Facility

GRAVITATIONAL RADIATION AND THE ULTIMATE SPEED IN ROSEN'S BIMETRIC THEORY OF GRAVITY*

Carlton M. Caves

W. K. Kellogg Radiation Laboratory

California Institute of Technology, Pasadena, California 91125

(NASA-CR-158718) GRAVITATIONAL RADIATION AND THE ULTIMATE SPEED IN ROSEN'S BIMETRIC THEORY OF GRAVITY (California Inst. of Tech.) 35 p HC A03/MF A01 CSCL 08N

N79-27743

Unclas G3/46 27445

*Supported in part by the National Science Foundation [AST76-80801 AC2] and the National Aeronautics and Space Administration [NGR 05-002-256].

ABSTRACT

In Rosen's "bimetric" theory of gravity the (local) speed of gravitational radiation v_g is determined by the combined effects of cosmological boundary values and nearby concentrations of matter. It is possible for v_g to be less than the speed of light. I show here that emission of gravitational radiation prevents particles of nonzero rest mass from exceeding the speed of gravitational radiation. Observations of relativistic particles place limits on v_g and the cosmological boundary values today, and observations of synchrotron radiation from compact radio sources place limits on the cosmological boundary values in the past.

LIST OF SYMBOLS

μ = Greek lower case mu

ν = " " " nu

η = " " " eta

α = " " " alpha

γ = " " " gamma

ε = " " " curly epsilon

β = " " " beta

δ = " " " delta

λ = " " " lambda

π = " " " pi

∇ = nabla

θ = Greek lower case theta

$\int$ = integral

$\mathcal{S}$ = upper case script S

ω = Greek lower case omega

ℓ = lower case script "ell"

Δ = Greek upper case delta

∞ = infinity

Ω = Greek upper case omega

ξ = " lower case xi

1. INTRODUCTION

Several years ago Nathan Rosen [1] proposed a new theory of gravity, the "bimetric" theory -- the two metrics being the physical metric $g_{\mu\nu}$ and a flat, "background" metric $\eta_{\mu\nu}$.[1] The theory is perhaps better described as a two-tensor, metric theory (see [2] for discussion). It is a metric theory in the sense that the physical metric obeys the Einstein Equivalence Principle: in the local, freely falling frames of $g_{\mu\nu}$ the nongravitational laws of physics reduce to those of special relativity. One immediate consequence is local conservation of nongravitational stress-energy $T_{\mu\nu}$ -- the matter-response equation:

$$T^{\mu\nu}{}_{;\nu} = 0 \quad . \tag{1.1}$$

The auxiliary, symmetric two-tensor $\eta_{\mu\nu}$ can be thought of as a second metric; it is constrained to be flat and is used in constructing the field equations for the physical metric (see Section 2B). In a series of papers Rosen and others have analyzed various consequences of the theory, including the maximum mass of neutron stars [3], cosmological models [4], equations of motion [5], gravitational radiation [6], and other topics [7].

The traditional testing ground for such a theory is the solar system, where observations at today's accuracies probe the theory's predictions to post-Newtonian order (see [8] for a review). Lee et al. [9] have calculated the post-Newtonian limit of Rosen's theory and shown that it is the same as that of general relativity, except for the preferred-frame PPN parameter α_2. [For a discussion of the Nordtvedt-Will Parametrized Post-Newtonian (PPN) Formalism and a description of the meaning of the PPN parameters, see chapter 39 of [10]; in particular, Box 39.5.] The values of α_2 and the Newtonian gravitational "constant" G are determined by the distant matter in the Universe, which reaches into the solar system through boundary conditions applied far outside it. An appropriate adjustment of the cosmological boundary values brings the theory into agreement with present limits on α_2 and on the time rate of change of G. Put the other way around, these limits place constraints on the possible boundary values. One way to test the viability of the theory is to construct cosmological models and ask whether the models can be made consistent with these constraints. In this paper I point out a new set of observations which yield particularly stringent constraints on the cosmological models in Rosen's theory.

The two metrics in Rosen's theory play different roles. Gravitational radiation propagates along "light" cones of the flat metric, while light propagates along "light" cones of the physical metric. The two "light" cones need not coincide, so the speed of gravitational radiation is, in general, different from the speed of light. Lee et al. [9] showed that the speed of gravitational radiation, as measured by an observer at rest in the universal rest frame far from any local concentration of matter, is determined solely by the cosmological boundary values. This speed v_{gc} is related to α_2 by

$$v_{gc}^2 = (1 + \alpha_2)^{-1} \quad . \tag{1.2a}$$

In the vicinity of a local source of gravity with (dimensionless) Newtonian potential $U \ll 1$ $(U > 0)$, the speed of gravitational radiation increases to

$$v_g = v_{gc}(1 + 2U) \tag{1.2b}$$

(see Section 2B). It is possible for v_g to be less than the speed of light.

I show here (Section 3B) that as a particle of nonzero rest mass is accelerated through the gravitational "light" cone, it emits an infinite amount of energy in gravitational radiation. It follows that, if $v_g < 1$, the speed of gravitational radiation is the ultimate speed for such particles; they cannot escape the gravitational "light" cone. As a result, observation of a relativistic particle with Lorentz factor γ provides a lower bound for v_g at the point of observation:

$$1 - v_g < \frac{1}{2} \gamma^{-2} \quad . \tag{1.3a}$$

If the Newtonian potential at the point of observation is known, one also obtains a lower bound for v_{gc}. This lower bound can be re-expressed as an upper bound on the value of $\alpha_2 \approx 2(1 - v_{gc})$:

$$\alpha_2 < \gamma^{-2} + 4U \quad . \tag{1.3b}$$

Equations (1.3) are the basis for obtaining observational constraints on v_g and α_2 (see Section 4).

In this paper I analyze the gravitational radiation emitted by particles moving at speeds near the speed of gravitational radiation. This analysis leads to the conclusion that, in Rosen's theory, particles of nonzero rest mass cannot exceed the speed of gravitational radiation. This conclusion is likely to have far wider applicability than just to Rosen's theory. The detailed analysis presented here does not depend critically on any special feature of Rosen's theory; one can make a strong case that a similar analysis holds in any theory of gravity which permits the speed of gravitational radiation to differ from the speed of light (see [11] for a brief review of such theories). Indeed, it seems likely that the "gravitational speed limit" is a feature of all such theories.

Another crucial test of Rosen's theory comes from observations of the change in orbital period of the binary pulsar [12]. Unless the two components of the binary system have identical ratios of gravitational binding energy to inertial mass, Rosen's theory predicts that the system will emit dipole gravitational radiation and that the radiation will carry away negative energy [13]. Observations of the binary pulsar are now good enough that Rosen's theory can be ruled out -- unless the two ratios are the same to within less than a percent [14].

Section 2 develops the formalism for analyzing gravitational radiation emission from weak-field systems in Rosen's theory. Section 3 builds upon this foundation to justify the claim that a particle of finite rest mass cannot exceed the speed of gravitational radiation. Section 3A calculates the energy spectrum of gravitational Cherenkov radiation emitted by a particle moving with uniform velocity $v > v_g$, and Section 3B analyzes the energy emitted as a particle is accelerated through the gravitational "light" cone. The result of these considerations is Eqs. (1.3), which Section 4 uses to obtain observational limits on v_g and α_2 (v_{gc}). Section 5 argues that these constraints apply to any theory of gravity with a variable speed of gravitational radiation.

2. FOUNDATION FOR ANALYZING EMISSION OF GRAVITATIONAL RADIATION

This section lays the foundation for analyzing emission of gravitational radiation from weak-field, linearized systems in Rosen's theory. The foundation will be laid in two pieces: the first piece is construction of coordinates which take into account matching to boundary values provided by an external gravitational field; the second piece is construction of equations governing generation of gravitational radiation and specifying the amount of energy the radiation carries.

A. Isolated Sources and Preferred Coordinates

Below I shall deal with "isolated" sources of gravity, such as the solar system or one of the ultrarelativistic particles of Section 3. Since such sources are not actually alone in the Universe, it is necessary to describe briefly what is meant by an "isolated" source.

The key feature of an isolated source is that the gravitational field can be split into two pieces: the field of the isolated source (the local field), which applies near the source; and the field of the rest of the matter in the Universe (the external field), which applies far from the source.

To understand the conditions necessary for such a split, consider the length scales characteristic of the source and the external field. The source is characterized by two lengths: its physical size R and the length Gm corresponding to its mass m. The external field is also characterized by two lengths: a typical radius of curvature a and the length L over which the external field varies appreciably. Let r_o be the distance from the source at which the curvature produced by the source becomes comparable to the external curvature:

$$r_o \equiv (Gma^2)^{1/3} \quad . \tag{2.1}$$

To get a clean split between the local and external fields, the source must be buried deep inside r_o ($R \ll r_o$), and r_o must be much smaller than the external scales ($r_o \ll \min\{a,L\}$). These two conditions translate into

$$Gm/R^3 \gg a^{-2} \quad , \tag{2.2a}$$

$$Gm \ll \min\{a\, ,\, L(L/a)^2\} \quad . \tag{2.2b}$$

The curvature produced by the isolated source is a large, but small-scale "bump" in the large-scale external curvature.

When Eqs. (2.2) are satisfied, the region around the source can be broken up into three parts, which provide a natural split of the gravitational field:

(i) the local-field region, in which the curvature of the isolated source dominates:

$$r \lesssim r_o \quad , \tag{2.3a}$$

where r is distance from the source;

(ii) the transition region, in which the external curvature dominates, but which is small enough that the external field is nearly homogeneous:

$$r_o \lesssim r \lesssim r_1 \equiv \varepsilon \cdot \min\{a,L\} \quad , \tag{2.3b}$$

where ε is a suitably chosen factor less than one;

(iii) the external-field region:

$$r \gtrsim r_1 \quad . \tag{2.3c}$$

The nearly flat transition region splits the gravitational field into local and external fields. The only connection between the two fields is the requirement that they match smoothly in the transition region; from the point of view of the local field, the external field establishes boundary conditions in the transition region.

The boundary conditions are made explicit by choosing a specific coordinate system. A particularly convenient set of coordinates can be constructed as follows. Consider the external gravitational field in the absence of the isolated source. Let an observer falling freely in this field construct Fermi normal coordinates $\{x^\alpha\} \equiv \{t,x^j\}$ in the vicinity of his world line [15]. In these coordinates the two metrics can be expanded about the observer's world line:

$$g_{\alpha\beta} = g^{(B)}_{\alpha\beta} + [\text{terms of order } R_{\alpha j\beta k}x^j x^k \sim (r/a)^2] + \cdots \; , \tag{2.4a}$$

$$\eta_{\alpha\beta}(t,x^j) = \eta_{\alpha\beta}(t,x^j=0) + \eta_{\alpha\beta,k}(t,x^j=0)x^k + \cdots \; , \tag{2.4b}$$

where $g^{(B)}_{\alpha\beta}$ is Minkowskian, i.e., $\| g^{(B)}_{\alpha\beta} \| = \text{diag}(-1,+1,+1,+1)$, and $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor derived from the physical metric. Now introduce the isolated source in the vicinity of the fiducial world line and use these coordinates to solve for its local field. The flat metric retains the form (2.4b), and the physical metric retains the form (2.4a) in the transition region outside the source.

Equations (2.4) display explicitly the boundary conditions to be applied in the transition region. In general relativity, which has only a physical metric, the external field influences the isolated source only through the Riemann and higher-order terms in Eq. (2.4a), which represent tidal and higher-order multipole forces on the isolated source. The situation is different in Rosen's theory because of the presence of the flat metric. Although the region around the source has been split cleanly into local and external parts, $\eta_{\alpha\beta}$ cannot be adjusted independently in the two regions; rather, the external field determines the form of $\eta_{\alpha\beta}$ in the transition region, and a particular choice of coordinates together with its flatness then determines $\eta_{\alpha\beta}$ in the local-field region. (The above choice of coordinates insures that $\eta_{\alpha\beta}$ is nearly constant in the local field region.) In general, the external field prohibits finding coordinates such that both $g_{\alpha\beta}$ and $\eta_{\alpha\beta}$ are nearly Minkowskian in the transition region. This lack of "meshing" allows the external field to reach into the vicinity of the isolated source and affect local gravitation physics. (Will [2, Section 5.3] gives a general discussion of the manner in which auxiliary tensor fields in metric theories of gravity couple local gravitation physics to an external field.)

Gravitational radiation emitted by the source is analyzed in the transition region. In order to separate the radiation from the external curvature, the wavelength λ of the radiation must be much smaller than external scales:

$$\lambda \ll \min\{a,L\} \quad , \tag{2.5}$$

a requirement which also guarantees that the wave zone of the radiation extends into the transition region. In Section 3 I will be interested in calculating gravitational radiation emission in the linear approximation. In this limit another consequence of (2.5) is that, in Eqs. (2.4), one can ignore both the tidal terms in $g_{\alpha\beta}$ and the spatial and temporal derivatives of $\eta_{\alpha\beta}$; these terms cannot affect radiation at wavelengths much smaller than their own characteristic lengths.

As a result, in calculating the gravitational radiation emitted by an isolated source in the linear approximation, one can always use coordinates with the following two properties:

<u>Property 1</u>. The physical metric $g_{\alpha\beta}$ is asymptotically Minkowskian in the transition region far from the source.

<u>Property 2</u>. The flat metric $\eta_{\alpha\beta}$ is a nearly constant matrix in the the local-field and transition regions; its slowly changing values are determined by the external field, and its temporal derivatives can be ignored.

I shall refer to a coordinate system which satisfies these two properties as a <u>preferred coordinate system</u>. Such coordinates are particularly useful for analyzing gravitational radiation emission: Property 1 insures that the coordinates provide a good reference frame for an observer in the transition region monitoring the emitted radiation; and Property 2 insures that the field equations and gravitational stress-energy assume a particularly simple form (see Section 2B). Properties 1 and 2 do not uniquely specify the coordinates; instead, they specify a family of preferred coordinate systems, the members of which are related by arbitrary Lorentz transformations and translations. Throughout the following I shall use preferred coordinates.

Now restrict attention to the sources considered in Sections 3 and 4 -- ultrarelativistic particles moving in a typical astrophysical environment. For such sources, the external field must include both the smoothed-out cosmological solution and the fields of nearby, large-scale density enhancements. A typical source might be a cosmic-ray proton near the Earth; then the nearby density enhancements include the Virgo cluster, the Local Group, the Galaxy, the solar system, and the Earth. To sufficient accuracy the gravitational fields of the large-scale density enhancements can be treated in the weak-field, slow-motion approximation. If the Universe is homogeneous and isotropic (assumed henceforth), the solution for the full external field — including the cosmological boundary values and nearby density enhancements — is that given in reference [9]. In the universal rest frame — the frame in which the cosmological fluid is at rest — the two metrics are given by

$$g_{00} = -1 + 2U \quad , \tag{2.6a}$$

$$g_{0j} = 0 \quad , \tag{2.6b}$$

$$g_{jk} = \delta_{jk}(1 + 2U) \quad , \tag{2.6c}$$

$$\|\eta_{\alpha\beta}\| = \mathrm{diag}(-c_0^{-1}, c_1^{-1}, c_1^{-1}, c_1^{-1}) \quad , \tag{2.6d}$$

where c_0 and c_1 are determined by the cosmological solution, and U is the Newtonian potential due to those nearby density enhancements which produce a significant deviation from the cosmological solution. The g_{0j} components have been neglected, since they are much smaller than U for slow-motion sources.

The external field (2.6) is used to construct preferred coordinates appropriate for analyzing an isolated source. For many purposes the most convenient set of preferred coordinates is obtained by using a (freely falling) fiducial observer who is initially at rest in the universal rest frame. In the resulting preferred coordinates $g_{\alpha\beta}$ is asymptotically Minkowskian in the transition region (Property 1), and $\eta_{\alpha\beta}$ is given by

$$\eta_{00} = - c_0^{-1}(1 + 2U) \quad , \tag{2.7a}$$

$$\eta_{0j} = 0 \quad , \tag{2.7b}$$

$$\eta_{jk} = c_1^{-1}\, \delta_{jk}(1 - 2U) \quad , \tag{2.7c}$$

where U (= constant) is evaluated in the vicinity of the isolated source. These preferred coordinates will be called the local universal rest (LURF); they will be used for all the calculations in Section 3.

B. Linearized Field Equations and Gravitational Stress-Energy

It is not necessary to give the full, nonlinear Rosen field equations here; only the linearized version will be needed. For the full equations the reader is referred to the original papers of Rosen [1] and to [9].

For a weak-field source, the physical metric in a preferred coordinate system is nearly Minkowskian in both the local-field and transition regions. In the usual way, define the metric perturbation $h_{\mu\nu}$ to be the deviation of $g_{\mu\nu}$ from Minkowskian:

$$g_{\mu\nu} = g_{\mu\nu}^{(B)} + h_{\mu\nu} \;, \quad g_{\mu\nu}^{(B)} \equiv \mathrm{diag}(-1,+1,+1,+1) \;; \tag{2.8}$$

and let $\bar{h}_{\mu\nu}$ be the trace-reversed metric perturbation:

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}^{(B)}\, h \quad , \tag{2.9}$$

where $h \equiv g^{(B)\mu\nu} h_{\mu\nu}$. The indices of $h_{\mu\nu}$ and $\bar{h}_{\mu\nu}$ are raised and lowered using $g_{\mu\nu}^{(B)}$.

To linear order in the metric perturbation, the field equations in any preferred coordinate system are given by

$$\eta^{\alpha\beta}\bar{h}_{\mu\nu,\alpha\beta} = -16\pi\ (-g/-\eta)^{1/2}\ G_o T_{\mu\nu} \quad . \tag{2.10a}$$

Here $\eta^{\alpha\beta}$ is the inverse of $\eta_{\alpha\beta}$; g and η are the determinants of $g_{\mu\nu}$ and $\eta_{\mu\nu}$, respectively; and G_o is a coupling constant with dimensions of the Newtonian gravitational constant (see [9]). In LURF coordinates, Eq. (2.10a) becomes

$$\nabla^2\bar{h}_{\mu\nu} - (1/v_g^2)\ \bar{h}_{\mu\nu,00} = -16\pi\ GT_{\mu\nu} \quad . \tag{2.10b}$$

Here $G = (c_0 c_1)^{1/2} G_o$ is the gravitational "constant" at the epoch of interest (as measured, e.g., by a Cavendish experiment performed far away from any local density enhancements), and v_g -- the speed of gravitational radiation in the LURF -- is given by Eqs. (1.2). Equation (1.2a) uses the results of [9] to relate v_{gc} to α_2 [$v_{gc}^2 = c_1/c_0 = (1 + \alpha_2)^{-1}$].

The linearized matter-response equations are obtained from Eq. (1.1):

$$T^{\mu\nu}{}_{,\nu} = 0 \quad . \tag{2.11}$$

Just as in general relativity or any other metric theory, gravitational effects disappear from the matter-response equations at linear order; the linear approximation is valid only so long as the motion of the source is governed by nongravitational forces.

To analyze gravitational radiation emitted by a source, one must be able to calculate the energy and momentum carried by the radiation. Rosen [1] has demonstrated the existence of a stress-energy complex $\Theta_\mu{}^\nu$ which is conserved with respect to the flat metric:

$$\Theta_{\mu}{}^{\nu}{}_{|\nu} = 0 \qquad , \tag{2.12a}$$

$$\Theta_{\mu}{}^{\nu} = (-g/-\eta)^{1/2}\,(T_{\mu}{}^{\nu} + t_{\mu}{}^{\nu}) \qquad . \tag{2.12b}$$

The tensor $t_{\mu}{}^{\nu}$ is interpreted as the gravitational stress-energy; it is a quadratic expression in first derivatives of $g_{\mu\nu}$ with respect to $\eta_{\mu\nu}$ $(g_{\mu\nu|\alpha})$. To lowest order in the metric perturbation, it is given in any preferred coordinate system by

$$t_{\mu}{}^{\nu} = \frac{1}{32\pi G_o}\,(-\eta/-g)^{1/2}\left(\delta_{\mu}{}^{\alpha}\eta^{\nu\beta} - \frac{1}{2}\,\delta_{\mu}{}^{\nu}\eta^{\alpha\beta}\right)\left(\bar{h}^{\gamma\delta}{}_{,\alpha}\bar{h}_{\gamma\delta,\beta} - \frac{1}{2}\,\bar{h}_{,\alpha}\bar{h}_{,\beta}\right) \quad , \tag{2.13}$$

where $\bar{h} \equiv g^{(B)\mu\nu}\bar{h}_{\mu\nu}$.

Equations (2.12) can be integrated to obtain conservation laws for the total 4-momentum. In a preferred coordinate system, the 4-momentum P_{α} of the source is defined by

$$P_{\alpha} \equiv \int (-g)^{1/2}\,(T_{\alpha}{}^{0} + t_{\alpha}{}^{0})\,d^3x \qquad . \tag{2.14}$$

P_{α} transforms like a 4-vector under Lorentz transformations among the preferred coordinates, and its indices are raised and lowered using $g^{(B)}_{\mu\nu}$. Now surround the source with a closed 2-surface $\mathcal{S}$ which lies in the transition region, and let $\underset{\sim}{n}$ be the unit outward normal (with respect to $g^{(B)}_{\mu\nu}$) to $\mathcal{S}$. The conservation law (2.12) relates the change of 4-momentum inside $\mathcal{S}$ to a flux of 4-momentum through $\mathcal{S}$:

$$\frac{dP^{\alpha}}{dt} = -\int_{\mathcal{S}} (T^{\alpha j} + t^{\alpha j})n_j\, dA \qquad . \tag{2.15}$$

Here the first index of $t_{\mu}{}^{\nu}$ has been raised using $g^{(B)}_{\mu\nu}$.

The important quantity for calculating energy loss due to gravitational radiation is t^{0j}, the energy flux in the radiation. Its form (to lowest order in the metric perturbation) is particularly simple in LURF coordinates:

$$t^{0j} = -\frac{1}{32\pi G}\left(\bar{h}^{\gamma\delta}{}_{,0}\,\bar{h}_{\gamma\delta,j} - \bar{h}_{,0}\,\bar{h}_{,j}\right) \quad . \tag{2.16}$$

Note that the field equations (2.10a) and the gravitational stress-energy (2.13) are not invariant under infinitesimal coordinate (gauge) transformations. This lack of invariance reflects the fact that a gauge transformation destroys Property 2 of the preferred coordinates, i.e., $\eta_{\alpha\beta}$ does not remain constant on local scales. In such coordinates, $g^{(B)}_{\alpha\beta|\gamma} \neq 0$ and terms containing $g^{(B)}_{\alpha\beta|\gamma}$ and $g^{(B)}_{\alpha\beta|\gamma\delta}$ appear in the linearized field equations and in the gravitational stress-energy.

3. GRAVITATIONAL RADIATION AND THE GRAVITATIONAL SPEED LIMIT

In Rosen's theory the speed of gravitational radiation is determined by the combined effects of the cosmological gravitational field and the gravitational fields of nearby, local concentrations of matter [Eqs. (1.2)]. Although the latter always tend to increase v_g, the cosmological field can force v_g to be less than the speed of light. It is this case -- $v_g < 1$ -- that I consider in this section; in particular, I investigate the gravitational radiation emitted by particles moving at speeds near v_g.

The motivation for doing so is provided by an analogy with electromagnetism. A charged particle, moving through a material medium at a speed faster than the speed of light in the medium, emits electromagnetic Cherenkov [16] radiation. In any real medium dispersion restricts the radiation to a finite range of frequencies; however, for an idealized, dispersionless medium, the energy emitted diverges. Similarly, in Rosen's theory, a particle which exceeds the speed of gravitational radiation ought to emit <u>gravitational</u>

Cherenkov radiation. Moreover, the gravitational "medium" is dispersionless (at least at high frequencies), so the electromagnetic analogy suggests that the energy emitted ought to diverge. If so, this result would suggest that particles cannot exceed the speed of gravitational radiation.

These ideas were first considered in a different context by Aichelburg, Ecker, and Sexl [17]. They considered a particle whose equation of motion apparently allows it to exceed the speed of light, but which is coupled to a field that propagates at the speed of light. They argued that radiation reaction prohibits accelerating the particle to speeds greater than the speed of light. They showed, for example, that if such a particle is charged, the electromagnetic radiation it emits diverges as it is accelerated through the light cone. The situation considered here is similar, and the analysis is patterned after their work. I shall first consider the gravitational Cherenkov problem and then analyze the power radiated in gravitational radiation as a particle is accelerated through the gravitational "light" cone.

A. Gravitational Cherenkov Radiation

Consider a particle with rest mass m_o moving with uniform velocity $\underset{\sim}{v}$ relative to the LURF; let $v > v_g$. In the case of interest, v_g is very close to the speed of light. Adopt LURF coordinates and solve for the gravitational field in the linear approximation. The solution of the field equations (2.10b) proceeds exactly as in the analogous electromagnetic problem (see, e.g., [18], Section 14.9). The metric perturbation $h_{\mu\nu}$ forms a shock front along a cone which extends back from the instantaneous position of the particle (see Fig. 1); the angle θ_C between the velocity $\underset{\sim}{v}$ and the normal to the cone is given by $\cos\theta_C = (v_g/v)$. Outside the cone $\bar{h}_{\mu\nu}$ vanishes; inside the cone,

$$\bar{h}_{00}(\underset{\sim}{x},t) = \frac{8G\gamma m_o}{|\underset{\sim}{x} - \underset{\sim}{v}t|[1 - (v/v_g)^2 \sin^2\alpha]^{1/2}} \quad , \tag{3.1a}$$

$$\bar{h}_{0j} = -\bar{h}_{00}\, v^j \quad , \tag{3.1b}$$

$$\bar{h}_{jk} = \bar{h}_{00}\, v^j v^k \quad , \tag{3.1c}$$

where α is the angle between the observation point $\underset{\sim}{x}$ and the velocity $\underset{\sim}{v}$, and $\gamma \equiv (1-v^2)^{-1/2}$ is the particle's Lorentz factor. The field (3.1) represents gravitational Cherenkov radiation propagating in the direction normal to the Cherenkov cone. By evaluating the energy flux using (2.16) and then following the procedure used for electromagnetic Cherenkov radiation, one obtains the energy d^2E radiated into an angular frequency interval $d\omega$ as the particle moves a distance $d\ell$:

$$\frac{d^2E}{d\omega d\ell} = Gm_o^2\omega(v^{-2} - 1) \qquad \text{for} \quad v > v_g \quad . \tag{3.2}$$

This expression is similar to the Frank-Tamm [19] result for electromagnetic Cherenkov radiation.

Equation (3.2) does not, of course, hold for all frequencies, and it is important to determine its region of validity. In using the formalism of Section 2, the above analysis neglects variations in the external gravitational field. However, since the particle is assumed to radiate for an infinite amount of time, these variations cannot be ignored; their effect is to modify Eq. (3.2) at low frequencies. To estimate the frequency at which such modification becomes important, consider a particle which radiates for only a finite time $T \sim (r_1/v)$ [see Eq. (2.3b)]. Then the particle's motion and the radiation it emits can be analyzed within the transition region, where the formalism developed in Section 2 is applicable. The emitted radiation is a pulse which lies just inside the Cherenkov cone (see Fig. 1). It is easy to show that, when the radiation is analyzed at a distance $\sim r_1$ from the particle's trajectory, the pulse has a duration $\Delta t \sim [(v/v_g) - 1](T/8)$. Thus the energy spectrum will be given by Eq. (3.2) for frequencies $\omega \gtrsim \omega_c \equiv (1/\Delta t)$. The wavelength corresponding to the critical frequency ω_c is

$$\lambda_c \sim \varepsilon(v_g^{-1} - v^{-1}) \cdot \min\{a,L\} \simeq \frac{1}{2}\,\varepsilon\,(\gamma_g^{-2} - \gamma^{-2}) \cdot \min\{a,L\} \quad , \qquad (3.3)$$

where $\gamma_g \equiv (1 - v_g^2)^{-1/2}$. I have confirmed this result by a detailed analysis of the radiation emitted by a particle which moves faster than v_g for only a finite time.

Variations in the external gravitational field can be regarded as producing "dispersion" in the propagation of gravitational radiation. This dispersion modifies the Cherenkov spectrum at frequencies below ω_c. Note that, as v approaches v_g, ω_c increases and dispersion affects more of the spectrum. Even when γ is not close to γ_g, the critical wavelength λ_c is typically rather small. For example, for a particle near the earth, the relevant external scale is of order the radius of the earth: $L \sim 10^9$ cm. Using the smallest value of v_g allowed by the limits obtained in Section 4 [see Eq. (4.2b)] and choosing $\varepsilon = 10^{-2}$, one finds $\lambda_c \sim (10^{-11}\text{cm})[1 - (\gamma_g/\gamma)^2]$.

The importance of the preceding analysis lies not so much in estimating the size of λ_c, but rather in demonstrating that, as long as $v > v_g$, there is a <u>finite</u> critical frequency above which the gravitational "medium" is dispersionless and Eq. (3.2) applies. In the purely classical analysis given above, the validity of Eq. (3.2) extends to arbitrarily high frequencies, and the spectrum diverges as $\omega \rightarrow \infty$. However, quantum mechanics often eliminates classical divergences, and one might expect a proper quantum-mechanical treatment to modify the classical spectrum at very high frequencies. In particular, conservation of energy might seem to require that the spectrum be cut off at a frequency $\omega_{max} = (\gamma m_o/\hbar)$ corresponding to emission of a graviton whose energy is equal to the particle's energy. Applying this cutoff to Eq. (3.2) (and assuming $\omega_{max} >> \omega_c$), one finds an energy loss rate

$$\frac{dE}{d\ell} = \frac{Gm_o^4}{2v^2\hbar^2} \sim 10^{-16} \text{ eV - cm}^{-1} \quad \text{for protons .} \tag{3.4}$$

This energy loss rate is so small that, if there is a cutoff at ω_{max}, the effects of gravitational Cherenkov radiation are negligible even on galactic distance scales.

However, the existence of the cutoff is by no means certain. The uncertainty arises because it is not clear that Rosen's theory, even in its linearized version, can be quantized; the linearized field equations (2.10a) are not those of a canonical field theory. The difficulties that thereby arise are perhaps most apparent in an examination of plane gravitational waves in Rosen's theory:

(i) The Riemann tensor derived from an arbitrary plane wave has six independent polarizations -- the most general polarization structure allowed in a metric theory. Even in the case $v_g = 1$, where the theory is Lorentz-invariant, these six polarizations form a nonunitary representation of the inhomogeneous Lorentz group; they cannot be associated with massless quanta of definite, Lorentz-invariant helicity (see [20] for a general discussion of these issues).

(ii) The time-averaged energy density in an arbitrary plane wave ($v_g \neq 1$), evaluated using (2.13), can be regarded as a quadratic form in the amplitudes of the ten independent potentials $\bar{h}_{\mu\nu}$. (None of these ten potentials can be removed from the energy density by a gauge transformation; see Section 2B.) When this quadratic form is diagonalized, one finds that four of the eigenvalues are negative. In other words, four of the ten degrees of freedom in the wave carry negative energy. The presence of negative-energy radiation has been noted previously in analyses of radiation emitted by binary systems in Rosen's theory [13]. In a theory with such negative-energy radiation, the stability of the vacuum is uncertain.

Any attempt to quantize Rosen's theory must confront these two problems. Even if they can be overcome, the presence of negative-energy radiation removes the raison d'etre for a cutoff at ω_{max}. The classical Cherenkov radiation (3.1) is made up of both positive- and negative-energy parts, the total energy emitted being a balance between the two. Quantum-mechanically, this Cherenkov emission might well be represented by multi-graviton processes in which both positive- and negative-energy gravitons are emitted. In such processes, conservation of energy imposes no restrictions on the frequency of the emitted gravitons.

Another potential quantum-mechanical cutoff is the Planck frequency $\omega_p \equiv (G\hbar)^{-1/2}$. If the Cherenkov spectrum (3.2) is cut off at ω_p, the energy loss rate becomes

$$\frac{dE}{d\ell} \simeq \frac{\gamma m_o}{2\gamma^3(\hbar/m_o)} \sim \frac{\gamma m_o}{\gamma^3(5\times 10^{-14}\,\mathrm{cm})} \qquad \text{for protons .} \tag{3.5}$$

Just as for the cutoff at ω_{max}, it is not clear that this cutoff should be imposed. However, even if it is, the loss rate (3.5) is large enough that the limits obtained in Section 4 are not affected.

The classical analysis of gravitational Cherenkov radiation hints at a serious problem in Rosen's theory. The divergence of the spectrum as $\omega \rightarrow \infty$ means that the energy emitted is infinite (and positive). This result strongly suggests that particles cannot exceed the speed of gravitational radiation. It is not clear that a quantum-mechanical treatment will eliminate the divergence, nor indeed that such a treatment can be given. Even as a purely classical analysis, the above calculation has serious difficulties: it is clearly inconsistent and, just as clearly, the linear approximation is not valid. However, there is little point in trying to patch up these difficulties. If particles cannot exceed the speed of gravitational radiation, a consistent calculation of gravitational Cherenkov radiation is not possible. More realistic and more relevant would be an examination of what happens as a particle is accelerated up to the speed of gravitational radiation.

Before turning to this problem, it is interesting to ask about the Cherenkov radiation emitted by photons and other zero rest-mass particles. The best that can be done using the above calculation is to model a free photon as the limit $v \rightarrow 1$, $\gamma \rightarrow \infty$, $\gamma m_o \rightarrow$ constant. Applying this limit to Eq. (3.2), one finds that free photons apparently do not produce any gravitational Cherenkov radiation.

B. Acceleration through the Gravitational "Light" Cone

Now consider a particle with rest mass m_o which has velocity $\underset{\sim}{v}(t)$ in LURF coordinates. The particle is being accelerated by interactions with other matter and nongravitational fields. The objective is to evaluate, in the linear approximation, the energy emitted in gravitational radiation as v approaches v_g. In doing so, one must remember that the total stress-energy is conserved [Eq. (2.11)]. This means that one cannot, in general, neglect the radiation emitted by the matter and nongravitational fields as they "recoil" from the interaction. However, I shall argue that in the case of interest here, this "recoil" radiation can be neglected.

Imagine the following scenario for accelerating the particle -- a scenario similar to those often envisioned for accelerating cosmic rays [21]. The particle is accelerated by a series of "collisions" with local concentrations of stress-energy. These "blobs" of stress-energy have masses much larger than m_o, and their velocities -- both center-of-mass and internal -- relative to the LURF are small. In each collision, the momentum exchanged is small compared to the particle's momentum. The subsequent motion of the "blob" occurs on time scales much longer than the collision time; clearly, the radiation emitted by the "blob" does not diverge. Now consider the final stage of the acceleration process, when after many collisions the particle has attained a velocity so close to v_g that one more collision can push its velocity above v_g. From the point of view of the particular "blob" involved, this collision is no different from the preceding ones. However, the radiation produced by the particle in this collision is beamed in the direction of its velocity, and the radiation diverges in that direction as v approaches v_g. Therefore, in analyzing the final stage of the acceleration process, one can neglect the "recoil" motion and calculate the radiation emitted by considering only the particle's motion. The results obtained will be valid when v is very close to v_g.

The field equations (2.10b) for a single-particle source can be solved in the same way as in electromagnetism (see, e.g., [18], Sec. 14.1). The $\bar{h}_{\mu\nu}$ have the same form as the Liénard-Wiechert potentials. The energy flux is evaluated using Eq. (2.16), and an integral over a sphere in the transition region gives the power radiated:

$$\frac{dE}{dt} = \frac{1}{3}\frac{Gm_o^2 v_g^3}{(1-\mu^2)^4}\left[\gamma^6(\underset{\sim}{v}\cdot\dot{\underset{\sim}{v}})^2(-11+26\mu^2-9\mu^4) + \gamma^4\dot{\underset{\sim}{v}}^2(1-\mu^2)(-11+12\mu^2)\right]$$

$$\longrightarrow Gm_o^2 v_g^3\left[2\gamma^6(\underset{\sim}{v}\cdot\dot{\underset{\sim}{v}})^2(1-\mu^2)^{-4} + \frac{1}{3}\gamma^4\dot{\underset{\sim}{v}}^2(1-\mu^2)^{-3}\right] \quad , \tag{3.6}$$

where $\mu \equiv \gamma/\gamma_g$, and where the last expression contains the leading-order terms in the limit $v \to v_g$. As anticipated, the power radiated diverges. In a real situation, of course, the power radiated cannot diverge; instead, radiation reaction diverges and prevents the particle from exceeding the speed of gravitational radiation.

This calculation suffers from some of the same difficulties as the Cherenkov calculation. The particle radiates substantial amounts of energy only at very high frequencies where quantum corrections might well be important. For the reasons given earlier, the effect of these corrections is uncertain, and I shall ignore them. A perhaps more serious objection is that the linear approximation is not valid; however, it seems unlikely that the nonlinear terms in the field equations can eliminate the divergences that have cropped up in both the preceding problems.

Despite its uncertainties, the analysis of Rosen's theory in this section leads one to the following tentative conclusion: if $v_g < 1$, the speed of gravitational radiation is the ultimate speed for particles of nonzero rest mass -- a "speed limit" enforced by the emission of gravitational radiation. Hence, observations of relativistic particles can be used to place limits on the speed of gravitational radiation [Eq. (1.3a)] and on the cosmological boundary values [Eq. (1.3b)].

4. OBSERVATIONAL CONSTRAINTS

The highest-energy particles in the vicinity of the earth are ultra-high-energy cosmic rays, which have been detected at energies exceeding 10^{20} eV (see [22] for a review of the observations). At these very high energies, cosmic rays are not observed directly; rather, they are detected by the air shower they produce as they enter the atmosphere. The energy assigned to the primary particle in a given event is somewhat uncertain, since it is derived from a model for the shower. However, an energy of 3×10^{19} eV seems reasonably firm.

This energy estimate, even if correct, is not a measurement of velocity. One obtains a velocity by using the familiar relation $E = \gamma m_o$. However, one might expect this relation to fail in Rosen's theory, because a particle's gravitational binding energy might diverge as v approaches v_g. Indeed, an analysis using the linear approximation suggests that the energy of a particle, as measured by an observer at rest in the LURF, diverges logarithmically:

$$E = \gamma m_o + \gamma q \, \Omega_o \log\{2v_g[1 - (\gamma/\gamma_g)^2]^{-1/2}\} \quad , \tag{4.1}$$

where Ω_o is the gravitational binding energy when the particle is at rest in the LURF, and q is a dimensionless quantity which depends on the structure of the particle. This divergence is one more reason why particles cannot exceed the speed of gravitational radiation.

The logarithmic divergence (4.1) is slow enough that it does not interfere with interpretation of the cosmic-ray observations. If a particle's speed is so close to v_g that the binding-energy term in (4.1) dominates, then Eq. (3.6) predicts that the particle will radiate away almost all its energy as gravitational radiation. It will not produce the observed shower of particles.

A more serious uncertainty results from the inability to identify the primary particle. The most likely candidates are protons or, perhaps, alpha particles; however, the possibility of heavier nuclei -- perhaps nuclei near iron -- has not been ruled out. For a proton at 3×10^{19} eV, the limit (1.3a) on the speed of gravitational radiation near the earth today is

$$1 - v_g \lesssim 5 \times 10^{-22} \quad . \tag{4.2a}$$

For an iron nucleus at the same energy, the limit is a bit weaker:

$$1 - v_g \lesssim 1 \times 10^{-18} \quad . \tag{4.2b}$$

Since v_g increases toward the galactic center, these limits also apply at any point closer to the galactic center than the earth.

Equations (4.2) actually hold not only at the earth but also in those regions traversed by the cosmic rays after their initial acceleration. Unfortunately, the point of origin of ultra-high-energy cosmic rays is uncertain. Their Larmor radii in the galactic magnetic field are much larger than the thickness of the galactic disk. This, together with the lack of anisotropy in the observed events [23], means that, if they are galactic in origin, they must come from a distance less than the thickness of the disk ($\sim$ 200 pc). It seems more likely that they are extragalactic, in which case Eqs. (4.2) probably apply out to a distance of at least 100 Mpc.

Earth-based observations of relativistic particles also provide an upper bound on the value of α_2 (v_{gc}) today [Eq. (1.3b)]. This limit is considerably less stringent than the limit on v_g because it is determined by the Newtonian potential at the earth, which is dominated by the galactic potential U_{gal}. A γ particle with $\gamma \gtrsim 10^3$ -- a medium-energy cosmic-ray proton or electron, a positron or electron produced in a high-energy collision at Fermi Lab or CERN or circulating in a storage ring at SLAC or DESY -- yields the same limit:

$$\alpha_2\Big|_{today} \lesssim 4U_{gal} \sim 3\times10^{-6} \quad . \qquad (4.3)$$

Here I have used a galactic mass of $1.4\times10^{11}M_{\odot}$ at a distance of 10 kpc. For positive values of α_2, this limit (valid only in Rosen's theory) is almost three orders of magnitude better than the best previous limit, obtained by searching for anomalous earth tides [24].

There is a possibility that the Newtonian potential of the Virgo cluster at the earth is as large as the galactic potential. However, there is considerable uncertainty in estimating the mass of the Virgo cluster, and the two potentials are comparable only for the largest estimates. In any case, including the potential of the Virgo cluster is not likely to degrade the limit (4.3) by more than a factor of two.

Compact radio sources at substantial red shifts provide information about the speed of gravitational radiation in the past. They emit a power-law radio spectrum which is thought to be incoherent electron-synchrotron radiation; the spectrum has a low-frequency turnover attributed to synchrotron self-absorption. The Lorentz factor of the electrons can be estimated from the brightness temperature T_b at the turnover frequency: $\gamma \sim (kT_b/m_e)$, where m_e is the rest mass of an electron. Jones, O'Dell, and Stein [25] have developed a detailed model for compact, nonthermal sources, including the effects of synchrotron self-absorption and synchrotron self-Compton radiation. Burbidge, Jones, and O'Dell [26] have applied the model to several compact sources, some of which have more than one component (see their Tables 1-3). For nine of the ten sources in their sample, they provide (for one or more of the components) a red shift, an angular diameter determined by VLBI, a size determined from the angular diameter by placing the source at its red-shift distance, and a Lorentz factor determined by the model. To estimate a Newtonian potential for each component, I have assumed a mass of $10^9 M_\odot$ — a mass larger than or of order those usually thought to be associated with active galactic nuclei; and I have assumed a constant (nonevolving) gravitational constant G. As an example, consider the source with the largest red shift in their sample — PKS 2134 + 00.4 at $z = 1.936$. The estimates for one of its two components are $\gamma \sim 590$ and $U \sim 1 \times 10^{-5}$, which implies $\alpha_2 < 4 \times 10^{-5}$ [Eq. (1.3b)]. Similar considerations for the other sources provide upper bounds on α_2 at a variety of red shifts; considering all these limits together, one can conclude that

$$\alpha_2 \lesssim 5 \times 10^{-4} \qquad \text{for} \qquad 0 \lesssim z \lesssim 2 \quad . \tag{4.4}$$

No other observation provides information about the value of α_2 (v_{gc}) in the past.

There are considerable uncertainties in estimating the Newtonian potentials which go into the limit (4.4). The masses and radii of the sources are uncertain; in addition, the gravitational "constant" does evolve in Rosen's theory, so that its value in the past depends on the cosmological model. Because of these uncertainties, the limit (4.4) has been chosen conservatively; with the above assumptions, all but one of the six sources at $z > 0.1$ provide a limit at least an order of magnitude stronger than (4.4).

5. CONCLUSION

Rosen [27] has recently modified his "bimetric" theory. In the modified version, the "background" metric $\eta_{\mu\nu}$ is no longer required to be flat;

instead, it is required to be a space of constant curvature. Cosmological models are affected by this modification, but local gravitation physics is not, except insofar as it is influenced by cosmological boundary values. The analysis in Sections 2 and 3 remains the same, and the limits obtained in Section 4 apply to the new version of the theory.

The analysis in this paper has been restricted to Rosen's theory, but the results obtained are likely to have far wider applicability. There are numerous metric theories of gravity which predict different speeds for gravitational radiation and light. Typically in such theories, the difference in speed is produced just as in Rosen's theory: light propagates along "light" cones of the physical metric, while gravitational radiation propagates along "light" cones of a flat, "background" metric. In all such theories, one expects the speed of gravitational radiation to have a form similar to that in Rosen's theory: $v_g = v_{gc}(1 + 2\xi U)$, where v_{gc} is determined by cosmological boundary values and ξ is a constant of order unity. The important question is whether emission of gravitational radiation restricts particles to speeds less than v_g. Although detailed calculations are necessary in each theory, one can give a general argument, based on the analysis in Rosen's theory, for the existence of the "gravitational speed limit."

Whenever a particle exceeds the speed of propagation of a "radiation" field to which it is coupled, one expects a shock wave to form. One can think of numerous examples, such as the shock wave produced by supersonic motion in an acoustic medium and electromagnetic Cherenkov radiation. In these familiar examples, the radiation does not diverge because the shock front is not absolutely sharp; it is spread out over some length d characteristic of the medium through which the radiation is propagating. This "blurring" of the shock front cuts off the radiation at frequencies $\gtrsim v_{wave}/d$. In the gravitational case, the "medium" is spacetime itself, or more accurately, the "background" structure on spacetime which determines the speed of gravitational radiation. The gravitational "medium" has no small-scale structure to blur the shock front. Thus, there is no high-frequency cutoff (unless quantum mechanics introduces one), and the radiation does diverge.

This argument makes it seem quite likely that any theory with a variable speed of gravitational radiation must confront the limits obtained in Section 4. If so, Eqs. (4.3) and (4.4) can be used to constrain the cosmological boundary values in any such theory. [In general, the α_2 of these limits is not a PPN parameter; it is simply a parameter related to v_{gc} by Eq. (1.2a).] In addition, Eqs. (4.2) provide a general, theory-independent lower bound on the speed of gravitational radiation near the earth.

ACKNOWLEDGMENTS

I thank Kip S. Thorne and Mark Zimmermann for frank and productive discussions. I also acknowledge the support of a Feynman Fellowship at Caltech during part of the time this research was being carried out.

FOOTNOTES

[1]Throughout I use the summation convention, Greek indices running from 0 to 3 and Latin indices from 1 to 3. The signatures of the metrics are +2. A semicolon (;) denotes a covariant derivative with respect to $g_{\mu\nu}$, a vertical bar (|) a covariant derivative with respect to $\eta_{\mu\nu}$, and a comma an ordinary partial derivative. Units are chosen so that the speed of light $c = 1$.

REFERENCES

1. N. ROSEN, J. Gen. Rel. and Grav. 4 (1973), 435; Ann. Phys. (N.Y.) 84 (1974), 455.

2. C. M. WILL, The Theoretical Tools of Experimental Gravitation, in "Experimental Gravitation: Proceedings of Course 56 of the International School of Physics 'Enrico Fermi'" (B. Bertotti, Ed.), p. 1, Academic Press, New York, 1974.

3. J. ROSEN and N. ROSEN, Astrophys. J. 202 (1975), 782; 212 (1977), 605; I. GOLDMAN and N. ROSEN, Astrophys. J. 225 (1978), 708.

4. D. BABALA, J. Phys. A 8 (1975), 1409; I. GOLDMAN and N. ROSEN, J. Gen. Rel. and Grav. 7 (1976), 895; Astrophys. J. 212 (1977), 602.

5. M. ISRAELIT, J. Gen. Rel. and Grav. 7 (1976), 623; 7 (1976), 857.

6. I. GOLDMAN, J. Gen. Rel. and Grav. 9 (1978), 575.

7. I. GOLDMAN, J. Gen. Rel. and Grav. 7 (1976), 681; I. GOLDMAN and N. ROSEN, J. Gen. Rel. and Grav. 8 (1977), 617; D. FALIK and R. OPHER, Astron. and Astrophys. 71 (1979), 332; J. Gen. Rel. and Grav. 10 (1979), 227.

8. C. M. WILL, The Confrontation between Gravitation Theory and Experiment, in "Gravitational Theories Since Einstein" (S. W. Hawking and W. Israel, Eds.), Cambridge University Press, Cambridge, 1979.

9. D. L. LEE, C. M. CAVES, W.-T. NI, and C. M. WILL, Astrophys. J. 206 (1976), 555.

10. C. W. MISNER, K. S. THORNE, and J. A. WHEELER, "Gravitation," Freeman, San Francisco, 1973.

11. K. S. THORNE, How to Test Gravitation Theories by Means of Gravitational-Wave Experiments, in "Colleques Internationaux C.N.R.S. No. 220 — Ondes et Radiations Gravitationelles," p. 213, Institut Henri Poincare, Paris, 1974.

12. R. A. HULSE and J. H. TAYLOR, Astrophys. J. (Letters) 195 (1975), L51.

13. C. M. WILL and D. M. EARDLEY, Astrophys. J. (Letters) 212 (1977), L91; C. M. WILL, Astrophys. J. 214 (1977), 826.

14. J. H. TAYLOR, L. A. FOWLER, and P. M. McCULLOCH, Nature 277 (1979), 437.

15. F. K. MANASSE and C. W. MISNER, J. Math. Phys. 4 (1963), 735; W.-T. NI and M. ZIMMERMANN, Phys. Rev. D 17 (1978), 1473.

16. P. A. CHERENKOV, Dokl. Akad. Nauk. SSSR 2 (1934), 451.

17. P. C. AICHELBURG, G. ECKER, and R. U. SEXL, Nuevo Cimento 2B (1971), 63.

18. J. D. JACKSON, "Classical Electrodynamics," 1st ed., Wiley, New York, 1962.

19. I. FRANK and I. TAMM, Dokl. Akad. Nauk. SSSR 14 (1937), 109.

20. D. M. EARDLEY, D. L. LEE, A. P. LIGHTMAN, R. V. WAGONER, and C. M. WILL, Phys. Rev. Letters 30 (1973), 884; D. M. EARDLEY, D. L. LEE, and A. P. LIGHTMAN, Phys. Rev. D 8 (1973), 3308.

21. E. FERMI, Phys. Rev. 75 (1949), 1169.

22. A. A. WATSON, Proc. 14th Int. Cosmic Ray Conf. 11 (1975), 4019.

23. A. M. HILLAS, Phil. Trans. R. Soc. London A 277 (1974), 413.

24. K. NORDTVEDT, JR., and C. M. WILL, Astrophys. J. 177 (1972), 775. R. J. WARBURTON and J. M. GOODKIND, Astrophys. J. 208 (1976), 881.

25. T. W. JONES, S. L. O'DELL, and W. A. STEIN, Astrophys. J. 188 (1974), 353.

26. G. R. BURBIDGE, T. W. JONES, and S. L. O'DELL, Astrophys. J. 193 (1974), 43.

27. N. ROSEN, J. Gen. Rel. and Grav. 9 (1978), 339.

FIGURE CAPTION

Figure 1. A "snapshot", taken at time t, of the Cherenkov cone produced by a particle moving with uniform velocity $v = 2v_g$ along the z-axis. The particle is at the apex of the cone. The angle θ_C between the normal to the cone and the z-axis is given by $\cos\theta_C = (v_g/v)$. The shaded region is the pulse of Cherenkov radiation produced by a particle which radiates from $t = 0$ to $t = T$.

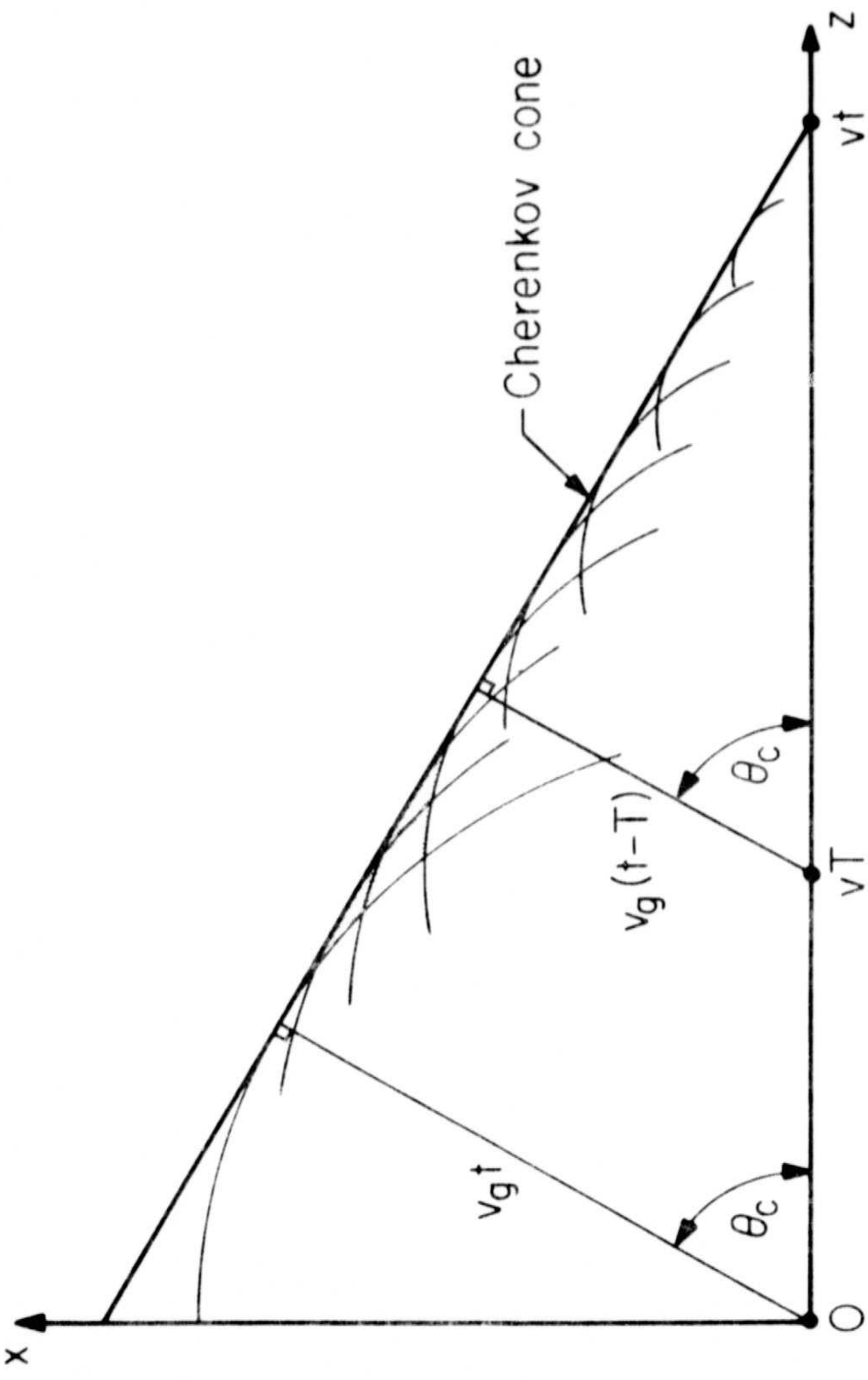

Fig. 1