MULTIPLE OBJECT TRACKING WITH NON-UNIQUE DATA-TO-OBJECT ASSOCIATION VIA GENERALIZED HYPOTHESIS TESTING

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ABSTRACT

A generalized hypothesis testing approach is applied to the problem of tracking several objects where several different associations of data with objects are possible. Such problems occur, for instance, when attempting to distinctly track several aircraft maneuvering near each other or when tracking ships at sea. Conceptually the problem is solved by first associating data with objects in a statistically reasonable fashion and then tracking with a bank of Kalman filters.

The objects are assumed to have motion characterized by a fixed but unknown deterministic portion plus a random process portion modeled by a shaping filter. For example, the object might be assumed to have a mean straight line path about which it maneuvers in a random manner. Several hypothesized associations of data with objects are possible because of ambiguity as to which object the data comes from, false alarm/detection errors, and possible uncertainty in the number of objects being tracked.

The statistical likelihood function is computed for each possible hypothesized association of data with objects. Then the generalized likelihood is computed by maximizing the likelihood over parameters that define the deterministic motion of the object. This forms the basis of the generalized hypothesis testing approach.

The computational burden is dominated by combinatoric considerations. Procedures are addressed that relieve the computational burden.

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1. Introduction

This paper deals with the problem of tracking multiple objects where it is unclear which measurements can be associated with which objects. Such problems arise, for instance, when tracking aircraft maneuvering near each other or when tracking ships at sea. The problem is approached by first performing data-to-object association in a statistically reasonable fashion and then tracking with a bank of Kalman filters.

A recent survey paper Reference 1 describes the various Bayesian and likelihood approaches that have been developed to handle multiple object tracking problems. This paper follows the likelihood line of development pioneered by Sittler in Reference 2 and carried forward by various researchers, notably by Morefield in Reference 3. The contribution of this paper is the use of a generalized likelihood approach to deal with unknown system parameters. The conceptual approach is applicable to a wide variety of parameters including process noise covariances and time constants in shaping filter models for object motion. The specific theory developed here applies to unknown parameters that enter measurements linearly. Examples of such parameters are position and velocity coordinates at track initiation or parameters describing a fixed but unknown mean path for object motion.

The next section describes the problem in more detail and states previous results. This is followed by a section presenting new results with a discussion of computational consideration and a section presenting an analytical example.

2. Previous Results

Object motion is usually modeled with a state space shaping filter where different classes of objects having different models may be considered. It is assumed that the objects move independently of each other. Several sensors of different types may be in use with associated sensor error models. The parameters of the models in previous work have usually been assumed known. This paper includes unknown parameters entering the measurements z_k linearly through initial conditions and forcing terms according to the following model for an object in a given class:

$$
\begin{aligned}
x_{k+1} &= \Phi_k x_k + G_k w_k + B_{2k} b_2 \\
z_k &= H_k x_k + v_k \\
x_o &= B_1 b_1 + x_{or} \\
k &= 0, 1, \ldots, M
\end{aligned}
\qquad (2\text{-}1)
$$

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \text{ - Fixed but unknown parameter vector}$$

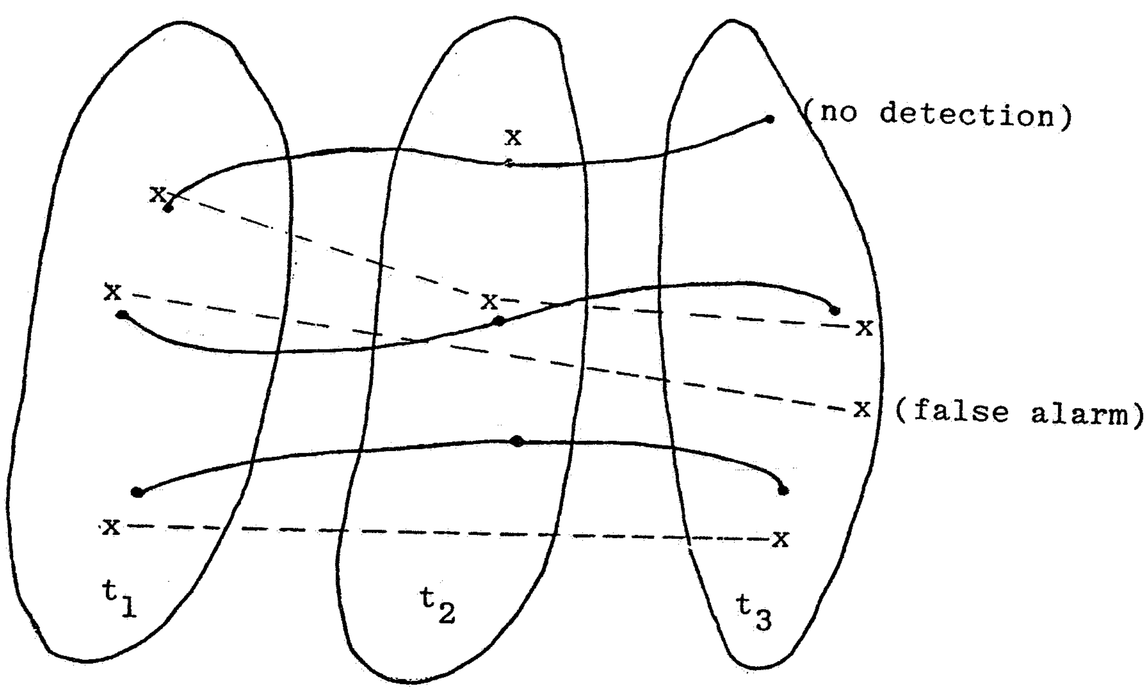
w_k - Zero mean, white, Gaussian process noise with covariance Q_k

v_k - Zero mean, white, Gaussian measurement noise with covariance R_k

x_{or} - Zero mean and Gaussian with covariance $P_{x_{or}}$

For the remainder of this section $b_1 = b_2 = 0$, but b_1 and b_2 are nonzero in the next section.

The data association problem is displayed by Figure 2-1. Given



—o— Object path with dot at measurement time t_i

x Position measurement

--- Hypothesized data association

Figure 2-1. Data Association Hypotheses

the data, there are a number of reasonable data associations. Many factors may have to be considered. Different sensors or the same sensor at different times may have a different coverage region, false alarm and detection errors may occur, and the number of objects may be initially unknown. In order to focus attention on new results, it is assumed in the remainder of the paper that the sensor coverage region includes all objects at each measurement time and that there are no false/alarm detection errors. Further, it is assumed that no *a priori* probabilistic information is available as to the correctness of a given data association.

Previous results compute the statistical likelihood function with a bank of Kalman filters as shown in Figure (2-2) and maximize the likelihood over hypothesized data associations. The hypothesizer selects

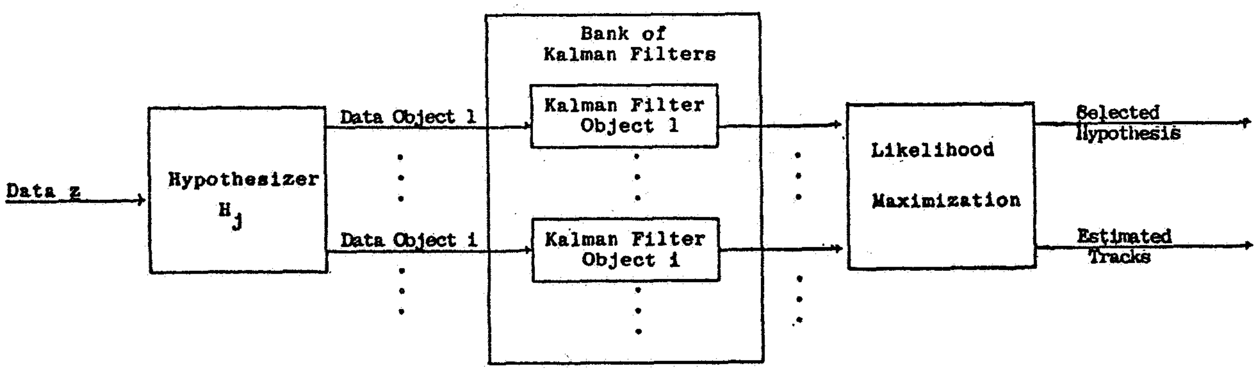


Figure 2-2. Likelihood Maximization

the data association, and then the data is whitened by a bank of Kalman filters using appropriate object class models. Consequently, negative twice the log likelihood can be computed efficiently by the equation

$-2\ln p(z;H_j)$

$$= \sum_i \sum_k \left(\ln|P_{\tilde{z}^j_{i,k}}| + \tilde{z}^{j^T}_{i,k} P^{-1}_{\tilde{z}^j_{i,k}} \tilde{z}^j_{i,k}\right) \qquad (2\text{-}2)$$

$$+ m \ln 2\pi$$

$\tilde{z}^j_{i,k}$, $P_{\tilde{z}^j_{i,k}}$ - Residual and residual covariance for hypothesis H_j, object i, at time k

m - Total number of scalar measurements.

The data dependent part of equation (2-2) is chi-square with degrees of freedom m (χ^2_m). Then the likelihood is maximized by minimizing equation (2-2). In this manner a statistically reasonable hypothesis is selected and tracking is performed by the bank of Kalman filters.

3. New Results

The contribution of this paper is the inclusion of fixed but unknown parameters in the object motion model. The generalized likelihood approach provides a solution by maximizing the likelihood over the parameters before maximizing over hypotheses. This is accomplished by replacing the Kalman filter for object i in Figure 2-2 by the equation shown in block diagram form in Figure 3-3 where the i and j subscripts are suppressed. The Kalman filter is based on the model of equation 2-1 with $b = 0$ This is indicated by subsripting filter computed quantities with "fb = 0" The filter residuals and residual covariance are used in a set of auxilliary equations to compute a maximum likelihood estimate $\hat{b}_k$ with estimate error covariance $P_{\tilde{b}_k}$ These results are used in turn to compute the portion of the likelihood

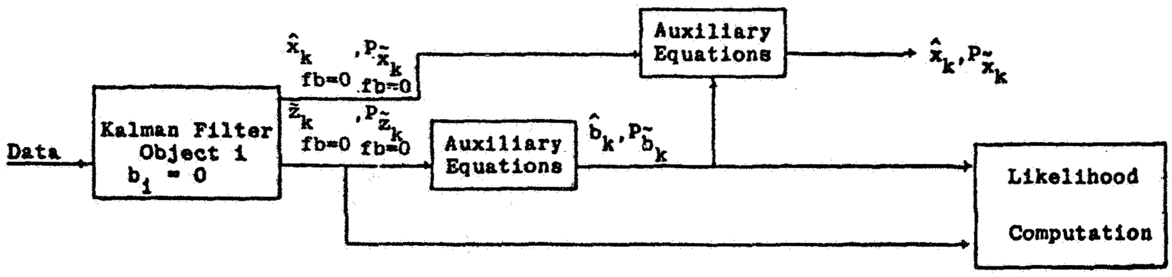


Figure 3-3. Generalized Likelihood Computation

for the object under consideration. Then using the filter estimate for the state and the estimate for b recovered from the filter residuals, the minimum variance and unbiased estimate $\hat{x}_k$ of the state and its estimate error covariance $P_{\tilde{x}_k}$ are computed.

The estimate for b and the estimate uncertainty are given by

$$\hat{b}_k = \Big(\sum_{\ell=0}^{k} T_{1\ell}^T H_\ell^T P^{-1}_{\tilde{z}_\ell \,|\, fb=0} H_\ell T_{1\ell} \Big)^{-1} \sum_{\ell=0}^{k} T_{1\ell}^T H_\ell^T P^{-1}_{\tilde{z}_\ell \,|\, fb=0} \tilde{z}_\ell \big|_{fb=0} \tag{3-1}$$

$$P_{\tilde{b}_k} = \Big(\sum_{\ell=0}^{k} T_{1\ell}^T H_\ell^T P^{-1}_{\tilde{z}_\ell \,|\, fb=0} H_\ell^T T_{1\ell} \Big)^{-1}$$

$$T_{1,\ell+1} = \Phi_\ell (I - K_\ell H_\ell) T_{1\ell} + [0_{n\times n_1} \,\vdots\, B_{2\ell}]$$

$$T_{10} = [B_1 \,\vdots\, 0_{n\times n_2}]$$

K_ℓ - Kalman gain at ℓ^{th} time

n_1, n_2 - Number of elements in b_1 and b_2

n - Number of elements in x.

Note that the above equations can be implemented recursively. The minimized negative twice log likelihood for the given object is expressed as

$$-2\ln p(z;\hat{b}_M) = \Big(\sum_k \ln|P_{\tilde{z}_k}|_{fb=0} + \tilde{z}_k^T\big|_{fb=0} \, P_{\tilde{z}_k}^{-1}\big|_{fb=0} \, \tilde{z}_k\big|_{fb=0}\Big)$$

$$- \hat{b}_M^T P_{\tilde{b}_M}^{-1} \hat{b}_M + m \ln 2\pi \tag{3-2}$$

Now, including the i and j subscripts, the generalized negative twice log likelihood over all objects is

$$-2 \ln p(z;H_j,\hat{b}_{1M}^j,\cdots\hat{b}_{iM}^j,\cdots)$$

$$= \sum_i\Big(\sum_k \ln|P_{\tilde{z}_{ik}^j}|_{fb=0} + \tilde{z}_{ik}^{j\,T}\big|_{fb=0} \, P_{\tilde{z}_{ik}^j}^{-1}\big|_{fb=0} \, \tilde{z}_{ik}^j\big|_{fb=0}\Big) - \hat{b}_{iM}^{j\,T} P_{\tilde{b}_{iM}^j}^{-1} \hat{b}_{iM}^j$$

$$+ m \ln 2\pi \tag{3-3}$$

The data association is accomplished by minimizing the above equation over H_j. It can be shown that the data dependent part of equation 3-3 is chi-square with degrees of freedom equal the total number of scalar measurements minus the total number of elements in all the b_1 and b_2 vectors under hypothesis H_j.

The generalized likelihood has been computed but valid state estimates for the objects still remain to be specified. It can be shown that the following equations give the minimum variance unbiased estimate and its uncertainty for a given object, again suppressing the i, j subscripts:

$$\hat{x}_k(+) = \hat{x}_k(+)_{fb=0} + T_{2k}\,\hat{b}_k$$

$$P_{\tilde{x}_k}(+) = P_{\tilde{x}_k}(+)_{fb=0} + T_{2k}\,P_{\tilde{b}_k}\,T_{2k}^T \tag{3-4}$$

$$T_{2k} = (I-K_k\,H_k)\,T_{1k}$$

It is worthwhile to compare the new results of this section with the previous results of Section 2 where the noninformative prior approach is used to deal with the parameters b. The distribution of the data dependent part of the negative twice log likelihood is chi-square in the total number of scalar measurements for the previous results but is chi-square in the number of scalar measurements minus the total number of elements in all the b_1 and b_2 vectors for the new results. However, it can be shown that the actual numerical value of the data dependent parts is the same. On the other hand, comparison of equations (3-3) and (2-2) shows that the model dependent parts are different. Further, it can be shown that the state estimate obtained from the Kalman filter in the previous results is the same as that given by equation (3-4). There are clearly basic similarities and basic differences between the two approaches and caution must be exercised in selecting which approach is most appropriate.

Numerical considerations are of major importance to the application of the new results as they are to previous results. It is easily verified that even for a modest number of objects and measurement times that a combinatoric explosion occurs in the number of hypothesized data associations. Much use can be made of pre-processing techniques to greatly limit the number of hypotheses. For example, if two position measurements require an object to exceed its maximum possible velocity then those measurements can not both come from the same object. Also examination of filter residuals directly can be used to exclude clearly unlikely data associations. In general, however, such preprocessing does not appear to stop the combinatoric explosion, but only to slow it. Thus, suboptimal approaches appear to be required for most realistic problems. For example, data association can be performed at each measurement time saving only tracks from the "best" hypotheses up to a maximum number of tracks. It is more efficient to save tracks than hypotheses since two hypotheses may have many tracks in common. Another technique is to again perform data association at each point in time but consider only hypotheses that can be generated by back tracking N steps in time. Some mathematical programming techniques are also applicable such as integer programming techniques (Reference 3). Ultimately, the best technique is determined by the details of the specific problem at hand. In general, what is desired as an algorithm with storage requirements that do not grow in time and with execution time that grows only linearly in time.

4. Analytical Example

A simple example is presented to provide some "feeling" for the generalized likelihood approach. The example is actually somewhat more general than the theory of the preceding section in that a power spectral density parameter is to be estimated from the data.

The objects under observation move in a plane as random walks about straight mean paths with constant mean velocity where parameters describing the mean path are fixed but unknown. The power spectral density of the process noise will be considered known in one case and unknown in another. Measurements are taken at a constant interval Δt and are accurate enough to be considered noise free. It is assumed that all objects are observed at each measurement time and that no false alarm or detection errors occur.

The analysis model for the i^{th} object under the j^{th} hypothesis is

$$\begin{bmatrix} r_{1,i,k+1} \\ r_{2,i,k+1} \end{bmatrix} = \begin{bmatrix} r_{1,i,k} \\ r_{2,i,k} \end{bmatrix} + \begin{bmatrix} u_{1,i} \\ u_{2,i} \end{bmatrix} \Delta t + \begin{bmatrix} w_{1,i,k} \\ w_{2,i,k} \end{bmatrix}$$

$$\begin{bmatrix} z^j_{1,i,k} \\ z^j_{2,i,k} \end{bmatrix} \quad \begin{bmatrix} r_{1,i,k} \\ r_{2,i,k} \end{bmatrix} \qquad (4\text{-}1)$$

$k = 0, 1, \cdots, M$

$r_{1,i,o}, r_{2,i,o}, u_{1,i}, u_{2,i}$ - Fixed but unknown parameters

$w_{1,i,k}, w_{2,i,k}$ - zero mean, white Gaussian each having variance $Q_i\ \Delta t$ and uncorrelated with each other

The first measurement estimates initial position exactly so it will no longer be considered unknown. Using Kalman filter equations, the negative twice log likelihood for the object of equation (4-1) is found to be

$$-2 \ \ln p\ (\cdots z^j_{i,k}, \cdots;\ u_{1,i},\ u_{2,i},\ Q_i) + \text{constant}$$

$$= \sum_{k=1}^{M} 2 \ \ln Q_i\ \Delta t + \frac{(z^j_{1,i,k} - z^j_{1,i,k-1} - u_{1,i}\Delta t)^2}{Q_i \Delta t} + \frac{(z^j_{2,i,k} - z^j_{2,i,k-1} - u_{2,i}\Delta t)^2}{Q_i \Delta t} \qquad (4\text{-}2)$$

Now minimize equation (4-2) with respect to $u_{1,i}$ and $u_{2,i}$ to obtain the maximum likelihood estimates

$$\hat{u}^j_{1,i} = \frac{1}{M\Delta t} \sum_{k=1}^{M} z^j_{1,i,k} - z^j_{1,i,k-1}$$

$$\hat{u}^j_{2,i} = \frac{1}{M\Delta t} \sum_{k=1}^{M} z^j_{2,i,k} - z^j_{2,i,k-1} \qquad (4\text{-}3)$$

These estimates are valid whether Q_i is known or not. Now assume Q_i is unknown and minimize equation (4-2) to obtain

$$\hat{Q}^j_i = \frac{1}{2M\Delta t} \sum_{k=1}^{M} (z^j_{1,i,k} - z^j_{1,i,k-1} - \hat{u}^j_{1,i}\Delta t)^2$$

$$+ (z^j_{2,i,k} - z^j_{2,i,k-1} - \hat{u}^j_{2,i}\Delta t)^2 \qquad (4\text{-}4)$$

Now consider the case where the Q_i are known and form the generalized negative twice log likelihood over all objects minimized over $u_{1,i}$ and $u_{2,i}$

$$-2 \ \ell n \ p \ (z;\cdots,\hat{u}^j_{1,i},\hat{u}^j_{2,i},\cdots) + \text{constant}$$

$$\frac{1}{\Delta t} \sum_i \frac{1}{Q_i} \sum_{k=1}^{M} (z^j_{1,i,k} - z^j_{1,i,k-1} - \frac{1}{M}\sum_{\ell=1}^{M}(z^j_{1,i,\ell} - z^j_{1,i,\ell-1}))^2$$

$$+ (z^j_{2,i,k} - z^j_{2,i,k-1} - \frac{1}{M}\sum_{\ell=1}^{M}(z^j_{2,i,\ell} - z^j_{2,i,\ell-1}))^2 \qquad (4\text{-}5)$$

For the case of Q_i unknown, the generalized likelihood is maximized by minimizing the following expression

$$\prod_i \sum_{k=1}^{M} \left(z^j_{1,i,k} - z^j_{1,i,k-1} - \frac{1}{M}\sum_{\ell=1}^{M}\left(z^j_{1,i,\ell} - z^j_{1,i,\ell-1}\right)\right)^2 + \left(z^j_{2,i,k} - z^j_{2,i,k-1} - \frac{1}{M}\sum_{\ell=1}^{M}\left(z^j_{2,i,\ell} - z^j_{2,i,\ell-1}\right)\right)^2 \qquad (4\text{-}6)$$

Equations (4-5) and (4-6) have a term that sums over time in common. This common term is simply a measure of the variation of position change about mean position change for a given object. Both equations qualitatively attempt to perform data association by minimizing the variation in position change over all objects. However, in equation (4-5) where Q_i is known, the variation in position change is weighted heavily for small Q_i. This is intuitively reasonable since a small variation would be expected. It might seem reasonable that if Q_i is unknown that equation (4-5) should still apply but with $Q_i = 1$. However, the variation in position change for objects that truly have a small Q_i would be swamped out by that for objects with large Q_i. The multiplication of equation (4-6) keeps this swamping effect from occurring.

References

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