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POSTSEISMIC SURFACE DEFORMATIONS DUE TO LITHOSPHERIC AND ASTHENOSPHERIC VISCOELASTICITY

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ABSTRACT

This paper proposes a model for post-seismic surface deformations by attributing them to lithospheric and asthenospheric viscoelasticity. The model predicts that the deformations due to lithospheric viscoelasticity depend on the ratio of the effective shear modulus acting long after the lithospheric viscoelastic relaxation to that acting immediately following the earthquake. While such deformations are generally smaller than those associated with asthenospheric viscoelasticity, they occur on a shorter time scale and may be in opposite direction to both the motion occurring at the time of the earthquake and that occurring as the asthenospheric relaxation occurs.

POSTSEISMIC SURFACE DEFORMATIONS
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There is considerable interest in the time dependent surface deformations following an earthquake. The description of such deformations is important for understanding postseismic stress relaxation and redistribution, the occurrence of aftershocks, and in developing models of earth structure and rheology. Deformations that occur on a scale which is long compared to the time for elastic rebound in an earthquake are presumably due to anelastic properties of the earth. We intend to model some of these anelastic properties by using linear viscoelastic elements to describe the earth's rheology. An important principle associated with the linear theory is the correspondence principle which allows the time dependent viscoelastic deformation to be deduced by applying certain mathematical transformations to the solution for the associated elastic problem. The application of the correspondence principle to a homogeneous half-space reveals that when the viscoelastic properties in shear and dilatation are identical there is no additional surface deformation beyond the purely elastic displacement. Thus for time dependent postseismic displacement to occur with the context of a linear theory either there must be discontinuities in the properties of the half-space or the responses in shear and dilatation must be different. The latter view was adopted by Singh and Rosenman (1974) who calculated surface deformations associated with a displacement discontinuity in a half-space which is elastic dilatational and Maxwell or Voigt deviatoric. Conversely, Nur and Mavko (1974) considered a dislocation in an elastic layer lying over a viscoelastic half-space. Physically the elastic layer is associated with the earth's lithosphere and the viscoelastic half-space with the underlying asthenosphere. They considered both a thrust fault which was represented by an edge dislocation and a strike-slip fault which was represented by a screw dislocation. Referring to the strike-slip case they found that the time-dependent displacement parallel to the direction of the initial fault motion is such as to increase the total displacement. The relative magnitude of the viscoelastic versus elastic displacement increases with distance from the fault. Furthermore, the viscoelastic displacement increases as the depth of the screw dislocation approaches the lithosphere-asthenosphere boundary. The time constant associated with the motion is several years, this being the time constant for the viscoelastic response of the asthenosphere.

In addition to the viscoelasticity of the asthenosphere, there is evidence to suggest that a shorter time scale viscoelasticity can be associated with the lithosphere. Rock mechanics experiments, for example, suggest viscoelastic response times of 10^3 - 10^5 seconds (Robertson, 1964). Episodic creep events along the San Andreas fault have similar time constants (Crough and Burford, 1977) and might be associated with lithospheric viscoelasticity as might some aftershocks.

With these thoughts in mind we propose the following model for postseismic deformations. The lithosphere's instantaneous elasticity and subsequent partial flow is modeled by a standard viscoelastic solid with shear stress, σ, -strain, ϵ, relation (Flugge, 1967)

$$\left(1 + p_1 \frac{d}{dt}\right) \sigma = \left(q_0 + q_1 \frac{d}{dt}\right) \epsilon \qquad (1)$$

The instantaneous shear modulus is $\frac{q_1}{p_1}$. The quantity $\tau_L = \frac{q_1}{q_0}$ is the retardation time at constant stress for the lithospheric layer. The asthenosphere is modeled as a Maxwell viscoelastic element with shear stress-strain relation

$$\left(1 + p_2 \frac{d}{dt}\right) \sigma = q_2 \frac{d\epsilon}{dt} \qquad (2)$$

Here the shear modulus is $\frac{q_2}{p_2}$ and the stress relaxation time for constant strain is $\tau_a = p_2$. We shall have need of the following parameters $\alpha = \frac{q_1}{p_1} \frac{p_2}{q_2}$ and $\beta = \frac{q_1}{q_0 p_1} - 1$. Numerically α has a value near unity with the values 0.25 – 4.0 being used by Rybicki (1971); for β a value $\beta = 3$ is typical (Dieterich, 1972). For the model of the fault we choose the screw dislocation representation of a strike-slip fault used by Rybicki (1971) and Nur and Mavko (1974). We shall find it convenient to assume $\tau_a \gg \tau_L$ although this simplification is for physical clarity rather than mathematical necessity.

The starting point for the analysis is the equation for the displacements associated with a dislocation, ΔU, in an elastic layer with shear modulus μ_1 overlaying a half-space with shear modulus μ_2. With the coordinate system shown in Figure 1, the only non-vanishing displacement is U_2 where (Rybicki, 1971)

$$U_2(X_3 = 0; X_1 \geq 0) = \frac{\Delta U}{\pi}\left\{-\frac{\pi}{2} + \tan^{-1}\frac{X_1}{D} - \sum_1^{\infty}\left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right)^n \tan^{-1}\left[\frac{2X_1 D}{4n^2H^2 - D^2 + X_1^2}\right]\right. \qquad (3)$$

To obtain the viscoelastic solution we replace μ_1 and μ_2 by the corresponding Laplace transformed viscoelastic parameters $\bar{\mu}_1(s)$ and $\bar{\mu}_2(s)$ where s is the transform variable. We then replace ΔU by $\frac{\Delta U}{s}$ since the solution is valid only for $t > 0$ and the Laplace transform for this step function in time is $\frac{1}{s}$. The resulting modified equation 1 gives the Laplace transform for the viscoelastic deformation. Taking the inverse transformation yields the desired time dependent displacements. To carry out this procedure we use the standard results (Flugge, 1967)

$$\bar{\mu}_1(s) = \frac{1}{2}\,\frac{q_0 + q_1 s}{1 + p_1 s}$$

$$\bar{\mu}_2(s) = \frac{1}{2}\,\frac{q_2}{1 + p_2 s}$$

Then

$$G^n(s) \equiv \left[\frac{\bar{\mu}_1(s) - \bar{\mu}_2(s)}{\bar{\mu}_1(s) + \bar{\mu}_2(s)}\right]^n \frac{1}{s} = \left[\frac{a_1 s^2 + a_2 s + a_3}{(s-s_1)(s-s_2)}\right]^n \frac{1}{s} \qquad (4)$$

where

$$a_1 = \frac{\alpha - 1}{\alpha + 1} \qquad a_2 = \frac{\alpha - 1 - \beta}{\tau_L(\alpha + 1)} \qquad a_3 = \frac{\alpha}{\tau_a \tau_L(\alpha + 1)}$$

$$s_1 = -\frac{(\alpha + 1 + \beta)}{\tau_L(\alpha + 1)} \qquad s_2 = -\frac{\alpha}{(\alpha + 1 + \beta)\tau_a}$$

The simple forms for s_1 and s_2 result from the assumption that $\tau_a \gg \tau_L$. Taking the inverse Laplace transformation by the method of residues we find

$$L^{-1}\left[G^n(s)\right] = 1 + \frac{1}{(n-1)!}\left[F_1^{\,n-1}(s_1) + F_2^{\,n-1}(s_2)\right] \qquad (5)$$

where

$$F_1^{\,n-1} = \frac{d^{n-1}}{ds^{n-1}}\left[(A\frac{s}{s_1} - B)^n e^{st} \frac{s_1}{s}^n\right]$$

$$F_2^{\,n-1} = \frac{d^{n-1}}{ds^{n-1}}\left[(B\frac{s}{s_1} - 1)^n e^{st} \frac{s_2}{s}^n\right]$$

and

$$A = a_1 = \frac{\alpha - 1}{\alpha + 1} \qquad B = \frac{a_2}{s_1} = \frac{\alpha - 1 - \beta}{\alpha + 1 + \beta}$$

We are most interested in determining the surface displacements at three times. At time $t = 0$ we recover the instantaneous elastic displacements, at a later time $\tau_a \gg t \gg \tau_L$ the deformation associated with viscoelasticity of the lithosphere has occurred and at yet a later time $t \gg \tau_a$ the deformation associated with viscoelasticity of the asthenosphere has also taken place. In these three limits equation 5 becomes particularly easy to evaluate and

$$U_2(t = 0) = \frac{\Delta U}{\pi}\left\{-\frac{\pi}{2} + \tan^{-1}\frac{X_1}{D} - \sum_1^{\infty} A^n \tan^{-1}\frac{2X_1 D}{4n^2H^2 - D^2 + X_1^2}\right\}$$

$$U_2(\tau_a \gg t \gg \tau_L) = \frac{\Delta U}{\pi}\left\{-\frac{\pi}{2} + \tan^{-1}\frac{X_1}{D} \cdot \sum_1^{\infty} B^n \tan^{-1}\frac{2X_1 D}{4n^2H^2 - D^2 + X_1^2}\right\} \qquad (6)$$

$$U_2(t \gg \tau_a) = \frac{\Delta U}{\pi}\left\{-\frac{\pi}{2} + \tan^{-1}\frac{X_1}{D} - \sum_1^{\infty} \tan^{-1}\frac{2X_1 D}{4n^2H^2 - D^2 + X_1^2}\right\}$$

Notice that the deformations associated with the lithosphere viscoelasticity depend critically on the parameter B which in turn is strongly influenced by the quantity $1 + \beta$. This latter quantity is the ratio of the effective shear modulus for $t \gg \tau_L$ to that for $t = 0$. As this ratio increases so do the deformations.

As an illustration of the deformations predicted by these equations we consider the case $\alpha = 1$, $\beta = 3$, $D = H/2$. The results are presented in Figure 2. The displacements that occur on the time scale of the lithospheric viscoelasticity act to reduce the initial elastic displacements while those occurring on the time scale of the asthenospheric viscoelasticity increase the displacements. At first it might seem unlikely that the lithospheric viscoelasticity acts to reduce the initial elastic displacements. In fact, in an earlier paper concerned with the role of viscoelasticity in fault dynamics we argued that viscoelasticity could be responsible for forward post-seismic creep and aftershocks along a fault (Cohen, 1978). The present behavior can be explained by a simple one dimensional argument using the standard linear solid shown in Figure 3. Prior to the earthquake shear stress is slowly loaded on the system with the result that just prior to the earthquake the shear force is

$$F_i = \frac{\mu_a \mu_b}{\mu_a + \mu_b} L$$

where L is the initial stretch of the element. During the earthquake there is a sudden drop in the shear stress and stretch. Only the spring with elastic constant μ_a can respond on the time scale of an earthquake so immediately after the earthquake the shear force is

$$F_f = \mu_a \left[\frac{L}{1 + \mu_a/\mu_b} \right] - \mu_a \Delta X$$

where ΔX is the displacement occurring in the earthquake. In the fault motion model there is sliding against friction during the earthquake and after the earthquake friction prevents further motion until the stress partially recovers to an amount exceeding the frictional resistance. In the present model for the motion off the fault there is no friction and the initial dislocation is relaxed by the sliding so that $L = \Delta X$. Thus the elastic shear stress is negative immediately after the earthquake and viscoelastic flow acts to reduce the initial elastic displacement. We are, therefore, ignoring any additional forward displacement induced by post-seismic creep along the fault.

The increased displacement occurring when the asthenosphere adjusts to the earthquake is the more usual viscoelastic rebound discussed by Nur and Mavko (1974) and is due to asthenospheric flow and associated drag on the lithosphere.

Although we have not yet investigated whether present geodetic or seismic data permits a definitive test for the validity of the concepts presented here, there are a number of potentially significant implications of the theory if it is correct. One of the most important is that surveys taken during the days to months following an earthquake may be misleading if interpreted in terms of the elastic rebound occurring at the time of the earthquake. Furthermore failure to achieve closure in a network survey might be attributed to the continued motion occurring as the network is traversed. On the positive side, repeated simultaneous determinations of the location of survey sites in the weeks following a major earthquake might produce much information about the earth's rheology and tectonic processes.

The present theory has suggested that viscoelasticity in both the asthenosphere and lithosphere may contribute to post-seismic surface deformations. A detailed quantitative model will require additional consideration of the three dimensional nature of the problem with the possibility that the dilational and deviatoric responses may be different. The model can be developed by applying the procedure used in this paper to the elastic deformation equations of Rundle and Jackson (1977). The labor involved in such calcuations is more arduous than that involved in those we have reported here; we are in the process of working out the details.

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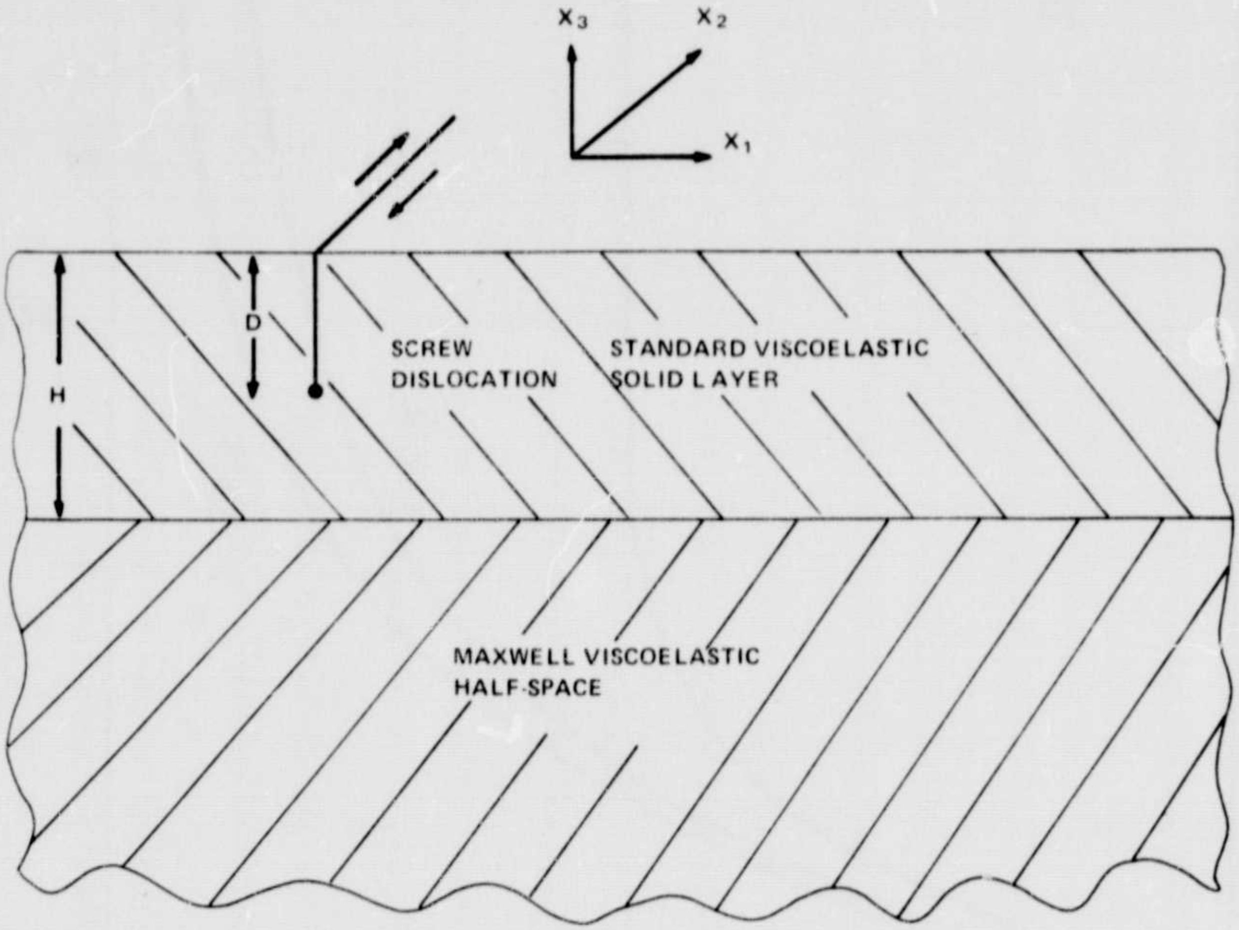
FIGURE CAPTIONS

Figure 1. Model for Lithospheric and Asthenospheric Viscoelasticity. A Strike-Slip Fault is Modeled as a Screw Dislocation in the Lithospheric Layer.

Figure 2. Computed Surface Deformations as a Function of Distance from the Fault for Various Times Following an Earthquake.

Figure 3. Rheological Element for Standard Viscoelastic Solid.



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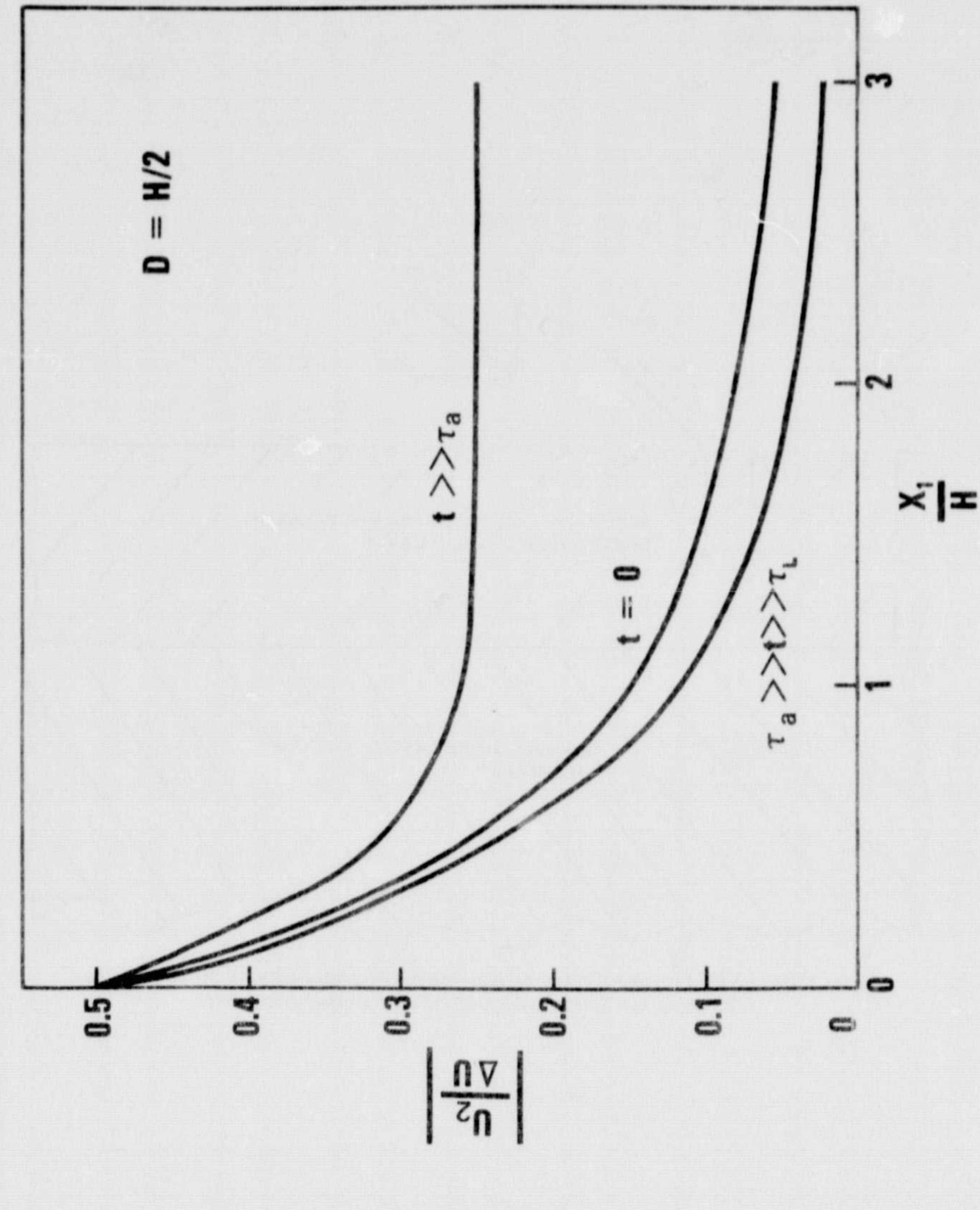

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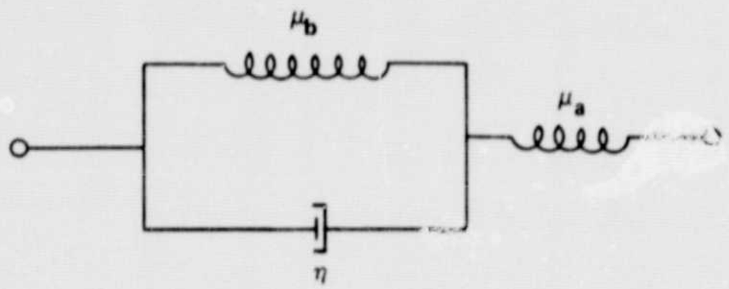

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