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EVOLUTION OF THE ROTATION OF AN ARTIFICAL EARTH SATELLITE UNDER THE INFLUENCE OF A PERTURBING MOMENT WHICH IS CONSTANT IN FIXED AXES

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16. Abstract

Reference is made to the change in the modulus of the vector of the kinetic moment of a satellite, noted during the determination of the actual orientation of an artificial earth satellite. The current study seeks to explain this change as a result of the presence of a small perturbing moment, which is constant in fixed axes relative to the satellite. It is also shown that the averaged equations in this problem can be integrated.

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EVOLUTION OF THE ROTATION OF AN ARTIFICIAL EARTH SATELLITE UNDER THE INFLUENCE OF A PERTURBING MOMENT WHICH IS CONSTANT IN FIXED AXES

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During the determination of the actual orientation of the artificial earth satellite "Prognoz", a slow change was noted in the modulus of the vector of the kinetic moment of the satellite in that section of the flight with a disconnected system of orientation. Proposed in the study is an explanation of this phenomenon by the presence of a small perturbing moment, which is constant in a coordinate system which is fixed with the satellite. It is shown that the averaged equations are integratable in this problem. An evaluation is obtained of the perturbing moment for the "Prognoz-6" artificial earth satellite. /2*

A small perturbing moment, which is constant in a coordinate system which is fixed with the satellite, evokes evolution of the rotation of the satellite. Set forth in section 1 are the reasons which make it possible to assume the presence of such a moment on the "Prognoz" artificial earth satellite. Examined in section 2 is the problem of the evolution of the rotation of the artificial earth satellite under the influence of a perturbing moment which is constant in fixed axes. Given in section 3 is an evaluation of the magnitude of the perturbing moment, according to data on the actual orientation of the "Prognoz-6" artificial earth satellite. /3

1. Orientation of artificial earth satellites of the "Prognoz" series is accomplished using a gas reactive system. The reorientation sequence consists of the coincidence of

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*Numbers in the margin indicate pagination in the foreign text.

the structural axis of the satellite, which is close to the main central axis, with the direction to the sun, and the warp of the satellite around this direction. Then, the engines are turned off. The very high orbit of the "Prognoz" artificial earth satellites ($h_\pi \sim 700$ km, $h_a \sim 200{,}000$ km) and their rapid warp ($\omega \sim 3$ degrees/sec) provide a rotation which is close to an unperturbed rotation; however, because of the movement of the sun, according to a celestial sphere, the reorientation must be repeated approximately once every 11 days.

A slight difference in rotation, from the movement /4
according to Euler-Poinsot, was noted in the interval between adjacent reorientation sequences during the determination of the actual orientation of the "Prognoz" artificial earth satellites, according to the data of measurements. Specifically detected was a slow increase in the modulus of the vector of the kinetic moment of the satellite. One can attempt to explain this effect by the effect of a perturbing moment which is constant in fixed axes, with this moment being evoked by the ever-present small amount of release of gas in the gas reactive system. The other possible explanation of the noted evolution is the effect of radiation pressure.

2. The movement of a solid body relative to the center of mass, under the influence of a perturbing moment $\vec{M}$, is described by the equations [1]

$$\begin{aligned} A\frac{dp}{dt} + (C-B)qr &= M_1, \\ B\frac{dq}{dt} + (A-C)rp &= M_2, \\ C\frac{dr}{dt} + (B-A)pq &= M_3. \end{aligned} \qquad (I)$$

Here, A, B, C are the main central moments of inertia of the body, p, q, r are the components of the vector of angular velocity of the body in the main central axes of inertia, and M_1, M_2, M_3 are the components of the moment in those same axes.

The kinetic energy of rotation T and the modulus of the vector of the kinetic moment L are computed according to the formulas

$$2T = Ap^2 + Bq^2 + Cr^2 , \qquad L^2 = A^2p^2 + B^2q^2 + C^2r^2 . \tag{2}$$

If the perturbing moment is absent (Euler-Poinsot case), then T and L are intervals of movement, and the change in p, q, r with time is described by the known formulas [1]

$$p = \pm\sqrt{\frac{2TC-L^2}{A(C-A)}}\, cn(\tau, k) , \quad q = \sqrt{\frac{2TC-L^2}{B(C-B)}}\, sn(\tau, k) , \quad r = \pm\sqrt{\frac{L^2-2TA}{C(C-A)}}\, dn(\tau, k) , \tag{3}$$

$$\tau = t\sqrt{(C-B)(L^2-2TA)/(ABC)} , \quad k^2 = (B-A)(2TC-L^2)/((C-B)(L^2-2TA)) , \tag{3'}$$

and the time t is calculated from the moment, when q=0, q>0.

Here, for definiteness, it is assumed that A<B<C and

$$L^2 > 2TB . \tag{4}$$

/5

Inequality (4) distinguishes two areas in the space p, q, r, one of which corresponds to the plus sign in (3), and the other—to the minus sign [1]. If, on the other hand, the following is satisfied:

$$L^2 < 2TB, \qquad (4')$$

then, it is necessary to permute A and C, p and r in the formulas (3).

The change in L and T in the perturbed problem is described by the equations.

$$\frac{dT}{dt} = pM_1 + qM_2 + rM_3,$$
$$\frac{dL^2}{dt} = 2(ApM_1 + BqM_2 + CrM_3). \qquad (5)$$

We will examine the perturbing moment, which is constant in fixed axes. We will assume that the perturbation is small: $|\vec{M}|/T \ll 1$. Then, the evolution of L and T with time can be approximately described by averaging the equations (5) according to unperturbed Euler-Poinsot movement [2]. We will designate this averaging by a bar above. Then, from (3), we obtain /6

$$\overline{M_1 p} = M_1 \bar{p} = 0, \quad \overline{M_2 q} = M_2 \bar{q} = 0,$$
$$\overline{M_3 r} = M_3 \bar{r} = \pm M_3 \sqrt{\frac{L^2 - 2TA}{C(C-A)}}\, \frac{\pi}{2K(k)}, \qquad (6)$$

where K is the complete elliptical integral of the first order.

Thus, it follows from (5) that the evolution of T and L is approximately described by the equations

$$\frac{dT}{dt} = M_3 \bar{\tau} \quad , \quad \frac{dL^2}{dt} = 2CM_3 \bar{\tau} \ . \tag{7}$$

System (7) has the integral

$$H = 2CT - L^2 = \text{const} \ . \tag{8}$$

Therefore, the solution of (7) can be reduced to squaring

$$\int_{T_0}^{T} \frac{dT}{CM_3 \bar{\tau}} = t - t_0 \ , \tag{9}$$

where, in place of $\bar{\tau}$, is is necessary to substitute its expression in terms of T and H, according to (3'), (6), and (8).

If, in place of (4), (4') is satisfied, then we obtain the following in place of (7)

$$\frac{dT}{dt} = M_1 \bar{p} \quad , \quad \frac{dL^2}{dt} = 2AM_1 \bar{p} \ ,$$

and the following will be the integral of the averaged system

$$H' = 2AT - L^2 = \text{const} \ . \tag{10}$$

In the plane L^2, T, movement takes place according to /7 ray (8) in area (4), and according to ray (10) in area (4') (fig. 1). The direction of movement according to the ray is determined by the sign in formulas (3), and the sign of the components of $\vec{M}$. In the two areas of space p, q, τ, determined by inequality (4), the movement according to ray (8) takes place in different directions (and similarly for (4')). The law of movement according to the ray is determined by squaring.

Note. If the moment $\vec{M}$ changes slowly with time, then all of the preceding discussions hold. Thus, in this case as well, movement takes place according to rays (8), (10).

3. Selected for the evaluation of the magnitude of the perturbing moment was a 10-day interval (10/2/77-10/11/77), in which the rotation proved to be practically axial, according to the results of determination of the orientation.

In this case, the second equation of (7) evidently gives

$$L = L_0 + M_3(t - t_0).$$

The constants L_0, M_3 were determined by the method of least squares according to "measurements" of L, which are results of the determination of the orientation in nonoverlapping 100-minute intervals. The error in the determination of L was assumed to be uncorrelated.

The obtained evaluations for M_3 and σ_{M_3} are as follows

$$\hat{M}_3 = 2{,}57 \text{ dynes}\cdot\text{cm}, \quad \hat{\sigma}_{M_3} = 3{,}8\cdot 10^{-2} \text{ dynes}\cdot\text{cm}.$$

Presented in figure 2 are the measurements involved in the processing, and the straight line obtained as a result. One can maintain that the hypothesis on the presence of a perturbing moment which is constant in fixed axes describes the actual movement well.

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the evolution of rotation of the "Prognoz" artificial earth satellites, and submitted data on the actual orientation.

REFERENCES

1. Landau, L. D., Lifshits, Ye. M., Teoreticheskaya fizika [Theoretical Physics], Vol. 1. Mechanics, Moscow, "Nauka", 1973.

2. Volosov, V. M., Morgunov, B. I., Metod osredneniya v teorii nelineynykh kolebatel'nykh sistem [The Method of Averaging in the Method of Nonlinear Oscillatory Systems], MGU Publishing House, 1971.

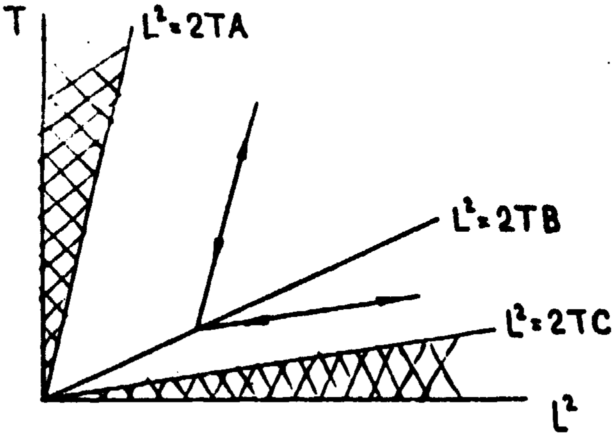

Fig. 1
The cross-hatched sections of the plane do not correspond to any p, q, τ.

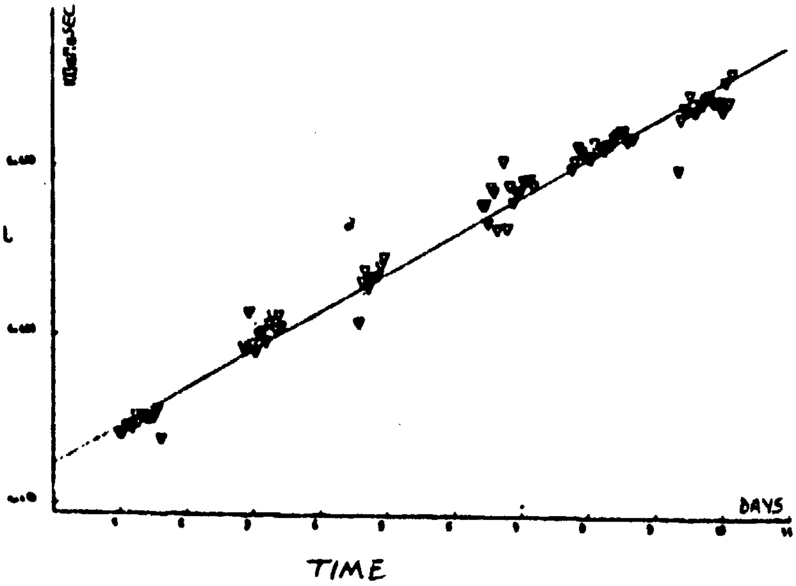

Fig. 2