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UPGRADED VISCOUS FLOW ANALYSIS OF MULTIELEMENT AIRFOILS*

Guenter W. Brune and Joseph W. Manke

The Boeing Company

SUMMARY

A description of an improved version of the NASA/Lockheed multielement airfoil analysis computer program is presented. The improvements include several major modifications of the aerodynamic model as well as substantial changes of the computer code. The modifications of the aerodynamic model comprise the representation of the boundary layer and wake displacement effects with an equivalent source distribution, the prediction of wake parameters with Green's lag-entrainment method, the calculation of turbulent boundary layer separation with the method of Nash and Hicks, the estimation of the onset of confluent boundary layer separation with a modified form of Goradia's method, and the prediction of profile drag with the formula of Squire and Young. The paper further describes the modifications of the computer program for which the structured approach to computer software development was employed. Important aspects of the structured program development such as the functional decomposition of the aerodynamic theory and its numerical implementation, the analysis of the data flow within the code, and the application of a pseudo code are discussed.

Computed results of the new program version are compared with recent experimental airfoil data. The comparisons include global airfoil parameters such as lift, pitching moment and drag coefficients, and distributions of surface pressures and boundary layer velocity profiles.

INTRODUCTION

In the past, high lift design and technology rested in the hands of a few experienced aerodynamicists. Design methodology and criteria were heavily influenced by the analytical inviscid flow methods and the experimental data available. With the advent of high-speed computers and the appearance of improved models for turbulent flows, many complex problems, including high-lift design and analysis, were attacked theoretically.

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One such approach to high-lift, multielement airfoil analysis was developed at Lockheed-Georgia under the sponsorship of the NASA-Langley Research Center (ref. 1). This program was among the first attempts at analyzing the complex viscous flow about slotted airfoils and has received worldwide distribution and usage. A unique feature of this multielement airfoil program is the model of the confluent boundary layer flow (ref. 2).

Over the years, the original version of the program was modified extensively to improve its predictions for different types of high-lift airfoils. Many improvements, mainly in the area of the potential flow calculation, were made by researchers at the Langley Research Center (ref. 3). For this reason, the code is generally referred to as the NASA/Lockheed multielement airfoil program. A version for single element airfoils was recently extracted from the multielement airfoil code by researchers at North Carolina State University (ref. 4).

Widespread and steady usage of the computer program clarified its strengths and weaknesses. Both favorable and unfavorable aspects have been brought to the surface by continued attempts at using the program as an engineering tool. The more serious shortcomings were the lack of agreement between the documentation and the available version of the code and the high failure rate in applying the method for various configurations. However, the program was found to contain sufficient positive features to justify its choice as a starting point for additional theoretical work in the high-lift area.

This paper briefly describes the aerodynamic theory and the corresponding computer program of a new version of the multielement airfoil program; a detailed description can be found in references 5 and 6. Symbols are defined in an appendix.

MULTIELEMENT AIRFOILS

The flow around high-lift airfoils is characterized by many different inviscid and viscous flow regions. Their complex physics is illustrated by figure 1. In particular, the existence of confluent boundary layers and the regions of separated flow distinguish the high-lift airfoil problem from the aerodynamic problem of airfoils at cruise conditions. The various flow regions, including the outer potential flow, the ordinary laminar and turbulent boundary layers, viscous wakes, and the confluent boundary layer, are analyzed by the code. Furthermore, the prediction of transition from laminar to turbulent boundary layer flow and the prediction of the onset of boundary layer separation are a necessary part of the code. Cove separation and large scale separation phenomena, however, are not modeled.

PROGRAM MODIFICATIONS

The new program version differs from the baseline version (ref. 3) in the following areas:

1) The method used to represent the effect of the viscous flow on the outer potential flow, termed equivalent airfoil representation in the baseline version of the program, has been modified. It has been replaced by the surface transpiration method which uses a distribution of sources along airfoil surfaces and wake centerlines to model boundary layer and wake displacement effects.

2) The flow model of the potential core region has been changed. The new method performs independent boundary layer and wake calculations. These calculations utilize the ordinary laminar and turbulent boundary layer routines of the baseline version of the code, and in addition, the lag-entrainment method of reference 7 for wake flows. The revised flow model of the core region calculates the location of the wake centerlines.

3) An attempt is made to predict the onset of separation of the confluent boundary layer by a modified version of Goradia's confluent boundary layer method. In this method, the power law velocity profile of the wall layer is replaced by Coles' two-parameter velocity profile (ref. 8).

4) The drag prediction method of Squire and Young (ref. 9) has been incorporated into the program, replacing the previous pressure and skin friction integration scheme.

5) The original method used for the prediction of separation for ordinary turbulent boundary layer flow has been replaced by the Boeing version of the method of Nash and Hicks (ref. 10).

6) The modifications of the aerodynamic theory required a major overhaul of the computer code. Most parts of the code have been rewritten using a systematic approach to computer software design. This work was guided by a functional decomposition of the many aspects of the aerodynamic model and its numerical implementation. In addition, a detailed study was made of the data flow within the program, and the logic of the code was outlined prior to the actual program development using a pseudo code. The most important aspects of this work are briefly reviewed in this paper.

AERODYNAMIC FLOW MODELS

The aerodynamic theory of the new version of the computer program is outlined below with emphasis on modifications. The aerodynamic analysis and its numerical implementation assume two-dimensional, subsonic flow in which all boundary layers are attached to the airfoil surface.

Potential Flow

Inviscid, irrotational flow is calculated using the stream function approach of Oeller (ref. 11). Laplace's equation is solved subject to the boundary condition of a constant value of the stream function on airfoil surfaces. The method is of the panel type, see figure 2, with a constant strength vortex distribution on each panel of the airfoil surface. A formulation of the Kutta condition is imposed which requires the tangential velocities at the upper and lower surface trailing edge points to be equal. Compressibility effects are taken into account by employing the Karman-Tsien rule.

Viscous Flow Representation

Oeller's stream function method has been modified in order to account for displacement effects of boundary layers and wakes within the potential flow solution. An equivalent distribution of sources simulating the viscous flow displacement thickness is placed on the surface and wake centerline of an airfoil component. This is the surface transpiration method which, within the framework of thin boundary layer theory, is completely equivalent to the method of geometrically adding the displacement thickness to the basic airfoil geometry. Application of this technique is computationally efficient, since most of the aerodynamic influence coefficients do not change during the solution procedure.

The strength σ of the equivalent source distribution is obtained from

$$\sigma = \frac{d}{ds}(\delta^* U)$$

where δ^* denotes the displacement thickness of either boundary layer or wake, and the symbol U stands for the inviscid flow velocity on airfoil surface and wake centerline. The variable s represents arc length. The computed source distribution is discretized using panels with constant source strength on the surface and wake centerline of each airfoil component.

It should be emphasized that the employed flow model does not account for wake curvature effects. Consequently, constant strength vortex panels are only used on the surface of an airfoil and not on its wake centerline, see figure 2.

Wake Centerline

The capability of computing the position of wake centerlines has been added to the program as part of the revision of the flow model in the core region, see figure 1. A wake centerline is part of the stagnation streamline.

Since the potential flow problem is solved on the basis of a stream function approach, which in addition to the surface velocity provides the value of the stream function for each stagnation streamline, it is convenient to also use the stream function formulation to trace wake centerlines. This is done in an iterative procedure beginning with an assumed initial position of a wake centerline. During each step of this iteration the locations of all panels of the wake centerline are updated simultaneously by solving a linearized form of the following stream function equation:

$$\psi_{m} = \psi\ (\vec{x})$$

Here, ψ_m denotes the known value of the stream function at a stagnation streamline. The variable $\vec{x}$ represents the array of unknown panel corner point coordinates of the wake centerline.

Laminar Boundary Layer

Laminar boundary layer characteristics are calculated with the compressible method of Cohen and Reshotko (ref. 12) who reduced the problem to simple quadrature. Application of a compressible method seems to be necessary for slotted high-lift airfoils, since laminar boundary layers often exist in the slot between neighboring airfoil components, where even at low free stream Mach numbers the flow is highly compressible with velocities approaching and frequently exceeding sonic conditions.

Laminar separation is predicted with the criterion of Goradia and Lyman (ref. 13) which is an empirical correlation of the local values of the Mach number gradient with the momentum thickness Reynolds number. Either laminar stall or the occurrence of laminar short bubble separation is predicted. In the case of laminar short bubbles, subsequent turbulent reattachment of the separated laminar boundary layer is assumed, but neither the length of the separation bubble nor the details of the flow within the bubble are modeled.

The computer program provides two options for transition from laminar to turbulent boundary layer flow. The user can either specify fixed transition points, such as the location of trip strips, or can compute free transition. In the latter case, a standard two-step approach is employed.

Turbulent Boundary Layer

Two different integral methods determine the characteristics of ordinary turbulent boundary layers. The method of Truckenbrodt (ref. 14) is used during the iterative solution procedure. It is an incompressible approach based on the momentum and energy integral equations. Goradia (ref. 1) introduced the idea of constraining the shape factor $\bar{H}$, defined as the ratio of energy dissipation thickness to momentum thickness, in order to avoid premature separation of the turbulent boundary layer during the first cycles of the iteration. This approach avoids failures of the boundary layer integration at separation and can be viewed as an artificial way of modeling separated flows. The integral method of Nash and Hicks (ref. 10), which accounts for the history of turbulent shear stresses, is applied at the end of the iterative solution procedure for the purpose of computing boundary layer separation. Displacement thickness and skin friction obtained from the method of Nash and Hicks are not utilized.

Wake Flow

The properties of turbulent wakes are analyzed with the lag-entrainment method of reference 7. The method is formulated in terms of the momentum integral equation, the entrainment equation, and an empirical equation for the streamwise rate of change of the entrainment coefficient. The entrainment equation is derived from the definition of the entrainment coefficient, which represents the change of mass flow within the wake layer. An incompressible version of Green's treatment of wake flow is used, neglecting the effects of curvature on the mean flow and the turbulence structure of the wake.

Confluent Boundary Layer

The program computes confluent boundary layers with the model of Goradia (ref. 2). In this model, the confluent boundary layer downstream of the core region is divided into two regions. In the first region, turbulent mixing of wake and boundary layer is incomplete. The mean velocity profile clearly shows the remainder of the wake profile, see figure 3. In the second region, the effect of the wake is not visible in the mean velocity profile which is similar to that of a wall jet. Downstream of the second region, the confluent boundary layer degenerates into an ordinary turbulent boundary layer.

Goradia formulated an integral method by subdividing the confluent boundary layer into several layers and assuming the validity of boundary layer equations and self-similarity of the mean velocity profile in each of these layers. The method is incompressible, neglects curvature effects, and relies to a large extent on empirical information about shear stress, the rate of growth of various layers, and velocity profiles. Furthermore, the model ignores multiple wakes and multiple potential cores that might exist near the trailing edge of a high-lift airfoil consisting of more than two components. The shape factor H of the layer adjacent to the airfoil surface, termed the wall layer, is constrained in order to avoid program failures in regions of separated flow. For this reason, Goradia's confluent boundary layer method is applied during the iteration procedure, when unrealistic potential flow pressure distributions can cause premature boundary layer separation.

The described flow model has been modified to predict separation of confluent boundary layers. The power law velocity profile of the wall layer has been replaced by Coles' two-parameter profile (ref. 8), which is known to provide a realistic representation of ordinary turbulent boundary layers near separation. The shape factor of the wall layer is not constrained in this modification, but most other features of Goradia's confluent boundary layer model including its empirical content are retained.

Iterative Solution Procedure

The solution is iterative, since most of the individual flow problems and their coupling are nonlinear. The computer program uses a conventional cyclic iteration procedure in which each cycle consists of the following steps:

Step 1: A potential flow solution for the multielement airfoil is calculated. During the first cycle of the iteration, the calculation is performed without any representation of viscous flow displacement effects.

Step 2: The positions of the wake centerlines are computed.

Step 3: Solutions of all viscous flow problems including laminar and turbulent boundary layers, confluent boundary layers, and viscous wakes are calculated with the potential flow velocities and wake centerline locations obtained in the previous steps as input data. At the end of this computational step, the displacement thicknesses of all boundary layers and wakes are available.

Step 4: A source distribution representing the displacement effect of all viscous layers is computed.

The computer program does not rely on a convergence criterion. Instead, five iteration cycles are always executed and the user of the program must judge the quality of the solution.

In order to assist the iteration scheme in arriving at a converged solution, the source strength σ computed in Step 4 of each cycle is modified by adding 2/3 of σ computed in this iteration cycle to 1/3 of σ computed in the previous iteration cycle.

Forces and Moments

At the end of each iteration cycle, airfoil lift and pitching moment coefficients are computed by an integration of surface pressure and skin friction. Profile drag is obtained by applying the formula of Squire and Young (ref. 9). The total profile drag of a multielement airfoil is assumed to be the sum of the contributions of its components. The drag of each airfoil component in turn is calculated from the values of boundary layer momentum thickness and surface velocity at the component trailing edge.

COMPUTER CODE

The new version of the code is written in the CDC FORTRAN extended 4 (FTN4) language and will run under the CDC Network Operating System (NOS). The CDC overlay system is used to assure that the code will execute in a field length less than 100 K octal.

The programming methodologies used to design and develop the new version of the computer code include a functional decomposition of the aerodynamic theory, a data flow analysis, and a control flow analysis.

Each of these related design tasks was performed several times in an iterative manner to produce a final design for the new version of the computer code before changes or improvements to the baseline code were made. The final design resulted in major changes of the baseline version of the program in the following sections: upper level control routines, geometry preprocessing routines, and the potential flow solution routines. The final design was also used to integrate the new aerodynamic models into the baseline code. Table 1 lists all subroutines in the baseline version of the code and indicates the type of changes made to incorporate them in the new version.

The functional decomposition of the code was based on the engineering specification of the aerodynamic models and the numerical techniques necessary for their solution. Figure 4 shows the upper level decomposition chart where the major functions are defined in engineering terms. The complex physics of the flow about multielement airfoils is reflected in these charts.

The data flow analysis of the code was done for each module identified in the functional decomposition by specifying the input and output data for the module as well as its own decomposition. Control of the data flow within a module is maintained by requiring that the input to any of its submodules must be either an input to the module or the output of another of its submodules.

The control flow analysis of the new code was done with the aid of a pseudo code, which is a small set of simple logic and loop statements which suffice to describe the control within a module of the functional decomposition. Although the submodules of a module can be used in any sequence and any number of times to complete the function of the module, it is an aim of the design process to keep the control within a module as simple as possible. All new subroutines in the code include as comment cards the pseudo code for the module which they implement.

TEST-THEORY COMPARISONS

In the following comparison of theoretical and experimental airfoil data, three versions of the NASA/Lockheed multielement airfoil program are referred to:

Version A: This is the baseline version of the computer program with minor modifications. The baseline version was available from the NASA in June 1976.

Version B: This version, described in reference 15, differs from version A in two areas. Profile drag is predicted by the Squire and Young formula. Separation of ordinary turbulent boundary layers is calculated using the method of Nash and Hicks.

Version C: This is the version of the program described in this paper.

A large number of airfoil configurations were analyzed using the program versions listed above. Only a few results of this program evaluation, concerning the GA(W)-1 airfoil and a Boeing multielement high-lift airfoil (figure 5), are discussed in this paper. A detailed report of this evaluation is contained in reference 6.

GA(W)-1 Airfoil

This airfoil was chosen to test the program capability of predicting performance characteristics of single airfoils. Figure 6 contains the theoretical lift, pitching moment, and drag curves and their comparison with experimental data of McGhee and Beasley (ref. 16). Both Version A and the new program Version C predict identical lift and moment curves that in turn agree well with measured GA(W)-1 data up to the onset of trailing edge stall at about 8 degrees angle of attack. Trailing edge stall is not modeled by any of the program versions.

Considerable differences between all drag polars are shown in figure 5. Version A, utilizing an integration of surface pressure and skin friction in the prediction of profile drag, gives the highest drag coefficients. Version C, applying the Squire and Young formula, offers drag values that are lower than the corresponding experimental drag coefficients. The lack of agreement of the three drag polars emphasizes the fact that even for single airfoils at low speed the problem of obtaining accurate drag computations is not yet solved.

Surface pressures of the GA(W)-1 at 8 degrees angle of attack, computed by program Version C and plotted in figure 7, agree well with their experimental counterparts. In this figure, the symbols S and LS refer to theoretical points of turbulent separation and laminar short bubbles, respectively. The symbol FT indicates the experimental trip strip location which is specified as a fixed transition point in the computer simulation. A laminar short bubble with subsequent turbulent reattachment of the boundary layer is indicated near the upper surface leading edge, and turbulent boundary layer separation is predicted theoretically near the upper surface trailing edge. The latter prediction is confirmed by the experimental pressure distribution which shows a constant pressure downstream of the theoretical point of separation.

Boeing High-Lift Airfoil

The Boeing four-element high-lift airfoil was used as the main test case for multiple airfoils. It consists of a wing section with a leading edge flap and a double-slotted trailing edge flap. Global airfoil parameters and detailed distributions of surface pressures and boundary layer data are available for comparisons. The data were obtained in the Boeing Research Wind Tunnel (BRWT) on a model with a 2 foot unextended wing chord and a 5 foot span. Careful blowing of the wall boundary layers was applied in order to closely approximate a two-dimensional flow pattern across the whole span of the airfoil.

The lift curve, pitching moment, and drag polar of this airfoil at a Reynolds number of two million, based on the wing reference chord, are given in figure 8. The experimental lift coefficients are balance data that are within 1.5% of the lift obtained by pressure integration. The profile drag of the airfoil is the result of wake rake measurements taken at a fixed spanwise position relatively free from interference effects of flap supporting brackets and pressures taps. The maximum spanwise variation of the measured airfoil drag is indicated in figure 8.

All attempts failed when using program Version A to obtain a converged solution for this airfoil. Program Version B arrived at converged solutions between 8 and 20 degrees angle of attack, but underpredicted the lift by a considerable amount. The theoretical predictions of program Version C match the experimental lift coefficients well at angles of attack below the onset of trailing edge stall, which is theoretically predicted by the program to take place at about 16 degrees.

The theoretical values of the profile drag of Version C are relatively close to the measured profile drag. In judging the quality of this drag comparison, the reader should recall the problems of two-dimensional high-lift testing and the uncertainties in applying the Squire and Young formula to multielement airfoils.

Figure 9 contains comparisons of theoretical and experimental surface pressures of wing and main flap at 8.4 degrees angle of attack. This figure confirms the earlier finding that Version C indeed provides the best theoretical results. Differences between the theory of Version C and experiment are noted in cove regions and on the upper surface of the wing near the leading edge. The latter problem is due to the failure of the program to accurately simulate the flow on the lower surface of the leading edge device.

Figure 10 shows boundary layer velocity profiles at several chordwise stations on the upper wing surface. The experimental velocity profiles reveal that very little confluence of the wake behind the leading edge device and the wing boundary layer has taken place and that an initially existing weak confluent boundary layer above the wing has degenerated early into an ordinary turbulent boundary layer. This feature of the flow field is very well simulated by Version C, but not by Versions A and B.

CONCLUSIONS

The following conclusions about the reliability and quality of the predictions of the new program are drawn:

1) The reliability of the program executions has been greatly improved. All test cases have produced converged solutions within a few iteration cycles. This improvement is a consequence of the application of the structured approach to computer programming where much attention was paid to the functional decomposition of the aerodynamic model, its numerical implementation, and the data flow within the code.

2) The accuracy of the program predictions has been improved. This is due to several major modifications of the aerodynamic model - above all, due to the different representation of the viscous flow displacement effects and the improved model of the potential core region.

3) The computed results are consistent with the basic assumptions of the aerodynamic model. Best results are obtained in cases where most of the flow is attached to the airfoil's surface, but the quality of the predictions gradually deteriorates with increasing trailing edge stall and cove separation.

4) The usefulness of the confluent boundary layer method of Goradia and its modification developed for the purpose of predicting confluent boundary layer separation have not yet been tested. Configurations were chosen for most of the program evaluation with little confluence of wakes and boundary layers.

5) The performance of the program needs to be tested for configurations at off optimum shape design.

6) The evaluation of the computer program was hampered by the shortage of reliable experimental high-lift data. Additional wind tunnel testing of some of the more important high-lift airfoil configurations will increase confidence in their predicted performance.

APPENDIX

SYMBOLS

C	airfoil reference chord
C_d	drag coefficient
C_ℓ	lift coefficient
C_m	pitching moment coefficient about the quarter chord point
C_p	surface pressure coefficient
$\tilde{H}$	ratio of energy dissipation thickness and momentum thickness
M_∞	free stream Mach number
R_N	Reynolds number formed by free stream velocity and airfoil reference chord
s	arc length
U	inviscid surface velocity or wake centerline velocity
U_∞	free stream velocity
u	boundary layer velocity parallel to airfoil surface
X	x - coordinate of global axis system
$\vec{x}$	array of panel corner points
Y	coordinate normal to airfoil surface
α	angle of attack
δ^*	displacement thickness
σ	source strength
ψ	stream function
ψ_m	stream function value at a stagnation streamline

Abbreviations

FT	fixed transition
LS	laminar short bubble
S	separation

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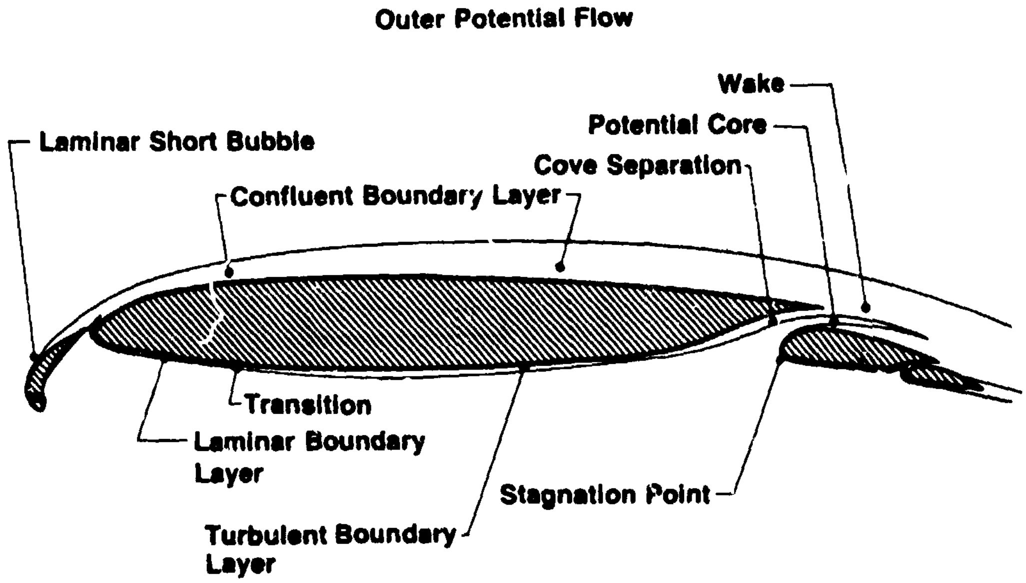

Figure 1.- Flow regions of multielement airfoils.

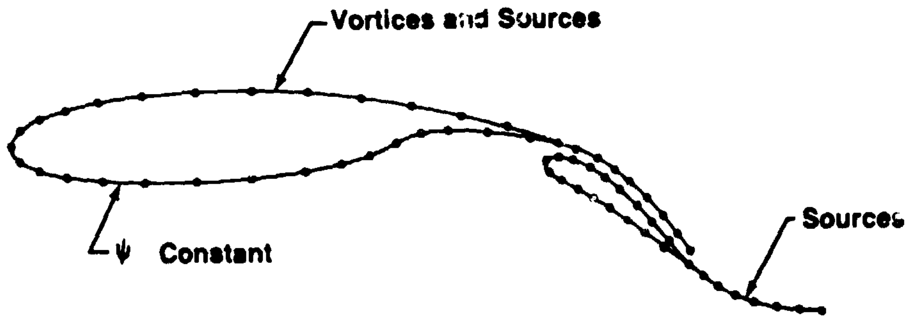

Figure 2.- Potential-flow singularities.

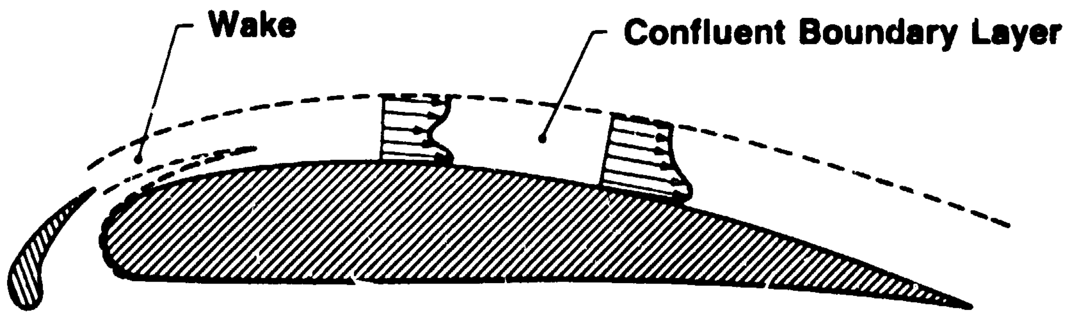

Figure 3.- Mean velocity profiles of confluent boundary layer.

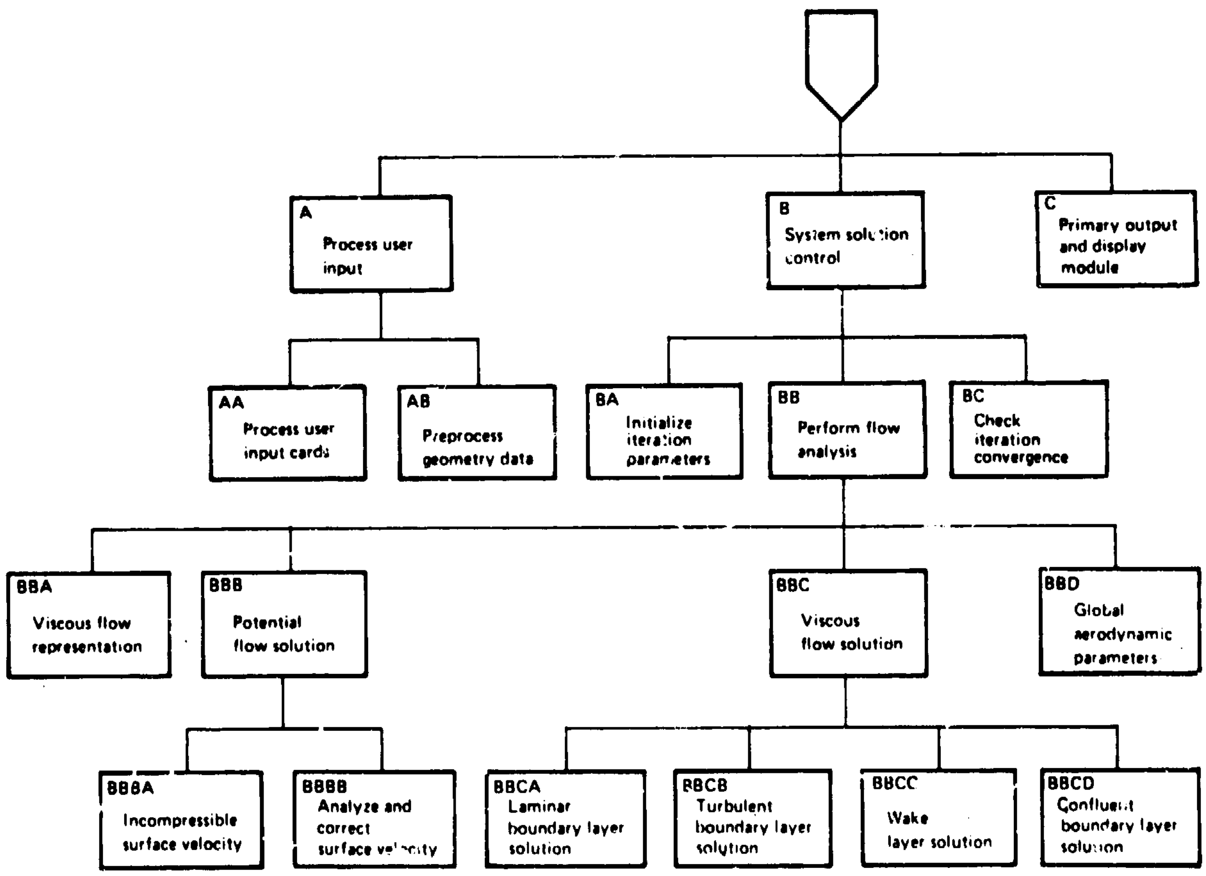

Figure 4.- Functional decomposition of NASA-Lockheed program.

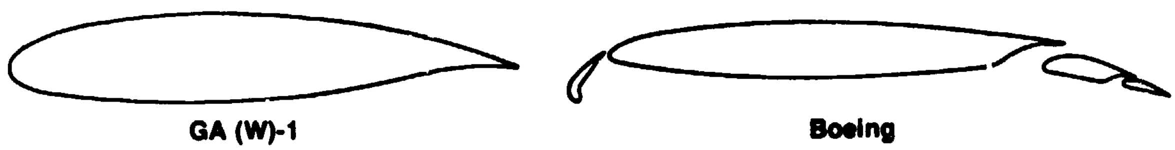

Figure 5.- Airfoil geometries.

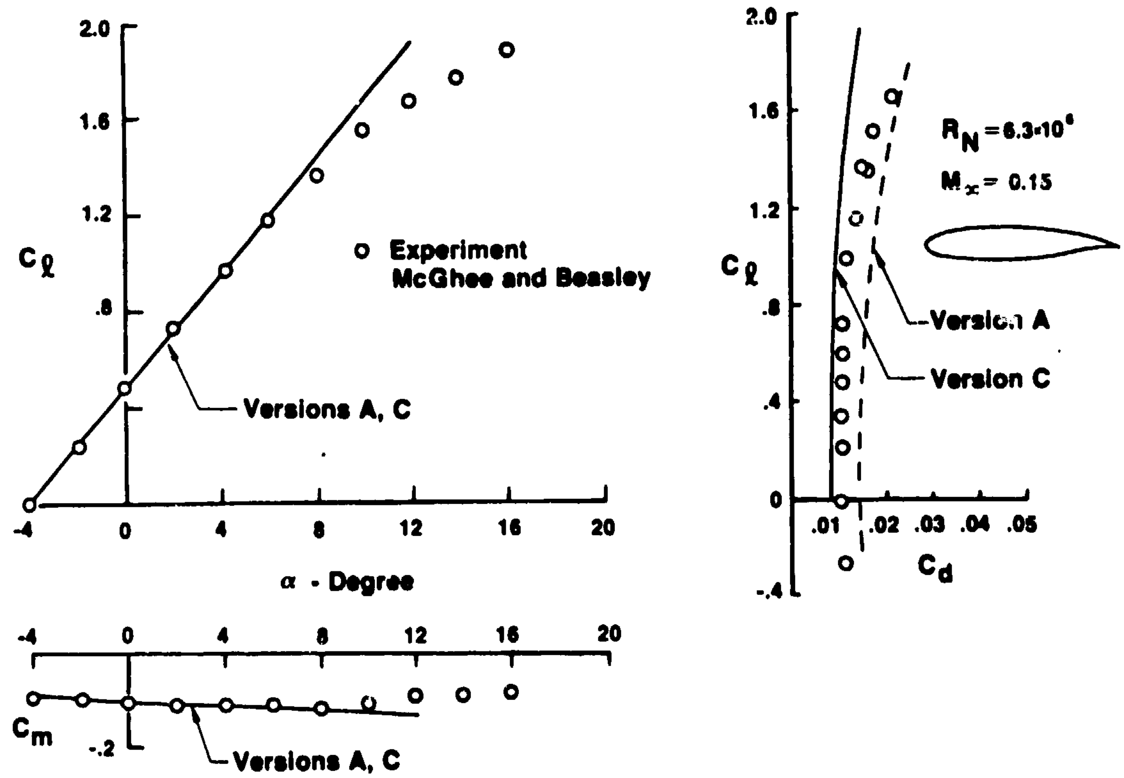

Figure 6.- Lift, drag, and pitching moment of GA(W)-1 airfoil.

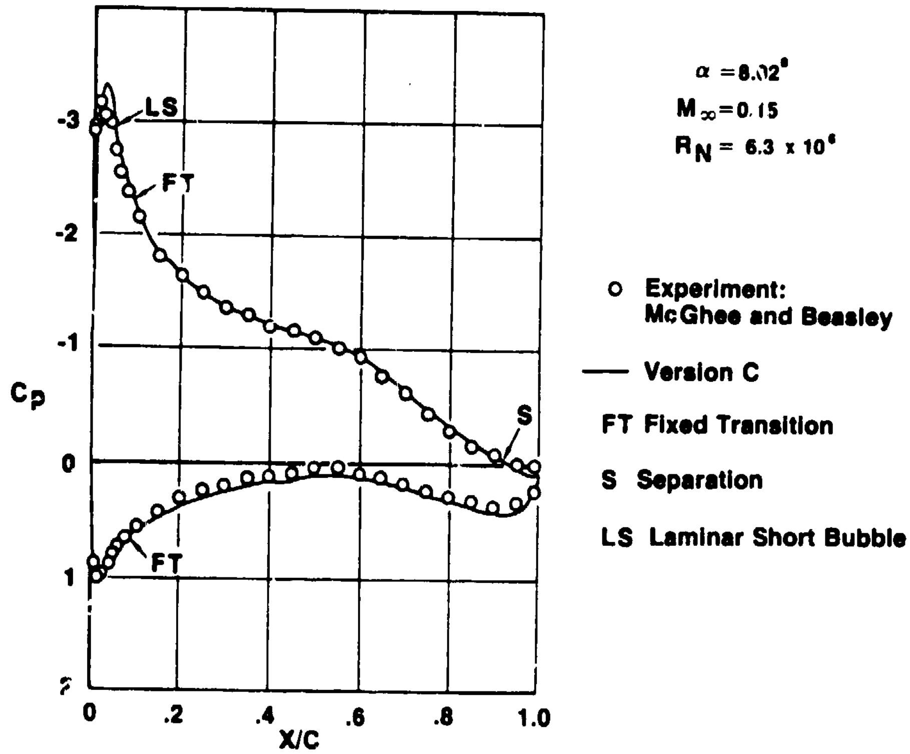

Figure 7.- Surface pressure of GA(W)-1 airfoil.

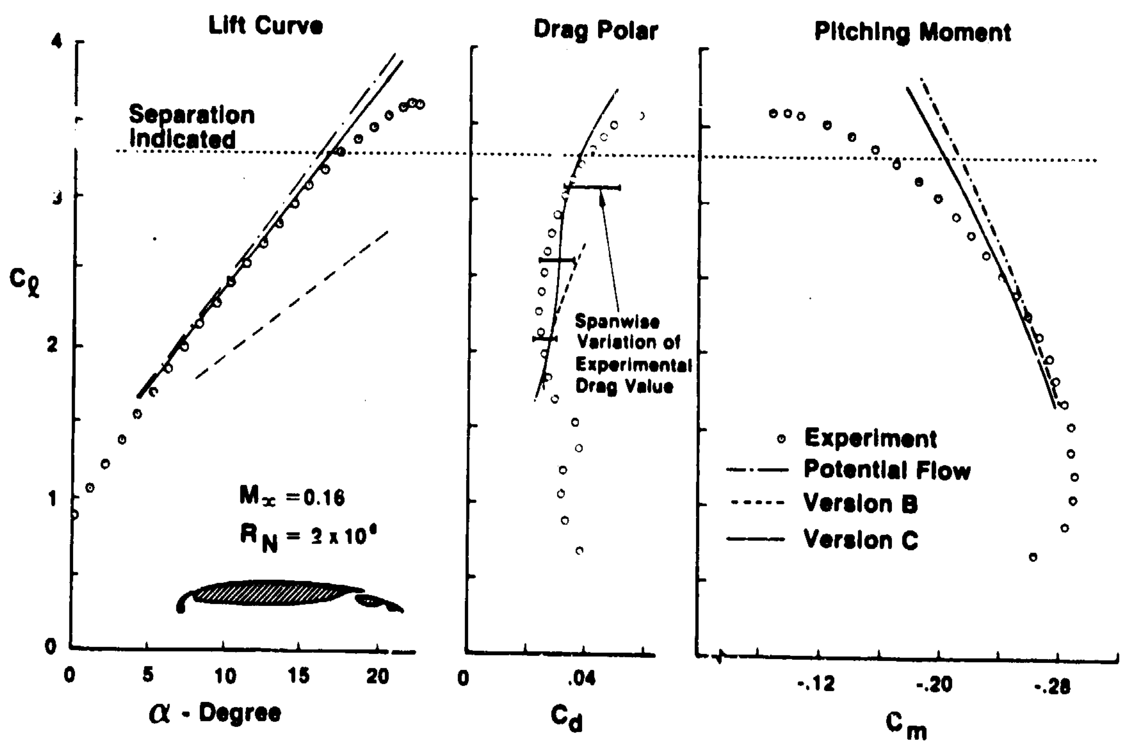

Figure 8.- Characteristics of Boeing four-element airfoil.

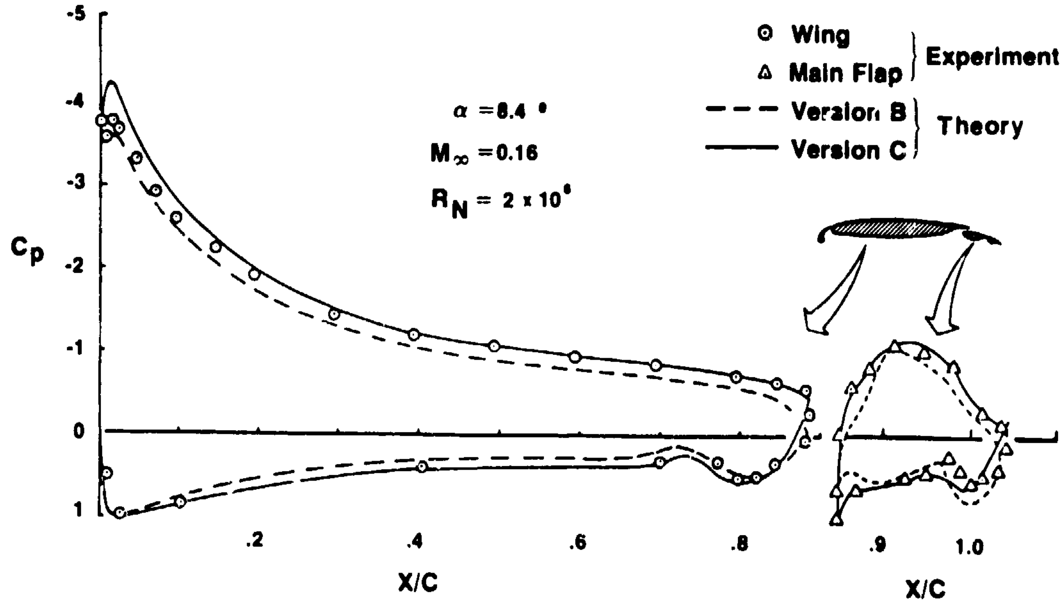

Figure 9.- Surface pressure of Boeing four-element airfoil.

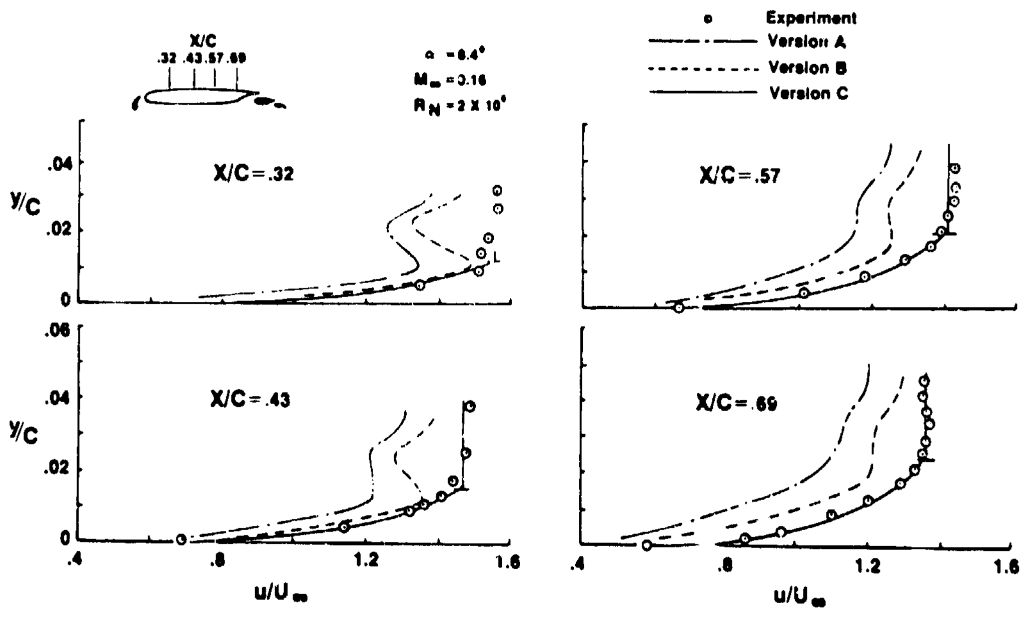

Figure 10.- Bo.r.dary-layer profiles on upper surface of Boeing four-element airfoil.