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MATHEMATIC MODELING OF THE EARTH'S SURFACE AND THE PROCESS OF REMOTE SENSING

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It is shown that real data from remote sensing of the earth from outer space are not best suited to the search for optimal procedures with which to process such data. To work out the procedures we propose to use data synthesized with the help of mathematical modeling. Subject to the condition that these data are similar to the real data, the processing procedure worked out for them will also work well in practice. A criterion for similarity to reality is formulated. It is shown that a simple method for staisfying it exists. The basic principles for constructing methods for modeling the data from remote sensing are recommedded. A concrete method is worked out for modeling a complete cycle of radiation transformations in remote sensing. The crux of this method is a suitable procedure for modeling a multi-dimensional random field. We describe a computer program which realizes the proposed method. Some results from calculations are presented which show that the method satisfies the requirements imposed on it. The required statistical characteristics of the terrestrial surface and the radiation transformations are assigned to the program and it produces a frame for remote sensing which is ready for processing.

/2 *

INTRODUCTION

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The data from remote sensing are not the best raw material for experiments. In remote sensing (RS) the sole observable quantity, from which information can be extracted concerning the condition of the terrestrial suface, is the reflectance of the surface. (Here we do not consider natural radiation and polarization, since taking them into account changes nothing in principle.) As is well-known, reflectance depends on a multiplicity of factors:

*Numbers in margins indicate pagination in original foreign text.

soil, the vegetation phase of flora, humidity, etc, These factors are extremely variable, are often difficult to measure directly, and are completely uncontrollable. It proves to be impossible to obtain two images of parts of the surface for which these factors are identical, or in which only one of them is different. Cf. [1, 2, 3]. Therefore we cannot study the influence of individual factors on the quality with which the frame for some precedure is processed, and the investigator turns out to be in the position of a person who attempts to evaluate the voice of a singer by listening to his singing in a chorus. Due to the difference in the original data it is impossible to compare the results of processing obtained by differing groups of investigators. Cf. [1, 4]. Because of the large number of factors it has not been possible to find the reasons for the difference in the behavior of the processing procedures for various regions [5, 6, 7].

Note that in the exact sciences, for example, in physics, the ability to repeat, and the strictness with which conditions are controlled, are unalterable requirements for an experiment. If a physicist were asked to work in a situation in which the characteristics of the patterns (frequency) and the conditions of an experiment (temperature, magnetic field) do not depend on him, cannot be reproduced at will, and are rapidly changing, he would doubtless regard such conditions as unacceptable. This is precisely the situation when we have occasion to work with remote sensing. /4

Certain methods of processing: [5, 8, 9, 10, 11] are used for training in the procedures for recognizing a priori information from regions which are selected as standard for an entire frame. This information is adequate only for a certain neighborhood of the standard regions. In such methods it is absolutely necessary to estimate the influence of the degree of adequacy and the completeness of the a priori information on the quality of the work [1]. But in order to do this, it is necessary to have a priori information from the entire frame, which is impossible in practice.

The traditional methods for working out the procedures in RS are reminescent of an attempt to learn English or French not from textbooks, but from the works of Joyce or Proust. Even if it is necessary to read precisely these authors, it is not necessary to learn the language from these works. And in RS, although it is necessary to process the real data, it is better to learn processing from some type of textbook. In the present paper we also propose a method for creating such textbooks. We refer to mathematical modeling of data, so that the results resemble the real data in their basic features, but have controlling mathematical characteristics. After checking out the procedures on synthesized data it will be possible to carry out a "precise adjustment" to the real data.

Modeling of data. Advantages. A Quality criterion. Replacing natural data by artificial data makes it possible to change some characteristics of the frame being processed without touching the others. Moreover, precise information about the entire frame is available to the experimenter. However, we must ensure a connection between the results of testing some processing procedure and reality. Indeed, in typical models of signals and their representations, the data is obtained in analytical form (e.g., /5
additive Gaussian noise is chosen) and analytical evaluations of quality are obtained, but they do not guarantee quality under real conditions. See [1, 12, 14]. It is clear that a criterion is necessary for the quality of the modeling which ensures convergence of the process of optimization for the procedure to a procedure that is optimal for the real data. Such a criterion is formulated in the following section.

The author knows of only one paper on modeling in RS. This is [13] in which the change in the radar image of a locality is simulated when the observation angle and the form of the projection are changed. The basic purpose of [13] was to detect parts of the locality which were concealed from the beam from a radar station. The reflection coefficient at each point is assumed given.

Therefore [13] corresponds to one unit of the given method: modeling the process of scanning and conversion of radiation into a signal. The possibilities for applying modeling. The various applications of artificial data can be divided into 4 groups. In the first place, it is possible to perserve unchanged the modeled frame and to select the optimal paramters of the photography (height of the flight, size of the frame, settling on the locality, the height of the sun, etc.). This might also be done with real data but for the impossibility of keeping the landscape unchanged while it is being photographed several times in different ways. Secondly, it is possible to change the properties of the noise (static, the noise of the receiver) and to determine the domain of effectiveness of the processing procedure. In the third place and this is the most interesting group of applications, it is possible to change the set of objects on the terrestrial surface and their characteristics: humidity, soil type, contamination, etc., and for each set of characteristics to compare the various processing procedures. Then we shall clarify to which changes in the framework under observation the quality of the processing is most sensitive. Moreover, we shall be able to decompose the region of changed characteristics into subregions, inside of each of which some precedure is optimal. Knowledge of such a decomposition makes it possible in practice to choose the most suitable processing procedure. In the fourth place, since the entire a priori /6
information is at our disposal, we can change its amount from 0% to 100% and can clarify which minimum is necessary to ensure an acceptable quality of work. This makes it possible to determine the structure of the ground measurements and the data bank.

In the general case it is possible to investigate some procedure with respect to all four directions. This makes it possible to check it out in the laboratory, so that in the case of real work it is only necessary to do a little polishing, which can be carried out by the method of self-instruction. Clearly, it is impossible to undertake the investigations enumerated with respect to real data. Up to now such attempts have only lead to the accumulation

of piles of useless information. However, as is clear from what has been said, the use of the methods of modeling might place the processing of RS data on a more or less rigorous basis.

The method proposed in the present paper makes it possible to carry out all the indicated investigations.

1. THE PRINCIPLES OF MODELING IN RS

The transition from recognition to measurement. For the time being, the processing of data in RS is limited basically to recognition of ground objects and the construction of thematic maps. The search for effective recognition procedures is incomplete (to a significant degree due to the problems enumerated above), but the role of RS is not restricted to recognition. The basic purpose of RS is the measurement of the most important characteristics of objects (productivity, pollution, etc.), since this is most important economically. The measurement must be sufficiently precise so as to be competitive with the traditional methods for obtaining the same information. There are still no methods for evaluating the quality of measurement procedures, not the procedures themselves, but assuming that there is a future for them, we have oriented the modeling twward them. The ideology of the parameters which is proposed in the following section serves as a basis for modeling the characteristics being measured. It seems that in this or the /7
other form is should inevitably appear, even in the measurement procedures themselves. The description of objects with the help of the parameters. Presently the frames in RS are simply described by the luminosity at each point, i.e., directly by an observable quantity. Such a phenomenological description is characteristic of all sciences in their early stages. But it would be practically impossible to study, e.g., hydrodynamics or the molecular theory of heat if we were only able to describe motion phenomenologically, going into all the details of the complicated changes in positions. Fortunately the characteristics of bodies which determine their motion uniquely (mass, velocity, acceleration, external force) are known.

A quite similar conversion is also possible for RS. It is well-known that humidity, the phase of vegetation and the soil strongly influence the reflectivity of agricultural crops, and that depth, pollution, etc., affects the reflectivity of reservoirs, [14, 15]. We may assume that the specification of a sufficiently large set of such characteristics ("parameters") with great precision determines its reflectivity. (For brevity we shall call reflectivity albedo, although it depends on wave length and the observation angle.) Once having measured the dependence of the albedo of objects on their parameters, it is then possible by describing the frame in the language of the parameters to recover its observable appearance. Of course, for different objects, the parameters are different.

It is necessary to impose two requirements on the parameters. First, they must uniquely determine (within the limits of the specified precision) the albedo of an object. Otherwise, the description of the frame with their help will not be complete. Second, they must be intrinsic, i.e., they must correspond to the characteristics generally accepted in geography, the agricultural sciences, etc. Then it will be possible to use the information about the parameters accumulated by these sciences.

The quantities which RS serves to measure: productivity, pollution, etc., affect the albedo (otherwise it would be impossible to measure them), and this means that they fall into the category of parameters. A parametric representation makes it possible to introduce these quantities into the processing in explicit form and to describe the framework as a system of stochastic equations relative to the parameters. The quality of a processing procedure is naturally characterized by the precision with which the parameters are measured. Note that the paameters are spectrally invariant, and therefore the values of the intensity in different channels are easily associated in terms of them. /8

It seems that a parametric description arises naturally in RS. The proposed method is entirely based on such an ideology.

The statistical approach. In RS the statistical treatment of data now predominates. Accordingly we shall assume that the treatment procedures produce the statistical characteristics of the parameters being measured, e.g., their histograms of the distribution of probabilities. Experiments with the method of modeling consits in changing the desired statistical characteristics of the parameters and transformations of radiation (the "input data" of the method) and recording the corresponding changes in the output of the processing procedures. The criteria for the similarity of the modeled data to reality also have to be formulated in statistical terms.

Specific criteria for the quality of the modeling. We shall call the precision with which the statistical characteristics of the parameters being measured are recovered with the aid of the processing procedure the quality of the procedure. To model a frame the necessrry input information is: the desired statistical characteristics of the parameters, etc. The synthesized data corresponding to some real region of the surface are data obtained by modeling in accordance with the input information gathered in this region. Naturally the modeling is considered successful if the quality of the processing procedure is considered identical for both the real and the corresponding modeled data. Now we shall formulate the criterion for the quality of the modeling.

Let U be a region of the surface. Let us photograph it from /9
outer space and process the photograph by some procedure T. The procedure will provide histograms of the probability distributions of the parameters i: $P_1^i = T(U)$. Let us collect on U the input data for the modeling U_s and with respect to them and with the help of the method M, let us model the frame $W = M(U_s)$. Processing this frame with the same procedure T will give some histograms P_2^i. Now let us compare P_1^i and P_2^i and ascertain whether they reflect the same (i.e., the "true") distribution of the probabilities. For this purpose numerous criteria have been developed (the sign tests of Wilcoxson, Kolmogorov, etc.). When these criteria are satisfied

for all i or for some function of particular criteria, we shall consider that the quality criterion for the modeling is satisfied for the procedure T and the frame of U.

Two defects remain: it is necessary to verify the criterion anew for each new procedure and for all input data, while drawing on the corresponding real frames (and where do they come from?). It is desirable to verify the method only once, even if this involves a complex method, so that this verification ensures satisfaction of the quality criterion for sufficiently broad sets of processing procedures and input data. We shall show that this is possible.

We shall assume that the description of the region of the terrestrial surface U and the transformations of the radiation include a frame observable from outer space and a priori informatinn about the parameters of ground objects and the atmosphere, and also the laws for transforming the parameters into the observed radiation intensity. This description (which possesses some redundancy) generates a space of frames A. It is possible, for example, to specify the parameters and the intensity values of the radiation in all the spectral channels, for each point of the frame A is compact, i.e., the number of points is finite and the values of the radiation intensity and the parameters are bounded.

Let us introduce a metric d into A. As is well-known, any compact set can be covered by an ε-network with a finite number of points U_i such that for any $U \in A$ there exists a U_i so that

$$d(U_i, U) \leq \varepsilon \qquad (1.1)$$ /10

Note that the criteria for the agreement of the distributions, which are based on quadratic functions, possess metric properties.

Since continuous transformations do not affect compactness, we may choose any description of the frame from a broad class which preserves compactness. We shall select the description which has the most convenient ε-network. We shall denote the set $\{U_i\}$ by

S and we shall use it as input data for modeling. We shall describe the method of modeling by the operator M which maps S onto A. The operator M is stochastic, but here we shall require that each of its realizations satisfy the quality criterion. The result of processing is the subset S, and to the procedure of data processing corresponds the operator T, from A in S. An illustration of the structure described is given in Figure 1 (p. 27).

The criterion introduced above for the frames U and W with the significance level α is described by the condition

$$d[T(U), T(W)] \leq \alpha, \quad W = M(U_S). \tag{1.2}$$

U_S is the projection of U onto S, i.e., input data collected on the frame U and analogously for W_S. Let $d(W_S, U_S) = \beta$. Let us impose on the procedure T the Lipschitz condition relative to A with constant K (which can be evaluated analytically), uniform on the class of the procedures under consideration:

$$d(T(U_1), T(U_2)) \leq K d(U_1, U_2). \tag{1.3}$$

This means that the procedure does not yield strongly differing results for more or less close frames. Then

$$d[T(U), T(W)] \leq K d(U, W) \leq K[d(U, U_S) + d(U_S, W_S) + d(W_S, W)] \leq K(2\varepsilon + \beta). \tag{1.4}$$

It is clear that if α is given, then provided that ε is chosen sufficiently small, i.e., the volume of input data is sufficiently large, so that on the class of the procedures used $2K\varepsilon < \alpha$, we obtain a sufficient condition for the satisfaction of the quality criterion in the form $K(2\varepsilon + \beta) \leq \alpha$. That is, for a given class of procedures and a threshold α we select a volume of input data, after which the quality criterion reduces to the condition /11

$$\beta \leq \frac{\alpha}{K} - 2\varepsilon, \tag{1.5}$$

in other words, to testing the precision of the reproduction of

the input data in the modeled framework. This test is purely mathematical and does not require real data. The latter are used only in the selection of ε.

Thus we have shown that it is possible to simplify the criterion. This is not a rigorous proof, but rather a proof scheme.

Some practical questions. As a priori input data for modeling we have selected the following.

1. The marginal density of the probability of the parameters for ground objects $p_{a^j_i}(x)$ where a^j_i is the ith parameter of the jth object. The coefficients of the correlation between the different parameters at one point of the surface $r^j_{i_1,i_2}$ and the homogeneous, isotropic functions of the correlation between the values of a single parameter at different points of the surface (x_1, y_1) and (x_2, y_2): $r^j_i\left[\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right]$.

2. The analogous densities of the probability and the correlations for the parameters of the atmosphere: $p^\ell_K(x)$, $r^\ell_{K_1,K_2}$ and $r^\ell_K\left[\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right]$. Here K indexes the parameters of the atmosphere in the spectral channel ℓ.

3. It is necessary to measure, and once for all build into the method, the functions which describe the dependence of the spectral reflection factor and the indicatrices of scattering on the parameters for all ground objects, and likewise for the dependence of scattering and absorption in the atmosphere on its parameters.

4. The remaining input data (the characteristics of scanning, of the receivers of radiation, the general location of objects) are less important.

These input data were chosen because they are already being measured in different sciences, and to measure any others is difficult in practice. Starting from an experiment it was assumed that such data form a sufficiently dense ε-network on the set of /12

frameworks. Of course, the quantity ε must be determined experimentally, and it is limited by the attainable quality of the modeling.

Note that the criterion for β is only a sufficient condition. Within its scope, it is possible to improve the agreement with reality, proceeding from empiricial considerations. We shall assume that the most important objective is to model precisely the densities $P_{a^{j_i}}(x)$ and less importantly, the correlations $r^{j}_{i}\left[\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right]$, and then $P^{i}_{x}(x)$ for the atmosphere, and finally, the correlations of the parameters of the atmosphere. The data in group 3 are assumed to be precise (this depends only on the quality of the ground measurements). This ranking in terms of importance proves to be essential.

The proposed method possesses a definite generality, but basically it is oriented toward photography of agricultural crops. This is the most economically effective application of RS. In geographical problems, for example, there exist geometrical structures to which statistical ideology is not applicable.

In the following two sections we describe a concrete realization of the method of modeling based on the concepts discussed above.

2. A DESCRIPTION OF THE METHOD

The basic steps. The method consists of the following basic steps.

1. Modeling a plausible arrangement of the objects or reading it out from an external lay-out.

2. Modeling the values of the parameters at each point. This is a random field with characteristics which are given in the input data.

3. Transformation of the parameters into the albedo in accordance with given functions. Modeling the field of the radiation

which is incident on the earth, and transforming it into the field of the escaping radiation with regard to the albedo.

4. Modeling the field of the atmospheric parameters. The corresponding transformation at each point of the frame. /13

5. The imitation of scanning and the transformation of radiation into the RS singal.

A more detailed block diagram of the method is presented in Figure 2 (p. 28). Let us discuss in the order of importance (steps 2, 4, 3, 5, 1) the basic problems and the solutions.

Modeling the parameters: requirements. The random field of the parameters a^j_i where $j = 1, \ldots, M$ indexes the objects, and $i = 1, \ldots, N_j$, the parameters is multidimensional. We shall require that the field have given marginal distributions of the probabilities $P_{a^j_i}(x)$ and have given correlations $r^j_{i_1, i_2}$ and $r^j_i\left[\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right]$. Here restrictions must be imposed on the form of the distributions.

This requirement must also be satisfied in case it is necessary to confer previously assigned values on certain parameters at certain points. Indeed, such a parameter as the height of a locality can be completely fixed in advance. Moreover, we may fix a part of the parameters, and alter the other part. Finally, when investigating the influence of standard information it is possible to fix the parameters on the standard region and alter them on the remaining territory.

In practice the field of the parameters is not stationary, since their statistical characteristics have drift along the framework. This is important since drift limits the maximum size of a region which can be processed in one reception and determines the density of the network of points in which it is necessary to collect a priori ground information for the processing procedures. Therefore

it is necessary to model a nonstationary field also.

Finally, the quickness in the response of this step is also important. It is potentially the most voluminous, since at each point it is necessary to compute the correlations of the parameters with respect to its neighborhood (possibly, quite large).

In toto we make noticeably more stringent demands on a random field than usual [16, 17]. We propose that the modeling procedure satisfy them.

Modeling the parameters: realization. If N_j is the number of parameters a^j_i for an object j, we shall select $N = \max(N_j)$ and we shall model an N-dimensional random process. For points with /14
different objects its components correspond in general to different parameters, but this complication is compensated for by the number of components from $\sum_j N_j$ to $\max(N_j)$ which provides great computational economy. In order that it may be possible to work effectively with arbitrary $P_{a^j_i}(x)$ let us perform the following transformation. In place of the desired parameters let us model the field of the normally (0, 1) distributed random quantities ξ^j_i. Let us denote their probability distribution functions by $G_{\xi^j_i}(x)$, and the correlations by $\rho^j_{i_1, i_2}$ and $\rho^j_i\left[\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right]$. Then we note that the transformation

$$a^j_i = F^{-1}_{a^j_i}\left[G_{\xi^j_i}(\xi^j_i)\right] \quad \text{where } F_{a^j_i}(x) = \int_{-\infty}^{x} P_{a^j_i}(x)\,dx \,. \tag{2.1}$$

yields precisely the desired distribution of the probabilities $P_{a^j_i}(x)$. The correlations r are found from

$$r = \frac{M[a^j_{i_1} \cdot a^j_{i_2}] - M[a^j_{i_1}]\,M[a^j_{i_2}]}{\sqrt{\mathcal{D}[a^j_{i_1}]\,\mathcal{D}[a^j_{i_2}]}} \tag{2.2}$$

where M and $\mathcal{D}$ are the mathematical expectation and dispersion operators, respectively, and

$$M[a^j_{i_1} a^j_{i_2}] = \frac{1}{2\pi\sqrt{1-\rho^2}} \int\int_{-\infty}^{\infty} F^{-1}_{a^j_{i_1}}[G_{\xi^j_{i_1}}(\nu)] \cdot F^{-1}_{a^j_{i_2}}[G_{\xi^j_{i_2}}(\mu)] e^{-\frac{\nu^2 - 2\rho\nu\mu + \mu^2}{2(1-\rho^2)}} d\mu d\nu. \quad (2.3)$$

Thus we obtain a convenient modeling scheme.

1. Given $f_{a^j_i}(x)$ to find $M[a^j_i]$ and $\mathcal{D}[a^j_i]$.

2. Given r to solve the system 2.2, 2.3 for ρ.

3, To model the field with Gaussian (0, 1) joint porbability density and given correlations ρ.

4. To carry out at each point of the field the inverse of the transformation 2.1.

The laborious steps 2.2 and 2.3 are performed only once, independent of the number of points in the frame, but the modeling at each point of the Gaussian quantities is carried out quite rapidly. For the $F_{a^j_i}(x)$ there are standard approximations and typical forms, for which step 4 (i.e., 2.1) also takes up an acceptable amount of time.

Generally speaking, equations 2.2 and 2.3 may not have solutions. But if the distributions $F_{a^j_i}(x)$ are not too far removed from normal distributions, or if the correlations are not too close to unity, this risk does not threaten. Cf. [17]. /15

In such a scheme it is easy to take account of a nonstationary condition. For this purpose transformation 2.1 is carried out with a new $F_{a^j_i}$, taking account of drift. It is true that ρ was computed only for a single $F_{a^j_i}$, so that the r obtained by the inverse transformation will have correlations somewhat different from the given one. But, as is clear from 2.2 and 2.3, the error has the same order of smallness as the nonstandardness of $F_{a^j_i}$; thus it can be reduced to an arbitrary value of the frame processing "in the squares". Moreover, we have arranged the

characteristics with respect to importance and we shall give preference to precise reproduction of the drift of the distributions over precise reproduction of the correlations.

There remains the problem of fixing certain values of a^j_i. We shall solve it by a method which in general significantly increases the flexibility of the modeling for a random field. We propose to replace the traditional raffle of all N components of the process at once at each point by a successive modeling of conditional distributions. In fact,

$$P(\xi^j_1,\dots,\xi^j_N|S)=P(\xi^j_1|S)\cdot P(\xi^j_2|\xi^j_1,S)\cdot\dots\cdot P(\xi^j_N|\xi^j_1,\dots,\xi^j_{N-1},S)\ , \quad (2.4)$$

where S denotes the values $\xi^j_1,\dots,\xi^j_N$ in the neighborhood of the given point. If certain values ξ^j_i are fixed, we must simply bypass them when modeling, and the given form $P(\xi^j_1,\dots,\xi^j_N)$ will be maintained automatically, In the neighborhood of ξ^j_i only those points are included which contain the very same object as the neighborhood under consideration. Note that some hierarchy is inherent in the parameters (the topography depends on nothing, the soil depends only on the topography, etc.) which the successive modeling approaches.

The key feature in the sequential scheme is the possibility of calculating in advance the form of the dependence of $P(\xi^j_K|\xi^j_1,\dots,\xi^j_{K-1},S)$ on S and $\xi^j_1,\dots,\xi^j_{K-1}$. This is impossible for arbitrary distributions. But in the present case the joint distribution, /16
and therefore all the conditional distributions ξ^j_i are normal. Then, in accordance with [18], for the jointly normal (0, 1) $\{x_i\}$, $P(x_K|x_1,\dots,x_{K-1})$ is normal with

$$M[x_K]=-\sum_{l=1}^{K-1}\frac{\Lambda_{Kl}}{\Lambda_{KK}}x_l\ ;\ \mathcal{D}[x_K]=\frac{\Lambda}{\Lambda_{KK}}\ , \quad (2.5)$$

where Λ is the determinant, and Λ_{Ki}, the cofactor of the element λ_{Kl} in the matrix of the second moments of the joint distribution $\|\Lambda\|$. That is, in the distributions 2.4 the dispersion is known,

and the calculation of the mathematical expectation reduces to a summation with given weights with respect to $\xi^j_1, \ldots, \xi^j_{K-1}$ and S, which is extremely simple. Therefore the combination of the transformation 2.1 with a sequential scheme is successful.

Next between stages 2 and 3 in the modeling of a random field it is necessary to calculate $\Lambda_{K\ell}$. For ξ^j_K the columns of $\|\Lambda\|$ correspond a) to the quantity ξ^j_K itself; b) to $\xi^j_1, \ldots, \xi^j_{K-1}$ at the very same point; c) to ξ^j_K in the neighborhood of S; d) to $\xi^j_1, \ldots, \xi^j_{K-1}$ in S. The computation of $\Lambda_{K\ell}$ reduces to the inversion of $\|\Lambda\| : \Lambda_{K\ell} = \lambda^{-1}_{K\ell} \in \|\Lambda\|^{-1}$. The inversion of a large matrix is quite laborious. But it is logical to assume that the parameters of the group b) are connected to c) only indirectly in terms of a). In the general case, for three groups of quantities Y_1, Y_2, Y_3 the condition for Y_1 to depend on Y_3 in terms of Y_2 only is written as

$$P(Y_1, Y_3 | Y_2) = P(Y_1 | Y_2) \cdot P(Y_3 | Y_2). \qquad (2.6)$$

Then according to the theorem of Bayes

$$P(Y_1 | Y_2, Y_3) = \frac{P(Y_1, Y_2, Y_3)}{P(Y_2, Y_3)} = \frac{P(Y_1 Y_3 | Y_2) P(Y_2)}{P(Y_2, Y_3)} = \frac{P(Y_1 | Y_2) P(Y_3 | Y_2) P(Y_2)}{P(Y_2, Y_3)} = P(Y_1 | Y_2) \qquad (2.7)$$

Thus d) can be discarded, and $\Lambda_{K\ell}$ can be calculated with respect to the submatrix consisting of a), b) and c). Further simplifications are not possible.

Thus a method has been constructed which meets the demands of the preceeding section. Intrinsic modeling reduces to the computation of the linear form (2.5).

Modeling the atmosphere: problems. 1. As is well-known, there is as yet no unique model for the transformation of radiation in the atmosphere. This means that it is necessary to construct a model such that a change in the model causes only a change in the coefficients which occur in the method. /17

2. Due to the limitations on computing time, it is impossible to take into account all the components of the atmosphere which affect radiation.

3. The behavior of radiation depends on the distribution of the atmospheric components along a ray. In practice this information is rarely available.

4. From each point the scattered radiation is spread over a large neighborhood in view of the great length of the path of the ray and the multiplicity of the scattering. Computer time does not permit calculation of this entire neighborhood.

Modeling the atmosphere: realization. In the statistical approach these problems are significantly moderated. The task reduces to computing the change in the distribution of the probability and in correlations of the radiation during passage through the atmosphere when the distributions of the probability and the correlations for the parameters of the atmosphere are given. The following computational method is proposed which is capable of accommodating various concrete models of the atmosphere.

The parameters of the atmosphere are the density of the air d and the concentration of the "effective" absorbers q_c and the diffusers p_ℓ of radiation in each spectral channel ℓ $(\ell = 1, \ldots, N)$.

$$p_\ell = \frac{\sum_{i=1}^{n} t_i \int_{\Omega_\ell} g_i(\lambda)\, d\lambda}{\sum_{i=1}^{n} \int_{\Omega_\ell} g_i(\lambda)\, d\lambda}; \quad q_\ell = \frac{\sum_{i=1}^{m} u_i \int_{\Omega_\ell} f_i(\lambda)\, d\lambda}{\sum_{i=1}^{m} \int_{\Omega_\ell} f_i(\lambda)\, d\lambda} \tag{2.8}$$

Here t_i and u_i are the concentrations of each of the n diffusers and the m absorbent components of the atmosphere; $\Omega_\ell = \lambda_\ell - \lambda_{\ell-1}$ is the ℓth spectral channel; λ the wave length; g and f the coefficients of the dispersion and absorption. The atmosphere is assumed to be plane and infinitely thin, so that p_ℓ and q_ℓ are the mass of a

unit area. Generally speaking, this approximation is extremely crude, and the statistical characteristics are slightly distorted, if, as necessary, we select the functions expressing the dependence of the scattering and absorption on d, ρ_i and φ_i. These functions $c_i(d, \rho_i)$ and $l_i(d, \varphi_i)$ must be determined from a concrete model. In the first approximation we may set

/18

$$c_i = \frac{1}{\Omega_i} M\left[\sum_{i=1}^{n} \int_L \int_{\Omega_i} g_i(\lambda) t_i(z) d(z) \, d\lambda \, dz\right]$$
$$l_i = \frac{1}{\Omega_i} M\left[\sum_{i=1}^{m} \int_L \int_{\Omega_i} f_i(\lambda) u_i(z) d(z) \, d\lambda \, dz\right] \tag{2.9}$$

The integration is performed along a column of atmosphere of height L. The random variables are t, u and d. For a thin layer LZ the intensity of the previous radiation $I_1(LZ)$ in the channel i is

$$I_i(\Delta z) = I_i(0)\left[\Omega_i - \sum_{i=1}^{n} d \cdot t_i \cdot \Delta z \int_{\Omega_i} f_i(\lambda) d\lambda - \sum_{i=1}^{m} d \cdot u_i \cdot \Delta z \int_{\Omega_i} g_i(\lambda) d\lambda\right] \cdot \frac{1}{\Omega_i} = I_i(0)\left[1 - \rho_i \cdot d \Delta z c_i - \varphi_i d \Delta z l_i\right]$$

Analogously, if $I_i(x, y)$ is the radiation at the "entrance" to the atmosphere, then in our model we shall define the radiation at the "entrance" as

$$I_i'(x, y) = I_i(x, y)\left[1 - d(x, y) \rho_i(x, y) c_i\left(d(x, y), \rho_i(x, y)\right) - d(x, y) \varphi_i(x, y) l_i\left(d(x, y), \varphi_i(x, y)\right)\right]. \tag{2.10}$$

$d(x, y)$, $\rho_i(x, y)$ and $\varphi_i(x, y)$ are modeled as the random fields d^{ij}.

The scattered radiation $J_i(x, y)$ is computed as follows. The distribution of the probability $P_{c_i(x,y)}(\eta)$, the correlations r_{c_i, c_k} and $r_{c_i}\left[\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right]$ are computed with respect to $d(x, y)$ and $\rho_i(x, y)$. Then the field of the random variable $J_i(x, y)$, with characteristics which depend on $c_i(x, y)$ and $M[I_i(x, y)]$, is raffled:

$$P_{J_i(x,y)}(\eta) = F\left[P_{c_i(x,y)}(\eta)\right] \cdot M\left[I_i(x, y)\right]$$
$$r_{J_i(x,y)} = G\left[P_{c_i(x,y)}(\eta)\right] \cdot H(r_{c_i}) \cdot M\left[I_i(x, y)\right] \tag{2.11}$$

Here the absence of local characteristics is connected with the assumption concerning the great length of the path and the multiple

scattering of the radiation. The total radiation incident on a point of the earth (x, y) or incident from the earth onto a satellite is

$$I_\ell''(x,y) = I_\ell'(x,y) + \vartheta_\ell(x,y)\chi\{\psi, M[d(x,y)], M[\sigma_\ell(x,y)]\} \quad (2.12)$$

Here χ^0 is the indicatrix of atmospheric scattering which depends on the angle ϕ between the line of sight and the direction of the average radiation current. /19

Thus modelling the atmosphere reduces to the following steps.

1. Raffling the field of the characteristics of the atmospheric parameters with respect to these characterisiics.

2. Modeling the field of the scattered radiation by means of formulas (2.11).

3. Calculating the input radiation $I_\ell''(x, y)$ by means of formulas (2.10), (2.12) and the input radiation ($I_\ell(x, y)$

By suitably choosing $\sigma_\ell(d, \rho_\ell)$ and $\sigma_\ell(d, \eta_\ell)$, it is possible to obtain any needed distribution of the probabilities of the output radiation. As shown above, we attach less importance to the correlations. The given model is plausible, but it needs to be tested experimentally.

Transformation of the parameters into albedo and the escaping radiation. Here we take into account the indicatrix of the scattering, the illumination of the soil by radiation passing through vegetation, the presence of shadows, etc. The transformation is described by the following formulas.

$$I_\ell'(x,y) = I_\ell(x,y)\chi[\beta_j h_{1\ell}\mathcal{L}(x,y,\xi_0,\eta_0) + (1-\beta_j)h_{2\ell}] + \vartheta_\ell(x,y)\Psi(x,y,\xi_0,\eta_0), \quad (2.13)$$

where $h_{1\ell} = F_\ell'(a_1^j, \ldots, a_{N_j}^j, j)$; $h_{2\ell} = G_\ell'(a_1^j, \ldots, a_{N_j}^j, j)$; $\mathcal{L}(x,y,\xi_0,\eta_0) = H_\ell'(a_1^j, \ldots, a_{N_j}^j, j, \xi_0, \eta_0)$; $\Psi(x,y,\xi,\eta) = \int_0^{2\pi}\int_0^{\pi}\mathcal{L}(x,y,\xi-\nu,\eta-\mu)\chi[\psi(\xi,\eta)]\,d\nu\,d\mu$.

$\beta_j = \frac{\beta_{1j}\beta_{2j}}{\cos\tau}$ is the projective covering of the soil by the object j;

β_{1j} is the density of the planting which has a given probability distribution; $\beta_{2j}(\{a_i^j\})$ is the projective covering when $\beta_{1j} = I$; τ is the inclination angle of the locality; $a_1^j, \ldots, a_{N_j}^j$ are the parameters of the object j; ξ, η are angles in the plane connected with the sun (cf. [14]); ξ_0 and η_0 are ξ and η for the satellite to point (x, y) line of sight; $\varphi(\xi, \eta)$ is the angle between (ξ, η) and the direction (x, y) to the sun; χ is the indicatrix of scattering for the object, and χ that for the atmosphere, ψ is their convolution, which plays the role of the indicatrix of the object with respect to the scattered light; $h_{1\ell}$ is the albedo of the object; $h_{2\ell}$ the albedo of the soil. The functions F'_ℓ, G'_ℓ, H'_ℓ are determined experimentally and are once and for all incorporated into the method. $I_\ell(x, y)$ is the direct, and $J_\ell(x, y)$ the scattered solar radiation incident on the earth; $I'_\ell(x, y)$ is the total radiation escaping from a point in the direction of the satellite; $\alpha = 1$ in the sun, and $\alpha = 0$ in shadow. Thus the transformation consists of the following steps. /20

1. Calculation of the field of the incident light and the indicatrix of the atmosphere.

2. Calculation of the object's albedo and the albedo of the soil by means of the given functions F'_ℓ and G'_ℓ and the values of parameters $a_1^j, \ldots, a_{N_j}^j$ at each point.

3. By means of the values $a_1^j, \ldots, a_{N_j}^j, \xi_0, \eta_0$, calculating in the direction toward the satellite the scattering indicatrix of the object and its convolution with the indicatrix of the atmospheric scattering.

4. For "pure soil" and the "pure vegetation", calculating by means of the direct and the scattered incident radiation, the indicatrices and the albedo, the intensity of the radiation escaping to the satellite.

5. Calculating the projective covering at each point from β_1, the density of planting, the parameters of the object and the slope

of the locality.

6. After mixing the radiation of the object and the soil in proportion to the projective covering, we obtain the radiation which escapes from the point to the satellite.

Then mixing is carried out at points between which the boundary of objects passes. For example, for the boundary between (X_1, Y_1) and (X_1, Y_2):

$$I''_i(X_1,Y_1) = I'_i(X_1,Y_1)\cdot\beta + I'_i(X_1,Y_2)(1-\beta);\quad I''_i(X_2,Y_2) = I'_i(X_2,Y_2)\cdot\beta + I'_i(X_2,Y_2)(1-\beta). \qquad (2/14)$$

The mixing ratio β is a random variable which is uniform on [0, 1].

Here we take account of the basic phenomena described, for example, in [14].

Scanning and conversion of radiation into a signal. We shall discuss briefly the modeling step under consideration, since this question has been thoroughly studied.

Here the scanning parameters: the diameter of the intake pupil, the response and the characteristic signal-to-noise ratio of the receiver, the parameters of the preamplifier, etc., serve as input data. By the method of finite differences, we solve the equation $C\frac{dU(t)}{dt} + \frac{U(t)}{R} = i(t)$ for the input signal U(t), where C and R are the load parameters, and i(t) is the input excitation: $i(t) = i_{signal}(t) + i_{noise}(t) = \alpha I_i[x(t), y(t)] + F_{\omega}[I_i(x(t), y(t))]\beta(\Delta f, t)$; α is the receiver response and F_{ω} is its characteristic signal-to-noise ratio; β is a random Gaussian process with M(β) = 0 corresponding to the band frequencies Δf. The set of discrete readings U(t) forms the electrical representation of the frame in the given channel. It also serves as the output of the entire modeling method. /21

Simulation of a probable landscape. This is a rather large program unit intended to generate an arrangement of objects, which

would not be just a meaningless collection of points, but rather would suggest something to the eye. Here objects are chosen which have a characteristic form: rivers, reservoirs, settlements, forests, fields, etc. In the input data only the desired general form of the landscape is specified, and the simulated arrangement is random in character, which imparts naturalness to it.

The second unit is a key factor in satisfying the quality criterion for the modeling. Therefore it is realized more carefully and is described in greater detail. The remaining units must be regarded as a first approximation. Specialists in the respective fields will no doubt improve significantly the quality of these units.

3. THE PROGRAM AND SOME COMPUTATIONAL RESULTS

The computer program which realizes the proposed method was written for the electronic computer YeS-1040. It has a capacity of nearly 1000 instructions. The basic computations are performed with a frame of dimensions 30 x 30 cells with 10 objects (5 agricultural crops, a forest, a meadow, roads, water, settlements) and with a maximum of 7 parameters (for agricultural crops: the height of topography, the soil, the vegetative phase, humidity, productivity, contamination, falling [i.e., of a plant under its own weight]). The real relationships between the parameters and the albedo, the indicatrix, etc., were taken from [14, 15], and also from a collection of journals. Very many of the relationships did not turn out successfully. Then we used their heuristic evaluation. One of the versions of a data bank served as input data. Therefore interest was manifested in the volume of data under various conditions. The data bank is presented in Figure 3. The largest files involve the relationships of the albedo and the indicatrix in terms of the parameters.

The five units described in the program are completely independent, and operate consecutively over the files which represent /22

the frame. The computer time of the program is shown in Figure 4. It was found by extrapolation from the known operating speed of each unit. Clearly, to check out the processing procedure it is not necessary to use very large frames. In general, the given method is designed for laboratory experiments, and the problem of rapidity of action is not very critical for it.

As has been shown, satisfaction of the quality criterion for modeling consists of two parts: selection of an ε which is sufficiently small relative to the real data, and sufficiently precise reproduction of the input data (the criterion with respect to β). Intrinsically, modeling is characterized by the criterion β. We also checked it out. The input data were chosen with normal distributions (since they can be characterized by two parameters all told) and arbitrary correlation coefficients. The agreement of the simulated distributions and correlations with the input distributions and correlations were vierified respectively by means of the χ^2 and Fisher criteria. It was assumed that they make up part of the metric d. The nonstationariness of the mathematical expectation of the input distributions contributed 10%. The results of a test with the corresponding significance levels for the deviations are presented in the table.

Clearly, the agreement is good on the whole. This means that intrinsically the modeling satisfied the requirements imposed on it. In (1.2) α must be chosen on the basis of practical considerations and an ε-network must be constructed which ensures the satisfaction of (1.5).

CONCLUSIONS

The necessity for mathematically modeling the terrestrial surface and the process of remote sensing is shown. The basic principles of the methods of modeling are introduced, in particular, the description of objects in terms of their essential parameters. A criterion for the quality of modeling is introduced and a practical method for satisfying it is proposed. A flexible scheme is suggested /23

for modeling a random field with given characteristics, and a generalized model of the atmosphere is also given.

On this basis a full cycle of transformations of solar radiation into RS is modeled. A computer program is formulated and tested. The testing shows that the modeling procedure complies with the requirements imposed on it. Moveover, the following problems are posed:

1. To find a more stringent evalatuion for the quality criterion.

2. To test on concrete special models the general models for the transformations of radiation proposed here.

3. Practically, using a large number of RS frames, to select input data which ensure a good approximation to real frames (ε-network).

4. With the aid of field investigations, to measure all the necessary relationships: the dependence of albedo and the indicatrix on the parameters, etc.

5. Finally, to test the processing procedures on synthesized and real data and to be convinced with one's own eyes of the effectiveness of modeling.

In the present paper these questions are only touched upon due to the limited competence of the author or the necessity for extensive field investigations. It is to be hoped that if modeling interests experimenters, then these problems will be solved with expedition.

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TABLE

Results of testing the reproduction of input data when modeling the field of parameters (Fixed values equal to M $+\sqrt{D}$ provided 10% of the readings. The number of readings oscillated between 50 to 200 for various parameters).

Designation of parameter	Input Data			Modeling Results											
	Stationary Cases											Nonstationary Cases			
	M	D	r	M	σ(M)	σ	σ(σ)	r	σ(r)	Criterion χ^2	Fisher's Criterion PA	Correlation r	Error r σ(r)		Fisher's Criterion PA
Falling	0,3	0,008	0	0,28	0,02	0,006	0,003	0,05	0,07	10%	34%	-0,18	0,12		2,8%
	0,3	0,008	0,5	0,30	0,02	0,009	0,002	0,48	0,05	26%	55%	0,60	0,10		17%
	0,3	0,008	-0,5	0,27	0,02	0,009	0,003	-0,58	0,05	29%	13%	-0,44	0,11		17%
Productivity	0,5	0,01	∞	0,53	0,015	0,010	0,004	-0,1	0,09	23%	72%	-0,14	0,10		4%
	0,5	0,01	0,5	0,50	0,018	0,011	0,003	0,51	0,06	45%	63%	0,47	0,09		11%
	0,5	0,01	-0,5	0,49	0,009	0,012	0,004	-0,47	0,07	15%	7%	-0,39	0,10		3,5%
Contamination	0,1	0,004	0,2	0,06	0,09	0,011	0,008	0,17	0,03	3%	24%	0,18	0,08		2,6%
Humidity	0,2	0,003	0	0,18	0,013	0,007	0,003	-0,04	0,02	7%	9%	-	-		-
Vegetation phase	0,7	0,01	0,6	0,7	0,015	0,012	0,004	0,55	0,06	44%	38%	-	-		-
Soil	0,5	0,003	0,6	0,48	0,009	0,003	0,001	0,61	0,04	11%	26%	-	-		-
Height of topography	0,3	0,01	0,2	0,27	0,007	0,012	0,002	0,24	0,05	18%	34%	-	-		-

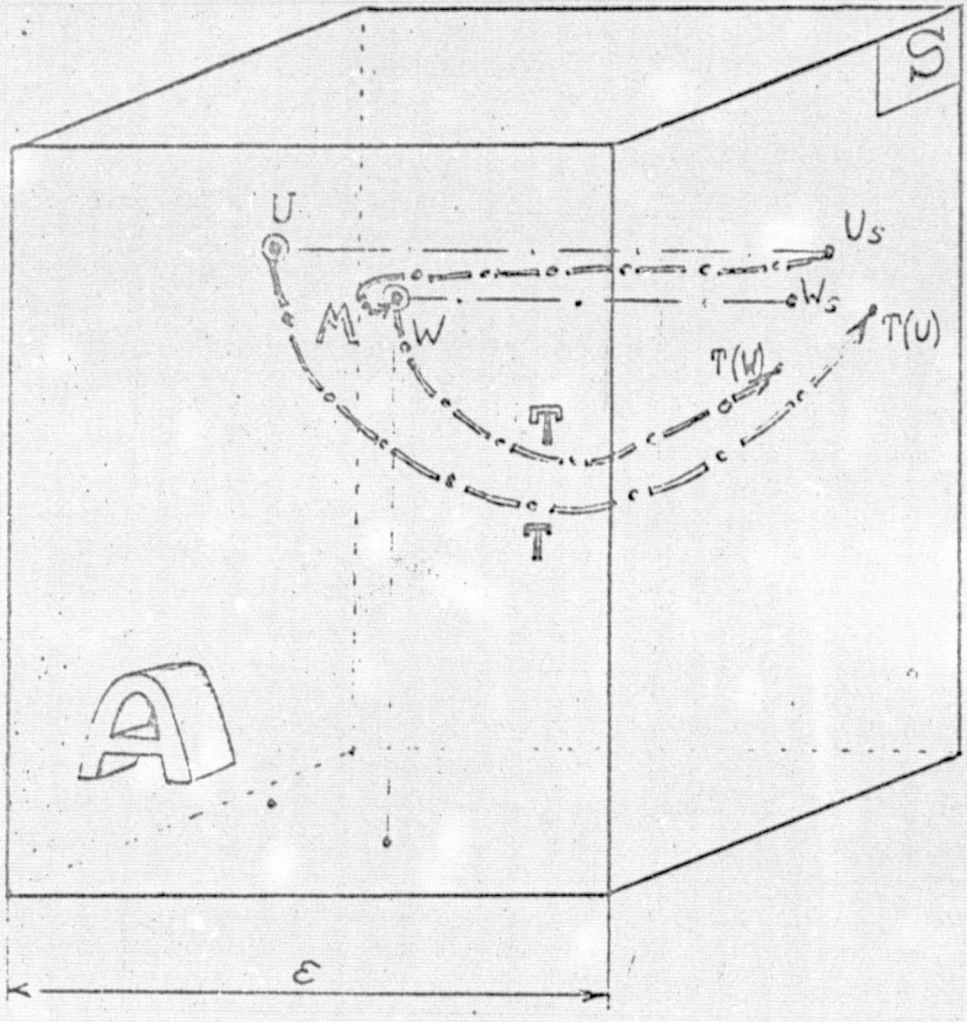

Figure 1. Functional structure of the modeling problem

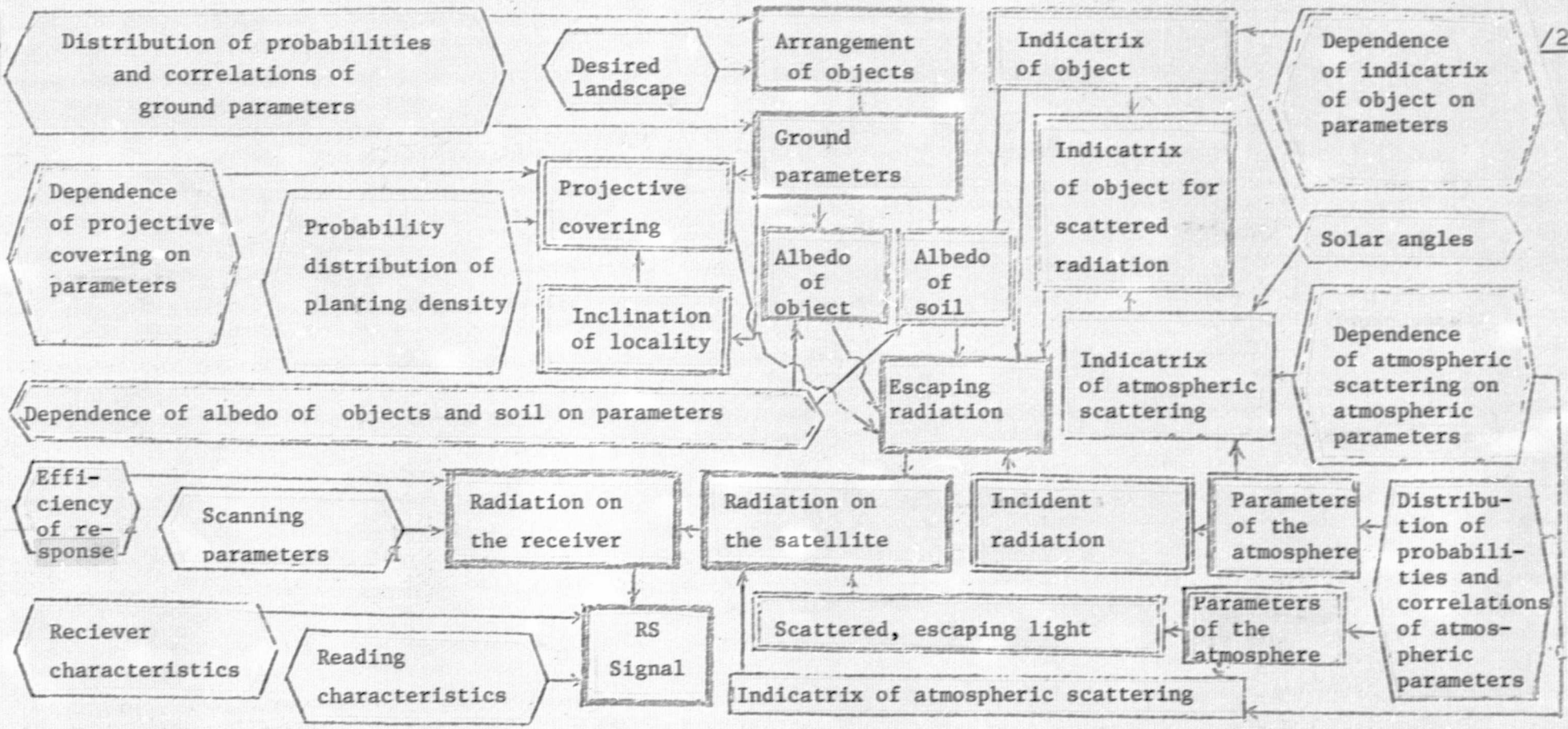

Figure 2. Block diagram of the modeling method; : input data; : constant input data measured in nature; : intermediate steps; : the principal file representing the frame; : results of the operation of the program units. The characteristics of the atmosphere on the path of the incident radiation are outlined by dotted lines, and the corresponding quantities on the path of the escaping radiation, by dot-and-dash lines.

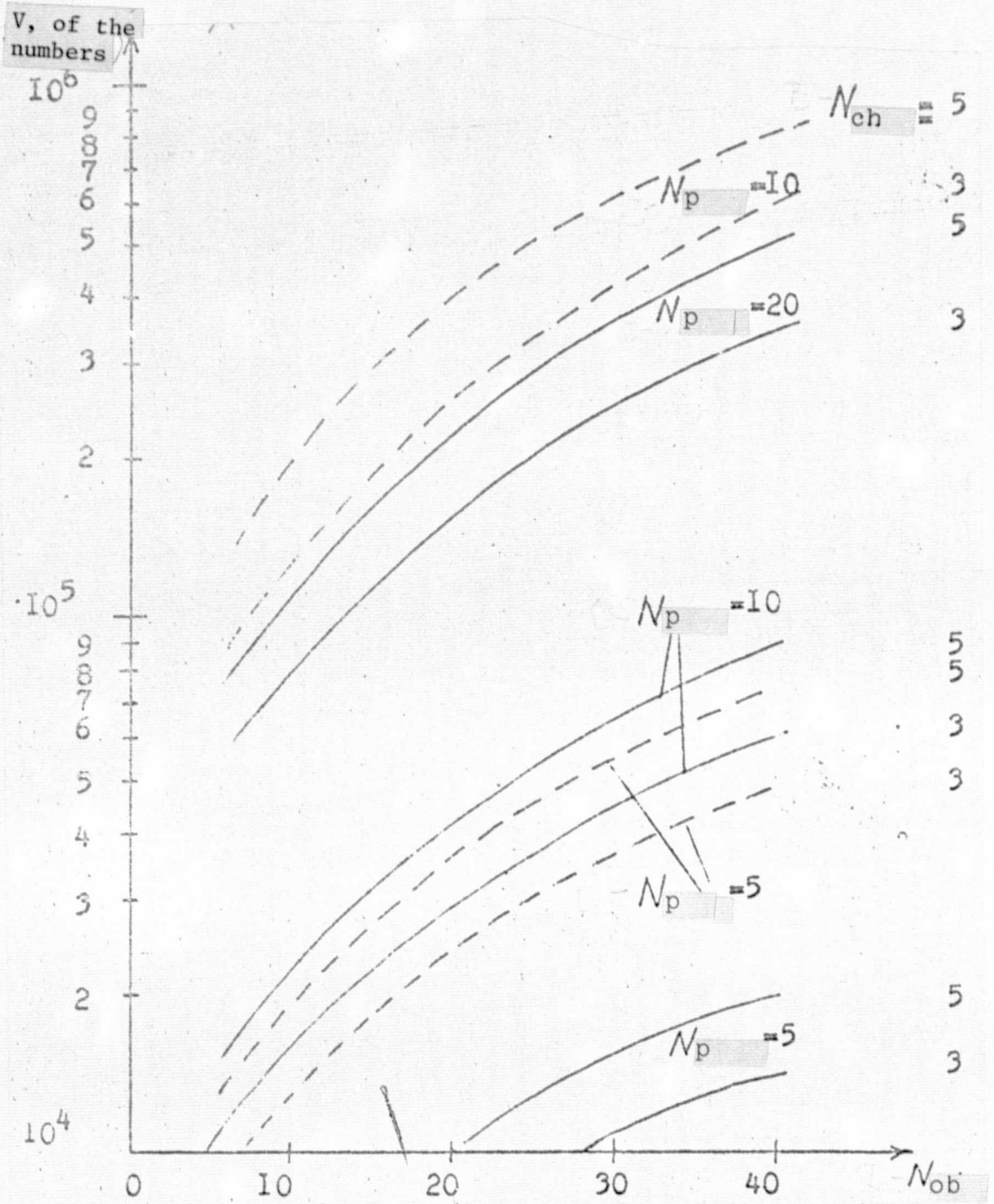

Figure 3. Dependence of the volume of input data V on the number of parameters of the ground objects N_p, on the number of objects N_{ob}, the number of spectral channels N_{ch}, and the degree of polynomial used to approximate the different nonlinear functions N_d. On the solid lines, $N_d = 3$, and on the dotted lines, $N_d = 5$.

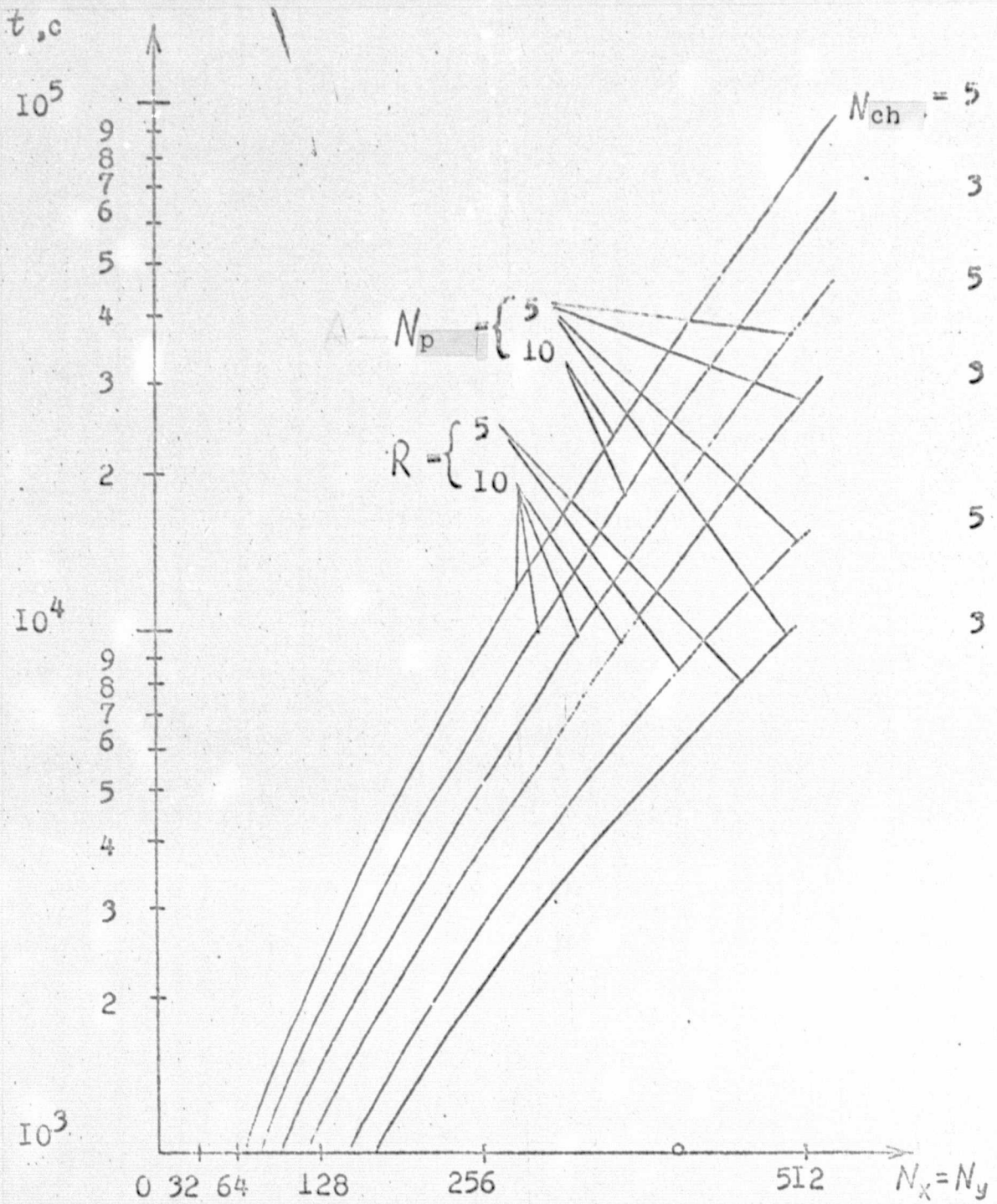

Figure 4. The dependence of computing time t on the dimensions of the frames N_x, N_y, the number of parameters of the ground objects N_p, the radius of the correlational neighborhood for the parameters R, and the number of spectral channels N_{ch}. The degree of approximating polynomial $N_d = 3$.