables, and the general nature of the variables themselves. Furthermore, notwithstanding the apparent incompleteness of the descriptive power, or in the understanding of the N-S equations as a mathematical model of the general fluid flow system, the mathematical problem of solving those equations, for any other than the simplest cases, remains a rather formidable and often yet impossible task. From the few solutions of the N-S equations available, very little information on very important flow phenomena such as flow instability, transition and turbulence are yet possible. These and similar misgivings lead one to the inevitable opinion that it would be unlikely to formulate a satisfactory unified model of general fluid flow capable of describing such fluid flow phenomena as turbulence, within the framework of deterministic classical mechanics and thermodynamics such as the ordinary understanding of the N-S equations would represent. In other words, if the N-S equations are to be employed generally for the description of fluid flow systems, it seems that all the variables in the N-S equations must be understood to be statistical, not deterministic, quantities.

In the face of these realities theoretical fluid mechanics research turned to statistical methods, especially, and often exclusively, for the description of the turbulence phenomenon. Initially these statistical methods consisted in direct formulation and application of probability distribution functions to describe the turbulent velocity, pressure and temperature fluctuations in simple turbulent flows (refs: 5, 7, 21). More recently, powerful methods (refs: 2, 8, 12, 14) based upon statistical mechanics and thermodynamics have been developed, in which new models of the fluid flow system (usually as a set of interacting particles) are conceptualized and in which mechanical characteristics are viewed as probabilities and their values appear as mathematical expectations.

But although these statistical mechanical approaches show definite promise toward eventually resolving the fluid flow problem, there appears to be still a long way to the realization of that goal. Fundamental problems, notoriously those of determining the proper stress-strain relation in the fluid flow system and the proper "history" effects to facilitate closure of the set of descriptive equations of the fluid flow system, remain virtually untouched, and continue to palgue every effort, however sophisticated, at mathematically modeling the general fluid flow system.

From all practical considerations it seems quite valid to conclude that theoretical fluid mechanics research has for a while now been stuck in a dead-end alley and does not appear to possess any sensible exit-direction. And the time seems overdue to re-evaluate the progress in theoretical fluid mechanics research, as well as to systematically set up the fluid flow problem towards a complete practical solution.

The urgent need for a systematic approach in theoretical fluid mechanics research cannot be overemphasized. Hitherto, the trend in this area of human endeavor has been characterized by what may be described metaphorically as the "band-wagon" mentality: someone at some point introduces some "new" method and everyone jumps into his band-wagon with modifications and extensions but with little or no understanding of the fundamental philosophy involved; and when the new method is seen to lead nowhere, a lull appears as the crowd waits for another "new" method to emerge. Such an appraoch is completely unwarranted and has been very expensive in terms of monetary cost and individual frustrations, not to mention its fruitlessness in terms of a basic understanding and solution of the fluid flow problem--a problem of extreme importance to man's technological and general development.

A systematic approach to fluid mechanics research should lead to:

(a) a better perspective of the supremal fluid flow problem,

(b) the identification of the infimal problems fundamental to a complete solution of the supremal problem, and

(c) a clear recognition of the possibilities or impossibilities of complete solutions to the fluid flow problem, as well as of the nature and type of mathematical tools ideally suited to tackling the mathematical modeling of the fluid flow system.

And the results of such efforts should infuse definite and needed direction to theoretical fluid mechanics research.

In 1969 this investigator undertook this needed systematic investigation of the general fluid flow problem, and by 1971 had fully constated to himself that any complete practical solution of the fluid flow problem must involve the application of a valid and complete "general-system" theory. It does indeed appear that the fluid flow system represents a real model of the general dynamical system in nature, to the extent that anyone who can effectively analyse the fluid flow system can also, with only slight modifications to his technique, effectively describe the dynamical characteristics of any other natural system. Following extensive research in search of a "general-system" theory complete and valid for all natural systems, this investigator established in 1973 the foundation for the formulation of such a theory. Elements of this "general-system" theory are presented in reference (1). As will ultimately be seen, general-system theory either resolves or points to a definite direction for the resolution of all the fundamental problems hitherto encountered in the complete description of fluid flow systems.

The present work represents a preliminary application of the underlying principles of this investigator's "general system" theory to the description and analyses of the fluid flow system. An attempt is made herein to establish practical models, or elements thereof, of the general fluid flow system from the point of view of the general system theory fundamental principles. Results thus obtained are applied to a simple "experimental fluid flow system," as test case, with particular emphasis on the understanding of fluid flow instability, transition and turbulence.

This report, however, is a presentation only of the fundamental aspects of the stated work. In later and more detailed efforts by this investigator, each of the major findings reported herein is taken separately and considered in depth.

The fundamental philosophy that will be employed throughout this and future work is that any equations that are to be used must be derived by objective application of the universal and invariant general system theory to the fluid flow system; and any assumptions made must be explicitly indicated. Nothing is taken to be sacred unless it conforms specifically to the universal invariance principles of the general system theory. That, it seems, is the only way by which we can be sure to detect and correct all those fundamental errors which have been propagated through the history of fluid mechanics research and which have stymied the development of a true understanding of fluid flow.

The "experimental fluid flow system" chosen as the test case in this work is the very simple flow satisfying the following requirements:

(a) incompressible and time-independent flat plate boundary layer flow governed by the equations:

$$\partial u/\partial x + \partial v/\partial y = 0 \tag{1.1}$$
$$u\partial u/\partial x + v\partial u/\partial y = \partial(\varepsilon\partial u/\partial y)/\partial y \tag{1.2}$$

(b) ε is the relevant system particle viscosity divided by the mass density; that is, if the relevant system particle is the fluid molecule, then $\varepsilon = \nu = \mu/\rho$. It is assumed, with good reason, that the usual molecular viscosity should not vary in the "experimental fluid flow system";

(c) the following boundary conditions are satisfied.

$$\left.\begin{array}{l} u = v = 0 \text{ at } y = 0 \\ u \to U_1 \text{ as } y \to \infty \end{array}\right\} \tag{1.3}$$

where u, v, x and y have the usual meaning.

2. SEA MODEL OF THE FLUID FLOW SYSTEM

2.1 Introduction

What I refer to as the Statistical Energy Approach (SEA) is that scalar approach in the analysis of systems which is based upon total energy as the primary and indeed sole relevant system variable and which is founded upon the theory of the "general-system," as enunciated by this investigator in reference (1), with statistical mechanics as its primary analytical tool.

General system theory provides the "physical foundation of this approach and permits the extension of a generalized mechanics to the description of the basic dynamics of any system, once the system's "relevant" elements (i.e. system particles) have been identified. Statistical mechanics, on the other hand, provides the analytical tool by which the interactions of the system particles and the consequent states of the system may be studied.

Details of statistical mechanical techniques for the analysis of the interactions of sets of particles are usually commonly available in modern textbooks on that subject; this investigator especially recommends the presentations given by Khinchin (11) and Balescu (2). But details of the general system theory from which the physics of SEA derives are beyond the scope of the present work. It should suffice, nonetheless, to note that the basic tenets of SEA are generally accepted in modern physics and statistics--especially in modern general kinetic theory. Sadly, however, the potential value of these tenets in general system analyses is not very much appreciated by many of their users.

Within the framework of SEA, the analysis of any system reverts conveniently to the problem of determining the evolution or involution of the system's total energy field based upon applicable initial and boundary conditions; any other relevant system variable may be recovered by appropriate, and usually relatively simple, secondary calculations based upon modal analysis of the system's total energy field. The basic governing equation is, of course, the equation of conservation of total energy for a system particle interacting with its "relevant" environment. However, it is in the meaning and the identification of the system particle that SEA is most sensitive and most prone to errors. Nevertheless, the conception of the system particle can be made exact.

The model of a system as a group of interacting system particles arranged in accordance with some generalized structure principle and transforming in accordance with some generalized mechanics is what we refer to, herein, as the SEA, or statistical energy, model. I shall now briefly present the SEA model of the fluid flow system.

2.2 The Physical Model

Essentially, the physical SEA model of the fluid flow system is as follows: that the fluid flow system is at any instant a statistical field of energy states space-time stratified in an order either of monotonically increasing or of monotonically decreasing state total-energy values. We may further understand this model when we consider in more detail what it is we refer to as the energy state. From the basic SEA model of the general system, any system is a set of interacting particles, with prescribed initial and boundary conditions. And we define the system particle (that is, the "relevant" system element) not necessarily as a discrete entity but rather as a statistical subspace of the subject-system comprising all those physical or ordinary units of the subject-system which possess identical mean total energy and whose motion is described by exactly the same statistical law. The term "physical or ordinary" is of course, in relation to the scale of perception of the subject-system. Thus, what one accepts as a system's particle depends on the scale at which the one proposes to describe the system.

In the fluid flow system the physical or ordinary system units are usually the fluid molecules (or, in the case of dissociated fluids, atoms or ionized particles). The fluid flow system particle which we shall simply call the "fluid particle" is thus any statistical subspace of the fluid flow system containing molecules which possess identical mean total energy and whose motions are described by exactly the same statistical law. In other words, a fluid particle can be either a single fluid molecule or a group of fluid molecules, or in dissociated fluids, a single atom or ionized particle or a group of atoms or ionized particles.

Struminskiy (20) has argued quite convincingly that indeed theoretical as well as experimental data indicate that the main difference between turbulent flows involves groups of molecules rather that individual molecules; that is to say, fluid particles in laminar flows may consist of single molecules while in turbulent flows fluid particles would be groups of molecules. Generally, of course, the number of fluid units (molecules, or other) within any fluid particle will depend upon the local total energy field. The mathematical characterization of the fluid particle size in a fluid flow system is discussed later.

Let us, however, continue with our physical argumentation. We know, for example, that in the absence of any externally imposed energy field Brownian-type motion should prevail among all the molecules of any fluid system; that is, all the fluid units would have identical mean total energy and execute statistically similar motion and, therefore the fluid system would be one large fluid particle. Thus, a stagnant fluid system with no energy input would contain only one fluid particle and therefore, would possess only internal dynamics--the motion of the units within the fluid particle. A system can possess external dynamics only if it contains more than one system particle. If, however, an external energy field is imposed upon a fluid system, then flow may occur if the imposed energy field is either non-uniform or is non-uniformly constrained in the fluid flow system; this will result in local variation of the mean total energies of the fluid units which will manifest as a stratification of the fluid system on the basis of mean total energy--that is, as a shearing of the fluid. In such a flow situation, those fluid units possessing identical mean total energy group to form the flow energy states. Within each such flow energy state, one can, at least conceptually, distinguish and group together those fluid units whose motion obeys the same statistical laws--these are the fluid particles.

Quite obviously, a fluid particle can only be probabilistically described, since by definition it is a subspace of a fluid space containing those ordinary or physical units (molecules in this case) of the fluid which possess statistically similar motion as well as identical mean total energy. Furthermore, we now have a better understanding of definite differences among fluid systems. For instance, we know that a stagnant fluid with no energy input constitutes only one fluid particle and thus possesses only internal dynamics; a sheared flow must be defined as one which contains at least two energy states each of which contains fluid particles; and a non-sheared flow must contain only one energy state with at least two fluid particles.

In conclusion to this presentation of the physical conception of the SEA model of the fluid flow system, I must re-emphasize that what is being described is a statistical, not a deterministic, field of energy states. Such a statistical field is best conceptualized as an instantaneous cloud of non-uniform property density. Thus, any point in a fluid flow system is always enclosed by a fluid particle; and a fluid particle cannot be conceptualized as a fixed subspace. Therefore, contrary to possible criticism that the SEA model implies discreteness of energy states in contradiction to accepted continuum concepts in fluid systems, a careful study of the foregoing discussion should obviate the fact that the SEA model merely emphasizes that a non-uniform energy field, if imposed upon an undissociated fluid system, for example, will introduce a stratification of the fluid molecules on the basis of mean total energies: and that as a result of this stratification or "shearing" of the fluid, groups of, rather than individual, fluid molecules will become the relevant characteristic units of the fluid system.

2.3 Characterization of the Flow Energy State

From foregoing discussions of the physical SEA model of the fluid flow system, a flow energy state is seen to be merely a set of interacting fluid particles all of which possess identical mean total energies. Thus, one of the characteristic descriptive variables for any arbitrary energy state, $_j$, must be the state energy density, e_j--defined as the total energy per unit volume in that energy state. The value of e_j should be uniform throughout the flow energy state, $_j$, but, of course, only probabilistically so; for e_j is clearly a statistical variable. Each fluid particle within the energy state, $_j$, would contain molecules (or other appropriate physical units) whose total energy would correspond with e_j and whose mean total energy would also be identical.

A flow energy state may further be described as a volume of "similar" fluid particles. By the term "similar" we must certainly mean that the fluid particles possess identical mean total energy and that the motions of the fluid particles obey exactly the same statistical law. As a fluid volume, a flow energy state could be described in terms of characteristic length scales--for example: its thickness, its length and its width or, in terms of corresponding spherical or cylindrical coordinates. In this work, we shall employ the symbol, Λ_j, to represent the characteristic spatial size of the arbitrary flow energy state, $_j$; again, Λ_j, must be a statistical variable since a flow energy state is only probabilistically describable.

Finally, in a field of interacting flow energy states it would be necessary to distinguish the locations of the energy states relative to some reference frame. In this work we shall choose as our reference frame the energy state in the field which possesses the lowest energy density value; and we shall call this reference energy state the "relative zero" energy state with j=0. Furthermore, we shall employ the symbol, ℓ_j, to describe the location of any other energy state relative to the zero energy state. More precisely, and for computational facility, the local value of ℓ_j will be defined as the coordinates of any point of interest in the energy state $_j$ relative to whatever coordinate system by which the transformed energy field has been described and in which the origin of coordinates lies in the lower boundary of the relative zero energy state. In other words, ℓ_j will be employed as a local position vector radial to the lower boundary of the relative zero energy state and centered at the location of the flow field being investigated. The vector ℓ_j must again be a statistical variable for any flow energy state $_j$.

2.4 SEA Representation of the Fluid Flow Problem

We have already observed that the fluid flow problem concerns, essentially, the qualitative and quantitative description of relevant property fields (such as velocity, etc.) of fluid flow systems with appropriate elucidation of associated dynamical phenomena such as flow instability, transition and turbulence. We seek now to re-formulate this fluid flow problem in the parlance of the statistical energy approach (SEA).

The SEA model represents a fluid flow system as a field of interacting flow energy states; and each flow energy state is described as a field of interacting "similar" fluid particles, where, by the term "similar" we mean that the fluid particles possess identical mean total energy and their motions obey exactly the same statistical law. Thus, more generally a fluid flow system is modeled by SEA as a field of interacting fluid particles some of which are "similar" and some "non-similar." Quite clearly this latter conceptualization of the fluid flow system readily permits tested methods of statistical mechanics to be objectively applied in the analyses of fluid flow. We further noted that a flow energy state may be characterized by the three statistical variables e_j, Λ_j, and ℓ_j, which describe respectively: the total energy density, the spatial size and the position vector of the energy state.

Within the framework of the statistical energy approach the fluid flow problem therefore reduces to one of describing the statistical field e_j for any fluid flow system, and then recovering any desired property field of the fluid flow system from appropriate secondary calculations based upon modal analyses of the energy density field e_j.

In this work we shall refer to the instantaneous energy density field e_j, as the flow system response energy field. Generally the flow system response energy field is a space-time distribution involving the variables Λ_j, and ℓ_j or any other variables by which the flow energy state has been characterized.

The statistical energy approach thus splits the supremal fluid flow problem into two infimal problems, namely:

(a) Computation of the flow system response energy field; and

(b) Computation of any desired flow system property field by secondary analyses of the response energy field.

And in each case, statistical mechanics is the basic and self-suggesting mathematical tool.

Finally, we note that the concept of the fluid particle introduced by SEA must necessarily alter our usual formulations of transport coefficients in fluid flow systems. For instance, in the transport of momentum we may speak of molecular viscosity, per se, only in cases in which there exists only internal dynamics, that is, when there exists only one very large fluid particle; otherwise, we must define and employ a "particle" or "eddy" viscosity and not molecular viscosity. It is clear that such a particle or eddy viscosity would always be space-dependent. Even in the so-called laminar flows the appropriate viscosity is not necessarily the usual molecular viscosity, although it seems, from practical experimental results, that the appropriate laminar flow particle viscosity equals the molecular viscosity in magnitude and distribution.

2.5 Comparative Remarks About SEA and Other Statistical Approaches

At this point I must emphasize that the statistical energy approach described in this work is quite different from current and perhaps more familiar statistical mechanical approaches to turbulence studies, such as are described in references (2, 8, 12, 13, 14, and 15). There are three major differences that are easily discerned.

First, the basic models of the fluid flow system differ between SEA and conventional statistical mechanical methods. Most, if not all, contemporary statistical mechanical analyses of fluid flow model the fluid flow system as a set of interacting fluid molecules; SEA models the fluid flow system as a set of interacting "groups" of fluid molecules--the fluid particles--some of which are similar and some of which are non-similar. To this investigator's knowledge, only a recent work by Struminskiy (20) and perhaps some other subsequent work by the same and possibly follower-authors have recognized the need for the concept of fluid particles. Furthermore, as a result of this SEA model, the concept of molecular viscosity gives way to that of a space-time dependent "particle-viscosity"; and this realization automatically resolves the long-lived stress-strain relation problem in fluid dynamics.

Secondly, the techniques used to simplify the mathematical analyses differ between this work and most other statistical mechanical analyses of the fluid flow system. In this work probability theory and the methods of stochastic analyses are explicitly emphasized in contradistinction to the general and special kinetic theory approaches and approximations employed by contemporary models.

Thirdly, and most importantly, unlike most, if not all, other statistical mechanical analyses of the fluid flow system, SEA is a strictly scalar energy method. The experiences of many investigators over many ages of men have shown scalar energy methods to be more general and more powerful than those methods of analysis based upon vector concepts of force, momentum and acceleration. Nonetheless, investigators had hitherto very reluctantly avoided the use of energy methods primarily because, even though the methods were simple, they gave only global results and in the absence of general techniques for the decomposition of energy into other relevant properties such as velocity, acceleration, force, temperature, density and so on, these energy methods could not be employed in any detailed work. However, the general system theory enunciated by this investigator has now removed, or, if you will, pointed to a definite direction for the removal of,the aforementioned difficulty; it is now possible to recover most desired property-fields from a given total energy-field. Thus, the scalar energy method has been emancipated and has regained its formidability.

The statistical energy approach has been applied in different forms, and with remarkable success, to problems in acoustics and structural analysis (16). In fact, a number of large system dynamical analyses (19) are shifting towards SEA especially because of the mathematical simplicity of the method. This investigator strongly anticipates that SEA will become a standard system's analytical method once more investigators assure themselves of the validity yet simplicity of the modal analyses of the system response energy field suggested in this work for the recovery of desired property fields of systems.

Finally, SEA very clearly stands higher in the hierarchy of descriptive methods than hitherto employed methods of fluid flow analysis and as such, offers a more likely avenue, than other contemporary methods, to a better understanding and containment of the turbulence problem as well as of the problems of flow instability and transition. To illustrate this reality we shall test the SEA model by applying it to our simple, though, quite practical, "experimental fluid flow system."

3. THE TURBULENCE PARAMETER, ϕ

SEA, reference (1), identifies the turbulence field of any system as the net pure fluctuation field of that system, characterized by a mean turbulence energy, e'. And in "general-system" analyses the mean value or mathematical expectation, e', of the turbulence energy field, is shown to be a computable fraction, $\phi(\theta)$, of the system's total kinetic energy, e. Obviously this phi-parameter, which we shall generally call the turbulence parameter, is extremely important in the SEA description of fluid flow turbulence. We shall investigate in this chapter, the general nature of this turbulence parameter, $\phi(\theta)$, with respect to the fluid flow system. First, we list the relations and constraints defining $\phi(\theta)$, as derived in reference (1).

$$e' = \phi(\theta) \cdot e$$

$$\phi(\theta) \simeq 2\theta\exp(-0.736\theta^2); \text{ and} \tag{3.1}$$

$$0 \leq \phi(\theta) \leq 1$$

In the above relations, e' is the local mean turbulence energy, e is the local total kinetic energy and θ is a dimensionless quantity which can readily be shown to be directly proportional to the local flow fluctuation Reynolds number.

From relations (3.1) we deduce, as illustrated in figure (2), that $\phi(\theta)$ has a maximum value equal to unity when $\phi \simeq 0.82$, and that as $\theta \to 0$ or as $\theta \to 4, \phi(\theta) \to 0$. Thus, if in any flow system the θ-value lies close to unity, most, if not all, of that system's total kinetic energy will reside in the turbulence mode. For a standing fluid we therefore expect that θ will assume its critical value of 0.82 and should be space-time independent. On the other hand, in fluid flow systems we expect that:

$$0 \leq \theta \leq 0.82; \text{ or}$$
$$\theta > 1 \tag{3.2}$$

Since we know that in practical fluid flow systems the unit fluctuation Reynolds number is usually greater than unity and since θ is directly proportional to the fluctuation Reynolds number, we may speculate that in real flow systems the constraint, $\theta > 1$, should hold.

3.1 The Phi-Equation

Since the total kinetic energy, e, of a fluid flow system is simply the sum of the mean kinetic energy $\bar{e}$, and the turbulent kinetic energy e', the relation between e' and e, (3.1), yields the following relation between $\bar{e}$ and e.

$$\bar{e} = (1-\phi)e \tag{3.3}$$

We may now establish the equation of evolution of $\phi(\theta)$ by subtracting the equation of transport of the mean kinetic energy, $\bar{e}$, from the equation of transport of the total kinetic energy, e, using the relation (3.3) above.

For our simple "experimental fluid flow system" it can readily be shown that the transport equation for total kinetic energy could be written as follows:

$$u\partial e/\partial x + v\partial e/\partial y = \varepsilon\partial^2 e/\partial y^2-(\varepsilon/2y-\partial\varepsilon/\partial y)\ \partial e/\partial y \tag{3.4}$$

Substituting for $e \equiv \bar{e}/(1-\phi)$ in equation (3.4) and subtracting the mean kinetic equation, as is customarily done in fluid mechanics research, we obtain the equation of evolution of $\phi(\theta)$ as follows:

$$u\partial\phi/\partial x + v\partial\phi/\partial y = \varepsilon\partial^2\phi/\partial y^2 + \{\varepsilon/(1-\phi)\}\ (\partial\phi/\partial y)^2 - \{\varepsilon/2y - \partial\varepsilon/\partial y - (\varepsilon/e)\ \partial e/\partial y\}\ \partial\phi/\partial y \tag{3.5}$$

Equation (3.5) corresponds exactly to the classical "prey-predator" equation commonly employed in the study of ecosystems and the stability of natural systems. This should be expected, since ϕ may be viewed as the normalized predator (or turbulence) population feeding upon the prey (or mean flow), and thereby critically dependent on the quantity and quality of the mean flow.

The following boundary conditions must hold:

$$\begin{aligned} \phi(\theta) &\to 1 \text{ as } y \to 0 \\ \phi(\theta) &\to 0 \text{ as } y \to \infty \end{aligned} \qquad (3.6)$$

3.2 Analyses of the Phi-Equation

Let us now investigate some of the more obvious implications of the evolution equation for the turbulence parameter, ϕ, and see to what extent such implications conform with or contradict experimental observations of fluid flow systems.

We shall rewrite equation (3.5) in a transformed form using the Levy-Lees transformation for a flow over curved bodies as fully discussed and employed in reference (18); and we shall use exactly the notations of reference (18). Thus, the (x, y) space transforms into the $\{\xi(x), \eta(x, y)\}$ space, and the (u, v) velocity field transforms into the (F, V) dimensionless velocity field; r_o describes the body radius and t describes the transverse curvature (r/r_o), while j describes the flow type (j=0, for planar flow; j=1, for axisymmetric flow). Using the above transformation and with primes indicating partial differentiation with respect to η equation (3.5) transforms to the following:

$$\phi'' + C_2\phi'^2 + C_1\phi' + C_0 = 0 \qquad (3.7)$$

where: $C_2 = 1/(1-\phi)$

$C_1 = -\{T_1 + T_2 - T_3 - T_4 - T_5\}$

$C_0 = K(\partial\phi/\partial\xi)$

$$T_1 = (\sqrt{2\xi})/(2y(\rho U_e r_o^{\,j} t^j))$$

$$T_2 = V\nu_e(\rho_e/\rho)^2/(\varepsilon t^{2j})$$

$$T_3 = \varepsilon'/\varepsilon$$

$$T_4 = 2F'/F$$

$$T_5 = (\rho/\rho_e)'/(\rho/\rho_e)$$

$$K = -2\xi\nu_e F(\rho_e/\rho)^2/(\varepsilon t^{2j})$$

Let us now separately inspect the coefficients C_0, C_1, and C_2 occurring in equation (3.7).

C_2 is the coefficient of the source of non-linearity in equation (3.7); it is a function only of ϕ and is definitely non-zero; thus, the evolution of $\phi(\theta)$ transverse to the flow direction will always be non-linear.

C_1 does not explicitly contain ϕ, but is composed of five terms which seem to effectively import the influences of various flow conditions as follows:

T_1 : The basic term; also importing the effect of externally-impressed stream curvature

T_2 : imports the impact of the transverse flow (i.e. transpiration at the boundaries, etc.) and the influence of externally-impressed stream curvature; also the direct influence of viscosity and compressibility are carried by this term

T_3 : imports the direct effects of shear and thermal stratification

T_4 : imports the effect of shear; and

T_5 : imports the effects of compressibility and thermal stratification.

Thus, in terms of the impact or influence of flow type and flow boundary and initial conditions on the turbulence field of a flow system, it is clearly the coefficient C_1 that is critical.

C_o contains essentially the influence of the streamwise gradient of ϕ.

We may conclude from the above that the evolution of ϕ intricately involves the impacts of flow type and flow boundary and initial conditions, as would be expected in real flow situations. However, it remains to be determined if the impacts of flow type and flow conditions on the turbulence parameter follow the directions usually observed in real flow situations. To make this determination we must attempt an analytical solution of equation (3.7) and then inspect such a solution, or we may numerically experiment on equation (3.7) in the computer, varying the different impacts imported by the coefficients C_2, C_1, and C_0. To an extent we shall execute both alternatives in this report.

An analytical solution of equation (3.7) may be initiated by transforming the equation into the classical Abel's equation, with functional coefficients, and employing Kamke's, reference (10), solution to the Abel equation. These efforts yield the following integro-differential equation:

$$\phi' = -\left\{C_1\phi(1-\phi/2 - \phi^2/6-\cdots) + (1-\phi)\S\right\} \tag{3.8}$$

where: $\S = \int K\{(\partial\phi/\partial\xi)/(1-\phi)\}\{1/(C_1\xi)\}d\xi$

The solution may be completed for $\phi(\xi,\eta)$ if $\S$ can be explicitly integrated. Nonetheless some deductions may yet be made concerning the evolutions of ϕ by the inspection of the partial solution given in equation (3.8).

We clearly see from equation (3.8) that the ϕ-profile at any ξ-station will be described by a function of η decreasing from a maximum at fixed boundaries to a minimum in the free stream: the slope of the curve being critically determined by the coefficient C_1.

The ξ-evolution of the turbulence parameter is more difficult to explicitly determine from equation (3.8), but the general nature of the solutions to the "prey-predator" equation, to which class equation (3.7) belongs, suggests that ϕ will evolve like "logistic" curves, from initially low values through a transitional range of growing values to a final range of high, but constant or only slowly changing, values. This evolutionary trend conforms with physical reality.

Furthermore, by inspecting the signs of the terms T_1 through T_5 of the coefficient C_1 in the partial solution, equation (3.8), it is easy to see that the directions of the impacts of flow type and flow conditions also conform with physical reality.

Thus, the turbulence parameter, ϕ, appears to be a very plausible conception.

3.3 Numerical Solution of the Phi-Equation

A preliminary numerical solution of the transformed equation (3.7) is attempted herein as follows:

Using the notation in reference (18), the following approximations are made.

$$\phi''_{m+1,\,n} = Y_1\phi_{m+1,\,n+1} - Y_2\phi_{m+1,\,n} + Y_3\phi_{m+1,\,n-1} \tag{3.9}$$

$$\phi'_{m+1,\,n} = Y_4\phi_{m+1,\,n+1} - Y_5\phi_{m+1,\,n} - Y_6\phi_{m+1,\,n-1} \tag{3.10}$$

$$(\phi'_{m+1,n})^2 \simeq \phi'_{m,n} \cdot \phi'_{m+1,\,n} \tag{3.11}$$

$$\phi^2_{m+1,\,n} \simeq \phi_{m,\,n} \cdot \phi_{m+1,\,n} \tag{3.12}$$

$$(\partial\phi/\partial\xi)_{m+1,\,n} \simeq \{(X_1X_2 - X_3)\phi_{m+1,\,n} - (X_2X_5 - X_3X_4)\phi_{m,\,n}\}/(2\Delta\xi_2 \cdot X_5) \tag{3.13}$$

Equation (3.7) can then be rewritten as follows:

$$-A_n\,\phi_{m+1,\,n+1} + B_n\,\phi_{m+1,\,n} - C_n\,\phi_{m+1,\,n-1} = D_{m,\,n} \tag{3.14}$$

where: $A_n = -(Y_1 + G^*Y_4)$

$B_n = \{C_0^* \,(X_1X_5 - X_3)/(2X_5\Delta\xi_2) - Y_2 - G^*Y_5\}$

$C_n = (G^*Y_6 - Y_3)$

$$D_{m,\,n} = C_0^{\,*}(X_2X_5 - X_3X_4)\phi_{m,\,n}/(2X_5\Delta\zeta_2)$$

$$G^* = \phi'_{m,\,n}/(1-\phi_{m,\,n}) + (C_1)_{m+1,\,n}$$

$$C_0^{\,*} = -\{2\xi\nu_e(\rho_e/\rho)^2F/(\varepsilon t^{2j})\}_{m+1,\,n}$$

Equation (3.14) has the tridiagonal matrix form and can readily be solved to yield $\phi_{m+1,\,n}$.

The numerical approximations employed herein are clearly not the best possible; the convergence obtained has not been satisfactory. Nonetheless, figures (3) and (4) show a typical evolution of ϕ in a shear flow. A more detailed numerical analysis of equation (3.7) will be performed in a future work.

4. THE TURBULENCE FLOW FIELD

4.1 The Mean Turbulence Energy Field

As already indicated in the preceding chapter, "general-system" (g-s) theory gives the mathematical expectation, e' of the turbulence energy field of any system as a computable fraction, ϕ, of that system's total kinetic energy field, e. Since the total kinetic energy field of a system is defined in g-s theory as the sum of the turbulence kinetic energy, e', and the mean flow energy $\bar{e}$, the following relations result:

$$e' = \phi \cdot e = \Psi \cdot \bar{e} \tag{4.1}$$

$$\Psi = \phi/(1-\phi) \tag{4.2}$$

The above relations imply that once the total kinetic or the mean flow energy field of a fluid flow system is known, the corresponding mean turbulence kinetic energy field may be considered known, if the field of the turbulence parameter, ϕ, is also known.

Usually, however, we may know the total velocity field or the mean velocity of a flow system and we would want to explicitly compute the mean turbulent velocity field in some given coordinate directions. In order to accomplish this task we would have to exploit equations (4.1) and (4.2) in search of practical and valid formulations by which we may recover velocity fields from total kinetic energy fields in any desirable coordinate direction. Let us now attempt that task for the mean turbulent velocity field.

Let us represent the total, mean and turbulence velocity fields of a fluid flow system by the vectors V, $\bar{V}$, and V', respectively. In three-dimensional cartesian coordinate system these velocity fields become

$$\begin{aligned} V &\equiv (u, v, w) \\ \bar{V} &\equiv (\bar{u}, \bar{v}, \bar{w}) \\ V' &\equiv (u', v', w') \end{aligned} \tag{4.3}$$

In terms of these velocity fields, the energies, e , $\bar{e}$, and e' may be expressed as follows:

$$
\begin{aligned}
e &\equiv \rho(u^2 + v^2 + w^2)/2 \\
\bar{e} &\equiv \rho(u^2 + v^2 + w^2)/2 \\
e' &\equiv \rho(u'^2 + v'^2 + w'^2)/2
\end{aligned}
\qquad (4.4)
$$

And from equations (4.1), (4.2), and (4.4) we obtain the relations:

$$
(u'^2 + v'^2 + w'^2) = \phi(u^2 + v^2 + w^2) \equiv (\phi/(1-\phi))(\bar{u}^2 + \bar{v}^2 + \bar{w}^2) \qquad (4.5)
$$

A general expression for the turbulence components in the principal directions of the reference frame may be given as follows:

$$
u'^2_j = \phi(\gamma_{ij} u_i^2) \equiv (\phi/(1-\phi))(\gamma_{ij} \bar{u}_i^2);\ i,\ j,\ = x,\ y,\ z \qquad (4.6)
$$

where:

$u_x = u$, $u_y = v$, $u_z = w$; $u'_x = u'$, $u'_y = v'$, $u'_z = w'$; $\bar{u}_x = \bar{u}$, $\bar{u}_y$, $= \bar{v}$, $\bar{u}_z$, $= \bar{w}$; and the γ-terms represent the fractional redistribution of the turbulence energy from one principal coordinate direction to another. Obviously, the following constraint applies:

$$
(\gamma_{xx} + \gamma_{xy} + \gamma_{xz}) = (\gamma_{yx} + \gamma_{yy} + \gamma_{yz}) = (\gamma_{zx} + \gamma_{zy} + \gamma_{zz}) = 1 \qquad (4.7)
$$

As a first approximation in this work we had assumed that:

$$
\left.\begin{aligned}
\gamma_{ij} &= 1 \text{ for } i = j \\
\gamma_{ij} &= 0 \text{ for } i \neq j
\end{aligned}\right\} ;\ i,\ j = x,\ y,\ z \qquad (4.8a)
$$

We have found this assumption not to be acceptable since we know that w'^2 may be quite different from zero even in flows which are two-dimensional in the mean. We could correct for such an assumption by complicating the $\phi(\theta)$-function; but that would be highly undesirable.

The γ-terms may be formulated in terms of the total and mean velocity fields by invoking an analogy between the γ-tensor, apparent from the above relation, and the well-known strain tensor. This exercise is, however, rather delicate; it will be given full attention in a later work.

For the purpose of the remainder of this work we shall make a second approximation, namely, that:

$$\left.\begin{aligned} \gamma_{ij} &= a < 1;\ \text{for all } i = j \\ \gamma_{ij} &= (1-a)/2;\ \text{for all } i \neq j \end{aligned}\right\} ;\ i,\ j,\ = x,\ y,\ z \qquad (4.8b)$$

This assumption automatically satisfies the constraints (4.7) and appears, in fact, to be a reasonable approximation in many practical fluid flow systems. The quantity, a, appears to be a system constant. We shall experiment with a few values of a, in the range $0.3 \leq a \leq 0.6$; but in a later and more detailed work we shall attempt to find appropriate formulations for a.

The foregoing results indicate that once the total or the mean velocity field of a fluid flow system is known, then only a knowledge of the ϕ-parameter is required to obtain the corresponding mean turbulence field for the system.

4.2 The Probability Law of Turbulence Intensity

General-system theory indicates that if we define a turbulence intensity, $I = e'/e$, the local ratio of the turbulence kinetic energy to the total kinetic energy, then the probability density function of I is given as follows:

$$\begin{aligned} f_I(i) &= f_X(g_1)\,|dg_1/di| + f_X(g_2)\,|dg_2/di|,\ 0 < i < 1 \\ &= 0 \qquad\qquad ;\ \text{elsewhere} \end{aligned} \qquad (4.9)$$

where: $g_1 = \theta\{1 + (1-i^2)^{\frac{1}{2}}\}/i$

$g_2 \quad \theta\{1 - (1-i^2)^{\frac{1}{2}}\}/i$

$$\begin{aligned} f_X(x) &= 128x^5/\{\exp(5\ x) - 1\}\ ;\ o < x < \infty \\ &= 0 \qquad\qquad ;\ \text{elsewhere} \end{aligned}$$

and θ is, as previously defined, proportional to the local fluctuation

Reynolds number.

Since $\phi = 2\theta\exp(-0.736\theta^2)$, our earlier speculation that the constraint, $\theta < 1$, must hold yields, that:

$$\theta \approx \{-\ln(0.5\phi)\}^{\frac{1}{2}} \qquad (4.10)$$

Thus, once we have computed the ϕ-field for a fluid flow system we may not only compute the mean turbulence field but also, from knowledge of $f_I(i)$, we may completely describe the statistical field of the turbulence.

4.3 The Spectrum of Turbulence Energy

Finally, general-system theory also provides a spectral representation of the turbulence energy field.

If in any energy state of a flow system the most probable turbulence frequency is ω_c, then the turbulence kinetic energy, E_z, which resides in the turbulence frequency $\omega \equiv z \cdot \omega_c$ in that energy state is given by general system theory as follows:

$$(E_z/e) \approx 256\theta z^6/\left[(z^2 + \theta^2)\{\exp(5z) - 1\}\right] \qquad (4.11a)$$

or $$(E_z/\bar{e}) \approx 256\theta z^6/\left[(1-\phi)(z^2 + \theta^2)\{\exp(5z) - 1\}\right] \qquad (4.12a)$$

Employing equation (4.10) we have that:

$$(E_z/e) \approx 256z^6/D \qquad (4.11b)$$

$$(E_z/\bar{e}) \approx 256z^6/\{D(1-\phi)\} \qquad (4.12b)$$

where: $D = \{(z^2 - \ln 0.5\phi)/(-\ln 0.5\phi)^{\frac{1}{2}}\}\{\exp(5z) - 1\}$

Quite obviously, these spectral forms conform with observed reality; the bulk of the turbulence kinetic energy is seen to reside in the lower frequency fluctuational modes.

5. THE MEAN FLOW FIELD

In the context of "general-system" theory the mean motion of a fluid flow system (i.e. the mean flow field) is the instantaneous sum of the pure translational and the pure rotational motion of that system. This is the usually observed mean motion of the fluid flow system.

Let us again denote the total motion of the fluid flow system by the velocity field, $V \equiv (u, v, w)$, in three-dimensional cartesian coordinate system; and let us denote the mean (or observed) motion by $\bar{V} \equiv (\bar{u}, \bar{v}, \bar{w})$; the pure rotational motion by the velocity V_R and the pure translational motion by the velocity, $V_Q \equiv (q_x, q_y, q_z)$.

The mean flow energy, $\bar{e}$, is then given as follows:

$$\begin{aligned} \bar{e} = \tfrac{1}{2}\rho\bar{V}^2 &\equiv \rho(V_Q^2 + V_R^2)/2 \\ &\equiv \rho(1-\phi)V^2/2 \end{aligned} \tag{5.1}$$

The pure rotational velocity field may be written as: $V_R = \vec{\omega}x\vec{r}$, where in this case $\vec{r} \equiv \{r_x \cdot \underline{i} + r_y \cdot \underline{j} + r_z \cdot \underline{k}\}$ is the vector of the local fluid particle rotation. Since the general motion of a system particle within any energy state may usually be replaced by the motion of the center of volume, A, of that particle plus a pure rotation and a pure vibration of the particle about A such that the total kinetic energy density of the particle corresponds to the total kinetic energy density value in the energy state, it becomes obvious that in this case $\vec{r}$ may be considered identical with the fluid particle characteristic radius, $\vec{r}_c$, already discussed in reference (1); $|r_c|$ essentially represents the characteristic radius of that fluid particle whose surface passes through the space-point of interest. Reference (1) suggests the following formulations for $\vec{r}_c$:

$$r_x^2 = m^2\bar{x}^2/(Re_{\bar{x}}|u^*|) \tag{5.2a}$$

$$r_y^2 = m^2\bar{x}\bar{y}/(Re_{\bar{x}}|v^*|)$$

$$\text{or:}\quad r_y = m\bar{y}\{(1-\phi)u^*\}^{\frac{1}{2}}/\{Re_x^{\frac{1}{2}}|\partial u^*/\partial y|\} \tag{5.2b}$$

$$r_z^2 = m^2\bar{x}\bar{z}/(Re_{\bar{x}}|w^*|) \tag{5.2c}$$

where u^*, v^*, w^* are the component total velocities non-dimensionalized with U_{max}, and $Re_{\bar{x}} = (U_{max}x/\nu)$; m is a numerical constant and $\bar{x}$, $\bar{y}$, and $\bar{z}$ are respectively the effective streamwise, transverse and lateral coordinates in three-dimensional rectangular cartesian coordinate system.

The pure rotational velocity may also be written in three-dimensional cartesian coordinate system as follows:

$$V_R = (\omega_y r_z - \omega_z r_y)\cdot \underline{i} + (\omega_z r_x - \omega_x r_z)\cdot \underline{j} + (\omega_x r_y - \omega_y r_z)\cdot \underline{k} \tag{5.3}$$

where:

$$\begin{aligned} \omega_x &= (\partial w/\partial y - \partial v/\partial z)/2 \\ \omega_y &= (\partial u/\partial z - \partial w/\partial x)/2 \\ \omega_z &= (\partial v/\partial x - \partial u/\partial y)/2 \end{aligned} \tag{5.4}$$

Thus, the observed mean velocity field may be written as follows:

$$\begin{aligned} \bar{u} &= q_x + \omega_y r_z - \omega_z r_y \\ \bar{v} &= q_y + \omega_z r_x - \omega_x r_z \\ \bar{w} &= q_z + \omega_x r_y - \omega_y r_x \end{aligned} \tag{5.5}$$

and:

$$(\bar{u}^2 + \bar{v}^2 + \bar{w}^2) = (1-\phi)(u^2 + v^2 + w^2) \tag{5.6}$$

From equation (5.6) we may generalize the following relations between the observed and the total velocity fields in fluid flow systems:

$$\bar{u} = \{(1-\phi\gamma_{xx})\cdot u^2 - \phi\gamma_{xy}v^2 - \phi\gamma_{xz}\cdot w^2\}^{\frac{1}{2}}$$

$$\bar{v} = \{-\phi\gamma_{yx} u^2 + (1-\phi\gamma_{yy}) \cdot v^2 - \phi\gamma_{yz} \cdot w^2\}^{\frac{1}{2}}$$
$$\bar{w} = \{-\phi\gamma_{zx} \cdot u^2 - \phi\gamma_{zy} \cdot v^2 + (1-\phi\gamma_{zz}) \cdot w^2\}^{\frac{1}{2}} \tag{5.7a}$$

where ϕ and the γ-terms are as previously defined in the preceding chapter. If, according to our second approximation of the preceding chapter, we take $\gamma_{ij} = a$, for all $i = j$ and $\gamma_{ij} = (1-a)/2$ for all $i \neq j$, then equations (5.7a) reduce as follows:

$$\bar{u} = G_1\{u^2 - G_2(v^2 + w^2)\}^{\frac{1}{2}}$$
$$\bar{v} = G_1\{v^2 - G_2(u^2 + w^2)\}^{\frac{1}{2}} \tag{5.7b}$$
$$\bar{w} = G_1\{w^2 - G_2(u^2 + v^2)\}^{\frac{1}{2}}$$

where:

$$G_1 = (1 - \phi a)^{\frac{1}{2}} \text{ and } G_2 = \phi(1 - a)/\{2(1-\phi a)\}$$

If, therefore, the total velocity field of the fluid flow system is known, the observed mean velocity field may readily be computed from a knowledge of ϕ and a. If we want to compute the actual pure translational and pure rotational velocities we must employ equations (5.3) and (5.4).

It is easy to see from the foregoing that in the so-called boundary layer flows, the observed mean velocity field would be mostly rotational velocity; that is, pure translation is very small in the boundary layer compared to pure rotation.

Alternatively, the quantities $\bar{u}$, $\bar{v}$, and $\bar{w}$, may be computed from the evolutionary equation for $\bar{e}$ as obtained from equations (3.4) and (3.5) in a preceding chapter. For our "experimental fluid flow system" we would have the following equations for $\bar{e}$ and its components:

$$u\partial\bar{e}/\partial x + v\partial\bar{e}/\partial y = \varepsilon\partial^2\bar{e}/\partial y^2 - \{\varepsilon/2y - \varepsilon/(1-\phi) \cdot \partial\phi/\partial y - \partial\varepsilon/\partial y\}\partial\bar{e}/\partial y \tag{5.8a}$$
$$u\partial\bar{u}^2/\partial x + v\partial\bar{u}^2/\partial y = \varepsilon\partial^2\bar{u}^2/\partial y^2 - \{\varepsilon/2y - \varepsilon/(1-\phi) \cdot \partial\phi\partial y - \partial\varepsilon/\partial y\}\partial\bar{u}^2/\partial y \tag{5.8b}$$
$$\bar{e} = (\bar{u}^2 + \bar{v}^2)/2 \tag{5.8c}$$

6. THE COMPUTATION OF THE FLUID FLOW SYSTEM

6.1 A Hierarchy of Descriptive Philosophy

The fluid flow system may be described in any one of several ways depending on the scale of fundamentality of property-fields that one is interested in. This freedom in description sets up what I have called a hierarchy of descriptive philosophy, based upon the elementarity of the chosen basic property-field.

In physical reality the fluid flow system is most fundamentally an energy system; that is, its sole basic property-field is "energy." Any description of the fluid flow system as an energy field may therefore be referred to as a "single-field" or "general system" approach; it would be the highest physical description of the fluid flow system.

Lower down in the hierarchy of descriptive philosophy we may choose to describe a fluid flow system as a mixture of "forces" and "energy"; such a description would conform with the classical Hamiltonian mixed-field approach.

But by far the most popular and, therefore, the dominant description of the fluid flow system in contemporary fluid mechanics research is the Newtonian mixed-field approach in which the system is viewed as a mixture of "momenta" and "force" fields. Clearly, the Newtonian approach is lower in the hierarchy of descriptive philosophy than the Hamiltonian approach.

The reason for a descent in the hierarchy of descriptive philosophy is usually the desire to obtain more detailed information in an explicit form and on as many property-fields of the subject-system as possible. For instance, the general system approach would usually yield only the total energy field of a subject system while the Hamiltonian approach would provide the force field as well as the total kinetic energy fields; the Newtonian approach, on the other hand, would explicitly yield the velocity, mass-density, temperature, pressure and other property-fields of the same subject system.

But what one gains in explicit "detail" the one loses in explicit "complexity" of the chosen method. The Newtonian mixed-field approach requires more complicated equations, often with severe closure problems, for the description of the fluid flow system than does the Hamiltonian mixed-field approach; and, as we have seen in ref (1) of this report, the general system approach describes the fluid flow system by one very simple equation--the equation of conservation of total energy.

6.2 The General-System Approach

In employing SEA (or the general system approach) for the computation of the fluid flow system, essentially only the following three procedures are required:

(i) transform the ordinary space of the fluid flow system by some set of transformations into the general system space, for instance as discussed in references (17) and (22),

(ii) solve the simple fundamental equation of the general system in the transformed or general system space; and

(iii) transform the solutions from (ii), above, back into the ordinary space of the fluid system.

What one obtains in (iii), above, is the response total energy field in the ordinary space of the fluid flow system. In order to recover any other property-field from this total energy field one must employ the results of the analyses of energy suggested by this investigator, some of which have been presented in this report, and all of which are based upon this investigator's general system theory.

The foregoing notwithstanding, a fundamental difficulty in the full application of the general system approach still remains unresolved at the time of this report. This concerns the elicitation of a simple set of transformations for transforming the ordinary space of any system into the general system space. A number of relevant transformations have been suggested by different investigators such as in references (17) and (22), but these transformations still make more demands on computer time and space than the present investigator strongly believes is necessary; after all, the aim of the exercise of the general system approach is to attain maximum simplicity with maximum accuracy and not just to find another alternative approach. At the moment, given a fairly large computer space and time (though much smaller than what would be required by other contemporary methods to provide the same or equivalent amount of results) this investigator's general system approach can yield very detailed results on any fluid flow system; but such computer space and time are not easily available.

6.3 A Derived Newtonian Approach

An alternative way to test or demonstrate this investigator's general system approach is to deduce from it the relevant descriptive equations in a lower scale of description, such as the popular Newtonian mixed-field scale, for instance, and then to examine the correspondence of the deduced description with experimentally verifiable knowledge on real fluid flow systems. In doing so we have come to the conclusion that the system of N-S equations of fluid mechanics is in itself a complete and correct description of the dynamics of fluid flow in a vector space, provided that:

(a) all the dependent variables in the N-S equations are understood to be total variables admitting only of statistical description;

(b) the viscosity term, ε, is understood to be not the usual kinematic viscosity based upon molecular transport of momentum, but rather the generalized "fluid particle" or "eddy" viscosity based upon the transport of momentum by the conceptual fluid particles of fluid flow systems; molecular viscosity would be only a special case of this eddy viscosity; and

(c) the conventional understanding of pressure is modified to explicitly distinguish between the internal pressure of a fluid flow system and the internal gravitational forces in the fluid system; this is especially important in the generalization of the Bernoulli equation.

For the purpose of practical computations in this report the N-S equations will, therefore, be accepted with the viscosity term, ε, given by the following simple form:

$$\varepsilon = \nu(1 + T \cdot \varepsilon_{\tau}/\mu) \qquad (6.1)$$

where T is proportional to the integrated value of the ϕ-parameter at any x-station and ε_{τ} is any one of the currently employed eddy viscosity models in turbulent fluid flow analyses. In effect, T will serve in this case as the usual intermittency factor for a flow in which transition is imagined to begin at the leading edge or entrance region.

Although equation(6.1) is not exact, it should suffice to prove the validity or invalidity of the concepts suggested by the general system theory as applied to our "experimental fluid flow system"

To compute the "experimental fluid flow system" we shall execute the following procedures:

(i) solve the following system of coupled equations for the velocity and ϕ-profiles, for all x-stations in the fluid flow system:

$$\partial u/\partial x + \partial v/\partial y = 0 \qquad (6.2)$$

$$u\partial u/\partial x + v\partial u/\partial y = \partial(\varepsilon\partial u/\partial y)/\partial y \qquad (6.3)$$

$$\varepsilon = \nu(1 + T\ \varepsilon_{\tau}/\mu) \qquad (6.4)$$

$$u\partial\phi/\partial x + v\partial\phi/\partial y = \varepsilon\partial^2\phi/\partial y^2 + \{\varepsilon/(1-\phi)\}(\partial\phi/\partial y)^2 - \{\varepsilon/2y - \partial\varepsilon/\partial y - (\varepsilon/e)\partial e/\partial y\}\partial\phi/\partial y \qquad (6.5)$$

$$T = \mathcal{L} \int_0^\delta \phi \partial y \tag{6.6}$$

$$e \equiv u^2 + v^2 + w^2 \approx u^2 \text{ (in this case)} \tag{6.7a}$$

and ϵ_T is given by the usual mixing length eddy viscosity model and the following boundary conditions are observed:

$$\left.\begin{array}{l}\phi \to 1 \\ u \text{ and } v \to 0\end{array}\right\} \text{as } y \to 0$$
$$\phi \to 0 \text{ and } u \to U_1; \text{ as } y \to \infty \tag{6.7b}$$

(ii) At each x-station compute the mean flow field as follows:

$$\bar{u} = G_1(u^2 - G_2v^2)^{\frac{1}{2}} \tag{6.8}$$

$$\bar{v} = G_1(v^2 - G_2u^2)^{\frac{1}{2}} \tag{6.9}$$

where $G_1 = (1 - \phi \cdot a)^{\frac{1}{2}}$, $G_2 = \phi(1 - a)/\{2(1 - \phi \cdot a)\}$, and a is experimentally varied between 0.3 and 0.6.

(iii) At each x-station compute the turbulent flow field as follows:

$$u'^2 = G_3 \ (u^2 + G_4v^2) \tag{6.10}$$

$$v'^2 = G_3 \ (v^2 + G_4u^2) \tag{6.11}$$

$$w'^2 = G_3G_4(u^2 + v^2) \tag{6.12}$$

where $G = \phi.a$, $G_4 = (1 - a)/2a$ and a is as previously defined.

These three steps complete the computation of the flow field. Other aspects of the flow system may be determined in the usual manner from the mean and the turbulent velocity fields.

Time has not permitted us to include in this report the results of the actual demonstration of the above computations for our "experimental fluid flow system." In a later report these results will be presented.

7. CONCLUDING REMARKS

A primary conclusion from this research work is that the fluid flow system is sensibly, fully and practically described by the "general-system" theory enunciated by this investigator. This statement rests firmly on at least the following theoretical and practical observations:

First, although the explicit derivation of the governing equations of fluid flow from the fundamental equation of the "general-system" is not included in this report, the similarity between the latter and the Navier-Stokes (N-S) equations is rather obvious. Indeed, the N-S equations can readily be shown to derive from the fundamental equation of the general system; hence our confidence in the N-S equations as a valid model of fluid dynamics. But, although the N-S equations may now enjoy our full confidence, we note that without the benefit of the insight into their nature generated by "general-system" theory and the SEA model, the N-S equations may easily become grossly misunderstood and misused.

Secondly, from current theoretical and experimental knowledge of the fluid flow system, the SEA model presented in this work appears to be inherently consistent and complete, vis-a-vis, the provision of deep insight into the real physics of fluids. The model explains the real mechanics of fluid flow in greater detail than do most other contemporary models of the fluid flow system. But above all, by obviating very clearly the relationship among the different types and the different modes of fluid flow, the SEA model greatly simplifies the analyses of fluid flow systems. In this regard, the nature of turbulence, the nature of the "observed" mean flow and the nature of the relationship between the turbulence field and the observed (or measured) mean flow field deserve particular attention.

Turbulence is clearly identified to be the pure fluctuational mode of the fluid particles of the fluid flow system. And, contrary to classical affirmations such as that the turbulence field of the fluid flow system is not generally a unique function of the mean velocity profile, the SEA model demonstrates that indeed the mean turbulence field can be expressed as a computable unique function of the mean flow field. General system philosophy clearly exposes that the errors in earlier prediction methods which led to the affirmation that the turbulence field is not generally a unique function of the mean flow field were not at all due to the reliability of those methods on the mean flow field for the description of the turbulent field; rather, the errors were due to the fact that most, if not all currently available theoretical methods in fluid mechanics, are founded either upon no fundamental philosophical considerations at all, or upon erroneous philosophical wiseacring. For instance, the examination of experimental data should not automatically lead to some intuitive or empirical formulation to describe the "observed" trend; without consideration of virtually all possible varieties of the observed system such intuitive formulations would usually be at best restrictive and would often be completely erroneous, for the simple reason that dynamical system behavior is often counter-intuitive. To create a realistic relation from physical observations it is critical that one formulate a valid philosophical base--a consistent and complete foundation or viewpoint that includes the observed data. Contained within or derived from such a philosophical base would be a theory upon which the empirical or exact relations valid in the observed system may be founded.

The error-situation being clarified above is analogous to the case in medicine where the symptoms of a disease have been focussed upon and elaborately described while the true nature of the disease and the mechanics by which it generates those symptoms have completely eluded the doctor. Most of what currently available theoretical models of fluid flow describe are observational--optical and instrumental--illusions; hardly any model, before SEA, described the real nature of the fluid flow system. This rather pompous statement is borne out by the fact that, with very little effort, virtually every hitherto given description of fluid flow, upon which numerous theoretical models hinge, can be reconstructed as observational illusions from the point of view of the SEA model presented herein.

I would not be belaboring my criticism of the "symptomatic-approach" commonly practised in fluid mechanics research were it not for the inherently adverse impacts of that approach. Especially, one major line of inquiry, arising from the aforementioned "symptomatic-approach," and one which has, more than any other, seriously stymied the development of turbulence theories, is the continued combination, in all turbulence models, of the pure fluctuation and the pure dissipative energy fields. This error and its destructive impacts are not easy to see outside the viewpoint of the SEA model. It is true that the dissipative energy field appears to be derived from the fluctuation energy field, but it is also true that these two fields are characteristically very different. We have shown in reference (1) that the gross production and gross dissipation of fluctuation energy in any system are extremely complicated quantities to theoretically describe. What we measure as the turbulence field is only the net fluctuation energy field--a much simpler quantity to compute. The dissipative energy field manifests as the pressure and temperature fields of any system and should be considered separately from the turbulence field: only their interaction or interdependency need be noted.

Finally, let us remember that the Statistical Energy Approach (SEA) is a general approach valid not just for the description of the fluid flow system but for the description of any conceivable natural system. It is based upon an internally consistent and universally complete philosophical viewpoint transcending the limitations of subjectivity.

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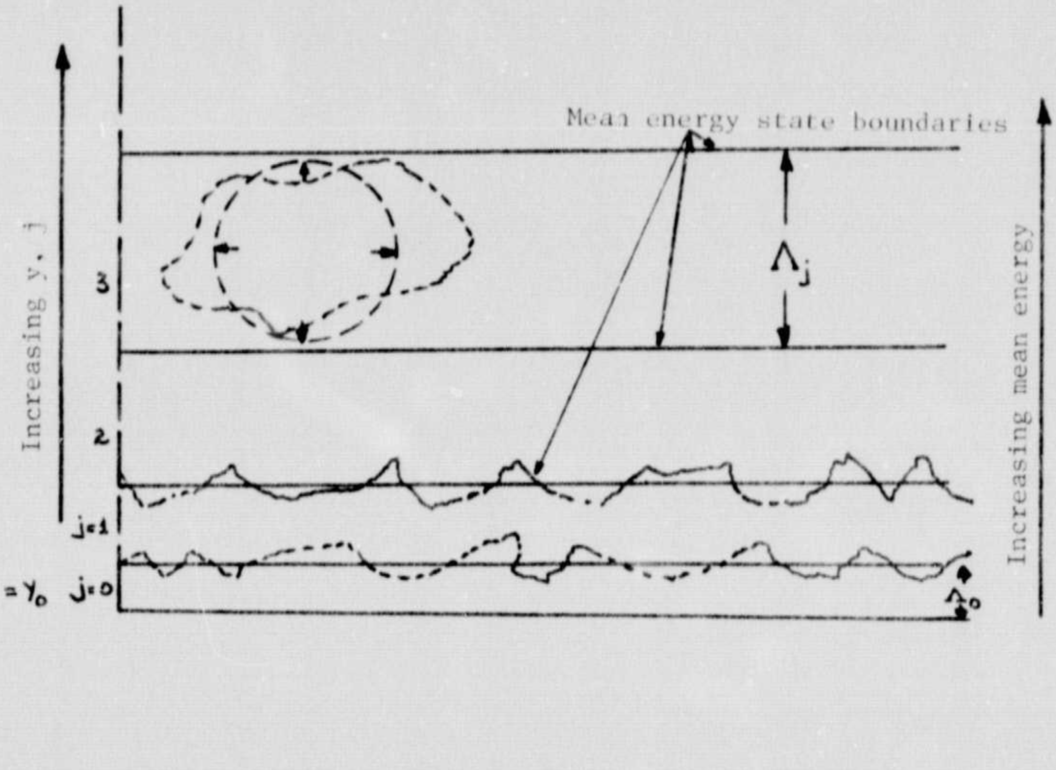

Fig. 1: Schema of the statistical energy state concept

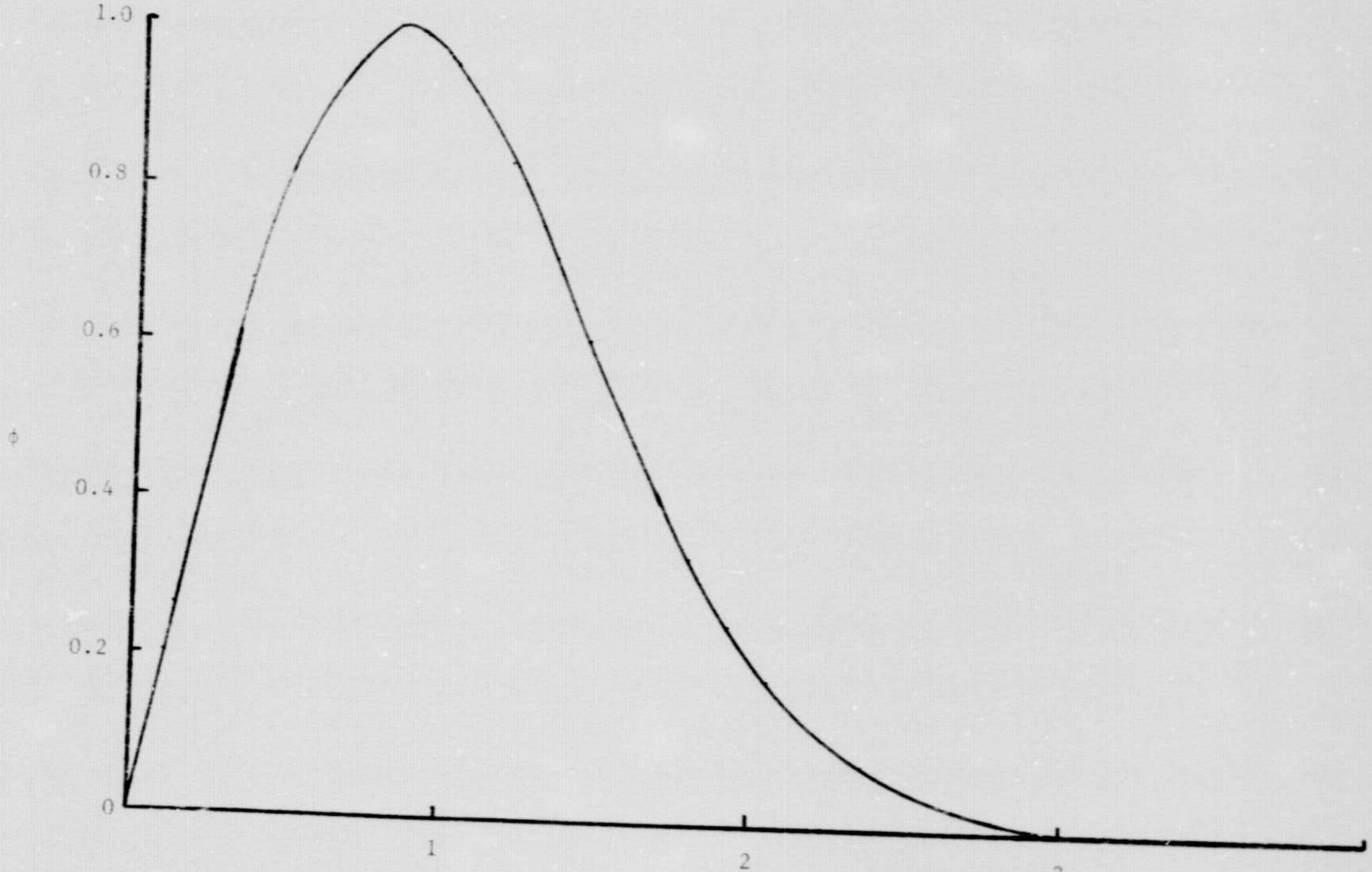

Fig. 2: Permissible Ranges of the Theta and Phi-Parameters

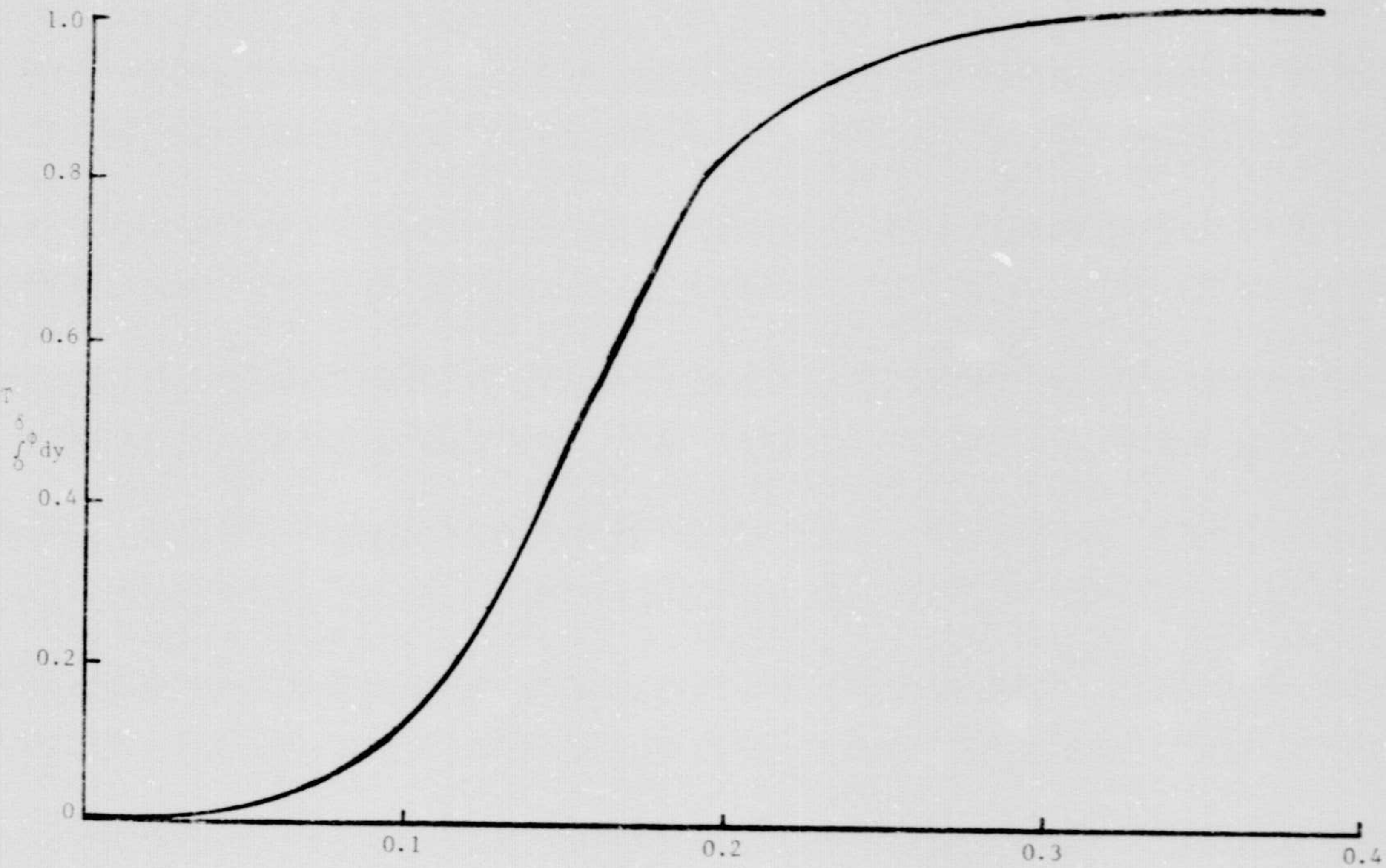

Fig. 3: Evolution of the Phi-Parameter In Test Shear Layer

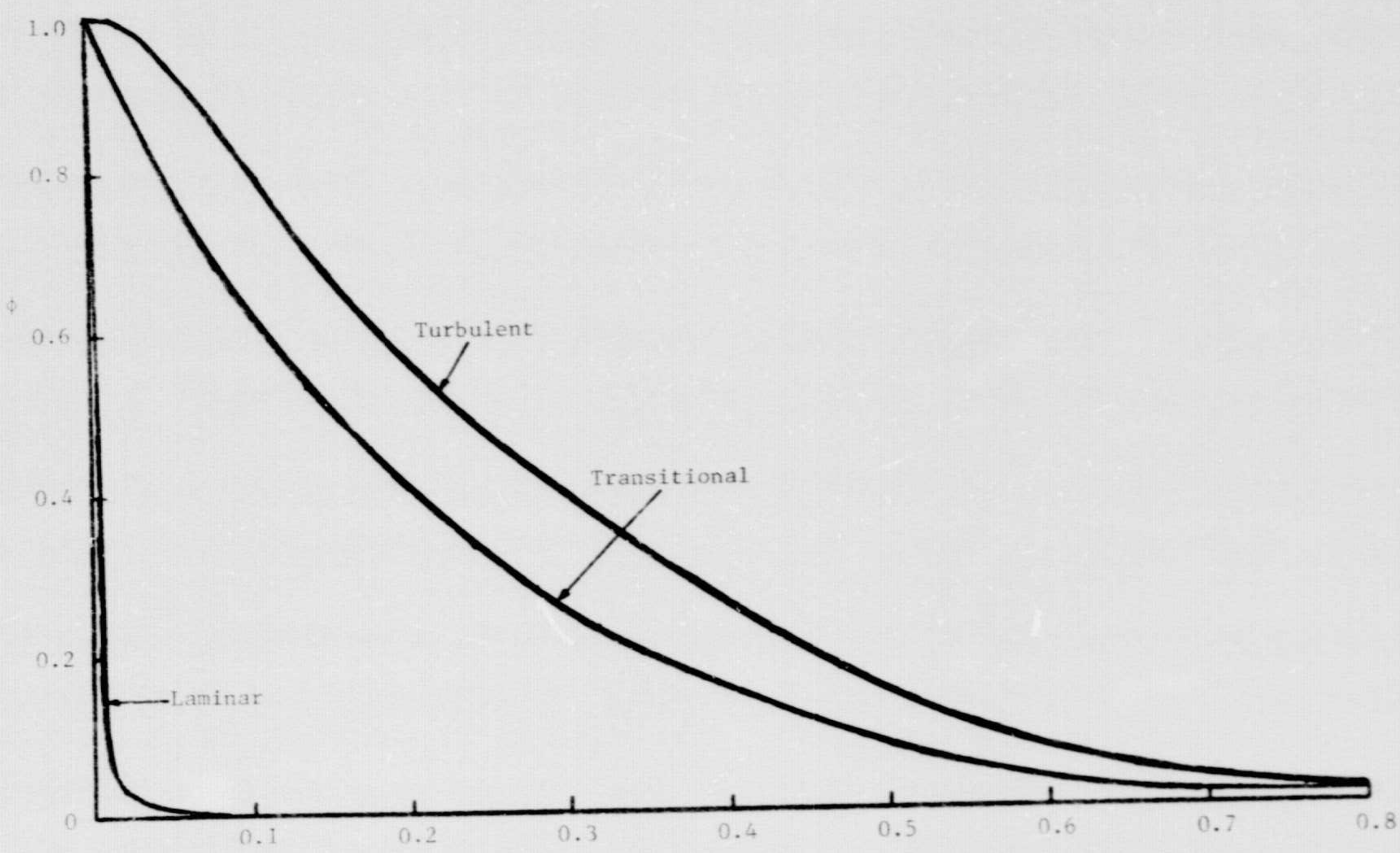

Fig. 4: Typical Phi-Parameter Profiles In Test Shear Layer