NASA TECHNICAL MEMORANDUM NASA TM-75693

TWO-DIMENSIONAL NONSTATIONARY MODEL OF THE PROPAGATION OF AN ELECTRON BEAM IN A VACUUM

S.L. Ginzburg and V.F. D'yachenko

Translation of "Dvumernaya nestatsionarnaya model' rasprostraneniya elektronnogo puchka v vakuume," Academy of Sciences USSR, Institute of Applied Mathematics imeni M.V. Keldysh, Moscow, Preprint No. 51, 1979, pp 1-22

(NASA-TM-75693) TWO-DIMENSIONAL NONSTATIONARY MODEL OF THE PROPAGATION OF AN ELECTRON BEAM IN A VACUUM (National Aeronautics and Space Administration) 14 p HC A02/MF A01 CSCL 09C G3/33

N80-10435

Unclas 45946

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON D.C. 20546 OCTOBER 1979

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-75693		

4. Title and Subtitle	5. Report Date
TWO-DIMENSIONAL NONSTATIONARY MODEL OF THE PROPAGATION OF AN ELECTRON BEAM IN A VACUUM	October 1979
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
S.L. Ginzburg and V.F. D'yachenko, USSR Academy of Sciences, Keldysh Institute of Applied Mathematics, Moscow	10. Work Unit No.
9. Performing Organization Name and Address	11. Contract or Grant No.
Leo Kanner Associates Redwood City, California 94063	NASw-3199
	13. Type of Report and Period Covered
	Translation
12. Sponsoring Agency Name and Address	14. Sponsoring Agency Code
National Aeronautics and Space Administration, Washington, D.C. 20546	

15. Supplementary Notes

Translation of "Dvumernaya nestatsionarnaya model' rasprostraneniya elektronnogo puchka v vakuume," Academy of Sciences USSR, Institute of Applied Mathematics imeni M.V. Keldysh, Moscow, Preprint No. 51, 1979, pp 1-22

16. Abstract

A two-dimensional nonstationary model of the propagation of a relativistic electron beam injected into vacuum is considered. Collision effects are ignored. There are no external fields. Results obtained by computer simulation of the Maxwell-Vlasov equations showed there are two types of the electron current propagation. If the injected current is less than some critical value, the beam remains regular, laminar. Otherwise, surface reflecting electrons arise. Part of them return; the rest leave along a cone.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified		

NASA-HQ

TWO-DIMENSIONAL NONSTATIONARY MODEL OF THE PROPAGATION OF AN ELECTRON BEAM IN A VACUUM

S.L. Ginzburg and V.F. D'yachenko
USSR Academy of Sciences, Keldysh Institute of Applied Mathematics, Moscow

1. The dynamics of a relativistic electron beam in its inherent electromagnetic field, injected into vacuum, are considered, without taking account of particle collisions, within the framework of the Maxwell-Vlasov equations [1] /4*

$$\frac{1}{c}\frac{\partial \bar{E}}{\partial t} - \mathrm{rot}\,\bar{H} + \frac{4\pi}{c}\bar{j} = 0, \quad \mathrm{div}\,\bar{E} = -4\pi e n$$
$$\frac{1}{c}\frac{\partial \bar{H}}{\partial t} + \mathrm{rot}\,\bar{E} = 0, \quad \mathrm{div}\,\bar{H} = 0 \tag{1}$$

$$\frac{\partial f}{\partial t} + \bar{v}\frac{\partial f}{\partial \bar{x}} - e\left(\bar{E} + \frac{1}{c}[\bar{v}\times\bar{H}]\right)\frac{\partial f}{\partial \bar{p}} = 0 \tag{2}$$

Here, $\bar{E}(t,\bar{x})$, $\bar{H}(t,\bar{x})$ are the electric and magnetic fields, $f(t,\bar{x},\bar{p})$ is the electron pulse distribution function, $\bar{p}$, $\bar{v}=W_{\bar{p}}$ is the velocity, $W(\bar{p}) = \sqrt{m^2c^4+c^2\bar{p}^2}$ is the electron energy, m is its rest mass, e is the elementary charge, c is the velocity of light. The electron density $n(t,\bar{x})$ and current density $\bar{J}(t,\bar{x})$ are expressed by the integrals over impulse space

$$n = \int f d^3p, \quad \bar{J} = -e\int \bar{v} f d^3p \tag{3}$$

The electrons are injected from a circular plane cathode, of radius L along the normal to it, with energy

$$W_0 = \gamma mc^2 \tag{4}$$

and producing a current

$$I_0 = \sigma \frac{mc^3}{4e} \tag{5}$$

the density of which

$$j_0 = \sigma \frac{mc^3}{4\pi e L^2} \tag{6}$$

*Numbers in the margin indicate pagination in the foreign text.

is constant (γ and σ are assigned constants).

There are no external fields. The plane which contains the cathode is assumed to be equipotential and, consequently, here, the tangential component of the electric field equals zero.

In cylindrical coordinates r, z, ψ, where z = 0 is the plane of the cathode, and r = 0 is the axis of symmetry, we obtain two dimensional problem (1)-(3), with $\partial/\partial\phi \equiv 0$, relative to $E_r(t,r,z)$, $E_z(t,r,z)$, $H(t,r,z)$ and $f = \delta(P_\phi)F(t,r,z,P_r,P_z)$, with initial data at t=0, /5

$$E_r = E_z = H_\varphi = F = 0\,, \tag{7}$$

with the boundary conditions at z=0

$$eF = \begin{cases} j_o\,\delta(p_r)\,\delta(W-W_o) & \text{at } r<L,\ p_z>0 \\ 0 & \text{at } r>L,\ p_z>0 \end{cases} \tag{8}$$

$$E_r = 0 \tag{9}$$

and with intrinsic conditions on the r=0 axis

$$E_r = H_\varphi = 0\,, \quad F(p_r) = F(-p_r) \tag{10}$$

The following two dimensionless parameters in the problem thus formulated prove to be decisive

$$\gamma = \frac{W_o}{mc^2}\,, \quad \sigma = \frac{4e}{mc^3} I_o = \frac{4\pi e L^2}{mc^3} j_o \tag{11}$$

the injected electron energy and current.

2. The problem formulated was solved by computer. To integrate Vlasov equation (2), a macroparticle model was used. The latter consists of presentation of the electron gas as a discrete set of macroparticles, each of which is a group of electrons with the same coordinates and impulse. Of course, the electron motion is described by the equations

$$\frac{d\bar{x}}{dt} = \bar{v}\,, \quad \frac{d\bar{p}}{dt} = -e\left(\bar{E} + \frac{1}{c}[\bar{v}\times\bar{H}]\right). \tag{12}$$

The electron density is calculated by the formula

$$n(t,\bar{x}) = \sum_m N_m \hat{\delta}(\bar{x}_m(t)-\bar{x})\,, \tag{13}$$

where summing is carried out over all macroparticles, and $\hat{\delta}$ is a deltoid function, with a carrier on the order of the dimensions of the calculation cell. The current density is calculated similarly. /8

The calculation region is a cylinder $0<r<r_1$, $0<z<z_1$, on the outer boundaries of which the following conditions are laid down

$$E_r - H_\varphi = 0 \quad \text{at} \quad z = z_1$$
$$E_z + H_\varphi = 0 \quad \text{at} \quad r = r_1 \,, \qquad (14)$$

which simulates the absence of electromagnetic radiation from the outside.

Electrons leaving the boundaries $z=0$, $z=z_1$, $r=r_1$, are excluded from the calculation, and those striking the $r=0$ axis are reflected from it, in accordance with (10). Quite large values of r_1, z_1 are chosen, so as not to affect the result.

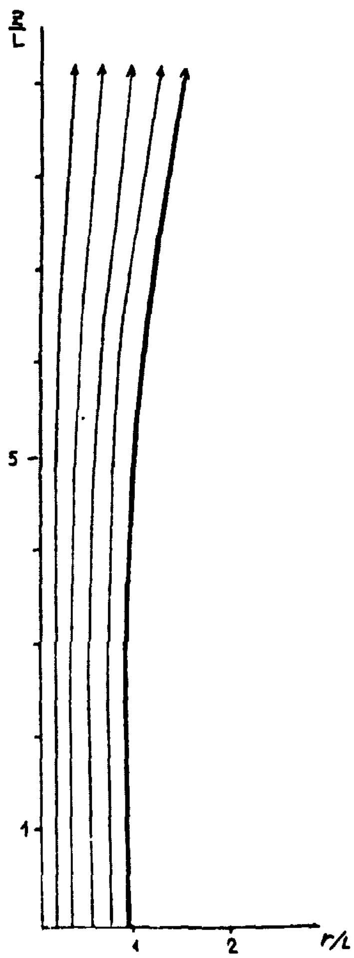

Fig. 1. Current at $\gamma=2$, $\sigma=0.2$

3. We proceed to description of the calculation results. First and foremost, we note that, in all (with respect to γ, σ) versions, practically steady state behavior is observed. We dwell on transient details below, and we now give the general characteristics of these behaviors.

The two schemes of movement presented in Figs. 1 and 2 are typical. The current lines for two versions, which differ in the magnitude of the injected current ($\sigma=0.2$ and $\sigma=2$, respectively), with the same energy $\gamma=2$, are presented here.

In the first case, we have a uniform, expanding beam, propagating along the z axis. In the second case, the movement pattern is completely different. There is an envelope in the family of trajectories, the $z=z_*(r)$ surface, which reflects electrons ("pseudocathode"), and a nonuniform, multivelocity flow.

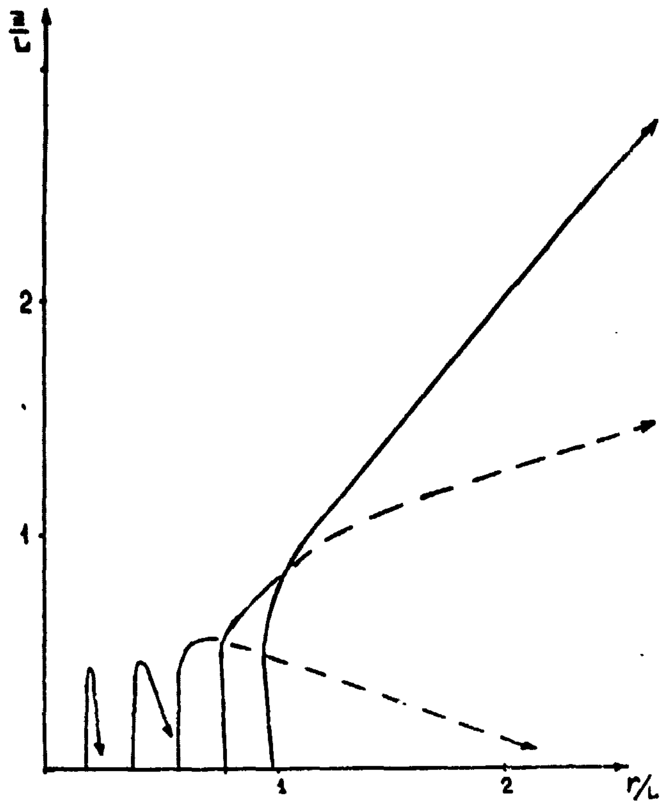

Fig. 2. Current at $\gamma=2$, $\sigma=2$

The electrons flow past the $r/z\sim1$ cone, and a substantial fraction of them returns to the cathode.

4. A priori, it is clear that conditions are produced in the pseudocathode behavior, for the development of the known two beam instability [2]. There are two counterbeams, forward and reflected, in which the density of the fluxes is high in the reversal region and, consequently, high frequencies should be excited. /11

On the other hand, there are stabilizing factors, steady state injection from the nearby cathode, constant current drift and a decrease in density with distance from the axis.

As the calculation results show, the interaction of all these factors results in an oscillatory process, the amplitude of which is small, and the average state corresponds to the steady state behavior pointed out above.

The transient effects intensify with increase in current. Therefore, we turn to the version with $\gamma=2$, $\sigma=4$ to describe them. $E_z(z)$ curves on the $r=0$ axis at different moments of time are presented in Fig. 3. The points where E_z passes through zero correspond to the pseudocathode location z_*. E_z vs. time is shown in Fig. 4, at a point located on the $r=0$ axis near the pseudocathode ($z\sim0.25L$), for the same version, $\gamma=2$, $\sigma=4$. In this area, because of the large field gradient in the direction of movement of its profile, the amplitude of the oscillations is comparable to the size of the field itself.

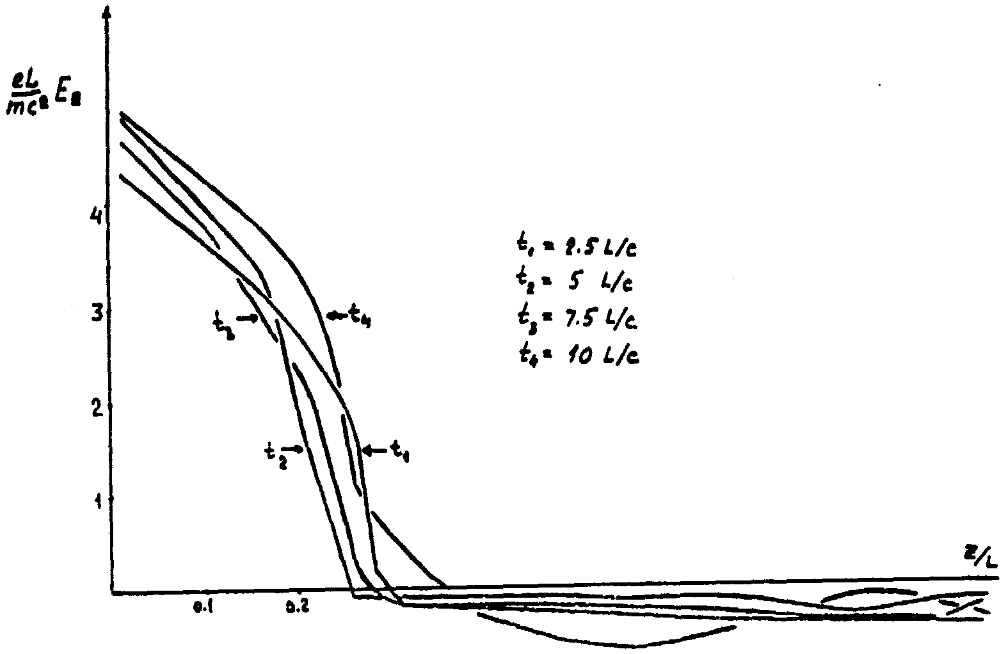

Fig. 3. E_z profiles on axis at $\gamma=2$, $\sigma=4$.

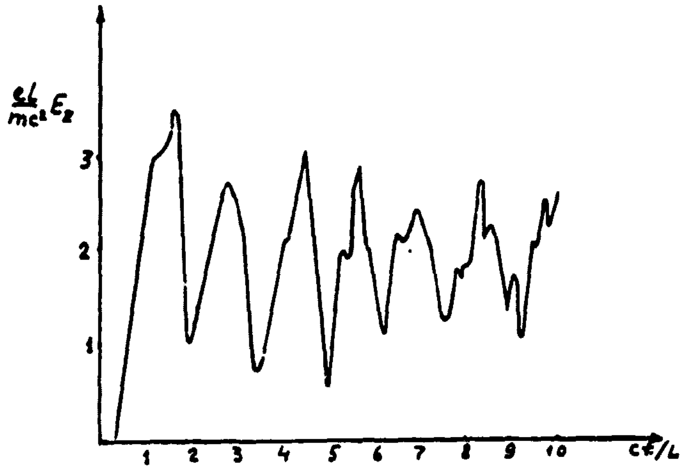

Fig. 4. $E_z(t)$ in pseudocathode area, at $\gamma=2$, $\sigma=4$.

As a result of the pseudocathode pulsations, a small fraction of the electrons periodically prove to be above it, and they are dumped in the space beyond the cathode.

These pulsations give rise to transient phenomena in the entire flow. Electrons which reach the pseudocathode along one current line acquire various velocities upon reflection from it and, then, moving practically rectilinearly, they are located on curves, the shapes of which are similar to the jet from an oscillating hose. The movement of such jets basically determines the structure of the weak turbulent flow which arises.

For this reason, in description of the steady state characteristics of the behaviors by means of the current lines, we have represented the latter provisionally in the figures, by a dashed line which shows the average movement.

It can be said that the transient nature of the process is reduced /17
to the transient nature of the pseudocathode.

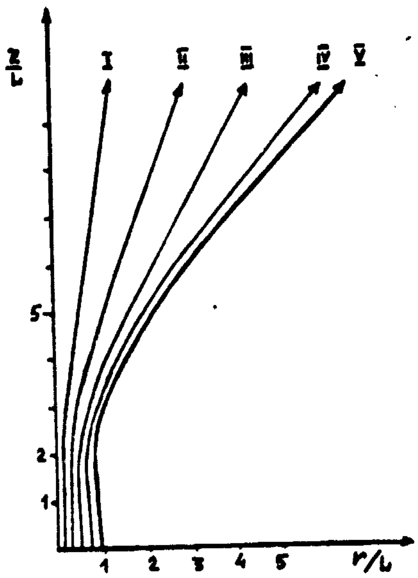

Fig. 5. Flow at γ=2, σ=0.5

5. We know dwell in greater detail on the dependence of the average steady state properties of the flow on current $_0$. We consider versions with injected electron energies γ=2.

The results of calculation of the version with σ=0.5 are presented in Fig. 5. Although, the trajectories intersect in a narrow boundary layer, on the whole, the beam remains uniform, with practically constant current density over the cross section, which rises slightly at the boundary itself. Compared with the σ=0.2 version (Fig. 1), the rate of broadening of the beam has increased appreciably.

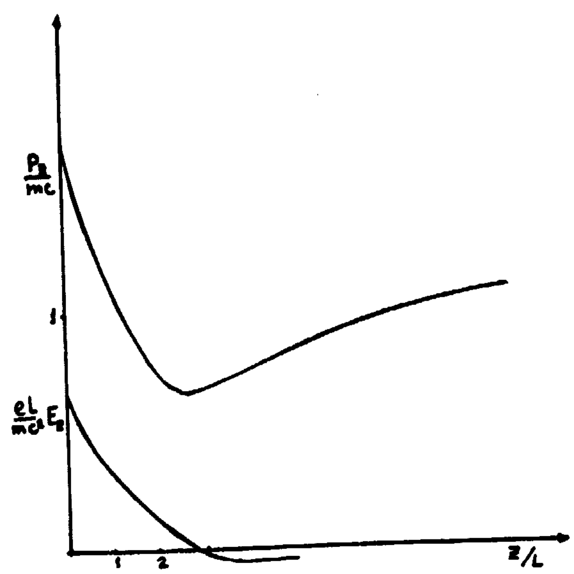

Fig. 6. E_z and P_z profiles on axis, at γ=2, σ=0.5

Curves of the $E_z(t)$ and pulse $P_z(z)$ fields along the z axis are given in Fig. 6, for the same version σ=0.5. The point where E_z=0, P_z=min corresponds to the minimum potential. Upon passing it, the flow is accelerated. However, electron energy $W(P_z)$ remains less than the initial $W_0=\gamma mc^2$.

On the same terms, the variant with σ=0.65 is shown in Figs. 7 and 8. Uniformity is preserved only in the

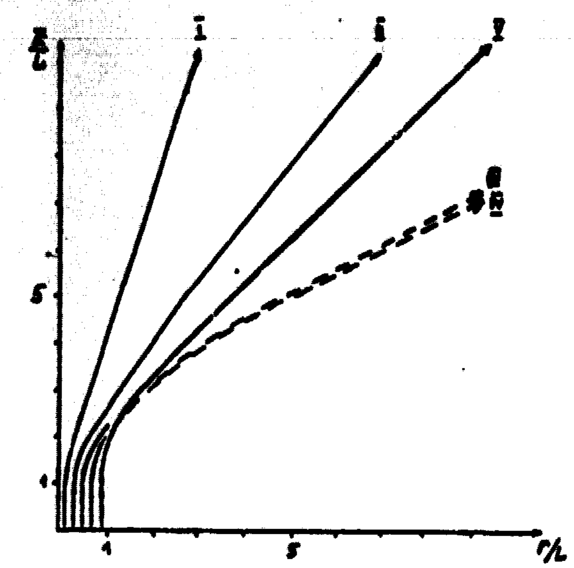

Fig. 7. Flow at $\gamma=2$, $\sigma=0.65$

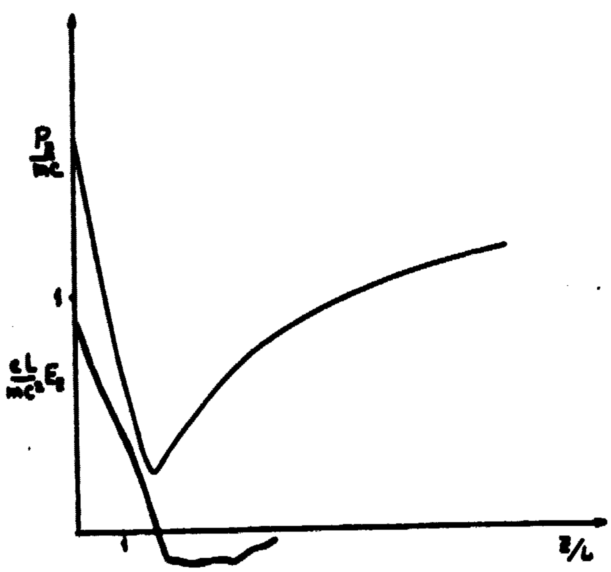

Fig. 8. E_z and P_z profiles on axis, at $\gamma=2$, $\sigma=0.65$.

axial region. A pseudocathode forms in the outer layers. The outer boundary of the injected beam (current line V) proves to be inside the flow. Eighty percent of the current moves outside it.

The version $\sigma=0.75$ is presented in Fig. 9. The pseudocathode has formed completely and reached the axis. The outer layer of the beam becomes the inner one (line V). The flow "was turned inside out". The main portion of the current, 84%, flows in a cone between lines II and V. A negligible fraction of the current ($\sim$4%) leaks through the pseudocathode in a transient manner. A reverse current develops.

As we see, a qualitative rearrangement of the structure and shape of the flow occurs over a small interval of change of σ from 0.5 to 0.75. A critical current value can be spoken of, in /19
this case, $\sigma_{cr}\sim 0.6$.

With further increase in current, the flow pattern does not change qualitatively. The pseudocathode subsides and becomes flatter. The reverse current fraction increases. At $\sigma=2$ (Fig. 2) 34% of the electrons return on the z=0 plane and, at $\sigma=4$, 56%.

In the outer zone, the flow retains the shape of a cone with the same angle $\sim 45^{\circ}$. Of course, the fraction of the current traveling in

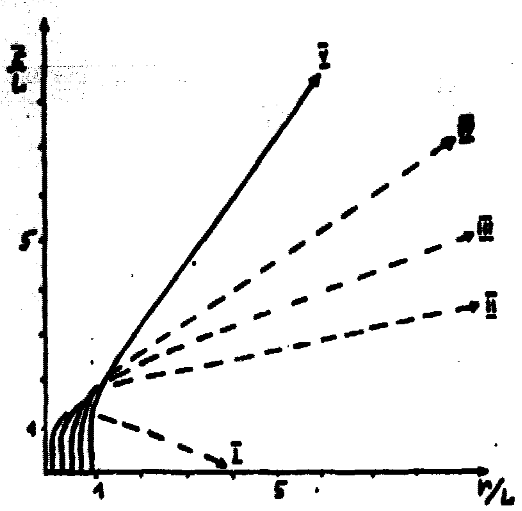

Fig. 9. Flow at $\gamma=2$, $\sigma=0.75$.

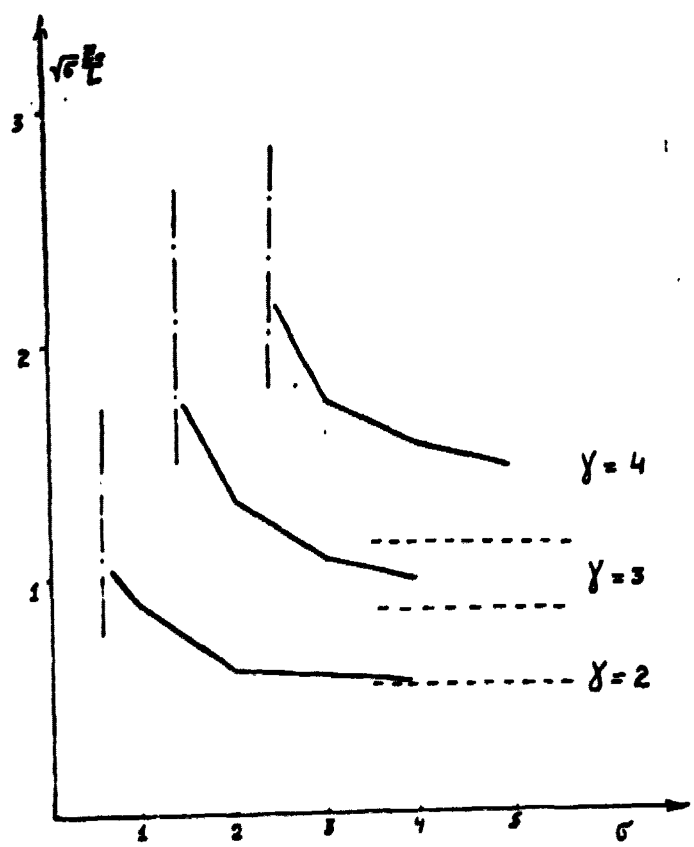

Fig. 10.

it decreases.

6. We have reported the calculation results of versions with injection energy $\gamma=2$. At other γ, the quantitative characteristics of the solution are different, of course, but all the effects described are preserved. Fig. 10 gives an idea of the presence of the pseudocathode and its height z_* (at $r=0$), in versions with other γ.

Specific information on z_* and σ_{cr} vs. γ also can be obtained, by means of analytic estimates.

7. In the limit, as $\sigma\to\infty$, the pseudocathode approaches the cathode without restriction, $z_*/L\to 0$. But, with $z_* \ll L$, the finite dimensions of the cathode should not affect the processes in the axial region. Therefore, if the increase of σ occurs through an increase in cathode radius L, with the density of the injected current preserved, for determination of the asymptotes of the solution in the axial zone, a unidimensional version of our problem, $\partial/\partial r\equiv 0$, which corresponds to an infinite cathode, can be used.

This problem was investigated in 3 . The steady state version of it has an analytical solution, which can be written in the form

$$eF = j_0\,\delta(p_r)\,\delta\left(W - \gamma mc^2 + e\int_0^z E_z\,dz\right) \qquad (15)$$

$$LeE_z = 2\sqrt{\sigma mc^3 p_z}\,, \quad E_r = H_\varphi = 0 \qquad (16)$$

in which the relationship between $W(P_z)$ and z is determined, with $0<z<z_*$, by the integral

$$\sqrt{\sigma}\,\frac{z_* - z}{L} = \frac{1}{2}\int_1^{W/mc^2} (x^2-1)^{-3/4}\,dx \qquad (17)$$

and z_* itself /20

$$\sqrt{\sigma}\,\frac{z_*}{L} = \frac{1}{2}\int_1^{\gamma} (x^2-1)^{-3/4}\,dx \qquad (18)$$

Precisely these values for the corresponding γ are represented in Fig. 10 by a dashed line. The asymptote is expressed completely clearly.

8. Simple reasoning permits an estimate of the critical current $\sigma_{cr}(\gamma)$ to be obtained, at which a change in type of behavior occurs.

We find the conditions of the existence of a steady state, single velocity flow, unbounded along the z axis.

The motion of an electron in a steady state field with potential ϕ, $\nabla\phi = -\bar{E}$, occurs with retention of its total energy, $W - e_\phi = \text{const}$. Since, at the time of injection, all the electrons have the same energy $W_0 = \gamma mc^2$, and the cathode plane is equipotential, $\phi|_{z=0} = \phi_0$,

$$W - e\varphi = W_0 - e\varphi_0\,, \quad W \geq mc^2 \qquad (19)$$

for all electrons in the entire flow.

The potential satisfies the Poisson equation

$$\Delta\varphi = 4\pi e n \qquad (20)$$

and it is finite at infinity.

For a single velocity flow, the current density

$$j = env. \tag{21}$$

By virtue of the assumed boundedness of the beam with respect to r, at each z, evidently, $\phi \to \phi_0$ as $r \to \infty$. This gives a basis for presenting the radial portion of the Laplace operator in the form

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \varphi}{\partial r} = \alpha \frac{\varphi_0 - \varphi}{R^2} \tag{22}$$

where α is a dimensionless, indeterminate coefficient, and R is the effective radius of the beam, such that /21

$$\pi R^2 j = I_0 = \frac{mc^3}{4e}\sigma \tag{23}$$

is the total current.

With (19), (21)-(23) taken into consideration, we rewrite equation (20) in the form

$$R^2 \frac{\partial^2 W}{\partial z^2} = \frac{4eI_0}{v} + \alpha (W - W_0) \tag{24}$$

On the assumption that $R/z <$ const, it is easy to determine that a necessary condition of the existence of a bounded solution of equation (24), as $z \to \infty$, is the possibility of the reduction of its right side to zero. Otherwise, either $W \to \infty$, or the solution exists only in a finite interval, since W cannot be less than mc^2.

By making the right side of (24) equal to zero, with fixed I_0 and α, we obtain an equation for W. Simple analysis shows that it has a real root, only in the event

$$\sigma \leq \alpha (\gamma^{2/3} - 1)^{3/2} \tag{25}$$

This formula gives an estimate of the value of the critical current $\sigma_{cr}(\gamma)$. If $\alpha \approx 1.3$, it is in good agreement with the calculation results (Fig. 10). In particular, this justifies the "simplicity" of the reasoning used.

9. The authors thank V.S. Imshenitskiy, K.V. Krasnobayev, O.V. Lokutsiyevskiy and K.V. Khodatayev, the initiators of this work and constant participants in fruitful discussions, as well as N.M. Khokhlova and Ye.V. Shakhanova for assistance in the work.

REFERENCES

1. Davidson, R., Teoriya zaryazhennoy plazmy [Charged Plasma Theory], Mir Press, Moscow, 1978. /22

2. Briggs, R., Dvukhpuchkovaya neustoychivost' [Two Beam Instability], Mir Press, Moscow, 1974.

3. Poukey, J.W., N. Rostoker, "One dimensional model of relativistic electron beam propagation," Plasma Physics 13/10, (1971).