General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

UNSTEADY FLOW MODEL FOR

CIRCULATION-CONTROL AIRFOILS

Balusu M. Rao

June 1979

FOREWORD

The work described in this technical report was performed by Analytical Methods, Inc. for NASA Ames Research Center, Moffett Field, California, under Contract NAS2-10132. The research program was undertaken under the technical cognizance of Mr. Gerald Shockey and Mr. Jeffrey Cross. Dr. Frank A. Dvorak was the Program Manager.

ABSTRACT

An analysis and a numerical lifting surface method are developed for predicting the unsteady airloads on two-dimensional circulation control airfoils in incompressible flow. The analysis and the computer program are validated by correlating the computed unsteady airloads with test data and also with other theoretical solutions.

Additionally, a mathematical model for predicting the bending-torsion flutter of a two-dimensional airfoil (a reference section of a wing or rotor blade) and a computer program using an iterative scheme are developed. The flutter program has a provision for using the CC airfoil airloads program or the Theodorsen hard flap solution to compute the unsteady lift and moment used in the flutter equations. The adopted mathematical model and the iterative scheme are used to perform a flutter analysis of a typical CC rotor blade reference section. The program seems to work well within the basic assumption of the incompressible flow.

NOTATION

C_J	jet blowing momentum coefficient
D	semi-width of a panel
h, H	position of the wake jet
k, K	doublet strength
ℓ	semi-chord length of an airfoil
p	frequency of oscillation
t, T	time
x, X	coordinate along the chord line of an airfoil
U	free stream velocity
w, W	downward displacement of an airfoil
z	coordinate for plunging motion
α	angle of attack, coordinate for pitching motion
γ, Γ	vorticity
ω	reduced frequency
ρ	density

TABLE OF CONTENTS

INTRODUCTION

In the last few years there has been a considerable increase in activity in the prediction of unsteady airloads on the circulation-control and jet flap airfoils. Williams et al. (Ref. 1) and Platzer (Ref. 2) presented a review of the experimental and analytical studies of jet flap airfoils. Several investigators conducted a series of experimental studies on airfoils with trailing-edge jet flaps. Simmons and Platzer (Ref. 3) measured the frequency response of a trailing-edge flap on a two-dimensional NACA 0012 airfoil. Two values of jet momentum coefficients ($C_J = 0.14$ and 0.158) and several values of reduced frequencies were investigated at constant jet deflection amplitudes and the airfoil at zero angle of attack. Trenka (Ref. 4) considered the jet flap and wing oscillations on a two-dimensional NACA 0015 airfoil with a 20% flap over the span. Takeuchi (Ref. 5) measured the oscillatory forces and moments on a semi-span wing model of a NACA 63_2A015 airfoil with a trailing-edge jet flap. Using false walls, various aspect ratio wings ranging from 2.1 to infinity were simulated. Kretz (Ref. 6) investigated the harmonic oscillations of a jet flap on a NACA 0012 at $C_J = 0.25$, 0.50 and 1.0 for a reduced frequency range from 0 to 1.26.

Some progress has been made towards the development of theoretical solutions for jet flaps under unsteady flow conditions using potential flow theory. It has been a standard procedure to assume pure harmonic motions of jet and airfoil. Three types of motions are of interest; namely: (i) the stationary airfoil with oscillating jet flap; (ii) the pitching motion of the airfoil; and (iii) the plunging motion of the airfoil. For harmonic motions of the airfoil or jet flap, the pressure distribution, and hence the aerodynamic loads, lag the motion. The prediction of this lagged response taking the jet interaction into account has not been too successful even under potential flow conditions. Spence (Ref. 7) suggested that for low values of reduced frequency (less than 0.2), the wall jet effects may be neglected and Theodorsen's trailing-edge, hard-flap solution may be a good blowing representation. At low blowing coefficients, there is a reasonable correlation between the experimental and analytical (Theodorsen) results for isolated airfoils. Theodorsen's solution can easily be modified to include the wake effects of a helicopter rotor blade in hover using Loewy's mathematical representation of a wake model.

Spence (Ref. 7) obtained a solution for jet flaps in unsteady flow, modifying the governing flow equation and the jet dynamic equation using a weak jet approximation. However, his solution is valid only for small values of jet momentum coefficient. Potter (Ref. 8) solved the problem as an initial value problem by reducing the governing equation into a finite-difference form and using point vortex distributions. The jet is initially specified as a straight horizontal line with zero vorticity, after which time the jet exit angle is either instantaneously deflected or continuously oscillated, and the shed vortices are assumed to propagate downstream with the free-stream velocity. Conservation of circulation is maintained for each time step, and the jet shape is determined by taking into account all changes in bound airfoil vorticity in computing the downwash and the resultant velocities at each jet vortex point. The adoption of an averaging scheme over several neighboring vortices provided a satisfactory approach to correct the erratic vortex motion with increasing time steps, and gave satisfactory results for jet momentum coefficients smaller than 0.1.

Schmidt (Ref. 9) presented a complete summary and evaluation of the experimental investigations (Refs. 10 through 13) on the circulation control airfoils under unsteady flow conditions conducted at the Naval Postgraduate School. Bauman (Ref. 10) developed and tested the operation of a control valve in the pressure supply line just upstream of the airfoil cavity so as to permit the superposition of a harmonic perturbation upon a mean value of cavity pressure. Rickelsimer (Ref. 11) developed a numerical algorithm for identifying Fourier components in multi-channel, discretized, truncated signal information. This technique enabled the determination of amplitude and phase angle of the various data channels relative to the reference clock channel (unsteady cavity pressure) and also the in-phase and out-of-phase lift coefficient components from unsteady pressure data. Kail (Ref. 12) investigated the behavior of the Coanda sheet formed by the tangential jet flow over the airfoil's rounded trailing edge and noted that the superposition of an oscillatory blowing component upon the steady tangential jet flow did not produce any discernible change in the average value of lift augmentation. Englehardt (Ref. 13) developed the hardware and software implementation of a microprocessor based on a high-speed digital data acquisition and reduction system that was tailored for use in time-varying signal analysis of the unsteady pressure information derived by oscillating the cavity pressure of a circulation-control airfoil. Schmidt (Ref. 9) analysed the results for the CCA model at one angle of attack, one tunnel airspeed, with the harmonic oscillation of the cavity pressure at four frequency values using one average value of momentum blowing coefficient, and arrived at the following conclusions:

2

a. The airfoil behaved in a linear manner.

b. The mean or average value of the lift coefficient was not altered during unsteady blowing.

c. The frequency response traits of the Coanda sheet region suggest the presence of a transportation lag between the pressure cavity and the blowing slot.

d. The overall airfoil lift transfer function from harmonic circulation-control variations had a behavior quite similar to that of a simple pole in classical control theory.

e. A frequency-dependent pitching moment about the 54 percent chord center of pressure location for circulation control type lift was evident with the attributes of aerodynamic damping being shown.

Williams et al (Ref. 1) presented a discussion on the appropriate mathematical modeling of the circulation-control airfoil under unsteady flow conditions. With a rate of change of angle of attack, heave or velocity at constant duct pressure, they have noted that the airfoil will produce a corresponding lagged response in pressure distribution and lift and have hypothesized that these unsteady effects may be treated by neglecting the wall jet dynamics, especially for low values of reduced frequency. Under this assumption, the CC airfoil problem can then be stated in terms of a bluff, elliptical type of airfoil with a small trailing-edge vane or flap, a variation of whose position produces a corresponding change in circulation on the airfoil and vorticity in the wake. For oscillating angles of attack or heave, the flap position is assumed to be controlled only by the blowing coefficient so that the problem can be treated classically. Also presented in Reference 14 was some limited test data which demonstrated that the oscillating mechanical flap is a good blowing representation.

In conclusion, at the present time, there do not exist any analytical and/or computational methods for predicting the unsteady airloads on circulation-controlled airfoils for high blowing coefficients. For the present development, a numerical lifting surface-method will be adopted. Rao and Jones have successfully applied this method for predicting steady and unsteady airloads over a variety of thin lifting surfaces in subsonic flow, such as helicopter blades, cascade of blades, and delta wings (Refs. 14, 15, 16 and 17). In all of their work, they have used potential flow and small perturbation assumptions at small incidence angles. For circulation-control airfoils, the effect of the unsteady jet interaction can be incorporated by prescribing the jet displacement taking into account the jet dynamics.

In the present study, a numerical lifting-surface method is developed for predicting the unsteady airloads on circulation-control airfoils, for arbitrary combinations of pitching and plunging motions, longitudinal velocity variations, and variations in blowing pressure and jet deflection angle. The analysis and the computer program are tested by correlating the computed unsteady airloads with test data and the approximate prediction based on Theodorsen's method.

In addition, a simple two-dimensional bending-torsion flutter analysis and a computer program are developed using the unsteady airloads program developed in the present study. This analysis and the computer programs are used to perform a flutter analysis of a typical CC rotor blade section.

UNSTEADY AIRLOADS ANALYSIS

The Governing Flow Equations

The governing equation for incompressible flow relating the downwash on the airfoil to the vorticity distribution from the leading edge to infinity and the jet dynamic equation relating the jet displacement to the local vorticity distribution (see Figure 1) are expressed in a non-dimensional form as:

$$W(X,T) = - \frac{1}{2\pi} \int_{-1}^{\infty} \frac{\Gamma}{\zeta - X} \, d\zeta \qquad (1)$$

and

$$\left(i\omega + \frac{d}{dX}\right) \Gamma_J = - 2\mu \frac{d^3 H_J}{dX^3} \qquad (2)$$

where

$$X = x/\ell, \quad T = U t/\ell, \quad W = w/U, \quad \Gamma = \gamma/U, \quad H = h/\ell, \quad \omega = p\ell/U,$$

$\mu = C_J/4$, and ℓ is the semichord.

4

Figure 1. Mathematical Flow Model.
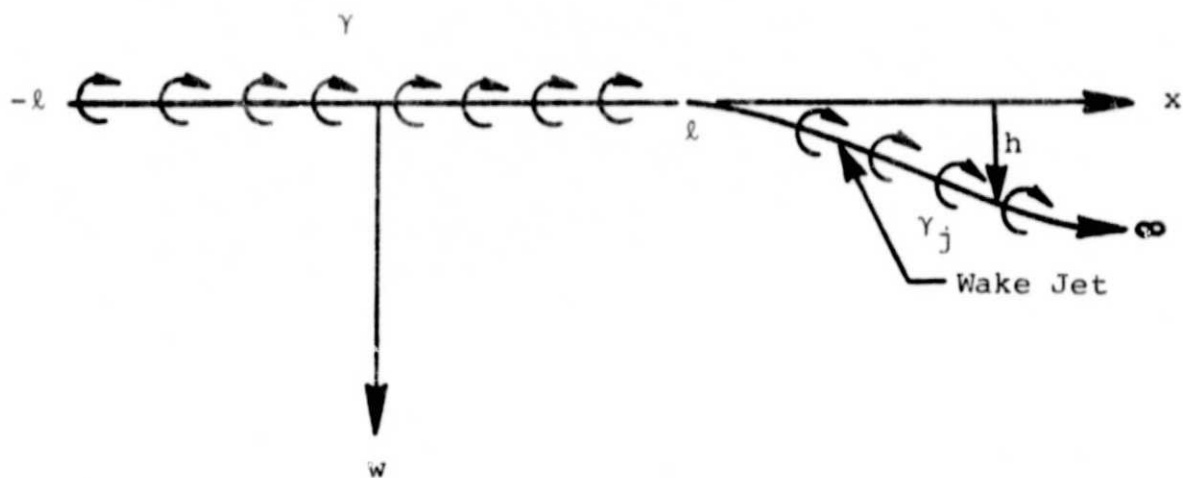
w   Downward displacement

γ   Vorticity on the airfoil

γ_j Vorticity in the wake

For an oscillating airfoil or jet it is assumed $h_J = 0$, and $h'_J = h'_{Jc}$ is constant at the trailing edge of the airfoil. These boundary conditions can be matched by assuming the following form of the wake shape:

$$H_J = \frac{iH'_{Jc}}{\omega} \left[e^{-i\omega(X-1)} - 1 \right] \tag{3}$$

Numerical Lifting-Surface Method

The numerical procedure uses doublet distributions on the airfoil surface. Hence Eqns. (1) and (2) are rewritten in terms of doublet distributions as:

$$W(X,T) = \frac{1}{2\pi} \left[\frac{K_t}{X-1} + \int_{-1}^{1} K \frac{\partial}{\partial \zeta} \left(\frac{1}{\zeta - X} \right) d\zeta \right.$$

$$\left. - \int_{1}^{\infty} \frac{\partial K/\partial \zeta}{\zeta - X} d\zeta \right] . \tag{4}$$

and

$$\left(i\omega + \frac{d}{dX} \right) K_J = - 2\mu \frac{d^2 H_J}{dX^2} \tag{5}$$

where

$$K(X) = \int_{-1}^{X} \Gamma dX' \qquad \text{or} \qquad \Gamma(X) = dK/dX,$$

and K_t $(K_J(X)ex = 1)$ is the doublet strength at the trailing edge of the airfoil.

Equations (3) and (5) are combined to yield

$$K_J = \left[K_t + i2\mu H'_{Jc}\omega(X - 1) \right] e^{-i\omega(X-1)} . \tag{6}$$

6

$$W(X,T) = \frac{1}{2\pi}\left[\frac{K_t}{X-1} + \sum K_n\left(\frac{1}{X_n + D_n - X} - \frac{1}{X_n - D_n - X}\right)\right]$$

$$+ i\omega \int_1^\infty \frac{K_t - 2\mu H'_{Jc} + i\mu H'_{Jc}\omega(\zeta-1)}{(\zeta - X)}\, e^{-i\omega(\zeta-1)}\, d\zeta. \tag{7}$$

The last expression in Eqn. (7) can be analytically integrated for any given value of X. The expansion of this wake integral is presented in the Appendix.

A typical airfoil surface is represented by a number of chordwise panels (N) over each of which the doublet strength (K_n) is assumed to be a constant. K_t can be expressed as a function of K_N using Eqns. (3), (5) and (6):

$$K_t = \frac{K_N + i2\mu D_N H'_{Jc}(2 - e^{-i\omega D_N})}{e^{-i\omega D_N} + 2i\omega D_N} \tag{8}$$

where D_N is the semi-width of the last panel, N. For chosen N values of X at the collocation points of each of the N strips, using Eqns. (7) and (8), a set of N linear equations are obtained. For the known boundary conditions, $W(X_n)$, the solution of these linear equations yields the K distribution from which the airloads can be computed.

The above analysis is valid for any arbitrary motion where W and K values are time-dependent. However, for aeroelastic analysis applications, it is a standard practice to assume a simple harmonic motion. For a two degree-of-freedom simple harmonic motion in the plunge and pitch of the airfoil,

$$z = z'e^{ipt} = z'e^{i\omega T} \tag{9}$$

$$\alpha = \alpha'e^{ipt} = \alpha'e^{i\omega T} \tag{10}$$

where Z' ($= z/\ell$, assumed to be positive downward) and α' are the amplitudes of the airfoil motion due to blade bending and torsion, respectively. The boundary condition at any collocation point, i, then becomes

$$W(X,T) = \left[i\omega Z' + (1 + i\omega X_i) \, \alpha' \right] e^{i\omega T}$$

$$= W(X_i) \, e^{i\omega T} \tag{11}$$

where

$$W(X_i) = i\omega Z' + (1 + i\omega X_i) \, \alpha'.$$

Similarly, for a simple harmonic motion, $K = K'e^{i\omega T}$, where K' ($= K'(X)$) is a function of X only. From these conditions, Eqns. (7), (8), (9) and (10) can be replaced with quantities that are functions of X only, and the doublet distribution (amplitudes) can be obtained for the assumed boundary conditions given by Eqn. (11).

The unsteady lift and moment coefficients are then expressed as

$$\frac{L'}{\rho U^2 \ell} = C_{\ell z} \, Z + C_{\ell \alpha} \, \alpha \tag{12}$$

$$\frac{M'}{\rho U^2 \ell^2} = C_{mz} \, Z + C_{m\alpha} \, \alpha \tag{13}$$

where

$$C_{\ell (\cdot)} = K_{t (\cdot)} + i2\omega \sum_{n=1}^{N} D_n \, K_{n(\cdot)} \tag{14}$$

8

and

$$C_{m(\cdot)} = -K_{t(\cdot)} + 2 \sum_{n=1}^{N} D_n K_{n(\cdot)}$$

$$-i2\omega \sum_{n=1}^{N} D_n K_{n(\cdot)} X_n \qquad (15)$$

The parentheses, $(\cdot)$, in Eqns. (14) and (15) can be replaced either by z or α and the appropriate boundary conditions are applied.

TWO-DIMENSIONAL FLUTTER ANALYSIS

Flutter is an oscillating instability produced by the negative damping terms of the aerodynamic forces at speeds known as critical flutter speeds. The classical type of flutter is associated with potential flow and it is almost universal to make the assumption of <u>linearity</u>, according to which the forces brought into play by any small deviation and of its time derivative. The flutter problem can be reduced to two basic problems--mechanical and aerodynamic. The first involves consideration of the motion of a flight structure as a continuous vibrating system acted on by external forces and internal damping, which then becomes one of writing the equations of motion for such a system. The second basic problem is that of determining the nature of the oscillatory aerodynamic forces, which are independent of the static forces that are needed to maintain the system in an equilibrium position.

Equations of Motion

Although an actual flight vehicle should, strictly speaking, be considered a single elastic unit, it becomes necessary from the engineering viewpoint to make certain simplifying assumptions. Theodorsen (Ref. 18) and Theodorsen and Garrick (Ref. 19) approached the flutter problem by considering a wing of infinite aspect ratio, moving with small oscillatory amplitudes at constant velocity. Thus the aerodynamic forces can be determined by considering the problem as one of two-dimensional flow. Furthermore, instead of the actual distributed mass and geometric properties of the wing, Theodorsen and Garrick assumed at that time that the results for the actual wing could conservatively be obtained by considering the motion of unit span of the wing

at some representative position; this has been often chosen at the 3/4 semispan. This approach is termed the two-dimensional flutter problem.

To simplify the problem further, the assumption is made that the actual motion of a rotor blade system can be considered a combination of fundamental bending and fundamental torsion about the elastic axis. The system can then be replaced by an equivalent system containing an airfoil section of unit span restrained by springs against independent vertical (bending) motion and torsion. Then the equations of motion for a two degree-of-freedom (bending torsion) system of an airfoil are

$$M \ell \ddot{Z} + S \ddot{\alpha} + K_z \ell Z = -\rho U^2 \ell \ (C_{\ell z} Z + C_{\ell \alpha} \alpha) \qquad (16)$$

$$S \ell \ddot{Z} + I \ddot{\alpha} + K_\alpha \alpha = \rho U^2 \ell^2 \ (C_{mz} Z + C_{m\alpha} \alpha) \qquad (17)$$

where,

$$M = \int dm = \text{blade mass per unit span,}$$

$$S = \int r \, dm = \text{blade static moment about elastic axis per unit span,}$$

$$I = \int r^2 \, dm = \text{blade mass moment of intertia about elastic axis per unit span,}$$

ℓ = semi-chord length of the blade,

Z = flapping (bending) motion,

α = pitching motion,

K_z = generalized bending stiffness of the blade, and

K_α = generalized torsional stiffness of the blade.

Assuming simple harmonic motion,

$$Z = Z'e^{ipt} = Z'e^{i\omega T} \qquad (18a)$$

$$\alpha = \alpha'e^{ipt} = \alpha'e^{i\omega T} \qquad (18b)$$

and separating the aerodynamic derivatives into real and imaginary parts,

$$C_{\ell z} = C_{\ell z R} + iC_{\ell z I} \qquad (19a)$$

$$C_{\ell \alpha} = C_{\ell \alpha R} + iC_{\ell \alpha I} \qquad (19b)$$

$$C_{mz} = C_{mzR} + iC_{mzI} \qquad (19c)$$

$$C_{m\alpha} = C_{m\alpha R} + iC_{m\alpha I} \qquad (19d)$$

Eqns. (16) and (17) reduce to

$$(\ell K_z - M\ell p^2 + \rho U^2 \ell C_{\ell z})Z' + (\rho U^2 \ell C_{\ell \alpha} - Sp^2)\alpha' = 0$$

$$- (\rho U^2 \ell^2 C_{mz} + S\ell p^2)Z' + (K_\alpha - \rho U^2 \rho^2 C_{m\alpha} - Ip^2)\alpha' = 0.$$

$$(20)$$

Equations (20) are a set of two linear, homogeneous equations in Z' and α'. For this system to have a nontrivial solution, the coefficient determinant of Z' and α' must be equal to zero.

$$\begin{vmatrix} (\ell K_z - M\ell p^2 + \rho U^2 \ell C_{\ell z}) & (\rho U^2 \ell C_{\ell \alpha} - Sp^2) \\[2mm] -(C_{mz}\rho U^2 \ell^2 + S\ell p^2) & (K_\alpha - \rho U^2 \ell^2 C_{m\alpha} - Ip^2) \end{vmatrix} = 0$$

<div align="right">(21)</div>

This determinant is referred to as the flutter determinant and the solution gives the flutter frequency. However, a direct solution cannot be obtained since the aerodynamic derivatives are functions of the reduced frequency. Since the aerodynamic derivatives are complex quantities, the determinant can be expressed in two parts, real and imaginary. After expanding the flutter determinant and substituting appropriate values of Eqns. (19), the real and imaginary parts of the flutter determinant become,

$$(\ell K_z K_\alpha)\bar{p}^2 - \left[\ell K_z\left(I + \frac{\rho \ell^4}{\omega^2} C_{m\alpha R}\right) + K_\alpha\left(M\ell - \frac{\rho \ell^3}{\omega^2} C_{\ell zR}\right)\right]\bar{p}$$

$$+ \left[(M\ell I - S^2\ell) - \frac{\rho \ell^3}{\omega^2}(IC_{\ell zR} - M\ell^2 C_{m\alpha R} + S\ell C_{mzR} - S\ell C_{\ell \alpha R})\right.$$

$$\left. - \frac{\rho^2 \ell^7}{\omega^4}(C_{\ell zR} C_{m\alpha R} - C_{\ell zI} C_{m\alpha I} + C_{\ell \alpha I} C_{mzI} - C_{\ell \alpha R} C_{mzR})\right] = 0$$

$$\left[K_\alpha C_{\ell zI} - \ell^2 K_z C_{m\alpha I}\right]\bar{p} + \left[S\ell C_{\ell \alpha I} - IC_{\ell zI} + M\ell^2 C_{m\alpha I}\right.$$

$$- S\ell C_{mzI} + \frac{\rho \ell^4}{\omega^2}(C_{\ell \alpha R} C_{mzI} + C_{\ell \alpha I} C_{mzR} - C_{\ell zR} C_{m\alpha I}$$

$$\left. - C_{\ell zI} C_{m\alpha R})\right] = 0.$$

<div align="right">(22)</div>

where $\bar{p} = 1/p^2$. If the real and imaginary parts are set to zero, solutions p_1 and p_2 can be obtained for the real part, and p_3 for the imaginary part. For an assumed value of ω, if one of the solutions, p_1 or p_2, of the real part is equal to the solution, p_3, of the imaginary part, then this ω corresponds to the flutter frequency. However, the aerodynamic derivatives

12

are functions of the Mach number and the reduced frequency. The flutter problem can only be solved after the aerodynamic derivatives are evaluated. It is necessary to assume a Mach number and a reduced frequency and test whether flutter occurs at these values. If the test results are negative, then it is necessary to iterate on reduced frequency until a flutter case occurs at these values.

DESCRIPTION OF THE COMPUTER PROGRAM

The main program incorporates the iterative scheme for a two-dimensional bending-torsion flutter analysis as described in the previous section. The geometric and structural data of a reference section of a wing of a rotor blade are read in as input information to the program. For any assumed value of reduced frequency (ω), the unsteady airload and moment coefficients are obtained by calling one of the two subroutines, either AEROCC or AEROTH. Then in the main program the flutter determinant is formulated, the solution of which yields two real roots and one imaginary root. If one of the two real roots is not equal to the imaginary root, then the assumed reduced frequency does not correspond to a flutter condition and another value of ω is assumed, and the process is repeated iteratively until the flutter frequency is obtained. From the flutter frequency, the flutter velocity and the Mach number are computed.

Subroutine AEROCC is the unsteady airload prediction program for circulation-control airfoils. The numerical lifting surface method described in a previous section was adopted. The boundary conditions are assumed to correspond to a simple harmonic motion of the airfoil in plunge and pitch, with the fixed jet blowing momentum coefficient as a parameter. For a specified C_J, the steady airload and moment coefficients are computed as a function of reduced frequency and are used in the main program in evaluating various terms in the flutter equations.

Subroutine AEROTH is the unsteady airload prediction program based on Theodorsen's hard flap solution for incompressible two-dimensional flows (Ref. 18). The theory and the computational procedure adopted is not presented in this report since it is available and well presented in the literature. The flap/chord ratio and the reduced frequency are the parameters used in the program. The program involves the generation of Hankel functions as a function of ω. The purpose of

13

this subroutine is to provide a cross check and also an alter-nate means to compute airloads in place of a circulation-control airload prediction.

The input/output specifications and other relevant infor-mation for the computer program are included in the Program User's Manual.

RESULTS AND DISCUSSION

The numerical procedure adopted for the computation of airloads on circulation-control airfoils was based on a lifting-surface method in which a thin airfoil is divided into N strips over each of which the doublet strength is assumed to be a con-stant. The computed results for a typical case of an oscillating jet are shown in Table 1. From these results, it can be seen that the program yields a convergent solution even for the case in which the airfoil is represented by only 20 panels.

Figures 2 and 3 present the lift response of an oscillating jet and its comparison with several theoretical and test results. The oscillatory jet deflection angle (amplitude) was assumed to be 6°, based on test results to correspond with $C_J = 0.1$ under steady flow conditions. Figure 2 presents the comparison of the lift amplitude ratio ($|C_L|/|C_{Lo}|$) variation with the reduced frequency (ω). As can be seen from this figure, the amplitude ratio decreases as the frequency increases for most of the theoretical and test results, including the results of the present study. However, for the test results of Takeuchi (Ref. 5) and the theoretical solution of Spence (Ref. 7), the ampli-tude ratio increases as ω increases. Takeuchi used false walls in his tests and the discrepancy may be due to these walls and tunnel interference. Spence's solution was based on a weak jet approximation and is valid for only very weak jet momentum coefficients. The present results are in good agreement with those of Theodorsen's hard flap solution (Ref. 18), and are in reasonable agreement with Potter's theory (Ref. 8) and the test results of Simmons and Platzer (Ref. 3) and Kretz (Ref. 6).

Figure 3 presents the comparison of the phase lag angle with ω. Once again Takeuchi's and Spence's solutions are com-pletely out of phase from the other results. The present re-sults compare reasonably well with Theodorsen's and Potter's solutions for ω values less than 1. They deviate substantially

Table 1. Lift Amplitude Ratios and Phase Angles for an Oscillating Jet (C_J = 0.14 and Jet Deflection Angle = 7.3°).

NUMBER OF PANELS

ω	20		24		30	
	$\left\|\dfrac{C_L}{C_{Lo}}\right\|$	ϕ	$\left\|\dfrac{C_L}{C_{Lo}}\right\|$	ϕ	$\left\|\dfrac{C_L}{C_{Lo}}\right\|$	ϕ
0.05	0.923	-0.126	0.923	-0.126	0.923	-0.124
0.10	0.857	-0.176	0.857	-0.175	0.858	-0.173
0.15	0.804	-0.197	0.805	-0.196	0.806	-0.194
0.20	0.762	-0.205	0.763	-0.204	0.764	-0.201
0.25	0.728	-0.204	0.728	-0.203	0.730	-0.200
0.30	0.700	-0.198	0.701	-0.197	0.702	-0.193
0.35	0.677	-0.190	0.678	-0.188	0.679	-0.184
0.40	0.658	-0.179	0.658	-0.177	0.659	-0.173
0.45	0.641	-0.167	0.642	-0.165	0.643	-0.161
0.50	0.627	-0.155	0.627	-0.153	0.628	-0.150
0.55	0.615	-0.143	0.615	-0.142	0.616	-0.138
0.60	0.604	-0.131	0.604	-0.130	0.605	-0.126
0.65	0.594	-0.120	0.595	-0.119	0.595	-0.115
0.70	0.586	-0.110	0.586	-0.109	0.587	-0.105
0.75	0.578	-0.099	0.579	-0.099	0.579	-0.096
0.80	0.572	-0.090	0.572	-0.090	0.572	-0.087
0.85	0.566	-0.082	0.566	-0.082	0.566	-0.080
0.90	0.561	-0.074	0.561	-0.075	0.561	-0.073
0.95	0.557	-0.067	0.557	-0.068	0.557	-0.066
1.00	0.553	-0.060	0.553	-0.062	0.553	-0.061
1.05	0.550	-0.054	0.549	-0.057	0.549	-0.057
1.10	0.547	-0.049	0.547	-0.052	0.547	-0.053
1.15	0.545	-0.045	0.545	-0.048	0.545	-0.049
1.20	0.543	-0.040	0.543	-0.045	0.543	-0.047
1.25	0.542	-0.037	0.542	-0.041	0.542	-0.044

ω - Reduced Frequency (pℓ/U) ϕ - Phase Angle (radians)

C_L - Lift Coefficient

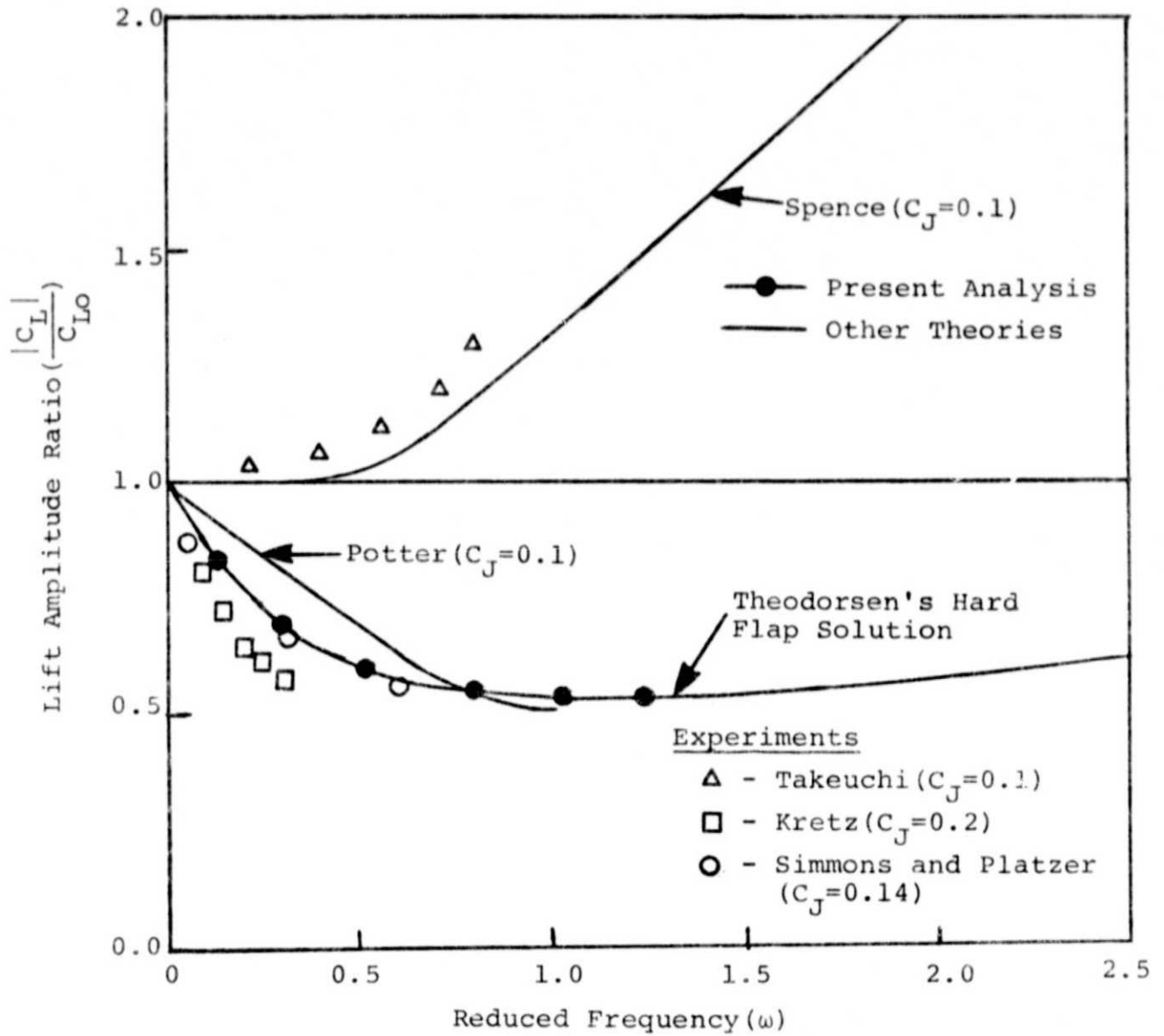C_{Lo} - Steady Lift Coefficient (ω = 0)

15

Figure 2. Comparison of Results for Lift Amplitude Ratio of an Airfoil with an Oscillating Jet at the Trailing Edge.
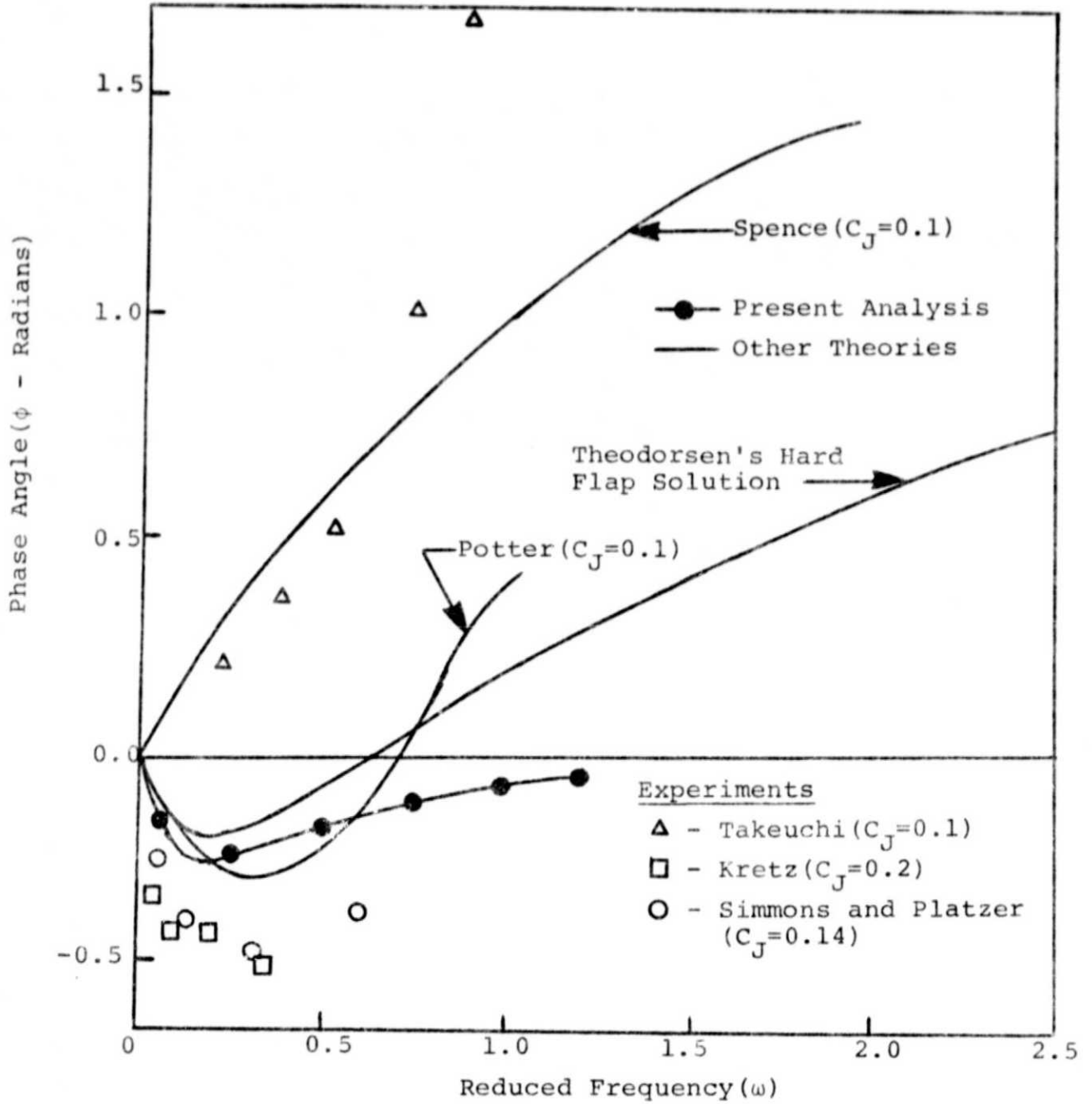
Figure 3. Comparison of Results for Lift Phase Angle of an Airfoil with an Oscillating Jet at the Trailing Edge.

17

from the rest results as well as from the theoretical solutions for higher values of ω. However, note that for a typical classical flutter analysis, only lower values of reduced frequency ($\omega \leq 0.5$) are of interest. Also, the test conditions of the several experimental results are different and, hence, a good correlation is not really expected.

The unsteady airloads program is computationally very efficient and it takes only about 0.02 cp seconds on a CDC 7600 for one value of ω. This is a very desirable feature, since a typical flutter analysis involves an iterative scheme requiring the computation of unsteady airloads for several values of ω.

The Flutter Program is checked out by applying it to a CCR blade case. The following data at 0.75 radius section of a CCR blade are taken from Reference 20:

Reference chord length	1.467 ft.
Mass per unit span	0.3737 slugs/ft.
Elastic axis location	0.35c
Moment of inertia per unit span	0.0776 slugs/ft.^{2}/ft.
First static moment per unit span	0.0547 slugs/ft.^{2}/ft.
First bending frequency	4.8 cps
First Torsional frequency	44.5 cps
Alternate torsional frequency	15.0 cps

The flutter analysis was performed for this airfoil assuming $C_J = 0.1$ at a jet (or flap) deflection angle of 6^0. The computed flutter speeds using each of the two airload programs, AEROCC and AEROTH, are tabulated below.

Altitude	Torsional Frequency (cps)	Flutter Speed (ft./sec.)	
		AEROCC	AEROTH
Sea level	44.5	1,248	1,252
10,000 ft.	44.5	1,448	1,454
Sea Level	15.0	376	379
10,000 ft.	15.0	435	439

This example once again demonstrates that the circulation-control airloads program developed under the present study is in agreement with Theodorsen's hard-flap solution. Also note that the airloads computed in this program were based on the incompressible flow assumption, and hence the higher flutter speeds

obtained using this program are in gross error and are beyond the range of the Program's validity.

CONCLUSIONS

A fast and efficient computer program for predicting the unsteady airloads in incompressible flow was developed and correlated. The results for CC airfoils under harmonic motion are in good agreement with those of Theodorsen's hard-flap solution. A two-dimensional bending-torsion flutter computer program was developed and used by performing a flutter analysis of a typical CC rotor blade section.

In the present analysis, the compressibility and the rotor wake effects are not taken into account. As the flutter Mach number increases, it is important to include the compressibility effects, and usually they tend to decrease the flutter speed. In a typical hovering rotor, the wakes of the other blades are directly underneath the reference blade and may have a significant effect on the flutter speed. The rotor wake effects can be incorporated in the present numerical lifting-surface method by adopting the mathematical model of Loewy (Ref. 21). The compressibility effects can be incorporated by adopting the analysis of Jones and Rao (Ref. 22).

REFERENCES

1. Williams, R.M., Rogers, E.O., and Leitner, R.T., "X-Wing Technology Summary - Volume II: Documentation of Analysis Routines for the Circulation-Control X-Wing VTOL Concept", DTNSRDC TM-16-76-72, June 1976.

2. Platzer, M.F., "The Oscillating Jet Flap", Naval Postgraduate School, NPS-57P176031, March 1976.

3. Simmons, J.M. and Platzer, M.F., "Experimental Investigation of Incompressible Flow Past Airfoils with Oscillating Jet Flaps", J. Aircraft, Vol. 8, No. 8, August 1971, pp. 587-592.

4. Trenka, A.R. and Erickson, Jr., J.C., "The Determination of Unsteady Aerodynamics of a Two-Dimensional Jet Flap Wing", Report No. AC2260-5-1, Cornell Aeronautical Laboratory, April, 1970.

5. Takeuchi, K., "Two- and Three-Dimensional Unsteady Jet-Flapped Wings", M.S. Thesis, Pennsylvania State University, June 1970.

6. Kretz, M., "Commande asservie des forces aerodynamiques instationnaires", Report DE07-44E5, Giravions Dorand, December 1973.

7. Spence, D.A., "The Flow Past a Thin Wing with an Oscillating Jet Flap", Phil. Trans. Soc. of London, Ser. A, No. 1085, 1965, pp. 445-477.

8. Potter, G.E., "A Numerical Solution for an Unsteady Jet Flapped Airfoil", M.S. Thesis, Pennsylvania State University, 1972.

9. Schmidt, L.V., "Circulation-Control Airfoil Study", Progress Report No. 4, Naval Postgraduate School, NPS-67SX-77111, November 1977.

10. Bauman, J.L., "Development of a Control Valve to Induce an Oscillating Blowing Coefficient in a Circulation Control Rotor", M.S. Thesis, Naval Postgraduate School, December 1976.

11. Pickelsimer, B.M., "Data Reduction for the Unsteady Aerodynamics on a Circulation-Control Airfoil", M.S. Thesis, Naval Postgraduate School, March 1977.

12. Kail, K.A., "Unsteady Surface Pressure and Near-Wake Hotwire Measurements of a Circulation-Control Airfoil", Eng. Thesis, Naval Postgraduate School, December 1977.

13. Englehardt, C.D., "Data Acquisition System for Unsteady Aerodynamic Investigation", M.S. Thesis, Naval Postgraduate School, June 1977.

14. Rao, B.M. and Jones, W.P., "Application to Rotary Wings on a Simplified Aerodynamic Lifting Surface Theory for Unsteady Compressible Flow", Proceedings of the AHS/NASA Ames Specialists Meeting on Rotorcraft Dynamics, February 1974.

15. Rao, B.M. and Schatzle, P.R., "Analysis of Unsteady Airloads of Helicopter Rotors in Hover", Presented at the AIAA 15th Aerospace Sciences Meeting, AIAA Paper No. 77-159, January 1977.

16. Rao, B.M. and Jones, W.P., "Unsteady Airloads on a Cascade of Staggered Blades in Subsonic Flow", Presented at the 46th Meeting of the AGARD Propulsion and Energetics Panel, AGARD-CP-177, September 1975.

17. Rao, B.M. and Jones, W.P., "Theoretical Prediction of Airloads on thin Delta Wings in Incompressible Flow", TEES-3029-75-01, Texas A & M University, January 1975.

18. Theodorsen, T., "General Theory of Aerodynamic Instability and the Mechanism of Flutter", NACA TR-496, 1934.

19. Theodorsen, T. and Garrick, I.E., "Mechanism of Flutter, A theoretical and Experimental Investigation of the Flutter Problem", NACA TR-685, 1940.

20. Wilkerson, J.B., "A CCR Blade Parameters", David W. Taylor Naval Ship Research and Development Center, Private Communication, May 1979.

21. Loewy, R.G., "A Two-Dimensional Approximation to the Unsteady Aerodynamics of Rotary Wings", J. Aero. Sciences, Vol. 24, February 1957.

22. Jones, W.P. and Rao, B.M., "Compressibility Effects on Oscillating Rotor Blades in Hovering Flight", AIAA Journal, February 1970.

APPENDIX: EXPANSION OF WAKE INTEGRAL

$$\int_1^\infty \frac{K_t - 2\mu H'_{Jc} + i2\mu H'_{Jc}\omega(\zeta - 1)}{(\zeta - X)} e^{-i\omega(\zeta-1)} d\zeta$$

$$= e^{i\omega(1-X)} \left[\left\{ K_t - 2\mu H'_{Jc}(1 + i\omega - i\omega X) \right\} A \right.$$

$$\left. + i2\mu H'_{Jc}\omega B \right]$$

where

$$A = \int_1^\infty \frac{e^{-i\omega(\zeta - X)}}{(\zeta - X)} d\zeta$$

$$= -\gamma - \log \omega (1 - X) + \frac{\omega^2(1 - X)^2}{2 \cdot 2!} - \frac{\omega^4(1 - X)^4}{4 \cdot 4!} + \dots$$

$$- i \left[\frac{\pi}{2} - \omega(1 - X) + \frac{\omega^3(1 - X)^3}{3 \cdot 3!} - \dots \right]$$

and

$$B = \int_1^\infty e^{-i\omega(\zeta-X)} d\zeta$$

$$= \frac{1}{\omega} \int_{\omega(1-X)}^\infty e^{-i\zeta} d\zeta = \frac{1}{\omega} \int_0^\infty e^{-i\zeta} d\zeta - \frac{1}{\omega} \int_0^{\omega(1-X)} e^{-i\zeta} d\zeta$$

$$= - \frac{\sin \omega (1 - X)}{\omega} + \frac{1}{\omega} \left[1 - \cos \omega (1 - X) \right]$$