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TEMPERATURE DEFORMATIONS OF THE MIRROR OF A RADIO TELESCOPE ANTENNA

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V. I. Kostenko

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1. Resolving Equations and Their Solution /3*

Figure 1 shows a general form of a mirror consisting of 19 identical parts -- "lobes." The position of an arbitrary point A on the middle surface of the "lobe" is determined by the angles θ and φ, and the angles θ_0 and θ_1 correspond to parallel edges of the "lobes" and φ_k and φ_{k+1} correspond to the meridional edges (Fig. 2). The mirror consists of 19 "lobes", i.e., k 12. Therefore, we obviously have

$$\varphi_{k+1} = \varphi_k + \frac{\pi}{6}. \tag{1.1}$$

In the case of arbitrary thermal influence, the thermoelasticity equations for the "lobes" may be written in the form [1, 2]:

$$\begin{gathered} L\{\tilde{U}\} - \left[1 - ic\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{1}{\sin^3\theta}\right]\frac{\partial^2 T}{\partial\varphi^2} = \bar{f}_1(\theta, \varphi), \\ T - icL\{T\} + \left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{1}{\sin^3\theta}\tilde{U} = R_2 f_2(\theta, \varphi), \end{gathered} \tag{1.2}$$

where

$$L = \frac{1}{R_1 R_2 \sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\frac{R_2^2\sin\theta}{R_1}\cdot\frac{\partial}{\partial\theta}\right) + \frac{1}{R_2\sin^2\theta}\cdot\frac{\partial^2}{\partial\varphi^2}, \tag{1.3}$$

$$\left.\begin{aligned} \bar{f}_1 &= \frac{1}{R_1 R_2 \sin\theta}\left\{\frac{\partial}{\partial\theta}\left[\left(\bar{f}_2\cos\theta - \bar{f}_3\sin\theta\right)R_2^2\sin^2\theta\right] + \frac{\partial\bar{f}_4}{\partial\varphi}R_1 R_2^2\sin^2\theta\right\}, \\ \bar{f}_2 &= \frac{iEhc}{R_1 R_2 \sin\theta}\left[\frac{\partial}{\partial\theta}\left(\frac{R_2\sin\theta}{R_1}\cdot\frac{\partial\varepsilon_T}{\partial\theta}\right) + \frac{R_1}{R_2\sin\theta}\cdot\frac{\partial^2\varepsilon_T}{\partial\varphi^2}\right], \\ \bar{f}_3 &= \frac{iEhc}{R_1^2}\cdot\frac{\partial\varepsilon_T}{\partial\theta}, \qquad \bar{f}_4 = \frac{iEhc}{R_2^2\sin\theta}\cdot\frac{\partial\varepsilon_T}{\partial\varphi}, \end{aligned}\right\} \tag{1.4}$$ /4

$\tilde{U}$, $\tilde{T}$ -- the V. V. Novozhilov functions [2];

R_1, R_2 -- main radii of curvature for the lobe;

*Numbers in the margin indicate pagination of original foreign text.

i -- imaginary unit;

E -- Young modulus;

ν -- Poisson coefficient,

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$$c^2 = \frac{h^2}{12(1-\nu^2)}, \tag{1.5}$$

$$\varepsilon_T = \alpha T, \tag{1.6}$$

where α is the linear expansion coeffieicnt; T -- temperature.

The meridional edges of the "lobe" are free. The functions

$$\left.\begin{aligned} \tilde{U} &= \tilde{U}' \cdot \cos b\varphi, \\ \tilde{T} &= \tilde{T}' \cos b\varphi, \end{aligned}\right\} \tag{1.7}$$

satisfy the corresponding conditions at these edges. We assume that within the limits of each "lobe" the temperature is distributed according to the law

$$T = T_0' + T_1' \cos b\varphi, \tag{1.8}$$

where the quantities T_0' and T_1' for each meridion are selected in accordance with the given temperature field for the mirror:

$$T = T_0 + T_1 \cos\varphi, \tag{1.9}$$

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$$\left.\begin{aligned} T_0 &= \frac{1}{2}(T_{01} + T_{02}), \quad T_1 = \frac{1}{2}(T_{01} - T_{02}), \\ T_{01} &= 80^\circ\ \mathrm{C}, \quad T_{02} = -120^\circ\ \mathrm{C}. \end{aligned}\right\} \tag{1.10}$$

The system of equations (1.2) for the first component on the right side of (1.9) is reduced to the equation [3] (symmetric case)

$$\frac{\partial^2 \sigma_0}{\partial \theta^2} + \left[\left(2\frac{R_1}{R_2} - 1\right) ctg\,\theta - \frac{1}{R_1}\frac{dR_1}{d\theta}\right]\frac{d\sigma_0}{d\theta} + i\frac{R_1^2}{cR_2}\sigma_0 = 0, \tag{1.11}$$

and for the second component -- to the equation [1] (anti-symmetric case)

$$\begin{aligned} &\frac{d^2\tilde{W}}{d\theta^2} + \left[\left(2\frac{R_1}{R_2} - 1\right) ctg\,\theta - \frac{1}{R_1}\frac{dR_1}{d\theta}\right]\frac{d\tilde{W}}{d\theta} + \left[-\frac{3b R_1^2}{R_2^2 \sin^2\theta} + \right. \\ &\left. + i\frac{R_1^2}{cR_2}\right]\tilde{W} = \frac{Ehc\alpha T_1' R_1^2}{R_2^2}\left[\frac{r_0}{c(\varkappa - 1)\sin^2\theta} - bi\right]. \end{aligned} \tag{1.12}$$

The following notation is used in (1.11) and (1.12):

$$\sigma_0 = Eh\vartheta_0 - i\frac{V_0}{c}, \tag{1.13}$$

$$\tilde{W} = Eh\Psi + i\frac{\sqrt{12(1-\nu^2)}}{h}V, \tag{1.14}$$

where ϑ_0 and V_0 are the revolution angle of the normal and the stress function in the case of symmetric deformation of the "lobe"; Ψ and V -- displacement function and stress function in the case of anti-symmetric deformation of the "lobe"; h -- thickness of the "lobe",

$$R_1 = \frac{R_0}{\cos^3\theta}, \qquad R_2 = \frac{R_0}{\cos\theta}, \tag{1.15}$$

where R_0 are the radii of curvature at $\theta = 0$, /6

$$r_0 = R_2 \quad \text{at} \quad \theta = 30^\circ,$$
$$\varkappa = R_2/R_1 \quad \text{at} \quad \theta = 30^\circ,$$

i.e.,

$$r_0 = 2R_0/\sqrt{3}, \quad \varkappa = 3/2. \tag{1.16}$$

Applying the method of reference equation [4] to the equations (1.11) and (1.12), we obtain their solution:

$$\sigma_0 = \eta(\theta)\left[(a_1 - ia_2)\,\mathrm{ber}\,\lambda\theta - (a_2 + ia_1)\,\mathrm{bei}\,\lambda\theta + (a_3 - ia_4)\,\mathrm{ker}\,\lambda\theta - (a_4 + ia_3)\,\mathrm{kei}\,\lambda\theta\right], \tag{1.17}$$

$$\tilde{W} = \eta(\theta)\left[(C_1 - iC_2)\,\mathrm{bei}_\delta\,\lambda\theta + (C_2 + iC_1)\,\mathrm{ber}_\delta\,\lambda\theta - (C_3 - iC_4)\,\mathrm{kei}_\delta\,\lambda\theta - (C_4 - iC_3)\,\mathrm{ker}_\delta\,\lambda\theta\right] +$$
$$+ \frac{Ehc\alpha T_1'\left[\frac{3\delta r_0}{c(\varkappa-1)\sin^4\theta} + \frac{\delta R_2}{c} + i\left(\frac{r_0 R_2}{c^2(\varkappa-1)\sin^2\theta} - \frac{2\delta}{\sin^2\theta}\right)\right]}{\frac{12\delta^2}{\sin^4\theta} - \left(\frac{R_2}{c}\right)^2}, \tag{1.18}$$

where

$$\lambda^2 = \frac{R_0}{c}, \quad \eta(\theta) = \frac{2\theta}{\sin 2\theta}, \tag{1.19}$$

$\mathrm{ber}\,x, \mathrm{bei}\,x, \mathrm{ker}\,x, \mathrm{kei}\,x$ -- Thomson functions;

$C_1, a_2, a_3, a_4 \text{ и } C_1, C_2, C_3, C_4$ -- integration constants.

Separating the real and imaginary parts of the solution (1.17) and (1.18) with allowance for (1.13) and (1.14), we will have the resolving functions

$$\left.\begin{aligned} v_c &= \frac{\eta(\theta)}{Eh}(a_1 \operatorname{ber}\lambda\theta - a_2 \operatorname{bei}\lambda\theta + a_3 \operatorname{ker}\lambda\theta + a_4 \operatorname{kei}\lambda\theta), \\ V_\theta &= -c\eta(\theta)(a_2 \operatorname{ber}\lambda\theta + a_1 \operatorname{bei}\lambda\theta + a_4 \operatorname{ker}\lambda\theta + a_3 \operatorname{kei}\lambda\theta), \end{aligned}\right\} \qquad (1.20)$$

$$\left.\begin{aligned} \psi &= \frac{\eta(\theta)}{Eh}(c_1 \operatorname{bei}_6\lambda\theta - c_2 \operatorname{ber}_6\lambda\theta - c_3 \operatorname{kei}_6\lambda\theta - c_4 \operatorname{ker}_6\lambda\theta) + \\ &+ \frac{6\alpha c T_1' \left[\frac{8\sqrt{3}}{\sin^4\theta} + \frac{1}{\cos\theta}\right]}{R_0\left[\frac{1296\, c^2}{R_0^2 \sin^4\theta} - \frac{1}{\cos^2\theta}\right]}, \\ V &= c\eta(\theta)(-c_3 \operatorname{bei}_6\lambda\theta + c_1 \operatorname{ber}_6\lambda\theta + c_4 \operatorname{kei}_6\lambda\theta - \\ &- c_3 \operatorname{kei}\lambda\theta) + \frac{4Eh^2\alpha T_1'\left[\frac{\sqrt{3}}{3\cos\theta} - \frac{54\, c^2}{R_0^2}\right]}{\sqrt{12(1-\nu^2)}\left[\frac{1296\, c^2}{R_0^2 \sin^2\theta} - tg^2\theta\right]}. \end{aligned}\right\} \qquad (1.21)$$ /7

2. Displacement of the Lobe

Figure 3 shows the displacements of points on the middle surface of the "lobe" whose positive directions are shown in this figure and have the form [3]:

$$\left.\begin{aligned} u_\rho &= u_{\rho(0)} + u_{\rho(1)}\cos\varphi, \\ u_z &= u_{z(0)} + u_{z(1)}\cos\varphi, \\ W &= W_0 + W_1\cos\varphi, \end{aligned}\right\} \qquad (2.1)$$

where

$$\left.\begin{aligned} u_{\rho(0)} &= f_1(\theta), \\ u_{z(0)} &= \int_{\theta_0}^{\theta} f_2(\theta) - f_3(\theta)\, d\theta + D_1 \\ W_0 &= u_{\rho(0)}\sin\theta + u_{z(0)}\cos\theta, \end{aligned}\right\} \qquad (2.2)$$

$$\left.\begin{aligned} u_{\rho(1)} &= f_4(\theta) - \int_{\theta_0}^{\theta}\left[f_5(\theta) + f_6(\theta) - \frac{u_{z(1)}}{\cos^2\theta}\right] d\theta - D_2, \\ u_{z(1)} &= f_7(\theta) - \int_{\theta_0}^{\theta} f_8(\theta)\, d\theta - R_0\, tg\,\theta\, D_2, \\ W_1 &= u_{\rho(1)}\sin\theta + u_{z(1)}\cos\theta. \end{aligned}\right\} \qquad (2.3)$$

In (2.2) and (2.3), we use the notation /8

$$f_1(\theta) = -\frac{c\sin\theta\,\eta(\theta)}{Eh}\Big[\cos^2\theta\,(a_2\,ber'\lambda\theta + a_1\,bei'\lambda\theta + a_4\,ker'\lambda\theta + a_3\,kei'\lambda\theta) - \nu\,ctg\,\theta\,(a_2\,ber\,\lambda\theta + a_1\,bei\,\lambda\theta + a_4\,ker\,\lambda\theta + a_3\,kei\,\lambda\theta)\Big] + R_0\alpha T_0'\,tg\,\theta, \qquad (2.4)_1$$

$$f_2(\theta) = \frac{R_0\,\eta(\theta)}{Eh\cos^2\theta}(a_1\,ber\,\lambda\theta - a_2\,bei\,\lambda\theta + a_3\,ker\,\lambda\theta - a_4\,kei\,\lambda\theta), \qquad (2.4)_2$$

$$f_3(\theta) = -\frac{c\sin\theta\,\eta(\theta)}{Eh\cos^3\theta}\Big[ctg\,\theta\,(a_2\,ber\,\lambda\theta + a_1\,bei\,\lambda\theta + a_4\,ker\,\lambda\theta + a_3\,kei\,\lambda\theta) - \nu\cos^2\theta\,(a_2\,ber'\lambda\theta + a_1\,bei'\lambda\theta + a_4\,ker'\lambda\theta + a_3\,kei'\lambda\theta)\Big] + \frac{\alpha R_0 T_0'\sin\theta}{\cos^3\theta}, \qquad (2.4)_3$$

$$f_4(\theta) = \frac{tg\,\theta}{Eh}\Big\{c\cos\theta\,\eta(\theta)(-c_2\,bei_6'\lambda\theta + c_1\,ber_6'\lambda\theta + c_4\,kei_6'\lambda\theta - c_3\,ker_6'\lambda\theta) + (1-\nu)\,c\cdot\cos\theta\cdot ctg\,\theta\,\eta(\theta)\times(-c_2\,bei_6\lambda\theta + c_1\,ber_6\lambda\theta + c_4\,kei_6\lambda\theta - c_3\,ker_6\lambda\theta) - \frac{Eh^2\alpha T_1'}{3\sqrt{1-\nu^2}}\cdot\Big[\cos^4\theta - 2(1-\nu)\,ctg^2\theta\Big]\Big\} + R_0\alpha T_1'\,tg\,\theta, \qquad (2.4)_4$$

$$f_5(\theta) = \frac{2(1+\nu)c\,\eta(\theta)}{Eh\cos^2\theta\sin\theta}(-c_2\,bei_6\lambda\theta + c_1\,ber_6\lambda\theta + c_4\,kei_6\lambda\theta - c_3\,ker\,\lambda\theta) - \frac{4(1+\nu)h\alpha T_1'}{3\sqrt{1-\nu^2}\cos\theta\sin^2\theta}, \qquad (2.4)_5$$

$$f_6(\theta) = \frac{1}{Eh\cos^2\theta}\Big\{\cos^3\theta\,c\cdot\eta(\theta)(-c_2\,bei_6'\lambda\theta + c_1\,ber_6'\lambda\theta + c_4\,kei_6'\lambda\theta - c_3\,ker_6'\lambda\theta) - (1+\nu)\,ctg\,\theta\cos\theta\cdot c\cdot\eta\cdot\theta\times$$

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$$\times(-c_2\,bei_6\lambda\theta + c_1\,ber_6\lambda\theta + c_4\,kei_6\lambda\theta - c_3\,ker_6\lambda\theta) - \frac{Eh^2\alpha T_1'}{3\sqrt{1-\nu^2}}\Big[\cos^4\theta + 2(1-\nu)\,ctg^2\theta\Big] + \frac{R_0\alpha T_1'}{\cos^2\theta}, \qquad (2.4)_6$$

$$f_7(\theta) = \frac{R_0\,tg\,\theta\,\eta(\theta)}{Eh}\,c_1\,bei_6\lambda\theta + c_2\,ber_6\lambda\theta - c_3\,kei_6\lambda\theta - c_4\,ker_6\lambda\theta - \frac{6c\alpha T_1'\,tg\,\theta\left(\frac{6\sqrt{3}}{\sin^4\theta} + \frac{1}{\cos\theta}\right)}{\frac{1296\,c^2}{R_0^2\sin^4\theta} - \frac{1}{\cos^2\theta}}, \qquad (2.4)_7$$

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$$f_3(\theta) = \frac{1}{Eh\cos^2\theta}\Big\{\cos^2\theta\,\eta(\theta)(c_1 bei_6'\lambda\theta + c_2 ber_6'\lambda\theta -$$
$$- c_3 kei_6'\lambda\theta - c_4 ker_6'\lambda\theta + ctg\,\theta\,\eta(\theta)(c_1 bei_6\lambda\theta +$$
$$+ c_2 ber_6\lambda\theta - c_3 kei_6\lambda\theta - c_4 ker_6\lambda\theta) +$$
$$- \frac{6c\alpha T_1' ctg\,\theta\left(\frac{\theta\sqrt{5}}{\sin^4\theta} - \frac{1}{\cos\theta}\right)}{\frac{1296\,c^8}{R_0^2\sin^4\theta} - \frac{1}{\cos^8\theta}}\Big\} - \frac{\alpha T_1'}{\cos^2\theta}, \qquad (2.4)_8$$

The prime over the Thomson functions designates their derivative with respect to θ; D_1, D_2, D_3 -- constants to be determined.

3. Determination of the Constants α_i, c_i (i = 1, 2, 3, 4) and D_j (j = 1, 2, 3)

The internal contour ($\theta = \theta_0$) of the "lobe" is rigidly fastened, and the external contour is free. Therefore, we have the following conditions on their contours:

For determining the constants α_i:

$$\left.\begin{aligned} &\varepsilon_{2(0)} = \vartheta_0 = 0 \quad \text{at} \quad \theta = \theta_0\,, \\ &T_{1(0)} = M_{1(0)} = 0 \quad \text{at} \quad \theta = \theta_1\,. \end{aligned}\right\} \qquad (3.1)$$

To determine the constants α_i: /10

$$\left.\begin{aligned} &\varepsilon_{2(1)} = \Psi = 0 \quad \text{at} \quad \theta = \theta_0\,, \\ &T_{1(1)} = M_{1(1)} = 0 \quad \text{at} \quad \theta = \theta_1\,. \end{aligned}\right\} \qquad (3.2)$$

where [3]:

$$\left.\begin{aligned} &\varepsilon_{2(0)} = \frac{1}{Eh}\left(\frac{1}{R_1}\frac{dV_0}{d\theta} - \nu\frac{ctg\,\theta}{R_2}V_0\right) + \alpha T_0', \\ &T_{1(0)} = \frac{ctg\,\theta}{R_2}V_0\,, \\ &M_{1(0)} = -\frac{Eh^3}{12(1-\nu^2)}\left(\frac{1}{R_1}\frac{d\vartheta_0}{d\theta} + \nu\frac{ctg\,\theta}{R_2}\vartheta_0\right), \end{aligned}\right\} \qquad (3.3)$$

$$\varepsilon_{2(1)} = \frac{1}{Eh}\left[\frac{1}{R_1}\frac{dV}{d\theta} + (1-\nu)\frac{V\cos\theta}{R_2\sin\theta}\right] - \frac{h^2}{12R_2^2}\frac{\Psi\cos\theta}{\sin\theta} + \alpha T_1', \qquad (3.4)$$

$$\left.\begin{aligned} &T_{1(1)} = \frac{V\cos\theta}{R_2\sin\theta} - \frac{D}{R_2}\left[\frac{1}{R_1}\frac{d\Psi}{d\theta} + (1+\nu)\frac{\Psi\cos\theta}{R_2\sin\theta}\right] - \frac{Eh^2\alpha T_1'}{12(1-\nu)R_2^2}, \\ &M_{1(1)} = D\left[\frac{1}{R_1}\frac{d\Psi}{d\theta} + (1+\nu)\frac{\Psi\cos\theta}{R_2\sin\theta} + \frac{(1-\nu^2)V\cos\theta}{EhR_2^2\sin\theta} + (1+\nu)\alpha\frac{T_1'}{R_2}\right] \end{aligned}\right\} \qquad (3.5)$$

$$D = \frac{Eh^3}{12(1-\nu^2)}.$$

Substituting (3.3) into (3.1) with allowance for (1.20), and substituting (3.4) into (3.2) with allowance for (1.21), we obtain:

a) the system of equations for α_i:

$$\left.\begin{aligned} &A_{11}a_1 - A_{12}a_2 + A_{13}a_3 - A_{14}a_4 = A_0, \\ &A_{21}a_1 + A_{22}a_2 + A_{23}a_3 + A_{24}a_4 = 0, \\ &A_{31}a_1 + A_{32}a_2 + A_{33}a_3 + A_{34}a_4 = 0, \\ &A_{41}a_1 - A_{42}a_2 + A_{43}a_3 - A_{44}a_4 = 0, \end{aligned}\right\} \qquad (3.6)$$

b) the system of equations for c_i:

$$\left.\begin{aligned} &B_{11}c_1 + B_{12}c_2 - B_{13}c_3 - B_{14}c_4 = B_0, \\ &B_{21}c_1 + B_{22}c_2 + B_{23}c_3 + B_{24}c_4 = B_1, \\ &B_{31}c_1 + B_{32}c_2 + B_{33}c_3 + B_{34}c_4 = B_2, \\ &B_{41}c_1 + B_{42}c_2 + B_{43}c_3 + B_{44}c_4 = B_3, \end{aligned}\right\} \qquad (3.7)$$

where

$$\left.\begin{aligned} &A_{11} = \mathrm{ber}\,\lambda\theta_0,\ A_{12} = \mathrm{bei}\,\lambda\theta_0,\ A_{13} = \mathrm{ker}\,\lambda\theta_0,\ A_{14} = \mathrm{kei}\,\lambda\theta_0, \\ &A_{21} = -\frac{\nu\,\mathrm{ctg}\,\theta_0}{R_2^0}\,\mathrm{bei}\,\lambda\theta_0 - \frac{1}{R_1^0}\,\mathrm{bei}'\lambda\theta_0, \\ &A_{22} = -\frac{\nu\,\mathrm{ctg}\,\theta_0}{R_2^0}\,\mathrm{ber}\,\lambda\theta_0 - \frac{1}{R_1^0}\,\mathrm{ber}'\lambda\theta_0, \\ &A_{23} = -\frac{\nu\,\mathrm{ctg}\,\theta_0}{R_2^0}\,\mathrm{kei}\,\lambda\theta_0 - \frac{1}{R_1^0}\,\mathrm{kei}'\lambda\theta_0, \\ &A_{24} = -\frac{\nu\,\mathrm{ctg}\,\theta_0}{R_2^0}\,\mathrm{ker}\,\lambda\theta_0 - \frac{1}{R_1^0}\,\mathrm{ker}'\lambda\theta_0, \\ &A_{31} = \mathrm{bei}\,\lambda\theta_1,\ A_{32} = \mathrm{ber}\,\lambda\theta_1,\ A_{33} = \mathrm{kei}\,\lambda\theta_1,\ A_{34} = \mathrm{ker}\,\lambda\theta_1, \\ &A_{41} = \frac{\nu\,\mathrm{ctg}\,\theta_1}{R_2^1}\,\mathrm{ber}\,\lambda\theta_1 - \frac{1}{R_1^1}\,\mathrm{ber}'\lambda\theta_1, \\ &A_{42} = \frac{\nu\,\mathrm{ctg}\,\theta_1}{R_2^1}\,\mathrm{bei}\,\lambda\theta_1 - \frac{1}{R_1^1}\,\mathrm{bei}'\lambda\theta_1, \\ &A_{43} = \frac{\nu\,\mathrm{ctg}\,\theta_1}{R_2^1}\,\mathrm{ker}\,\lambda\theta_1 - \frac{1}{R_1^1}\,\mathrm{ker}'\lambda\theta_1, \\ &A_{44} = \frac{\nu\,\mathrm{ctg}\,\theta_1}{R_2^1}\,\mathrm{kei}\,\lambda\theta_1 - \frac{1}{R_1^1}\,\mathrm{kei}'\lambda\theta_1, \end{aligned}\right\} \qquad (3.8)$$

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$$A_0 = \frac{\alpha E h T_0'}{c\,\eta(\theta)}. \qquad (3.9)$$

$$\left.\begin{aligned} &B_{11} = \mathrm{bei}_6\,\lambda\theta_0,\ B_{12} = \mathrm{ber}_6\,\lambda\theta_0,\ B_{13} = \mathrm{kei}_6\,\lambda\theta_0,\ B_{14} = \mathrm{ker}_6\,\lambda\theta_0, \\ &B_{21} = \frac{c}{R_1^0}\,\mathrm{ber}_6\,\lambda\theta_0 + \frac{(1-\nu)c\,\mathrm{ctg}\,\theta_0}{R_2^0}\,\mathrm{ber}_6\,\lambda\theta_0 - \frac{h^2\,\mathrm{ctg}\,\theta_0}{12(R_2^0)^2}\,\mathrm{bei}_6\,\lambda\theta_0, \\ &B_{22} = -\frac{c}{R_1^0}\,\mathrm{bei}_6\,\lambda\theta_0 - \frac{c(1-\nu)\,\mathrm{ctg}\,\theta_0}{R_2^0}\,\mathrm{bei}_6\,\lambda\theta_0 - \frac{h^2\,\mathrm{ctg}\,\theta_0}{12(R_2^0)^2}\,\mathrm{ber}_6\,\lambda\theta_0, \\ &B_{23} = -\frac{c}{R_1^0}\,\mathrm{ker}_6'\lambda\theta_0 - \frac{c(1-\nu)\,\mathrm{ctg}\,\theta_0}{R_2^0}\,\mathrm{ker}_6\,\lambda\theta_0 + \frac{h^2\,\mathrm{ctg}\,\theta_0}{12(R_2^0)^2}\,\mathrm{kei}_6\,\lambda\theta_0, \\ &B_{24} = \frac{c}{R_1^0}\,\mathrm{kei}_6'\lambda\theta_0 + \frac{c(1-\nu)\,\mathrm{ctg}\,\theta_0}{R_2^0}\,\mathrm{kei}_6\,\lambda\theta_0 + \frac{h^2\,\mathrm{ctg}\,\theta_0}{12(R_2^0)^2}\,\mathrm{ker}_6\,\lambda\theta_0. \\ &B_{31} = \frac{c\,\mathrm{ctg}\,\theta_1}{R_2^1}\,\mathrm{ber}_6\,\lambda\theta_1 - \frac{h^2}{12(1-\nu^2)R_1^1R_2^1}\,\mathrm{bei}_6\,\lambda\theta_1 - \frac{(1+\nu)h^2\,\mathrm{ctg}\,\theta_1}{12(1-\nu^2)(R_2^1)^2}\,\mathrm{bei}_6\,\lambda\theta_1, \end{aligned}\right\} \qquad (3.10)$$

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$$B_{32}=-\frac{c\,ctg\,\theta_1}{R_2^1}\,bei_6\lambda\theta_1-\frac{h^2}{12(1-\nu^2)R_1^1R_2^1}\,ber_6\lambda\theta_1-\frac{(1+\nu)h^2\,ctg\,\theta_1}{\sqrt{12(1-\nu^2)}(R_2^1)^2}\,ber_6\lambda\theta_1,$$

$$B_{33}=-\frac{c\,ctg\,\theta_1}{R_2^1}\,ker_6\lambda\theta_1-\frac{h^2}{12(1-\nu^2)R_1^1R_2^1}\,kei_6'\lambda\theta_1+\frac{(1+\nu)h^2\,ctg\,\theta_1}{\sqrt{12(1-\nu^2)}(R_2^1)^2}\,kei_6\lambda\theta_1,$$

$$B_{34}=\frac{c\,ctg\,\theta_1}{R_2^1}\,kei_6\lambda\theta_1-\frac{h^2}{12(1-\nu^2)R_1^1R_2^1}\,ker_6\lambda\theta_1+\frac{(1+\nu)h^2\,ctg\,\theta_1}{\sqrt{12(1-\nu^2)}(R_2^1)^2}\,ker_6\lambda\theta_1,$$

$$B_{41}=\frac{\nu}{R_1^1}\,bei_6'\lambda\theta_1+\frac{(1+\nu)\,ctg\,\theta_1}{R_2^1}\,bei_6\lambda\theta_1+\frac{(1-\nu^2)c}{R_1^1R_2^1}\,ber_6'\lambda\theta_1+\frac{(1-\nu^2)c\cdot ctg\,\theta_1}{(R_2^1)^2}\,ber_6\lambda\theta_1,$$

$$B_{42}=\frac{\nu}{R_1^1}\,ber_6'\lambda\theta_1+\frac{(1+\nu)\,ctg\,\theta_1}{R_2^1}\,ber_6\lambda\theta_1-\frac{(1-\nu^2)c}{R_1^1R_2^1}\,bei_6'\lambda\theta_1-\frac{(1-\nu^2)c\,ctg\,\theta_1}{(R_2^1)^2}\,bei_6\lambda\theta_1,$$

$$B_{43}=-\frac{\nu}{R_1^1}\,kei_6\lambda\theta_1-\frac{(1+\nu)\,ctg\,\theta_1}{R_2^1}\,kei_6\lambda\theta_1-\frac{(1-\nu^2)c}{R_1^1R_2^1}\,ker_6'\lambda\theta_1-\frac{(1-\nu^2)c\,ctg\,\theta_1}{(R_2^1)^2}\,ker_6\lambda\theta_1,$$

$$B_{44}=-\frac{\nu}{R_1^1}\,ker_6\lambda\theta_1-\frac{(1+\nu)\,ctg\,\theta_1}{R_2^1}\,ker_6\lambda\theta_1+\frac{(1-\nu^2)c}{R_1^1R_2^1}\,kei_6'\lambda\theta_1+\frac{(1-\nu^2)c\,ctg\,\theta_1}{(R_2^1)^2}\,kei_6\lambda\theta_1,$$

$$B_0=-\frac{6Ehc\alpha T_1'\left(\frac{8\sqrt{3}}{\sin^4\theta_0}+\frac{1}{\cos\theta_0}\right)}{R_0\eta_1(\theta_0)\left(\frac{1295\,c^2}{R_0^2\sin^4\theta_0}-\frac{1}{\cos^2\theta_0}\right)},$$

$$B_1=\frac{Eh\alpha T_1'}{\eta(\theta_0)}\left[\frac{h\cos\theta_0}{3R_1^0\sqrt{1-\nu^2}}+\frac{2(1-\nu)h\cdot ctg\,\theta_0}{3R_2^0\sqrt{1-\nu^2}\sin^2\theta_0}+\right.$$

$$\left.+\frac{ch^2\left(\frac{8\sqrt{3}}{\sin^4\theta_0}+\frac{1}{\cos\theta_0}\right)ctg\,\theta_0}{2R_0(R_2^0)^2\left(\frac{1295\,c^2}{R_0^2\sin^4\theta_0}-\frac{1}{\cos^2\theta_0}\right)}-1\right], \qquad (3.11)$$

$$B_2=\frac{Eh^3\alpha T_1}{12(1-\nu)(R_2^1)^2\eta(\theta_1)}\times$$

$$\times\left[-\frac{8(1-\nu)\sqrt{3}\,ctg\,\theta_1\cdot c\left(\frac{\sqrt{3}}{3\cos\theta_1}-\frac{24c^2}{R_0^2}\right)R_2^1}{\sqrt{1-\nu^2}\left(\frac{1296\,c^2}{R_0^2\sin^2\theta_1}-tg^2\theta_1\right)h}+\frac{\sqrt{3}R_0}{27c}-1\right],$$

$$B_3=\frac{(1+\nu)Eh\alpha T_1'}{R_2^1\,\eta(\theta_1)}\cdot\left[-\frac{6c\cdot ctg\,\theta_1\,\frac{8\sqrt{3}}{\sin^4\theta_1}+\frac{1}{\cos\theta_1}}{\frac{1295\cdot c^2}{R_0^2\sin^4\theta_1}-\frac{1}{\cos^2\theta_1}}-\right.$$

$$\left.-\frac{c\sqrt{1-\nu^2}\,h\cos^2\theta_1}{3(1+\nu)R_1^1}+\frac{2c\sqrt{1-\nu^2}\cdot h\,ctg\,\theta_1\cos\theta_1}{3R_2^1\sin^2\theta_1}-1\right],$$

$$R_1^0 = \frac{R_0}{\cos^3\theta_0}, \qquad R_2^0 = \frac{R_0}{\cos\theta_0}, \qquad R_1^1 = \frac{R_0}{\cos^3\theta_1}, \qquad R_2^1 = \frac{R_0}{\cos\theta_1}. \tag{3.12}$$

We may determine the constants D_j from the conditions on the internal fixed contour of the "lobe":

$$U_{2(0)} = 0 \quad \text{at} \quad \theta = \theta_0, \tag{3.13}$$

$$U_{2(1)} = U_{\rho(1)} = 0 \quad \text{at} \quad \theta = \theta_1. \tag{3.14}$$

These conditions give:

$$\left.\begin{aligned} D_1 &= 0, \\ D_2 &= R_2^0 \sin\theta \cdot \varepsilon_{2(0)}^0, \\ D_3 &= (\Psi)_0, \end{aligned}\right\} \tag{3.15}$$

where

$$(\Psi)_0 = \frac{\eta(\theta_0)}{Eh}\left(c_1 \,\mathrm{bei}_6 \lambda\theta_0 + c_2 \,\mathrm{ber}_6 \lambda\theta_0 - c_3 \,\mathrm{kei}_6 \lambda\theta_0 - c_4 \,\mathrm{ker}_6 \lambda\theta_0\right) + \frac{6c\alpha T_1\left[\dfrac{8\sqrt{3}}{\sin^4\theta_0} + \dfrac{1}{\cos\theta_0}\right]}{R_0\left[\dfrac{1296c^2}{R_0^2\sin^4\theta_0} - \dfrac{1}{\cos^3\theta_0}\right]},$$ /14

$$\left.\begin{aligned} \varepsilon_{2(0)}^0 &= \frac{1}{Eh}\left[\frac{1}{R_1^0}\left(\frac{dV}{d\theta}\right)_0 + (1-\nu)\frac{(V)_0 \,\mathrm{ctg}\,\theta_0}{R_2^0} - \frac{h^2 \,\mathrm{ctg}\,\theta_0}{12(R_2^0)^2}(\Psi)_0\right] + \alpha T_1', \\ (V)_0 &= c\eta(\theta_0)\left(-c_2 \,\mathrm{bei}_6 \lambda\theta_0 + c_1 \,\mathrm{ber}_6 \lambda\theta_0 + c_4 \,\mathrm{kei}_6 \lambda\theta_0 - c_3 \,\mathrm{ker}_6 \lambda\theta_0\right) + \frac{2Eh^2\alpha T_1'\left[\dfrac{\sqrt{3}}{3\cos\theta_0} - \dfrac{54c^2}{R_0^2}\right]}{\sqrt{3(1-\nu^2)}\left[\dfrac{1296c^2}{R_0^2\sin^2\theta_0} - \mathrm{tg}^2\theta_0\right]}, \\ \left(\frac{dV}{d\theta}\right)_0 &= c\eta(\theta_0)\left(-c_2 \,\mathrm{bei}_6' \lambda\theta_0 + c_1 \,\mathrm{ber}_6' \lambda\theta_0 + c_4 \,\mathrm{kei}_6' \lambda\theta_0 - c_3 \,\mathrm{ker}_6' \lambda\theta_0\right) - \frac{Eh^2\alpha T_1' \cos\theta_0}{3\sqrt{1-\nu^2}}. \end{aligned}\right\} \tag{3.16}$$

Thus, the displacements of the "lobes" are completely determined analytically.

4. Numerical Results

The formulas (2.1) with allowance for (2.2) and (2.3) were used to calculate the value of the displacement of the "lobes" of the reflecting surface (mirror) of an antenna as a function of the angle θ and φ for the temperature field (1.8), which were made in accordance with the given field (1.9) for each calculated value of φ. The following initial data were assumed for a mirror made of the material AMg6-M,

$$E = 7{,}2\cdot 10^3 \ (\mathrm{kg/mm^2}), \qquad \alpha = 23{,}8\cdot 10^{-6} \quad (1/\mathrm{degree})$$

$$\nu = 0{,}33; \qquad h = 2{,}5 \ (\mathrm{mm}); \qquad R_0 = 1550\sqrt{3}\ (\mathrm{mm});$$

$$\theta_0 = 5^{\circ}20', \qquad \theta_1 = 30^{\circ}.$$

The calculation results for the normal displacement W of the "lobes", whose location with respect to the origin of the angle φ is shown in Fig. 4, are given in the table.

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TABLE /16

Displacement	θ = 5°20'	θ = I0°	θ = I5°	θ = 20°	θ = 25°	θ = 30°	φ	"Lobe" Number
W (мм)	0	0,09	0,25	0,45	0,72	I,I5	I5°	I, XI
	0	0,09	0,23	0,42	0,66	I,09	25°	
	0	0,07	0,20	0,38	0,60	I,0I	35°	
	0	0,05	0,I5	0,28	0,44	0,82	45°	
	0	0,05	0,I5	0,28	0,44	0,82	45°	II, X
	0	0,03	0,09	0,I5	0,24	0,49	55°	
	0	-0,03	-0,09	-0,I4	-0,26	-0,50	65°	
	0	-0,06	-0,I6	-0,30	-0,45	-0,82	75°	
	0	-0,06	-0,I6	-0,30	-0,45	-0,82	75°	III, IX
	0	-0,08	-0,22	-0,37	-0,6I	-I,08	85°	
	0	-0,II	-0,26	-0,48	-0,77	-I,30	95°	
	0	-0,I3	-0,32	-0,58	-0,92	-I,50	I05°	
	0	-0,I3	-0,32	-0,58	-0,92	-I,50	I05°	IV, VIII
	0	-0,I6	-0,37	-0,68	-I,07	-I,68	II5°	
	0	-0,I8	-0,42	-0,77	-I,I8	-I,85	I25°	
	0	-0,2I	-0,47	-0,84	-I,30	-2,00	I35°	

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Displacement	θ = 5°20	θ = I0°	θ = I5°	θ = 20°	θ = 25°	θ = 30°	φ	"Lobe" Number
W (мм)	0	-0,2I	-0,47	-0,84	-I,30	-2,00	I35°	V, VII
	0	-0,23	-0,5I	-0,92	-I,39	-2,08	I45°	
	0	-0,24	-0,55	-0,97	-I,46	-2,27	I55°	
	0	-0,26	-0,60	-I,0I	-I,5I	-2,23	I65°	
	0	-0,26	-0,60	-I,0I	-I,5I	-2,23	I65°	VI
	0	-0,27	-0,64	-I,05	-I,55	-2,27	I75°	
	0	-0,27	-0,65	-I,06	-I,58	-2,30	I80°	
	0	-0,27	-0,64	-I,05	-I,55	-2,27	I85°	
	0	-0,26	-0,60	-I,0I	-I,5I	-2,23	I95°	
	0	0,09	0,25	0,45	0,72	0,I3	345°	XII
	0	0,I2	0,28	0,49	0,76	I,20	355°	
	0	0,I3	0,30	0,5I	0,79	I,24	0°	
	0	0,I2	0,28	0,49	0,76	I,20	5°	
	0	0,09	0,25	0,45	0,72	I,I3	I5°	

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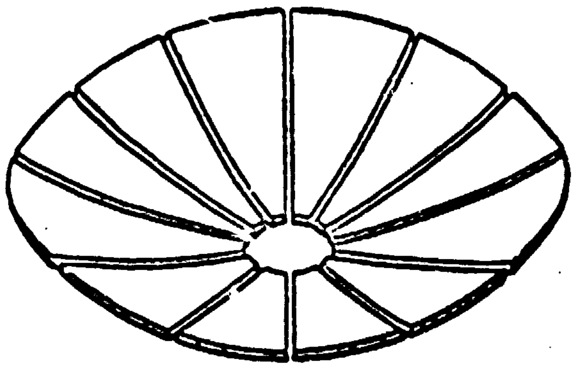

Fig. 1. General form of the antenna mirror

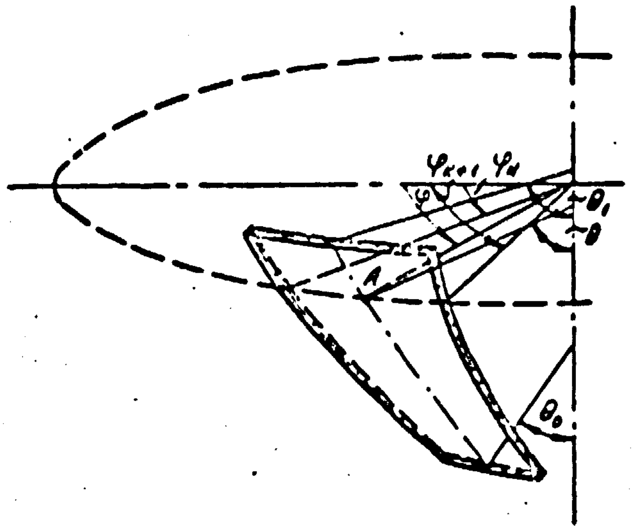

Fig. 2. Coordinates of an arbitrary point on the middle surface of the "lobe."

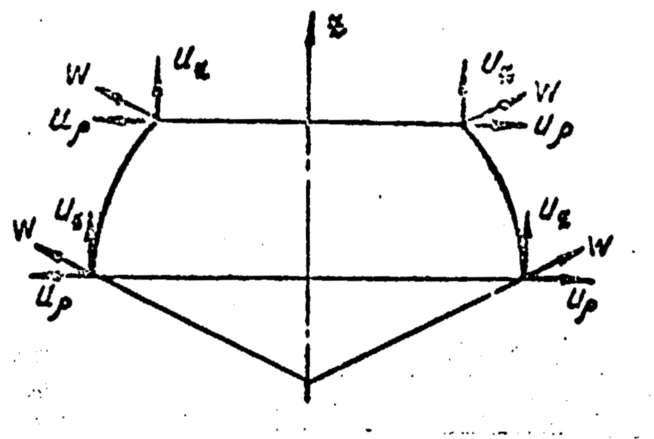

Fig. 3. Displacement of points on the middle surface of the "lobe."

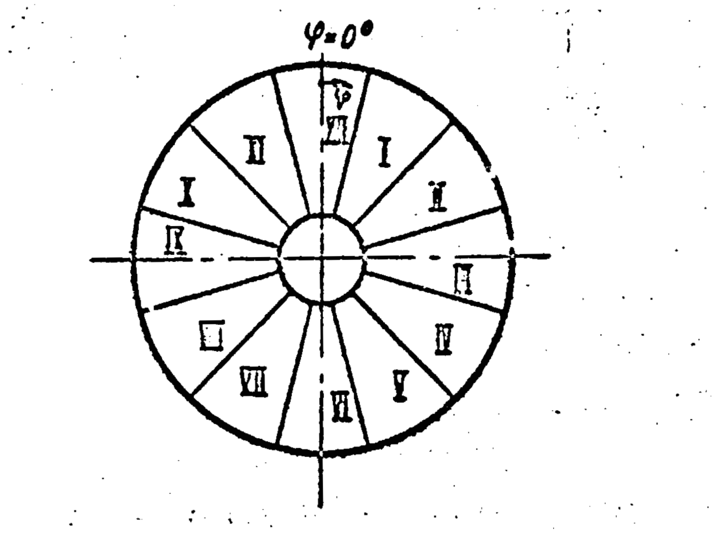

Fig. 4. Location of "lobes" with respect to the origin $\varphi = 0$