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FLUID DYNAMIC PROBLEMS ASSOCIATED WITH AIR-BREATHING PROPULSIVE SYSTEMS

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by
W. L. CHOW

Final Report
Prepared for
Research Grant NASA NGL 14-005-140

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FOREWORD

This is the final report of the step funded Research Grant NASA NGL 14-005-140. This research program was originally suggested and initiated by Mr. M. Beheim, Chief, Wind Tunnel and Flight Test Division, Lewis Research Center, NASA, and was carried out under the supervision and management of Mr. D. Bowditch. Messrs, B. H. Anderson, and L. J. Bober served as technical monitors for this research project.

ABSTRACT

A brief account of research activities on problems related to air-breathing propulsion is made in this final report for the step funded research grant NASA NGL 14-005-140. Problems studied include the aircract ejector-nozzle propulsive system, non-constant pressure jet mixing process, recompression and reattachment of turbulent free shear layer, supersonic turbulent base pressure, low speed separated flows, transonic boattail flow with and without small angle of attack, transonic base pressures, Mach reflection of shocks, and numerical solution of potential equation through hodograph transformation.

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1. INTRODUCTION

In early spring of 1969, in response to the suggestion from the technical staff of the Wind Tunnel and Flight Test Division, NASA Lewis Research Center, a research proposal was prepared by the author of this report and submitted to NASA headquarters to initiate research on a variety of fluid dynamic problems related to modern airbreathing propulsive systems. While the immediate task was to cooperatively develop a comprehensive universal computer program with the technical personnel from NASA Lewis Research Center to evaluate the performance of aircraft ejector systems, research of many other basic problems related to airbreathing propulsion was really the purpose of initiating such a long-range research program. Indeed, after this research grant was awarded, upon recommendation by the technical management in NASA Lewis Research Center, this research grant was immediately modified into a step-funded program (NASA Grant NGL 14-005-140).

During the last ten years, a variety of basic and important problems has been investigated. While part of the effort was aimed at its immediate practical application, other activities were directed toward problems of a more basic nature which often times laid the foundation for other important and future activities. As a result of the shift in the mission of the cooperating technical group in NASA Lewis Research Center, this research grant will be terminated at the end of May 1979.

The present report serves to present a brief account of activities carried out within the past ten years under the sponsorship of this NASA grant, the achievements accomplished, and other investigations which are directly initiated, and related as a result of these research accomplishments. No attempt will be made to review these research activities in detail. References of publications will be mentioned and cited wherever possible. To avoid a separate listing of publications resulting from this research effort, publications with formal acknowledgement of NASA support will be marked by "(NASA)," while other publications, as a result of promotion and initiation from this effort, will be marked by "(related)" in the section of REFERENCES of this report at the end of their listing.

This research program also has its educational aspects of merit. It provided certain opportunities for graduate students to pursue their Ph.D. dissertation research. APPENDIX A lists the Ph.D. theses produced from this comprehensive research program.

A brief review of these research activities is given in the next section.

2. RESEARCH CONDUCTED IN PROBLEMS RELATED TO AIRBREATHING PROPULSION

Problems studied in the area of airbreathing propulsion within the last ten years can be categorized as follows:*

A. Ejector-nozzle as an aircraft propulsive system

B. Jet mixing process under the influence of a pressure gradient

C. Recompression and reattachment of turbulent free shear layers

D. Supersonic turbulent base pressure problem

E. Turbulent base pressure of low speed flow past bluff bodies

F. Transonic flow past boattailed afterbodies with and without small angle of attack

G. Transonic base pressure problems

H. Perturbation of the corresponding inviscid body geometry in separated flows

I. Mach reflection of shocks and related free jet flows

J. Numerical solution of potential equation through hodograph transformations

It should be noted that while many of the above research activities were conducted under the direct support of NASA, other programs were essentially promoted and initiated as a result of the accomplishments achieved from these earlier efforts and are now supported by the U.S. Army Research Office. Brief descriptions of these projects are discussed individually in the following section.

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*The sequence of listing more or less reflects the sequence of development of the research program.

2.1 Ejector-Nozzle as Aircraft Propulsive System

The supersonic ejector system was originally studied [1,2,3]* by analyzing the detailed inviscid-viscid interaction between the primary and secondary streams. For its application as an aircraft propulsive system, the primary nozzle usually has a conical shape which would introduce considerable distortion of the sonic line at the exit of the nozzle. The effort to study the realistic jet flow configuration related to this type of primary nozzle was carried out by Brown and Chow [4,5]. When these analyses were later incorporated into the considerations of the practical aircraft ejector propulsive system by Anderson [6,7], the distortion of the sonic line and the mixing between the primary and secondary streams were clearly identified to be very important processes for consideration. The final results obtained for a variety of configurations and flow conditions were in excellent agreement with the experimental data. Typical results obtained and reported [6] are shown in Figs. 1 and 2. No additional research effort was deemed necessary in this area.

An attempt to improve the one-dimensional analysis for the secondary flow by the method of integral relations has also been made and reported [8].

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*Numbers in brackets refer to entries in REFERENCES.

2.2 Jet Mixing Process under the Influence c Pressure Gradient

In a majority of separated flow problems, the fluid entrained within the wake region usually flows with a small velocity which cannot support any pressure gradient. Thus, the turbulent mixing process occurring along the wake boundary is under an essentially constant pressure condition. On the other hand, for the mixing between two streams within confined boundaries such as that occurring within the ejector system, pressure varies considerably along the mixing region that one must consider the influence of the pressure gradient. A simple way of estimating its effect would be the adoption of a quasi-constant pressure mixing process to predict the velocity profile under this situation. The validity of such an approach can never be established until a precise and detailed analysis of such a process has been made. This analysis has been carried out by employing Meksy's asymptotic method of integrating the "non-similar" boundary layer equation [9,10]. It has been found, indeed, that the quasi-constant pressure treatment provides a good estimation of such a process. Figure 3 shows the typical results obtained [9] which substantiate the validity of quasi-constant pressure approximation. In addition, the relaxation of any non-similar mixing flows toward the asymptotic similar flows has been beautifully illustrated [11] as the typical behavior of the governing parabolic equation. Other studies of turbulent jet mixing process as an important component of separated flows have also been documented [12,13].

2.3 Recompression and Reattachment of Turbulent Free Shear Layers

For all separated flow problems, recompression of free shear layer occurs downstream of the turbulent mixing region. Part of the flow entrained within the mixing layer is turned back, as a result of recompression, to form the recirculating wake flow while the rest will proceed downstream. Eventually, the flow reattaches on the lower wall. In the absence of mass bleed into the wake, the stagnation streamline must be the jet boundary streamline which divides the fluid of the upstream flow from that trapped within the wake. Previous suggestion of the "discriminating criterion" by Korst [14]--the stagnation pressure of the jet boundary streamline is identical to the pressure behind the recompression shock--provided the closure condition for the supersonic base pressure problem. However, the experimental evidence on the pressure distribution did not substantiate this hypothesis. Detailed analysis of such a process is necessary to clarify this situation. An effort was made to formulate such an analysis to evaluate this process in a detailed manner [15]. An integral approach was adopted for the viscous flow, and a coupling relationship was introduced to correlate the dividing streamline velocity and its slope to assure that the reattachment profile could be ultimately reached. This approach was, indeed, very fruitful. It not only provided an adequate description of the flow process but was also recognized immediately after the successful completion of this analysis that it can be applied to separated flow problems in any other flow regimes. Figure 4 presents the schematic flow field within the recompression flow region as reported in Ref. [15].

2.4 Supersonic Turbulent Base Pressure Problem

It is well known that the base pressure problems are governed by the phenomenon of viscid-inviscid interaction. It stresses the fact that the solution of the problem can be obtained only through simultaneous considerations of both the viscous and inviscid flows. It is true that the inviscid flow still guides the viscous flow in the sense of the boundary layer concept. However, the configuration of the corresponding inviscid flow, particularly the pressure level within the wake region, relies on the viscous flow mechanisms of mixing, recompression, reattachment, and flow redevelopment. For supersonic turbulent flow problems, the base pressure is low, and the Mach number of the external free stream is large prior to recompression. In conjunction with the fact that streamlines have large curvatures as a result of turning, the pressure difference across the viscous layer during recompression, reattachment, and the subsequent redevelopment, will no longer be negligible.

The study of this viscid-inviscid interaction related to supersonic base pressure problems has been carried out by using an integral analysis for the viscous layer. The aforementioned pressure difference across the viscous layers were also properly accounted for [15]. Furthermore, by interpreting the process of flow redevelopment after flow reattachment as a process of relaxation of this pressure difference [16], it can be shown that the fully relaxed (or fully developed) state is a saddle point singularity for the system of equations governing the flow redevelopment. This provides the closure condition for the base pressure problem for the supersonic flow regime. The two-dimensional results [16] of base pressures for $M_\infty = 2$ are shown in Fig. 5. Extension to the axisymmetric supersonic base pressure problems has also been carried out [17,18]. Fairly good agreement with the experimental data has been obtained. Figures 6 and 7 show some of the typical results obtained and reported.

2.5 Turbulent Base Pressure of Low Speed Flow past Bluff Bodies

Separated flow problems resulting from low speed flow past bluff bodies are well known. Since the viscous layer cannot cope with the eventual pressure rise if the flow were to follow the configuration of a blunt based body, the flow must separate, forming a wake behind the body. This separation not only effectively modifies the corresponding inviscid flow configuration, it also energizes the slowly moving viscous layer through the ensuring turbulent jet mixing process to prepare itself for the subsequent recompression and reattachment. For two-dimensional flow in the low speed flow regime, flow separation is invariably coupled with the unsteady vortex shedding. However, with the proper insertion of a solid plate along the centerline of the wake, as shown by Roshko [19], this process of vortex shedding can be effectively suppressed. Early analysis of this type of problem was based exclusively on the free streamline theory. The mathematical approach usually treats the base pressure as a free parameter whose value is supplied from experimental observations. It should be noted that since the governing inviscid flow equation is elliptic, any reasonable analysis with the experimentally observed base pressure as a parameter should yield excellent agreement with the data on the pressure distribution. The heart of the problem lies in the determination of the base pressure which is really governed by the viscid-inviscid interaction.

A flow model has been suggested and developed [20,21] to study the steady, two-dimensional, low-speed separated flow problems. For the convenience of arriving at solutions through conformal mapping, a free streamline model with idealized processes was suggested to account for the behavior of the corresponding inviscid flow. The complete determination of the inviscid flow relies upon two discrete parameters. Although the attached viscous flow is guided by the inviscid flow in the sense of the boundary layer concept, the viscid-inviscid interactions of these problems is manifested by the fact that these two discrete parameters, required to establish the corresponding inviscid flow, are to be determined from viscous flow considerations. Analysis for a backward facing step [21] was carried out. It was learned that the point of reattachment behaves as a saddle point singularity of the system of equations for the viscous flow recompression. This condition, together with the condition at the point of reattachment, allowed the determination of the aforementioned two discrete parameters. The obtained results on pressure distribution showed good agreement with the available experimental data. This method of analysis was also applied to wedges of arbitrary angles [22,23] and again yielded favorable results. Some of the typical results are shown in Figs. 8 and 9. Extension to an approaching jet flow of finite thickness has also been carried out [24] which would be useful in the study of flow fields within fluidic devices.

2.6 Transonic Flow past Boattailed Afterbodies with and without Angles of Attack

To evaluate the performance of an airbreathing propulsive system for a supersonic aircraft operating at subsonic cruise condition, it is necessary that the flow field associated with transonic flow past boattailed afterbody be successfully analyzed. This type of problem is difficult because the flow is transonic, and the governing inviscid flow is of the mixed type. In addition, the boattail is usually immersed in a thick boundary layer, built up along the forebody, that the effect of the viscous interaction is no longer negligible. An effort was carried out to study a transonic flow past a specific afterbody at zero angle of attack [25,26]. The inviscid flow field was established through numerical calculations of the full compressible potential equation, and the turbulent boundary layer was calculated on the basis of an integral approach. The viscous interaction was accounted for through corrections of the corresponding inviscid body configurations. It has been learned that even with a relatively slender body configuration, the transonic small disturbance equation cannot produce accurate results for the boattail when the results were compared with the experimental data. The final results obtained from the full potential equation with the viscous effect taken into account are in excellent agreement with the experimental data when the flow is not separated and show good qualitative agreement when the flow separates in the rear portion of the boattail. Figures 10 and 11 present the model and the results obtained in this investigation as reported in Ref. [25]. They also show that the viscous effects tremendously modify the flow field and this type of the problem should be classified as strong-interaction.

This study of transonic flow past the same boattailed afterbodies was later extended to cases with small angle of attack [27]. This leads immediately to considerable complexities and difficulties. Since the flow properties also depend upon the meridional coordinate, it is a three-dimensional flow problem. The computational effort multiplies with the number of meridional planes considered. Due to the uneven growth of the boundary layer and thus uneven displacement effect, the effective inviscid body configuration becomes non-axisymmetric even if the original boattailed afterbody is axisymmetric. When the cylindrical system of coordinates is employed to describe the flow, additional manipulations must be applied for all points along the axis ahead of the body since the term V_r/r approaches infinity there. Approximate calculations have been carried out by ignoring the effect of viscous cross flow (i.e., quasi-axisymmetric viscous flow). Although the results clearly indicated again the importance of the viscous effects, accurate results cannot be obtained without considering the full three-dimensional character of the viscous effect. This would require immediate attention if this research effort is to be continued. Figures 12 and 13 present the results obtained from this effort indicating again the important influence from the viscous effect and their comparison with the experimental data.

2.7 Transonic Base Pressure Problems

Concurrent with the improvement of analyzing the viscous flow recompression in separated flows, the capability of calculating the inviscid transonic flow field has also been developed. A natural incorporation of these accomplishments led to the development of a model to study the transonic base pressures [28,29]. It was stipulated that the corresponding inviscid body geometry is characterized by a certain parameter and the viscid-inviscid interaction is manifested by the fact that this parameter will be determined by the viscous flow analysis. The corresponding inviscid flow can be established from numerical calculations of the full potential equation, and the same integral analyses on mixing, recompression and reattachment are employed for the viscous flow. Again, it has been found that the point of reattachment behaves as a saddle point singularity for the system of equations governing the process of viscous flow recompression. It has been also learned that the definition of base pressure within this flow regime should be carefully clarified. This analysis was later also extended to study transonic base pressure problems in the axisymmetric configuration [30,31]. The results so obtained agreed reasonably well with the available experimental data. Some of these results are shown in Figs. 14 and 15.

2.8 Perturbation of the Corresponding Inviscid Body Geometry in Separated Flow

In view of the method developed for studying separated flows in the incompressible and transonic flow regimes, the viscid-inviscid interaction is accounted for from the fact that the inviscid flow guides the viscous flow in the sense of the boundary layer concept, and the characteristic parameter required to establish the inviscid flow is determined from the viscous flow analysis. The calculations must begin by selecting this parameter and later to be iterated upon until, for example, the condition at the point of reattachment is satisfied. However, it is immediately recognized that the corresponding inviscid flow compatible to the already established viscous flow is not the one determined from this characteristic parameter. This inviscid flow field is only compatible if the characteristic Reynolds number approaches infinity. Due to the effect of finite Reynolds number, the compatible inviscid flow should have a configuration which is approximately away from the established viscous dividing streamline by a distance of the "displacement thickness" of the viscous layer above the dividing streamline. It is thus obvious that the results established so far constitute the first approximation of the solution. To obtain a closer approximation, the corrected corresponding inviscid flow field must be re-established. Since the transonic inviscid flow has been established from numerical calculations, solutions corresponding to the corrected inviscid configuration can be established accordingly. For incompressible two-dimensional flow, the method of conformal mapping can no longer be applied and other methods, such as the surface source-sink method, must be employed. It may be conjectured, however, that this type of iteration of the inviscid flow configuration should be a rapidly convergent process since it is well known that the influence of the Reynolds number within the turbulent (high Reynolds number) flow regime is small. In fact, it is expected that the base pressure results from the first approximation should be quite accurate. This concept of perturbation of the corresponding inviscid body geometry for separated flows [32] may even be extended to study separated flow problems in the supersonic flow regime.

2.9 Mach Reflection of Shocks and Related Free Jet Flows

The flow field of a compressible free jet with curved shocks, whether in two-dimensional or axisymmetric configuration, is always of considerable interest. It is a problem of a basic nature, exhibiting mechanisms of inviscid interaction. It is also related to practical application such as tracking of jet aircraft or missiles through heat-seeking devices.

It is well known from the inviscid flow theory that under specific conditions, a shock cannot be regularly reflected from a solid boundary, and Mach reflection of shock would occur. An attempt has been made to study the generation of such a Mach reflection and also the accompanied free jet flow field. The early study of ejector system [1] suggested that this type of flow must be governed by the inviscid flow interaction. It was thus stipulated that the occurrence and the location of a Mach reflection configuration must be such that the central core flow downstream of the triple point shall reach a sonic state at a section of minimum area, and the viscous effects occurring along the slip line and the free jet boundary can only introduce a modifying influence. This was ineed a fruitful effort to locate the Mach stem [33] and was later extended to locate the Mach disk in the axisymmetric flow under under-expanded nozzle flow condition [34]. It was also later found that, under over-expanded nozzle flow condition, whether it is of the two-dimensional or axisymmetric geometry, the location and the size of the Mach stem can be estimated from a simple approximate theory irrespective of whether the flow behind the reflected curve shock is supersonic or subsonic [35]. All these calculations produced results which agreed with the experimental data. The nozzle pressure ratios required to produce various flow regimes of the free jet flow, including that required for the occurrence of Mach reflection at different nozzle Mach numbers, have also been presented and discussed [35]. It was also obvious that there exists a flow regime of strong curved shock (provided that the inviscid flow field is not significantly modified by the viscous effects) for a nozzle-free jet flow. The asymptotic flow conditions of these flows have also been successively analyzed [36]. Figures 16, 17, and 18 present some of the results obtained and reported [34,35,36].

2.10 Numerical Solutions of Potential Equation through Hodograph Transformations

Along with the development of high speed digital computers, finite difference calculations of flow fields are becoming increasingly popular. Calculations of transonic flow provided good examples of the progress made in the study of flow field through numerical computations. Transonic flow problems were considered to be the most difficult problems years ago because of their mixed character, and these problems are now solved as a matter of routine through numerical treatment even when the viscous displacement effects must be accounted for [25]. However, it is this author's experience that the convergence and accuracy of results from numerical calculations depend upon the coordinate systems and the method of calculation employed.

One important advantage that the numerical calculation of the hodograph equation offers is the linearity of the equation governing the compressible flow. As an initial attempt to employ the hodograph equation, the discharge of compressible flow through the channel was calculated [37]. It reproduced Von Mises results of Vena-contracting coefficients for incompressible flow. These calculations have been extended to compressible flow regime and can produce results for any subsonic approaching flow with any convergent angle.

It was found later that this method of calculation can solve many problems in the area of open channel flow where the effects of gravitational field control the phenomenon. The problem of discharge from a channel with gravitational effect has been solved in this manner [38]. The problem of free overfall has also been examined [39]. Figures 19 and 20 present some of the results reported. Other calculations on sluice gate and sharp crested weir also produced excellent results [40]. All of these investigations substantiated the efficacy of numerically solving the hodograph equation. Extension to compressible flow with curved solid boundaries will be carried out in the future.

3. CONCLUDING REMARKS

Within the last ten years, the author of this report had the opportunity of collaboration with the technical group of the Wind Tunnel and Flight Test Division, Lewis Research Center, NASA, and studied many basic and yet practical problems related to jet or rocket propulsion. This author also believes that this cooperative arrangement serves as a good example of a model of interaction between federal agencies and academic institutions.

Research on some of the technical problems discussed in this report has been brought into successful conclusion as a result of this program. Many other problems were initiated in the later stage of the program and will need considerable additional attention and cultivation. Thus, the present report reflects only the present status of these problems.

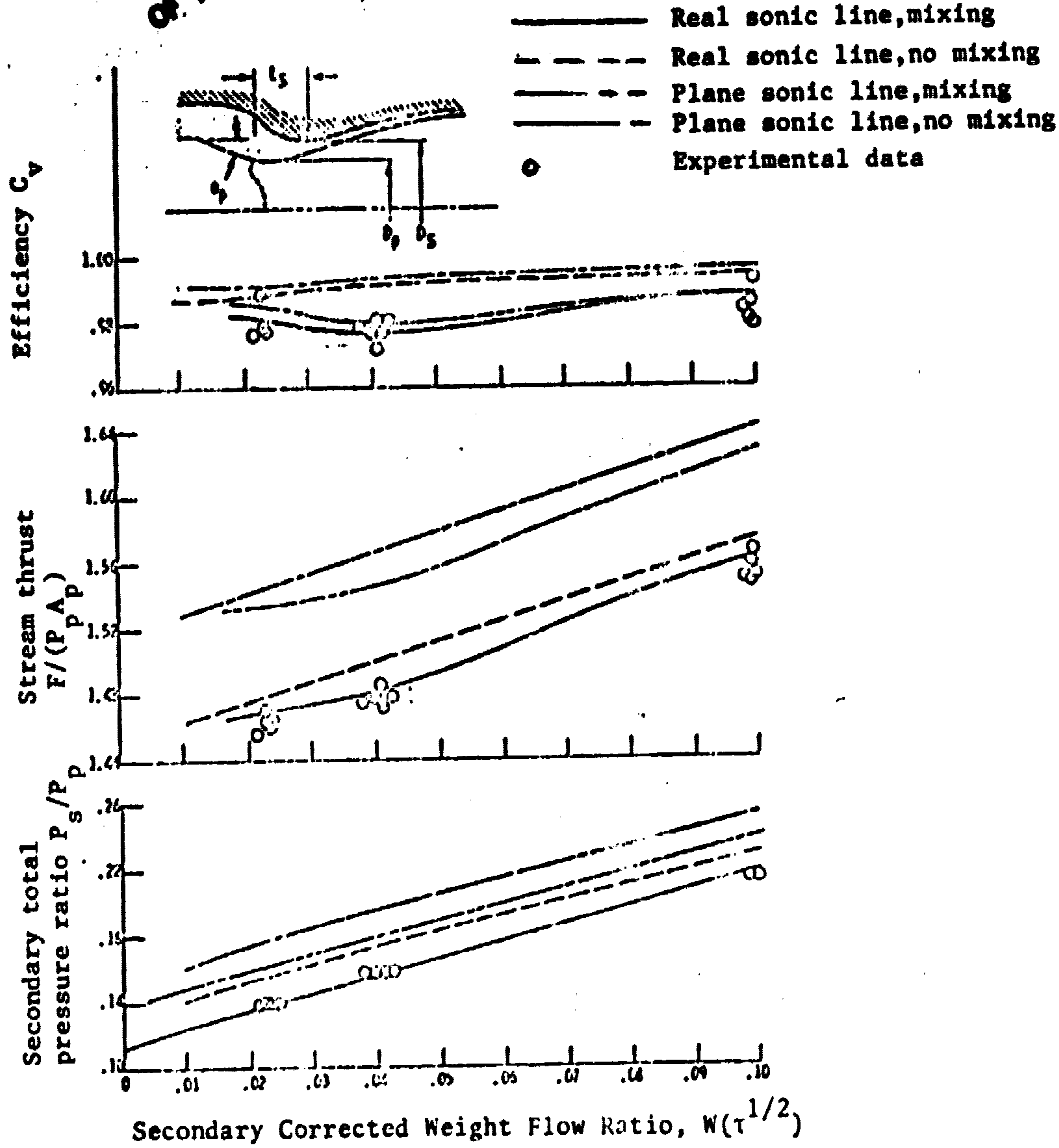


Figure 1 Influence of Sonic Line and Mixing Process on Performance of a Convergent-Divergent Contoured Flap Ejector (adapted from Fig. 2 of Ref. 6)

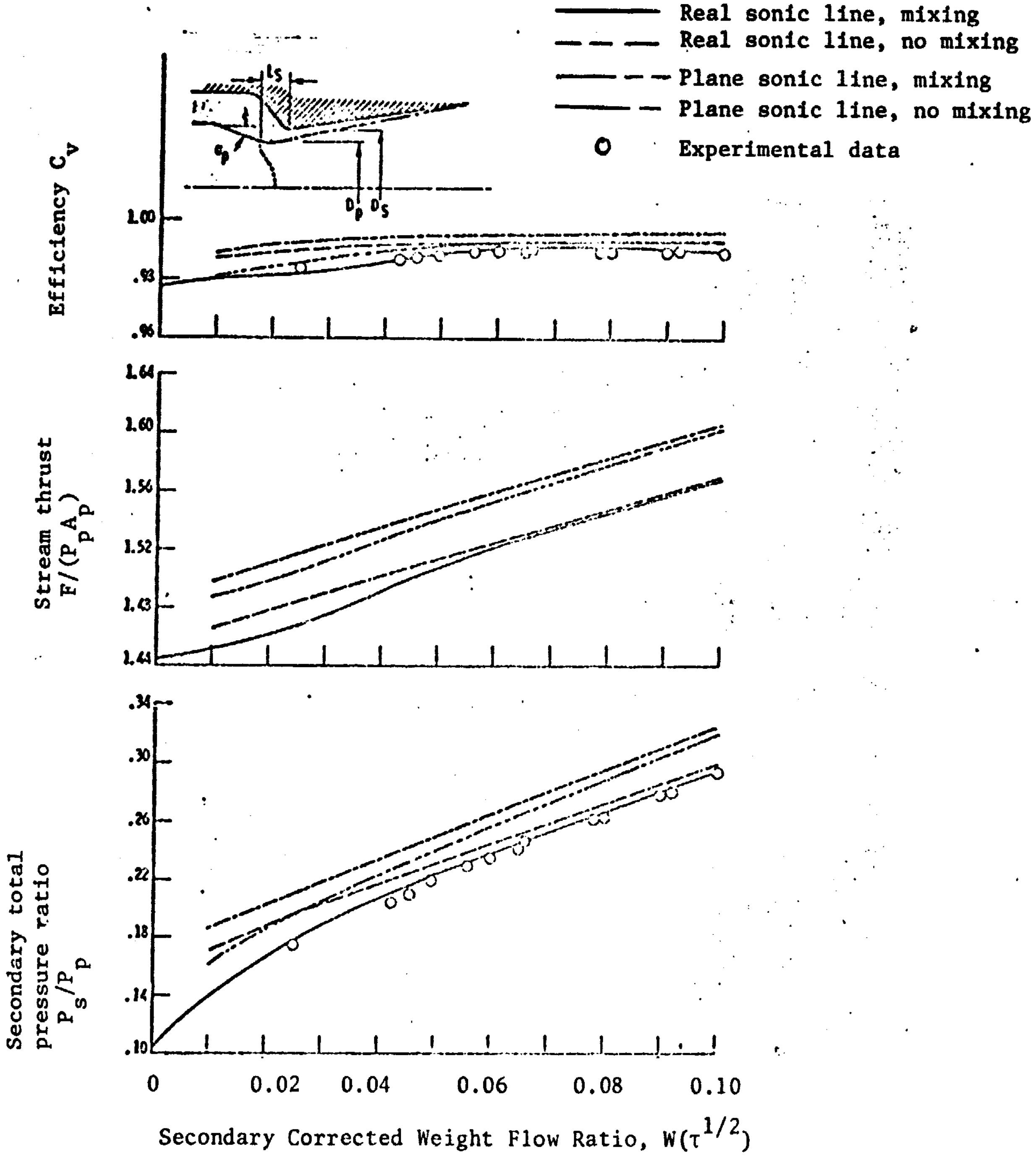


Figure 2 Influence of Sonic Line and Mixing Process on Performance of a Convergent-Divergent Conical Ejector (adapted from Fig. 5 of Ref. 6)

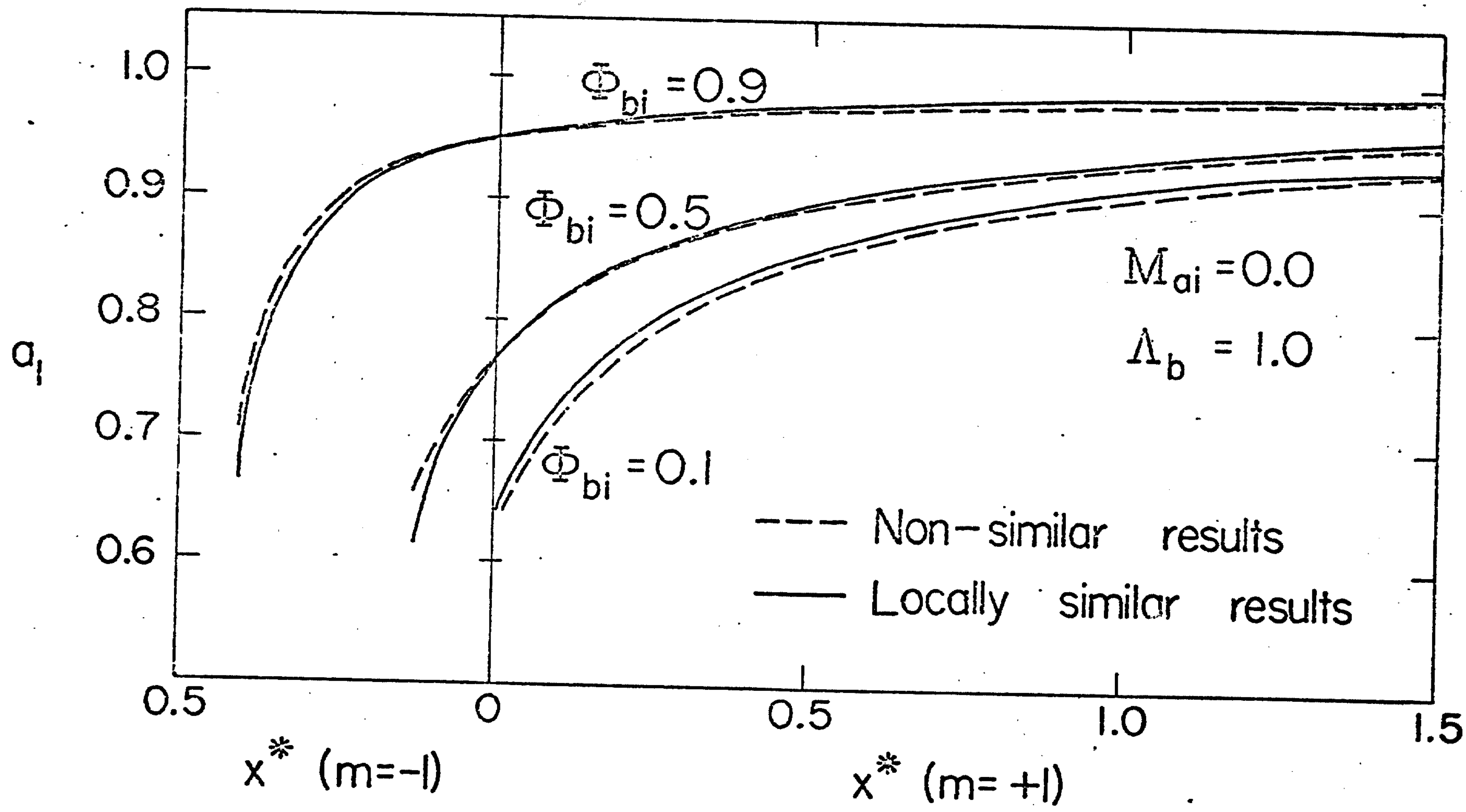


Figure 3 Comparison of the Dividing Streamline Velocity in Non-Similar and Locally Similar Mixing Flows with Pressure Gradient (adapted from Ref. 9)

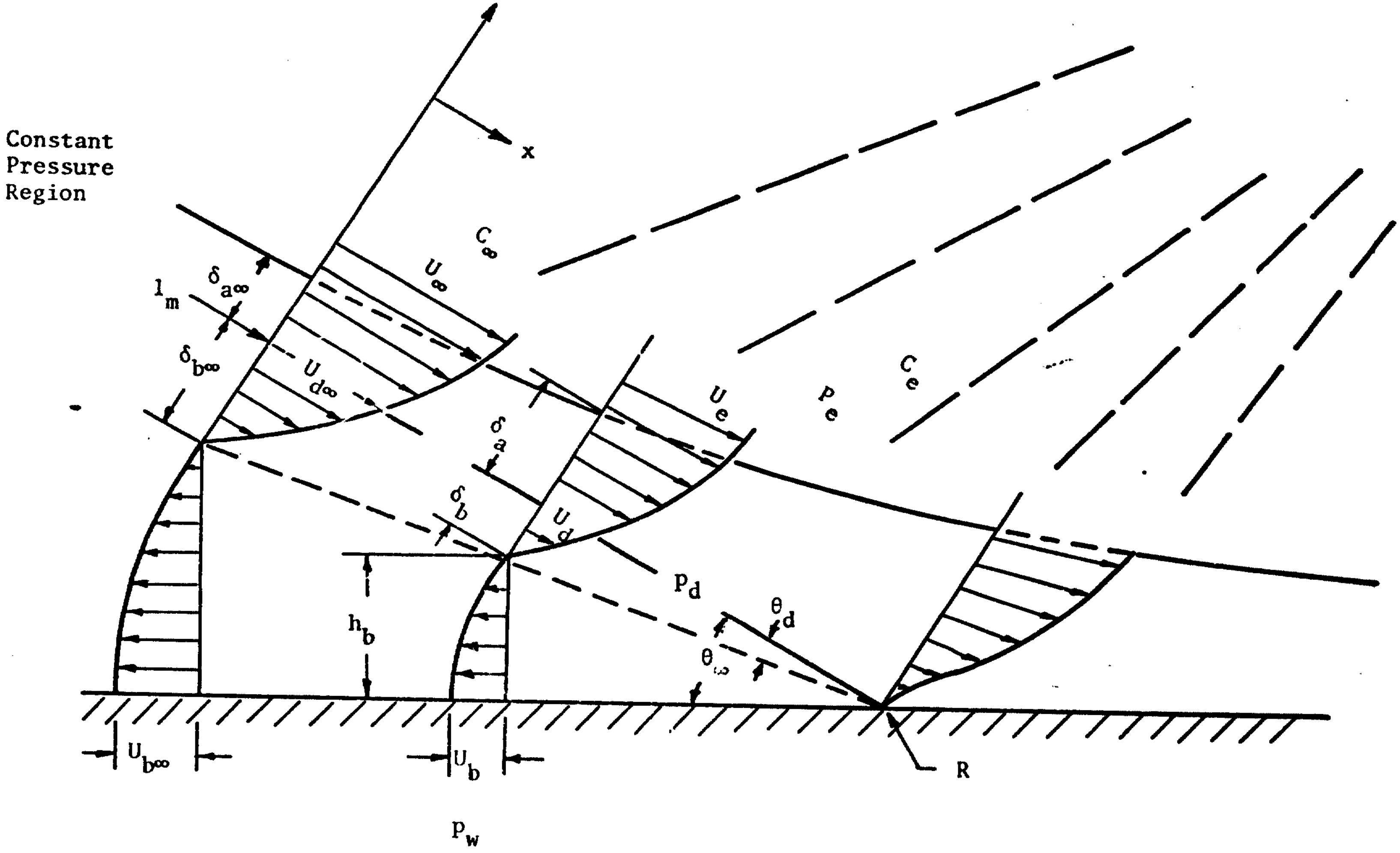


Figure 4 Recompression and Reattachment of a Compressible Free Shear Layer

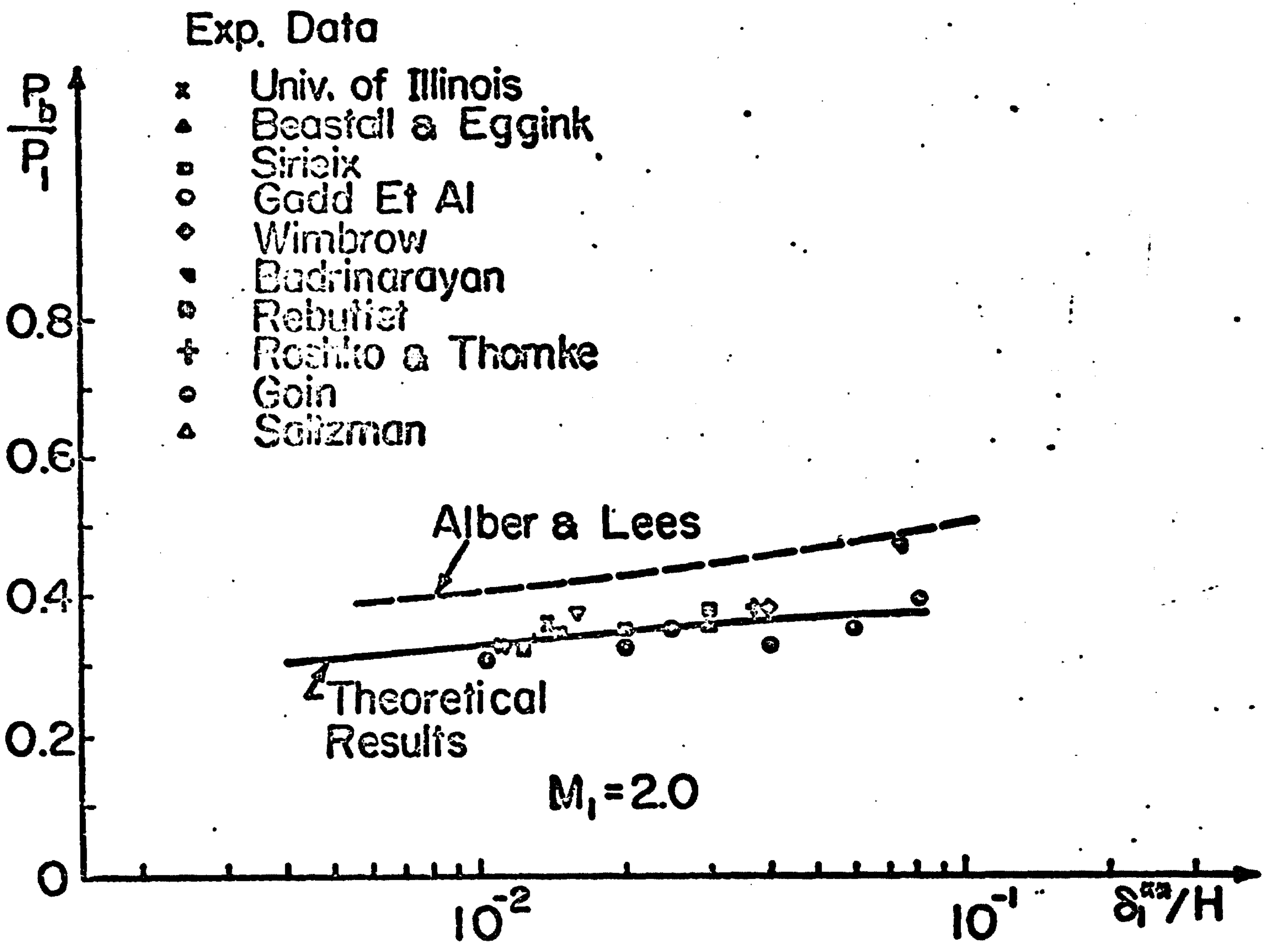


Figure 5 Base Pressure of Supersonic Two-Dimensional Flow past a Backstep (adapted from Ref. 16)

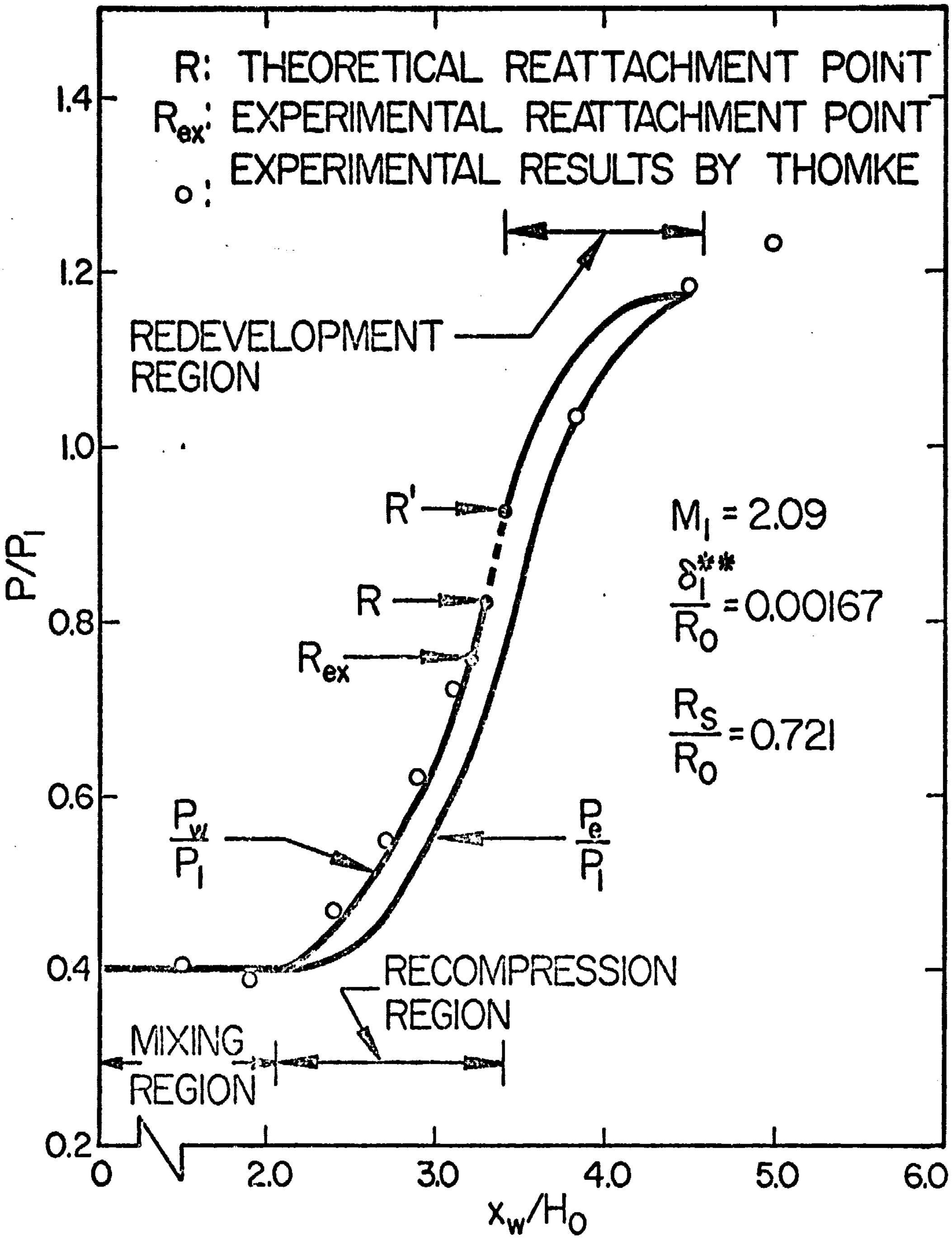


Figure 6 Pressure Distribution on the Wall of a Backstep in Axisymmetric Supersonic Flow (adapted from Ref. 18)

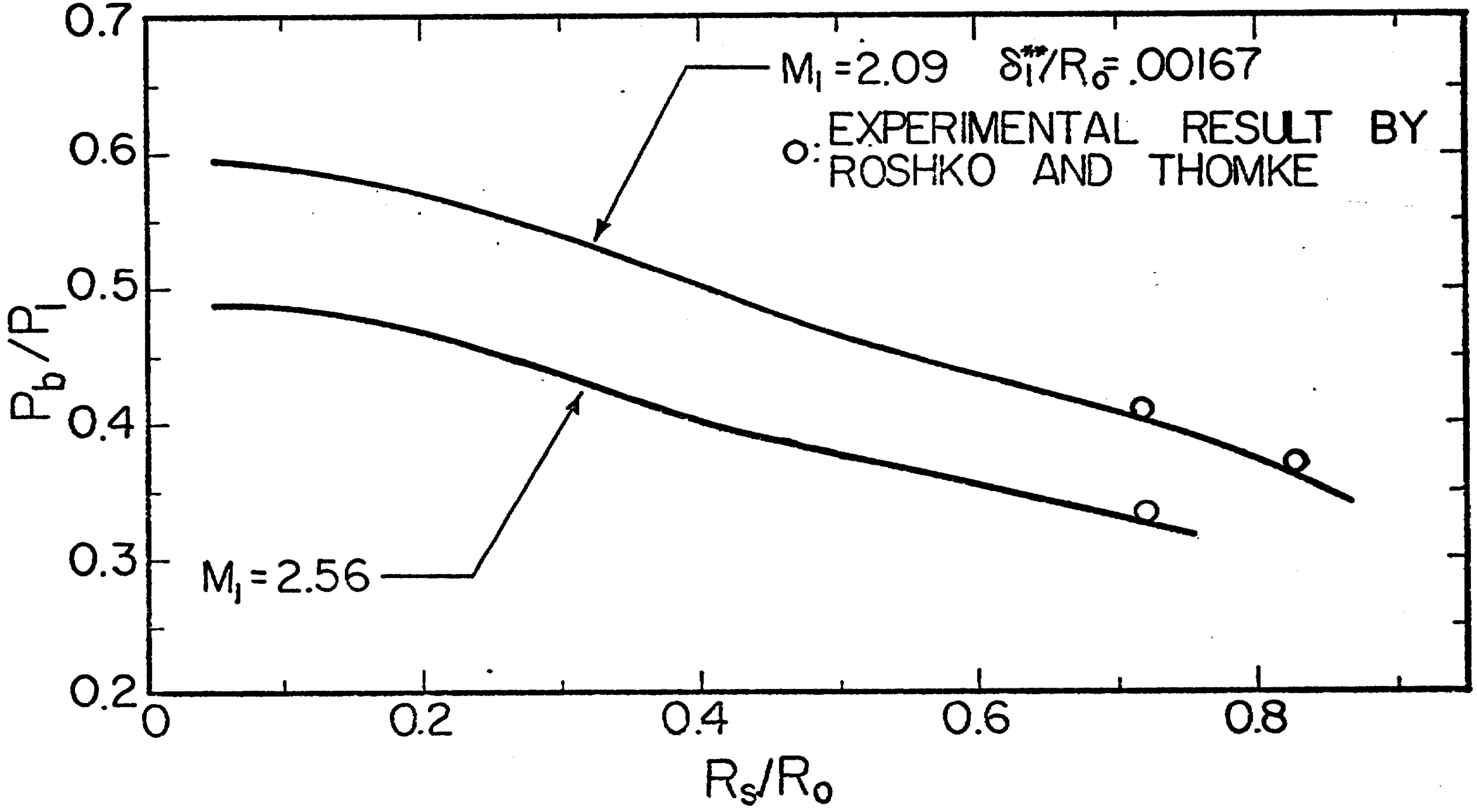


Figure 7 Base Pressure as Influenced by the Sting Radius Ratio in Axisymmetric Supersonic Flow (adapted from Ref. 18)

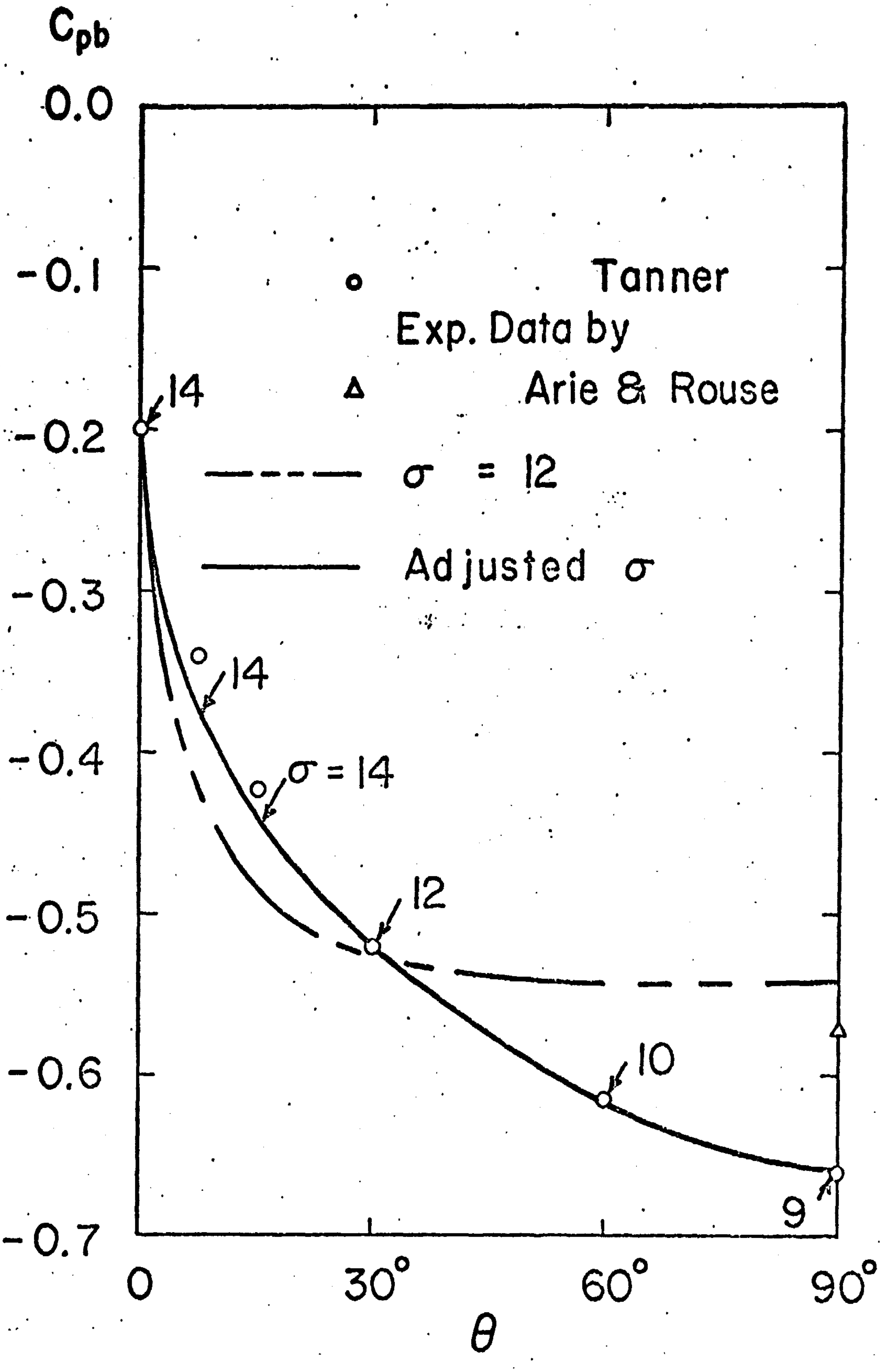


Figure 8 Base Pressure of Incompressible Wedge Flow (adapted from Ref. 22)

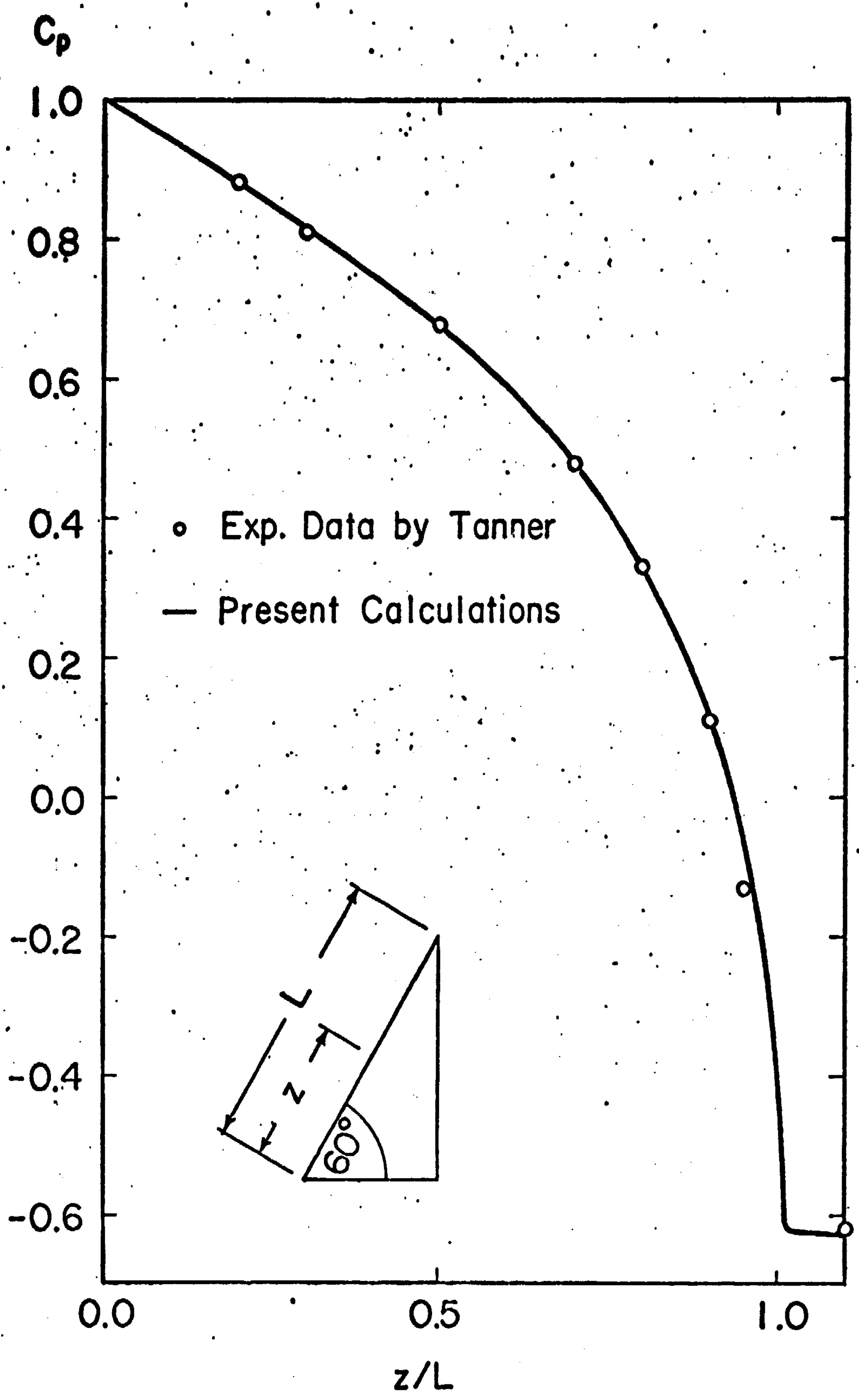
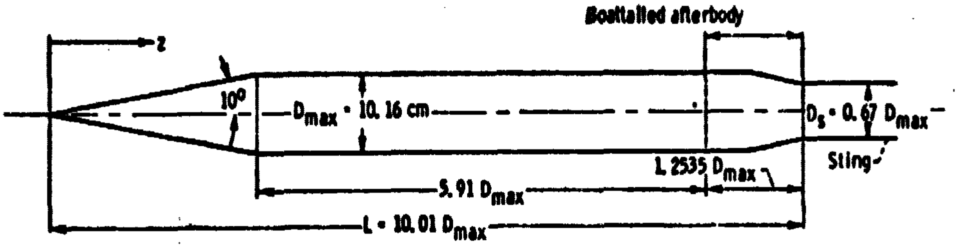
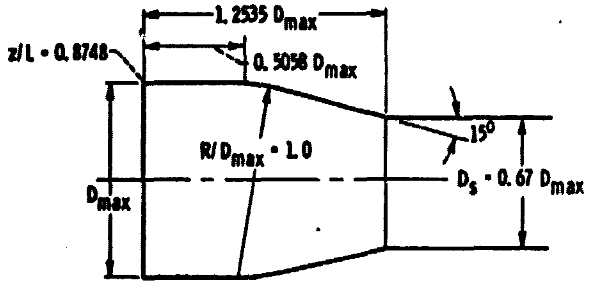


Figure 9 Pressure Distribution on the Surface of a Wedge and Couple (adapted from Ref. 22)



(a) Wind tunnel model.



(b) Boattail detail.

- Geometry used for numerical calculations.

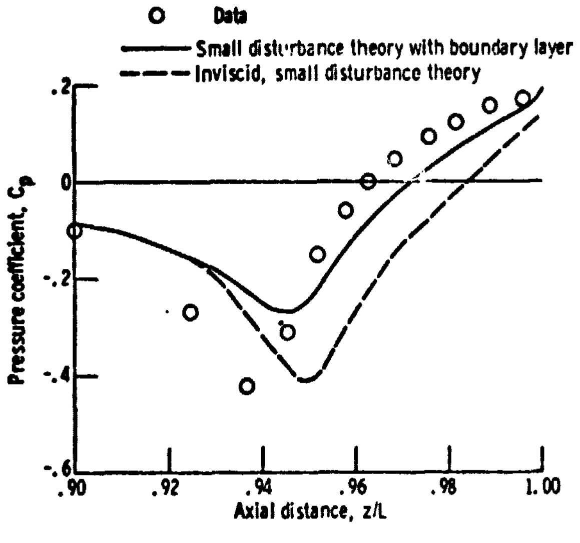


Figure 10 The Boattail Afterbody Model and the Inadequacy of the Small Disturbance Equation (adapted from Ref. 25)

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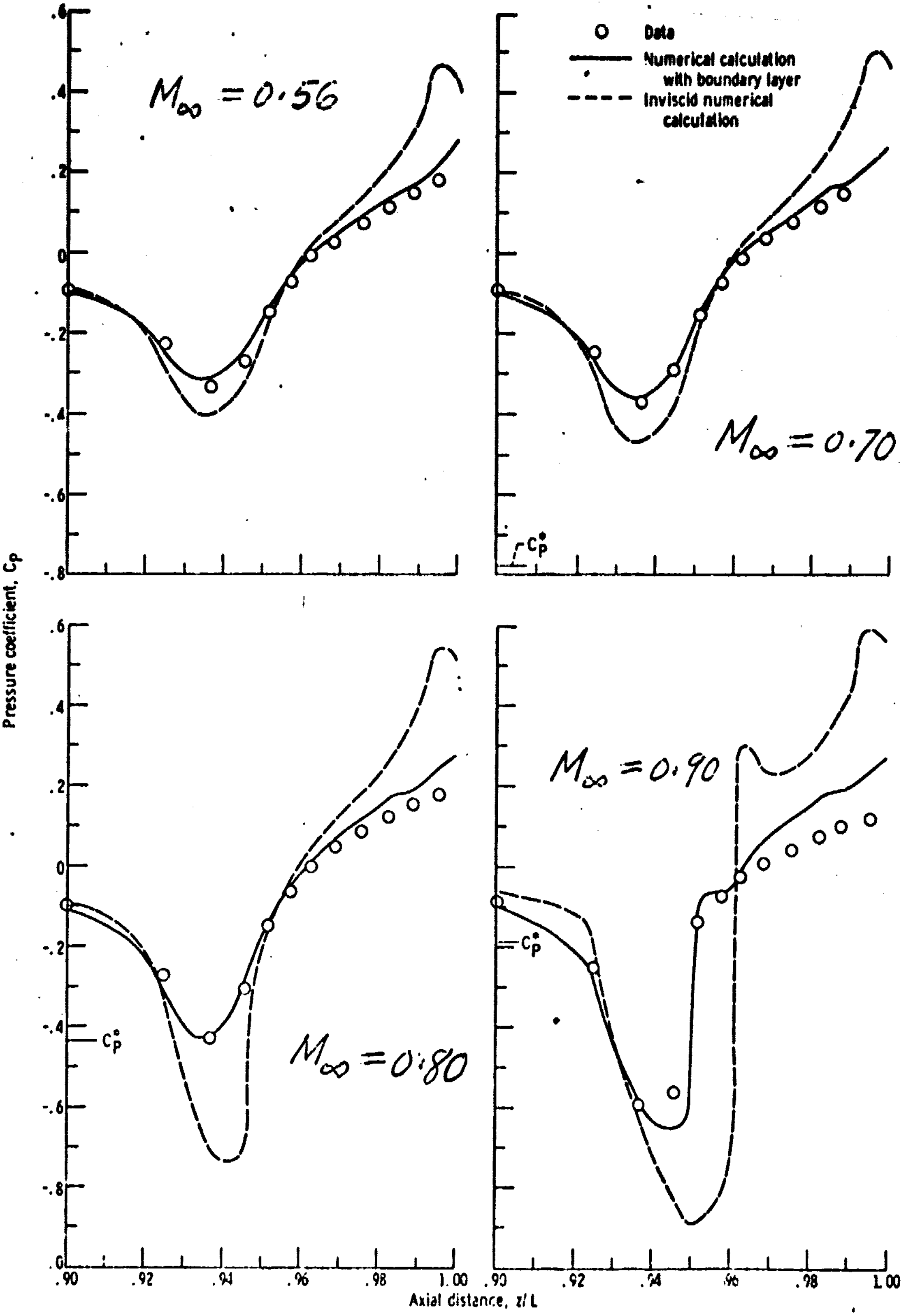

Figure 11 Results of Calculation and their Comparison with Experimental Data (adapted from Ref. 25)

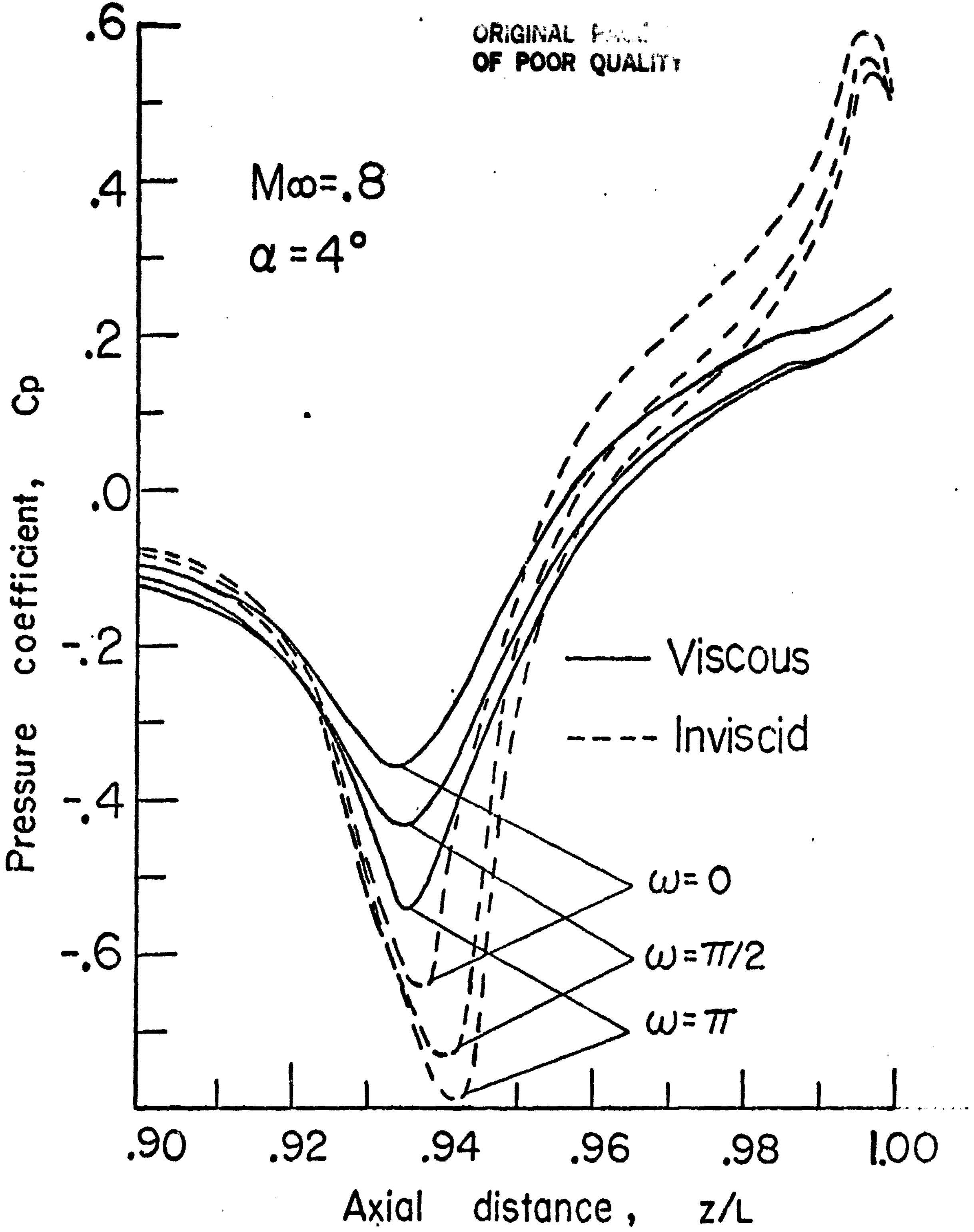

Figure 12 Results of Transonic Boattail Flow with Angle of Attack (adapted from Ref. 27)

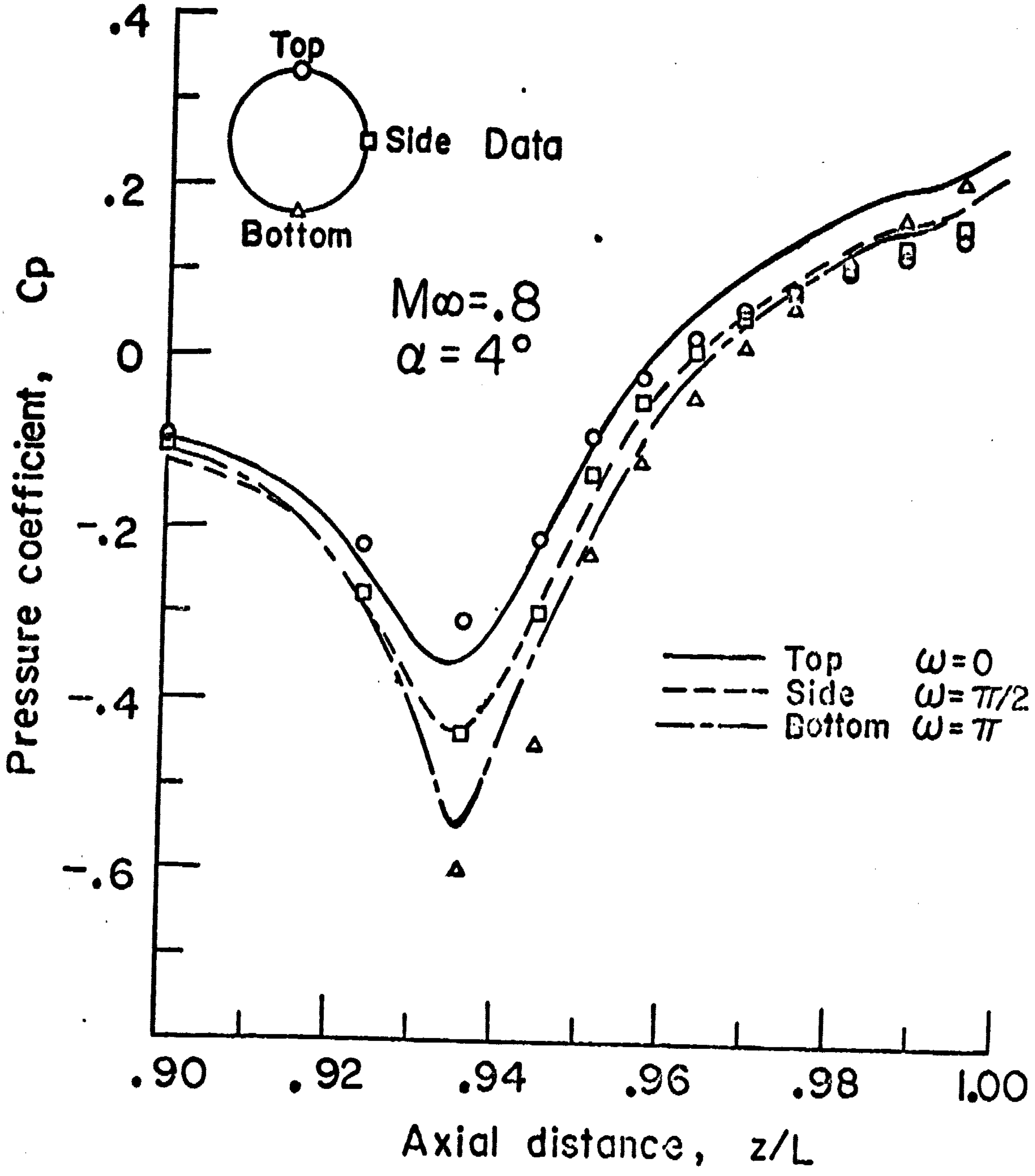

Figure 13 Comparison of Results with Experimental Data for Transonic Boattail Flow (adapted from Ref. 27)

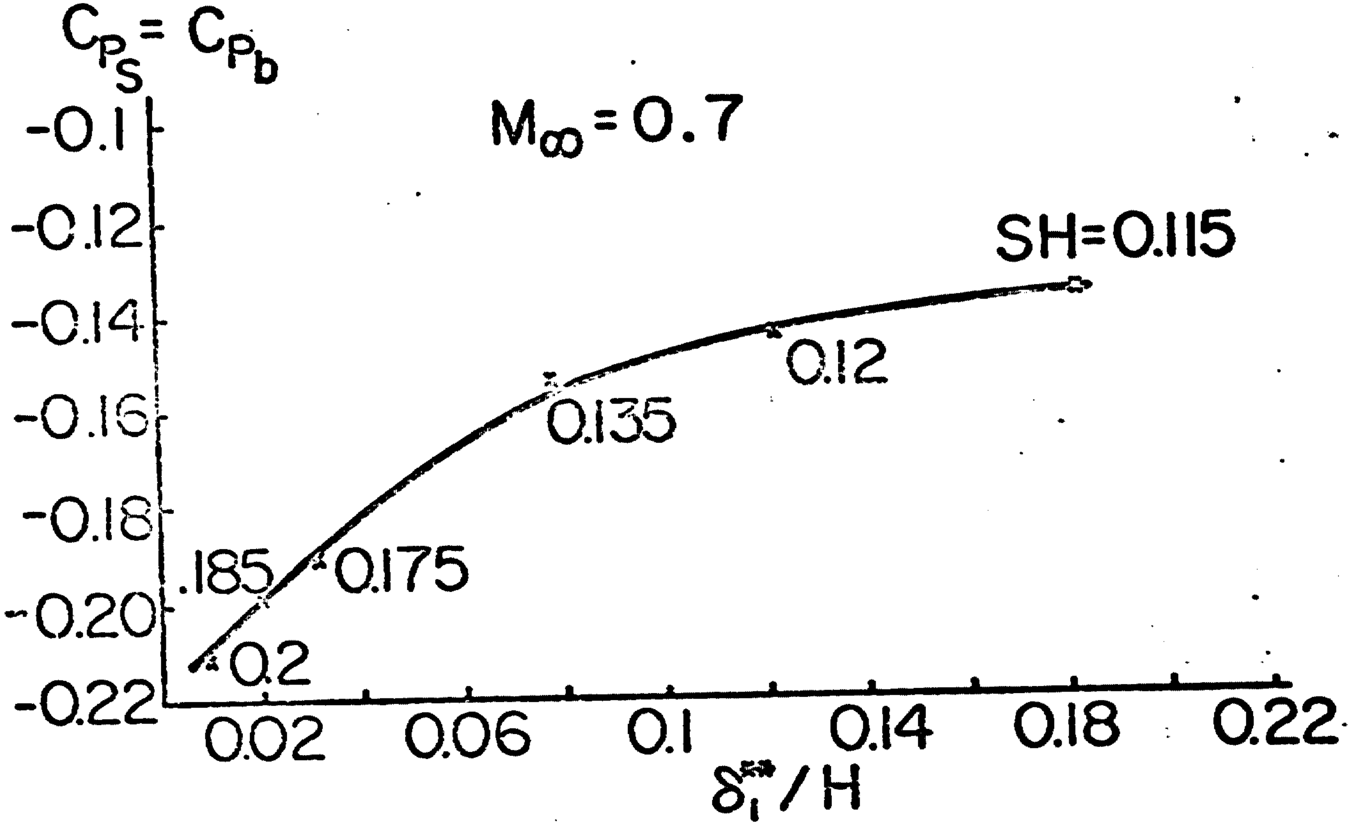

Figure 14 Results of Transonic Flow past a Two-Dimensional Backward Facing Step (adapated from Ref. 29)

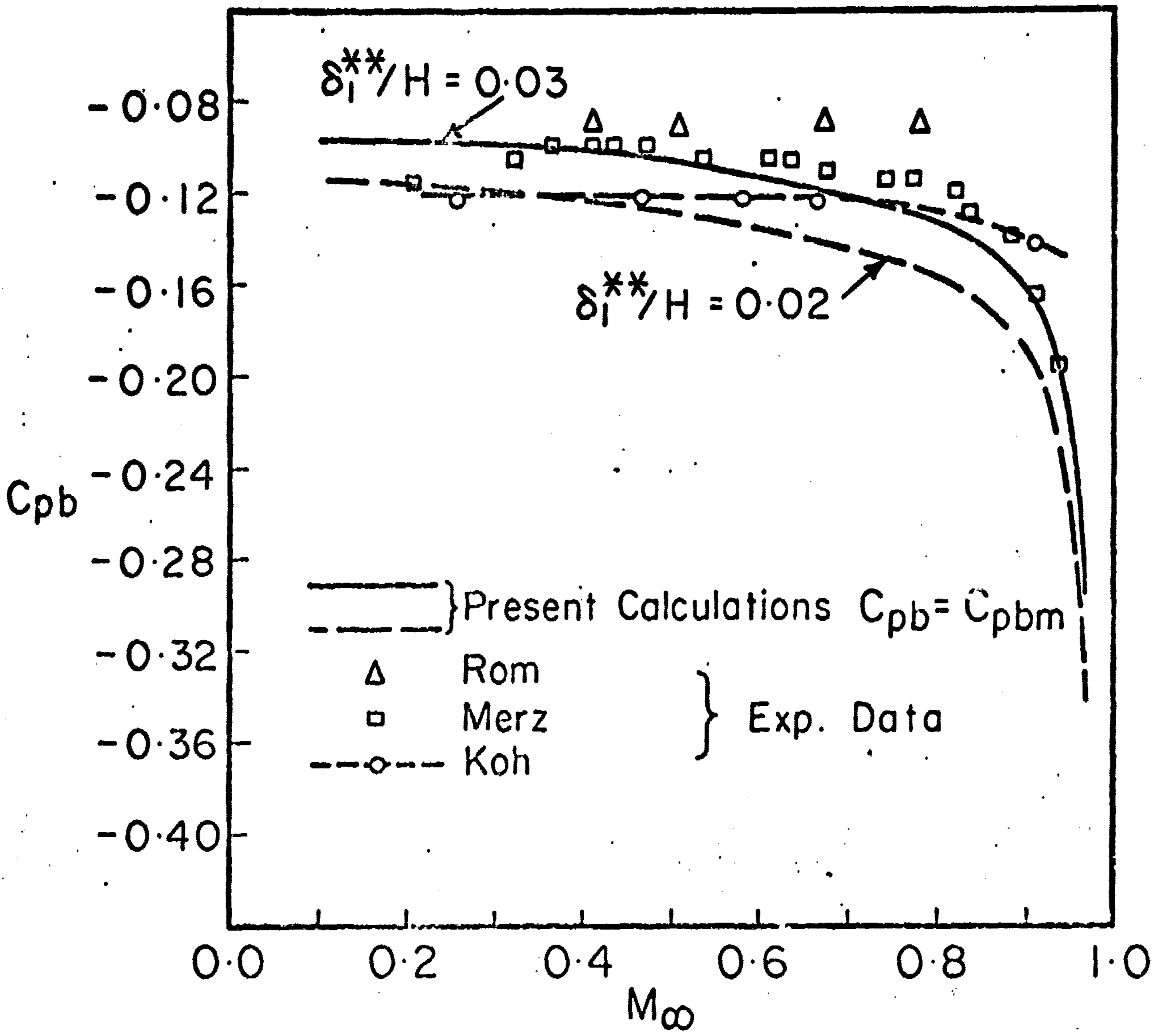

Figure 15 Results of Axisymmetric Transonic Flow and Base Pressure and their Comparison with Experimental Data (adapted from Ref. 30)

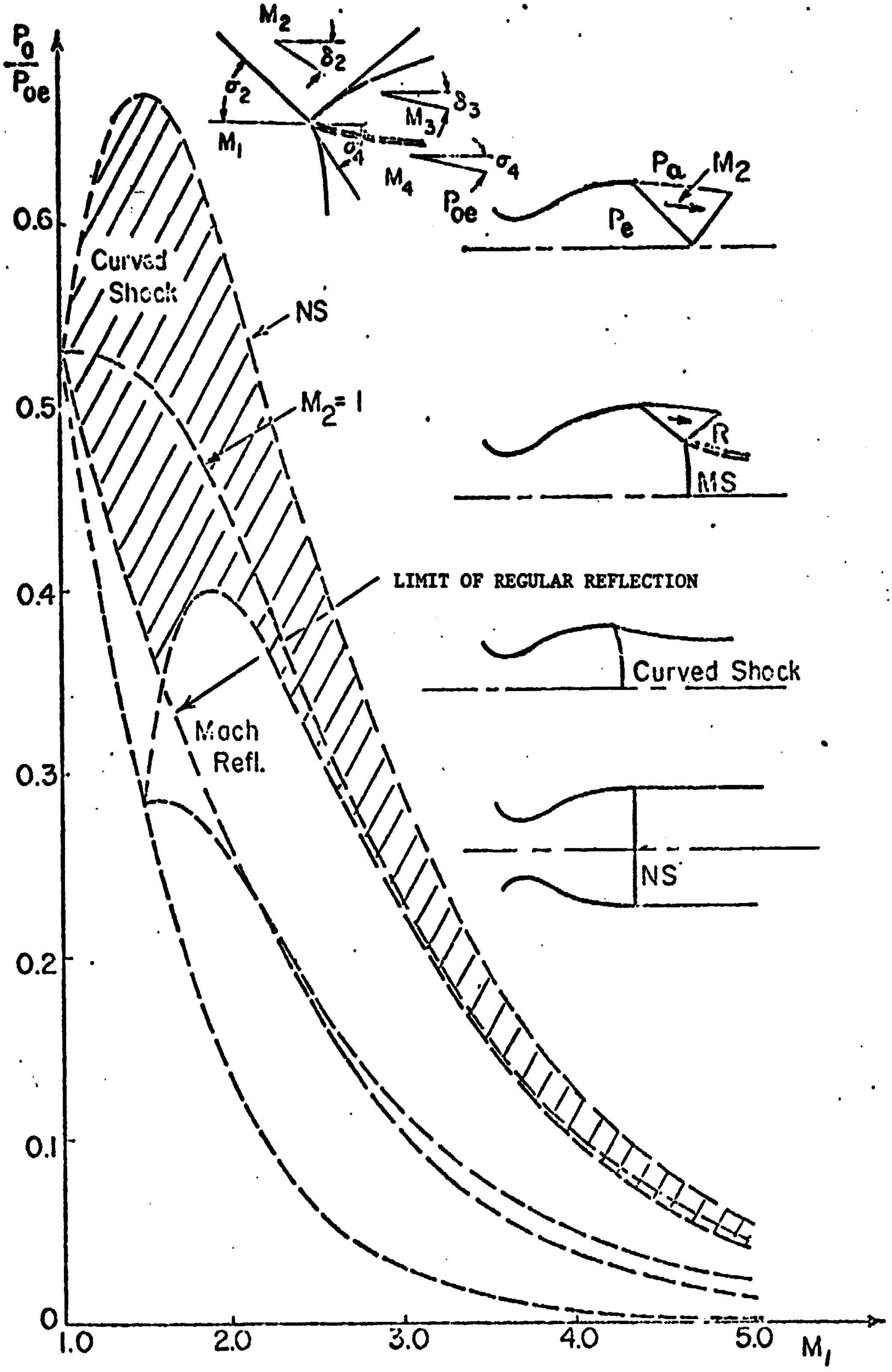

Figure 16 Influence of the Ambient Pressure on the Nozzle Free Jet Flow (adapted from Ref. 35)

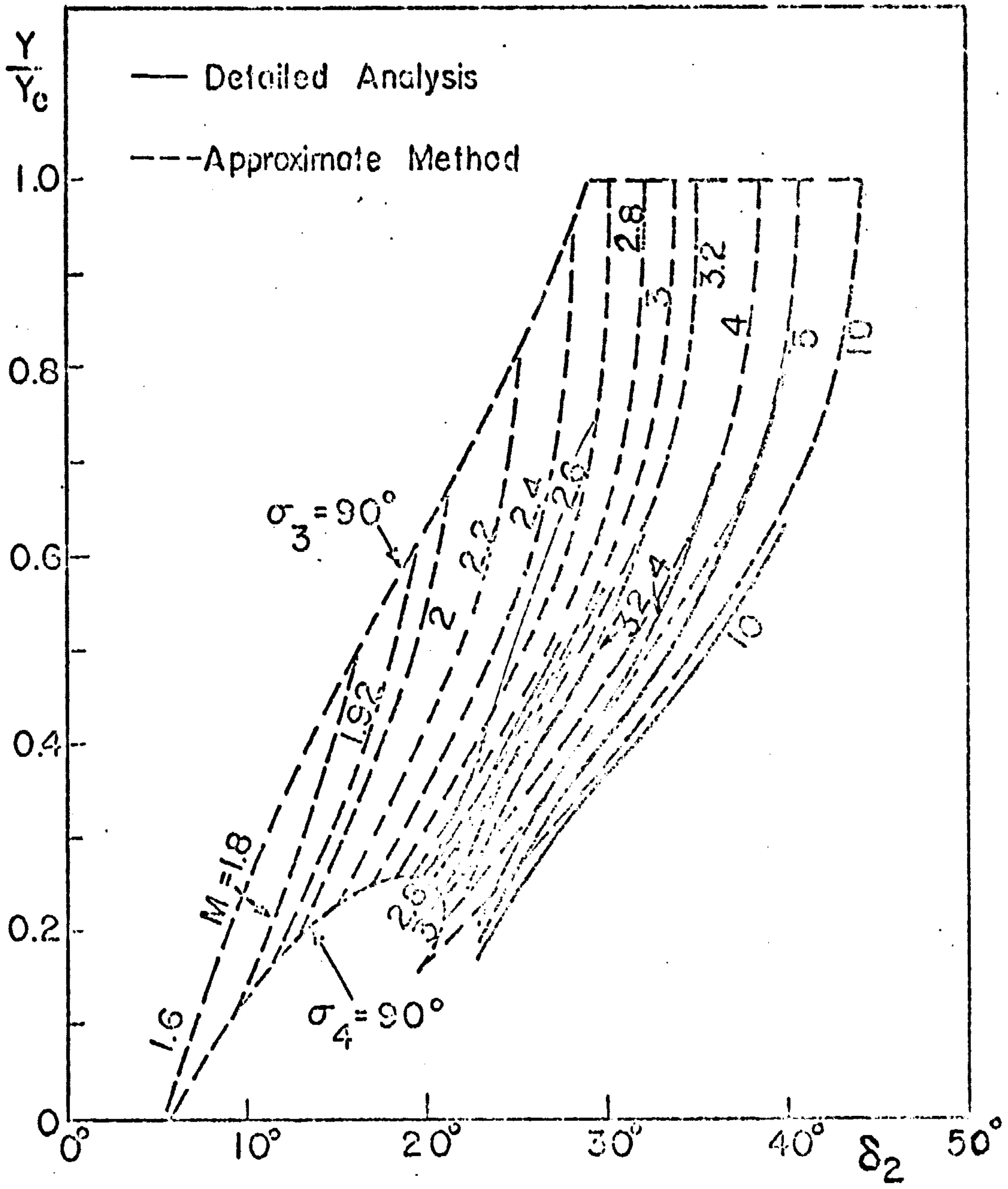

Figure 17 Mach Stem Height from Approximate Analysis
(adapted from Ref. 35)

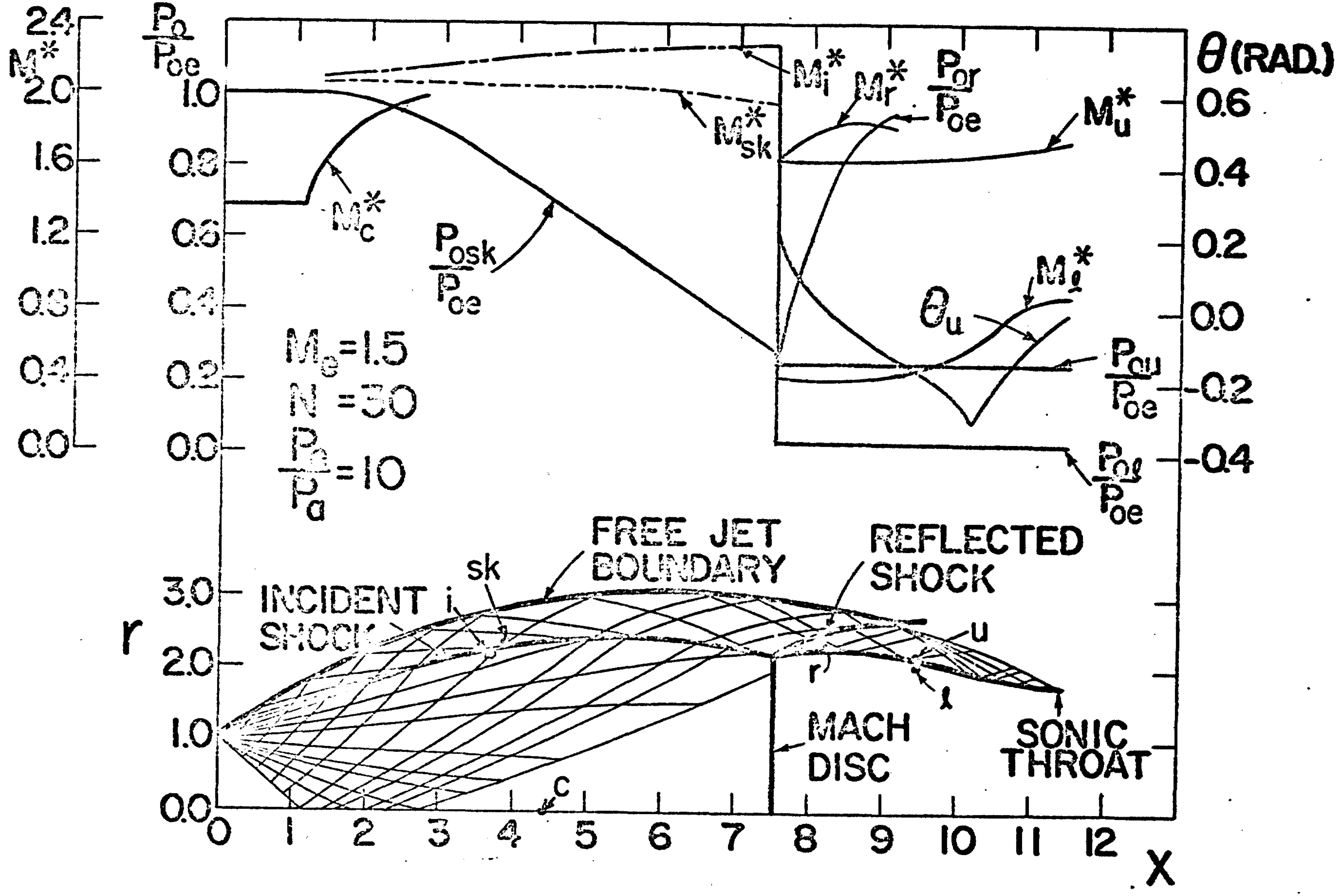

Figure 18 Calculation of Mach Disk (adapted from Ref. 34)

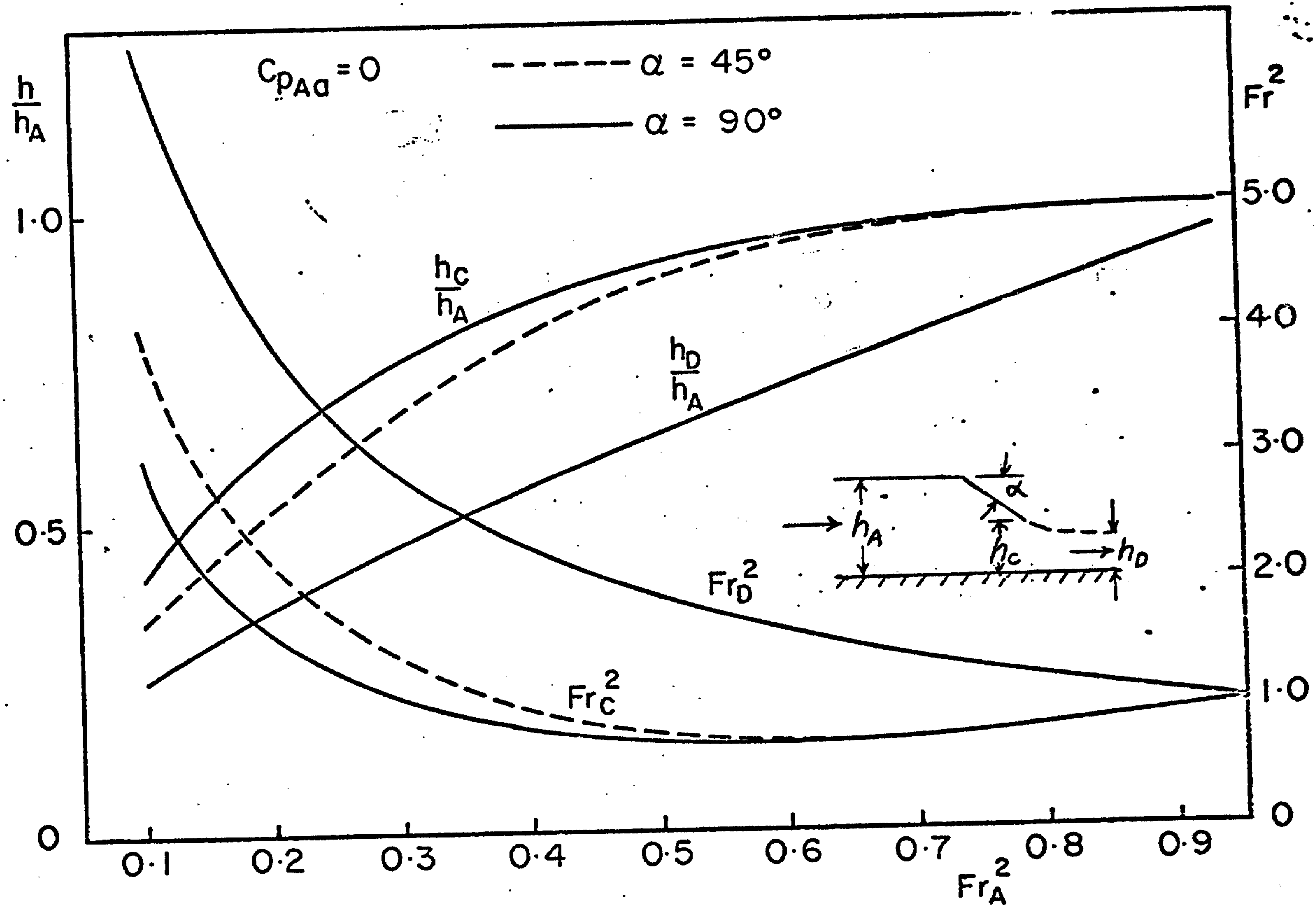

Figure 19 Discharge of Incompressible Fluid from a Channel under the Influence of Gravitation (adapted from Ref. 38)

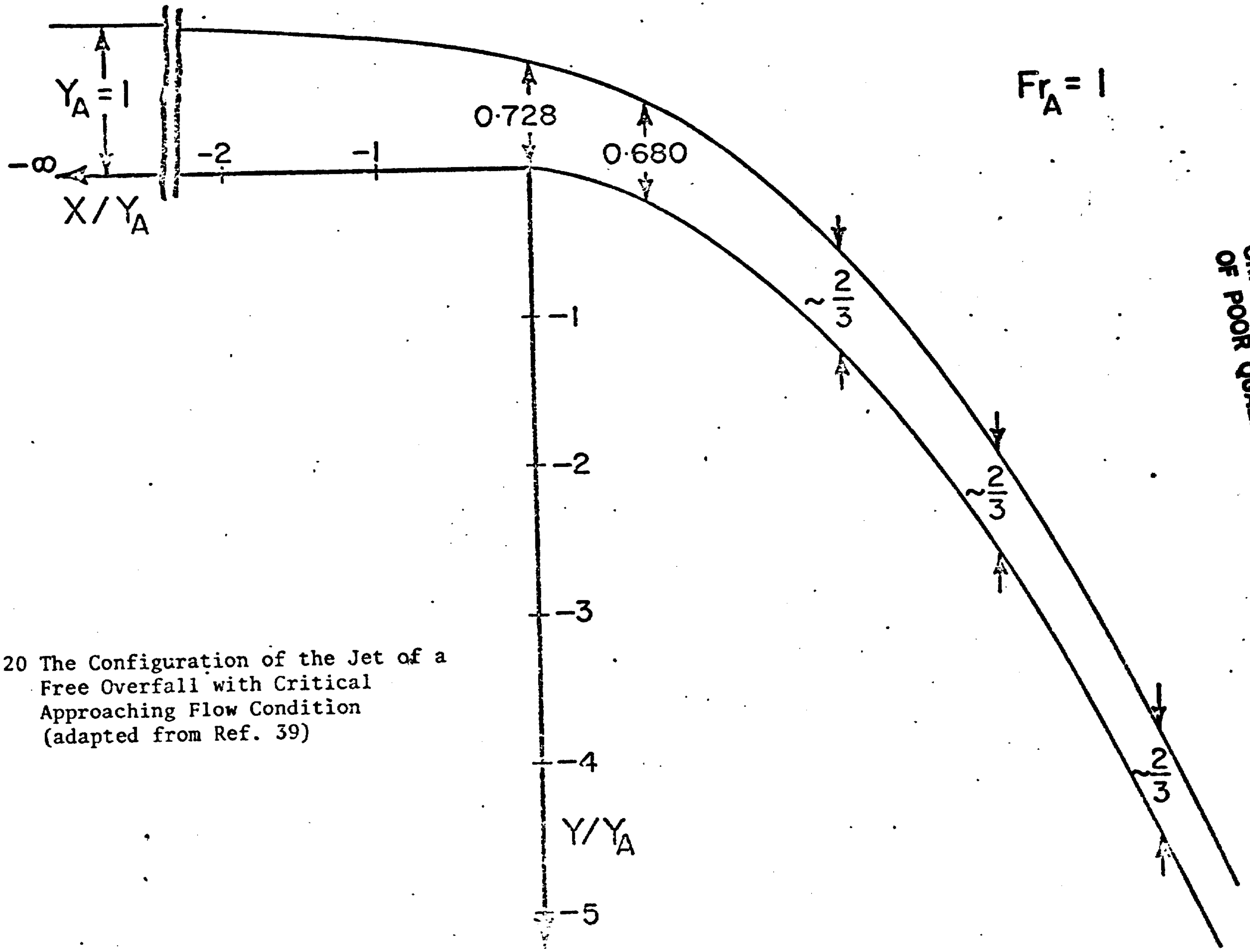

Figure 20 The Configuration of the Jet of a Free Overfall with Critical Approaching Flow Condition (adapted from Ref. 39)

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APPENDIX A

List of Ph.D. thesis produced under this research program

Department of Mechanical and Industrial Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801

Brink, D. F., "Jet Mixing under the Influence of a Pressure Gradient," August 1970.

Howlett, L. D., "A Study of Nozzle Flow Problems by the Method of Integral Relations," January 1972.

Spring, D. J., "Supersonic Laminar Flow Reattachment and Redevelopment behind a Two-Dimensional Rearward Facing Step," June 1972.

Chang, I. S., "Mach Reflection, Mach Disc, and the Associated Nozzle Free Jet Flows," October 1973.

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Liu, J.S.K., "Base Pressure Problems associated with an Axisymmetric Transonic Flow past a Backward Facing Step," October 1977.