Application of Global Positioning System to Determination of Tectonic Plate Movements and Crustal Deformations

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The NAVSTAR Global Positioning System is intended to provide observers with instantaneous measurements of position and velocity in three dimensions to 10 m accuracy in 1985. In order to obtain position in three dimensions, "pseudo-range" measurements are computed to four satellites on the basis of the difference in the time of transmission and the time of receipt of signals transmitted by the satellite. Since the time of receipt of the signals is biased due to the error in the observer's clock, the computed ranges are called "pseudo-ranges" and the measurements are made to four satellites so that the error in the observer's clock as well as the three components of position can be computed. In order to test the concept in 1979, six satellites are being launched into orbits which will allow an observer in the Southeastern part of the United States to receive simultaneous data from four satellites for several hours each day. Both test and operational satellites will be in circular twelve hour orbits at an inclination of 63 degrees. The operational system will have eight satellites in each of three orbit planes to provide world-wide continuous coverage.

A number of proposals have been made to utilize the GPS satellites to obtain positions to a few centimeters accuracy for use in crustal motion research. In each instance, orbit and satellite clock errors are minimized by computing the relative position of ground sites. Because of the high altitude of the satellites, these errors will not be significant even for stations separated by hundreds and perhaps thousands of kilometers, depending on whether predicted, fitted, or specially computed ephemerides are used (Anderle, in press). MacDoran (in press) proposes to make measurements on the spread spectrum signal broadcast by the satellite. Shapiro (private communication) would make measurements on an additional signal source he proposes to place aboard the satellites. Measurements could also be made on side-tone signals if these were provided by the satellite. In this report I will address use of the signals transmitted by the satellite and recorded by prototype equipment which is now being tested. Each of these approaches has certain advantages and disadvantages:

Proponent	Signal	Advantages	Disadvantages
Mac Doran	Spread Spectrum	Precision, Code not required	Untested, Antenna size
Shapiro	Unknown	Precision, low cost	Untested, special satellite hardware
Easton	Sidetone	Low cost, navigation option	Special satellite hardware
Anderle	Recovered code	Equipment developed	Precision or antenna size
Anderle	Reconstructed carrier	Equipment developed, precision	Oscillator error

The satellite transmits on two L band frequencies centered at 1575 (L_1) and 1227 (l_2) Mhz in order to provide a capability for correction for first order ionospheric effects. The carriers are fully suppressed by a bi-phase pseudo-random modulation at a 10 megabit rate. This "P code" is supplemented by a lower bit rate, shorter code sequence, "C/A code" on the L_1 signal which permits rapid acquisition of the signal. The code is also generated by the receiver, where it is shifted in time to obtain maximum correlation with the received signal. The time of maximum correlation is recorded as the time of receipt of the signal. Subtraction from broadcast time of transmission of the signal gives the apparent travel time to the signal, and multiplication of the difference by the velocity of light yields the pseudo range to the satellite. Once the correlation of the codes is made, the carrier signal is reconstructed and Doppler measurements are made.

Eight receiver channels would be required to maintain continuous lock on the L_1 signal from four satellites as well as the (L_1-L_2) signals. To reduce receiver size, power, weight and cost, fewer channels are used. Since the initial search for the signal correlation can take tens of seconds, five channels is the minimum number used when high data rates are desired; four of the channels maintain continuous lock on the L_1 signals from four satellites while the fifth channel rapidly sequences through the (L_1-L_2) signals. Simpler receivers have two channels which lock on the L_1 and L_2 signals from a satellite, then switch to the next satellite, or a single channel which sequences through the L_1 then L_2 signals on each satellite in turn. The lowest cost receiver uses a single channel which sequences through only the L_1 C/A signal on each satellite. The primary output of these receivers, depicted in figure 1, is the range obtained by correlation of the pseudo random noise signal. Doppler is counted to obtain velocity and reduce the noise of the PRN measurements. The NAVSTAR Geodetic Receiver (NGR) built by the Naval Surface Weapons Center and Stanford Telecommunications Incorporated (figure 2) has two channels and records range at L_1 and Doppler at L_1 and L_2, sequencing through satellites according to computer commands.

The measurement precision depends on antenna gain and the receiver hardware. Examples of pre-

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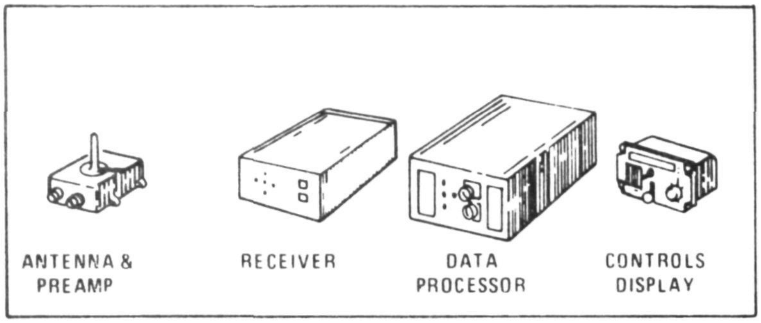
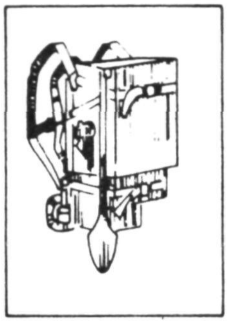
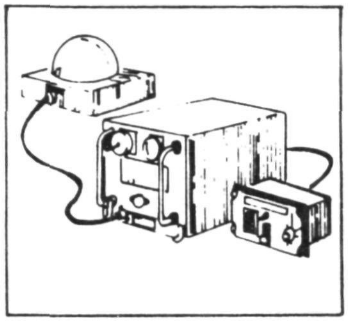
PHASE I USER EQUIPMENT







	X-SET HDWE • MAGNAVOX • TEXAS INST	HI AJ • COLLINS	Y SET • MAGNAVOX	MANPACK • MAGNAVOX • TEXAS INST	Z-SET • MAGNAVOX
•	4 CHANNEL CONTINUOUS TRACKING	4 CHANNEL CONTINUOUS TRK IMU AIDED	1 CHANNEL SEQUENTIAL TRACKING	1 OR 2 CHANNEL SEQUENTIAL TRACKING	1 CHANNEL SEQUENTIAL TRACKING
•	L_1 L_2 P OR C/A	L_1 L_2 P OR C A	L_1 L_2 P OR C A	L_1 L_2 P OR C A	L_1 C/A ONLY
•	HI PERFORMANCE	HI AJ	MED PERFORMANCE LOWER COST	LOWEST SIZE AND WEIGHT	LOWEST COST
• TIME TO FIRST FIX	180 SEC	150 SEC	300 SEC	240 SEC	300 SEC
J/S (PRECISION TRK)	43 dB	70 dB (?)	40 dB	40 dB	10 dB
• SIZE	9700 IN ^{3}	13500 IN	9700 IN ^{3}	1000 IN ^{3}	1100 IN ^{3}
WEIGHT	235 LB	335 LB	220 LB	30 LB	28 LB
POWER	860 W	1000 W	800 W	30 W	75 W

FIGURE 1

cision for two antenna gains are given below based on the equations given in figure 3 for monitor stations (similar to X sets) used to obtain data for orbit computations; the precision is also given for the NGR.

NAVSTAR GEODETIC RECEIVER

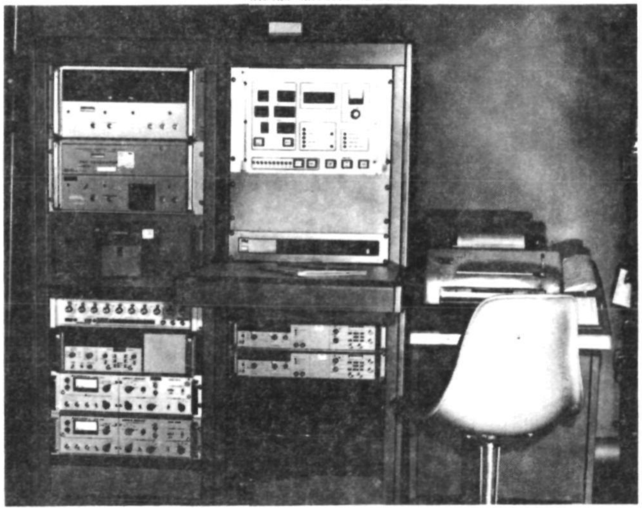


Table 1

Sample Measurement Precisions

Equipment	Monitor Station (6 sec. average)	NGR
Range Precision		
0 db antenna	36 cm	65 cm
21 db antenna	0.4 cm	5 cm
Doppler Precision		
0 db antenna	0.14 cm	0.17 cm
21 db antenna	0.08 cm	0.01 cm

The above figures are based upon a receiver power of 163 dbw, which is 3 dbw poorer than that observed for the first two satellites launched. The zero db antenna is a small stub which does not have to be aimed. A 21 db antenna is equivalent to a one meter dish which must track each satellite, although the required aiming accuracy is only about 10 degrees. If the measurement precision at each of the L-band frequencies is the same, the precision of the range corrected for ionospheric refraction will be a factor of four worse than that fundamental precision after application of the correction algorithm. However, by tracking the (L_1-L_2) the refraction correction can be obtained without significant degradation of the basic precision, as shown in figure 3. The precision inferred from monitor station data which has geen aggregated to 15 minute intervals is currently 20 cm. This is higher than expected for aggregated data, according to table 1; however, the value of 20 cm was obtained as the residuals of fit to five to ten hours of data. Figure 5 shows that oscillator variations integrate to range errors of 50 to 100 cm after 5 to 10 hours, which easily accounts for the result obtained. Hermann (private communication) analyzed 15 minute segments of data obtained on two satellites during an initial demonstration of the NGR. He found the noise level of the range measurements matched the expected 65 cm for one satellite. The noise of frequency measurements on each satellite at L_1 matched that expected for the frequency counter used in the demonstration, although the equipment was only capable of validating the precision of the phase measurements to the 20 cm level. Frequency measurements at L_2 were considerably worse than expected, probably due to interfering signals encountered in the area.

The difference in positions of two stations based on pseudo-range measurements to four satellites is equivalent in information content to an inter-ferometric determination of relative station positions. In such solutions, the precision of the position determination is a factor of two to three worse than the precision of the range measurements due to the Geometric Dilution of Precision (GDOP) arising from the simultaneous solution for position and time bias. Thus the expected precision for solutions for relative station positions based on X-set data over a six hour time span would be $\sqrt{2}\times3\times35/\sqrt{3600} = 2.5$ cm, where $\sqrt{2}$ arises from differencing data from two stations, 3 is a GDOP, 36 cm is the precision of six second data, and 3600 is the number of observations in a six hour interval. About the same

Precision of Monitor Station Data*

$$\text{Range Variance, } \sigma_r^2 = \frac{B_c}{S/N_o}\left(\frac{c}{2\pi f}\right)\left[\frac{1}{2T_1}+\frac{M}{16T_v}+\frac{M}{32T_vT_1}\sum_{i=1}^{16T_v/M}\left(\frac{4T_1-1}{4T_1}\right)^{i-1}\right]+\frac{q^2}{12}$$

$$\text{Variance in Refraction Data, } \sigma^2_{M_1-M_2} = \frac{25}{T_{12}\,S/N_o}\left(\frac{c}{2\pi f}\right)^2+\frac{q^2}{12}$$

$$\text{Range Difference Variation, } \sigma_r^2 = \frac{2B_L}{S/N_o}\left(\frac{c}{2\pi f_c}\right)^2+\frac{q_c^2}{12}+\left(\frac{\Delta t}{T_I}\right)^2$$

where

B_c = code channel bandwidth ~ 25 hz

S/N_o = signal to noise ratio in hz ~ $10^{db/10}$hz

c = velocity of light
f = code frequency ~10.23 mhz
T_1= L_1 code loop time constant ~ 10 sec

M = number of operating channels
T_v = nominal sampling interval ~ 10 sec

q = c/64 f = 1/64th of a code chip
T_{12} = min (T_{12}, $T_{Ts/3}$), where T_2 = L_1-L_2 code loop constant ~160 sec, T_{Ts} = time slot of MS tracking sequence ~ 60 sec

B_L = costas loop bandwidth for Doppler ~ 3.33 hz

f_c = L_1 carrier frequency ~1575.42 mhz

q_c = c/64 f_c = 1/64th of a carrier cycle

t = integrated doppler measurement interval ~ 6 sec
T_I = time interval per unit change in range due to ionosphere ~197 sec/m

*interface control document 12436, MCS/NSWC

FIGURE 3

level of accuracy was obtained by Goad (private communication) in a computer simulation which considered the geometric and other factors in the solution in a more realistic fashion.

Because of the higher precision of phase measurements and the successful use of such measurements of Navy Navigation Satellites in geodetic solutions, the Naval Surface Weapons Center is exploring the use of such data from GPS satellites for positioning. In this approach phase measurements and integral Doppler counts are made over successive time intervals during the passage of a satellite between the start and end of the interval can be computed. For the lower altitude, polar, Navy satellites, the range difference observations obtained during one pass of the satellite over the station provide a determination of station latitude and the range to the satellite at the time of closest approach of the satellite to the station. Data from two passes at different longitudes are required to resolve the two slant range measurements into station longitude and height to acceptable precision. For the higher altitude GPS satellites, a three-dimensional solution for station position can be obtained from observations made on a single pass of the satellite over the station, but not to the precision possible using data from two passes. The range differences observed during a pass can be treated in two ways: the range differences over successive time intervals can be assumed to be uncorrelated, or, if the cycle count is not reset at the time of readout, the range differences can be accumulated to provide a series of range measurements subject to a range bias common to the observation set. Simulations by Anderle (in press) indicate that biased range data during a pass yields a precision in the determination of two components of station position which is about equal to the precision of the range difference measurement. Uncorrelated range difference data provides a precision which is a factor of three or more worse than the precision of measurement. Due to the high measurement precision possible, this would still be a highly useful data type which would permit observations of two or more satellites sequentially with a two channel receiver. However, range difference data, treated as either correlated or uncorrelated, set severe requirements on the oscillator used to make the measurements. Figure 4 (Fell, private communication) shows that a Cesium oscillator will produce systematic errors which would reach 20 cm during a fit of mean frequency offset to eight hours of range difference data. The figure is a sample result for one sequence of data. Fell also gives the root mean square of the error for a large set of sequences of such data in figure 5 as a function of time within the pass; the rms error ranges from 10 to 15 cm, with the maximum error in the center and at the ends of the pass. The effect of this systematic error

Precision of NAVSTAR Geodetic Receiver*

$$\text{Range Variance, } \sigma_r^2 = \frac{c^2 B_L(\text{DLL})}{\left(S/N_o\right) f^2} \left[.905 + \frac{1.612\ B_I}{S/N_o} \right]$$

$$\text{Doppler Variance, } \sigma_r^2 = \left(\frac{c}{2\ f_c}\right)^2 \frac{B_L(\text{PLL})}{S/N_o} \left[1 + \frac{B_I}{2\ S/N_o} \right]$$

where

c = velocity of light

$B_L(\text{DLL})$ = one-sided loop noise bandwidth = 2 hz

$B_L(\text{PLL})$ = one-sided phase loop bandwidth = 16 hz

S/N_o = signal to noise ratio in hz ($10^{\text{db}/10}$)

f = code frequency ~ 10.23 mhz

B_I = IF bandwidth = 10^3 hz

f_c = carrier frequency ~ 1574.42 mhz

* Stanford Telecommunications Incorporated

FIGURE 4

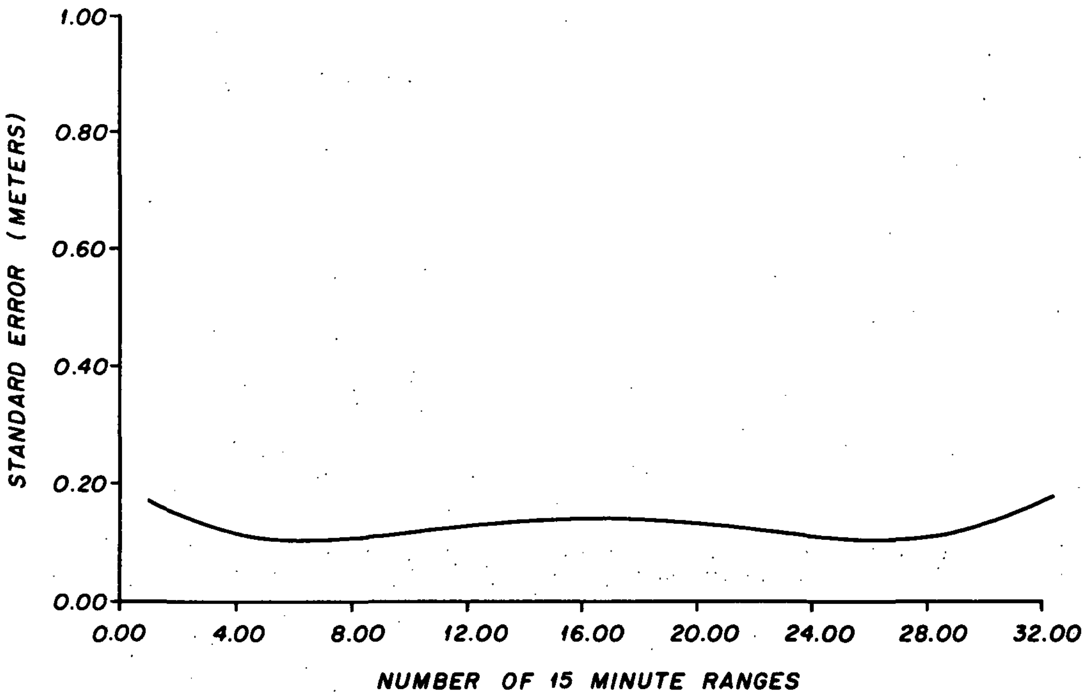


FIGURE 5

on station position is being computed. But it will clearly dominate the effects of the measurement error.

To summarize, pseudo-range measurements to four GPS satellites based on correlation of the pseudo random code transmissions from the satellites can be used to determine the relative position of ground stations which are separated by several hundred kilometers to a precision at the centimeter level. This precision is attained within 12 hours using a small antenna or much more rapidly with a directional antenna. Carrier signal measurements during the course of passage of two satellites over a pair of stations would also yield centimeter precision in the relative position, but oscillator instabilities would limit the accuracy by an as yet undetermined amount. Measurement precisions of code and carrier signals have generally been consistent with test conditions, but have not been tested to design levels yet. The accuracy of solutions based on either type of data would be limited by unmodeled tropospheric refraction effects which would reach five centimeters at low elevation angles for widely separated stations.

References

Anderle, R. J., Geodetic Applications of the NAVSTAR Global Positioning System, Proceedings of the Second International Symposium on Problems Related to the Redefinition of North American Networks, April 1978, Washington, D.C. (in press).