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NUMERICAL MODELLING OF THE FORMATION PROCESS
OF PLANETS FROM PROTOPLANETARY CLOUD

N. N. Kozlov and T. M. Eneyev

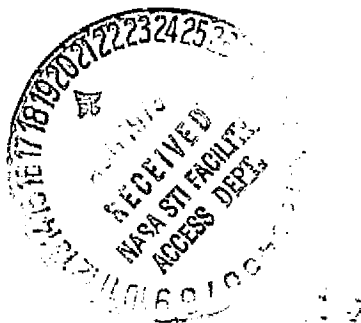| 1. Report No. NASA TM-75642	2. Government Accession No.	3. Recipient's Catalog No.
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16. Abstract

Evolution of the plane protoplanetary cloud, consistingof a great number of gravitationally interacting and uniting under collision bodies (protoplanets) moving in the central field of a large mass (the Sun or a planet), is considered. It is assumed that gravitational interaction between bodies takes place only under their binary close approach. It is also assumed that between the two close approaches the bodies move in Keplerian orbits, and orbits of all bodies being circular at the initial moment of evolution. It is shown that in the course of protoplanetary cloud evolution the ring zones of matter expansion and compression occur with the subsequent development leading to formation of planets, rotating about their axes mainly directly. The principal numerical results have been obtained through digital simulation of planetary accumulation.

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NUMERICAL MODELLING OF THE FORMATION PROCESS OF PLANETS FROM PROTOPLANETARY CLOUD

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1. INTRODUCTION

Modern nebular theories of the origin of planets of the solar system have been formulated fairly precisely into two main problems of planetary cosmogony -- the problem of the origin of a protoplanetary cloud and the problem of formation of planets from this cloud. Very probably no such sharp division exists between the two problems indicated, however, it is reasonable and advantageous, at least in a first approximation, to consider both problems independently of each other. This article presents a study and development of the second problem, that is, the problem of formation of planets from a protoplanetary cloud which is already formed around the Sun.

A gas and dust protoplanetary cloud which is already formed around the Sun has, obviously, passed through a number of stages in the course of its evolution. One can propose that the initial stage of evolution of the cloud was characterized by a dust component in the cloud thanks to the mutual non-elastic collision of its particles and also from friction of the gas, and that it gradually began to settle in an equatorial plane forming a more or less thin disk with high density of matter [1,2]. The opinion of scientists varies as to the character of the last stages of development of the structure of a dust disk. According to one concept, during a gradual decrease in thickness of the disk, coagulation of the dust particles occurred combining into larger and larger blocks until the bodies were of asteroid dimensions [3]. The basis of another concept is the theory of gravitation instability of a dust disk. The dust disk gradually thinning, bringing its density to

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* Numbers in the margin indicate pagination in the foreign text.

the critical Rosh density is broken down into many dust clots in which the internal gravitational forces are larger than the perturbing (influx) forces of the Sun. The dust clots,continuing to evolve with compression and partially combining with each other, also in the end form more or less dense bodies of asteroid dimensions [1]. Thus, both concepts agree that in a certain interval of time, in the course of evolution of a protoplanetary cloud, planet-like bodies with relatively small dimensions moved in its equatorial plane; these bodies had almost circular orbits initially. The moment of evolution of the cloud, that is, the moment when in its equatorial plane a cluster was formed of more or less dense masses moving in a circular orbit; this moment is the initial point of the study whose results are presented in this article. /6

The study of the process of evolution of a cluster of gravitationally interdependent colliding masses moving around the Sun is extremely important and has great significance for planetary cosmogony, since it is possible that as a result of this process, the solar planetary system was formed.

The Kant LaPlace ideas as to the accumulation of planets from dust or gaseous matter of a protoplanetary cloud rotating around the Sun first appeared in 1943-44 in the works of O. Yu. Shmidt and K. Veytsekker; then in 1946 O. Yu. Shmidt developed a fairly detailed mechanism for the process of accumulation of planets based on a gradual increase in the embryo planet, by exhausting the matter surrounding the latter whose particles moved in a very eccentric orbit [4]. Since then, the problem of the mechanism of accumulation of planets has attracted the continuous attention of scientists and a considerable number of them have continued to develop the system proposed in 1946 by O. Yu. Shmidt. In this connection one should note that the Shmidt system, having undergone

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fairly significant changes and modifications over three decades, has still retained its main characteristics [5]. The mechanism of exhaustion by the embryo (or embryos) of the planets of particles of matter of a cloud moving around the Sun in orbit with fairly large eccentrics (approximately 0.2) is the basis for processes of accumulation of planets studied by scientists who more or less support the O. Yu. Shmidt system. It is not our purpose to give a historical survey in this article of works which to one or another degree touch on the O. Yu. Shmidt system, but we will mention two very interesting works done in the last decade inasmuch as these works are similar to the work presented in this article in their research method.

In 1970, S. H. Dole published an article in which the results of a numerical modelling process of accumulation of planets which he had done on a computer were presented [6]. The model of a proto-planetary cloud adopted in this work consisted of two components which were, respectively, dust and gas. The essential feature of the cloud model, then, was that the particles of dust in them had orbits with the prescribed fairly large eccentricities. In this model of a cloud, according to a definite rule, with random form, there was placed a "nucleus" -- an embryo of a planet, of appropriate mass and dimensions. It was proposed that the particles of /7 dust collided with the "nuclei" of planets connected to it. It was also proposed that during collision of the "nucleus" combining of them occurred. Upon reaching bodies of certain dimensions, the possibility of accretion of gas on them was reached. The process of accumulation was traced until all of the dust in the cloud had disappeared.

In [6], several examples of numerical modelling of the formation of planetary systems were presented. In these examples, one can track fairly presicesly the commensurability in positioning

of planetary orbits and also certain other characteristic pec-
uliarities which exist in the solar planetary system.

In 1977, R. Isaacman and C. Sagan published a work which is
a continuation of the studies begun in [6]. New examples were
calculated according to the method developed in reference [6];
using these, the effect of the change (in a very broad range)
was studied for a number of parameters of the cloud model [7].
The results of work [7] also were very interesting and produced
new material for improvement of our concepts as to the processes
which occurred during the formation period of the solar system.
One should note that, in spite of the large number of numerical
experiments done, neither in [6] nor in [7] was a planetary system
successfully reached which is fairly close to the numerical value
of the basic parameters (mass and orbit) for the solar system.
The authors of references [6] and [7] draw an important conclusion
on the basis of this as to the possible very large morphologic
diversity of planetary systems in the Universe.

With all that has been published on the character of results
in works [6] an [7] in the end, a number of questions remain in-
volving certain characteristics used in the model [6] and [7] of
the properties of a protoplanetary cloud. We will point out two
properties of the cloud adopted for the model in [6] and [7] which,
in our opinion, need further study and foundation. As was already
noted earlier, the eccentricities of the dust particles of the
cloud have a fairly large significance and the dynamics of their
occurrence is not completely clear, mainly stabilization around
a given value. On the other hand, spontaneous birth of a "nucleus"
-- the embryo of a planet is the most important property of a
model of a cloud and here it is necessary to create and substantiate
a quantitative theory of this process.

The research whose results are presented in this article was first undertaken for a study including questions other than the two listed above. During the research, gradually a new theory of accumulation of planets from a protoplanetary cloud was formulated; this will be presented in succeeding sections of the <u>/8</u> article.

As was already noted above, the evolution of a system consisting of bodies of finite dimensions with spherical and symetrical distribution of mass, gravitationally interacting with each other and combining (flowing together) on contact is presented in the article. It is assumed that all bodies move in a single plane and in a single direction around a central massive body (Sun) and then that the initial orbits of the bodies are circular. Evolution of such a system can be, in principle, with as high precision as desired, described by a system of ordinary differential equations whose sequence in solution continuously decrease as a result of combining of bodies coming into contact. Unfortunately, the direct use of such a system of equations for describing evolution of a cluster of bodies, using numerical and analytical methods, is practically impossible in the case considered. Elementary evaluations indicate that a model which is more or less satisfactory for a protoplanetary cloud (in this case a cluster of bodies) must consist of tens of thousands of particles. Numerical methods for solving differential equations which rest even on very modern computer equipment, do not describe a more or less long evolution of this system. The same can be said of analytical methods. Therefore, the initial model described above of the cloud must be simplified. In order to simplify the cloud model the following two important postulates will be used later on:

1) The mass of a protoplanetary cloud is negligibly small in comparison with the mass of the Sun.

2) During evolution, the close approach of only two pairs

of bodies occurs. Then, by a close approach one means that the gravitational forces of interaction here between the two bodies are somewhat larger than the forces of attraction of each of the bodies to the Sun.

The first postulate makes it possible to state that from one close approach to another, the body moves along a Keplerian orbit. The second postulate makes it possible to eliminate the complex picture of simultaneous interaction of several bodies (more than two); however, this requires a certain additional analysis. The point is that the second postulate, like the first, does not make it possible to use any simple formulas which describe the results of close approach of two bodies. Gravitational interaction of two small bodies in the field of a third massive body can be studied using the so-called problem of three bodies which however does not have an analytical solution. The numerical solution of this problem, with each close convergence of a pair of bodies, with the existing method, decreases the effectiveness of simplifying the model using the two postulates listed. Because of this, the necessity arises for constructing a simplified model of a pair of two interacting bodies which, in its final results, would be adequate for an initial model of their gravitaional interaction. For this purpose, we will present an analysis of solutions of the problems of three bodies in which both small bodies at the initial moment of time move in a circular orbit.

2. The Problem of Three Bodies. Gravitational Cohesion.

Let us assume M_o, m_1, m_2 are three masses interacting with each other according to the law of universal gravity. Let us introduce an absolute connected by fixed stars to a system of coordinates whose origin we will place at the barycenter of the masses m_1 and m_2. In this case, the equations of motions for these

masses can be written in the following dimensionless form:

$$\begin{cases} \dfrac{d^2 \vec{\tau}_i}{dt^2} = -q\,\dfrac{\vec{\tau}_i}{|\vec{\tau}_i|^3\,(1+k)^2} + \dfrac{1}{|\vec{\tau}_o|^3\,(1+k)}\left[-\vec{\tau}_i\,(k+1+\psi_i+k\psi_2)+\vec{\tau}_o\,(\psi_i-\psi_2)\right] \\[4pt] \hspace{9cm}(2.1) \\[4pt] \dfrac{d^2\vec{\tau}_o}{dt^2} = -\left(q+\dfrac{1}{1+k}\right)\dfrac{1}{|\vec{\tau}_o|^3}\left[(\vec{\tau}_o-\vec{\tau}_2)(1+\psi_2)+k(\vec{\tau}_o-\vec{\tau}_i)(1+\psi_i)\right], \end{cases}$$

$$(2.1)$$

where

$$\text{ixe}\quad K=\frac{m_i}{m_2}\ ,\quad q=\frac{m_2}{M_o},\quad \psi_i=\psi\,(\tau_i,z_i);\quad i=1,2$$

$$\begin{cases} \psi(\tau,z)=\xi(\tau,z)\left[3+3\,\xi(\tau,z)+\xi^2(\tau,z)\right] \\[4pt] \hspace{7cm}(2.2) \\[4pt] \xi(\tau,z)=\sum_{1}^{\infty}P_n(z)\tau^n\ ,\quad \tau_i=\frac{|\vec{\tau}_i|}{|\vec{\tau}_o|}\ ,\quad z_i=\frac{(\vec{\tau}_i\cdot\vec{\tau}_o)}{|\vec{\tau}_i|\cdot|\vec{\tau}_o|}\ , \end{cases}$$

$$(2.2)$$

where $P_n(Z)$ -- is the Legendre polynomials of $\underline{n}$ magnitude
$\vec{r}_0, \vec{r}_1, \vec{r}_2$ -- are radius vectors corresponding to masses M_o,
$\underline{m}_1$ and $\underline{m}_2$, and according to definition of the system of coordinates
$\vec{r}_2 = -K\vec{r}_1$. We note that equations (2.1) are invariants in relation
to the following transform of variables $\underline{t}$ and $\vec{r}_1$

$$t'= c\sqrt{c}\,t,\quad \vec{\tau}_i'=c\,\vec{\tau}_i\qquad i=0,1,2\,.$$

Let us consider now an important case for the entire sequential
analysis where the following relationships take place

$$m_i \ll M_o \qquad\qquad m_2 \ll M_o$$

$$(2.3)$$

$$|\vec{\tau}_i| \ll |\vec{\tau}_o| \qquad\qquad |\vec{\tau}_2| \ll |\vec{\tau}_o|$$

$$(2.4)$$

then, the relationships (2.4) are fulfilled for the entire field of change of variables $\vec{r}_1$. In this case, equations (2.1) can be simplified. The effect of mass M_0 on motion of masses $\underline{m}_1$ and $\underline{m}_2$ relative to their common barycenter can be calculated with an adequate degree of precision using the perturbing members of the first magnitude from M_0, introduced in the right sections of equations (2.1), that is, in (2.2) it is adequate to assume $\underline{y} = \underline{z} \cdot \tau$. Further one can say that the indicated simplified equations of motion will be invariants in relation to transforms $\underline{g}$ and $\vec{r}_1$ of the following form

$$q' = \varkappa \cdot q \; , \quad \vec{\tau}_i' = \sqrt[3]{\varkappa} \cdot \vec{\tau}_i \; , \quad \varkappa = const$$

$$(2.5)$$

The parameter K is assumed then to be constant. The relationships (2.5) play an important role with the following construction of a model of a protoplanetary cloud.

Now let us consider an important special case corresponding to gravitational interaction of two bodies with equal masses ($\underline{k} = 1$), moving at the initial moment of time $\underline{t}_0$ in a close circular orbit lying in a single plane. Let us assume that the distance between the initial heliocentric (circular) orbits of the bodies equals $\Delta\tilde{\underline{a}}_0$. Let us study the character of change of a separate kinetic moment of these bodies relative to their common barycenter (that is, relative to the origin of the coordinates). Figure 1 shows a family of curves for a specific moment of bodies $\tilde{K}_{12}$ for different values $\Delta\tilde{\underline{a}}_0$. Here, along the ordinate axis, values of specific moment $\tilde{K}_{12}$, are given and along the axis of the abcissa, the values of $\tilde{r}$ -- the distances of bodies $\underline{m}_1$ and $\underline{m}_2$ from the origin of the coordinates (that is, from the barycenter). Each curve corresponds to a certain value of the parameter $\Delta\tilde{\underline{a}}_0$. The graph presented in Figure 1 has a universal characer. It is
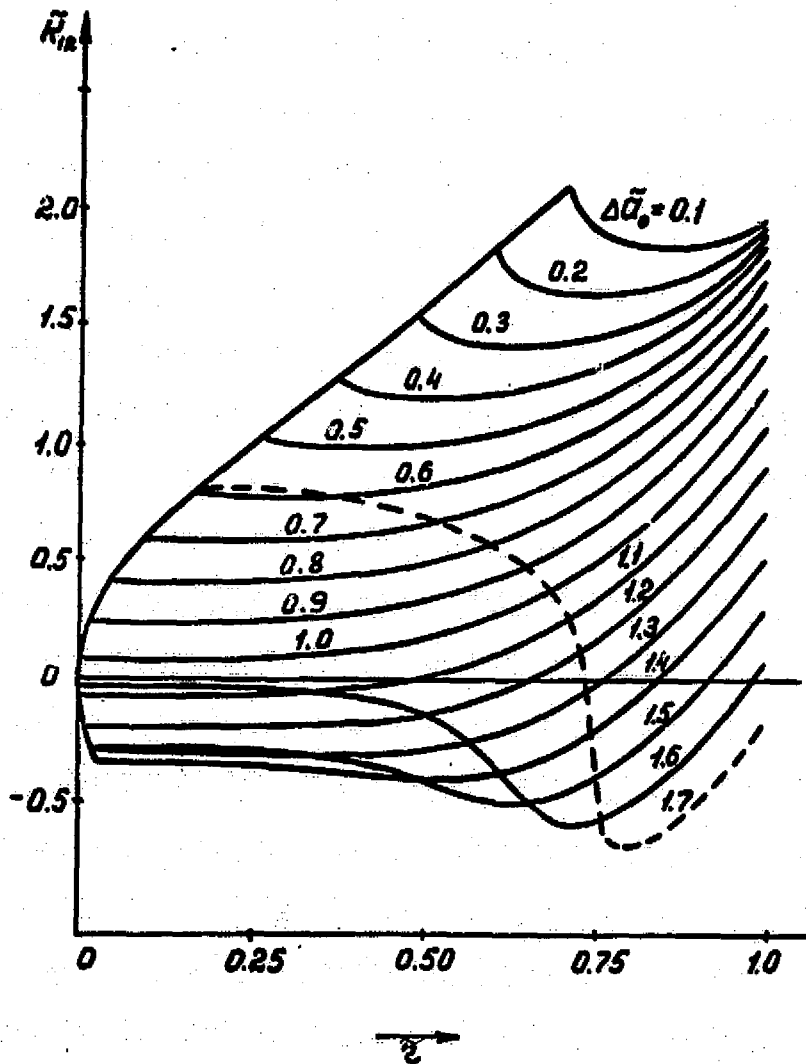
Figure 1. The specific kinetic moment of two gravitating bodies relative to their barycenter.

compiled in dimensionless form and the distance from the center of the body m_1 (or m_2) directed toward the center of the body M_O (Sun) is the unit of length assumed; here, the force of attraction of the body m_1 (m_2) equals the perturbing influx force from the M_O body. The appropriate dimensional values can be written as dimensionless using the formulas:

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$$\begin{cases} \Delta a_0 = \Delta \widetilde{a}_0 \cdot a \cdot \sqrt[3]{\frac{m}{M_0}} \\[2mm] \tau = 1.26 \cdot \widetilde{\tau} \cdot a \sqrt[3]{\frac{m}{M_0}} \\[2mm] K_{12} = 1.59 \; \widetilde{K}_{12} \cdot a^2 \cdot \left(\frac{m}{M_0}\right)^{2/3} \end{cases}$$

<div align="right">(2.6)</div>

where a is the mean distance of bodies m_1 and m_2 from M_0. We note that formulas (2.6) are the direct result of the relationship of invariance (2.5). The universal character of the graph in Fig. 1 makes it possible to use it for a broad range of masses m_1 and m_2, as soon as they satisfy the inequality (2.3). It stands to reason that due to the asymptotic character of the theory on whose basis the graph is constructed, that the higher the precision given them the smaller the value m/M_0 will be. However, calculations showed that for all masses of bodies possible in this formulation, which participate in accumulation of planets (to a mass on the order of the mass of Jupiter) that the graph in Fig. 1 provides completely satisfactory precision.

We note now the important property of curves $\widetilde{K}_{12}$ ($\widetilde{r}$). Almost all the curve families have two more or less different characteristic sections — sections of rapid change of specific moment $\widetilde{K}_{12}(\widetilde{r} > 0.5)$ and a section of stabilization of this moment ($\widetilde{r} < 0.5$). In other words, when two gratitationally independent bodies approach, their kinetic moment relative to the common center of mass, changing rapidly at first, begins at a certain distance ($\widetilde{r} \sim 0.5 -- 0.7$) stabilizes and remains practically constant up to maximum approach of both bodies. The curves $\widetilde{K}_{12}$ ($\widetilde{r}$) in Fig. 1 orbit to the left

at the moment when the bodies approach to the minimum distance. Consequently, the black curve which connects the left ends of curve $\tilde{K}_{12}$ ($\tilde{r}$), is the line of minimum distances. Another interesting feature of the curve $\tilde{K}_{12}$ ($\tilde{r}$) involves the sign of the moment K_{12} on a section of stabilization at the point of maximum approach of bodies. It is easy to see from Fig. 1 that for all values of $\Delta \tilde{a}_0$, which satisfy the inequality $0 < \Delta \tilde{a}_0 < 1.05$, we have $\tilde{K}_{12} > 0$ for the entire interval of change $\tilde{r}$. In the $1.05 < \Delta \tilde{a}_0 < 1.60$ range, on the stabilization section and when $\tilde{r} = \tilde{r}_{min}$ we have $\tilde{K}_{12} < 0$. Finally, for the range $1.60 < \Delta \tilde{a}_0 < 1.75$, for the section of stabilization again we have $\tilde{K}_{12} > 0$, although then the curve $\tilde{K}_{12}$ ($\tilde{r}$) has an intermediate section where $\tilde{K}_{12} < 0$(dashed line). In such a characteristic of curves $\tilde{K}_{12}$ ($\tilde{r}$) it is extremely important that for the majority of values of the parameter $\Delta \tilde{a}_0$ (70% of the range of change $\Delta \tilde{a}_0$) on stabilization sections and at $r = r_{min}$, we have $\tilde{K}_{12} > 0$. As will be shown later, this characteristic of the gravitational interaction of bodies moving around the Sun on almost a circular orbit stipulated, in the final analysis, a predominantly forward rotation of the planets around their axes. The main feature of curves $\tilde{K}_{12}$ ($\tilde{r}$) is stabilization with a predominantly positive sign of $\tilde{K}_{12}$ completely caused by gravitational interaction of two bodies, and with the absence of such interaction, no such characteristics would occur. For a comparison in, Fig. 2 curves /15 of $\tilde{K}_{12}$ ($\tilde{r}$) constructed on the same scale as the curves of Fig. 1 are presented but with the absence of gravitational interaction of the bodies m_1 and m_2. Actually the curves $\tilde{K}_{12}$ ($\tilde{r}$) in Fig. 2 have a completely different character from the analogous curves in Fig. 1. Here, more or less expressed sections of stabilization are absent and most important with maximum approach of bodies, the kinetic moment of $\tilde{K}_{12}$ has a negative sign.
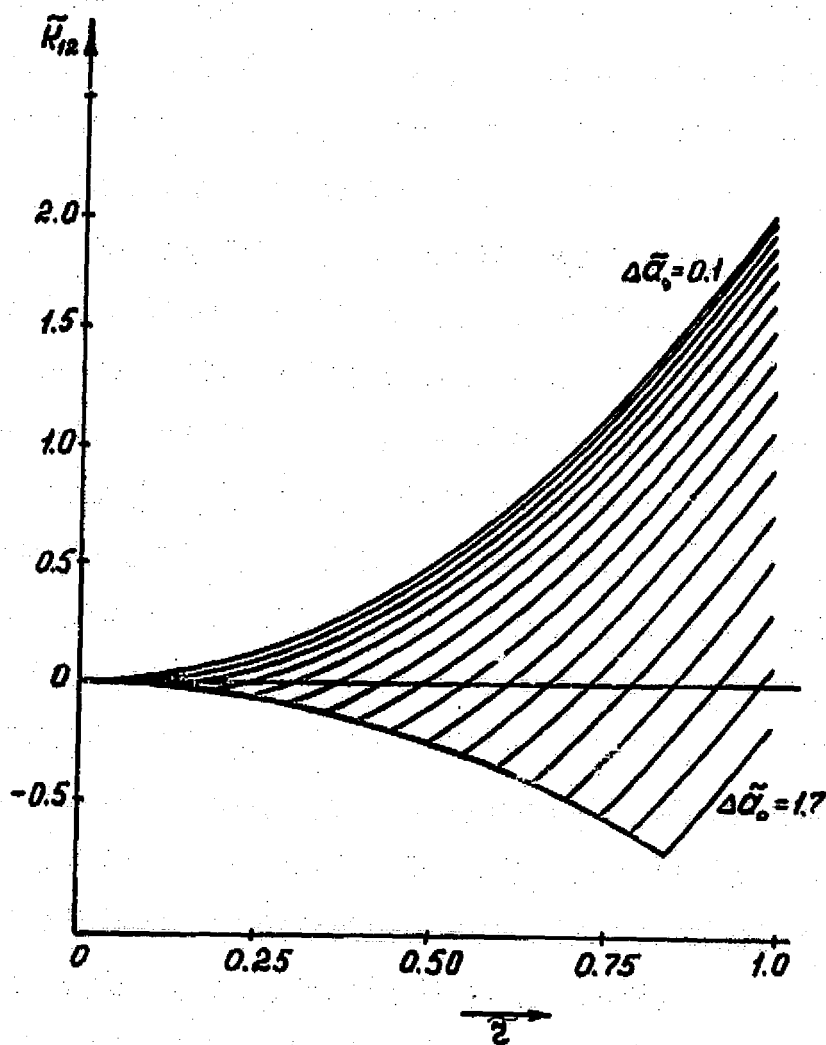
Figure 2. The specific kinetic moment of two bodies relative to their barycenter with the absence of gravitational interaction.

A case of gravitational interaction of two identical bodies (K = 1) was considered above. A similar analysis can be made for two bodies of different mass (K ≠ 1). Then, the main characteristic features of gravitational interaction of bodies described

above for $K = 1$ are retained in a general case of $K \neq 1$.

Relying on the effective stabilization of the kinetic moment given above we will construct a simplified model of gravitational interaction of two bodies. The essence of this simplification is based on a simple and well known principle of classical mechanics according to which motion of a center of mass in a mechanical system can be studied independent of its motion relative to its center of mass since the kinetic moment of the system relative to the center of mass remains constant. This principle, in a given case, can be used for the stabilization section of kinetic moment $\tilde{K}_{12}$. Following the principle indicated one can consider that in the stabilization period $\tilde{K}_{12}$, motion of the center of mass of bodies $\underline{m}_1$ and $\underline{m}_2$ is absolutely equivalent to the motion of the center of mass of one body with mass $(\underline{m}_1 + \underline{m}_2)$ and then the motion of the center of mass of this body around M_o (Sun) occurs according to a Keplerian orbit. The state of the bodies $\underline{m}_1$ and $\underline{m}_2$, then, can be defined as the state of a special type of bond -- adhesion, during which both bodies, in spite of their motion relative to each other, can be considered as a single aggregate. This state later on we will call the state of gravitational adhesion or gravitational cohesion, because it is caused by gravitational interaction of the $\underline{m}_1$ and $\underline{m}_2$ bodies. Thus, in the period of gravitational cohesion, the set of bodies m_1 and m_2, independent of their relative position and velocities, can be considered as a single body of mass $\underline{m}_1 + \underline{m}_2$ and with a center of mass moving around the Sun in a Keplerian orbit. On the other hand, in this same period of gravitational /16 cohesion, motion of the bodies $\underline{m}_1$ and $\underline{m}_2$ relative to each other can be considered independently from their motion around the body M_o and without taking into account its perturbing effect; in certain cases, it can be considered Keplerian also in relative motion. Finally, before a certain conventional moment corresponding to the origin of gravitational cohesion, motion of the approaching bodies $\underline{m}_1$ and $\underline{m}_2$ also can be considered as Keplerian

relative to a heliocentric system of coordinates. Thus, a model
of gravitational interaction of two bodies moving in a field of
gravity of a third massive body is simplified in both character-
istics. As is clear from what has been presented above, the
basic achievement of this model is that for each stage of inter-
action of the m_1 and m_2 bodies, their motion can be calculated
according to a comparatively simple final formula for the theory
of conical cross sections. Using the model indicated, now one
can move on to the construction of a common mathematical model
of a protoplanetary cloud.

3. Effective Body. Mathematical Model of a Cloud.

The dynamics of gravitational interaction of two bodies in
the case of a simplified model can be described using a graph
similar to the graph in Fig. 1. For this purpose, we will con-
struct a new graph which is a synthesis of the characteristic
sections of the graphs in Figs.1 and 2. Each line of the family
presented in Fig. 1 we will put in as a broken line consisting
of two lines -- straight, tangential to the corresponding line
of the family and parallel to the axis of the abcissa and
corresponding to the same line of the family of curves taken
from the graph in Fig. 2. A synthesized graph is presented in
Fig. 3. It is easy to see that the new graph, in spite of its
approximate character both from a qualitative and quantitative
point of view, completely satisfactorily describes the main
characteristics of the "precise" graph of Fig. 1. Actually,
the lines of the families shown in Fig. 3 have sections of rapid
change of the kinetic moment K_{12} and sections of its stabilization.
On the other hand, the values of K_{12} of the synthesized graph on
the sections for stabilization differ no more than 10% from their
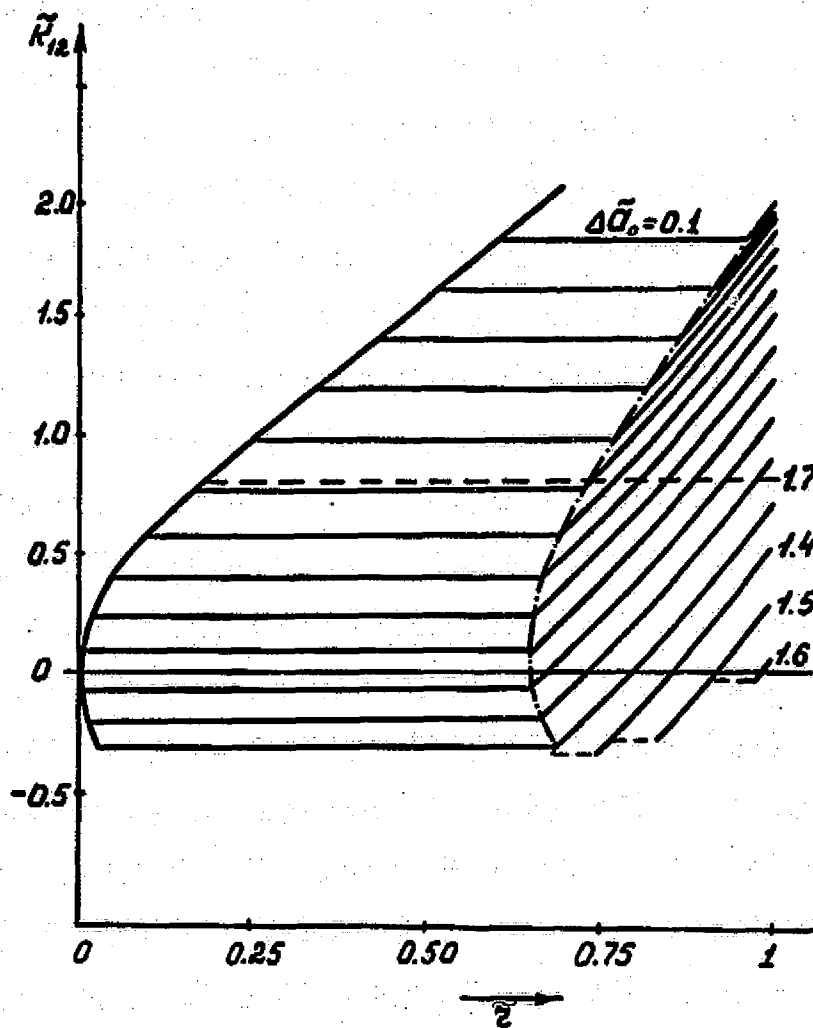values at minimum distances for a "precise" graph.

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Figure 3. A synthesized graph for specific kinetic
moment of two gravitational bodies.

Let us look at the dynamics of the approach of the bodies
m_1 and m_2 corresponding to any of the lines in the family in Fig.
3. The right section of the line (from the point of the break)
corresponds to Keplerian motion of both bodies in a heliocentric
system of coordinates. The left part of the line (beginning with
the point of the break) corresponds to Keplerian motion in a

heliocentric system of coordinates of the barycenter of bodies m_1 and m_2 (section of gravitational cohesion). This same section of the line corresponds to the osculating motion of the m_1 and m_2 bodies relative to its barycenter. The break point of the lines when $\tilde{r} = \tilde{R}_e$ is transitional -- the barycenter of the m_1 and m_2 bodies changes after the motion regime due to stabilization of its kinetic moment relative to the central body M_o (Sun). It is very important that the kinematic parameters of the barycenter (most of all velocity) at the break point can be calculated according to the theory of inelastic impact of two spheres with radius $\tilde{R}_e$ moving, before impact, on a heliocentric orbit of the bodies m_1 and m_2. Consequently the bodies m_1 and m_2 themselves in such a simplified model can be replaced by two nonelastic spheres with radius $\tilde{R}_e$, moving before the moment of contact on a Keplerian heliocentric orbit. It is obvious that similar dynamics of the approach of bodies m_1 and m_2 will occur for any other line in the family of Fig. 3 where, for each such point, it will have a contact radius $\tilde{R}_e$ of the m_1 and m_2 bodies. In other words, the contact radius is a function of the initial distance $\Delta\tilde{a}_o$ between the heliocentric orbits of bodies m_1 and m_2, that is, $\tilde{R}_e = \tilde{R}_e(\Delta\tilde{a})$.

After contact of the radii R_e of bodies m_1 and m_2, a period of gravitational cohesion begins for these bodies whose length t_k also depends on the lines of the family (or, that is to say, on $\Delta\tilde{a}_o$) and simultaneously with this, on the actual (physical) dimensions of bodies m_1 and m_2. The latter factor is particularly significant for constructing a mathematical model of a protoplanetary cloud. This means that the duration of gravitational cohesion of bodies as much as the results of it affect the character of evolution of the cloud. This result can be twofold. The bodies m_1 and m_2 approaching each other at a minimum possible distance can then move away from each other. These same bodies can combine (join) with each other in the period of gravitational cohesion.

In the first case, the period of gravitational cohesion ends
with gravitational uncoupling of the bodies m_1 and m_2. In the
second case, this period is completed by accumulation or combin-
ing of the m_1 and m_2 bodies into a new body with mass $m_1 + m_2$.
It stands to reason that the common mathematical model of a proto-
planetary cloud must envisage both possible sources of gravitational
cohesion. Undoubtedly, however, a special (maximum) case of a   /19
model in which gravitational cohesion must end with joining of
the bodies, is of definite and considerable interest. During
the study whose results are presented in this article, this maximum
case was first subjected to a thorough and detailed analysis. The
analysis of the maximum case indicated was of principle and pri-
mary value for the theory of the origin of the planets and satellites.
It seemed that the accumulation of most planets of the solar system
(except possibly Neptune) and also obviously the planetary satellites,
can best be described by just this special (maximum) model in
which each gravitational cohesion results in combining of bodies.
The model indicated will be an example for study in subsequent
sections of the article.

Here, we propose that during evolution of a protoplanetary
cloud at separate moments of time, that between these bodies there
occur gravitational cohesions each of which ends by the joining
of these bodies. Let us make still another simplifying modification
of the evolutionary collision process in a protoplanetary cloud.
For this purpose we will take two important and, according to what
has been said earlier, thoroughly obvious assumptions:

1) Each body of the cloud is surrounded by a conventional
spherical surface having an effective radius Re which depends on
the mass of the body m and on the mean distance of it a from the
center of the body M_o (Sun).

2) During contact of the bodies with these conventional spherical surfaces, instantaneous combination of them occurs. In other words, the period of gravitational cohesion is equal to zero.

We will give more details of the assumptions made.

On the basis of the first assumption, one can assume the phenomenon noted earlier of a shift in the state of motion of the barycenter of bodies m_1 and m_2 at the transition points of the break of curves of the family in Fig. 3. Additional simplification in this case involves ignoring the relationship of dimensionless radii of contact of the bodies $\tilde{R}_e$ (corresponding to the point of break) to the parameter $\Delta \tilde{a}_0$ and replace them with a single "weighted average" radius $\tilde{R}_{av}$. Transition from dimensionless radii $\tilde{R}_e$ to dimensional radius R_e gives the relationship of the latter to the mass of bodies m and to their average distance from the Sun, $\underline{a}$. /20 Actually, thanks to the invariance of equations(2.1) in relation to the transform (2.5) just as in the case of formulas (2.6), one can easily find

$$ R_e = 1.26 \cdot \tilde{R}_{av} \cdot a \sqrt[3]{\frac{m}{M_o}} \ , $$

(3.1)

where $\tilde{R}_{av}$ -- is the "average weighted" radius $\tilde{R}_e (\tilde{R}_{av} \sim 1)$, $\underline{a}$ -- is the average distance of the body m from M_o(Sun).

The second assumption is the direct result of the character of the evolutionary process in the special model of the cloud used. Actually, inasmuch as the contact of conventional spherical surfaces of bodies results in their gravitational cohesion, and the latter must end in physical combining of the bodies, the

dynamics of relative motion of the bodies in the period of gravitational cohesion does not affect the general picture and the final results of evolution of a protoplanetary cloud. Then, the actual period of gravitational cohesion can be fairly long -- during the period of cohesion of both bodies a more or less significant number of rotations can be made around the common center of mass, gradually slowing under the effect of influx forces at the approach to each other. It seems rapid combining of them with approaching combinations of kinematic parameters of heliocentric orbits, however, in any case from the moment of the origin of gravitational cohesion can occur and the character and time of interaction of two bodies does not affect the subsequent evolution of the protoplanetary cloud; therefore, cohesion time can be ignored.

Thus, in final form, the mathematical model of a protoplanetary cloud is a set of certain conventional bodies in spherical shape moving along a Keplerian orbit and combining upon contact, interacting with each other according to the law of absolute inelastic impact. Conventional bodies indicated will be called effective bodies henceforth. Each effective body has a radius R_e, calculated according to formula (3.1), and also corresponding to the effective area of a cross section S_e and effective volume V_e. It is easy to see that the actual physical body occupies only part of the volume of an effective body acting like a nucleus. Then, if the actual body is in a compact solid phase state, then the radius and volume of it can be very small in comparison with the corresponding radius and volume of the effective body. Thus, for example, the radius of the effective body of Earth equals approximately 10^6 km. The mass of the effective body, as follows from /21 what has been said, equals the mass of its nuclei, that is, the mass of the actual physical body m.

The mathematical body described above of a protoplanetary cloud can be completely adequate both from a qualitative and a quantitative point of view to describe the common evolution of an actual cloud; however, it is difficult to use for a direct numerical experiment on imitating the evolution indicated for digital computers due to too large a number of evolutionary bodies participating (approximately $10^8 + 10^{12}$). In connection with this, the necessity has arisen for creating a certain generalized model of a protoplanetary cloud capable, with corresponding laws of similarity, of describing the evolution of an actual cloud using a relatively small number of bodies. The mathematical model described earlier of a cloud becomes then a special case of this generalized model.

The generalized mathematical model of a protoplanetary cloud also is a set of conventional bodies moving along a Keplerian orbit and combining on contact; here, the effective radius of these bodies is determined according to the formulas

$$R = \lambda \cdot a \cdot \sqrt[3]{\frac{m}{m_*}}$$

(3.2)

$$\lambda = \frac{\sqrt{S_o}}{\sqrt{N}} \cdot const ,$$

(3.3)

where N -- is the total number of bodies contained in the cloud and S_o -- is the total effective area of the bodies at the initial moment of time occurring per unit of the area of a protoplanetary disc (in this case S_o = const.). The generalized model of a cloud in which the dimensions of the bodies are determined according to formula (3.2) is universal and useful with the presence of the corresponding theory of similarity for an adequately precise quantitative description of processes which occur in an actual protoplanetary cloud right up to computation of the actual parameters

of accumulated planets and their orbits. Such a generalized model
of a cloud henceforth will be called a gravitational model. More-
over, from now on along with a universal gravitational model we
will consider a simplified model of a cloud whose body radii, in
distinction from the preceding, do not depend on the value a. The
formula for the radii of bodies of such a model have the form /22

$$R = \lambda \sqrt[3]{\frac{m}{M}}$$

(3.4)

From now on, this model of a cloud will be called a liquid-
drop model. When using a liquid-drop model, it will always be
assumed that at the initial moment of time the masses of the
bodies m and, correspondingly, their radii R will be identical
and the effective area S_o will not depend on the distance R.
A liquid-drop model of a protoplanetary cloud, in spite of its
simplified character, was an extremely important and useful model
because, using it one could understand the basic principles of the
process of formation of the planets on simple examples, detect the
main effects of this process and, finally, obtain initial data for
calculating the parameters of a more complex gravitational model.

4. Basic Integral-Differential Equation.

The collision evolution process for the model of a protoplanetary
cloud presented above can be described by an integral-differential
equation which is a specialized modification and generalization
of appropriate equations in the modern theory of coagulation. For
this purpose, we will introduce into consideration the function of
distribution of bodies in a protoplanetary disk by mass and
distance

$$n = n(\zeta, \tau, t), \quad \zeta = \frac{m}{m_o},$$

(4.1)

where $\underline{n}$ is the number of bodies per unit area of the surface of the disk, and at single intervals of change of relative mass $\Delta\xi = 1$ and distance from the center of the disk $\underline{r} = 1$; $\underline{m}_o$ is the characteristic (for example, the mean quadratic) mass of the body in a protoplanetary disk at the initial moment of evolution. As was briefly noted earlier, the eccentricity of orbits of all bodies in a cloud at the initial moment equals zero. In this case, we will assume that the eccentricities remain equal to zero in the following period of evolution of the disk. When constructing the equation of coagulation of bodies on orbits, this assumption is fully verified by the results of analysis of numerical experiments. In this case, similarly to that done in the theory of coagulation, one can introduce the following equation for the functions $n(\xi, \underline{r}, \underline{t})$:

$$\frac{1}{\pi z}\frac{\partial n}{\partial t} = \int_0^\xi\int_0^{\tilde{R}} A(\zeta', \zeta - \zeta', z, \tilde{p})\cdot n(\zeta', z+p, t)\cdot n(\zeta-\zeta', z-\rho', t)\,d\rho\,d\zeta' -$$

$$- \frac{4}{\pi}n(\zeta, z, t)\int_{-0}^{\infty}\int_0^{R_1} A(\zeta, \zeta', z, \rho)\cdot n(\zeta', z+\rho, t)\,d\rho\,d\zeta' , \tag{4.2}$$

where

$$\tilde{p} = p + \rho' , \quad \tilde{R} = R_o(1 - \frac{\zeta'}{\zeta})(\sqrt[3]{\zeta} + \sqrt[3]{\zeta - \zeta'}) \tag{4.3}$$

$$\rho' = \frac{\zeta'}{\zeta - \zeta'}\rho, \quad R_1 = R_o(\sqrt[3]{\xi} + \sqrt[3]{\zeta'}). \tag{4.4}$$

The first member on the right in equation (4.2) corresponds to the number of bodies with mass ξ which form per unit of time on a unit of area of the disk due to combination of the masses ξ' and $\xi-\xi'$. The second member corresponds to the number of bodies with mass ξ which collide per unit of time on a unit of area with other bodies. The parameter R_o depends on the type of model

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of protoplanetary cloud and is determined according to the formula

$$R_\bullet = \lambda \sqrt[3]{\frac{m_\bullet}{M_\bullet}}$$

(4.5)

for a liquid-drop model, and according to formula:

$$R_\bullet = \lambda \cdot \tau \cdot \sqrt[3]{\frac{m_\bullet}{M_\bullet}}$$

(4.6)

for a gravitational model.

Both members of the right part of equation (4.2) involve important parameter A, ordinarily called the coefficient of coagulation which is the probability of collision and combining of bodies with different masses. So that equation (4.2) would be in improved form, it is necessary to point out the relationship of parameter A to the values of these masses and also the mutual positioning of their orbits and mean distance of the orbits from the body M_0 (Sun). This relationship can be easily found paying attention to the probability of the nature of parameter A. Let us assume bodies with radii R and R' moving in circular /24 orbit corresponding to a distance $\underline{r}$ and $\underline{r} + \underline{p}$ from the body M_0. Further let us assume for these two bodies the following inequality is true

$$\rho \leq R + R' .$$

(4.7)

In this case, the probability of contact of bodies per unit of time can be defined by the following formula:

$$A = \frac{V_{rel}}{2\pi\tau} ,$$

(4.8)

V_{rel} — is linear velocity of the shift of the bodies relative to each other. It is not difficult to show that V_{rel} can be

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defined according to the formula:

$$V_{rel} = \frac{3}{2} \sqrt{\mu} \; \frac{\rho}{\tau \sqrt{\tau}} \; ,$$

(4.9)

where μ is the gravitational constant for M_O. Substituting (4.9) in (4.8), we finally find:

$$A = \frac{3}{4\pi} \sqrt{\mu} \; \frac{\rho}{\tau^2 \sqrt{\tau}} \; .$$

(4.10)

Using (4.10), it is easy to find an expression for parameter A entering into both members of the right part of equation (4.2). Actually, using (4.10) and taking (4.3) into consideration, one can write:

$$A(\zeta', \zeta - \zeta', \tau, \bar{\rho}) = \frac{3}{4\pi} \sqrt{\mu} \; \frac{\rho}{\tau^2 \sqrt{\tau}} \left(1 + \frac{\zeta'}{\zeta - \zeta'} \right)$$

(4.11)

$$A(\zeta, \zeta', \tau, \rho) = \frac{3}{4\pi} \sqrt{\mu} \; \frac{\rho}{\tau^2 \sqrt{\tau}}$$

(4.12)

Equation (4.2) can be considerably simplified if one takes <u>/25</u> into consideration the small dimensions of the bodies in comparison with the characteristic dimensions of a protoplanetary disk. Considering that the contact between bodies of the cloud can occur only when the conditions of (4.7) are fulfilled, and also that R << r, one can obviously write the following:

(4.13)

(4.14)

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Substituting (4.13) and (4.14) in (4.2) and in the first member of the right part of (4.2) rejecting the component which contains the multiplier $\underline{p}^3$, after transformation we will have the following equation:

$$\frac{1}{B}\frac{\partial n}{\partial t} = \int_0^\zeta Q_1^2 \cdot \left(\frac{\zeta-\zeta'}{\zeta}\right) \cdot \left[\tilde{n}n' + \frac{2}{3}R_0 Q_1 \left(\frac{\zeta-\zeta'}{\zeta}\tilde{n}\frac{\partial n'}{\partial z} - \frac{\zeta'}{\zeta}n'\frac{\partial\tilde{n}}{\partial z}\right)\right]d\zeta' - 4n\int_0^\infty Q_2^2 \left(n' + \frac{2}{3}R_0 Q_2 \cdot \frac{\partial n'}{\partial z}\right)d\zeta',$$

$$(4.15)$$

(4.15)

where

$$n = n(\zeta,z,t), \quad n' = n(\zeta',z,t), \quad \tilde{n} = n(\zeta-\zeta',z,t) \qquad (4.16)$$

$$Q_1 = \sqrt[3]{\zeta} + \sqrt[3]{\zeta-\zeta'}, \quad Q_2 = \sqrt[3]{\zeta} + \sqrt[3]{\zeta'}. \qquad (4.17)$$

The parameter B inserted in the left part of equation (4.15), is determined according to formula

$$B = \frac{3}{8}\sqrt{\mu}\,\frac{R_0^2}{z\sqrt{z}} \cdot$$

(4.18)

The parameter R_0 is the radius of the body with mass $\underline{m}_0$ and is determined according to formula (4.5) or (4.6) depending on the type of model of protoplanetary cloud used. For improving the /26 formulation of problems on the evolution of a protoplanetary cloud in equation (4.2) it is necessary to add two integral relationships which express, respectively, the laws of conservation of matter and the moment of the quantity of motion of the cloud:

$$\begin{cases} M = 2\pi \iint\limits_{\cdot\cdot} m_o\, n\, \zeta\, \tau\, d\zeta\, d\tau = const \\ K = 2\pi\sqrt{\mu} \iint\limits_{\cdot\cdot} m_o\, n\, \zeta\, \sqrt{\tau}\, d\zeta\, d\tau = const \end{cases} \tag{4.19}$$

$$(4.20)$$

The relationships (4.19) and (4.20) must be fulfilled at any moment in time in the evolution process. Let us note further the relationship (4.20) has an approximate, asymptotic character -- it is more precise the smaller the initial dimensions of the bodies. The approximate character of the relationship (4.20)is due to the fact that the possibility of transition of part of the moment of orbital movement of a body to kinetic moment of their rotating motion during colliding interaction is not taken into consideration. In principle, such a calculation is completely possible although it is not obligatory in the first approximation.

5. A Qualitative Analysis of Solutions of the Basic Equation and the Effect of Annular Compression of Matter.

Let us present the desired solution of equation (4.15) in the following form:

$$n = n_o + R_o\, n_1$$

$$(5.0)$$

where R_o plays the role of the small parameter. Further let us assume that $\underline{n}_o$ is defined from equation:

$$\frac{1}{B} \frac{\partial n_o}{\partial t} = \int_0^\zeta Q_1^2\, \tilde{n}_o\, n_o'\, d\zeta' - 4 n_o \int_0^\infty Q_1^2\, n_o'\, d\zeta' \tag{5.1}$$

where $\underline{n}_o$, $\underline{n}_o'$ and $\tilde{\underline{n}}_o$ are defined according to formula (4.16).

In this case, for $\underline{n}_1$ in an asymptotic approximation from (4.15), it is easy to find the following linear equation relative to $\underline{n}_1$:

$$\frac{1}{\xi}\frac{\partial n_1}{\partial t} = \int_0^{\xi} Q_1^2\left(\frac{\zeta-\zeta'}{\zeta}\right)\left[n_0'\tilde{n}_1 + \tilde{n}_0 n_1' + \frac{2}{3}Q_4\left(\frac{\zeta-\zeta'}{\zeta}\tilde{n}_0'\frac{\partial n_1}{\partial t} - \frac{\zeta'}{\zeta}n_1'\frac{\partial \tilde{n}_0}{\partial t}\right)\right]d\zeta' - \int_0^{\infty} Q_2^2\left(n_0 n_1' + n_0' n_1 + \frac{2}{3}Q_2 n_0\frac{\partial n_1'}{\partial t}\right)\partial\zeta' \qquad (5.2)$$

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$$(5.2)$$

where $\underline{n}_1$, $\underline{n}_1'$ and $\tilde{\underline{n}}_1$ also are defined according to formulas (4.16). We note that equation (5.1) is a continuous analog of the well known Smolukhovskiy equation used in the theory of coagulation. It is easily deduced from (4.15) if one assumes that $\partial n_0/\partial r = 0$, being thus a special case of equation (4.15). In the case considered, however, generally speaking, $\partial n_0/\partial r \neq 0$ and, consequently, $\underline{r}$ is introduced into the function $n_0(\xi, \underline{r}, \underline{t})$ as a parameter.

Let us give the dependence of the initial radius of the bodies R_0 on $\underline{r}$:

$$R_{om}(\mathcal{r})$$

$$(5.3)$$

and also, let us give the initial effective area of a cross section of the body S_{om}. Let us further assume that the R_{om} and S_{om} given correspond to the solution $\underline{n}_{om}$ and $\underline{n}_{1m}$ of equations (5.1) and (5.2). Let us consider a new relationship $R_0(\underline{r})$, involving with (5.3) the following formula:

$$R_0 = \frac{1}{p}R_{om}$$

$$(5.4)$$

and we will assume that the initial values of new functions of distribution $n_0(\xi, \underline{r}, 0)$ and $n_1(\xi, \underline{r}, 0)$ involve $n_{om}(\xi, \underline{r}, 0)$

and $n_{1m}(\xi, r, 0)$ relationships:

$$n_{o}(\zeta, z, 0) = p^2 n_{om}(\zeta, z, 0), \quad n_{1}(\zeta, z, 0) = p^2 n_{1m}(\zeta, z, 0)$$

$$(5.5)$$

where p is a certain constant number. It is not difficult to
see that the new effective area of a cross section of the body
S_o equals the effective area of a cross section in case (5.3),
that is,

$$S_o = S_{om} .$$

$$(5.6)$$

In this case, a new solution of equations (5.1) and (5.2)
corresponding to formula (5.4) can be presented in the form:

$$n_o = p^2 n_{om} , \qquad n_1 = p^2 n_{1m} .$$

$$(5.7)$$

The curves of formulas (5.7) are easily tested by direct
substitution of them in equations (5.1) and (5.2). A decrease
in the characteristic initial radius of the bodies R_o in p
times with conservation of the total effective area of the cross
section S_o and with retention of the similarity of loss of
distribution of bodies according to relative mass ξ and the
distance r, it must unavoidably lead to an increase by p^2 times
the total quantity of bodies per unit of area of the proto-
planetary disk which is reflected in formulas (5.7). Later on,
transformation of the initial structure of the protoplanetary
disk described by the relationships (5.4) and (5.5) will be called
the transform of similarity.

An analysis of formulas (5.4) -- (5.7) causes an important
result both in the part concerning the content aspect of the

problem considered and in the method part concerning the direct
use of computation algorithms and interpretation of the numerical
results obtained. In order to show this, we will continue a
qualitative study of the character of change of the evolutionary
process in a protoplanetary disk with similarity transform of
the initial structure of the disk described by formulas (5.4)
-- (5.6). As was pointed out earlier, a transform of solutions
of equations (5.1) and (5.2) is described by formulas (5.7).
Using these formulas, we find the appropriate loss of trans-
formation of the characteristics of the evolutionary process
which are most important in this case, namely the law of trans-
formation of surface density of matter in the protoplanetary disk
σ. The surface density of matter σ, being an averaged charac-
teristic quantity of matter, existing per unit of area of a
protoplanetary disk, is defined by the following obvious formula:

$$\sigma = m_o \int_o^\infty \zeta n \, d\zeta \, .$$

(5.8)

Taking into consideration (5.0), one can also write

$$\sigma = \sigma_o + \sigma_1 \, ,$$

(5.9)

where

(5.10)

$$\sigma_o = m_o \int_o^\infty \zeta \, n_o \, d\zeta$$
$$\sigma_1 = R_o \, m_o \int_o^\infty \zeta \, n_1 \, d\zeta .$$

(5.11)

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From (5.9) it follows that

$$\frac{\partial \sigma}{\partial t} = \frac{\partial \sigma_o}{\partial t} + \frac{\partial \sigma_i}{\partial t} \; .$$

(5.12)

On the other hand, inasmuch as the solution of equation (5.1) corresponds to the stationary state of matter (in the concept of the absence of flow of mass from one field of space to another), relating to this solution of the component of surface density of matter σ_o, it must remain constant during evolution at any point on the protoplanetary disk, and, consequently must be identical to $\partial\sigma_o/\partial t \equiv 0$ for all $\underline{t}$ of the time range for evolution studied. In view of this, taking into consideration (5.12) and (5.11), we find

$$\frac{\partial \sigma}{\partial t} = R_o \, m_o \, \frac{\partial}{\partial t} \int_0^\infty \zeta \, n_i \, d\zeta \; .$$

(5.13)

Further, taking into account the structure of solution of the basic equation (4.15), given by (5.0) it is natural to use the following initial conditions for functions $n_1(\xi, \underline{r}, \underline{t})$:

$$n_i (\zeta, \tau, 0) = 0 \; .$$

(5.14)

In this case, from (5.11) it follows that:

$$\sigma_i (t=0) = 0 \; .$$

(5.15)

Taking into consideration (5.14) and (5.15), it is not difficult to introduce the following formula for the charactersitic initial mass of the bodies $\underline{m}_o$:

$$m_o = \frac{\pi \, R_o^2 \, \sigma_o}{S_o} \; , \qquad \tau = 1 \; .$$

(5.16)

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Substituting (5.16) in (5.13), we finally find

$$\frac{\partial \sigma}{\partial t} = \frac{\pi R_o^3 \sigma_o}{S_o} \frac{\partial}{\partial t} \int_0^\infty \zeta n_z \, d\zeta .$$

(5.17)

Using formula (5.17) we study the law of transformation of dynamics of development of surface density σ with a change in the initial state of the protoplanetary disk according to the formula of similarity transforms (5.4) and (5.5).
Substituting (5.4), (5.6) and (5.7) in (5.17), after cancellation,/30 we have

$$\frac{\partial \sigma}{\partial t} = \frac{1}{P} \frac{\partial \sigma_m}{\partial t} ,$$

(5.18)

where

$$\frac{\partial \sigma_m}{\partial t} = \frac{\partial \sigma_{zm}}{\partial t} , \quad \sigma_{zm} = \frac{\pi R_{om}^3 \sigma_o}{S_o} \int_0^\infty \zeta n_{zm} \, d\zeta .$$

(5.19)

Formula (5.18) has an important value for establishing the character of change of evolution of surface density with transformation of similarity (5.4), (5.5). Actually, from (5.18) it follows that the evolution of surface density $\sigma(\underline{r},\underline{t})$ has a false spatial similarity to evolution of initial density $\sigma_m(\underline{r},\underline{t})$, only it occurs p times more slowly. Then, this change in density Δσ is achieved in a transform of similarity (5.4) -- (5.5) for a larger time. Further, from the law of conservation of matter (4.19) taking into consideration that $\partial \sigma_0 / \partial t \equiv 0$, we find the identity:

$$\int_0^\infty \sigma_z (z,t) \, z \, dz \equiv 0$$

(5.20)

for all t. If one excludes the trivial case $\sigma_1(\underline{r},\underline{t}) \equiv 0$ for

all t from the consideration, then completion of the identity
(5.2) is possible only with conditions of simultaneous existence
of sections according to r, where $\sigma_1 > 0$ and $\sigma_1 < 0$. In other
words, in the course of evolution in a protoplanetary disk,
according to (5.20) the annular zones of rarefaction and
thickening of matter must be insignificant. This is seen
particularly well if one returns to formula (5.9) and considers
the case where $\sigma_0 =$ const, that is, when σ_0 does not depend on
r. Turning again to (5.18) and taking (5.20) into consideration,
one can formulate the following important law of the evolution of
the protoplanetary disk type cloud.

In a disk type protoplanetary cloud, in the course of an
evolutionary collision process accompanied by the combining
of bodies coming into contact, annular zones of rarefaction
and thickening of matter form whose number and dimensions, with
the appropriate times for comparison, and with other conditions
being equal, do not depend on the initial dimensions of the   /31
protoplanets but depend only on the effective initial area of
their cross section.

The law formulated has an asymptotic character, its conclusion
is based on a qualitative analysis of the solutions of linearized
equation (5.2) and therefore, strictly speaking, its use is limited
to the initial stage of evolution of a protoplanetary cloud.
Nevertheless, as will be apparent later on from analysis of the
results of numerical experiments, the value of the law indicated
clearly is outside the frame work for description of the initial
stage of evolution of a cloud and is the basis for assuming that
this law is the basis of a more general law of the formation of
planetary systems.

The effect of annular contraction of matter of a protoplanetary

cloud, directly involves the differential rotation of the latter
around the Sun and is caused almost completely by this rotation.
When differential rotation is absent, the effect of annular
contraction does not occur, having conceded its ordinary process
of the combination of bodies studied in the theory of coagulation.
A planetary system could not have been formed as a result of this
process.

The presence of differential rotation of the matter of a
cloud, being the direct consequence of Kepler's laws, results
first of all in significant relative angular velocities of motion
of neighboring bodies in different annular zones of the cloud.
In the zones with the highest relative angular velocities, com-
bination of bodies occurs more rapidly, showing large bodies in
this connection and instead of them, a high radial gradient accord-
ing to the mass of bodies in the cloud. The relatively large
bodies formed in this way, when combining with smaller bodies
in mixed zones, absorb (draw in) their matter in this zone,
creating thus an increased surface density for it. Thus, a zone
of compression is formed and next to it a zone of rarefaction of
matter. During the indicated process of absorption of small
bodies by the large, it is a fairly favorable picture of re-
structuring of the initial density of the character of distribution
of matter in a cloud (which, at the initial moment is distributed
in the form of a thin film in the equatorial plane of the cloud),
in an essentially volumetric character. It is interesting to note
that the effect described above of annular compression of matter,
in principle absolutely does not involve chaotic radial components
of the velocity of bodies, caused by eccentricities of their orbit /32
and is realized completely at zero eccentricities. The accumulation
mechanism for the formation of planets from a protoplanetary
cloud considered in this article differs considerably from other
well known mechanisms which rely mainly on the presence of chaotic

components of a velocity which are very large as is indicated above.

The surface density of matter of the initial structure of the cloud σ_m can be presented in the form:

$$\sigma_m = \sigma_o (z) + \sigma_{\ell m} (z, t),$$

(5.21)

where $\sigma_{lm}(r,t)$ is defined by formula (5.19). In this case, taking into consideration relationship (5.18) for density σ, it is not difficult to show the formula:

$$\sigma = \sigma_o (z) + \sigma_{\ell m} (z, \frac{t}{\rho}).$$

(5.22)

Comparing (5.21) and (5.22) we see that the state of the cloud with the initial structure (5.3) achieved in time $\underline{t}_m$, will be achieved and with a structure obtained using the transformation of similarity (5.4) -- (5.5), for time $\underline{t}$, where

$$t = \rho t_m .$$

(5.23)

One should note that, although from a theoretical point of view, the applicability of (5.23) is limited to the initial phase of evolution of a protoplanetary cloud, the numerical experiments with their different models showed practical universality of this formula for the entire evolutionary process. In particular, if T_m is the full time of evolution of the initial model of a protoplanetary cloud, then the full time of evolution T for transformation according to (5.4) -- (5.5) of the model is determined by the formula

$$T = \rho T_m .$$

(5.24)

6. The Kinetic Moment of Rotating Motion and the Theory of Similarity

In the course of the evolutionary collision process in a protoplanetary disk, transition occurs of part of the orbital kinetic moment of the bodies to the rotating moment of their motion around their axes. In order to study the principle of this transition, we will first consider the impact interaction of two bodies moving in close circular orbits.

Let us assume as we did earlier two bodies with radii R and R' and masses m and m' move in a circular orbit, respectively, at distances r and $r + p$ from the body M_o and then, as previously, the inequality p < R + R' is true. One can show that at the moment of contact, both bodies acquire an additional rotating moment ΔK_s relative to their common center of mass (barycenter), expressed by the formula:

$$\Delta K_s = \frac{m \cdot m'}{m + m'} \cdot \frac{\sqrt{\mu}}{2 r \sqrt{r}} \cdot (R + R')^2 \cdot (2 - 3\beta^2),$$

(6.1)

where

$$\beta = \frac{p}{R + R'} \quad , \quad 0 \leq \beta \leq 1.$$

(6.2)

Formula (6.1) has an asymptotic character -- it comes from the assumption that R/r<<1. Adding the increment of rotating moment ΔK_s, acquired by the protoplanetary cloud as a result of the paired interaction of this body, one can find the total rotating moments of the planets which form in the final stage of the

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evolutionary collision process. Unfortunately, a single over-
all formula giving the total of accumulation of the rotating
moment for each of the planets formed and which would encompass
the entire evolutionary process of their accumulation, is very
difficult to obtain in analytical form for a variety of reasons.
This causes, in turn, difficulties in studying the effect of
transformation of similarity (5.4) -- (5.5) for acquiring the
rotating moment K_s formed in the cloud by the planets. A study
of the effect of the transformation of similarity for rotating
moment K_s, however, is extremely important for constructing a
general theory of similarity of evolutionary processes in a
protoplanetary cloud and, therefore, an attempt was made to study
this effect without the direct use of a hypothetical general
formula for the total rotating moment of the planets. For this /34
purpose, a semi-empirical theory of similarity of rotating
moments of the planets was developed with the change in structure
of the protoplanetary disk, resting, on the one hand on analytical
formulas for increments of moments at separate stages of the
process of accumulation of planets, and, on the other hand, on
the results of specially organized numerical experiments which
simulate the processes of accumulation of planets in protoplanetary
disks caused by the similarities (5.4) -- (5.5) between these
laws.

The process of accumulation of a rotating moment in bodies of
a protoplanetary disk at the initial stage of its evolution can
be studied using modern equipment for the theory of coagulation.
Relying on the formula of impact interaction of two bodies (6.1)
and using the function of distribution $n(\xi, \underline{r}, t)$ introduced
earlier, one can similarly with equation (4.15) introduce a
formula for the total rotating moment of all bodies of a proto-
planetary disk accumulated by them as a result of impact

interaction for time T:

$$K_s = \frac{3\pi^2\mu}{8}\int\limits_0^{\infty}\int\limits_0^{\infty}\int\limits_0^{\infty}\int\limits_0^{T}\frac{G_i R_o^6}{r^2 S_o} Q_i^4 \zeta' \left(1-\frac{\zeta'}{\zeta}\right)^2 n(\zeta',r,t)n(\zeta-\zeta',r,t)drd\zeta'd\zeta dt.$$

$$(6.3)$$

(6.3)

Using formula (6.3), one can study the effect of transformation of similarity (5.4) -- (5.5) for the total rotating moment of a protoplanetary disk.

Let us assume K_{sm} -- is the total rotating moment accumulated by the bodies of the disk for time T; the relationship $R_{om}(r)$ given for this initial model of it with characteristic initial radius of the protoplanets is true. We will complete the transformation of similarity of the disk according to formulas (5.4) -- (5.5). Taking into consideration that, as formerly, in this case the following relationship will be true:

$$n(\zeta, r, t) = p^2 \cdot n_m(\zeta, r, t)$$

$$(6.4)$$

where $n_m(\xi, r, t)$ and $n(\xi, r, t)$ are functions of distribution of bodies, respectively, for an initial and a new model of a protoplanetary disk, and substituting (5.4), (5.5) and (6.4) in (6.3), after simple transforms we find:

$$K_s = \frac{K_{sm}}{p^2},$$

$$(6.5)$$

where K_s is the total rotating moment for the structure of a protoplanetary disk transformed according to formulas (5.4) and (5.5). Formula (6.5) is suitable for conversion of moments for the stages of evolution of a cloud when the bodies comprising it are adequately numerous and consequently, when it is possible to use continuous models of the theory of coagulation for describing

evolution. Unfortunately, the entire evolution of a protoplanetary cloud cannot be described to its end by these models. In its final stages, interaction of a few remaining large bodies requires the use of other discrete methods for its analysis and it requires moreover that quantitatively the effect of the final stage of evolution on the appearance of the planetary system formed is comparable (in certain relationships more significantly) to the effect of its basic period when there are many bodies. Due to this, we will return to formula (6.1) and attempt to use it for analysis of the final impact interactions of bodies of the protoplanetary cloud. Then, for simplicity, let us consider first the impact interaction of the last two bodies remaining in the accumulation zone of compression of matter and the planets formed as a result of this interaction. As before, we will study the effect of transformation of similarity (5.4) — (5.5) on rotating moment of the planets formed. However, before beginning this study we note that the strict requirements of spatial similarity of evolutionary protoplanetary disks with transformation of similarity (5.4) — (5.5) were obtained in an asymptotic approximation for the initial stage of evolution. Generally speaking, there are no theoretical bases for distribution of laws of such similarity in the case considered. Nevertheless, numerical experiments with different models of protoplanetary disks adequately verified that such similarity, apparently, occurs during the entire evolutionary process. Therefore, it is possible at least in working hypotheses to assume that the spatial similarity of interaction of bodies occurs in the final stage of evolution of the cloud. The adoption of the working hypotheses indicated makes it possible, obviously, to consider the masses of bodies m, m' and the parameter β introduced into (6.1) as unchanged during transformation of similarity. As to the radii of bodies R and R', this formula for their transformation will differ

from the formula of initial conversion (5.4). At the same time, the total volume of all bodies of a cloud during transformation of similarity (5.4) -- (5.5) decreases by p times. Consequently, the volume of the last pair of bodies found in the accumulation zones of the planet decrease by p times. In other words, if V_m and V'_m are the volumes of the bodies of masses m and m', for the initial structure of a protoplanetary cloud, and V and V' are the volumes of the bodies of these same masses with transformation according to (5.4) -- (5.5) structure, then the following relationship will occur:

$$V = \frac{V_m}{p} \quad , \quad V' = \frac{V'_m}{p} .$$

(6.6)

If R_m and R'_m are the radii of bodies for the initial structure of a cloud, then taking into account (6.6) it is easy to find:

$$R = \frac{R_m}{\sqrt[3]{p}} \quad , \quad R' = \frac{R'_m}{\sqrt[3]{p}} .$$

(6.7)

Substituting further (6.7) in (6.1), after simple transforms we find similarly to (6.5):

$$\Delta K_s = \frac{\Delta K_{sm}}{p^{5/3}} .$$

(6.8)

Thus, for two maximum stages of evolution of a protoplanetary cloud -- the initial and final, one could successfully obtain fairly simple formulas involving the increments of rotational moments of clouds with a different but similar initial structure. Formulas (6.5) and (6.8) can also be described in the following

form solved for p:

$$p = \left(\frac{K_{sm}}{K_s} \right)^{1/2} \quad , \quad p = \left(\frac{\Delta K_{sm}}{\Delta K_m} \right)^{3/2}.$$

(6.9)

Taking (6.9) into consideration, it is natural to assume that a relationship of similar type occurs for the entire evolutionary process in a protoplanetary cloud and not only for its separate stages. In other words, one can assume that a relationship exists in the form:

$$p = \left(\frac{K_{sm}}{K_s} \right)^{\alpha} ,$$

(6.10)

here K_{sm} and K_s are the total rotating moments of bodies of the protoplanetary cloud acquired in the entire time of evolution, respectively, for two different initial structures of the cloud. There is no strictly theoretical proof of the existence of relationship (6.10) at the present time. However, a number of numerical experiments with different models of a protoplanetary cloud, including experiments especially posed for this, very convincingly give evidence in favor of the existence of such a relationship, although this relationship would have an approximate character. Let us note that using numerical experiments, the approximate value of parameter α was reliably defined. It was an even inverse value of the mean arithmetic indices of the stages in formulas (6.5) and (6.8), that is, $\alpha = 3/4$.

Formula (6.10) makes it possible to complete constructing the theory of similarity for generalized liquid-drop and gravitational models of a protoplanetary cloud. Let us assume that R_{om} is the

initial radius of bodies of a given accumulation zone, T_m is the full time of evolution of the zone and K_{sm} is the rotating moment formed in the zone of the planet for a generalized model of a protoplanetary cloud and let us assume that K_s is the rotational moment of an actual planet, whose formation process is modeled. In this case, the real initial radii of the proto-planets R_o and the real time of evolution of the accumulation zone can subsequently be defined according to the formula:

$$ P = \left(\frac{K_{sm}}{K_s} \right)^{3/4} \quad , \quad R_o = \frac{R_{om}}{P} \quad , \quad T = P T_m $$

(6.11)

7. An Algorithm of the Computer Process of Modelling. Principle of Virtual Contacts

A study of the accumulation process of evolution of a proto-planetary cloud was made by numerical modelling of the process indicated on a digital computer. For this purpose, a very effective and economical algorithm was developed which makes it possible, with maximum possible precision, in the visible time and to the end, to trace the development of the complex process of combining of bodies in a protoplanetary cloud. On the basis of the algorithm, a specific principle was proposed called the principle of virtual contacts, which provides, in the final analysis, full solution of the problem in a strictly determinate formulation.

The main goal of the principle was to provide calculation of /38 a precise sequence of moments of collision for all bodies of the system. After this sequence is found, it requires a relatively short time for a calculation of the entire dynamics of the process.

We note that for a procedure of direct surplus of bodies for the entire system, one needs a machine time proportional to N^3 where N is the number of bodies in the system. When using the method of virtual contact, the necessity is avoided for a tremendous volume of computations involved in the need for a direct surplus. It is understood that in a large system of interacting bodies, the surplus procedure cannot be completely eliminated in the computation process. Therefore, the most effort when creating the method was directed towards making the surplus as limited as possible and, consequently as eccnomical as possible. Then it was important, that the final result of solving the problem by method of virtual contacts would coincide with the results of solving it by a direct surplus with a considerable decrease in the amount of machine time required.

A method of virtual contacts is based on prediction of the course of the collision process with subsequent corrections of the prediction after each collision. The prediction is the matrix of virtual (possible) contacts for all bodies of the system; this matrix is constructed at the initial moment on the basis of initial data. For this purpose, for each body, a matrix is constructed of all possible moments of binary collision with all other bodies which, in principle, can collide with a given body. From this matrix, for further calculation, the most recent collision in time is selected. A similar calculation is done for all the remaining bodies of the system. In summary, a matrix is formed consisting of the earliest moments of collision in time for all bodies. Further, this matrix is ordered according to the value of moments of collision in order of their increase. This is the <u>matrix of virtual contacts</u>. All of the moments of collision entering into this matrix are considered potentially possible but not all of them must be realized in actuality. an exception is the first element of the matrix of virtual contacts

inasmuch as it is the earliest moment of collision for the entire
system of bodies. Computation of the dynamics of the process of
accumulation leads first of all to correction of the matrix of
virtual contacts in order to find the true sequence of collisions.
This correction is made using a similar computation: for a body
which forms anew from all virtual contacts, one considers the
earliest one. Similarity calculation must also be done for
bodies which, in the process of evolution, have lost their
primary claims to collision as a result of their combination
with other bodies. In the end, the true moment of collision for
each step is obtained using a sequential excess of elements of a
continuously corrected matrix of virtual contacts (true collision
at this moment is the earliest collision corresponding to the
state of the system of the bodies for this moment). In this way,
at each step, a small volume of computation is realized without
an excess for the entire system of bodies as a whole. Making
corrections to the matrix of virtual contacts and finding the
actual moments of collision is achieved by the use of a specially
created logic; one of the most important elements of this logic
is a multiplier, variable in time, which makes it possible to un-
ambiguously determine the zone of virtual contacts. Such zones
for each body are relatively small and make up only a small part
of the entire system of bodies; this considerably speeds up cal-
culation of the process of accumulation.

A method of virtual contacts was realized on a BESM-6 EVM
[elektronnaya vychislitel'naya mashina, electronic computer],
for a case N = 25600. Due to the limitations of the operative
and external (rapid) memory of the BESM-6, a number of other
problems arose for the algorithm. The most important of these
involved optimum distribution of the blocks with the information
container changing in time with ordering of the matrices with

large dimensionality. Special machine algorithms for realization of a very important stage of the problem, computation of the dynamics of the accumulation process, were also created and developed. During testing and numerous modernizations, the method was carefully developed. The computation time for a single variant of the problem took a total of a few hours which is four magnitudes smaller than the time necessary for computing the same variant using a direct excess.

8. A Liquid-drop Model of a Protoplanetary Cloud. Results of Numerical Experiments.

A method of virtual contacts briefly described in the preceding section was used, primarily, for studying the evolution of a generalized liquid-drop model of a protoplanetary cloud. The model indicated was a disk shaped flat structure consisting of 25600 sphere shaped bodies identical both in dimensions and in /40 mass distributed evenly over the entire area of the disk. The initial circular orbits for each of the bodies were selected using a sensor for random numbers and then a method of selection guaranteed retention of the uniform identical densities of matter distributed over the area of the disk. The characteristic relative dimensions of the disk were selected in such a way that they would correspond with similar dimensions in the Earth group of planets. The values of distance from the center of the external edge of the disk a_{max} and its internal edge a_{min} satisfy the relationship:

$$\frac{a_{max}}{a_{min}} = 6$$

(8.1)

The following were taken as units for measurement: 1) for linear dimensions -- the distance of the internal edge of the disk a_{min} from the center of M_o, 2) for time -- the period of rotation T_{min} of the body relative to M_o for the distance a_{min}, 3) for

mass -- the total mass of a protoplanetary cloud (disk) M_d. Thus, one can write:

$$a_{min} = 1 \, , \qquad T_{min} = 1 \, , \qquad M_d = 1$$

(8.2)

Taking into consideration the first two equalities in (8.2), according to Kepler's law for a gravitational constant μ we find:

$$\mu = 4\pi^2$$

(8.3)

From the first and last equalities of (8.2) it also follows that

$$\sigma_o = 0.0091$$

The basic varying parameter in the numerical experiments with a liquid-drop model of a protoplanetary cloud was the initial total effective area of the bodies S_o. During numerical experiments it was clear that this effective area S_o is the chief parameter which determines the total appearance of the planetary system formed and, primarily, the number and positioning of the planets and their orbits.

The first example considered for a liquid-drop model of a cloud was an example in which, for effective area S_o, the value of $S_o = 0.126$ was taken, that is, it was assumed that the total initial effective area of the cross section of all bodies of the cloud consists of slightly more than 12% of the total area of the disk. In the course of the numerical experiment, the effective annular contraction of matter was confirmed; it was detected earlier with a qualitative study of solutions of the equation of coagulation (4.15). The protoplanetary disk during evolution was broken down into annular zones of contraction and rarefaction of matter and

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each contraction zone completed its own evolution by combining all of the bodies belonging to it into a single planet. In all twelve planets were formed in the plane of the disk moving in orbit with small eccentricities (e < 0.001). Between the large semiaxes of the planets the law of commensurability can be traced very clearly according to its character which is close to geometric progression. The ratios of large semiaxes of planets which succeed the preceding $\underline{a}_{i+1}/\underline{a}_i$, vary in average limits around a mean value equal to 1.16. The final results of the numerical experiment are presented in Fig. 4. In the upper graph one sees distribution of mass according to the planets formed; on the lower graph algebraic values are given for specific moments of the rotating motion of the planets around their axes. On both graphs, along the axis of the abscissa the distance from the central body M_o (Sun) is applied along the axis of the abscissa in the units of length presented above.
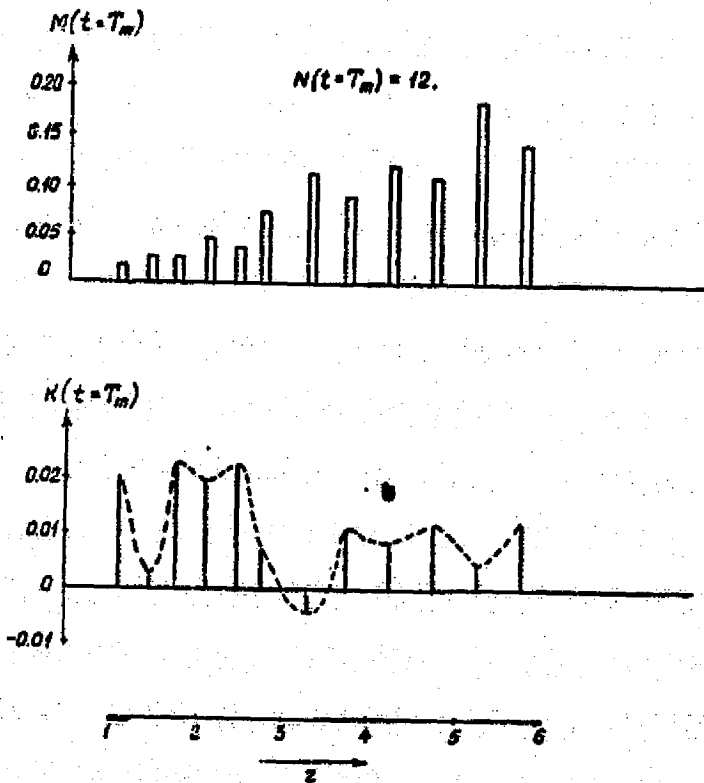
An analysis of the variant presented for calculation of the evolution of a cloud made it possible to draw important conclusions and to note the latest direction in numerical experiments. First of all, it became clear that in an actual protoplanetary cloud, the initial effective area of a cross section of a protoplanet

Figure 4. Distribution of mass and specific kinetic moments along a planet (variation 1.1, $S_o = 0.126$)

was very large and in its total expression was comparable to the general area of a protoplanetary disk. Actually, in real planetary (including satellite) systems, the number of planets in the range of relative distances used, between the internal and external edge of a protoplanetary disk do not exceed 4 or 5. In order to obtain the indicated number of planets, instead of twelve as was given in this example, it is necessary to considerably increase the initial effective area of the cross section of the body S_o. However, a result relating to the rotating motion of planets around their axes was most unexpected and interesting. From the lower graph on Fig. 4 we see that eleven of the twelve planets have acquired a forward (that is, like an orbital) rotation around their axes and only one (the seventh from the center) has backwards rotation. An explanation of the effect of the rotating motion of planets was fairly simple; it is in complete agreement with the mechanism of gravitational cohesion of interacting bodies presented above and an analysis of this effect will be given in one the succeeding sections in this article. Here, we note only that obtaining in this very first example a primarily forward rotation of the planets gave evidence of the correctness of the approach for selecting and constructing a mathematical model of a protoplanetary cloud and proved the usefulness of the concepts /43 on which this program of research was based.

With succeeding numerical experiments the initial effective area S_o was considerably increased. Six variations of the evolution of a protoplanetary cloud were calculated with the progressive increase of the parameter S_o from variation to variation. Table I gives the characteristics of these variations.

TABLE I

Var.	I.2	I.3	I.4	I.5	I.6	I.7
S_o	0.41	0.45	0.49	0.53	0.57	0.79

The results of numerical experiments are presented by analysis
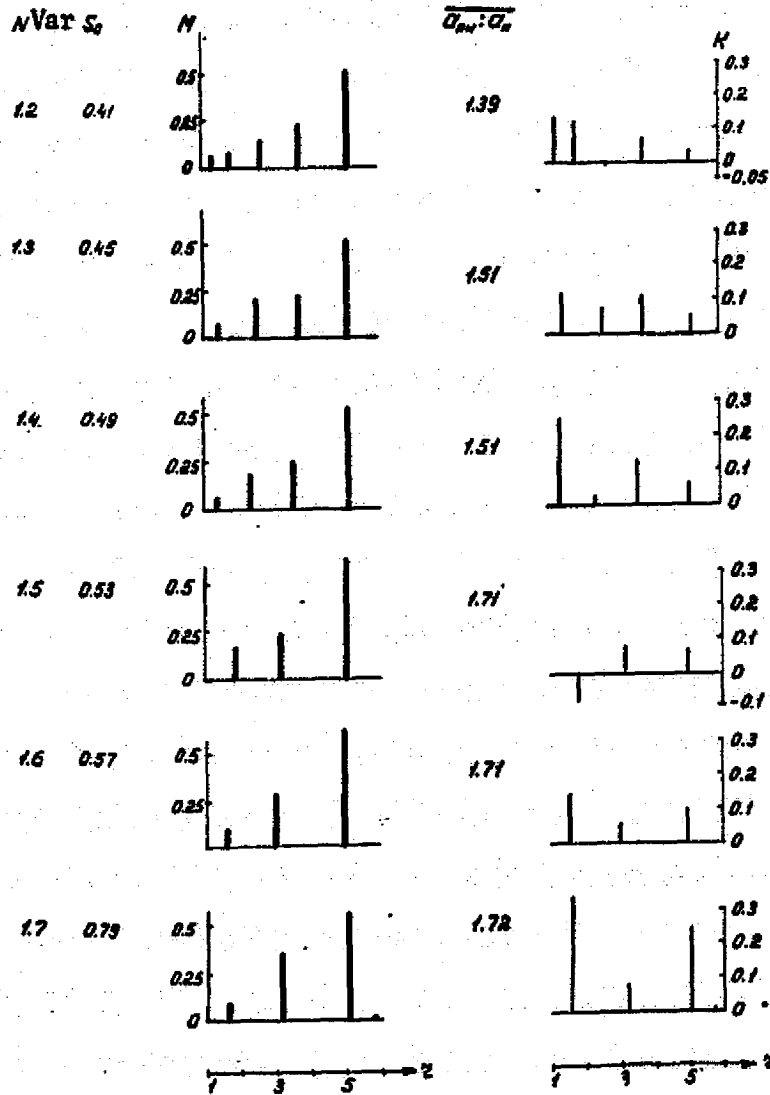in Fig. 5.

Figure 5. Distribution of mass and specific kinetic
moments for six variations of planetary systems
$(0.41 \lesssim S_0 \lesssim 0.79)$

The final results of all six variations of numerical experiments are presented on this drawing. Fig. 5 contains six horizontal pairs of graphs and each pair contains data on the distribution of mass by planet and on the rotational motion of the planets for the corresponding computation variation. A survey analysis of the variations presented in Fig. 5 confirms the existence of basic principles of an evolutionary process of accumulation of planets observed during a qualitative study of solutions of equation (4.15) and an analysis of the results of a numerical experiments of variation 1.1. Here, also, in all of the variations, the proto-planetary disk is divided sequentially into annular zones of contraction and rarefaction of matter and these zones completed its evolution by combining the bodies contained in them into a single planet. The circumstance then that the values S_o included in a range $0.4 < S_o < 0.8$ is extremely important; the number of planets which form during evolution of a protoplanetary cloud is close to or equal to their number in actual planetary and satellite systems. The "working" range indicated for the parameter S_o attests also to the especially tight compact position of effective bodies of protoplanets at the beginning of evolution of a protoplanetary cloud. This fact is extremely important when analyzing and determining the characteristics of a more complex gravitational model of a protoplanetary cloud. Further,turning to the right half of Fig. 5, we see from it that in all the variations, as in variation 1.1, predominantly forward rotation of the planets around their axes occurs. There is an exception in two cases relating respectively to variations 1.2 and 1.5. /45 Each of these variations occurs in a planet which has a forward rotating motion. Variation 1.5 is particularly interesting in this respect where a planet with backwards rotation has a negative specific moment of rotational motion which is very significant in absolute value. This negative moment was acquired by the planet during evolution as a result of a certain regular process whose

dynamics will be developed in one of the succeeding sections in the article. It is also important to note that in both variations (1.2 and 1.5) the values of the parameter S_o are close to their critical values which are maximum from the point of view of the numbers formed in the protoplanetary cloud of planets. From this point of view, variations 1.2 and 1.5 themselves are critical and the occurrence of planets in them with backward rotation is not random. Variations 1.3, 1.4, 1.6, and 1.7 contain planets only with forward rotational motion. In these variations, the effect of a decrease in specific rotational moment for the second from the origin of the axis of the abscissa of the planet (in comparison with the first planet) is interesting; here, this decrease is particularly great in variations 1.4 and 1.7.

Figures 4 and 5 show the final results of evolution of a protoplanetary cloud. The dynamics of evolution of a protoplanetary cloud developed in time, however, is very interesting. Such dynamics, in more or less detail, within the limits of the possibilities of the article will be presented for variation 1.7 in Figs. 6.1 -- 6.6. The dynamics of evolution of other variations, in their main characteristics, are very similar to the dynamics of variation 1.7. In Fig. 6.1 -- 6.2 and 6.3 -- 6.4, histograms are presented, respectively, of distribution of mass and specific rotating moments of accumulated bodies along the radial coordinate of the proto-planetary disk for different moments in time (an exception is Fig. 6.4 where, for a number of technical reasons, not histograms but individual specific moments are presented for the largest bodies belonging to a different zone of the histogram). In Figs. 6.5 -- 6.6 for all zones of the histograms in Figs. 6.1 -- 6.2, on the scale of the radial coordinated disk, the dimensions of the largest bodies are presented -- one on each zone of the histogram. In Figs. 6.1 -- 6.6, for the corresponding time, twelve characteristic phases of an evolutionary protoplanetary cloud are presented. In
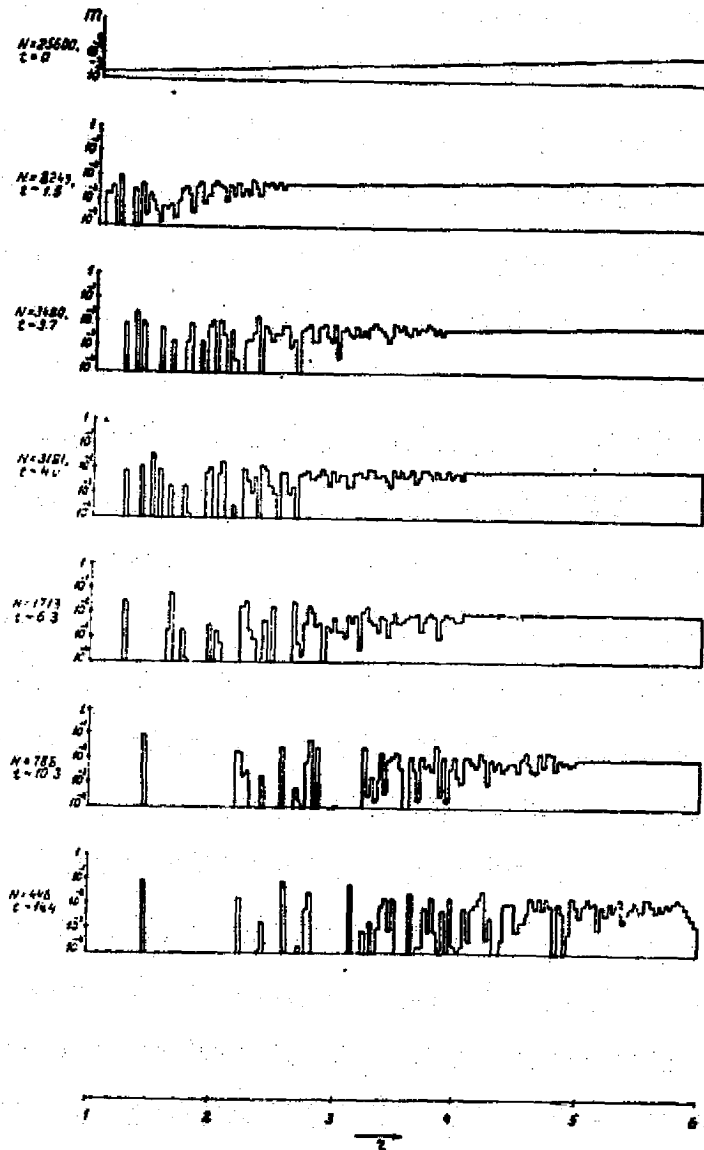
Figure 6.1. The dynamics of the accumulation process for the formation of planets in variation 1.7 (S_o = 0.79). Histograms of the mass.
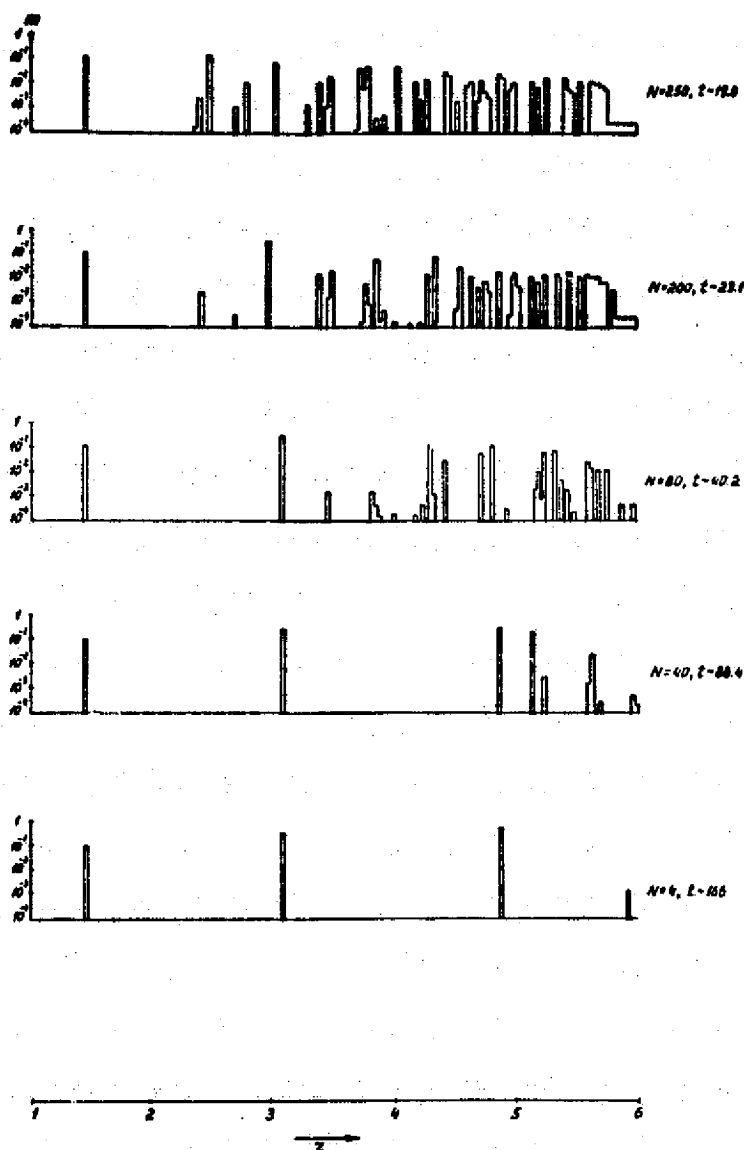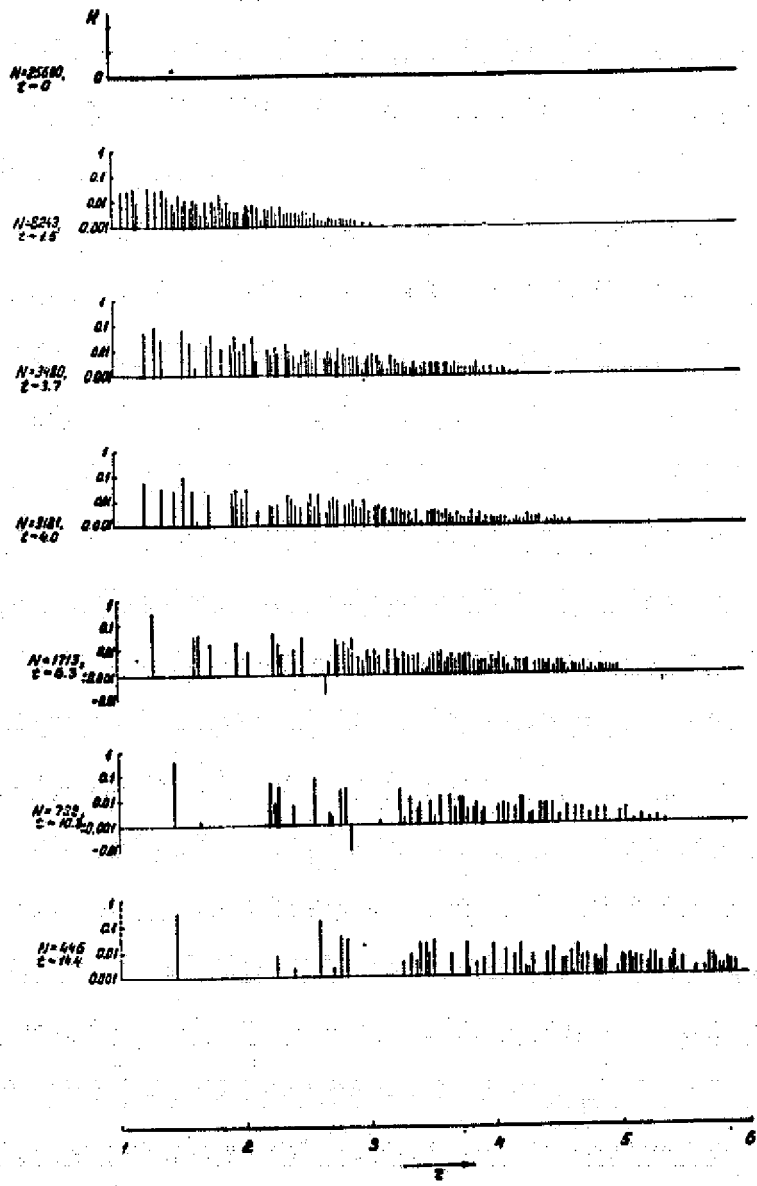
Figure 6.2.   Continuation of Fig. 6.1

Figure 6.3. Dynamics of the accumulation process of the formation of planets in variation 1.7 (S_o = 0.79). Histograms of specific kinetic moments.

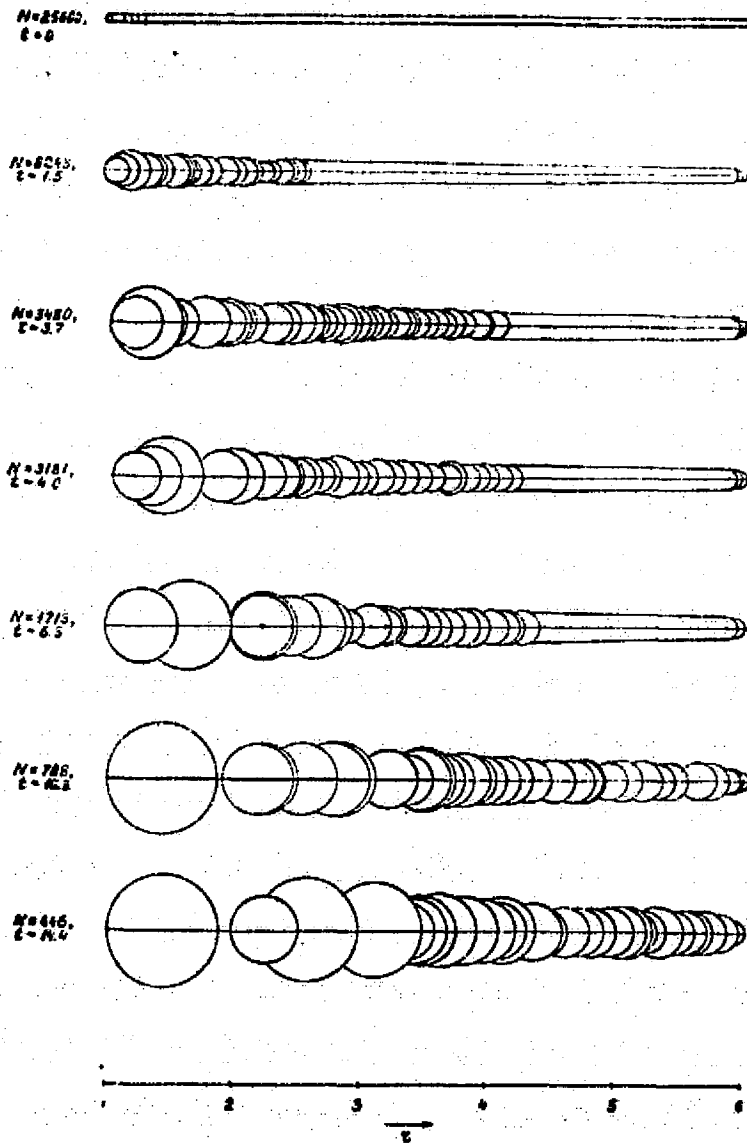Figure 6.4.   Continuation of Fig. 6.3.

Figure 6.5. The dynamics of the accumulation process of the formation of planets in variation 1.7 (S_o = 0.79). The dimensions of the largest bodies for the appropriate intervals of the histogram of mass and specific kinetic moments.
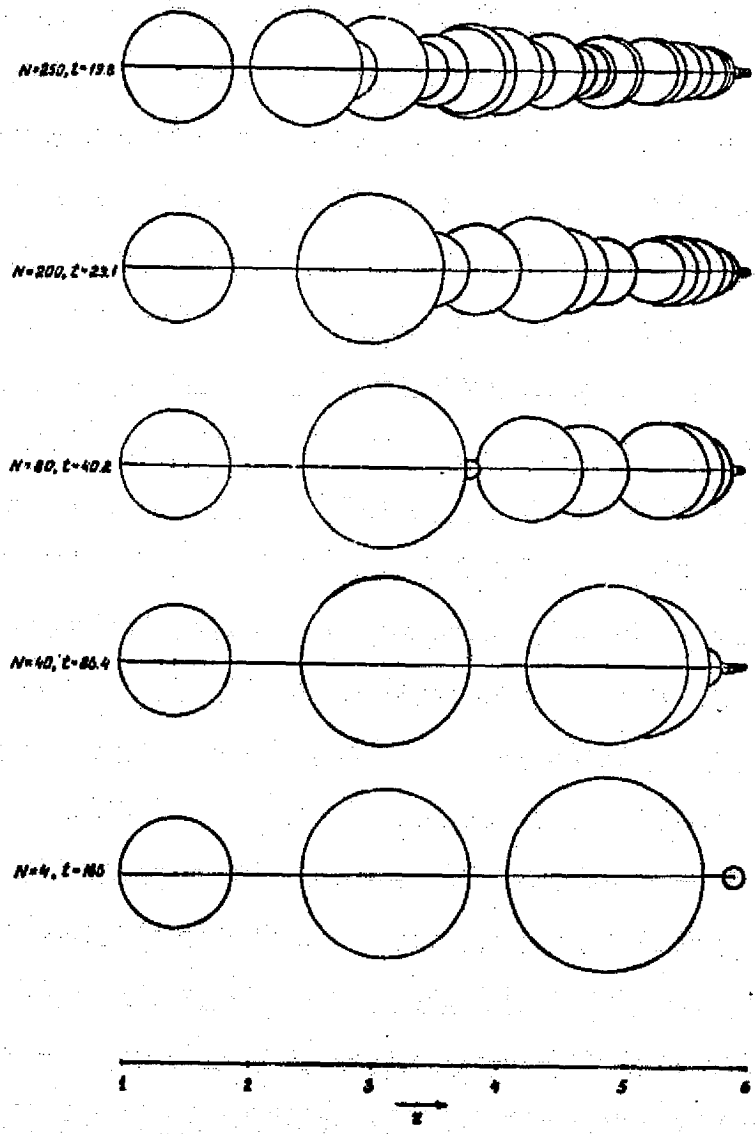
Figure 6.6.   Continuation of Fig. 6.5.

the drawings, for each phase, moreover, the number of bodies exist-
ing at a given moment in the cloud are indicated. The masses and <u>/52</u>
specific moments are applied along the axis of the ordinate in a
logarithmic scale.

Figures 6.1 -- 6.6 give a graphic concept of the dynamics
of an evolutionary protoplanetary cloud -- as to the sequential
phases of its development. Moreover, an analysis of the drawing
indicated brings up a number of questions involving the correct
interpretation of the results obtained; a discussion of these
results will be presented in the next section.

9. The Law of Formation of Planetary Systems. The Titius Bode
Law.

As was already noted in the preceding section, during analysis
of numerical experiments, in all of the variations the
<u>effect of annular contraction of matter</u>
of a protoplanetary cloud was detected which led in the final
analysis to the formation of a planetary system. Attention was
given to the possibility of identification of the effect indicated
with the effect of contraction of matter detected during qualitative
study of solutions of the equation of coagulation (4.15). Moreover,
this same analysis raised doubts as to the possibility of such an
identity; these doubts were based mainly on an analysis of the
dynamics of growth of dimensions of bodies during the accumulation
process. For clarifying what has been said, let us turn again to
the dynamics of the evolution of a cloud, variation 1.7.

Figures 6.5 -- 6.6 clearly show how the matter of the cloud,
first a thin film lying on both sides of its equatorial plane
(see upper graph in Fig. 6.5), during the course of evolution
gradually began to be agglomerated in drops with large dimensions
which were coverted at the center of condensation of the accumulation

zone. However, in the final stages of evolution of each zone, the dimensions of the drop became so large that the possibility arose for mutual interference between zones, with a breakdown, in the final analysis of the "correct" course of the evolutionary process. Then, by "correct" course of the evolutionary process we mean the course which, taking into account the laws of similarity (5.22), (6.11) would correspond to the course of the evolutionary process of an actual cloud. The possibility indicated was supported by an analysis of the course of evolution of the accumulation zone closest to the central body M_o. The final graph of Fig. 6.6 in which a planetary system which is fully formed is presented is evidence of this to some degree. It is apparent from this graph /53 that the distance between the surfaces of the planets, in the case where they are located on a single radial beam, is noticeably smaller than the diameters of the planets themselves. In connection with what has been said, a proposal has arisen that the accumulation process of the formation of planets which has been observed in a numerical experiment, for certain zones, to varying degrees, are involved not with the solution of the equation (4.15) found from analysis, by the effect of annular contraction of matter, but with the mechanism of its exhaustion in a way similar to that which was presented in 1946 by O. Yu. Schmidt. Here, in a given case, the mechanism of exhaustion of matter is caused not by the large values of eccentricities of the exhausted bodies but as is accepted in the O. Yu. Schmidt system, by the large dimensions of inter-acting protoplanets. The proposal indicated was shown as partially true and as a result numerical experiments resulting from it were conducted with a more complex gravitational model. However, the entire thorough analysis of all the numerical experiments made satisfactorily showed the veracity of the effect of annular con-traction of matter in the model processes of evolution of a proto-planetary cloud. This effect is fairly clearly apparent in all the accumulated zones of the protoplanetary disk even in those

very undesirably located close to the internal edge where, due to a relatively small number of bodies, it is shaded and at the final stages of evolution will break down as indicated above by a "parasitic" mechanism of exhaustion. Undoubtedly, it is true that the "parasitic" mechanism of exhaustion, being a truly geometric effect of rejection by large bodies of large areas in the disc, as a whole is caused by the approximate nature of the cloud model used, or, to speak more precisely, by the inadequately large (25600) number of bodies contained in it. An increase in the number of bodies in the mathematical model of a cloud undoubtedly leads to a decrease in the effect of the "parasitic" effect indicated, and, in the final analysis, with an adequately large number of bodies to its disappearance.

In order to study the effect of the "parasitic" mechanism of exhaustion of matter, on the one hand, and to be convinced of the significance of the effect of annular contraction of it on the other hand, special numerical experiments were conducted for studying the similarity of processes of accumulation of planets with a different number of initial bodies. One of these experiments related directly to a more detailed study of internal accumulation of the zone of variation 1.7. The process of evolution of a protoplanetary disk whose maximum radius coincided with the upper boundary of the accumulation zone of the first planet of variation 1.7 and with the maximum possible number of bodies (that is, 25600) was considered for this purpose. According- /56 ing to the dynamics of variation 1.7, the indicated upper boundary satisfied the value a_{max} = 2.07 (a_{min} = 1). In Figs. 7.1 and 7.2, a comparison is presented of the processes of accumulation in the first zone of variation 1.7 and in the entire field of the protoplanetary disk of a new variation 2.1. Fig. 7.1 presents a comparison of histograms of distribution of mass for both variations. In Fig. 7.2, by a similar method, the dimensions of the largest
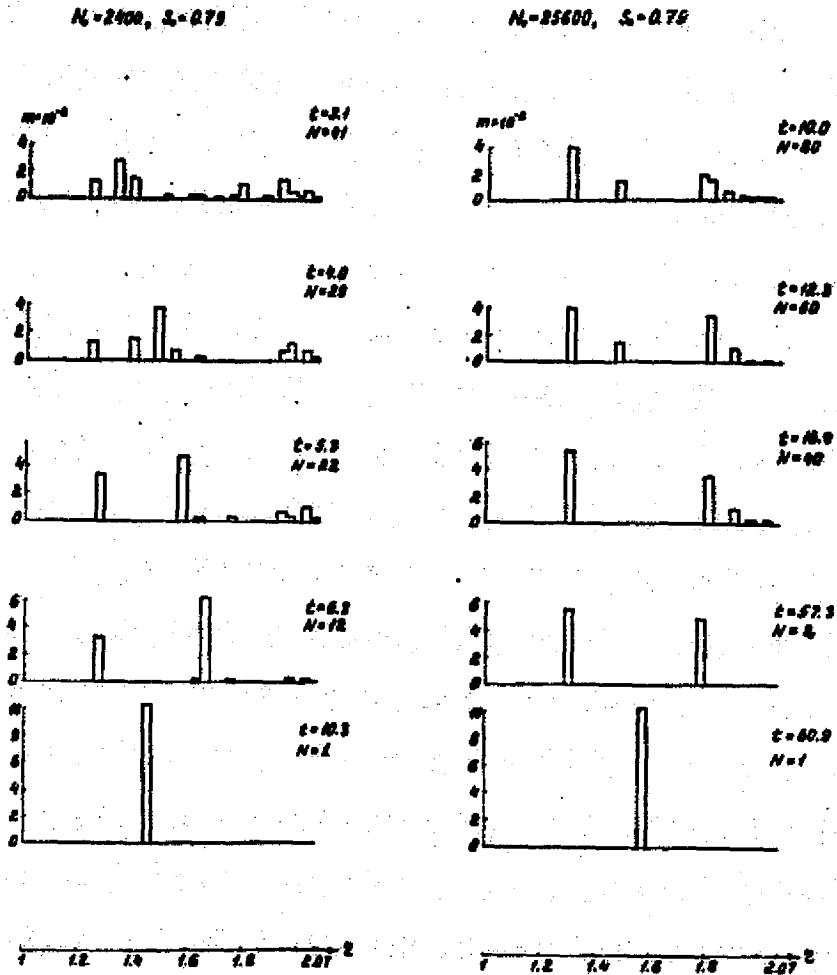
Figure 7.1. Histograms of mass for variations 1.7a (on the left) and 2.1 (on the right). An analysis of the similarity of the processes.
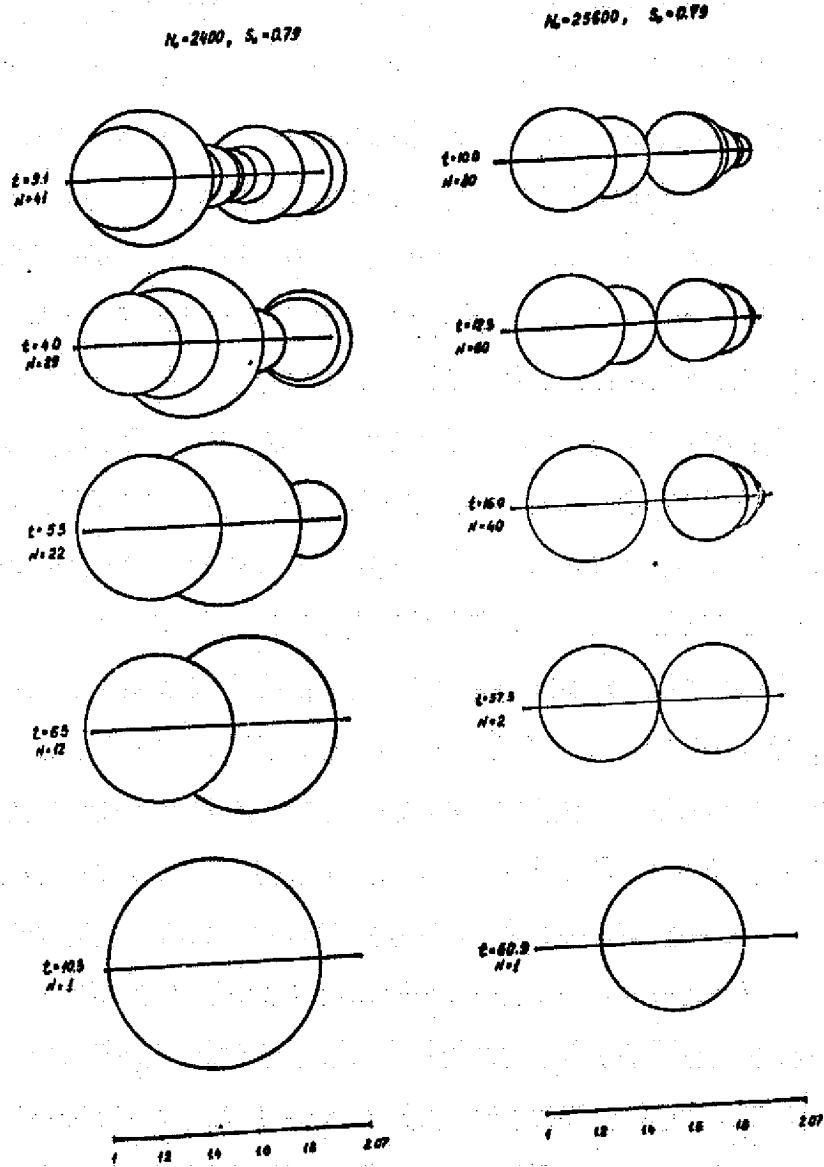
Figure 7.2. Dimensions of the largest bodies for the appropriate intervals of histograms in Fig. 7.1 and analysis of the similarity of the processes.

bodies relating to the appropriate zone of the histogram indicated
are compared. As is clear from the preceding, with uniform width
of the accumulation zone, both variations differ significantly in
the initial number of bodies accumulated, respectively, 2400 for
the first zone of variation 1.7 and 25600 for variation 2.1. A
comparison of both variations, both in the histograms of distribut-
ion of mass and in dimensions of bodies, clearly indicate the
significance of similarity in the evolution of a protoplanetary
cloud in both variations. Actually, by comparing the right and
left halves of Fig. 7.1 we see in both variations a uniform
process of concentration of mass in two more or less separate
regions of the cloud. The indicated concentration is expressed in
Fig. 7.2 where dimensions of the largest bodies are given for the
intervals of the histograms. It is interesting to note that the
fields of concentration of mass in variation 1.7a is somewhat
broader (more spread out) than the corresponding fields of variation
2.1. This was the expected because the model of the cloud in
variation 1.7a in this case is considerably rougher than the model
of variation 2.1 (radii of the initial bodies in variations 1.7a
is 3.1 times larger than the corresponding radii of variation 2.1,
and the bodies are 10 times less). During evolution, one observes
a tendency towards the formation of two planets in the accumulation
zone considered. The tendency indicated breaks off before the end
of evolution in both variations due to the "parasitic" mechanism
of exhaustion and differing from variation 1.7a, combining of the
last two planets in variation 2.1 occurred as the result of sig-
nificant eccentricity of the planets closest to the central body
($\underline{e}$ = 0.1). Thus, in variation 1.7, with a more complete model
(with a larger number of initial bodies) four large planets had
to form, and not three as in this case. This relates, apparently,
to the preceding variation. Too coarse a description of the in-
terior zone of the protoplanetary disk resulted in the fact that
due to the "parasitic" effect of exhaustion, the closest planets

in the final stage of evolution had to combine. In particular, /57
with a more detailed analysis of the dynamics of evolution variation
1.1, one observed that in the interior zone of the protoplanetary
disk, three planets had to form larger than were actually obtained.
Thus, with a more complete model, in variation 1.1, fifteen planets
were formed instead of twelve.

The value of variation 2.1 was not limited by the study on
the basis of its laws of similarity in the evolutionary processes.
In the numerical experiment with the boundaries of the accumulation
zone equal corresponding to $a_{min} = 1$, $a_{max} = 2.07$, as soon as two
planets were actually formed, the question arose as to the maximum
allowable relative width of the ring shaped cloud in which one
planet must form. For this purpose, a new series of numerical
experiments were undertaken with a liquid-drop model of a proto-
planetary cloud. At this time the external boundary of the
protoplanetary cloud a_{max} with constant values a_{min} and S_o were
taken as the variable parameter. Then, it was proposed that
$a_{min} = 1$, $S_o = 0.79$. A total of three additional variations of
a liquid-drop model of a cloud were considered. Table 2 gives
the characteristics of these variations. In the course of

TABLE 2

Var	2.2	2.3	2.4
a_{max}	1.88	1.72	1.50

numerical experiments with the variations indicated, the following
interesting result was obtained. Variation 2.2 in its main
characteristics did not differ from variation 2.1, as in variation
2.1, a tendency was detected toward concentration of matter in
two fields of the accumulation zone and, in the final analysis,

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the formation of two planets (if one does not take into account
the final "parasitic" effect). However, when transferring to
variation 2.3, a qualitative jump occurred in the character of
evolution of a protoplanetary cloud -- instead of the former,
a new tendency was apparent clearly for the formation of a
single planet. Then, one should calculate that $|\Delta a_{max}|$ with
transition from 2.2 to 2.3 is smaller than with transition
from 2.1 to 2.2 (0.16 and 0.19, respectively). Evolution of
both variations was illustrated in Fig. 8 where only the evolution
of dimensions of the bodies is presented for simplicity and   <u>/59</u>
visual clarity. Here, as before, the dimensions of the largest
bodies relating to the appropriate interval of the histograms of
mass are indicated. The drawing illustrates well the significant
difference in both variations. As to variation 2.4, qualitatively
it is no different from variation 2.3 which is clearly visible
on Fig. 8. Just as in variation 2.3, in 2.4 a clear tendencey
towards the formation of the single planet is apparent and this
formation is more compact (for example, a smaller "tail" zone
and a compressed cloud similar to that which was observed in 2.3).
Thus, the fact of the existence of the critical width of a ring
shaped protoplanetary disk was established in which a single planet
forms and the value appropriate to it for a given S_o (0.79) was
evaluated. It seems reasonable then that each effective cross
section of the body S_o corresponds to its characteristic critical
width of the disk.   For S_o = 0.79, this width can be evaluated
using the following approximate relationship:

$$\frac{a_{max}}{a_{min}} \approx 1.7$$

(9.1)

     Presenting the results of analysis made in this section,
we will formulate the main results of it. This result includes
the following. Numerical experiments with a liquid-drop
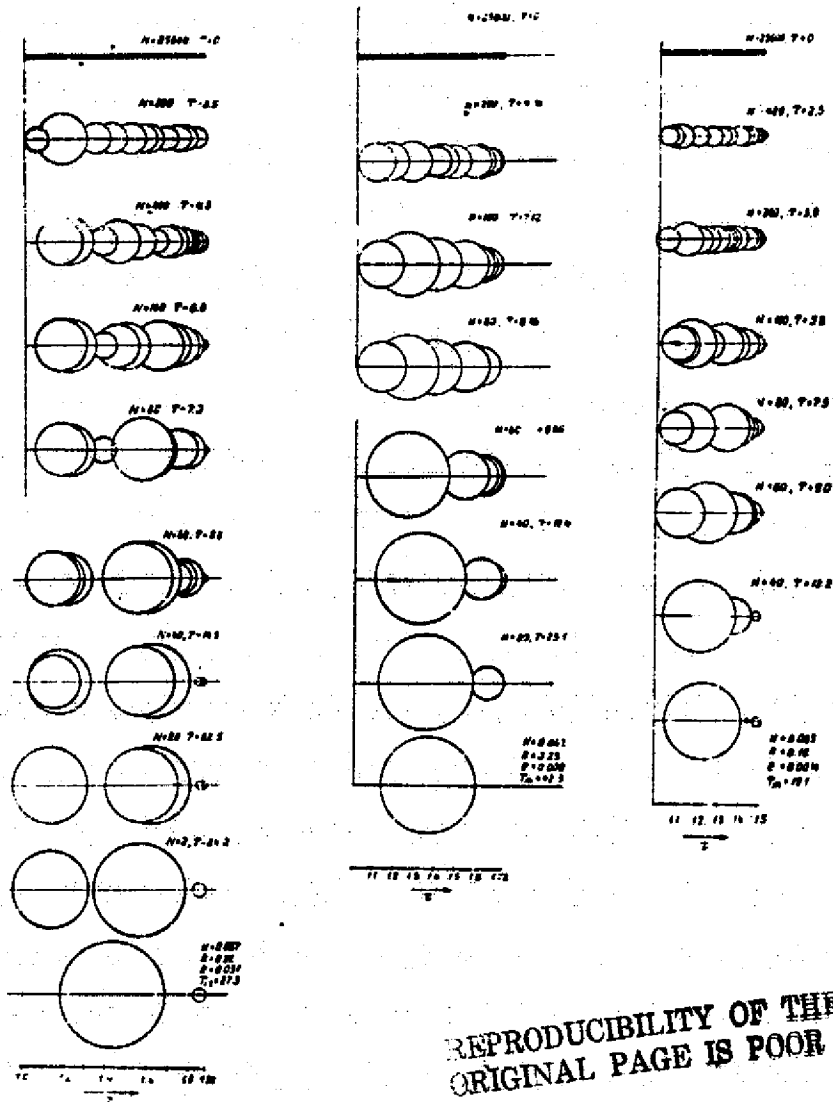
Figure 8. Variations of the formation of a single planet in accumulation zones with different widths: 1 < r ≤ 1.86 (variation 2.2), 1 < r ≤ 1.72 (variation [illegible] r ≤ 1.5 (variation 2.4)

model of a protoplanetary cloud, in spite of secondary inhibiting
phenomena involved with the coarseness of this model, confirmed
the significance of the effect of annular compression of the matter
of a cloud observed during theoretical study of the initial phase
of its evolution using equations of coagulation (4.15). Then, it
is possible with high reliability to confirm that the effect in-
dicated is present not only in the initial phase of evolution of
the cloud but also accompanies its evolution at all of its stages
to the final formation of a planetary system. The spatial parameters
of annular compression of matter are invariants in relation to
the initial dimensions of the protoplanet, with conditions of
their retention of the initial total effective area of the cross
section which exists per unit of area of the protoplanetary disk
and also when retaining the initial distribution of bodies accord-
ing to relative mass. For each initial total effective area of
the cross section of a protoplanet, a critical upper limit exists
for the relative width of the protoplanetary disk during which
no more than one planet is formed.

The numerical total results of the analysis presented above   <u>/60</u>
make it possible to move to consideration of one of the most
difficult questions of planetary cosmogony -- the question of
the <u>law of planetary distances</u>. In light of what has been presented
above, the question of moving to any plane of consideration in
which it has been considered up to the present time is advantageous.
Apparently, trying to find any single, universal, simple rule resting
on the physical essence of the problem considered is not promising,
one which temporarily guarantees a precise calculation of planet-
ary distances for any planetary systems. The character of the
processes leading to the formation of such systems is too complex
and depends too much on the initial parameters of the protoplanetary
cloud so that such a possibility could be realized. It seems
reasonable that one can take the approach indicated -- to find

certain simple empirical and approximate rules which have particularly been discussed and drawn attention to over the last two centuries beginning with the Titius Bode law. However, while it is not advantageous to look for a simple and precise law of planetary distances, it is expedient and correct to pose the question for finding any more general law, namely, a law of formation of planetary systems. Such a law, not producing directly formulas for calculation of planetary distances, must indicate simple and indisputable principles on whose basis both masses and distances of planets in a planetary system are formed. Such a law can now be formulated on the basis of existing material.

The law of formation of planetary systems. Let us assume R_0 and $\underline{m}_0$ are the initial characteristic radii and masses of the effective bodies when $\underline{r} = 1$, caused by relationships (4.5) or (4.6), S_0 -- is the initial total effective area of the cross section of protoplanets per unit of area of a protoplanetary disk, $\underline{n}(\xi \, \underline{r}, \underline{t})$ -- are functions of distribution of bodies according to relative mass $\xi = \underline{m}/\underline{m}_0$ and distance $\underline{r}$. Let us assume further for $\underline{t} = 0$, the following functions are given -- $S_0 = S_0(\underline{r})$, $\underline{n}_* = \underline{n}_*(\xi, \underline{r}, 0)$, where $\underline{n}_* = R_0^2 \underline{n}$. In this case, a protoplanetary cloud described by the parameters and functions indicated occurring in the course of evolution, the annular zones of compression and rarefaction of the matter itself, are formed into a planetary system, whose masses and large semiaxes do not depend on the initial characteristic radius of the protoplanet R_0.

The final accentuated part of the latter proposal comprises the essence of the law of formation of planetary systems. Relying on the law indicated, one can easily explain the main principles in the construction of planetary systems, in particular, the similarity of the solar system to satellite systems. Actually, inasmuch as the initial dimensions of a protoplanet, with other conditions being equal (in the first place, with equality of

effective cross sections of S_o), they do not affect the formation of _relative_ planetary distances, but gravitational constancy which characterizes only the scale of the phenomena also cannot affect them. Therefore, from the point of view of the relative proportions of orbits, all planetary systems must more or less be similar to each other although the masses of the centeral bodies and the dimensions of the initial protoplanets can differ by many magnitudes. It seems reasonable, when one assumes that the functions indicated earlier -- $S_o(\underline{r})$, $\underline{n}_*(\xi , \underline{r}, 0)$, must not differ too strongly for different systems.

Further, from the law presented above, an explanation exists observed in planetary and satellite systems, an approximate correspondence of the law of planetary distances and the law of geometric progression which, in particular, is expressed in the Titius Bode law. Actually, as is pointed out earlier, there exists a maximum critical value for the width of a protoplanetary disk in which only one planet forms. Because, due to the law considered, this width does not depend on the initial dimensions of the protoplanets, due to the dimensionless character of the distance of the internal edge of the disk ($\underline{a}_{min} = 1$), the indicated width has a relative character and with distance of the ring from the center, it must increase in absolute value. Introducing, in a rough approximation, a protoplanetary disk, we will compile from independently evolving rings adjacent to each other, with an initial structure satisfying the principle of similarity, and we will obtain, in this case, for planetary distances, a law of geometric progression. Actually, however, between separate accumulation zone (rings) of a protoplanetary disk, a more or less strong interaction exists and evolution of these zones, in the general case, cannot be considered independently. The interaction indicated results in the fact that even an approximate correspondence of planetary distances to the law of geometric progression can break down essentially which one observes in

actual planetary systems.

In light of what has been indicated above, now the concept
of the Titius Bode law has become clearer. This law, like other
laws however, resting on the principle of geometric progression,
corresponds to a very rough model of a protoplanetary cloud con-
sisting of a number of geometrically similar non-interacting
accumulation zones adjacent to each other. From the point of
view of the Titius Bode law, one must consider it as an approx-
imate rule and not as a law in the strict meaning of the word.

10. Forward and Backward Rotation of Planets. Rotation of
Venus and Uranus.

In the section presenting a survey of the results of the
numerical model of the processes of accumulation of planets from
a protoplanetary cloud, attention was given to the interesting
and important effect that most of the planets formed acquire a
forward (that is coinciding with orbital) rotational motion
around their own axes. This effect has an extremely stable
character and is observed in all, without exception, variations
of the numerical model of the processes of accumulation of
planets. Still earlier, in the section discussing the study
of the phenomenon of gravitational cohesion, also attention was
devoted to the interesting characteristic of gravitational
interaction of two bodies including stabilization in the process
of cohesion, of kinetic moment of both bodies relative to their
barycenter with a primarily positive sign of this moment. Earlier
it was established that gravitational interaction of two bodies,
in the case of their final joining, from the point of view of
their acquiring basic canonodic and dynamic characteristics
both of motion of the center of mass and motion around it,

is equivalent to absolutely inelastic impact of two conditional
sphere-shaped bodies which later on were called effective bodies.
It is significant and advantageous in this connection that the
mechanism of acquiring rotational motion, gravitationally inter-
acting and combining, be studied using an analysis of the collision
process of absolutely inelastic spherical bodies moving before
impact in a circular orbit.

Figure 9 shows a diagram of impact of two spheres moving in
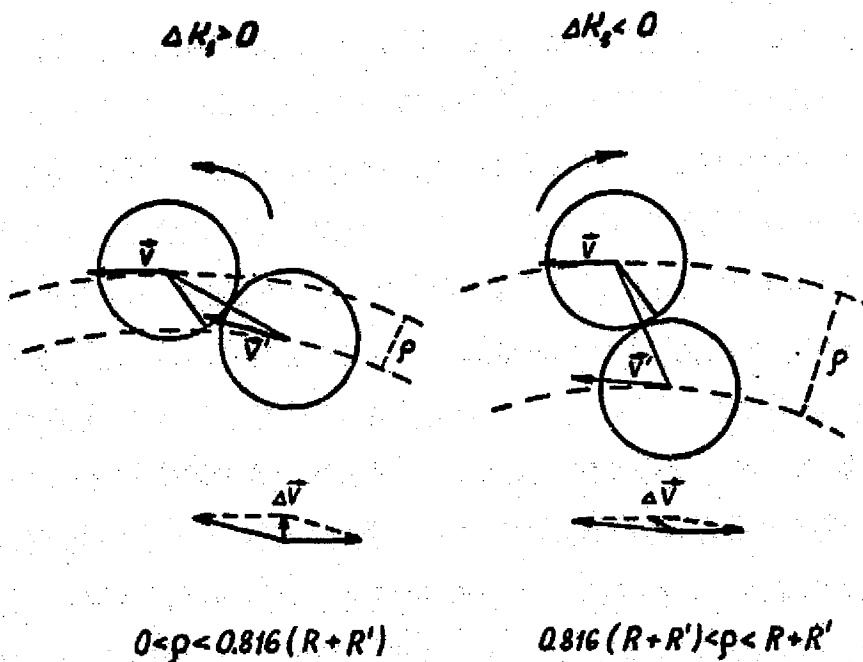a closed circular orbit. The plane of velocities of these bodies

Figure 9. Diagrams of frontal and glancing impacts

is presented in the same drawing. It is obvious that the increment of the moment ΔK_s significant for the sign acquired as a result of impact is the direction of relative velocity of overtaking (that is, close to M_o) body $\underline{m}'$. For acquiring a positive sign for ΔK_s, the indicated velocity must be directed counterclockwise relative to the center of the overtaking body $\underline{m}$. The latter however, depends on the radii of bodies R and R' and on the distance between their orbits $\underline{p}$. Naturally, in this connection, one attempts to find the critical value $\underline{p} = \underline{p}_*$ (with given R and R'), during which the vector of relative velocity of the overtaking body $\underline{m}'$ will, at the moment of contact, be directed at the center of the overtaking body $\underline{m}$. In this case, when $\underline{p} < \underline{p}_*$, the vector of relative velocity of the body $\underline{m}'$ will be directed counterclockwise relative to the center of the body $\underline{m}$ and then, obviously $\Delta K_s > 0$. When $\underline{p} > \underline{p}_*$, a contradictory picture will exist and consequently, as a result, we will find $\Delta K_s < 0$. In order to find the critical value $\underline{p}_*$, we will turn to formulas (6.1) and (6.2). Assuming $\Delta K_s = 0$, according to (6 1) and (6.2) we will find:

$$\rho_* = \sqrt{\tfrac{2}{3}}(R + R')$$

(10.1)

On the other hand, it is obvious that R + R' is the maximally allowable value $\underline{p} = \underline{p}_{max}$, during which contact of moving bodies in combined orbit is still possible. In this case, drawing attention to (10.1), one can write:

$$\rho_* = 0.816 \cdot \rho_{max}$$

(10.2)

and consequently, when

$$при \quad 0 < \rho < 0.816 \cdot \rho_{max} \qquad \Delta K_s > 0$$

(10.3)

when

$$0.816 \cdot \rho_{max} < \rho < \rho_{max} \qquad \Delta K_s < 0$$

(10.4)

From (10.3) and (10.4) it is easy to see that more than 80%
of the potential orbits permitting contact between two bodies
lead to a positive increment of rotational moment and only 20%
correspond to a negative increment of it. Qualitatively and
quantitatively this completely coincides with the results of
analysis of the phenomenon of gravitational cohesion.

From the analysis presented we see that two types of impacts
for interacting bodies exist -- the type resulting in a positive
increment of rotational moment of the combining bodies and the
type which creates its negative increment. Impact of the first
type will henceforth be called frontal, and impact of the second
type -- glancing. One should note that by itself the high percent
of potential orbits corresponding to frontal impact still does not /65
guarantee a positive moment at the end of the evolutionary process.
Combinations of orbits corresponding to glancing impact contain
a body with high relative velocities and, consequently, with high
probability of realization of contacts. The actual density of the
orbital flow of the bodies plays an important role in the total
course of accumulation. By the latter we mean the number of
actual orbits which occur per unit of length along the radius from
the central body M_o. High density of orbital flow favors the
predominance of frontal impacts. With low density one can have
a predominance of glancing impacts. As to density of orbital flow,
it is determined by the main mechanism of the evolutionary process --
by the effect of annular compression of matter --and directly involves
the degree of interaction of different accumulation zones among
each other. Among the accumulation zones, during evoution, an
unusual bond is established through attraction of the boundary
substance between them, which, in turn, affects the density of
orbital flow inside the zones themselves. In certain zones, the
main mass of matter breaks off this bond early and more or less

rapidly combining them creates a high orbital density of flow
and moreover a high final rotating moment for the planet. Other
zones subjected to the main mass of the matter, with a longer
period of pull by their neighbors, break off from them considerably
later and enter into the final stage of evolution with weaker
density of orbital flow; as a result of this, they create the planet
formed with a small positive or even negative rotational moment.
In a similar way, apparently, the backwards rotational motion
around their axes was formed in Venus and Uranus. In a common
gravitational model of a protoplanetary cloud, the picure of
distribution of accumulation zones of different types can be fairly
complex. As to the liquid-drop model, alternation of "strong"
and "weak" accumulation zones is characteristic for it. This
explains, in particular, the strange, at first glance, oscillating
character of distribution of moments and mass in the variation 1.1.
In common in both the liquid-drop and the gravitational model, in
all the numerical experiments, there was a decisive predominance
of frontal impacts with the very first accumulation process of
evolution of the cloud. This is clearly obvious, in particular,
in the example of variation 1.7 (see Fig. 6.3 -- 6.4). The   <u>/66</u>
significant role of glancing impacts was apparent in certain
accumulation zones even at the end of their evolution. Due to
this, the concept was presented in more detail in variation 1.5
in which the planet formed according to the model with a large
negative rotational moment. In spite of the flaws in this variation
(distortion, due to the final effect, of the picture of formation of
the first planet), it is of considerable interest inasmuch as it
demonstrates in precise form one of the possible regulating processes
which creates backward rotational motion of the planet formed.

    Figure 10 shows histograms of distribution of mass and
specific rotational moments of accumulated bodies along the radial

coordinate of the protoplanetary cloud and also on the scale
of the radial coordinate, the dimensions of the largest bodies
for corresponding intervals on the histogram.   The histograms
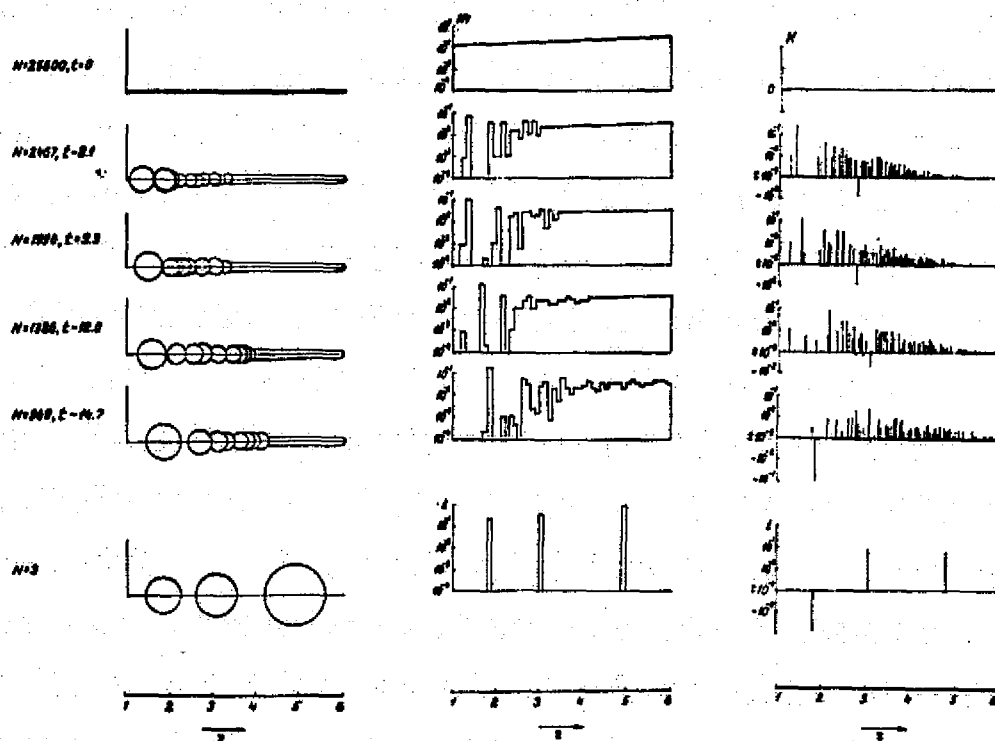

Figure 10.   The dynamics of formation of backward
rotation of a planet for variation 1.5 (S_o = 0.53)

and dimensions of the bodies are presented for five characteristic
moments of time of formation of the first planet of the cloud which

is closest to the central body M_o. It is clearly apparent from the drawing that when $\underline{t} = 8.1$, actually the nucleus of the first planet was formed with a very significant specific kinetic moment of rotational motion. This nucleus, however, maintains contact with the boundary substance of the neighboring accumulation zone. Continuing to maintain its contact, with $\underline{t} = 8.1$ to $\underline{t} = 14.7$, the nucleus indicated, by attempting a series of glancing impacts, gradually moved away from the central body M_o, decreased practically to zero the value of rotational moment and significantly increased its mass. Finally, when $\underline{t} = 14.7$, using the last glancing blow, the nucleus acquires a considerable negative rotational moment, increases its mass for the last time and, breaking off from the neighboring accumulation zone, forms the first planet. We note that besides what has been discussed, there exists at least one more regulating process in the impact interaction of bodies resulting in backward rotation of a planet. This process was detected when working with a gravitational model and differs from that considered by the appearance in the zone of accumulation, in the final stage of formation of the planet, of several large bodies with backward rotation.

At the conclusion of the section, we will stop to consider an important question involving rotational motion of planets. It is well known that in most planets, the axis of rotation is fairly strongly slanted from the perpendicular to the plane of the ellipse. As for Uranus, its inclination reaches 83°. In any theory of the origin of planets the facts indicated must be explained and the theory considered must not exclude it. Moreover, here certain /68 difficulties arise. The mechanism considered above for accumulation of planets does not favor a slope of the axes of their rotation from the normal to the plane of the orbit larger than the mutual slant of planes of the orbit of different planets. In particular,

according to the theory considered, it is very difficult (not to say impossible) to obtain combinations of interacting bodies which could be created with a similar slant to the axis of rotation such as Uranus has. One could say that it is necessary to find the cause for the slant of the axis in planets outside the mechanism of their accumulation from a protoplanetary cloud. One can apparently point out the cause at the present time.

V. V. Beletskiy, in reference [8] studied the question of evolution of the axis of rotation of a planet subjected, thanks to the perturbing body (Sun), to the effect of influx moment. The qualitative analysis of averaged evolutionary equations made it possible for Beletskiy to construct the following interesting phase picture of the behavior of the axis of rotation of a planet. Figure 11 shows integral curves of averaged evolutionary equations of rotating motion of a planet. In this drawing, along the axis of the abscissa the angle of slope of the axis of rotation of the planet to the plane of orbit Θ is applied along the ordinate axis as parameter Ω which is the ratio of angular velocity of rotation of the planet ω_s to angular velocity of motion of it along the orbit ω, that is, $\Omega = \omega_s/\omega$. We see from this drawing, then, all of the integral curves of the phase picture meet at the critical point $(0, \Omega_*)$ where Ω_* depends on the eccentricities e and then when $e = 0$, $\Omega_* = 0$. In this way, the influx moment always tries to direct the evolution of rotational motion, in the final analysis, so that it will become forward. Rotation of the planet first being backward $(\Theta = \Pi)$, under the effect of influx moment, will have to evolve toward forward rotation. Beletskiy, because of this, turns his attention to the possibility of explaining backward rotation of Venus by its capture in residence due to the effect of influx moments. In the light of the theory of the formation of a planetary system presented, however, this is
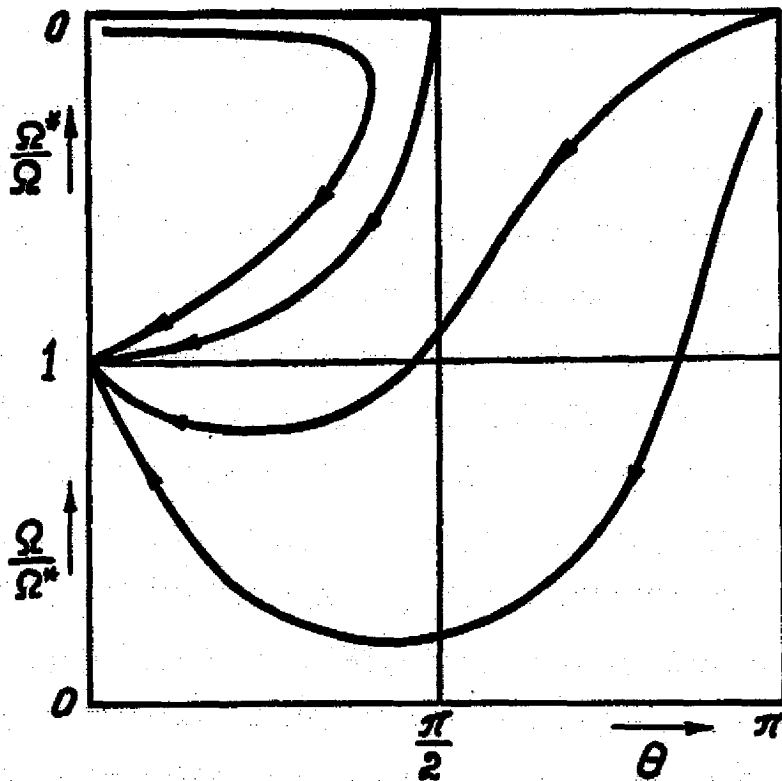
Figure 11. Integral curves of evolution of the slant of the axis of rotation of a planet under the effect of influx forces (according to Beletskiy [8])

not necessary. The backward rotation of Venus in a natural way is explained by the mechanism of accumulation of planets from a protoplanetary cloud. The absence of the evolution noted of the axis of its rotation is also explained by very slow rotation of the planet so that it is practically devoid of influx peaks. It is different in the case of Uranus. The rapid backward rotation /70

and powerful influx peaks of Uranus lead one to think that the
axis of its rotation, first close to normal to the plane of the
orbit, evolved strongly and approached the indicated plane.
The Beletskiy theory also explains the significant tilts of the
axes of planets with forward rotational motion. From Fig. 11 we
see that the axis of a planet which has high angular velocity
of forward rotational motion, during evolution can deviate from
the normal with the existing method, even approaching the plane
of the orbit ($\theta = \pi/2$). Unfortunately, at the present time it
is difficult to say in which phase of evolution the planets with
forward rotational motion were evolved, that is, the quantitative
side of the theory considered has still not been developed. One
can say however that it is very probable that the main part of
evolution, which has already been undergone by the planet indicated
(the same as, for example, Uranus) were completed very recently and
comparatively rapidly, so that possibly it is just in this period
when the planets have not completed formation as fairly dense
bodies. Future studies must be made to clarify this question.

11. The parameters of a protoplanetary cloud and gravitational instability. Calculation of parameters of a gravitational model of a cloud.

The results of numerical experiments presented make it possible
to make the first, although preliminary, evaluations of the
evolutionary parameters of a protoplanetary cloud. One should note
that when making the indicated evaluations, certain difficulties
arise, inasmuch as the liquid-drop model of a cloud is not completely
adequate as an actual prototype. A gravitational model of a cloud
has the adequacy, however, its use is not free from very considerable
difficulties in view of the lack of clarity in determining a
number of initial principles which describe the initial structure
of the cloud. For example, the character of initial relationships

from r is unclear for the effective area of the cross section and the radii of the protoplanets, that is, the relationships $S_o = S_o(r)$ and $R = R(r)$. It was proposed that numerical experiments with a liquid-drop model would have to clarify the question as to the character of the relationships indicated. These hypotheses were correct to a significant degree. Thus, numerical experiments fairly rapidly show the "working" range of the effective area of a cross section of bodies S_o which would correspond to the actual geometry of the solar planets and the satellite systems. As was <u>/71</u> noted earlier, this range corresponds to a very tight compact initial placement of effective bodies of the protoplanetary cloud. Speaking descriptively, the effective bodies of protoplanets, at the initial moment of evolution are more or less tightly pressed against each other. This fact made it possible later on, as an adequately good initial approximation, to form a hypothesis as to the independence of the total effective area of a cross section of the protoplanets on the radial distance r, that is, to assume that S_o = const. This same fact was the starting point for the search for dependencies on r for another important characteristic of the protoplanetary cloud, the initial radius of effective bodies of the protoplanets $R(r)$. However, before going on to a consideration of the question of character of the dependence $R(r)$, we will make certain estimates of the evolutionary parameters of an actual protoplanetary cloud. For this purpose we will use the theory of similarity constructed in section 6 for the results of numerical experiments.

We note, first of all, that using the variation of a liquid-drop model considered earlier (when R_o = const), the formation of the solar system or its separate parts cannot be modelled with adequate precision. This applies both to the Earth group of planets and to the group of planets called the giants. However, it was possible to model with complete satisfaction from the

quantitative point of view the process of individual formation of a number of planets relying on certain specific properties of the accumulation zones of a protoplanetary cloud. It was noted earlier that among the accumulation zones it is possible, from a certain point of view, to separate at least two types -- the "strong" zones and the "weak" zones. The accumulation zones of Mercury, Earth, Mars and Neptune belong to the first type. The second type includes the zones of Venus and Uranus. The question of accumulation zones of Jupiter and Saturn is somewhat different. The "strong" accumulation zone type, as was noted earlier, is characteristic for the well known autonomy of its method of development. The main mass of its matter interacts fairly weakly with the neighboring accumulation zones and therefore, for the type of zone considered, it is possible to have an isolated model of the accumulation process. Also the circumstance that the relative width of the "strong" type zone is comparatively small facilitates this circumstance considerably; in this case $a_{max}/a_{min} \sim 1.5$ (for a "weak" type zone, $a_{max}/a_{min} \sim 2$). Thanks to this circumstance, to a known degree, the question of the character of dependencies $R(\underline{r})$ loses its $\qquad$ /72 acuteness. In a first approximation, in this case, one can assume that $R_o = R(\underline{r}_{av}) = $ const, where $\underline{r}_{av} = 1/2 \ (\underline{a}_{min} + \underline{a}_{max})$. One should note further that the evolutionary process of formation of Mercury, unfortunately, is not suitable for purposes of evaluating the parameters of a protoplanetary cloud by using the theory of similarity constructed earlier. All of the data attest to the fact that the rotational moment of Mercury, due to its proximity to the Sun, lost its significant evolution for the time of existence of the solar system, and therefore cannot be used as the criterion of similarity. Three planets remain -- Earth, Mars and Neptune. Here, one should note that with a comparison of rotating moments, the real and the model, in this case, apparently, it was correct to consider the Earth-Moon system and not the Earth separately.

The fact is that independent of the solution of the question of the origin of the Moon, present day data convincingly attest to the fact that the initial, almost entire moment of the system of the Earth-Moon was concentrated in the Earth and only during a long period of influx evolution was it redistributed for the Moon. One should add to this that the hypothesis about the common origin of Earth and the Moon satisfies more the principle of the concept of the origin of the solar system considered as a whole than does the hypothesis as to the capture of the Moon by Earth.

In order to model the process of accumulation from matter of a protoplanetary cloud, the planets Earth, Mars and Neptune are closest to variation 2.4 for which $a_{max}/a_{min} = 1.5$. In variation 2.4, as a result of the accumulation process, a planet is formed with a large semiaxis of the orbit $a_{min} = 1.26$ and specific kinetic moment of rotating motion $K_{sm} = 0.083$. The time of evolution and the initial radii of effective bodies equal $T_m = 19.1$, $R_{om} = 0.00619$. The kinetic moment indicated above must be recalculated for the new distances of the actual modelled planets which can be done using the formula:

$$K_{sm}^{(i)} = K_{sm} \sqrt{\frac{a_i}{a_m}} \;,$$

(11.1)

where a_i is the large semiaxes of orbit of actual planets, $K_{sm}^{(i)}$ -- is the model specific kinetic moment of rotating motion recalculated for the appropriate semiaxis. The actual specific kinetic moments of the planet were calculated in a system where the Earth's year was used as the unit of time, the astronomical unit as the unit of length and the mass of the Earth as a unit of mass. /73 The combined data, as a result of using the theory of similarity for calculation of actual times of evolution of accumulation zones

T and actual radii of effective bodies of protoplanets of these zones R_o, are presented in Table 3.

TABLE 3

1) планета	K_s	$K_{sm}^{(i)}$	ρ	2) T (тыс.лет)	R_o (км)	$\tilde{R}_o \sim 6z^2$ Гуревич Лебединский 3)
4 Земля–Луна	$0.72.10^{-5}$	$0.74.10^{-I}$	1020	14	720	400
5 Марс	$0.33.10^{-6}$	$0.91.10^{-I}$	12000	300	95	76
6 Нептун	$0.38.10^{-4}$	0.41	1060	2400	21000	160000

Key -- 1) planet
2) T(thousand years)
3) Gurevich
Lebedinskiy
4) Earth-Moon
5) Mars
6) Neptune

In this table, the actual specific kinetic moments of rotational motion of planet K_s are sequentially presented as well as their model analog $K_{sm}^{(1)}$, obtained using variation 2.4 and formula (11.1) and calculated using the theory of similarity -- the parameters of similarity $\underline{p}$, actual full time of evolution T and actual radii of effective bodies of the protoplanets R_o. An analysis of Table 3 shows that the time of evolution of a protoplanetary cloud, on a cosmogonous scale, was very small. Effective bodies of the protoplanets at the beginning of evolution of the cloud were the smallest. However, this does not exhaust the information presented in the table. As will be seen below, it

potentially contains key data for determining a number of important
properties of a protoplanetary cloud.

The compactness of initial configuration was noted earlier
in the positioning of the effective bodies of the protoplanets.
Naturally the assumption occurred that this compactness is not
random and somehow involves important processes which occur in
the initial period of formation of a protoplanetary cloud. Here,
the logic of common conception which is the basis for the study
made, directed the search for this connection in the field of
problems of gravitational instability of a protoplanetary cloud
in the period of the dust components settling into its equatorial
plane. This problem, considered in all its aspects, is independent
and almost simultaneously was first studied in the works of
K. Edgeworth (1949) and L. E. Gurevich and A. I. Lebidinskiy (1950).
The most interesting results were obtained by L. E. Gurevich and
A. I. Lebidinskiy [1] who successfully made a significant step in
the study of the early phases of evolution of a protoplanetary
cloud. According to references [1,2], the dust component of the
cloud, thanks to mutual nonelastic collision of its particles,
and also their friction as gas, gradually began to settle to its
equatorial plane, forming a more or less thin disk with increased
density of the matter. Upon achieving a disk with certain
critical thickness, it broke down into many dust clots in which
the internal gravitational force was larger than the perturbing
force of the Sun. On the basis of the approach given in [1],
the following formula was successfully obtained for the maximum
radius (large semiaxis) of a clot:

$$\hat{R}_* = \gamma \sigma \tau^3$$

(11.2)

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where σ is the surface density of matter of the disk, γ = const.
The values $\hat{R}_0$ for different accumulation zones of a protoplanetary
cloud calculated according to formula (11.2) are located in the
last column of Table 3. The quantity of matter in each unit of
area of an accumulation zone is taken as the value of surface
density σ for each accumulation zone in the hypothesis that the
entire mass of corresponding planets "spread" evenly over the
entire area of the zone. The dimensions of the area of the zone
are calculated from the condition that

$$\frac{\alpha_{max}}{\alpha_{min}} = 1.5 .$$

From Table 3 we see that the dimensions of effective bodies
of a protoplanet and the dimensions of the clots obtained accord-
ing to formula (11.2) have the same magnitude for Earth and Mars
and differ only in magnitude for Neptune. The latter difference
can be explained by the fact that the maximum model used in this
work for a protoplanetary cloud in which each gravitational
cohesion ended in combining with the body, is not fully satisfactory
in the case of Neptune. According to Table 3, the time of
evolution of accumulation zones of Neptune is fairly large
($\sim 2.5 \cdot 10^6$ years). In this time, the clots which first occurred
could evolve strongly toward compression. As a result, the <u>/75</u>
nuclei of effective bodies of protoplanets (and which are inherently
protoplanets) can acquire fairly small dimensions at which a
large part of the gravitational cohesions ends not in combination
of bodies but in detachment. The bodies which had separated
acquired a marked eccentricity, disturbing the regular system
of frontal and glancing impacts described earlier. In the system
of impacts, a chaotic component occurred which slows down the
transition of the orbital kinetic moment to rotational. As a
result, Neptune could not acquire the entire potentially allowable

rotational kinetic moment (having discarded no more than 20%
of it). If this moment was completely rejected, then the divergence
in the radii of clots and effective bodies for the Neptune zone
would have had the same magnitude as in the case of Earth and
Mars.

Independently from the method of interpretation of data
related to Neptune, and giving attention to the roughness of
the mathematical model of a protoplanetary cloud, one should
recognize the existence on the whole of a close correspondence
between the dimensions of the clot and the effective bodies of
the protoplanets in all three examples given. This correspondence
could hardly be random. The dimensions of the clots and the
effective bodies of protoplanets are defined, in the final
analysis, from common initial samples (interior gravitational
forces are larger than external perturbing forces). On the other
hand, the principles on which the numerical values of dimensions
of clots and effective bodies are based vary considerably. In
the first case, this theory of gravitational instability of a
preplanetary cloud, in the second place -- the theory of occurrence
of rotational motion of a planet. A comparison of numerical
results obtained from both theories give evidence
in favor of their truth.

Thus, a comparison of the data in Table 3 and the results
of the theory of gravitational instability makes it possible to
find the missing link with which there is the possibility of
completing the description of the structure of a more common
gravitational model of a protoplanetary cloud. This link is
identification of the initial dimensions of effective bodies
of protoplanets with dimensions of clots, into which the pre-
planet gas dust cloud decomposes due to gravitational instability.

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Formula (11.2) becomes the formula for determining the initial radii of effective bodies of protoplanets of a gravitational model of a protoplanetary cloud.

12. Conclusion

At the conclusion of the article we will present the results and attempt to give an evaluation of the basic results obtained. Undoubtedly, the main effect observed during the study is the effect of annular compression of matter which occurs during evolution of a plane protoplanetary cloud. The existence of this effect is confirmed by two independent methods -- by qualitative study of equations of coagulation of bodies in orbit and by numerical modelling of the process of accumulation of protoplanets on a digital computer. A correlation is established between results obtained by the two different methods. During a qualitative study of the equation of coagulation, and using a comparison of the results of this study with the results obtained by numerical modelling, a theory of similarity was constructed which makes it possible to show the invariant principle of evolutionary processes, and, by kinetic characteristics of rotating motion of planets, to establish the connection between the parameters of a mathematical model and an actual protoplanetary cloud. Using this connection, the initial and evolutionary parameters of an actual protoplanetary cloud were successfully calculated. The theory of similarity made it possible to establish the independence of a proportion of the orbit and thus the total appearance of the planetary system formed from the initial dimensions of the protoplanets of the cloud when similarity is retained in the relative on-off time ratio of the initial configuration of the protoplanets in the cloud (or, in other words, in the relative compactness of this configuration). One should note that the last result applies mainly to the actual protoplanetary clouds whose bodies had small relative dimensions.

In the numerical model of the cloud, due to its coarseness, caused by an inadequate number of bodies, the proportions indicated broke down somewhat due to transition of a significant part of the orbital moment of motion to rotational and due to this transition, a more or less marked "collapse" of the orbits. Finally, the theory of similarity could formulate a common principle of formation of planetary systems and in this framework one could assume the concept and character of the Titius Bode law.

Further, the circumstance that annular compression of matter of a cloud occurred with very small eccentric orbits of accumulated bodies is very important. Even in a numerical model of the cloud, the overwhelming majority of bodies, in the course of evolution, had orbits with eccentricities which did not exceed 0.001. Even at the end of the accumulation process in a given zone, the last /77 few bodies, in some cases, could acquire a more or less important eccentricity ($e \sim 0.1$). In a real protoplanetary cloud, in most of the accumulation zones (except for the zone of Neptune), the eccentricities had, apparently, even smaller values (the values of eccentricities are the same as the values of rotational moments of the planets and increase along with growth of relative dimensions of the bodies participating in the accumulation process). In the same principle, annular compression of matter can exist, not disturbing the laws noted above for invariants of the total picture of the zones of compression and rarefaction of matter and with practically zero eccentricities. For this it is only necessary that the initial dimensions of the protoplanets be adequately small. All of this produces the accumulation process considered in this work in a considerably poorer way for similar processes of most of the preceding works which use a system of exhaustion by "nuclei" — by the embryos of planets of the matter of a protoplanetary cloud whose particles had very large eccentricities ($e \sim 0.2$).

From the preceding studies, the closest to the results of
this work are the interesting ideas of Alfven about jet streams
and their role in the structure of the asteroid band presented
by him in 1964 [9] and also touching on the ideas of the results
from reference [10]. In the opinion of Alfven, nonelastic
collision of particles rotating around the Sun which results in
averaging of their orbits played an important role in the formation
of large bodies in the protoplanetary cloud. Due to this, a
unique tendency occurred for mutual attraction of orbits which
force particles into a series of jet streams made up of particles
moving in a close orbit. As the mutual velocities of particles
decrease, due to inelastic collisions, the processes of their
combination began gradually to predominate over the processes of
breaking up and in the flow large bodies began to form. Alfven
considered the existence of an asteroid band and jet streams of
asteroids observed by him and by Arnold as one of the proofs for
his hypothesis. In reference [10] the phenomenon described
above of orbital focusing of particles was modelled using the
Boltzmann equation and by numerical solution of the latter on
a digital computer. The authors of reference [10] successfully
showed that a uniform plane system of small particles, during
evolution, due to nonelastic collisions can be transformed into
more or less dense annular flows in the system. In spite of the
difference in models in this work and [10], it is not exclu-   <u>/78</u>
ded that the mechanism of annular compression of matter studied
in this work and the mechanism of formation of the annular flow
in [10] have a common nature. Future studies must be made to
answer this question. In general, the mechanism of annular
compression of matter needs further careful study inasmuch as
it is completely possible that a number of its existing details
could be pointed out beyond the limits of the field of vision
of our study.

Another very important result of the work considered is an explanation of the forward and backward rotation of planets. The mechanism of the formation of primarily forward rotating motion in planets detected then is given a clear and simple explanation by the system of frontal and glancing impacts. From the preceding studies, the closest to the actual nature of the phenomenon indicated was presented in the work of A. V. Artem'yev [11] and A. V. Artem'yev and V. V. Radzievsky [12] conducted, respectively, in 1963 and 1965. In the opinion of these authors, the acquisition of a rotating moment by a planet is explair ' hv capture as a result of nonelastic collisions in its field ∪. gravitation, of particles which have come from outside with subsequent fall of the latter on the surface of the planet. Although the true mechanism for acquiring a rotational motion by planets differs considerably from that indicated, the authors have started on the right path inasmuch as the phenomenon of frontal and glancing impacts is similar to those described earlier in implicit form with predominance of the former over the latter. Generally speaking, frontal impacts predominate over glancing not in all cases -- in some, although in rare cases, the opposite picture is true and as a result, it is fully possible that a planet can acquire backward motion which apparently, is what occurred in the cases of Venus and Uranus.

The question of the aggregate state of matter of protoplanets during evolution of a cloud and also the planets themselves immediately after their formation applies directly to the problem of rotational motion of the planets in this case. That is to say, the system of frontal and glancing impacts, which guarantees a primarily forward rotation of the planets, can be realized only in a case where the gravitational cohesion of protoplanets, as a rule, ends in their combining in the entire range of allowable relative trajectories of cohesion whose parameters are presented in Fig. 1. This applies particularly to the trajectory of the

upper section of the graph in Fig. 1; here the maximum distances of protoplanets with approaches are characteristically very large /79 with, at the same time, a considerable kinetic moment of forward rotation of the protoplanets relative to their barycenter and which particularly provides predominantly forward rotation of the planets formed. As was noted earlier, for combining of proto-planets, it is not necessary that they be in direct contact on the first half loop of the approach. This contact can come later after a varying number of rotations around the common center of mass with slowing of relative motion due to influx forces. However, so that this slowing down would be effective and result in combining of bodies in the exemplary time necessary, so that their dimensions would be adequately great, the aggregate state made it possible to evolve large influx retarding moments during interaction of the bodies. This question requires a special study, but right now it is possible apparently with some assurance to state that the density of protoplanets during evolution, and also of the planets immediately after their formation, was several magnitudes smaller than the density characteristic for solid bodies. In other words, it is very probable that protoplanets and planets when they first existed were gas and dust bodies. This does not contradict the existing evaluation of the time for existence of protoplanets in the form of gas dust bodies, in the period of evolution of the cloud, if one takes into consideration the relatively short time period for this evolution [5]. If the conclusion as to the gas-dust structure of planets in the initial period of their existences is true, then this would lead to a consideration of a number of points of view for the question of the origin of satellite systems. Within the framework of this concept, it is natural to consider that the satellites' distance occurred from gas-dust disks separated as a result of the occurrence of rotational instability from gas and dust masses of the planet

rapidly compressing due to gravitational forces.

The results of numerical experiments relating only to the
liquid-drop model of a protoplanetary cloud are presented in this
article. The result of numerical experiments with a gravitational
model is planned for a separate publication. Here we will note only
that basically no new effects differing from those observed during
work with a liquid-drop model were found in the dynamics of the
gravitation model. Nevertheless, the gravitation model of a
protoplanetary cloud is extremely interesting inasmuch as only with
this model can one obtain the actual parameters of the solar system
and prove the correctness of certain important conclusions following
from the analysis of numerical experiments in a liquid-drop model
of a cloud. Thus, in the first numerical experiments with a   /80
gravitation model of a protoplanetary cloud of the Earth group
of planets, a strong pull from the accumulation zone of Venus was
detected much earlier than expected on the basis of the data of
the liquid-drop model; it occurred with a corresponding weakening
of density of the orbital flow in it and in one of the experiments,
backward rotation of this planet was obtained. One must expect
that the gravitation model of a protoplanetary cloud can answer
many important details as to the origin of the solar system which
are unclear at the present time.

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