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# Ocean Tidal Excitation of Polar Motion

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# OCEAN TIDAL EXCITATION OF POLAR MOTION


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ABSTRACT

An investigation has been conducted to ascertain the response of the rotational motion of the Earth to forcing functions produced by the water mass redistribution due to the ocean tides. In particular, the components of displacement of the rotation axis at the surface of the Earth were obtained. The investigation also addressed the larger question concerning the possibility of excitation of the Chandler wobble of the Earth.

In general, the results show the existence of a polar wobble as a response to each of the components of the ocean tides. The magnitude of the polar displacement depends on two factors: the amplitude of the tidal component and its period (in relation to the Chandler period).

The maximum periodic contributions are: the Doodson's component number 055.565 with a period of 18.613 years and 50 cm of polar displacement, the annual component 056.544 with 37 cm of polar displacement and the semi-annual 057.555 with 32 cm. The tidal components with daily and semi-daily periods yield very small polar displacements of the order of 0.01 cm. The combined effect of all the periodic components can yield as much as 90 cm of pole displacements.

The changes produced by the ocean tides in the products of inertia are periodic and regular, therefore, they cannot be the source of excitation of the Chandler wobble.

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# OCEAN TIDAL EXCITATION OF POLAR MOTION

## INTRODUCTION

The objective of this investigation is to ascertain the response of the rotational motion of the Earth to forcing functions produced by the water mass redistribution due to the lunisolar ocean tides. In particular, it is desired to obtain the components of displacement of the rotation axis at the surface of the Earth. The investigation also addresses the larger question concerning the possibility of excitation of the Chandler wobble of the Earth. The problem concerning the interaction between the angular velocity of rotation of the Earth and the induced oceanic tidal response has been treated by others, i.e.: Haubrich and Munk (1959), Dahlen (1976), and it will not be dealt with here.

One of the areas of interest in present day geophysics deals with the excitation of the polar motion of the Earth, that is, how do the different physical phenomena taking place inside and on the surface of the Earth affect the position of the rotation axis and the rate of rotation.

Among the many physical phenomena one must consider the redistribution of the mass of the ocean produced by the gravitational tides due to the Sun and the Moon. Such a redistribution of mass yields changes in the inertia tensor of the Earth which themselves are the causes of changes in the position of the rotation axis and the rate of rotation. All the changes involved are functions of time with various periodicities.

In order to ascertain the ocean tidal effects the Liouville equations of motion for a nonrigid body have been simplified by neglecting higher order terms and an analytic solution has been found for the $\omega_x$ and $\omega_y$ components of angular velocity. Such a solution involves the magnitude of the changes in the products of inertia $I_{xz}$ and $I_{yz}$ due to the various ocean tidal components as well as the frequencies of the tides and the natural frequency of rotation of the Chandler wobble.

The evaluation of the changes in $I_{xz}$ and $I_{yz}$ can be divided in two categories. Changes produced by ocean tides with periods of the order of a day and changes produced by long period ocean tides. The method of evaluation depends on the category and the effects produced by each are considerably different in order of magnitude.

The products of inertia due to the short period ocean tides have been obtained by making use of spherical harmonics representations of the tide heights. These representations are fits to numerical solutions of the Laplace tidal equations and as such represent non–equilibrium solutions. In particular, the harmonic coefficients used were those provided by C. Goad (1978), the LTE solution used is due to R. Estes (1977).

The long period ocean tides were treated differently since no harmonic expansions were available. The equilibrium response of an ocean covering the entire surface of the Earth was formulated, the tidal height for the real Earth is then obtained by means of the ocean function which is equal to one over the oceans and zero over the continents. In particular the (8 x 8) ocean function harmonic expansion due to Munk and MacDonald (1975) was adopted. In order to solve for the products of inertia it is then necessary to solve the integrals over the sphere of the product of three surface harmonics. Such a solution can be expressed in terms of the "3 – j" symbols often found in quantum mechanics.

In order to represent the lunisolar tidal potential the expansion by A. T. Doodson (1954) was adopted with the extensions and corrections due to Cartwright and Taylor (1971) and Cartwright and Edden (1973).

Finally, the solutions for the $\omega_x$ and $\omega_y$ components of angular velocity were mapped into components of displacement of the rotation axis at the surface of the Earth.

## 1. SOLUTION TO THE LIOUVILLE EQUATIONS

The Liouville equations of motion were first given by Liouville (1858). For purposes of this investigation the following assumptions can be made:

(1) the external moments are zero,

(2) the relative angular momentum terms vanish,

(3) the moments of inertia are constant and considerably larger than the products of inertia,

(4) the equatorial moments of inertia are equal,

(5) the $\omega_z$ component of angular velocity is a constant and much larger than $\omega_x$ and $\omega_y$.

Then, neglecting products of small quantities the following equations result.

$$\begin{aligned} A\dot{\omega}_x - (A - C)\,\omega_z\,\omega_y &= \dot{I}_{xz}\,\omega_z - I_{yz}\,\omega_z^2 \\ A\dot{\omega}_y + (A - C)\,\omega_z\,\omega_x &= \dot{I}_{yz}\,\omega_z + I_{xz}\,\omega_z^2 \end{aligned} \tag{1.1}$$

Equations (1.1) can be written as follows

$$\begin{aligned} \ddot{\omega}_x + n^2\omega_x &= A^{-1}\left\{\ddot{I}_{xz}\,\omega_z - n\omega_z^2\,I_{xz} - \dot{I}_{yz}\,(\omega_z^2 + n\omega_z)\right\} \\ \ddot{\omega}_y + n^2\omega_y &= A^{-1}\left\{\ddot{I}_{yz}\,\omega_z - n\omega_z^2\,I_{yz} + \dot{I}_{yz}\,(\omega_z^2 + n\omega_z)\right\} \end{aligned} \tag{1.2}$$

where

$$n = (C - A)\omega_z/A \tag{1.3}$$

Let

$$\begin{aligned} I_{xz} &= M_{xz}\cos(\zeta t - \phi_{xz}) \\ I_{yz} &= M_{yz}\cos(\zeta t - \phi_{yz}) \end{aligned} \tag{1.4}$$

Then

$$\begin{aligned} \omega_x &= K_1\cos(\zeta t - \phi_{xz}) + K_2\sin(\zeta t - \phi_{yz}) \\ \omega_y &= K_3\cos(\zeta t - \phi_{yz}) + K_4\sin(\zeta t - \phi_{xz}) \\ K_1 &= -M_{xz}\,\omega_z(\zeta^2 + n\omega_z)/[A(n^2 - \zeta^2)] \\ K_2 &= M_{yz}\,\omega_z\,\zeta(\omega_z + n)/[A(n^2 - \zeta^2)] \\ K_3 &= (M_{yz}/M_{xz})K_1 \\ K_4 &= -(M_{xz}/M_{yz})K_2 \end{aligned} \tag{1.5}$$

Equations (1.5) can be written as follows,

$$
\begin{aligned}
\omega_x &= W_x \cos(\zeta t - \phi_{\omega x}) \\
\omega_y &= W_y \cos(\zeta t - \phi_{\omega y}) \\
W_x &= \left\{K_1^2 + K_2^2 + 2K_1 K_2 \sin(\phi_{xz} - \phi_{yz})\right\}^{1/2} \\
W_y &= \left\{K_3^2 + K_4^2 + 2K_3 K_4 \sin(\phi_{yz} - \phi_{xz})\right\}^{1/2} \\
\phi_{\omega x} &= \arctan\left\{(K_1 \sin\phi_{xz} + K_2 \cos\phi_{yz})/(K_1 \cos\phi_{xz} - K_2 \sin\phi_{yz})\right\} \\
\phi_{\omega y} &= \arctan\left\{(K_3 \sin\phi_{yz} + K_4 \cos\phi_{xz})/(K_3 \cos\phi_{yz} - K_4 \sin\phi_{xz})\right\}
\end{aligned}
\tag{1.6}
$$

## 2. TIDAL POTENTIAL

The tidal potential can be written in different ways. In particular, the second degree zonal component in spherical coordinates is given by

$$V_2^0(R) = (R)^2 \; P_2^0(\cos\theta)\; q_2^0 \tag{2.1}$$

R: radius of the Earth,

$P_2^0$: second degree legendre polynomial,

$q_2^0$: a function which depends on the disturbing body.

Doodson's expansion for the long period lunisolar potential is given by (Bretreger, 1978),

$$V_2^0(R) = -G_D \; P_2^0(\cos\theta) \sum_i A_i \cos(\alpha_i) \tag{2.2}$$

Comparing Equations (2.1) and (2.2) it follows that

$$q_2^0 = -(G_D/R^2) \sum_i A_i \cos(\alpha_i) \tag{2.3}$$

where $G_D$ is Doodson's constant and $A_i$, $\alpha_i$ are the amplitudes and arguments for the various waves (see Appendix 2).

The principal terms of the low frequency tides are given by Cartwright and Edden (1973). For the purposes of this investigation the following terms will be considered: The 17 waves of group (0,5) with periods ranging from 91 days to infinity. The 2 terms with largest amplitude in group (0,6) with periods of 27 and 31 days. The term with largest amplitude in group (0,7) with period of 13 days. The group terminology is that of Doodson.

Table (2.1) below gives the tidal components with their amplitudes and periods.

Table (2.1)
Tidal Components

| Doodson's No. | Period (days) | Amplitude ($A_i$) |
|---|---|---|
| 055.555 | ∞ | 0.73806 |
| 055.565 | 6798.2530 | −0.06556 |
| 055.575 | 3399.1265 | 0.00064 |
| 055.765 | 1305.4098 | −0.00009 |
| 056.544 | 385.99876 | 0.00009 |
| 056.554 | 365.25964 | 0.01156 |
| 056.556 | 365.22476 | −0.00062 |
| 056.564 | 346.63545 | −0.00011 |
| 057.345 | 212.32434 | −0.00005 |
| 057.355 | 205.89383 | 0.00074 |
| 057.555 | 182.62110 | 0.07281 |
| 057.553 | 182.62982 | 0.00029 |
| 057.565 | 177.84369 | −0.00180 |
| 057.575 | 173.30987 | −0.00040 |
| 058.554 | 121.74934 | 0.00426 |
| 058.564 | 119.60730 | −0.00007 |
| 059.553 | 91.312730 | 0.00017 |
| 063.655 | 31.811921 | 0.01579 |
| 065.455 | 27.554564 | 0.08254 |
| 075.555 | 13.660791 | 0.15647 |

## 3. EQUILIBRIUM TIDES

The surface of an ocean covering the entire surface of the Earth is raised with respect to the ocean bottom by the following amount:

$$\xi_0 = \frac{(1 + k - h)}{g} V_2^T. \tag{3.1}$$

k, h: love numbers,

g: gravitational acceleration,

where $V_2^T$ denotes the disturbing potential evaluated at the surface of the Earth.

Let $\Omega$ denote the surface area covered by the oceans and define $\xi^1$ as follows:

$$\xi^1 \Omega = \iint_C \frac{(1 + k - h)}{g} V_2^T \, ds, \tag{3.2}$$

where the surface integral is taken over the area of the continents. $\xi^1$ represents the quantity which must be added to $\xi_0$ in order to satisfy conservation of mass. Note that,

$$\iint_C \frac{(1 + k - h)}{g} V_2^T \, ds = \iint_{Sphere} \frac{(1 + k - h)}{g} V_2^T \, ds - \iint_{Oceans} \frac{(1 + k - h)}{g} V_2^T \, ds \tag{3.3}$$

The first term on the right hand side of Equation (3.3) goes to zero, therefore, Equation (3.2) yields:

$$\xi^1 = \frac{-1}{\Omega} \iint_{Oceans} \frac{(1 + k - h)}{g} V_2^T \, ds. \tag{3.4}$$

The resulting height of the ocean surface is then given by:

$$\xi = \xi_0 + \xi^1. \tag{3.5}$$

Inserting Equations (3.1) and (3.4) into Equation (3.5) yields:

$$\xi = \frac{(1 + k - h)}{g} \left[ V_2^T - \frac{1}{\Omega} \iint_{\text{Oceans}} V_2^T \, ds \right]. \tag{3.6}$$

Let the "ocean function" be defined by:

$$f(\theta,\psi) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} P_n^m(\cos\theta) \begin{pmatrix} a_n^m \cos m \psi \\ b_n^m \sin m \psi \end{pmatrix}, \tag{3.7}$$

and the disturbing potential by:

$$V_2^T = R^2 \sum_{m=0}^{2} P_2^m(\cos\theta) \begin{pmatrix} q_2^m \cos m \psi \\ u_2^m \sin m \psi \end{pmatrix}. \tag{3.8}$$

Furthermore, consider only the terms up to n = 9 in Equation (3.7) and m = 0 in Equation (3.8).

Then,

$$\Omega = \iint_{\text{Sphere}} f(\theta,\psi) \, ds = 4\pi \, a_0^0, \tag{3.9}$$

$$\iint_{\text{Oceans}} V_2^T \, ds = \iint_{\text{Sphere}} f(\theta,\psi) \, V_2^T \, ds = \frac{4\pi}{5} R^2 \, a_2^0 \, q_2^0. \tag{3.10}$$

Equation (3.6) can then be written as follows:

$$\xi = \frac{(1 + k - h)}{g} R^2 \left[ P_2^0(\cos\theta) - \frac{a_2^0}{5a_0^0} \right] q_2^0. \tag{3.11}$$

The contribution to the products of inertia $I_{xz}$ and $I_{yz}$ are given by:

$$I_{xz} = \iint_{\text{Sphere}} f(\theta,\psi) \, xz \, dm, \tag{3.12}$$

$$I_{yz} = \iint_{\text{Sphere}} f(\theta,\psi) \, yz \, dm, \tag{3.13}$$

where

$$\begin{aligned} x &= R \sin\theta \cos\psi \\ y &= R \sin\theta \sin\psi \\ z &= R \cos\theta \\ dm &= \rho_w \, \xi(\theta,\psi) \, [R \, d\theta \cdot R \sin\theta \, d\psi]. \end{aligned} \tag{3.14}$$

$\rho_w$ : density of sea water

Making use of Equations (3.14), (3.11) and (3.7) allows Equations (3.12) and (3.13) to be written as follows:

$$I_{xz} = \rho_w R^6 \frac{(1 + k - h)}{g} \int_0^{2\pi} \int_0^{\pi} \sum_{n=0}^{9} \sum_{m=0}^{n} P_n^m(\cos\theta) \begin{pmatrix} a_n^m \cos m\psi \\ b_n^m \sin m\psi \end{pmatrix} \cdot$$

$$\cdot \left[ P_2^0(\cos\theta) - \frac{a_2^0}{5a_0^0} \right] q_2^0 \cdot \frac{1}{3} P_2^1(\cos\theta) \sin\theta \cos\psi \, d\theta \, d\psi,$$

$$I_{yz} = \rho_w R^6 \frac{(1 + k - h)}{g} \int_0^{2\pi} \int_0^{\pi} \sum_{n=0}^{9} \sum_{m=0}^{n} P_n^m(\cos\theta) \begin{pmatrix} a_n^m \cos m\psi \\ b_n^m \sin m\psi \end{pmatrix} \cdot \tag{3.15}$$

$$\cdot \left[ P_2^0(\cos\theta) - \frac{a_2^0}{5a_0^0} \right] q_2^0 \cdot \frac{1}{3} P_2^1(\cos\theta) \sin\theta \sin\psi \, d\theta \, d\psi.$$

Integration of Equations (3.15) yields the following (see Appendix 1):

$$\begin{aligned} I_{xz} &= K \, T_{xz} \, q_2^0 \\ I_{yz} &= K \, T_{yz} \, q_2^0 \end{aligned} \tag{3.16}$$

$$\begin{aligned} K &= (4\pi/g) R^6 \rho_w (1 + k - h) \\ T_{xz} &= (1/35)a_2^1 + (2/21)a_4^1 - (1/25)(a_2^0 \, a_2^1/a_0^0) \\ T_{yz} &= (1/35)b_2^1 + (2/21)b_4^1 - (1/25)(a_2^0 \, b_2^1/a_0^0) \end{aligned}$$

Making use of Equation (2.3) yields

$$\begin{aligned} I_{xz} &= -K \, T_{xz} \, G_D \, R^{-2} \sum_i A_i \cos(\alpha_i) \\ I_{yz} &= -K \, T_{yz} \, G_D \, R^{-2} \sum_i A_i \cos(\alpha_i) \end{aligned} \tag{3.17}$$

## 4. NON-EQUILIBRIUM TIDES

The method of solution employed for the equilibrium tides becomes less and less applicable as the period of the forcing function becomes shorter. Specifically, the equilibrium theory is approp riate to the extent that $(P_n/P_\zeta)^2 \ll 1$, where $P_n$ is a natural period of the system and $P_\zeta$ is the period of the forcing function.

The products of inertia due to the short period ocean tides have been obtained by means of existing spherical harmonics representations of the tide heights. These expansions are fits to numerical solutions of the Laplace tidal equations.

The spherical harmonic coefficients for the tide heights were obtained from C. Goad (1978), the LTE solution used is due to R. Estes (1977).

Table (4.1) gives the values of the coefficients used in this investigation.

The expression for the tide height is given by

$$
\begin{aligned}
\xi(\theta,\psi,t) = \sum_n \sum_{m=0}^{n} P_n^m(\cos\theta) & \left\{ \begin{pmatrix} A_n^m \cos \zeta_i\, t \\ C_n^m \sin \zeta_i\, t \end{pmatrix} \cos m\psi \right. \\
& \left. + \begin{pmatrix} B_n^m \cos \zeta_i\, t \\ D_n^m \sin \zeta_i\, t \end{pmatrix} \sin m\psi \right\}
\end{aligned}
\tag{4.1}
$$

The products of inertia are given by

$$
\begin{aligned}
I_{xz} &= \iint_S xz\, dm \\
I_{yz} &= \iint_S yz\, dm
\end{aligned}
\tag{4.2}
$$

Therefore, making use of Equation (4.1),

$$
\begin{aligned}
I_{xz} &= (4\pi/5) R^4\, \rho_w (A_2^1 \cos\zeta_i\, t + C_2^1 \sin\zeta_i\, t) \\
I_{yz} &= (4\pi/5) R^4\, \rho_w (B_2^1 \cos\zeta_i\, t + D_2^1 \sin\zeta_i\, t)
\end{aligned}
\tag{4.3}
$$

or

$$
\begin{aligned}
I_{xz} &= M_{xz} \cos(\zeta_i t - \phi_{xz}) \\
I_{yz} &= M_{yz} \cos(\zeta_i t - \phi_{yz}) \\
M_{xz} &= \alpha[(A_2^1)^2 + (C_2^1)^2]^{1/2} \\
M_{yz} &= \alpha[(B_2^1)^2 + (D_2^1)^2]^{1/2} \qquad (4.4) \\
\phi_{xz} &= \arctan(C_2^1/A_2^1) \\
\phi_{yz} &= \arctan(D_2^1/B_2^1) \\
\alpha &= (4\pi/5)\, R^4 \rho_w
\end{aligned}
$$

Table (4.1)
Coefficients for Short Period Tides

| Doodson No. | Darwin | Period (hr) | $A_2^1(10^2)$ | $C_2^1(10^2)$ | $B_2^1(10^2)$ | $D_2^1(10^2)$ |
|---|---|---|---|---|---|---|
| 273.555 | $S_2$ | 12.00 | -0.5642 | -0.2811 | -0.06755 | -0.3763 |
| 255.555 | $M_2$ | 12.42 | -1.274 | -0.8321 | -0.4521 | -0.9790 |
| 245.655 | $N_2$ | 12.66 | -0.2296 | -0.1622 | 0.1923 | -0.04628 |
| 145.555 | $O_1$ | 25.82 | 0.3666 | 3.070 | 1.974 | 2.322 |
| 165.555 | $K_1$ | 23.93 | -0.7530 | 2.110 | 3.676 | 3.961 |

## 5. NUMERICAL RESULTS

The results obtained by means of Equations (3.17) and (4.4) are given in Table (5.1) on the following page. The magnitudes $M_{xz}$, $M_{yz}$ are given in units of $10^{33}$ gm-cm$^2$. The values used for the constants are the following:

$$
\begin{aligned}
G_D &= 2.627723\,(10^4)\ \mathrm{cm^2/sec^2} \\
R &= 6.378388\,(10^8)\ \mathrm{cm} \\
\rho_w &= 1.030\ \mathrm{gm/cm^3} \\
A &= 8.016604490270\,(10^{44})\ \mathrm{gm\text{-}cm^2} \\
g &= 980\ \mathrm{cm/sec^2} \\
1 + k - h &= 0.7
\end{aligned}
$$

Table (5.1)
Tidal Contributions to the Products of Inertia, Epoch:
1899, Dec. 31, 12h

| Doodson No. | $M_{xz}$ ($10^{-33}$) | $\phi_{xz}$ (deg) | $M_{yz}$ ($10^{-33}$) | $\phi_{yz}$ (deg) |
|---|---|---|---|---|
| 055.555 | -1500.00 | 0 | -735.00 | 0 |
| 055.565 | 133.00 | 259 | 65.30 | 259 |
| 055.575 | -1.30 | 158 | -0.63 | 158 |
| 055.765 | 0.18 | 207 | 0.08 | 207 |
| 056.544 | -0.18 | -260 | -0.08 | -260 |
| 056.554 | -23.40 | -1 | -11.50 | -1 |
| 056.556 | 1.26 | 200 | 0.61 | 200 |
| 056.564 | 0.22 | 257 | 0.10 | 257 |
| 057.345 | 0.10 | -368 | 0.04 | -368 |
| 057.355 | -1.50 | -109 | -0.73 | -109 |
| 057.555 | -147.00 | 199 | -72.50 | 199 |
| 057.553 | -0.58 | -3 | -0.28 | -3 |
| 057.565 | 3.65 | 98 | 1.79 | 98 |
| 057.575 | 0.81 | -3 | 0.39 | -3 |
| 058.554 | -8.65 | 197 | -4.24 | 197 |
| 058.564 | 0.14 | 97 | 0.06 | 97 |
| 059.553 | -0.34 | 196 | -0.16 | 196 |
| 063.655 | -32.00 | 45 | -15.70 | 45 |
| 065.455 | -167.00 | -63 | -82.20 | -63 |
| 075.555 | -318.00 | 180 | -155.00 | 180 |
| 165.555 | 9.59 | 109 | 23.10 | 47 |
| 145.555 | 13.20 | 83 | 13.00 | 49 |
| 245.655 | 1.20 | -144 | 0.84 | -13 |
| 255.555 | 6.51 | -146 | 4.62 | -114 |
| 273.555 | 2.70 | -153 | 1.63 | -100 |

The values for the coefficients of the ocean function are those given by Munk and MacDonald (1975). The normalization factors were taken into consideration.

The values for the amplitudes $A_i$ are given in Table (2.1) for the equilibrium tides, for the short period tides Table (4.1) gives the values for the coefficients.

The phases $\phi_{xz}$ and $\phi_{yz}$ correspond to 1899 December 31 at 12h 0m 0sec ephemeris time (see Appendix 2).

Table (5.2) presents the tidal heights for the equilibrium tides as obtained from Equation (3.11) with $\theta = 0°$ and $\cos(\alpha_i) = 1$. These values therefore represent an upper bound.

Table (5.2)
Maximum Equilibrium Tide Heights

| Doodson No. | Tide Height (cm) |
|---|---|
| 055.555 | -14.30 |
| 055.565 | 1.27 |
| 055.575 | $-0.12\ (10^{-1})$ |
| 055.765 | $0.17\ (10^{-2})$ |
| 056.544 | $-0.17\ (10^{-2})$ |
| 056.554 | -0.22 |
| 056.556 | $0.12\ (10^{-1})$ |
| 056.564 | $0.21\ (10^{-2})$ |
| 057.345 | $0.97\ (10^{-3})$ |
| 057.355 | $-0.14\ (10^{-1})$ |
| 057.555 | -1.41 |
| 057.553 | $-0.56\ (10^{-2})$ |
| 057.565 | $0.35\ (10^{-1})$ |
| 057.575 | $0.77\ (10^{-2})$ |
| 058.554 | $-0.82\ (10^{-1})$ |
| 058.564 | $0.13\ (10^{-2})$ |
| 059.553 | $-0.33\ (10^{-2})$ |
| 063.655 | -0.30 |
| 065.455 | -1.60 |
| 075.555 | -3.04 |

The maximum x and y components of displacement for the position of the pole are given in Table (5.3). These values are obtained by means of the following relations,

$$x = (W_x/\omega_z)R$$

$$y = (W_y/\omega_z)R$$

where $W_x$, $W_y$ are given by Equations (1.6) and

$$\omega_z = (2\pi/86400) \text{ rad/sec.}$$

The value of "n" (Eq. 1.3) has been chosen so as to obtain a Chandler period of 428 days. These values represent maximum values since it has been assumed that $\cos(\alpha_i) = 1$. Table (5.4) shows the variation in $d = \sqrt{x^2 + y^2}$ as a function of "n" for the epoch Dec. 31, 1899, 12h ephemeris time. The value of "n" has been chosen so as to obtain Chandler periods of 400, 428, 444 and 460 days.

The last row of Table (5.4) gives the total pole displacement for the sum of all the periodic components, that is the component 055.555 is not taken into account since its effect can be nullified by a proper choice of coordinates.

Figures 5.1, 5.2 and 5.3 show the pole displacement due to the components 055.565, 056.554 and 057.555 respectively, the epoch is Dec. 31, 1899, 12h. In each case the time span corresponds to the period of the tidal component: 6798 days, 365 days and 182 days respectively.

Figures 5.4 and 5.5 show the x and y components of pole displacement generated by the sum of all the periodic components (055.555 excluded), the epoch is the same as above, time steps of 67.98 days were used.

Figure 5.6 is a plot of $\sqrt{x^2 + y^2}$ with x and y obtained as explained above. A Chandler period of 428 days has been assumed to generate the results shown in Figures 5.1 to 5.6.

Table (5.3)
Maximum Components of Pole Displacement
Chandler Period = 428 Days

| Doodson No. | x (cm) | y (cm) | $\sqrt{x^2 + y^2}$ (cm) |
|---|---|---|---|
| 055.555 | 510.000 | 250.000 | 568.0000 |
| 055.565 | 45.500 | 22.500 | 50.7000 |
| 055.575 | 0.450 | 0.220 | 0.5000 |
| 055.765 | 0.070 | 0.040 | 0.0800 |
| 056.544 | 0.300 | 0.320 | 0.4300 |
| 056.554 | 24.800 | 27.300 | 36.8000 |
| 056.556 | 1.320 | 1.460 | 1.9600 |
| 056.564 | 0.170 | 0.190 | 0.2500 |
| 057.345 | 0.010 | 0.020 | 0.0220 |
| 057.355 | 0.220 | 0.330 | 0.3900 |
| 057.555 | 17.200 | 26.900 | 31.9000 |
| 057.553 | 0.060 | 0.100 | 0.1100 |
| 057.565 | 0.440 | 0.690 | 0.8100 |
| 057.575 | 0.100 | 0.160 | 0.1800 |
| 058.554 | 0.520 | 0.920 | 1.0500 |
| 058.564 | 0.008 | 0.010 | 0.0120 |
| 059.553 | 0.010 | 0.020 | 0.0220 |
| 063.655 | 0.410 | 0.810 | 0.9000 |
| 065.455 | 1.850 | 3.700 | 4.1300 |
| 075.555 | 1.730 | 3.470 | 3.8700 |
| 165.555 | 0.010 | 0.010 | 0.0140 |
| 145.555 | 0.010 | 0.010 | 0.0140 |
| 245.655 | 0.001 | 0.001 | 0.0014 |
| 255.555 | 0.006 | 0.005 | 0.0078 |
| 273.555 | 0.002 | 0.002 | 0.0028 |

Table (5.4)
Pole Displacement vs. Chandler Period,
Epoch: 1899, Dec. 31, 12h

| | $\sqrt{x^2 + y^2}$ (cm) | | | |
|---|---|---|---|---|
| Doodson No. | 400 Days | 428 Days | 444 Days | 460 Days |
| 055.555 | 531.0000 | 568.0000 | 590.0000 | 611.0000 |
| 055.565 | 9.3000 | 10.0000 | 10.4000 | 10.8000 |
| 055.575 | 0.4300 | 0.4600 | 0.4800 | 0.5000 |
| 055.765 | 0.0640 | 0.0690 | 0.0730 | 0.0760 |
| 056.544 | 0.9100 | 0.3300 | 0.2500 | 0.2100 |
| 056.554 | 41.9000 | 23.9000 | 19.4000 | 16.4000 |
| 056.556 | 2.2700 | 1.3100 | 1.0700 | 0.9100 |
| 056.564 | 0.2700 | 0.1900 | 0.1700 | 0.1500 |
| 057.345 | 0.0140 | 0.0130 | 0.0120 | 0.0110 |
| 057.355 | 0.3500 | 0.3400 | 0.3300 | 0.3200 |
| 057.555 | 16.5000 | 15.4000 | 14.8000 | 14.3000 |
| 057.553 | 0.0550 | 0.0500 | 0.0480 | 0.0450 |
| 057.565 | 0.7000 | 0.6700 | 0.6600 | 0.6500 |
| 057.575 | 0.1360 | 0.1290 | 0.1260 | 0.1230 |
| 058.554 | 0.4400 | 0.4200 | 0.4100 | 0.4000 |
| 058.564 | 0.0164 | 0.0162 | 0.0161 | 0.0160 |
| 059.553 | 0.0107 | 0.0103 | 0.0102 | 0.0100 |
| 063.655 | 0.6530 | 0.6520 | 0.6519 | 0.6515 |
| 065.455 | 3.7100 | 3.7060 | 3.7050 | 3.7030 |
| 075.555 | 0.4100 | 0.4090 | 0.4040 | 0.4000 |
| Total – Permanent | 26.3000 | 16.2000 | 16.0000 | 16.6000 |

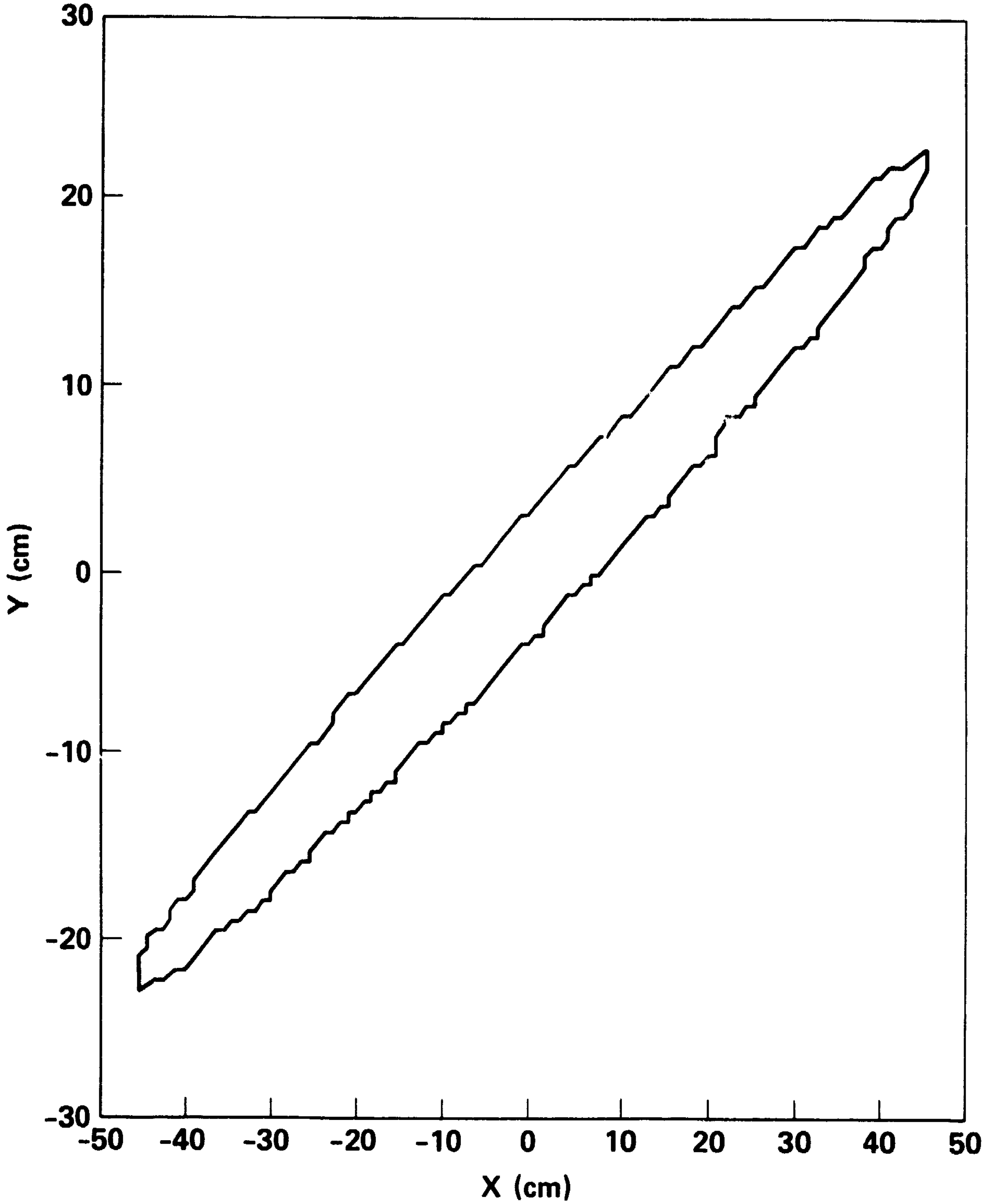


Figure 5.1. Pole Displacement, Component 055.565

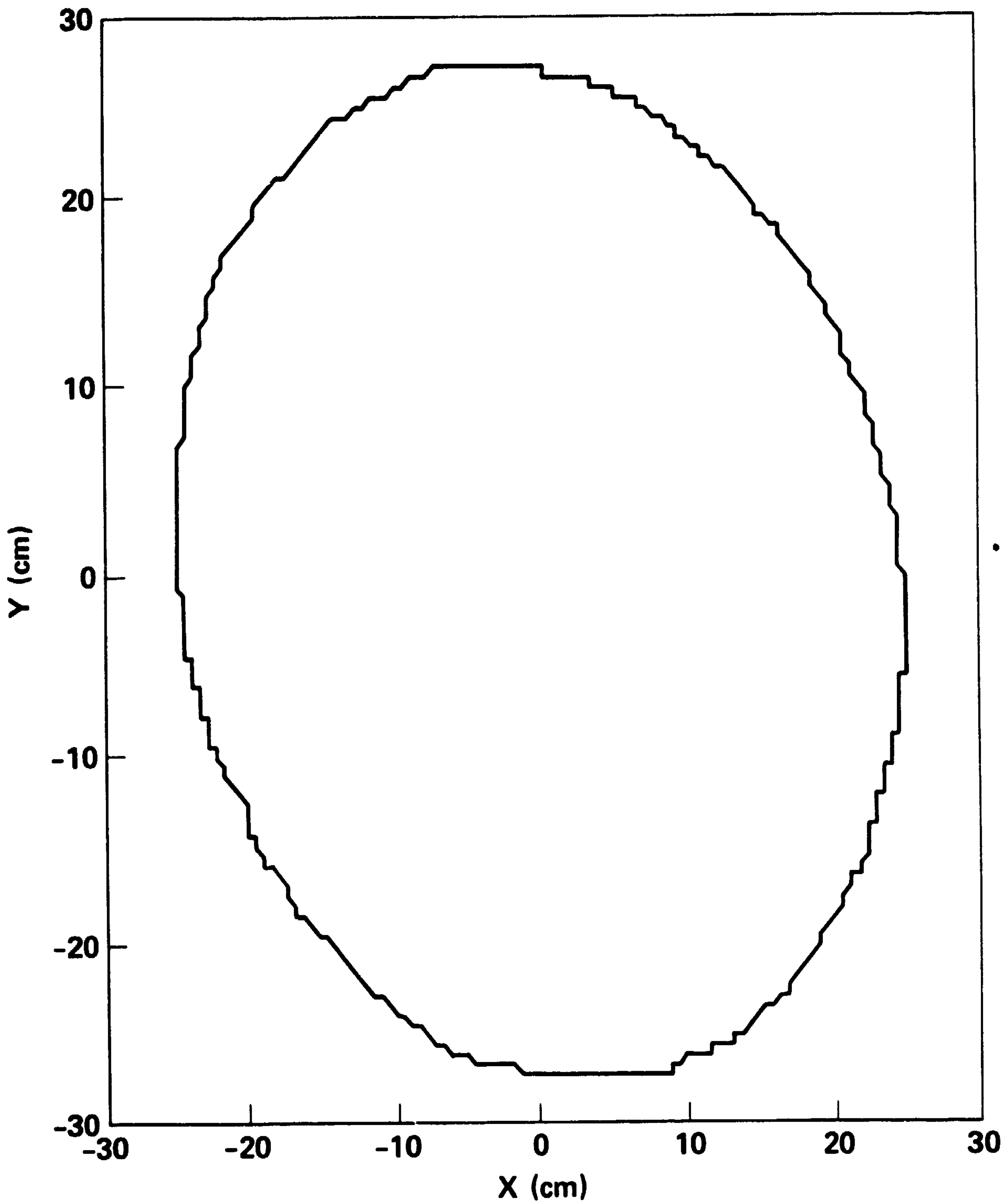


Figure 5.2. Pole Displacement, Component 056.554

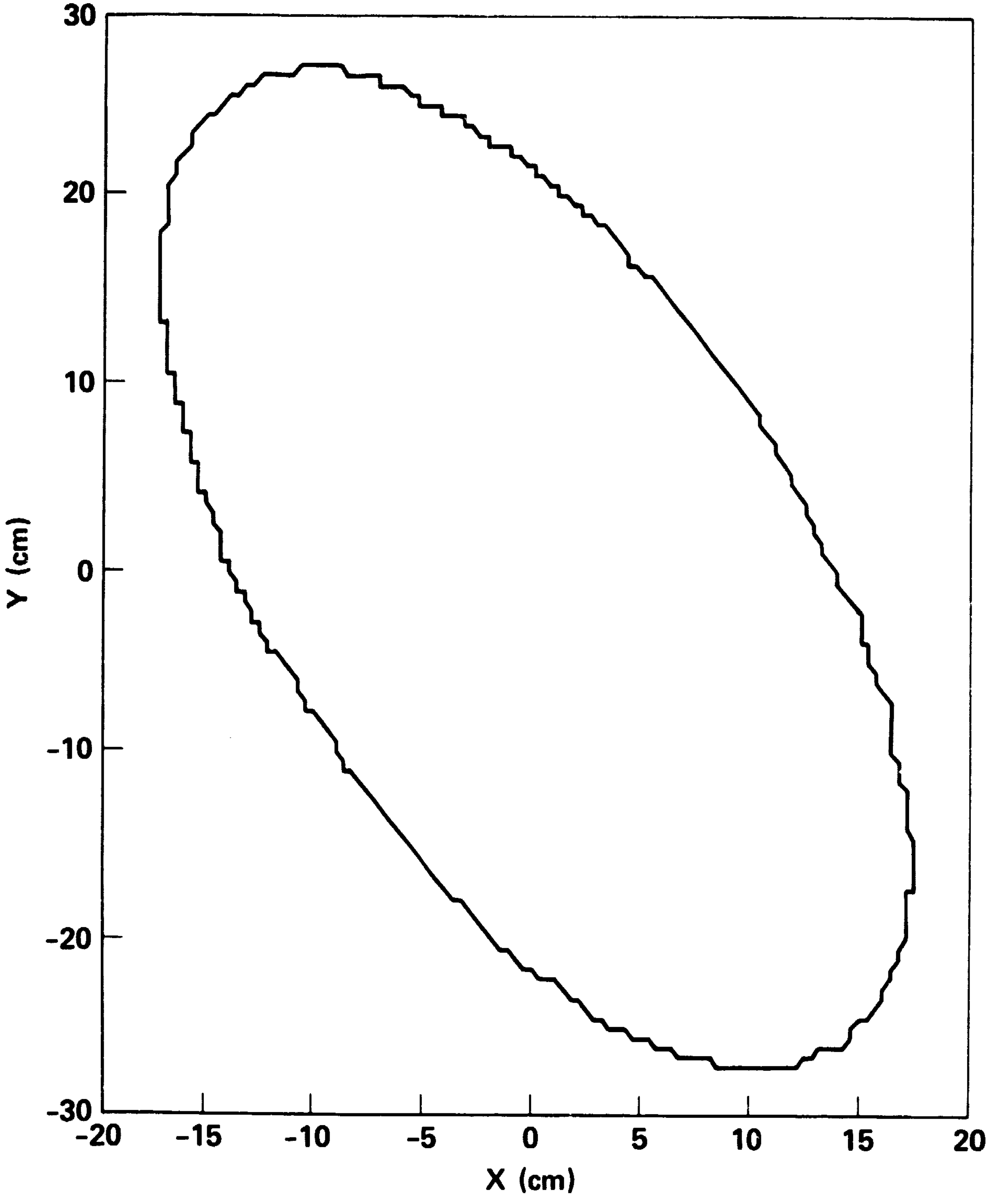


Figure 5.3. Pole Displacement, Component 05'.555

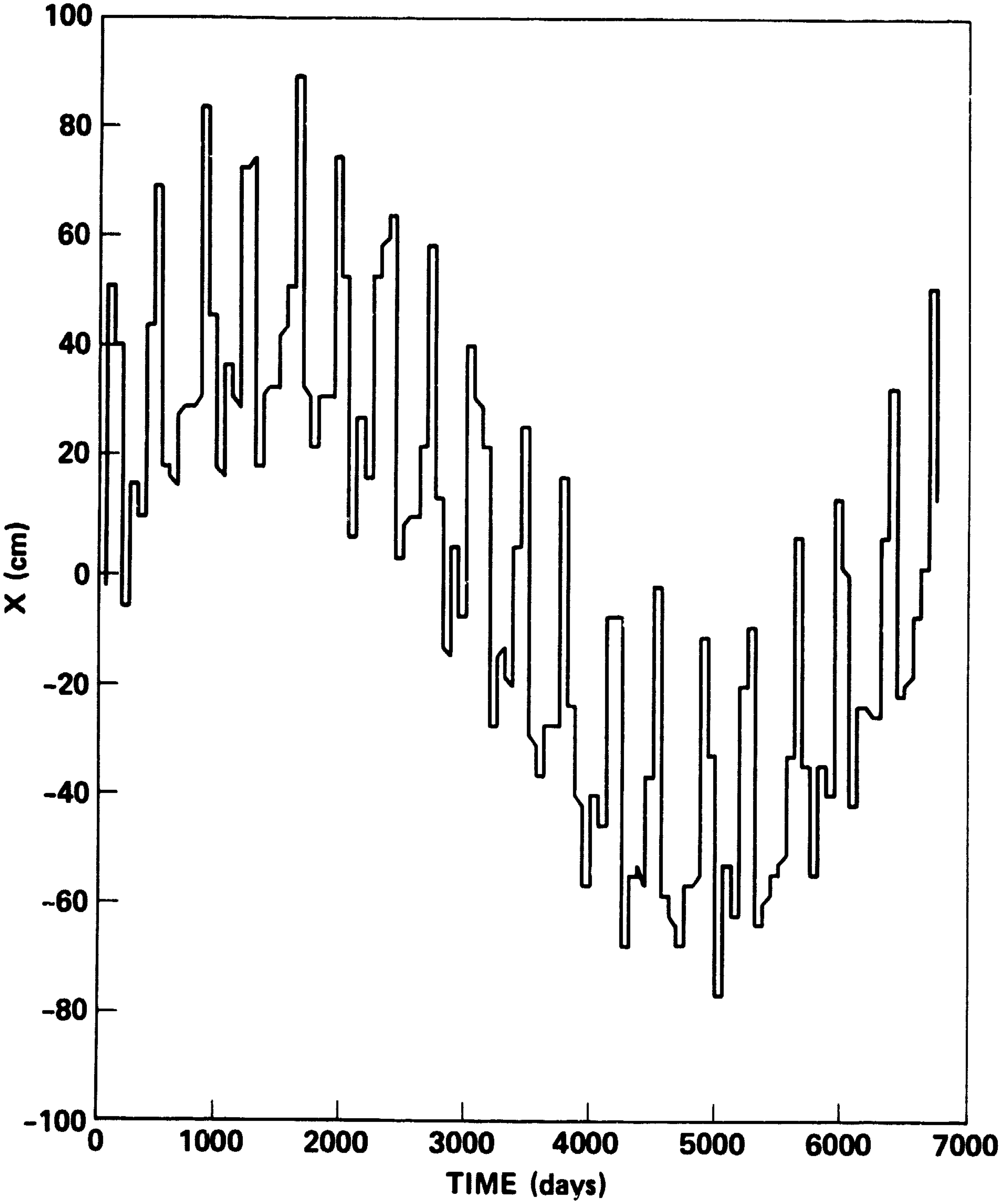


Figure 5.4. Total X Component of Pole Displacement vs. Time

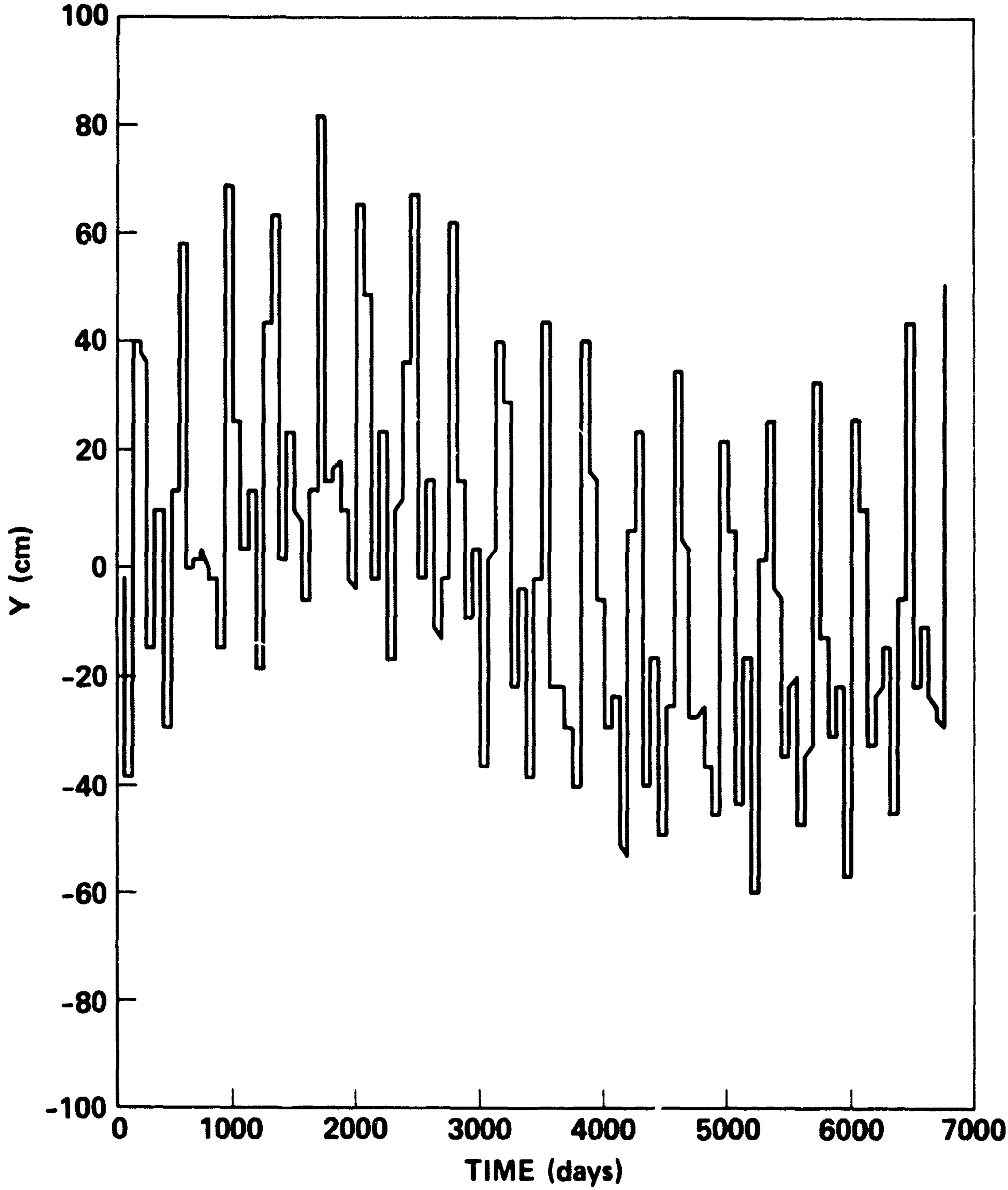


Figure 5.5. Total Y Component of Pole Displacement vs. Time

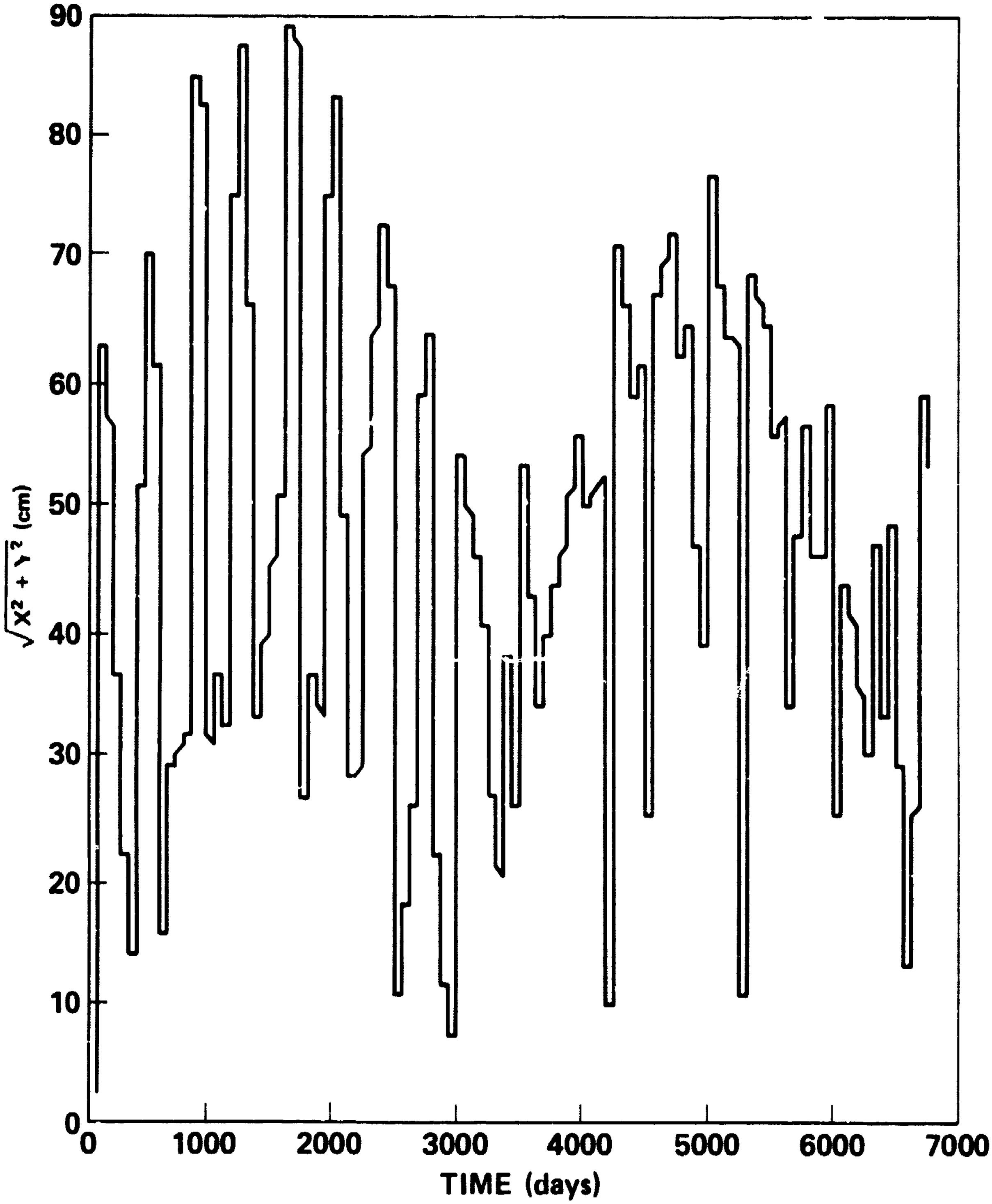


Figure 5.6. Total $\sqrt{X^2 + Y^2}$ vs. Time

## 6. CONCLUSIONS

The validity of most of the results is predicated upon the reality of the equilibrium response of the oceans to the long period components of the lunisolar potential. Within the context of this assumption and the simplifications made, the results indicate the following.

There exists a polar wobble as a response to each of the components of the tidal potential. The magnitude of the polar displacement depends on two factors: the amplitude of the tidal component and its period (in relation to the Chandler period). The maximum periodic contributions are: the component number 055.565 with a period of 6798 days (18.613 years) and 50cm of pole displacement, this is the component produced by the motion of the lunar ascending node; the annual component 056.554 with 37cm of pole displacement and the semiannual 057.555 with 32cm. The tidal components with daily and semidaily periods yield very small pole displacements, of the order of 0.01 cm. The combined effect of all the periodic components can yield as much as 90cm of pole displacement.

The tidal components with periods longer than the Chandler period yield larger pole displacements as the Chandler period is increased, the opposite is true for those components with periods shorter than the Chandler period. The changes produced by the ocean tides in the products of inertia are periodic and regular therefore they can not be the source of excitation of the Chandler wobble.

A possible future line of development is to consider the changes in all the components of the inertia tensor (not just $I_{xz}$ and $I_{yz}$) and to solve the fully nonlinear Liouville equations by means of numerical methods.

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# APPENDIX 1

# INTEGRAL OF THE PRODUCT OF 3 SURFACE HARMONICS

# APPENDIX 1

## INTEGRAL OF THE PRODUCT OF 3 SURFACE HARMONICS

The surface integral appearing in Equation (3.15) involves the product of three surface spherical harmonics. In general,

$$\int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} S_{\ell'm'} \; S_{LM} \; S_{\ell m} \, d\psi = I$$

$$S_{\ell m} = P_\ell^m(\cos\theta)\, e^{im\psi}$$

$$I = (-1)^{M+m} \, (4\pi) \left[\frac{(\ell' + m')!\,(L+M)!\,(\ell+m)!}{(\ell' - m')!\,(L-M)!\,(\ell-m)!}\right]^{1/2}$$

$$\begin{pmatrix} \ell' & L & \ell \\ -m' & M & m \end{pmatrix} \begin{pmatrix} \ell' & L & \ell \\ 0 & 0 & 0 \end{pmatrix}$$

The symbol $\begin{pmatrix} j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{pmatrix}$ denotes the 3 – j symbol of Wigner, for their evaluation and properties see Rotenberg et al., (1959).

# APPENDIX 2

# ANGULAR VARIABLES IN DOODSON'S TIDAL EXPANSION

# APPENDIX 2

## ANGULAR VARIABLES IN DOODSON'S TIDAL EXPANSION

The tidal expansion is given in terms of six parameters which constitute the frequencies appearing in the arguments on the right hand side of Equation (2.3). In terms of their periods they are the following:

$v_1$: Lunar day, period of 24h 50.47m,

$v_2$: Moon's mean longitude, period of 27.321582 days,

$v_3$: Sun's mean longitude, period of 365.242199 days,

$v_4$: Mean longitude of the lunar perigee, period of 8.847 years,

$v_5$: Mean longitude of the lunar ascending node, period of 18.613 years,

$v_6$: Mean longitude of perihelion, period of 20,940 years.

Melchior (1966) gives the following expressions:

$$v_2 = 270^\circ.43659 + 481267^\circ.89057T + 0^\circ.00198T^2 + 0^\circ.000002T^3,$$

$$v_3 = 279^\circ.69668 + 36000^\circ.76892T + 0^\circ.00030T^2,$$

$$v_4 = 334^\circ.32956 + 4069^\circ.03403T - 0^\circ.01032T^2 - 0^\circ.00001T^3,$$

$$v_5 = 259^\circ.18328 + 1934^\circ.14201T + 0^\circ.00208T^2 + 0^\circ.000002T^3,$$

$$v_6 = 281^\circ.22083 + 1^\circ.71902T + 0^\circ.00045T^2 + 0^\circ.000003T^3,$$

$$v_1 = t - v_2 + v_3,$$

$t$: mean solar time,

$T$: time expressed in Julian centuries,

$T = 0$ at 1899 December 31, 12h, 0m, 0s ephemeris time.