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Elementary, Analytic Models of Climate:

I. The Mean Global Heat Balance

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ABSTRACT

Climate models based on global radiative equilibrium are normally so complicated that they require extensive computer codes to provide adequate accuracy. However, by simply modifying the concept of a gray atmosphere, we can obtain a reasonably correct mean global temperature. This elementary model is then used to estimate the effects of changes in the abundances of minor infrared absorbers and changes in the solar constant or earth albedo. When applied to a Budyko-Sellers zonally averaged model, the quasi-gray model could give a physical basis for the latitude dependence of outgoing radiation and of opacity due to H₂O vapor content. The latter effect constitutes an important positive feedback on surface temperature.

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What approximate, analytic models lose in accuracy, compared with more elaborate computer-based analyses, they gain in clarity and adaptability. The model developed here is a case in point. It is a simple matter to change the CO_2 content of the atmosphere and find what change will occur in the global mean temperature. One can go a step further and find how the H_2O vapor content will be altered and include that feedback in the solution. An extension of this technique, of approximating the atmosphere as a quasi-gray one, could improve the utility of analytic, zonally averaged models, such as North's (1975) version of the Budyko (1969) and Sellers (1969) models.

The troposphere is not in radiative equilibrium nor is it even approximately gray (meaning that the absorption coefficient is independent of frequency). Further, it is not vertically homogeneous, since H_2O is distributed with a scale height of about 2 km, CO_2 with 8 km, and O_3 existing mainly in the stratosphere. Hence it is surprising that a gray, radiating homogeneous model has any value at all. Certainly this kind of model would not yield a very useful temperature-height profile. Such a simplified model can be useful, however, for exploring the importance of changes in the greenhouse effect, caused by changes in the atmospheric composition, and of changes in the incident solar flux.

1. The Gray Model

In the gray model only two temperatures are important: The effective emission temperature, T_e , of the planet is fixed by a balance between the incident solar flux and the planetary thermal emission. For a rotating planet of radius R

$$4\pi R^2 \sigma T_e^4 = (1 - \Lambda) \pi R^2 (\pi \mathfrak{F}) \quad (1)$$

where Λ is the omnidirectional albedo and $\pi \mathfrak{F}$ is the solar flux outside the atmosphere. The surface temperature, T_s , which is assumed to be the immediate source of radiation as far as the atmospheric heating is concerned, is related to T_e in a gray model by (Chamberlain, 1978, p 11)

$$T_s^4 = T_e^4 \left(1 + \frac{3}{4} \tau_g\right) , \quad (2)$$

where τ_g is the vertical opacity of the atmosphere, which we now estimate.

If the ground emits isotropically with intensity $B_\lambda(T_s)$, the flux transmission directly to cold space is

$$\begin{aligned} \sigma T_s^4 Q(\tau) &= \int_0^\infty d\lambda B_\lambda(T_s) \int_0^{2\pi} d\phi \int_0^1 e^{-\tau/\mu} \mu du \\ &= \sigma T_s^4 2E_3(\tau) \approx \sigma T_s^4 e^{-R\tau} \end{aligned} \quad (3)$$

where $E_3(\tau)$ is the exponential integral, which we approximate with a simple exponential. As τ varies from ∞ to 0 and the transmission function $Q = 2E_3(\tau)$ varies from 0 to 1, the coefficient R varies

from 1.2 to 2.0. For H_2O and O_3 we choose $\beta = 1.66$, which gives the exact value of E_3 when $Q = 1/2$; for CO_2 at $15 \mu m$, we take $\beta = \beta^* \approx 1.2$.

For a realistic atmosphere the same kind of considerations hold except that τ is a function of λ , and

$$Q \approx \sum_{j=1}^n p_j e^{-R\tau} \quad (4)$$

where the summation is over narrow wavelength intervals and p_i is the fractional flux in the i^{th} interval,

$$p_i = \frac{\pi B_{\lambda_i}(T_s) \Delta\lambda_i}{\sigma T_s^4} \quad (5)$$

To construct realistic temperature profiles, even for a truly gray atmosphere, one has to test the temperature gradient for adiabatic stability. In this investigation, that complication does not enter, since it would not change T_e and would have only a small effect on T_s .

2. Earth's Infrared Absorption

Between 0.54 and $8 \mu m$ H_2O has strong vibration-rotation bands and longward of $15 \mu m$ occur rotational lines from the ground vibrational state. The Earth's thermal radiation is mainly confined to the 5 to $30 \mu m$ region. The H_2O molecule is thus a strong

absorber over the thermal spectrum, except for the important 8 to 18 μm interval, where the Earth's emission peaks (at 290 $^{\circ}\text{K}$, $\lambda_{\text{max}} = 10 \mu\text{m}$). The 12 to 18 μm region is dominated by the CO_2 ν_2 -(bending) mode fundamental, leaving an 8 to 12 μm transparency window. A portion of this window is blocked by the 9.6 μm band of O_2 . Greenhouse heating of the atmosphere is thus dependent on composition in several distinct ways.

First, water vapor is dominant over a wide wavelength interval, although in much of this region it is not optically thick. Hence atmospheric cooling to space is critically dependent on H_2O abundance and even the vapor in the stratosphere is significant. (The latitudinal variation of H_2O is the main cause of the difference in temperature profiles between polar and tropical zones.)

Second, CO_2 fills a large part of the H_2O window, and current CO_2 abundances are enough to make the 15 μm region optically thick. Major CO_2 abundance changes will alter the greenhouse heating mainly by CO_2 absorption in the 10 μm bands.

Finally, minor constituents that absorb strongly in the 8 - 12 μm region can be critically important. Nitrous oxide (N_2O) and a wide array of hydrocarbons are potentially important, and the gray model can be used to estimate their effects.

The abundance of water vapor typically varies from 0.2 to 2 gm/cm^2 (or cm of precipitable water). We adopt a global average of 1 gm/cm^2 . With sufficient accuracy H_2O may be regarded as

completely opaque except in the near infrared and visual ($p_i \approx 0.03$); from 8.5 to 12.5 where $\tau \approx 0.10$ and $p = 0.29$; and 12.5 - 15 μm where $\tau < 1$ and $p = 0.14$. However, in the latter region H_2O does not contribute to atmospheric heating since it is dominated by CO_2 , and the H_2O therefore can be treated as transparent from 12.5 to 18 μm with $p = 0.26$ (Kondratyev, 1969, p 119). Thus Eq.(4) gives the absorption function due to water alone as

$$A(\text{H}_2\text{O}) = 1 - Q(\text{H}_2\text{O}) = 0.45 \quad (6)$$

As CO_2 increases, the mean opacity within a fixed wavelength interval also increases, but also the band effectively widens as weaker absorptions become stronger. Here we will not consider the broadening of the 15 μm band because a more important (optically thin) effect is the emission by the 10 μm bands. Thus the treatment of 15 μm itself is not quite correct, but the treatment of 15 μm + 10 μm together should give a good representation of CO_2 .

For CO_2 a volume mixing ratio of $f = 333$ ppm corresponds to an equivalent thickness at STP conditions of

$$\xi(\text{CO}_2) = 266 \text{ atm-cm} \quad (7)$$

An empirical formula for the mean transmission in the nearly saturated 15 μm band is (Kondratyev, 1969, p 126)

$$\langle Q(15\mu) \rangle = \exp(-\beta^* a \xi^b) \quad (8)$$

where the absorption data near $\xi = 300$ atm-cm yield $\beta^* = 1.2$,
 $a = 0.361$, and $b = 0.23$. Then the total absorption due to CO_2
at $15 \mu\text{m}$ (or more precisely between 12.5 and $18.2 \mu\text{m}$) is

$$A(\text{CO}_2) = 1 - Q(\text{CO}_2) = p(15\mu)[1 - \exp(-\beta^* a \xi^b)] = 0.27(0.791) = 0.214 \quad (9)$$

For the weak CO_2 band absorption we use the treatment in
Section 4, below, for minor constituents. The properties of the
two intercombination bands at $10 \mu\text{m}$ are listed in Table 1, and
from Eq. (24) we have $A(10\mu) = 0.025$.

For $\xi = 0.3$ atm-cm of ozone in the stratosphere we find
(Kondratyev, 1969, p 137) over the band (9.4 to $9.9 \mu\text{m}$) a mean
transmission of

$$\langle Q(9.6\mu) \rangle = \exp(-2.8\xi) \quad (10)$$

or

$$A(\text{O}_3) = 1 - Q(\text{O}_3) = p(9.6\mu)(1 - e^{-2.8\xi}) = 0.035(.57) = 0.020 \quad (11)$$

The computation of the various A's is illustrated in Fig. 1.

If the various substances do not overlap seriously in their
spectral regions of absorption, the A's can be added. Thus from
Eqs. (4), (6), (9), ~~and~~ ^{and (24)} (11), we have (with $\beta = 1.66$)

$$Q_{\text{net}} = e^{-\beta\tau_g} = 1 - [A(\text{H}_2\text{O}) + A(\text{CO}_2, 15\mu) + A(\text{CO}_2, 10\mu) + A(\text{O}_3)] = 0.29 \quad (12)$$

giving

$$\tau_g = 0.748 \quad (13)$$

With an albedo $\Lambda = 29\%$, Eq. (1) gives $T_e = 257^\circ\text{K}$. Then Eqs. (2) and (13) yield

$$T_s = 287.2 \quad (14)$$

which is close to the world average ($\sim 290^\circ\text{K}$). It is probably fortuitous that the agreement is so good, although the agreement does suggest that the gray model may provide some insight into climate changes due to changes in τ_g .

3. Variation of Ground Temperature with CO_2 Abundance

Differentiating Eq. (12) gives with Eqs. (9) and (24), below,

$$\begin{aligned} \beta e^{-\beta\tau} = \frac{dA(\text{CO}_2)}{d\tau} &= p(15\mu) \exp(-\beta^* a \xi^b) \beta^* a b \xi^{b-1} \left(\frac{1}{\xi} \frac{d\xi}{d\tau} \right) \\ &+ 1.08 \times 10^{-3} S_{\nu 0} \frac{d\xi}{d\tau} \end{aligned} \quad (15)$$

In terms of the mixing ratio or volume concentration

$$\begin{aligned} f \frac{d\tau}{df} = [\text{CO}_2] \frac{d\tau}{d[\text{CO}_2]} &= \frac{p(15\mu) \beta^* a b \exp(-\beta^* a \xi^b) \xi^{b-1}}{\beta e^{-\beta\tau}} + \frac{1.08 \times 10^{-3} S_{\nu 0} \xi}{\beta e^{-\beta\tau}} \\ &= 4.21 \times 10^{-2} + 7.5 \times 10^{-2} = 0.117 \end{aligned} \quad (16)$$

Similarly from Eq. (2) we obtain

$$\frac{dT_s}{d\tau} = \frac{3T_e}{16(1 + .75\tau_g)^{3/4}} = \frac{3T_s}{16(1 + .75\tau_g)} = 34.5 \quad (17)$$

and therefore

$$f \frac{dT_s}{df} = 4.0 \text{ } ^\circ\text{K} \quad (18)$$

if the CO₂ fractional concentration were changed by a small amount. If the CO₂ content were doubled, the gray model would give

$T_s = 290.0$ or $\Delta T_s = 2.8 \text{ } ^\circ\text{K}$. This value compares nicely to values obtained from more elaborate models. In reviewing a number of recent studies, Schneider (1975) concluded that a doubling of CO₂ would produce a ΔT_s of 1.5 to 3 $^\circ\text{K}$. As CO₂ is increased further, the temperature will continue to rise; although the 15 μm bands are already nearly saturated, the 10 μm bands are effectively closing the 8 - 12 μm window, as illustrated in Fig. 2.

4. Radiative Effects of Minor Constituents

The addition to the atmosphere of a minor constituent that absorbs in the 8 to 12 μm window could be important. In this region $\pi B_\lambda / \sigma T_s^4 \approx .060 - .070 / \mu$ and hence Eq. (5) is

$$p_i \approx 0.065 \Delta\lambda_\mu, \quad (19)$$

where $\Delta\lambda_\mu$ is the width of the absorption in microns. We will estimate the minimum amount of a trace gas that could produce a 1 $^\circ\text{K}$ change in global temperature.

For an optically thin amount of absorber, we have simply

Table 1
 ABSORBERS IN 8 - 12 μm WINDOW*

<u>Species</u>	<u>Band, λ (μm)</u>	<u>f (ppm)</u>	<u>S (cm)</u>
N_2O Nitrous oxide	ν_1 7.8	0.28	10^{-17}
CH_4 Methane	ν_4 7.66	1.6	7×10^{-18}
NH_3 Ammonia	ν_2 10.5	6×10^{-3}	$\sim 10^{-20}$ (?)
CF_2Cl_2 F-12	ν_1 9.1	1×10^{-4}	5×10^{-17}
	ν_6 8.7		3×10^{-17}
	ν_8 10.9		5×10^{-17}
CFCl_3 F-11	ν_1 9.2	1×10^{-4}	3×10^{-17}
	ν_4 11.8		6×10^{-17}
Hydrocarbons	--	$\lesssim 4 \times 10^{-3}$	$\lesssim 5 \times 10^{-17}$
CO_2	$2\nu_2-\nu_3$ 9.4		8.6×10^{-22}
	$\nu_1-\nu_3$ 10.4	333	2.7×10^{-21}

* For uniform mixing the integrated abundance is $\mathcal{N} = f \mathcal{N}_{\text{atm}}$
 $= f \times 2.15 \times 10^{25} \text{ cm}^{-2}$. Since the STP thickness is $\xi = \mathcal{N}/N_0$
 $= f \mathcal{N}_{\text{atm}}/2.687 \times 10^{19}$, we have

$$f(\text{ppm}) \approx 1.25 \xi (\text{atm}\cdot\text{cm}).$$

$$A_i = p_i [1 - e^{-\beta \tau_i}] \approx \beta p_i \langle \tau_i \rangle, \quad (20)$$

where $\langle \tau \rangle$ is the mean value in the band, or

$$\langle \tau \rangle = \frac{\mathcal{N}}{\Delta \lambda} \int \alpha \, d\lambda \quad (21)$$

which gives

$$\begin{aligned} A &= \beta (.065) \mathcal{N} \int \alpha \, d\lambda \\ &= .108 \mathcal{N} S_\lambda \end{aligned} \quad (22)$$

where \mathcal{N} is the integrated column density (molecule/cm²) and S_λ the integrated absorption coefficient or strength (in units of cm² micron). The more common units for strengths are cm (cross section in cm²; spectrum in cm⁻¹):

$$S_\nu = \int \alpha \, d\nu \quad (23)$$

Hence at 10 μm ,

$$\begin{aligned} A &= 0.108 \mathcal{N} S_\nu \lambda_\mu^2 10^{-4} \\ &= 1.08 \times 10^{-3} \mathcal{N} S_\nu \end{aligned} \quad (24)$$

The maximum band strengths are not likely to exceed 5×10^{-17} cm.

From Eq. (12) we have, for $A = 1 - Q_{\text{net}}$,

$$\frac{dA}{d\tau_g} = \beta e^{-\beta \tau_g} = 1.66 (0.25) = 0.415 \quad (25)$$

and with Eq. (17) the $T_s - A$ relation becomes

$$\frac{dT_s}{dA} = \frac{dT_s/d\tau_g}{dA/d\tau_g} = 83 \quad (26)$$

Thus for 1 °K changes in T_s , we are searching for increments in A the order of 1×10^{-2} , or from Eq. (24),

$$\Delta N \gtrsim \frac{1 \times 10^{-2}}{1.08 \times 10^{-3} \times 5 \times 10^{-17}} = 2 \times 10^{17} \text{ cm}^{-2} \quad (27)$$

The integrated column density of the atmosphere is $2.15 \times 10^{25} \text{ cm}^{-2}$ so we are concerned with mixing ratios of minor constituents greater than

$$\Delta f \sim 10 \times 10^{-9} \quad (28)$$

or a few parts per billion (ppb) for substances that have a single strong absorbing band in the 8 to 12 μm window.

The halogenated methanes (Freons) have been cited as a striking example of the effect we are examining here (Ramanathan, 1975). The two Freons F-11 and F-12 together have five bands with strengths $S_\nu \sim 3 \times 10^{-17} \text{ cm}$. The 1975 abundance was 0.1 ppb; an increase of 10^2 times would place these substances in the climate critical category.

Most substances will not have bands in the window with strengths close to the maximum. From Eq. (27) the pertinent quantity for bands in the window is

$$\Delta N \cdot S \gtrsim 10 \quad \text{or} \quad \Delta f(\text{ppm}) \cdot S \gtrsim 5 \times 10^{-19} \quad (29)$$

if an absorber is to produce a 1 °C change on future climate (see Table 1). Both N₂O and CH₄ sit in regions where there is already strong absorption and they have to be analyzed (as we did CO₂) allowing for the present near-saturation.

All of the substances in the Table except the hydrocarbons have been examined with a more elaborate radiative model (Wang, et al., 1976) and found to be marginally important, if their abundances increase. For N₂O, CH₄, and NH₃, factors of two are important.

More attention needs to be directed toward the global abundance and long term growth rate of hydrocarbons, which are among the primary urban pollutants, and whose emission is largely uncontrolled. The abundance in Table 1 applies to the total hydrocarbon population; the strength listed is the maximum allowable and probably is much smaller. Aldehydes (a hydrocarbon family) are partly destroyed near their urban source, where they participate in the production of oxidants. Away from high concentrations of NO_x, the aldehydes disappear by photodissociation and attachment to aerosols.

5. Destruction of Trace I.R. Absorbers

Many of the hydrocarbons are not only being emitted at increasing rates, but their natural sinks are becoming saturated. The principal sink for methane is hydroxyl:



Indeed, OH plays a crucial role in scavenging a number of the less soluble trace gases, such as CO, H₂S, SO₂, and the partially halogenated methanes, CH_xCl_yF_z and CH_xBr_y. A depletion of OH may thus aggravate a build up of infrared absorbing pollutants and their impact on the climate. At present there is a danger of such a depletion occurring because of the increasing production of CO, which is removed mainly by

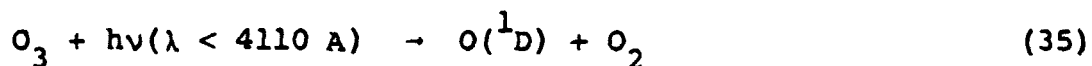


The present CO production from fossil fuel burning is 0.4×10^9 ton/yr, whereas the natural production is 1 to 3×10^9 ton/yr. The problem is amplified by the fact that the main natural source of CO is the oxidation of CH₄. Following reaction (30), oxidation proceeds by



Finally, formaldehyde is dissociated into H₂ + CO, or it may condense onto aerosols and ultimately rain out.

The principal source of tropospheric OH is photodissociation of tropospheric ozone:



followed by



Thus present levels of man-made CO emission are large enough to perturb the CO-OH-CH₄ cycle. Reactions that recycle OH, such as HO₂ + NO → OH + NO₂, may account for some 30 percent of the present OH.

6. Feedbacks: Comments on Water and the Stratosphere

Earth is the water planet and water affects climate differently in its different manifestations: vapor, clouds, sea, sea-ice, land-ice and snow, and foliage. Common to all models is the acknowledgement of the effect and inability to do anything useful about it. It is not unlikely that the response of water to a change in CO₂ is the dominant factor in climate change.

Also physical processes in the stratosphere can feed back on surface temperatures. The water abundance in the stratosphere is only several parts per million and is probably fixed by the vapor pressure at the tropical tropopause [although H₂O can be made in the stratosphere by reaction (30)]. A change in the content of water due to a change in stratospheric temperature will affect the earth's greenhouse (Chamberlain, 1977). An alteration of the ozone abundance will change the absorption at 9.6 μm, which is

the main absorber in the 8 - 12 μm window. Thus an increase in tropospheric CH_4 could affect stratospheric HO_x chemistry, the ozone abundance, and thence the climate.

7. Changes in the Sclar Flux or Albedo

Differentiating Eq. (1) and relating T_s to T_e by Eq. (2) for a constant opacity, we have

$$\frac{\Delta T_s}{T_s} = \frac{\Delta[(1 - \Lambda)\pi\bar{g}]}{4(1 - \Lambda)\pi\bar{g}} \quad (37)$$

Consequently a fractional change of δ percent in the solar constant or in $1 - \Lambda$ will produce a change in mean surface temperature of

$$\Delta T_s = \frac{1}{4} T_s \frac{\delta}{100} = 0.72 \delta \text{ } ^\circ\text{K} \quad (38)$$

In practice there will be an accompanying change in H_2O vapor and sea ice, affecting the opacity and feeding back on the albedo. Analysis of these effects is best handled with a latitude dependent (Budyko-Sellers) model. Accordingly, further discussion is deferred to a later paper.

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FIGURE CAPTIONS

Fig. 1 Thermal emission of the Earth's surface at $T = 290^{\circ}\text{K}$ (after Kondratyev, 1969, p. 119), with relative values of the absorption, $1 - \exp(-\beta\tau)$, indicated by shaded areas. The figure illustrates why trace amounts of absorbers in the $8.5 - 12.5 \mu\text{m}$ region can be critically important.

Fig. 2 Temperature versus CO_2 abundance for global radiative equilibrium with the quasi-gray model.

