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MODE I CRACK SURFACE DISPLACEMENTS FOR A ROUND COMPACT SPECIMEN SUBJECT TO A COUPLE AND FORCE

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ABSTRACT

Mode I displacement coefficients along the crack surface are presented for a radially cracked round compact specimen, treated as a plane elastostatic problem, subjected to two types of loading; a uniform tensile stress and a nominal bending stress distribution across the net section. By superposition the resultant displacement coefficient or the corresponding influence coefficient can be obtained for any practical load location. Load line displacements are presented for A/D ratios ranging from 0.40 to 0.95, where A is the crack length measured from the crack mouth to the crack tip and D is the specimen diameter. Through a linear extrapolation procedure crack mouth displacements are also obtained. Experimental evidence shows that the results of this study are valid over the range of A/D ratios analyzed for a practical pin loaded round compact specimen.

INTRODUCTION

ASTM Committee E-24 on Fracture Testing is currently considering the round compact specimen (edge-cracked disk) for incorporation into ASTM Standard Method of Test E3999 on Plane Strain Fracture Toughness of Metallic Materials. Pursuant to this task, a program to evaluate the round compact specimen was initiated by ASTM Task Group E24.01.12.

The objective of this paper is to aid in the establishment of the round compact specimen as an alternate standdard for the determination of the plane strain fracture toughness (K_{IC}) in international testing and certification standards. The round compact specimen geometry, when compared to existing standard specimens, is most efficient in testing round bar stock because of the low cost of fabrication. Displacement coefficients were obtained herein for the range of ratio of crack length-to-specimen diameter, A/D, from 0.40 to 0.95 (where A is the distance from the crack tip to the circumference). Displacement coefficients Δ_P and Δ_M apply to two types of specimen loading; Δ_P to a uniform distribution of stress across the net section, and Δ_M to a nominal bending stress distribution across the net section (fig. 1). While these types of load are in themselves impractical, the two coefficients can be combined to represent any practical case of loading by a pair of equal and opposite forces normal to the crack (pin loading). The appropriate displacement coefficient Δ is obtained from a linear combination of displacement coefficients Δ_P and Δ_M.

In obtaining the crack surface displacement solution, the same boundary conditions on the stress function and its derivative given in reference 1 were used. Through a linear extrapolation of the displacement coefficient function at the load line, the crack mouth displacement coefficient is obtained.

For small A/D ratios the differences between the actual distribution of loading forces in a pin-loaded specimen and the distribution assumed in the analytical model may be significant. Crack displacements often become sensitive to these differences as the pin hole nears the crack tip and/or the crack surface. To verify the applicability of the present results to the smallest A/D ratios of practical interest, calculated displacements are compared with experimental measurements. Fisher and Buzzard (ref. 2) experimentally obtained the influence coefficients across the crack surfaces along the load line and crack mouth for a given load line location (W/D = 0.7407) with A/D ratios varying from 0.398 to 0.909 (fig. 1). Comparisons of load point and load line displacements were made at A/D = 0.398 and 0.909. In addition, comparison is made with displacement coefficients for the standard rectangular compact specimen.

These results were obtained assuming a parabolic shear distribution along the load line. The boundary conditions on the stress function and its derivative are given in reference 3. Further comparisons are made with analytical results of other investigators for the round compact specimen given in references 4 and 5.

Displacement coefficients Δ_P and Δ_M are presented in tabular form as least squares best fit polynomials. An example is provided illustrating the use of these functions to obtain load line and crack mouth displacements.

NOMENCLATURE

A	crack length measured from crack mouth to crack tip
$a = A + W - D$	crack length measured from load line to crack tip
B	specimen thickness
D	disk diameter
E	Young's modulus
E'	effective modulus, equals $E/(1 - \nu^2)$ for plane strain or E for plane stress
E'Bv/P	influence coefficient
L	distance measured from load line to centerline of disk
M	resultant moment at nominal neutral axis position
P	applied pin load
$v = v_P + v_M$	resultant vertical displacement at location z across the crack surfaces
v_P	vertical displacement across the crack surfaces at z resulting from a fictitious uniform net section tension
v_M	vertical displacement across the crack surface at z resulting from a fictitious nominal net section bending
W	distance measured from load line to circumference of specimen $(L + D/2)$
x, y	Cartesian coordinates (fig. 1)

$z = \left[(A + W) - (x + D)\right]/(A - 0.15D)$	dimensionless location along the crack surface for $0.3 < z < 1$
$\delta z = (D - W)/(A - 0.15D)$	dimensionless distance from load line to crack mouth
$\Delta_P(z) = E'v_P/\sigma_P A$	displacement coefficient as a function of z due to σ_P
$\Delta_M(z) = E'v_M/\sigma_M A$	displacement coefficient as a function of z due to σ_M
$\Delta(z) = E'v/(\sigma_P + \sigma_M)A$	resultant displacement coefficient as a function of z
ν	Poisson's ratio
$\sigma_M = 6M/B(D - A)^2$	component of fictitious linear net section bending stress at crack tip resulting from moment M
$\sigma_P = P/B(D - A)$	component of fictitious uniform net section tensile stress resulting from applied load P

Subscripts:

M	value associated with net section bending
P	value associated with net section tension

APPROACH

For a given A/D ratio and at a given z location, the displacement coefficients are defined as follows:

$$\Delta_P(z) = E'v_P(z)/\sigma_P A \tag{1}$$

$$\Delta_M(z) = E'v_M(z)/\sigma_M A \tag{2}$$

$$\Delta(z) = E'v(z)/(\sigma_P + \sigma_M)A \tag{3}$$

Applying the superposition principle as shown in figure 1, $v = v_P + v_M$ we obtain

$$\Delta(z) = \left[\sigma_P/(\sigma_P + \sigma_M)\right]\Delta_P(z) + \left[\sigma_M/(\sigma_P + \sigma_M)\right]\Delta_M(z) \tag{4}$$

Since $\sigma_M = 6M/B(D - A)^2$ where $M = M_o + P(2L + A)/2$ and $\sigma_P = P/B(D - A)$.

Substitution into equation (4) together with some algebraic manipulation results in

$$\Delta(z) = \frac{PD(1 - A/D)\Delta_P(z) + 3\left[2M_0 + PD(2L/D + A/D)\right]\Delta_M(z)}{PD(1 + 6L/D + 2A/D) + 6M_o} \tag{5}$$

The influence coefficient $E'Bv/P$ can be obtained directly from equations (3) and (4) as

$$E'Bv/P = \frac{(A/D)(1 - A/D)\Delta_P(z) + 3(A/D)(2M_0/PD + 2L/D + A/D)\Delta_M(z)}{(1 - A/D)^2} \tag{6}$$

To obtain the crack south displacement coefficient the slope of $\Delta(z)$ is assumed constant at and beyond the load line location $x = 0.$, hence $zo = (A + W - D)/(A - 0.15D)$. Thus

$$d\Delta/dz\Big|_{zo} = \left[\sigma_P/(\sigma_P + \sigma_M)\right] d\Delta_P/dz\Big|_{zo} + \left[\sigma_M/(\sigma_P + \sigma_M)\right] d\Delta_M/dz\Big|_{zo}$$

and it follows that

$$\Delta_{mo} = \Delta\Big|_{zo} + d\Delta/dz\Big|_{zo} \delta z \tag{7}$$

where $\delta z = (D - W)/(A - 0.15D)$ and mo = mouth and zo = load line location. Similarly, the crack mouth influence coefficient is

$$E'Bv/P\Big|_{mo} = E'Bv/P\Big|_{zo} + d(E'Bv/P)/dz\Big|_{zo} \delta z \tag{8}$$

RESULTS AND DISCUSSION

The resultant solution of the cracked disk problem is obtained by superposition of two types of loading as shown in figure 1. The first type of loading is based on a constant net section stress $\sigma_P = P/\left[B(D - A)\right]$ resulting in the displacement coefficient Δ_P as a function of σ_P and A/D. The second solution is based on a nominal pure bending stress where $\sigma_M = 3\left[P(A + 2L) + M_o\right]/\left[B(D - A)^2\right]$ at the crack tip from which the displacement coefficient Δ_M as a function of σ_M and A/D is obtained. The values of Δ_P and Δ_M, table 1, are obtained using the boundary collocation method and stress function boundary conditions described in reference 1.

Table 2 contains comparisons of the present influence coefficient results (eqs. (6) and (8) with $M_o = 0$) with those of references 2 through 5. The results in reference 4 are for a point load at location $L = 0.25D$ and $y = 0.2D$ (fig. 1 and table 2A), thus, $W/D = 0.75$. The influence coefficient values obtained herein along the load line are in very good agreement with those published in reference 4. For the influence coefficient at the crack mouth good agreement was obtained for A/D ratios greater than 0.7.

For $W/D = 0.7407$, which is the value being considered for standardization, good agreement among the present results, those of reference 5, and the experimental results of reference 2 at the crack mouth and load line is obtained over the whole range of A/D ratios. The experimental results were obtained on a specimen geometrically similar to the proposed standard round compact specimen. The test specimen had the following dimensions: $D = 20.42$ cm, $B = 2.73$ cm, and $W = 15.12$ cm. The material used, 6061-T651 Aluminum, had a modulus $E = 689.48 \times 10^9$ N/m^2. A least squares best fit polynomial of sixth degree, $\ln(E'Bv/P) = C_0 + C_1X + C_2X^2 + C_3X^3 + C_4X^4 + C_5X^5 + C_6X^6$ (where $X = 1 + A/W - D/W$) is fitted to the experimental displacement data sets (1) measured at the crack mouth, and (2) measured across the load line. The experimental influence coefficient values in table 2B are then obtained using this fitting function for interpolation.

On comparing the load line experimental results with the present values, the percent variations ranged from -0.3 to +4.0 over the whole range of A/D ratios. At the crack mouth, the percent variation ranged from +2.3 to +6.8 over the whole range of A/D ratios. While the experimental test specimen approached plane stress conditions, the specimen thickness was 2.73 cm and a thinner test specimen should reduce these variations.

It should be appreciated that there can be a limitation on the applicability of the present results to the practical pin loaded cracked disk. When the crack tip is very close to the load line (small A/D ratios), differences in displacement coefficient values can occur since there is a significant difference between the actual distribution of loading forces and that assumed in the model.

An experiment, reference 2, was conducted to obtain the ratio of load point to load line displacement for $A/D = 0.398$ and 0.909. For $A/D = 0.398$ the ratio was 1.033. For $A/D = 0.909$, the ratio was 1.026. The experimental results indicate that the hole distortion is small and that indeed the compliance measured across the crack surface at the load line for relatively small A/D ratios can be used to accurately reflect load displacement.

As described earlier, the influence coefficients of table 2 were obtained by application of the superposition principle to those computed functions given in table 1, with load line $L/D = 0.2407$ and 0.2500. The results of table 1 are limited to an L/D maximum of 0.35 (ref. 1). The lower bound on L/D for crack load line displacements depends on the length of the crack and the lower bound of z which is 0.3.

As an example for $A/D = 0.40$ and $z = 0.30$, since $x = 0.$, L/D minimum $= 0.175$. For $A/D = 0.95$ and $z = 0.30$, since $x = 0.$, L/D minimum $= -0.21$. Here the negative value indicates that L (fig. 1) is measured to the right of the disk centerline.

SUMMARY AND CONCLUSIONS

The solution obtained provides crack displacements for any combination of bending moment and normal forces acting on the round compact specimen. These combinations of loads are not restricted to a single load line to specimen diameter ratio. The loading of the specimen is characterized by the statically equivalent combination of resultant forces and moment chosen to act through the mid-net section. The advantage of this approach is that the influence coefficient for any reasonable load line location can be obtained efficiently by superposition of two complementary fictitious cases, namely, net section tension where the value of the moment is zero and net section bending where the value of the resultant normal force is zero.

On comparing the load line experimental influence coefficient results with the present values, very good agreement was obtained over the whole range of A/D ratios (0.40 to 0.95), thus, verifying the accuracy of the mathematical model. Comparison of the crack mouth experimental influence coefficient results with the analytical results of this investigation showed good agreement over the whole range of A/D ratios.

EXAMPLE

The results in table 2 are for a round compact specimen with load P at load line location L (or equivalent location $x=0.$), $M_o = 0$ and $\sigma_P = P/B(D-A)$ and $\sigma_M = 3P(2L+A)/B(D-A)^2$.

For the following geometry, where $L = D/4$ ($W/D = 0.75$) and $A/D = 0.55$, to obtain the displacement across the crack at the load line and at crack mouth - it follows that $W/D = 0.75$ and $zo = 0.75$.

From equations (6) and table 1

$$E'Bv/P = \frac{0.55(0.45)\Delta_P\Big|_{zo} + 1.65(0.5 + 0.55)\Delta_M\Big|_{zo}}{(1. - 0.55)^2}$$

$$\Delta_P\Big|_{zo} = 1.10031 - 5.38109\ zo + 8.96387\ zo^2 - 9.41258\ zo^3 = -1.86426$$

$$\Delta_M\Big|_{zo} = 0.30182 + 3.85332\ zo - 1.46881\ zo^2 + 1.48341\ zo^3 = 2.99142$$

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Thus at the load line $E'Bv/P = 23.315$.

To obtain the crack mouth influence coefficient, differentiating equation (6) and evaluating the derivative at the load line one obtains

$$d(E'Bv/P)/dz\Big|_{zo} = \frac{0.55(0.45)d\Delta_P/dz\Big|_{zo} + 1.65(0.5 + 0.55)d\Delta_M/dz\Big|_{zo}}{(0.45)^2} = 25.978$$

$$d\Delta_P/dz\Big|_{zo} = -7.8190 \quad \text{and} \quad d\Delta_M/dz\Big|_{zo} = 4.1534$$

For this case $\delta z = 0.625$, and from equation (8) we obtain the crack mouth influence coefficient

$$E'Bv/P = 23.315 + 25.978\ (0.625) = 39.55$$

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TABLE 1. - COEFFICIENTS OF 3 DEGREE POLYNOMIAL LEAST SQUARES BEST FIT FOR CRACK SURFACE DISPLACEMENT COEFFICIENTS Δ_P AND Δ_M FOR RANGE OF $0.3 \leq z \leq 1$

A/D	$\Delta_P(z) = C_0 + C_1 z + C_2 z^2 + C_3 z^3$				$\Delta_M(z) = B_0 + B_1 z + B_2 z^2 + B_3 z^3$			
	C_0	C_1	C_2	C_3	B_0	B_1	B_2	B_3
0.40	0.63259	-2.79804	3.14766	-5.82857	0.52340	4.65676	-1.72455	1.73955
.45	.81814	-3.67342	5.39112	-7.34770	.43518	4.36949	-1.70006	1.73346
.50	.97526	-4.57193	7.38877	-8.56467	.36390	4.09767	-1.60570	1.63752
.55	1.10031	-5.38109	8.96387	-9.41258	.30182	3.85332	-1.46881	1.48341
.60	1.18337	-5.99026	9.94839	-9.82094	.25308	3.60392	-1.26286	1.27581
.65	1.22402	-6.38020	10.35781	-9.82159	.21419	3.35430	-1.01423	1.03953
.70	1.21929	-6.52059	10.15645	-9.40042	.18281	3.10465	-.72847	.78052
.75	1.16734	-6.40654	9.36121	-8.57212	.14603	2.92152	-.53120	.57055
.80	1.07257	-6.07930	8.06399	-7.38548	.12054	2.71632	-.29821	.35624
.85	.93559	-5.57558	6.34743	-5.87687	.08993	2.56627	-.15371	.19946
.90	.75158	-4.94125	4.33777	-4.15626	.05776	2.45278	-.07881	.09642
.95	.49699	-4.20252	2.14393	-2.11343	.03195	2.30750	.00978	.00863

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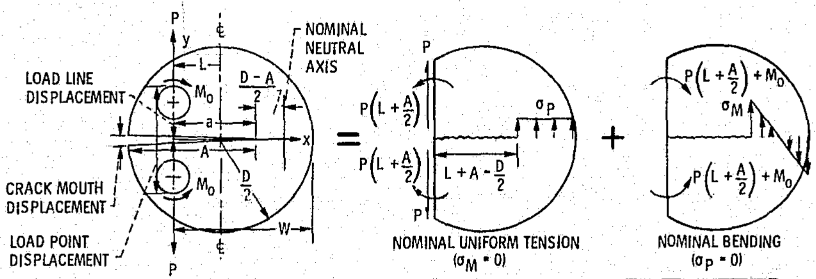

$$v = v_P + v_M$$

$$\Delta = \left(\frac{\sigma_P}{\sigma_P+\sigma_M}\right)\Delta_P + \left(\frac{\sigma_M}{\sigma_P+\sigma_M}\right)\Delta_M$$

Figure 1. - Application of superposition principle to obtain the resultant displacement at a given location L along the crack surface of a round compact specimen.

TABLE 2. - COMPARISON OF ANALYTICAL RESULTS WITH THOSE OF OTHER INVESTIGATORS [$a/W = (A/D + W/D - 1)/(W/D)$]

(a) W/D = 0.750

A/D	E'Bv/P						
	Analytical results collocation method		Newman (ref. 4)		Gregory (ref. 4)		Analytical results using collocation method for standard rectangular compact specimen
	Across crack at load line	At crack mouth	Across crack at load line	At crack mouth	Across crack at load line	At crack mouth	Across crack at load line
0.400	8.437	18.29	7.602	13.35	9.335	14.10	8.36
.450	12.02	23.33	------	------	12.88	------	------
.475	------	------	------	------	------	------	14.17
.500	16.77	30.15	15.86	25.51	17.62	26.40	------
.550	23.32	39.55	------	------	24.16	------	22.78
.600	32.68	52.92	31.76	48.44	33.53	49.36	------
.625	------	------	------	------	------	------	36.89
.650	46.67	72.74	------	------	47.53	------	------
.700	68.81	103.8	67.90	99.38	69.67	100.3	63.2
.750	106.5	156.1	------	------	107.4	------	------
.775	------	------	------	------	------	------	122.6
.800	177.8	254.3	176.9	250.3	178.6	250.8	------
.850	335.9	470.3	------	------	336.8	------	304.7
.900	800.0	1099	------	------	801.1	------	------
.950	3356	4536	------	------	3379	------	------

(b) W/D = 0.7407

A/D	E'Bv/P					
	Experimental results (ref. 2)		Analytical results collocation method		Specimen modelled with loading pin hole (ref. 5)	
	Across crack at load line	At crack mouth	Across crack at load line	At crack mouth	Across crack at load line	At crack mouth
0.400	7.91	17.03	7.89	17.89	------	------
.407	8.34	17.65	------	------	7.90	17.54
.450	11.17	21.61	11.35	22.78	------	------
.482	13.72	25.14	------	------	13.51	25.32
.500	15.46	27.57	15.94	29.46	------	------
.550	21.38	36.06	22.28	38.69	------	------
.556	22.18	37.22	------	------	22.45	38.29
.600	30.07	48.75	31.34	51.83	------	------
.630	37.28	59.26	------	------	37.95	60.72
.650	43.50	68.24	44.91	71.33	------	------
.700	65.11	98.92	66.39	101.9	------	------
.704	67.18	101.8	------	------	67.72	103.2
.750	101.7	149.7	103.0	153.5	------	------
.778	133.6	193.7	------	------	134.8	197.5
.800	169.5	243.0	172.3	250.3	------	------
.850	319.0	448.8	326.2	463.4	------	------
.852	327.7	460.9	------	------	334.6	474.6
.900	760.2	1050	778.0	1083	------	------
.950	------	------	3269	4477	------	------

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16. Abstract

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