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A SURVEY OF METHODS FOR THE CALCULATION OF FLOW AROUND BODIES OF REVOLUTION AT SUPERSONIC SPEEDS

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The theoretical determination of aerodynamic characteristics of bodies traveling at supersonic speed has been considered only recently in relation to flying problems but has been for a long time an important problem in ballistics. For this reason and because, for bodies of revolution, the problem is sensibly more simple, the bodies considered in the theoretical work are bodies having circular cross section, while small attention has been given heretofore to bodies having a cross section different from the circular, although these bodies are important for practical applications. A bibliography of information on this subject is presented at the end of this paper.

For bodies of revolution of good aerodynamic shape, in general the physical phenomena are well understood and the analysis of the flow phenomena can be made with good approximation when the boundary layer along the body does not separate. If the body analyzed is a sharp, slender body of revolution axially alined with the direction of the undisturbed stream, an axial-symmetrical shock is produced at the apex of the body (fig. 1). If the body is a cone of revolution, the generatrix of the shock is a straight line, while in the general case it is a curve that becomes more inclined with the direction of the undisturbed stream, moving away from the body and tending to become parallel to the Mach line.

Across the shock an increase of entropy occurs that corresponds to a variation of momentum in the flow, and, therefore, the shock produces a drag called shock drag. Because the variation of entropy changes with the inclination of the shock, the flow behind a curved shock is not any longer an isentropic flow. The entropy is constant along every streamline in the zone between two shocks, but a variation of entropy exists in a direction normal to the streamlines and, therefore, the flow is rotational flow.

Along the surface of the body, when the generatrix of the body is a curved line, the pressure decreases. If the radius of curvature of the generatrix of the body is small, in the zone in which the generatrix becomes parallel to the undisturbed stream the local pressure is lower than the free-stream pressure. If a cylindrical part of some extent follows the ogive, along the cylindrical part, the pressure increases and tends to become equal to the static pressure. If the back part of the body finishes with a tail as shown in figure 1, the pressure along the tail continues to decrease and at the end of the body another shock wave is produced.

At the surface of the body the boundary layer grows and a wake exists at the tail of the body. The wake changes the actual shape of the streamlines at the end of the body, and, therefore, in this zone a theoretical analysis of the phenomenon made with perfect-flow theory cannot reproduce correctly the physical phenomenon.

However, the phenomenon can be foreseen theoretically with good accuracy in the zone of the flow in which the effect of the boundary layer on the shape of the streamlines is negligible.

When the body has an angle of attack the phenomenon changes and the axial symmetry disappears; the theoretical analysis then becomes more difficult.

For small (infinitesimal) angles of attack the shock that is produced at the apex of the body remains approximately a surface of revolution, but its axis is not coincident with the axis of the body nor with the direction of the undisturbed stream. Because the phenomenon is not the same in every meridian plane passing through the axis of the body, a velocity component in a direction normal to the meridian plane exists, and a force component normal to the undisturbed stream can be found. This component produces a lift and a moment on the body. At the tail of the body when the body has an angle of attack, the wake produced by the boundary layer is not symmetrical with respect to the axis of the body. The wake has an effect on the value of the lift; therefore, because the lift is related to the boundary-layer phenomena, the lift of a body of revolution that ends with a point cannot be analyzed with good approximation if only perfect-flow theory is used.

If an open-nose body of revolution is considered, the analysis of the flow phenomena does not change if the flow enters the body with supersonic speed. In this case, at the lip of the body the phenomenon is two dimensional. The shock moving far from the lip decreases in intensity in a similar way to that of the pointed nose of revolution.

The theories used for the determination of pressure along a body of revolution are of two types: the small-disturbance theory and the characteristics theory. Both systems deal with adiabatic perfect flow, and the effect of viscosity and conductivity are neglected. The small-disturbance theory uses more simple hypotheses in the flow determination and allows in some cases analytical expressions for the aerodynamic phenomena, whereas the characteristics theory takes into account all the physical aspects of the phenomena for adiabatic perfect flow but does not solve practical problems of axial symmetrical flow in an analytical form and requires a numerical determination of every particular case considered.

The simplification accepted by the small-disturbance theory is that the variations of velocity components produced by the presence of the body in the stream are so small that the square terms of the disturbance velocities and of their derivatives can be neglected with respect to the first-order terms in the equation of motion. In this approximation the entropy remains constant throughout the stream and a velocity-potential function can be used. The equation of motion for potential flow and cylindrical coordinates (fig. 2), is expressed as:

$$\left(1 - \frac{u^2}{a^2}\right)\frac{\partial^2 \phi}{\partial x^2} + \left(1 - \frac{v^2}{a^2}\right)\frac{\partial^2 \phi}{\partial y^2} + \left(1 - \frac{w^2}{a^2}\right)\frac{1}{y^2}\frac{\partial^2 \phi}{\partial \varphi^2} - \frac{2uv}{a^2}\frac{\partial^2 \phi}{\partial x\, \partial y} - \frac{2uw}{a^2}\frac{1}{y}\frac{\partial^2 \phi}{\partial x\, \partial \varphi}$$

$$- \frac{2vw}{a^2}\frac{1}{y}\frac{\partial^2 \phi}{\partial y\, \partial \varphi} + \frac{v}{y}\left(1 + \frac{w^2}{a^2}\right) = 0 \qquad (1)$$

where ϕ is the total-velocity potential; u, v, and w are velocity components; and a is the speed of sound. Then, in the small-disturbance approximation the expression becomes:

$$\left(1 - M_1{}^2\right)\frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2} + \frac{1}{y^2}\frac{\partial^2 \phi'}{\partial \varphi^2} + \frac{1}{y}\frac{\partial \phi'}{\partial y} = 0 \qquad (2)$$

where M_1 is the Mach number of the undisturbed stream assumed parallel to the x-direction, and ϕ' is the potential function that represents the variations of velocity components produced in the stream by the movement of the body (disturbance velocities). The theory of small disturbances can be used in every case in which the more simple hypotheses are respected, and, therefore, for every free-stream Mach number considered. However, from a practical point of view it is necessary to remember that the theory cannot be used in the neighborhood of $M_1 = 1$, while the precision of the results decreases at high Mach numbers because for a given geometrical shape of the body the disturbance velocities increase in intensity when the Mach number increases.

The equation of motion in the simplified form (equation (2)) is a linear differential equation of second order with constant coefficients and therefore permits a superimposition of solutions that simplifies notably the problem. The solution of any problem in the approximation of equation (2) can be obtained by the superimposition of simple solutions, known from sound-wave theory, that in aerodynamics represent sources, sinks, and doublets. The problem of determining the flow properties of a given phenomenon is transformed in this way to the problem of determining the correct distribution of sources or doublets that respect the boundary conditions considered in the problem.

For bodies of revolution axially alined with the undisturbed stream, the solution of the problem can be obtained by considering a source-sink distribution along the axis of the body, the intensity of which depends upon the free-stream Mach number and upon the shape of the body. The intensity of the source-sink distribution can be obtained generally by a step-by-step calculation of simple form or in some cases in analytical form. The step-by-step calculation is usually required when the boundary conditions are exactly fulfilled by imposing the condition that the disturbance-velocity components at the surface of the body must produce a stream that, when superimposed on the undisturbed stream, must be exactly tangent to the body. For very slender bodies of revolution with a generatrix having finite curvature the step-by-step calculation can be avoided. In the neighborhood of the axis of the body the component of the disturbance velocity normal to the axis can be determined directly from the intensity of the source distribution along the axis, and vice-versa. Now, if the body is a slender body of revolution, the component of the disturbance velocity normal to the axis at the surface of the body is essentially equal to the same component in the neighborhood of the axis. But the component of the disturbance velocity normal to the axis at the surface of the body is given by the boundary conditions and depends upon the stream velocity and upon geometrical parameters of the body and, therefore, is known. In this case, the intensity of the source distribution can be obtained directly from the shape of the body without a step-by-step calculation.

The intensity of the source distribution is given in this approximation by

$$f(x) = \frac{V_1}{2\pi} \frac{ds(x)}{dx}$$

where $s(x)$ is the cross-sectional area of the body at the abscissa x. The value of the drag obtained by this approximation is independent of the Mach number.

This result depends upon the simplifications assumed; however, it gives an indication that the effect of variation of Mach number in the aerodynamic phenomena of slender bodies of revolution is not very large.

When the body of revolution has a small (infinitesimal) angle of attack, the solution is obtained by considering two potential functions, the potential function ϕ_1 that represents the phenomenon dependent on the component of the stream in line with the axis of the body, and the potential function ϕ_2 that considers the part of the phenomenon dependent on the component normal to the axis. The first potential function is identical in the approximation accepted to the function used in the axial symmetrical phenomena, while the second potential function corresponds to the potential of a doublet distribution placed along the axis of the body. The intensity of the doublet distribution must be determined as a function of the boundary conditions. Again, for very slender bodies of revolution having generatrices with finite curvature, the doublet distribution depends only on geometrical parameters of the body considered and can be determined directly. In this case, the lift coefficient is dependent only on the end section of the body and is independent of the Mach number. The lift obtained with the small-disturbance theory for bodies pointed at both ends is zero but is different from zero if the end section is different from zero. This result depends on the assumptions made; however, it shows that the lift is a function of the dimension of the wake and, therefore, of the phenomena in the boundary layer. Indeed, the cross section of the wake corresponds physically for the flow outside of the boundary layer to an end section of the body.

By use of the small-disturbance theory, it is possible to determine some general properties of bodies of revolution and to obtain some indications of the shapes of bodies having low shock drag. For example, if the length and diameter of an ogive body are fixed, the shape that corresponds to minimum drag has a blunt nose. The radius of curvature of the meridian curve at the nose is very small, and, therefore, the zone in which the nose is blunt is very small. This result is found also when more approximate treatment is used.

In order to give an idea of the approximation of the small-disturbance theory, comparison between the pressure coefficients $\Delta p/q$ given by this theory for different stream Mach numbers and the results obtained by exact-perfect-flow theory is shown in figure 3. The bodies considered are cones of revolution with different apex angles.

In the small-disturbance theory it is assumed that all the disturbances are transmitted along surfaces that have constant inclination with respect to the axis of the body. The inclination corresponds to the free-stream Mach angle. In this approximation no shock waves can be found.

When phenomena of bodies at an angle of attack are considered, the Mach cones move rigidly with the body or in some cases do not move with respect to the direction of the stream.

In order to obtain the effect of the presence of the shock and in order to determine with greater precision the phenomena outside of the boundary layer, the characteristics system must be used. The principal idea of the characteristics method is based on the fact that every small disturbance produced in a supersonic stream is transmitted only in the flow inside the Mach cone from the point in which the disturbance is produced; therefore, the Mach cone from the point in which the small (infinitesimal) disturbance is produced is a surface across which the phenomenon changes. In mathematical language this surface is a characteristic surface, because as the disturbance is small and the phenomenon continuous, the flow properties represented, for example, by the stream velocity components must be the same at the inside and outside surface of the Mach cone; since the analytical expression of the flow properties must change across the surface, therefore, the partial derivative of the flow properties (in our case the velocity components) must change in discontinuous form. The characteristic surfaces exist only if the flow is anywhere supersonic because only in this case the disturbances are transmitted along the surface correspondent to the Mach cone.

The characteristic surfaces are constituted by the envelope of all the Mach cones that have their vertices at the points in which a disturbance is produced. They separate the zone of the flow in which the perturbation is transmitted from the undisturbed zone. Because the Mach angle is not constant in the flow, the characteristic surfaces are curved and at the inclination any point is a function of the local velocity. When the body considered is a body of revolution at zero angle of attack, the phenomenon has axial symmetry and also the characteristic surfaces are surfaces of revolution. The velocity at any point is defined by two velocity components and the analysis of the phenomenon can be made by determining the motion in a meridian plane. In place of the characteristic surfaces the characteristic lines are considered. These lines are obtained by the intersection of the characteristic surfaces with the meridian plane.

At every point of the meridian plane the characteristic lines are inclined at the local Mach angle with the local direction of the velocity, and at every point two characteristic lines pass correspondent to the two directions in which a Mach line can be drawn with respect to the direction of the velocity. If μ is the local Mach angle and θ the local inclination of the velocity with respect to the axis of the body, a characteristic line is inclined at $\mu + \theta$ and the other at $\theta - \mu$. The value of μ and θ is different at different points of the flow; therefore, the characteristic lines are curved.

The characteristics theory consists in the analysis of the changes of flow properties along characteristic surfaces or, for bodies of revolution, along characteristic lines.

By use of the law of continuity, the law of conservation of energy, or the law of variation of momentum, a partial differential equation for the law of motion can be obtained. Because of the presence of curved shocks in the flow, the flow analyzed is rotational and, therefore, in general in the analysis the use of a potential function is not possible. For two-dimensional or axial symmetrical phenomena a special stream function can be used which permits the obtaining of a partial differential equation correspondent to the equation of motion for potential flow. This stream function for rotational flow, or differential expressions for the velocity components, the pressure, and the density can be used to obtain the equations of motion in differential form. The equation which defines the motion becomes much simpler if the variation of the flow properties is analyzed along the characteristic surfaces, because some terms of the differential equation disappear.

If a body of revolution axially alined with the free stream is analyzed, the variation of flow properties must be analyzed along the characteristic lines. Because across every point of the flow two characteristic lines pass, two equations are obtained which give in differential form the variation of the flow properties along the lines. Because the flow properties at any point are defined if two quantities (for example, the two velocity components) are known, the two equations permit the problem to be defined.

The equations that give the law of motion along the characteristic lines still are differential equations with variable coefficients but permit the numerical determination of the problems if a method of finite differences is used in place of the differential equation in order to determine the variation of flow properties along the characteristic line.

The equation of motion along the characteristic line can be given in the following form (see fig. 4):

$$\frac{dy}{dx} = \tan(\mu + \theta) \tag{3a}$$

$$\frac{dV}{V} - \tan \mu \, d\theta + dx\left[\frac{\sin^3\mu}{\cos(\theta + \mu)} \frac{dS}{dn} \frac{1}{\gamma R} - \frac{\sin \theta \sin \mu \tan \theta}{\cos(\theta + \mu)} \frac{1}{y}\right] = 0 \tag{3b}$$

and

$$\frac{dy}{dx} = \tan(\theta - \mu) \tag{3c}$$

$$\frac{dV}{V} + \tan \mu \, d\theta - dx\left[\frac{\sin^3\mu}{\cos(\theta - \mu)} \frac{dS}{dn} \frac{1}{\gamma R} + \frac{\sin \theta \sin \mu \tan \theta}{\cos(\theta - \mu)} \frac{1}{y}\right] = 0 \tag{3d}$$

where V is the intensity of velocity, θ the inclination of velocity with respect to the axis, $\frac{dS}{dn}$ the gradient of entropy in normal direction to the streamlines, and γ and R are constants.

From the equations (3), the flow property at a given point can be determined, when the flow properties at two points near to the point considered and along the characteristic lines are known (fig. 4).

If the flow properties in P_1 and P_2 near each other are known, the values of θ and μ are also known; therefore, from P_1 the tangent to the characteristic line given by equation (3a) and from P_2, the tangent to the characteristic line given by (3c) can be drawn, and a point P_3 can be determined. On the assumption that in the first approximation the coefficients in equations (3d) and (3b) are constants between P_1 and P_2 and P_2 and P_3, and applying equation (3b) between P_1 and P_3 and equation (3d) between P_2 and P_3 allows two equations to be obtained that permit determination of the variation $d\theta$ and dV between P_3 and P_1 (equation (3b)) and the variation dV and $d\theta$ between P_3 and P_2 (equation (3d)). Indeed, the variation dx is known, and all the coefficients are constants and known. When the values of V and θ at P_3 in the first approximation are determined, a second approximation can be obtained by assuming for the coefficients in equations (3b) and (3d), and for the direction of the characteristic in equations (3a) and (3b) the average values between the values at the points P_1 and P_2 and the values obtained in the first approximation for the point P_3. In equations (3b) and (3d), the term $\frac{dS}{dn}$ appears only if the flow is rotational flow. This term can be determined at every point because the entropy is constant along every streamline between shock waves, and the equation of the shock gives the variation of entropy that occurs when a streamline crosses a shock wave.

The system permits the determination of the phenomenon when shock exists and allows any required precision to be obtained, because the precision depends on the distance between the points P_3 and P_2 or P_3 and P_1, a distance that can be reduced to any value.

The system is numerical but the solution can be obtained in computing machines of large size that can give numerical results in a very short time. The only approximations introduced are that viscosity and conductivity can be neglected.

In order to apply the characteristic system to any body of revolution the flow must be determined initially along a characteristic line. If the body is a sharp-nosed body of revolution the calculation starts with the determination of the flow at the apex, flow that is conical flow; while if the body is an open-nosed body of revolution the calculations start by determining the shock at the lip of the nose with the two-dimensional theory. Figure 5 shows a comparison between experimental results and values determined with the characteristic system for a body of revolution of simple shape. Figure 6 shows a practical determination of the supersonic part of the flow inside a conical diffuser. The shock produced at the lip of the body increases in intensity and becomes normal at the axis. The increase in intensity of disturbances produced at the wall of circular tubes is a general phenomenon and is important for supersonic circular tunnels.

When the body is not a body of revolution or has an angle of attack, the characteristic system can still be applied but becomes much more involved, because it is necessary to determine the flow variations not along two characteristic lines, but along two characteristic surfaces. Practically, the determination of the flow properties at any point can be obtained by the analysis of the variations along the intersections with a meridian plane of two characteristic surfaces that pass at the point considered and along the intersection of one of two characteristic surfaces with a plane perpendicular to the axis. In this way a system of three linear equations is obtained, when the method of finite differences is used, that permits the determination of one of the flow properties, for example, of the velocity at the point considered.

The numerical solution becomes very involved, especially if the flow with shock is analyzed, and requires that the initial conditions can be determined at the front part of the body.

When a body of revolution with small angle of attack is analyzed, the hypothesis can be accepted that the effect of cross flow is very small and in this case a cross flow can be superimposed to the axial flow, the characteristic lines of the two flows are coincident, and the calculations can be simplified. For example, this system can be used easily in order to determine the flow around a cone of revolution in yaw.

In this approximation the velocity component in normal direction to every meridian plane of the body changes with the sine of the angle that defines the position of the meridian plane as in the small-disturbance theory.

This is the status of the analysis along bodies of revolution. Both theories can probably be extended to similar problems. The extension of the small-disturbance theory to bodies of cross section different from the circular but having constant shape must not be too difficult.

The use of the small-disturbance theory for interference problems seems also possible, and some results of this application are yet to be obtained.

For the small-disturbance theory a complete analysis of the approximations that can be obtained would be very useful. Discrepancy of opinions exists especially on the possibility for this theory of evaluating the Mach number effect on phenomena for bodies unclined at an angle of attack.

In order to give an idea of the precision that can be obtained from the theory of small disturbances, a comparison of the lift-coefficient-curve slope $dC_L/d\alpha$ given with the assumption of small disturbances and without this assumption is presented in figure 7 for different Mach numbers. The bodies are cones of revolution of different apex angles η_o.

The development of an analytical theory having higher approximation than the small-disturbance theory for bodies of revolution would be very useful but seems at present very difficult. The characteristics theory can be extended to the analysis of any shape of body having anywhere supersonic flow, if the initial conditions can be determined; therefore, the determination of general conical flow in more exact form is essential for the extension of the field of application of this theory.

Interference problems can be analyzed with the characteristics theory. This analysis, however, requires a large amount of numerical work in every application. This obstacle, which exists at the present time, can perhaps be eliminated by using for the numerical calculations large size computing machines. The characteristics system can probably be extended to viscous-flow phenomena or to phenomena with variable total energy.

BIBLIOGRAPHY

Small-Disturbance Theory

1. Von Kármán, Theodor, and Moore, Norton B.: Resistance of Slender Bodies Moving with Supersonic Velocities, with Special Reference to Projectiles. Trans. A.S.M.E., vol. 54, no. 23, Dec. 15, 1932, pp. 303–310.

2. Von Karman, Th: The Problem of Resistance in Compressible Fluids. Atti dei Convegni Volta, 5 Convegno di Sci. Fis., Mat. e Nat., R. Accad. d'Italia (Rome), second ed., 1940, pp. 210–269.

3. Ferrari, C.: Campi di corrente ipersonora attorno a solidi di rivoluzione. L'Aerotecnica, vol. XVII, fasc. 6, June 1937, pp. 507–518.

4. Tsien, Hsue-Shen: Supersonic Flow over an Inclined Body of Revolution. Jour. Aero. Sci., vol. 5, no. 12, Oct. 1938, pp. 480–483.

5. Brown, Clinton E., and Parker, Hermon M.: A Method for the Calculation of External Lift, Moment, and Pressure Drag of Slender Open-Nose Bodies of Revolution at Supersonic Speeds. NACA ACR No. L5L29, 1946.

6. Haack, W.: Geschossformen kleinsten Wellenwiderstandes. Bericht 139 der Lilienthal-Gesellschaft für Luftfahrtforschung, 1941, pp. 14–28.

7. Lighthill, M. J.: A Note on Supersonic Biplanes. R. & M. No. 2002, British A.R.C., 1944.

8. Jones, Robert T., and Margolis, Kenneth: Flow over a Slender Body of Revolution at Supersonic Velocities. NACA TN No. 1081, 1946.

9. Ferrari, Carlo: Sulla determinazione del proietto di minima resistenza d'onda. Nota I. Atti R. Accad. Sci. Torino, vol. 74, 1938–39.

10. Ferrari, Carlo: Sul problema del proietto di minima resistenza d'onda. Atti R. Accad. Sci. Torina, vol. 75, 1939–40.

Characteristic System for Bodies of Revolution in Axis with the Flow

11. Crocco, L.: Una nuova funzione di corrente per lo studio del moto rotazionale dei gas. Rend. della R. Accad. dei Lincei. Vol. XXIII, ser. 6, fasc. 2, Feb. 1936

12. Busemann, A.: Drücke auf kegelförmige Spitzen bei Bewegung mit Überschallgeschwindigkeit. Z.f.a.M.M., Bd 9, Heft 6, Dec., 1929, pp. 496–498.

13. Taylor, G. I., and Maccoll, J. W.: The Air Pressure on a Cone Moving at High Speeds. Proc. Roy. Sec. (London) ser. A, vol. 139, no. 838, Feb. 1, 1933, pp. 278–311.

14. Ferrari, C.: Campo aerodinamico a velocita iperacustica attorno a un solido di rivoluzione a prora acuminata. L'Aerotecnica, vol. XVI, fasc. 2, Feb. 1936, pp. 121–130.

15. Ferrari, C.: The Determination of the Projectile of Minimum Wave-Resistance. R.T.P. Translation No. 1180, British Ministry of Aircraft Production. (Part II from Atti R. Accademie Sci. Torino, vol. 75, Nov.-Dec. 1939, pp. 61–96).

16. Guderley, G.: Die Charakteristikenmethode für ebene und achsen-symmetrische Überschallströmungen. Jahrb. 1940 der deutschen Luftfahrtforschung, R. Oldenbourg (Munich), pp. I 522 – I 535.

17. Sauer, R.: Charakteristikenverfahren für räumliche achsensymmetrische Überschallströmungen. Forschungsbericht Nr. 1269, Deutsche Luftfahrtforschung (Aachen), 1940.

18. Tollmein, W.: Anwendung der rechnerischen Behandlung von Stossfront und Wirbelströmung auf einen aus Kegel und Zylinder zusammengesetzten Körper. Archiv. Nr. 44/14, Technischen Hochschule Dresden, Sept. 23, 1944.

19. Temple, G.: The Method of Characteristics in Supersonic Flow. R. & M. No. 2091, British A.R.C., 1944.

20. Ferri, Antonio: Application of the Method of Characteristics to Supersonic Rotational Flow. NACA TN No. 1135, 1946.

Characteristic System for Bodies of Revolution in Yaw

21. Ferrari, C.: Determination of the Pressure Exerted on Solid Bodies of Revolution with Pointed Noses Placed Obliquely in a Stream of Compressible Fluid at Supersonic Velocity. R.T.P. Translation No. 1105, British Ministry of Aircraft Production. (From Atti R. Accad. Sci. Torino, vol. 72, Nov.-Dec. 1936, pp. 140–163.)

22. Sauer, R.: Überschallströmung um beliebig geformte Geschoszspitzen unter kleinem Anstellwinkel. Luftfahrtforschung, Bd.19, Lfg. 4, May 6, 1942, pp. 148–152.

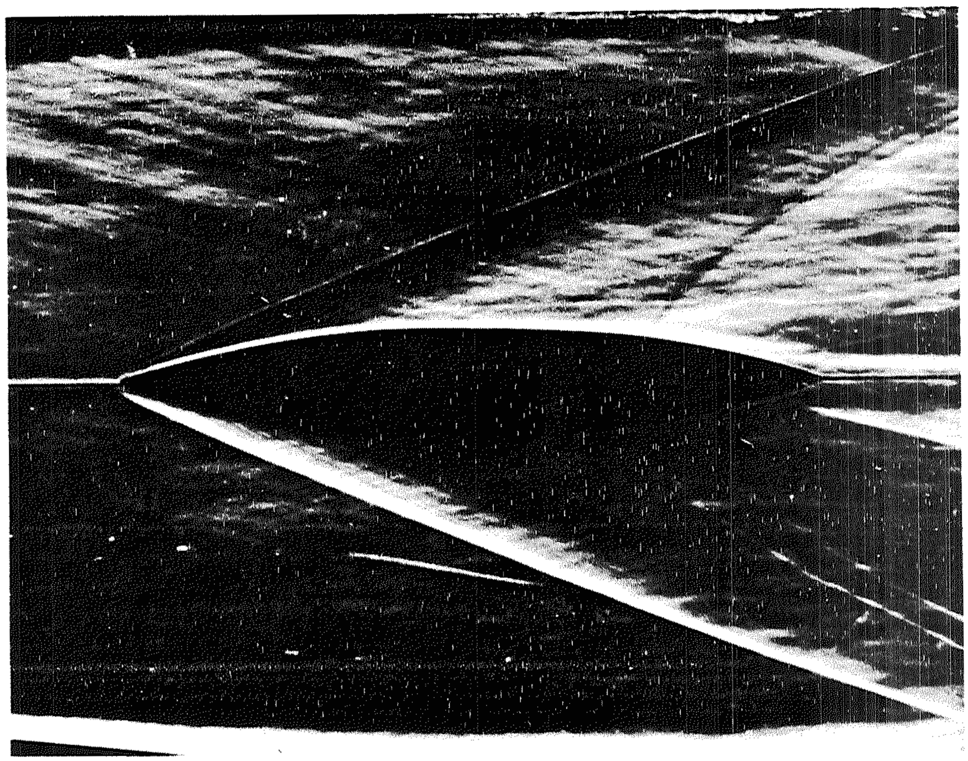

Figure 1.- Axial-symmetrical shock at apex of slender body of revolution.

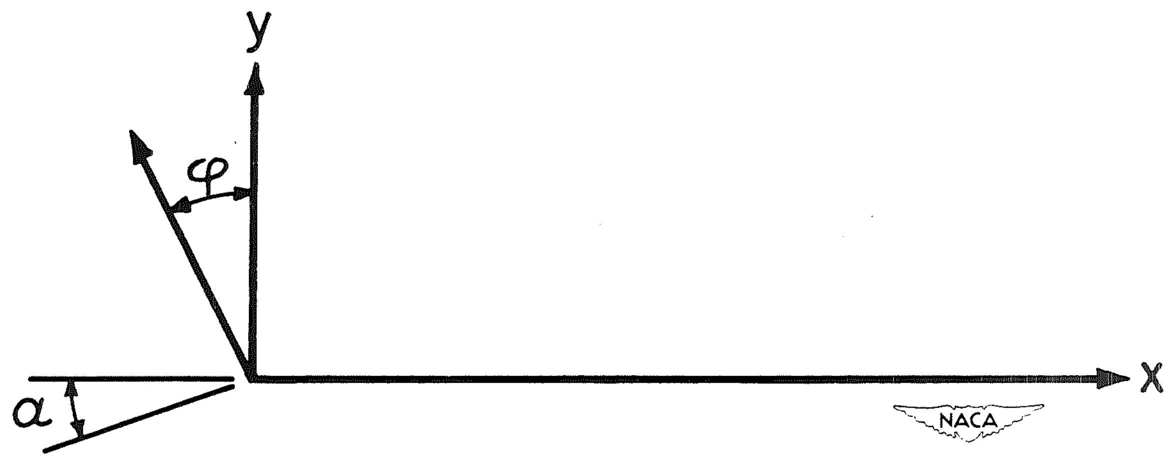

Figure 2.- Coordinate system.

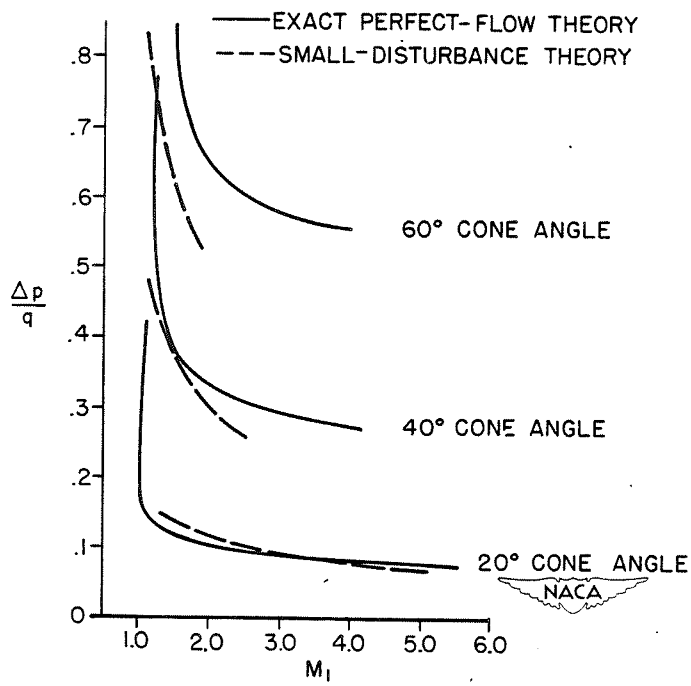

Figure 3.- Variation of $\Delta p/q$ with M_1 for various cone angles.