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University of Texas at Austin

Department of Astronomy

Austin, Texas 78712

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William H Jefferys

William H. Jefferys
Principal Investigator

Abstract

This report summarizes all the activity under the subject grant during the period 1977 December 1 - 1979 May 31. The accomplishments during the period of the grant were to fit a first order analytical theory of Enceladus-Dione to observational data, to develop a complete theory for Mimas-Tethys, to develop new software for the calculation of the disturbing function in the case of Titan and Hyperion, and to develop the disturbing function itself for Titan and Hyperion.

APPENDIX I

THEORY OF MIMAS AND TETHYS

William H. Jefferys and Lynne M. Ries
University of Texas at Austin
Austin, Texas 78712, U.S.A.

ABSTRACT

An analytical theory of Mimas and Tethys has been developed, retaining terms which produce perturbations of the order of ± 10 km. The theory uses a novel set of variables, and was developed with the Hori-Lie algorithm, using the algebraic manipulation language TRIGMAN. the perturbations have been implemented by means of FORTRAN subroutines produced by the computer.

I. Accomplishments Under this Grant.

a) Theory of Enceladus-Dione

Some 1,428 observations of Enceladus and Dione were fitted to the first order theory of Enceladus and Dione which was developed previously under NASA Grant NSG 44-012-282. The data consist of many right ascension and declination measurements on the two objects, as well as information on orbital parameters reported by Kozai in Annals of the Tokyo Astronomical Observatory Series 2, Vol. 5, no. 2 (1957). The theory was extended to the second order of the small parameters, due to the fact that a number of significant terms, both periodic and secular, exist in the second order which are important at accuracies which could be observed from space.

b) Mimas-Tethys

The theory of Mimas and Tethys was completed, and since the last semi-annual status report, a paper on this theory has been submitted to the Astronomical Journal. The theory of Mimas and Tethys represented a substantial advance over the Enceladus and Dione theory developed earlier. A new algorithm was developed for the calculation of the disturbing function for Mimas and Tethys, which was not only more accurate, but also consumed less computer time than the algorithm used for Enceladus and Dione. In addition, a new set of variables was developed, which had significant advantages over the variables used in the Enceladus and Dione investigation. Indeed, we went back and recalculated the Enceladus-Dione theory using the new variables and found them to be advantageous in that case as well. Since the Mimas-Tethys case involves a large amplitude libration, it was necessary to develop the theory to the 16th order in the small parameter of the libration. This is to my knowledge the first time that a 16th order development using the Hori-Lie algorithm has actually been applied to a practical problem. A copy of the paper on Mimas and Tethys, which describes the theory in detail is attached as Appendix I of this report.

c) Titan-Hyperion

Since the last report, we have been concentrating on the theory of Titan and Hyperion. This theory has presented some unexpected new challenges, which have tested the TRIGMAN algebratic manipulation system to its limits. Indeed, it turned out to be impossible with TRIGMAN alone to develop the entire disturbing function to the accuracy that would be required for a practical theory. The reason for this is that in the development of the disturbing function it is necessary to go to quite high order in the eccentricity of Hyperion in order to obtain a sufficiently accurate value of the motion of longitude of pericenter of Hyperion. Just to obtain accuracies comparable to the observed value, it is necessary to go to about the fourteenth or fifteenth power in the eccentricity. The development of terms in the disturbing function with such high powers of the eccentricity involves the subtraction of large quantities to produce small quantities, with consequent roundoff error. It turned out that about the tenth power of the eccentricity, the significance of the 60 bit coefficient word in TRIGMAN was completely lost. It therefore became necessary for a different strategy to be employed for the development of the disturbing function.

The short period and intermediate period terms in the disturbing function were developed as usual using the software which was used for Encealdus-Dione and Mimas-Tethys. The resonant and secular terms, on the other hand, were developed using entirely new software which has been developed during the past six months for this purpose. This involves calculating the particular terms that are required from a numerical Fourier analysis of special values of the disturbing function. In addition, certain derivatives of the disturbing function needed to be computed in the same way. These terms are then added to the short and intermediate period disturbing function to produce the final disturbing function. In addition, since solar terms are of importance, they are added to the expression for the disturbing function.

We now have in hand a definitive expression for the disturbing function, which is to be used for the theory of Titan and Hyperion. It is our intention to complete this theory in the near future, and to publish the results. We plan to do this on an unfunded basis.

II. Publications

One paper has appeared during the tenure of this grant. It is titled "Theories of Resonant Satellite Pairs in Saturn's System" by W. H. Jefferys and L. M. Ries, and was presented at I.A.U. Symposium #81 in Tokyo, Japan May 23 - 26, 1978. A second paper has been submitted for publication, and is attached to this report. A paper on the application of the observations of Enceladus and Dione to the theory is in preparation, and one on the theory of Titan and Hyperion will be prepared after the calculations are complete. Copies of these papers will be made available to NASA as soon as they have been prepared.

III. Staff

During the lifetime of this grant, the staff consisted of W. H. Jefferys (Project Director) 3.15 man-months (1.25 man months paid from grant funds, 1.9 man months contributed time); L. Ries, 7.75 man months; N. Otto, 1.0 man months. During the final reporting period W. H. Jefferys contributed 0.6 man months, and L. Ries was paid for 2.0 man months.

I. INTRODUCTION

The near future promises greatly improved precision in the observations of the satellites of the outer planets. Jupiter and Saturn flybys will produce greatly improved data, and the ± 0.002 arcsecond astrometric precision of Space Telescope, due to be launched in 1983, will also provide data of unprecedented accuracy. The ± 0.002 arcsecond figure corresponds to an error of about 10 km at Saturn's distance.

A program of regular observations of the satellites of the outer planets has been underway at McDonald Observatory since 1973 (Mulholland, Shelus and Abbott 1976, Mulholland and Shelus 1977, Benedict, Shelus and Mulholland 1978, Mulholland, Benedict and Shelus 1979). At the same time, work on improved orbital theories of Enceladus-Dione, Mimas-Tethys and Titan-Hyperion was commenced by the present authors (Jefferys and Ries 1975; Jefferys 1976; Jefferys and Ries 1979; hereinafter denoted Papers I, II and III, respectively). It has been the goal of these theories to provide terms to somewhat greater precision than that expected from the improved observations which will soon become available.

These theories are also of interest from a mathematical point of view. All possess a deeply resonant term in libration, and there are also shallowly resonant terms. In this paper we describe the theory of Mimas and Tethys.

Since there are some important differences between this theory and the theory of Enceladus and Dione which was described in Papers I and II, it will be described in some detail.

The theory of Mimas and Tethys, like the theory of Enceladus and Dione, has been computed with the aid of the automated algebraic manipulation system TRIGMAN (Jefferys 1970, 1972), using the Hori-Lie algorithm (Hori 1966), as formulated by Campbell and Jefferys (1970). This version of the Hori-Lie method is particularly suited for automated computation. In order to carry the theory out to a sufficiently high order, however, we have found it necessary to introduce a new set of variables, which has some interesting properties. In addition, our theory has some other unique features in the way the deep resonance is handled.

II. VARIABLES

In the theory of Enceladus and Dione, we employed variables A_j, E_j, I_j (in the notation of Paper I) that are linear functions of the usual elliptic elements (semimajor axis, eccentricity and sine inclination). These had the advantage of simplicity, but led to a rapid increase in the size of series whenever Poisson brackets were calculated. This was due to the fact that the partial derivatives of the variables A_j, E_j and I_j, with respect to the Delaunay variables, had to be expanded in powers of A_j, E_j and I_j themselves. This caused a rapid proliferation of terms in even the lowest orders, leading to a problem of "intermediate swell" which made it difficult to obtain the complete solution.

In the present work, we have used modified variables which have the property that the required partial derivatives consist of only a single term. The Delaunay variables are (Paper I)

$$\begin{aligned} L_j &= \nu_j (\mu_j a_j)^{\frac{1}{2}} \\ G_j &= L_j (1 - e_j^2)^{\frac{1}{2}} \\ H_j &= G_j (1 - i_j^2)^{\frac{1}{2}} \end{aligned} \qquad (1)$$

where $j = 1,2$ for the inner and outer satellites, respectively. Dropping the subscripts for convenience in what follows, we have

$$\frac{\partial a}{\partial L} = \frac{2L}{\mu\nu^2} = \frac{2a}{L}$$

$$\frac{\partial e}{\partial L} = \frac{G^2}{eL^3}, \quad \frac{\partial e}{\partial G} = -\frac{G}{eL^2} \tag{2}$$

$$\frac{\partial i}{\partial G} = \frac{H^2}{iG^3}, \quad \frac{\partial i}{\partial H} = -\frac{H}{iG^2}$$

For $e \ll 1$ and $i \ll 1$ we define new variables a^*, e^*, and i^* which satisfy the analogous relations

$$\frac{\partial a^*}{\partial L} = \frac{2a_0}{L_0}$$

$$\frac{\partial e^*}{\partial L} = \frac{1}{e^* L_0}, \quad \frac{\partial e^*}{\partial G} = -\frac{1}{e^* L_0} \tag{3}$$

$$\frac{\partial i^*}{\partial G} = \frac{1}{i^* L_0}, \quad \frac{\partial i^*}{\partial H} = -\frac{1}{i^* L_0}$$

where a_0 is the "nominal" value of the semimajor axis and $L_0 = \nu_0 (\mu_0 a_0)^{\frac{1}{2}}$. These equations have the solution

$$a^* = 2a_0 (L/L_0 - 1)$$

$$e^* = [2(L - G)/L_0]^{\frac{1}{2}} \tag{4}$$

$$i^* = [2(G - H)/L_0]^{\frac{1}{2}}$$

In order to write down the Hamiltonian we must express a, e and i in terms of the starred variables. This leads to:

$$
\begin{aligned}
L &= L_0(1 + a^*/2a_0) \\
a &= L^2/(\mu\nu^2) \\
e &= e^*(L_0/L)^{\frac{1}{2}} \; [1 - e^{*2} L_0/4L]^{\frac{1}{2}} \\
G &= L(1 - e^2)^{\frac{1}{2}} \\
i &= i^*(L_0/G)^{\frac{1}{2}} \; [1 - i^{*2} L_0/4G]^{\frac{1}{2}} \\
H &= G(1 - i^2)^{\frac{1}{2}}
\end{aligned}
\tag{5}
$$

For the calculation, we employ variables A^*, E^*, and I^* which are related to a^*, e^* and i^* through the equations

$$
\begin{aligned}
a^* &= a_0 + f_A A^* \\
e^* &= f_E E^* \\
i^* &= f_I I^*
\end{aligned}
\tag{6}
$$

in a manner similar to that used in Paper I. The Hamiltonian and all other quantities are expanded in powers of A^*, E^* and I^*, and the truncation parameters f_A, f_E and f_I are used, as in Paper I, to control truncation of small terms.

Our experience with these modified variables has been very gratifying. By cutting down on the amount of intermediate swell, the required computer time and field length have been considerably reduced, and the final results are quite manageable.

III. THE DISTURBING FUNCTION

Our method of calculating the disturbing function has also been modified from the method used in Paper I. In that work, an iterative scheme using the Newton-Raphson method was used to develop the quantity $1/\Delta$ (the reciprocal of the distance between the two satellites). In the present work, it was decided that an approach using Laplace coefficients would be superior. Developing r_1, r_2 and $\cos S$ as in Paper I, we write

$$\begin{aligned}
\Delta^2 &= r_1^2 + r_2^2 - 2r_1r_2\cos S \\
\Delta_0^2 &= a_1^2 + a_2^2 - 2a_1a_2\cos(\lambda_2-\lambda_1) \\
\Delta^2 &= \Delta_0^2 + \delta
\end{aligned} \tag{7}$$

and then expand $1/\Delta$ in powers of δ, using Laplace coefficients to obtain the required powers of $1/\Delta_0$. We have found that this method is not only faster than our original method, but also more accurate.

IV. NONRESONANT PERTURBATIONS

There are some surprisingly large nonresonant perturbations in the theory of Mimas and Tethys which appear not to have been computed before. For example, the argument $(l_1 + 2g_1 + 2h_1 - 2l_2 - 2g_2 - 2h_2)$, which is responsible for the deep resonance in the case of Enceladus and Dione, has a coefficient of over 8 minutes of arc in the theory of Mimas and Tethys, which produces perturbations in the position of Mimas of over 400 km. In addition, there are some smaller perturbations in twice this argument, and perturbations in l_1 which are due to the oblateness. To be sure, the principal perturbations in the Mimas-Tethys theory, due to the deep resonance, are much larger, but these other terms are large enough to be observed, possibly even from the ground.

In our work on Enceladus and Dione, it was pointed out that TRIGMAN, which is a Poisson series processor, is unable to handle literal divisors, which necessitated a fairly complex scheme for expanding the divisors in powers of small quantities. This expansion was necessary, as was pointed out in Paper I, in order to calculate all of the perturbations correctly. In this paper we have adopted a similar approach, but have streamlined it and made it more systematic. The final results of our new approach are the same as if we had employed the method of Paper I, but the method is easier to carry out, and has the additional advantage that it postpones to higher orders the terms which contain high powers of the expansion parameters.

The method we have employed in the present work is to adopt as the zero order Hamiltonian a function which, when differentiated with respect to the Delaunay variables, yields the numerical values of the corresponding mean motions, rather than a series expansion. The original Hamiltonian for the problem is then partitioned (according to powers of the small quantities) among first, second, third, and higher order terms, and the zero-order Hamiltonian subtracted from the first order. Thus, if the original Hamiltonian of the problem is $F = F_0 + F_1 + F_2 + \ldots$, and if F'_0 is a Hamiltonian which reproduces the numerical values of the mean motions, then the modified Hamiltonian that we use is given by

$$
\begin{aligned}
&\text{Order 0:} \quad F'_0 = F'_0 \\
&\text{Order 1:} \quad F'_1 = F_0 + F_1 - F'_0 \\
&\text{Order 2:} \quad F'_2 = F_2 \\
&\text{Order 3:} \quad F'_3 = F_3 \\
&\qquad\qquad\quad \cdots\cdot
\end{aligned}
\tag{8}
$$

Since the modified Hamiltonian is actually equal to the original Hamiltonian when all the terms are added up, differing only in the way it is partitioned among the various orders, the two systems must have the same solution. However, for the original Hamiltonian, the equation for the elimination of the kth order periodic terms by the Hori-Lie method takes the form

$$F_k + \{F_0, W_k\} = F_k^*$$

$$\text{where} \quad \{F_0, W_k\} = \Sigma\, n_j \frac{\partial W_k}{\partial y_j} \tag{9}$$

(in the notation of Paper I), where the mean motions n_j are functions; while for the modified Hamiltonian, the equation becomes

$$F'_k + \{F'_0, W'_k\} = F'^*_k$$

$$\text{where} \quad \{F'_0, W'_k\} = \Sigma\, n'_j \frac{\partial W'_k}{\partial y_j} \tag{10}$$

where the n'_j are the nominal values of the mean motions. As a result, with the modified equations, the divisors of the trigonometric terms are always numbers, which the Poisson series processor can handle, rather than expressions.

The only drawback to this scheme is that in order to obtain the solution to a given order (compared to the solution of the original Hamiltonian), it is necessary to carry the solution using the modified Hamiltonian one order higher. This turns out to be a small price to pay, as the series grow more slowly, and new terms are introduced into the determining function only as they are needed. The algorithm for performing a Hori-Lie transformation to arbitrarily high order is actually quite short when the method of Campbell and Jefferys (1970) is used - it takes only about a half a page of TRIGMAN code.

V. RESONANT PERTURBATIONS

After the elimination of the nonresonant part of the Hamiltonian, the method described in Paper II is used to handle the resonant perturbations. The basic approach is outlined in Paper III, and we develop it here in greater detail.

The critical argument in the Mimas-Tethys case is $2l_1 + 2g_1 + 3h_1 - 4l_2 - 4g_2 - 3h_2$, which is replaced by an angular argument, say θ. Conjugate to it, a new action variable, Θ, is introduced by the method of Paper II. The Hamiltonian is then expanded in powers of Θ to a sufficiently high order - in practice, only a few terms are actually needed, as Θ is very small. The resulting Hamiltonian has the form

$$
\begin{aligned}
F &= (A\Theta^2 - 2B\cos\theta)/2 + n_\theta \Theta + \ldots \\
&= (A\Theta^2 + B\theta^2)/2 + n_\theta \Theta + \ldots,
\end{aligned}
\tag{11}
$$

where A and B are functions of the other variables and n_θ, the mean motion of the critical argument, is identically zero. The condition $n_\theta = 0$ is in fact a <u>constraint</u> which must be satisfied in the final solution. It is one of two constraints which must be imposed to account for the two additional variables, θ and Θ , which have been introduced to handle the resonance. (The other constraint is equivalent to the condition that the mean value of θ must equal zero.)

In the second of Eqs. (11), we have shown the cosine expanded in powers of θ , and the lowest order term retained. In the theory of Enceladus and Dione, where the amplitude of the libration is small, such an expansion is quite adequate, and the Hamiltonian becomes that of a harmonic oscillator. In the theory of Mimas and Tethys, this is no longer true. The amplitude of the libration is very large, and many terms must be taken in the expansion of the cosine in order to attain sufficient accuracy.

The usual method of treating the Mimas-Tethys resonance is, in fact, to use elliptic functions. However, TRIGMAN is not able to handle elliptic functions, and we have chosen a different approach, that of treating the problem as a perturbed harmonic oscillator (that is, perturbed by the higher order terms in the expansion of the cosine).

We replace θ and Θ by a new canonically conjugate pair of variables p and q through the equations

$$\Theta = (2p\gamma)^{\frac{1}{2}} \cos q.$$

$$\theta = (2p/\gamma)^{\frac{1}{2}} \sin q, \qquad (12)$$

where $\gamma = (\bar{B}/\bar{A})^{\frac{1}{2}}$, and the bars indicate that the functions A and B have been evaluated at the nominal values of their parameters. The Hamiltonian now takes the form

$$\begin{aligned} F = {} & p[A(\bar{B}/\bar{A})^{\frac{1}{2}} + B(\bar{A}/\bar{B})^{\frac{1}{2}}]/2 \\ & + n_\theta (2p\gamma)^{\frac{1}{2}} \cos q \\ & + p[A(\bar{B}/\bar{A})^{\frac{1}{2}} - B(\bar{A}/\bar{B})^{\frac{1}{2}}] (\cos 2q)/2 \\ & + \text{terms of order } p^{3/2} \text{ and higher.} \end{aligned} \tag{13}$$

Notice that the first term, at the nominal values of A and B, is equal to $(\bar{A}\bar{B})^{\frac{1}{2}}$, which is the nominal value of the mean motion of the libration for small amplitudes. If it is evaluated away from the nominal values, the mean motion will, of course be different. In fact, the coefficient of p in the first term of Equation (13) consists of the leading terms of the expansion of $(AB)^{\frac{1}{2}}$, which is the actual mean motion of the small-amplitude librating argument about its nominal value $(\bar{A}\bar{B})^{\frac{1}{2}}$. The second term of Equation (13) is the one from which the principal part of the libration in longitude arises. As explained in Paper III, although the factor n_θ vanishes because of the constraint condition discussed above, its partial derivatives do not vanish, so that this term, and any term factored by a <u>single</u> power of n_θ, will give rise to nonvanishing perturbations in the angular variables.

The third term of Equation (13) also is factored by a term that vanishes, at least on the nominal orbit. This term can also give rise to perturbations in the angular variables, although they are much smaller than those arising from the second term.

The Hamiltonian must now be put into a form which is convenient for the elimination of the angular argument q by means of the Hori-Lie method. As with the nonresonant perturbations, we take for the zero-order Hamiltonian a term which yields the numerical value of the mean motion of the libration argument. The Hamiltonian is then partitioned among first, second, and higher orders as before, using the power of the variable p to determine the order of a term. The Hori-Lie algorithm is then applied, and the argument q eliminated from the Hamiltonian to the desired order.

Because of the amplitude of the libration, we have found it necessary to carry the solution out to the sixteenth power of $(p)^{\frac{1}{2}}$. However, this should not be considered quite the same as a sixteenth order theory. What we are actually doing is nothing more than expanding the elliptic functions in trigonometric series. The resulting determining function includes terms through the argument 7q, and powers of p up through the eighth.

VI. PERTURBATIONS IN COORDINATES

In order to apply the theory, it is neccessary to compute the perturbations in the observed coordinates. With the Hori-Lie method, this is very easy to do for any arbitrary function of the variables. The resonant and nonresonant determining functions are expressed in terms of transformed variables corresponding to the action-type variables A^*, E^*, I^* and p, and the angular variables l , g , h and q , from which the short and long period terms have been eliminated. The transformed action-type variables, together with the zero points of the transformed angular variables, are the constants of integration of the theory. They are close to the original variables A^*, E^* and I^* in terms of which the Hamiltonian was originally expressed.

To obtain perturbations in any other quantity, such as the radius vector or the longitude, it is only neccessary to express the desired quantity in terms of the variables A^*, E^* and I^*, and the angular variables l , g and h . One then applies to this expression the Hori-Lie algorithm with the nonresonant determining function to obtain the nonresonant perturbations. This function is then expanded in powers of Θ , which in turn is expressed in terms of p and q by means of Equation (12). (The latter operation generally adds few, if any terms to the series, since Θ is so small.)

The final step is to take into account the effect of the resonance by applying the Hori-Lie algorithm to this series, using the resonant determining function. The result is a series in the twice-averaged variables (which are constants and linear functions of the time). This operation can be carried out for any desired quantity which might be observed. We have done it for such observables as the radius vector, mean and true longitude, and latitude, as well as quantities such as the semimajor axis, inclination and eccentricity. Perturbations in virtually any desired quantity can be obtained easily and efficiently with our programs. In addition, partial derivatives in any quantity, with respect to the constants of the theory, are easy to provide.

It should be noticed that it is possible for "mixed" terms involving both the libration argument q as well as other angular arguments to appear in the solution. We find such terms in the present theory, and some of them are of significant size. For example, in the longitude of Mimas there is a term with an amplitude of about 3 minutes of arc involving both q and the shallow resonance $(l_1 + 2g_1 + 2h_1 - 2l_2 - 2g_2 - 2h_2)$.

The theory is available in the form of FORTRAN subroutines, automatically written by the computer, which are capable of calculating the perturbations in any of the functions mentioned above, such as longitude, latitude, radius vector, etc. Subroutines to calculate the partial derivatives of each of these functions with respect to the constants of integration are also provided for the purposes of differential correction. It would not be difficult to provide such subroutines for any other appropriate function if that turns out to be neccessary.

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