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Recent Advances in Computational Techniques

David E. Smith
Goddard Space Flight Center
Greenbelt, Maryland 20771

Abstract. The determination of very precise orbits and geodynamic parameters from laser tracking data requires the continual development and improvement of the software systems and computational techniques. Computational accuracies at the few centimeter level are presently required to match the performance of the present day laser ranging systems and altimeters and in the next few years the accuracies are expected to increase further. In this paper and the major error sources in orbit determination are briefly discussed and the present and future modeling activities needed to meet the accuracy requirements of the next few years are described.

Introduction

The precise computation of the motion of earth satellites has become a critical component of several space techniques for studying the earth. Perhaps the two most relevant at the present time are the computation of orbits for the determination of crustal motions and the computation of orbits for altimeter satellites. These and other applications require orbital accuracies in the centimeter range and challenge our present-day capability to accurately model many of the forces that perturb satellites and also our computational techniques. Laser ranging systems operated by NASA Goddard Space Flight Center are now at about the 5 cm accuracy level and systems capable of 2 cm are presently under development (Silverberg, this volume). However, our present ability to determine the orbits of satellites has not yet achieved the same level of accuracy and historically orbital accuracy has lagged behind the observational accuracy by several years because the improved observations are needed to improve upon the models for the spacecraft behavior. This situation is particularly true for the gravity field, which for most satellites and applications is still the largest source of orbital error.

Figure 1 shows the developmetn of orbit determination for arc lengths of three to five revolutions over the last several years in comparison with the improvement in the quality of laser range measurements. Figure 1 represents the situation for a typical low altitude spacecraft; such as Beacon Explorer C or GEOS-3 which have altitudes of about 1000 km or less. For most higher altitude satellites, such as Lageos at 6000 km, the orbital fit will be closer to the data quality. Another important consideration in interpreting Figure 1 is that the orbit fit curve is largely based on the experience in the San Andreas fault experiment (Smith, et al., 1979) in which two or three laser tracking stations within one or two thousand kilometers of each other were used. If these stations had been on different continents the fit might have been larger.

For these short orbital arcs of a few revolutions (3 to 8 hours) the largest source of error in the orbital computations is usually the gravity field but as the arc length increases to days, weeks, and months other sources of error usually begin to dominate. Figure 2 illustrates the way the orbit error typically grows as a function of arc length for the major perturbing forces. The vertical axis is arbitrary in scale and is only representative of the relative magnitudes. The exact variation of each of the curves will depend on the satellite, its orbit and the sophistication of the modeling of the perturbing force used in the orbit computation. For example, numerical errors in the computations are usually negligible if an analytical theory is being used for the spacecraft motion but can become extremely important if a numerical integration system is used because the error accumulates as the arc length increases.

At GSFC the major software system for the computation of precision orbits is the Geodyn program (Putney, 1977). This program system has the ability to determine orbits from a variety of tracking data types and is capable of estimating various geophysical parameters such as polar motion and earth rotation, tidal parameters, geopotential coefficients, as well as parameters associated with the perturbing forces, such as drag coefficients, and measurement errors, such as range biases.

In the following section the status of our modeling of the different perturbing forces at GSFC will be briefly described together with our plans for future improvements.

Orbital Perturbing Forces

Gravity

One of the areas that has shown greatest improvement during the last decade has been our knowledge of the gravity field of the earth. The inclusion of high precision range measurements from laser tracking systems and Unified S-Band doppler data and, more recently, altimeter data has enabled our definition of the gravity field to extend out to degree and order 36 with specialized altimeter and surface gravity solutions extending out to degree and order 180. These new fields (Goddard Earth Models - GEM) developed at GSFC have permitted improvements of at least an order of magnitude in the determination of short orbital arcs of satellites over the past decade. Figure 3 shows a comparison of the abilities of three gravity models, GEM's 1, 7 and 9, to fit five consecutive passes of laser data from a single tracking station. These five passes, obtained at GSFC in 1974 on the Beacon Explorer C satellite, when analyzed by the GEM 1 gravity field developed in 1970-71 (Smith, et al., 1973) could only be satisfied at the 2 meter level even though the data was of 10 cm quality.

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The same data analyzed a few years later with ghe GEM 7 model (Wagner, et al., 1977) could be satisfied to about the 50 centimeter level and more recently the GEM 9 (Lerch, et al., 1978) model fits to 12 cm. The improvement from GEM 1 to GEM 9 has been brought about largely by the inclusion in the later models of large quantities of laser tracking data on several satellites, particularly GEOS-3, but not the Beacon Explorer C data shown in Figure 3. The slight curvature of the GEM 9 results in Figure 3 show that some gravitational signature still exists in the data and that some improvement still remains to be made although this may well be the most difficult.

For longer arcs the gravity error increases to about the 50 cm to 1 meter level after one week with the GEM 9 field for low altitude satellites such as Lageos, reaching 50 cm after about one month.

The present plans at GSFC are to continue to improve our knowledge of the gravity field so that the locations of mobile and fixed laser stations can be determined to the few centimeter level for the precise measurement of crustal motions; and also for the precise analysis of the GEOS-3 and Seasat altimeter data.

Air Drag

As the orbital arc length increases the second most important perturbing force (after gravity) for low altitude satellites is usually the effect of air drag. The general form of the perturbation is

$$\text{accel.} = -\frac{1}{2} C_D \frac{A}{M} \rho \nu^2$$

where C_D is the drag coefficient, A is the spacecraft cross-sectional area, M is the spacecraft mass, ρ is the air density and ν is the spacecraft velocity. At the present time the density models used in computing the drag acceleration are based on the work of Jacchia and include variations in solar activity, diurnal terms, geomagnetic effects and semi-annual and seasonal latitude variations. In order to improve the responsiveness of the model to unmodeled changes in density we have introduced a time dependent parameter ($\dot{\rho}$) that enables us to account for systemated changes during the orbital arc. In addition, we are introducing a capability to estimate the drag coefficients (C_D) for specific periods during the orbital arc. Thus it will be possible to vary the drag coefficient from one day to the next and thereby modify the drag acceleration without any change to the density model. We believe this may accomodate density changes that last for short periods of time that are not represented in our models.

Another capability that exists in the GSFC Geodyn program is to accurately model the cross-sectional area of the spacecraft. This facility was introduced for the Beacon Explorer C, GEOS-3 and Seasat spacecraft because of their irregularity in shape and the need for very precise orbit calculations on these satellites. In all these cases drag was a major influence in the orbit computations and inclusion of a variable cross-sectional area could improve upon the computational accuracy. The technique is incorporated by computing externally to the main program the cross-sectional area as a function of angle of incidence and including this information in tabular form in the orbit program. At each integration step the appropriate area is deduced from the table and used in the drag calculations.

Solar Radiation Pressure

The form of the perturbation by solar radiation pressure is

$$\text{accel.} = -C_R \frac{A}{M} \frac{\text{(solar constant)}}{C}$$

where C_R is a constant that accounts for the type of reflection, specular diffuse, etc., that is taking place at the satellite, $\frac{A}{M}$ is the area (A) to mass (M) ratio and C is the velocity of light. The model used in the Geodyn program includes a solar flux varying with solar distance, and approximations for absorption and refraction at the terminator. The incident area is variable in the same manner as for air drag and takes into account spacecraft attitude, shape, shadowing and varying reflective properties over the spacecraft. Although these computations are reasonably precise we believe that errors are occurring at the umbra/penumbra/full sunlight boundaries where the numerical integration procedure jumps over one or even two boundaries in one step. The effect of this error is estimated to be a slight offset in the boundary location.

Albedo Radiation Pressure

At the present time we do not have an albedo model in our orbit computation program but one is under consideration. The basic form of the perturbation is

$$\text{Accel.} = -C_R \frac{A}{M} \frac{\text{(Albedo)}}{C} \left(\frac{R}{D}\right)^2$$

and is similar to that for solar radiation pressure except for the (R/D) term which shows that the acceleration follows the inverse square law (R is the earth radius, D is the radial distance of the satellite). The difficulty with evaluating this perturbing force is that the albedo is variable in both space and time and needs to be evaluated at every integration step for the entire surface of the earth observable from the spacecraft. This procedure is computationally very time consuming. Simplification of the albedo into only day/night effects, for example, will probably underestimate the effect and provide deceiving results. Our present considerations are directed towards the computation of the long-term effects of albedo by digitally representing seasonal albedo maps of the world derived from satellite meteorological data.

Earth and Ocean Tides

Our present modeling of earth tides in the Geodyn program is a single second degree spherical harmonic with one amplitude and phase. This model is used to compute the gravitational effects of earth tides on the satellite orbit and to compute the body tides on the locations of the tracking stations. Our ocean tide models only account for the displacement of the ocean surface (Hendershott model globally, Mofjeld in Northwest Atlantic) for the analysis of satellite altimeter data and do not include the gravitational effect on the satellite. However, considerable accomodation of the ocean tide effect on the orbit can be achieved by suitable modification of the solid earth tidal amplitude and phase. For example, for Beacon Explorer C we were able to model approximately 90 to 95% of the combined long period earth and ocean tidal perturbation of the satellite with a Love number (k_2) of 0.245, and a phase lag of 3.2 degrees (Smith, et al., 1972) used in the solid earth tide model. This accomodation of the oceans within the earth model ignores any frequency dependent terms in either the earth or ocean tides. Consequently, we are planning to allow for a frequency dependence of both the amplitude and phase of the solid-earth which we expect to permit complete accomodation of the ocean tides. In addition, we expect to incorporate a spherical harmonic representation of the major ocean tides (M_2, S_2, K_1, K_2, N_2, O_1, P_1, etc), derived from the numerical integration of Laplace's tidal equations, into our program system so that we can use these models for both altimetry and orbital analysis and also be able to use altimetry and orbital data to improve on the coefficients in the tidal expansions.

Numerical Problems

Numerical integration systems introduce errors of rounding and truncation into the orbit calculations that can become significant for long orbital arcs. In the Geodyn program system a typical step size within the integrator will be about sixty seconds but as the size of the gravity models has increased we are finding that this figure needs to be reduced to perhaps forty seconds in order to properly account for the high frequency terms; and the CPU time increases accordingly. Generally, with step sizes of the order of sixty seconds integration error can be kept to the order of a meter after about 30 days. This error is also predominantly along track and is an acceleration similar to drag. Thus, if drag is being adjusted in the orbit determination process then the integration error will be largely absorbed into the adjustment. In this case integration error is not a major contributor to orbit error.

Another aspect of numerical problems in orbit determination is the core size required to determine large spherical harmonic gravity fields and associated station coordinates. Some of our large gravity field and station coordinate solutions have nearly 2,000 unknowns and require several million bytes on an IBM 360/95 type computer. This storage is not always available and if the computations are attempted in smaller core the CPU and/or IO time increases. Nearly all our computations in satellite orbit, geodesy and geodynamics are requiring greater accuracy today than a few years ago and this means an increase in both CPU time and core storage.

Because of this need for additional precision and complexity in the computations we are giving consideration to the advantages and disadvantages of parallel processor computers and dedicated minicomputers for some of this work.

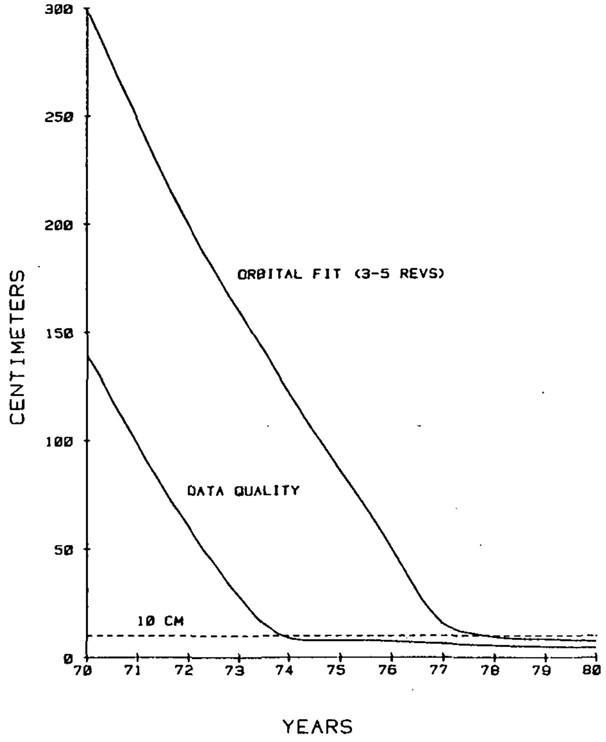

Fig. 1. Development of orbit determination capability.

Conclusions

The application of satellite geodetic techniques to problems in solid earth and ocean dynamics is requiring ever increasing accuracy in the computation of satellite orbits. This need presents considerable difficulty in the modeling of the many perturbing forces that influence the spacecraft motion. With increasing complexity and accuracy is the need for faster and larger computing facilities.

In this paper I have briefly described the status of the major GSFC orbit and geodynamic parameter estimation program (Geodyn) and the degree of complexity that we are finding necessary to meet orbital and geophysical accuracies. The computing of precision orbits at the centimeter level is difficult to obtain and even more difficult to maintain for any length of time and may have to be limited to satellites, such as Lageos, that are carefully configured to minimize the perturbing forces.

References

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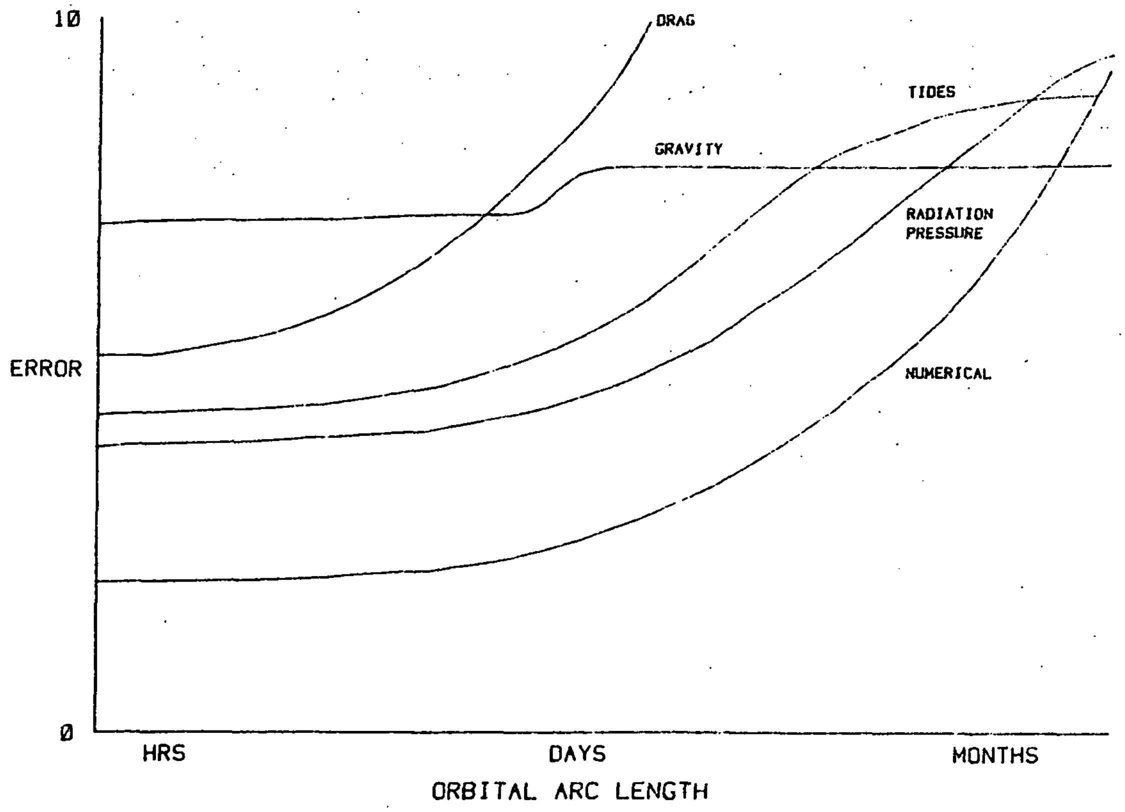

Fig. 2. Relative orbit error for low altitude spacecraft.

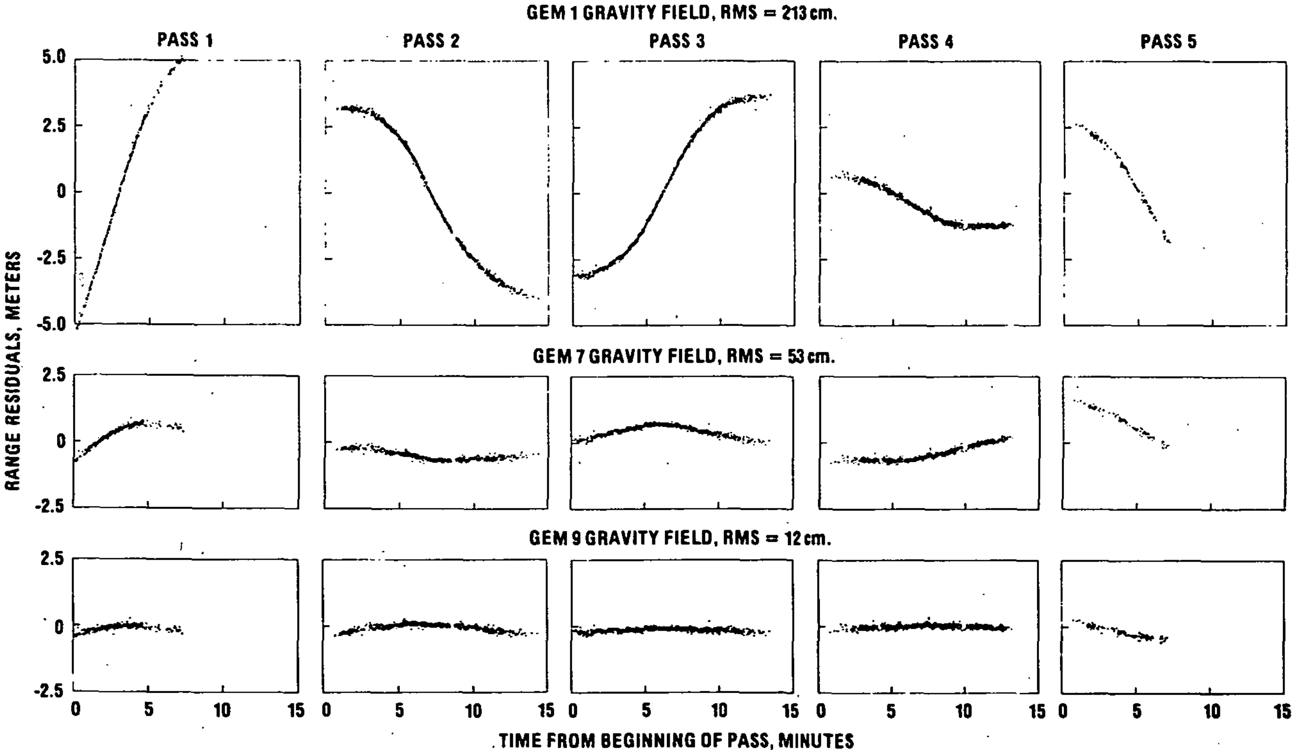

Fig. 3. Laser residuals from five pass orbit of Beacon Explorer C in 1974.

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