0

NASA TECHNICAL MEMORANDUM NASA TM-75666

ESTABLISHMENT OF THE SPECTRA OF KINETIC TURBULENCE

by

L. A. Bol'shov, A. M. Dykhne, V. P. Kiselev
and A. Kh. Pergament

Translation of: Ustanovleniye Spektrov Kineticheskoy Turbulentnosti, Academy of Sciences USSR, Moscow, Institute of Applied Mathematics imeni M. V. Keldysh, Preprint No. 31, 1979, pp. 1-30

(NASA-TM-75666) ESTABLISHMENT OF THE SPECTRA OF KINETIC TURBULENCE (National Aeronautics and Space Administration) 28 p
HC A03/MF A01 CSCL 20I

N80-10914

Unclas
G3/75 45938

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 AUGUST, 1979

TABLE OF CONTENTS

ANNOTATION

An analysis of kinetic equations which describe, in particular, the establishment of the spectra of Langmuir turbulence is presented. For the analytical nuclei and linear increment, a stationary spectrum, if it exists, is concentrated on a variety of dimensionality, the smallest dimensionality of the initial problem at points, on closed lines or surfaces. In the case where stationary distribution consists of many peaks, with a sharp inclusion of the increment of pumping in the system, secondary turbulence occurs. The position of peaks is established but their amplitudes complete undamped (in the absence of noise) oscillations. It is pointed out that establishing spectra can occur only during adiabatic inclusion of pumping. Then, one realizes a quasi stationary condition. It is significant that the adiabatic condition here is more rigid than the ordinary ($\gamma t >> 1$, γ -- the rate of pumping of oscillations, t -- the time of inclusion of instability) by several hundred times.

ESTABLISHMENT OF THE SPECTRA OF KINETIC TURBULENCE

by

L. A. Bol'shov, A. M. Dykhne, V. P. Kiselev
and A. Kh. Pergament

1. Introduction /5*

In a number of physical objects described in an approximation of a solid medium, with a fairly high level of excitation, complex motion occurs which is described in the language of mutually interacting degrees of wave freedom: the Langmuir turbulence in plasma, acoustic turbulence, noise in amplified media, spin waves in magnets, etc. Motion of this type, with not too great intensity of turbulent noise, can be described approximately as weak turbulence, as a kinetic equation for interacting waves with random phases. Interaction of the waves with nondecay by the principle of dispersion is accomplished in the lowest magnitude according to intensity of the wave by induced scattering on other exicitations (in the case of Langmuir turbulence in an isothermal plasma, scattering of the waves occurs basically in ions).

A kinetic equation for the number of occupations $n(\vec{k},t)$ is well known [1,2]

$$\dot{n}_k = 2\gamma_k n_k + n_k \int A_{kk'} n(\vec{k}',t)\, d^3k + \varepsilon_k \qquad (1)$$

here γ_k is an increment (or decrement) of surging of waves in a linear approximation, $A_{kk'}$ is a nucleus, which describes the process of the scattering; ε_k is the intensity of the source

*Numbers in the margin indicate the pagination in the foreign text.

caused by thermal noise and induction processes of a higher order.

In situations which are of interest from a point of view of heating plasma with an electric beam, or by electromagnetic radiation, the intensity of excited waves is much larger than the level of thermal noise. The latter added to (1) then is considerably smaller than the other members. There is interest in the study of the spectra of turbulence in time periods smaller than $\sim n/\varepsilon_k$. The behavior of spectra for large time periods is considered in references [3,6]. /6

The question of establishing the spectrum of Langmuir turbulence of plasma was studied within the framework of equation (1) in a number of works [3-6]. However, the study was incomplete [3], or contained a number of erroneous conclusions [5].

We wish to approach the study of the spectrum from both positions, with the possibility of not using the specifics of Langmuir turbulence, expressed in the actual form of the functions γ_k and $A_{kk'}$. We will try to answer the following questions:

1) What are the possible types of stationary spectra?

2) What are the characteristic traits of the spectra in the last stages of evolution?

3) Which can be nonstationary spectra and what are the causes for their establishment, or nonestablishment?

2. Stationary Spectra.

Stationary spectra, if such exist, satisfy the equation

$$n_k \Gamma_k = 0 \qquad (2)$$

Here the nonlinear increment

$$\Gamma_k = \gamma_k + \int A_{kk'} n_{k'} d^3k \qquad (3)$$

It is possible to solve equation (2) of three types.

(a) $n_k \neq 0$ with all $\vec{k}$ (solid spectrum).

(b) $n_k \neq 0$ in certain fields $\vec{k}$ -- space.

(c) $n_k \neq 0$ for multiple small dimensions (on surfaces, on curves and at separate points $\vec{k}$ -- space). We will consider these possibilities in more detail.

(a) Although examples exist in which there is the pos- /7 sibility (a) (for example [5]), this possibility must be designated as exclusive. Moreover, in this case when all $\vec{k}$ it must satisfy the integral equation $\Gamma_k = 0$. The solution of this equation, generally speaking, has a variable sign. The natural physical requirement for the positive state of n_k can be satisfied only with a special coordinated selection of the function γ_k and $A_{kk'}$.

(b) In this case, there must be $\Gamma_k = 0$ in fields where the spectrum is concentrated (that is, $n_k \neq 0$) and $\Gamma_k < 0$ (for stability of a given spectrum) in all other regions of $\vec{k}$-space. We note that the function Γ_k which satisfies this condition is an insignificant function of its independent variable $\vec{k}$. It is easy to see that any non-analyticity and even break in the function $n(\vec{k}')$ introduced into Γ_k cannot result in the non-analyticity of Γ_k (inasmuch as $n(\vec{k}')$ is integrated). Thus, the single cause of non-analyticity of Γ_k can be non-analyticity of the nucleus $A_{kk'}$ or the linear increment γ_k. The "striped" spectrum can be stabilized if $A_{kk'}$ or γ_k have characteristics as functions of their independent variables.

A model example with a nucleus can illustrate such a spectrum

$$A_{kk'} = \begin{cases} 1 \text{ when } k > k' \\ -1 \text{ when } k < k' \end{cases} \qquad (4)$$

If $\gamma(k)$ has one maximum at point $k=k_0$ and $\gamma(k_0)=-\dot{\gamma}(+\infty)$, then a spectrum is realized concentrated on semistraight

$$n_k = \begin{cases} -\frac{1}{2}\gamma'(k) \text{ when } k > k_0 \\ 0 \text{ when } k < k_0 \end{cases} \qquad (5)$$ /8

The peculiarity in n_k when $k=k_0$ involves the peculiarity of the nucleus.

It is interesting to note that for a nucleus which describes the scattering of the Langmuir waves in ions, with the use of which the Langmuir turbulence was studied in references [3-6,8], the $A_{kk'}$ nucleus can be both analytical (if the spectrum is concentrated in the $k \gg k_D\sqrt{m/M}$ field, in this case scattering is significant at large angles), and nonanalytical (if the spectrum is localized in the field of $k \ll k_D\sqrt{m/M}$, where small angle scattering of the plasmons plays a significant role).

(c) In the case of analytical functions $A_{kk'}$ and γ_k the most typical is the third variation in which the spectrum is concentrated to a multiplicity of dimensionality, the smallest dimensionality of the initial problem at points, on lines or surfaces. The existing lines or surfaces inevitably must be closed because in the opposite case the peculiarity of Γ_k is at the terminal points (lines) of the corresponding multiplicity.

Thus, the spectrum cannot be concentrated on sections of the beams (notwithstanding the statement contained in reference [5]). In the variation of the Γ_k function considered, on the aforementioned manifolds it has a maximum equal to zero and negative in all other space. In reference [5] the regular or singular character of the spectrum involves the width of the γ_k increment in relation to the width of the $A_{kk'}$ nucleus. As is apparent from what has been presented above, the character of the spectrum is defined exclusively by the analytical properties of the $A_{kk'}$ nucleus and the linear increment of γ_k.

During an analysis of establishing the spectrum of type /9 (c) in a case of Langmuir turbulence, it is necessary to keep in mind that with adequate constriction of the peaks they can become nonstationary as to modulation [7]. This circumstance limits the use of the picture considered here to large time periods.

Later on we will consider only the (c) type spectrum. When studying later stages of evolution of the spectra of this type, for simplicity, we will limit ourselves to a unidimensional case where the spectrum will be localized at N points $x_i (i=1,\ldots N)$

$$n(x) = \sum_{i=1}^{N} N_i \, \delta(x - x_i) \tag{6}$$

Then only when $x=x_i (i=1,\ldots N)$ $\Gamma(x)=0$ and from requirements for stability of the given spectrum $\Gamma'(x_i)=0$, and $\Gamma(x)<0$ at all remaining points. For determining a given spectrum it is adequate to solve the system:

$$\begin{cases} \gamma(x_i) + \sum_{k=1}^{N} A(x_i - x_k) N_k = 0 \\ \gamma'(x_i) + \sum_{k=1}^{N} A'(x_i - x_k) N_k = 0 \end{cases} \tag{7}$$

The peculiarity of system (7) involves the fact that not only the position of the peaks of x_i and their amplitude N_i are unknown but also the number of peaks N [illegible]. With weak instability (when the γ increment is small and positive only in a narrow field according to x) the distribution established consists only of one peak. As the increment increases the stability condition breaks down ($\Gamma(x)<0$ everywhere except at one point) and at this point where this occurs, a second peak of the established distribution occurs. With a further increase in γ, one can find a third peak or even more.

A numerical determination was made of this stationary distribution for the nucleus /10

$$\sigma(x) = x \exp(-x^2/2) \tag{8}$$

(this nucleus makes the transition in the direction of large x) and two model increments $\gamma(x)$

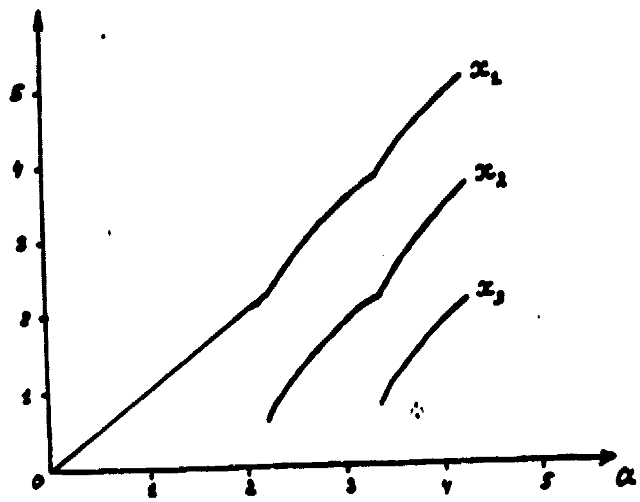
$$\begin{aligned} &1)\quad \gamma(x) = x(a-x) \qquad a>0 \\ &2)\quad \gamma(x) = 1/(1+x^2) - 1/B \qquad B>1 \end{aligned} \tag{9}$$

Stationary distribution consists of one peak when $0<a<2.21$ and $1<B<1.314$. When $2.21\leq a<3.3$ and $1.314\leq B<1.724$, the established spectrum contains two peaks. When $a=3.3$ and $B=1.724$, a third peak is apparent and so forth. The coordinates and amplitudes of the first three peaks in relationship to a and B are presented in Figures 1-2.

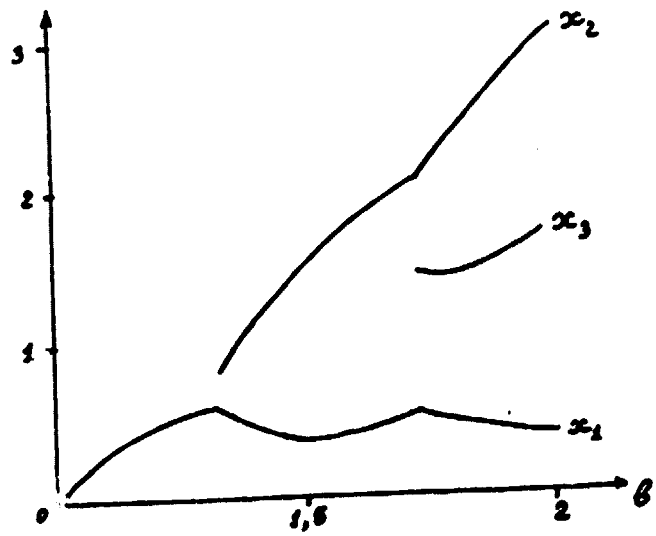
3. Evolution of the Structure of a Separate Peak.

Now we will study the process of the tendency toward stationary distribution of type (c). Here one can isolate

/22



1a

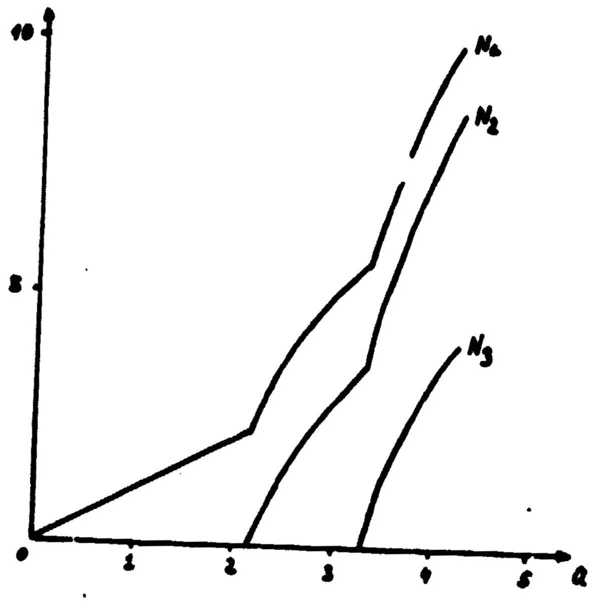


b

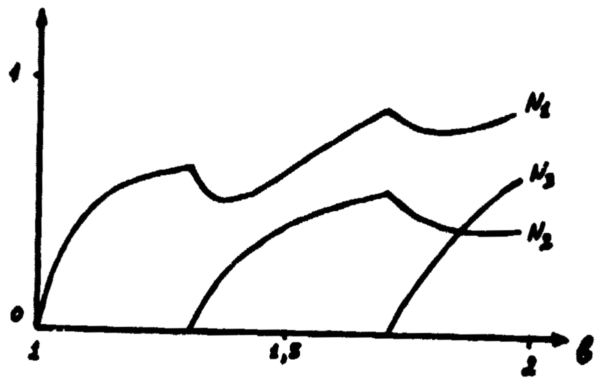
Figure 1. The relationship of coordinates of peaks of stationary distribution to the level of pumping with model increments /20

a) $\gamma(x)=x(a-x)$
b) $\gamma(x)=1/(1+x^2)-1B$

/23



a



b

Figure 2. Relationships of the amplitude of peaks of stationary distribution to the level of pumping with model increments. /20

a) $\gamma(x)=x(a-x)$
b) $\gamma(x)-1/(1+x^2)-1/B$

two questions. The first of these questions is on the structure of a separate peak in the later stages of evolution and the second question is on the behavior of the spectrum as a whole. When studying the structure of a separate peak, for simplicity, we will limit presentation to a case where stationary distribution contains a total of one peak. This situation occurs with a fairly weak instability when the increment γ (x) is small and positive in the narrow field of x. In this case, the stationary condition consists of one peak located at a point where $\gamma(x)=0$. For large time values, the distribution of n(x) which tends toward a δ-form, becomes fairly narrow. /11 This makes it possible to expand the G(x-x') nucleus to an integral and also the increment $\gamma(x)$ is close to the stationary position of the peak. For the Gaussian nucleus

$$G(x-x') = (x-x')\exp(-(x-x')^2/2) \qquad (10)$$

$$G(x) = x - x^3/2$$
$$\gamma(x) = -\alpha x - \beta x^2 \qquad (11)$$

The solution of equation (1) is written in the following form

$$n(x) = \frac{N_0}{\sqrt{2\pi}\, a} \exp\left(-\frac{(x-x_1)^2}{2a^2}\right) \qquad (12)$$

with calculation of (11), (12), equation (1) can be rewritten as

$$n(x)\left[\frac{\dot N_0}{N_0} - \frac{\dot a}{a} + \frac{(x-x_1)\dot x_1}{a^2} + \frac{(x-x_1)^2\dot a}{a^3}\right] = n(x)\Big[-\alpha x_1 - \beta x_1^2$$
$$+(x-x_1)(-\alpha-2\beta x_1) - \beta(x-x_1)^2 + N_0\Big((x-x_1)\Big(1-\frac{3a^2}{2}\Big) - \frac{N_0}{2}(x-x_1)^3\Big)\Big] \qquad (13)$$

Multiplying (13) by 1, $(x-x_1)$, $(x-x_1)^2$ and integrating by x we obtain

$$\dot N_0 = (\alpha x_1 - \beta(x_1^2 + a^2))\, N_0 \qquad (14)$$

$$\dot{x}_1 = \alpha a^2 - 2\beta x_1 a^2 - N_0 a^2 + 3N_0 a^4 \tag{15}$$

$$\dot{a} = -\beta a^3 \tag{16}$$

The established distribution corresponds to

$$N_{0s} = \alpha \qquad x_{1s} = 0 \qquad a_s = 0 \tag{17}$$

From (16) we have

$$a^2 = (2\beta t)^{-1} \tag{18}$$

Considering that N_0 and x differ little from stationary values, let us assume $N_1 = N_0 - N_{0s}$ and leaving only linear members in (14) and (15) along N_1 and x_1 we have with a calculation of (18)

$$\dot{N}_1 = \left(\alpha x_1 - \beta x_1^2 - \frac{1}{2t}\right)\alpha \tag{19}$$

$$\dot{x}_1 = -\frac{N_1}{2\beta t} - \frac{x_1}{t} \tag{20}$$

By introducing $z = x_1 t$ (21) from (19), (20), we find /12

$$\ddot{z} + \frac{\alpha^2}{2\beta t} z - \frac{\alpha z^2}{2t^3} - \frac{\alpha}{4\beta t} = 0 \tag{22}$$

from which

$$z \sim t^{1/4} \sin\left(\sqrt{2\alpha^2 t/\beta}\right) \tag{23}$$

that is

$$x_1 \sim t^{-3/4} \sin\left(\sqrt{2\alpha^2 t/\beta}\right) \tag{24}$$

$$N_0 - N_{0s} \sim t^{-1/4} \cos\left(\sqrt{2\alpha^2 t/\beta}\right) \tag{25}$$

Thus, the amplitude of the peak and the position of its center contain oscillating additions which decay with time.

The oscillations of the position of the center of the packet ($\sim t^{-3/4}$) decay most rapidly. The width of the packet decays like $t^{-1/2}$ and finally oscillation of the height of the peak ($\sim t^{-1/4}$) decays most slowly.

A numerical study was made of the process of making a permanent establishment in this case. Equation (1) has the form

$$\dot{n}(x) = n(x)\left(\gamma(x) + \int (x-x')\exp(-(x-x')^2/2)\,n(x')\,dx'\right) + \varepsilon_k \qquad (26)$$

where $\gamma(x)=x(1-x)$. The level of thermal noise ε_k was selected as the initial level of n(x). (It appeared that the behavior of solving equation (26) hardly depends on the initial level of n(x).) Calculations were made with different noise levels (during observation of the condition $\varepsilon_k \ll \max(\gamma)$). We attempted to show the characteristics of the establishment process not depending on the noise level.

With numerical integration according to time of the kinetic equation (26) the known explicit system with a second magnitude of precision was used which for equation

$$\dot{y} = F(y,t) \qquad (27)$$

can be writeen in the following form /13

$$\tilde{y}_{t+\frac{\tau}{2}} = y_t + F(y_t,t)\frac{\tau}{2}$$
$$y_{t+\tau} = y_t + F\left(\tilde{y}_{t+\frac{\tau}{2}}, t+\frac{\tau}{2}\right)\tau \qquad (28)$$

The integral in the right section of (26) is calculated according to the Simpson method and then preliminarily calculated values of the nucleus for different values of x are used. We note that n(x) was calculated as changing in time only in a certain limited field of values of x within which the full increment $\Gamma(x)$ is deliberately negative for any values of t and where one assumes $n(x)=\varepsilon_k$.

The process of showing the solution for a single peak permanent establishment has the following character. At first, in the field of the maximum increment, a broad peak develops (on the order of the width of the increment) which, having reached a fairly large value (approximately $100\gamma max$) begins to shift toward the large values of x (by virtue of selecting the sign of the nucleus). Having overshot the position of permanent establishment, the peak is damped and shifts toward small values of x, and begins to increase once more when x is smaller than the stationary value but larger than x corresponding to the maximum increment. Evolution n(x) in time is presented in Figures 3-4. Establishment of the position of the peak occurs most rapidly (when $\varepsilon_k=0.01$ for $\gamma t \sim 100$, when $\varepsilon_k=0.001$ for $\gamma t \sim 200$). The character of the appearance of the position of the center of the packet on a permanent establishment is shown in Figure 5. After establishing the position of the center of the packet, decay of the peak occurs with a simultaneous increase in its height (see Figure 7). Establishing the area of the packet occurs more slowly. With precision less than 1%, the area $S=\int n(x)dx$ appears on the permanent establishment when $\varepsilon_k=0.05$ for time $\gamma t \sim 200$, and when $\varepsilon_k=0.01$ for $\gamma t \sim 600$. Thus, the rate of appearance of the area of the packet on the permanent establishment strongly depends on the noise level. The half-width of a distribution /14
n(x), calculated according to the formula

$$a = \frac{\int n(x)\,dx}{\sqrt{2\pi}\, n_{max}} \qquad (29)$$

/24

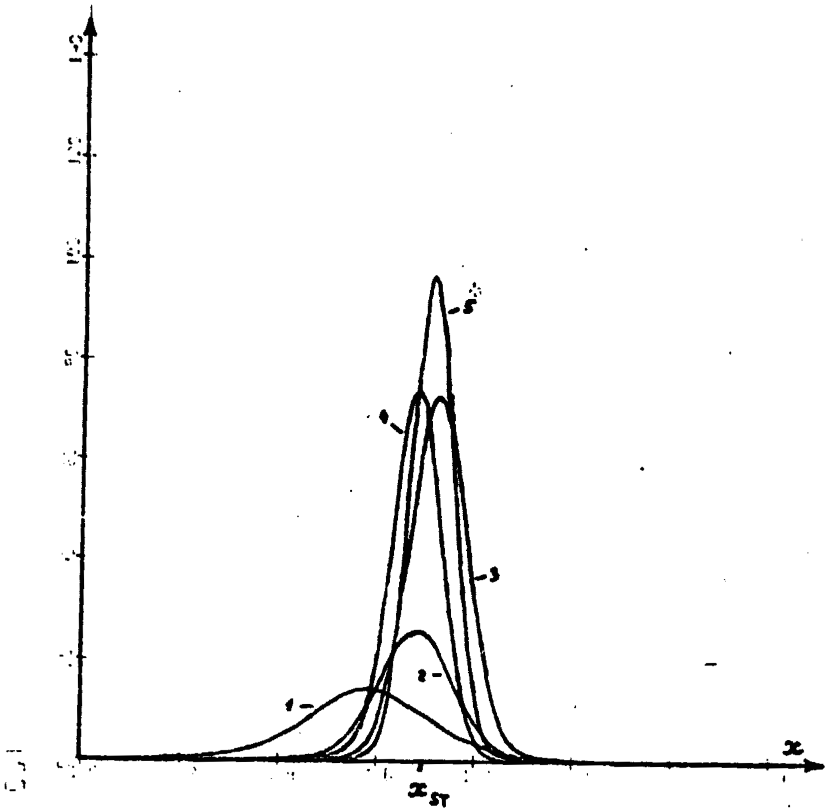


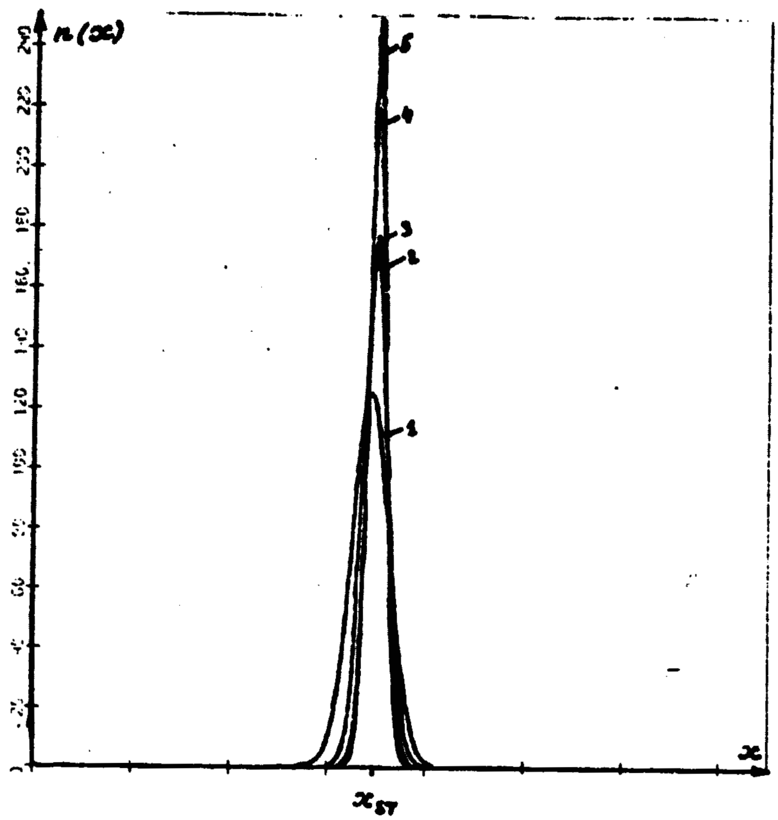
Figure 3. Evolution of n(x) for small time values. The curves correspond to times: 1-γt=15, 2-γt=30, 3-γt=45, 4-γt=60, 5-γt=75. /20

hardly depends on the level of noise and beginning with /14
$\gamma t \sim 50$ is subject to the law $t^{-1/2}$. (Figure 6 shows the relationship of $\ln 1/a$ to $\ln t$.)

One should note that the distribution of n(x) obtained numerically is essentially non-Gaussian. It is fairly symmetrical relative to the center, but its excess S1

$$S1 = \frac{\int (x-x_1)^4 n(x)\,dx \int n(x)\,dx}{\left(\int (x-x_1)^2 n(x)\,dx\right)^2} \quad (30)$$

appears to be several times larger than the excess for Gaussian



/25

Figure 4. Evolution of n(x) for large time values. The curves correspond to times: $1-\gamma t=100$, $2-\gamma t=200$, $3-\gamma t=300$, $4-\gamma t=400$, $5-\gamma t=500$. /20

distribution equal to three. Thus, the asymptotics of (18), (24), (25) catch only a qualitative character of the solution on permanent establishment. /14

It is significant that the area of the peak like the half-width is established slowly which, strictly speaking, causes doubt as to the existence in this case of the first stage of the process in which the energy balance is established between pumping and decay according to reference [8].

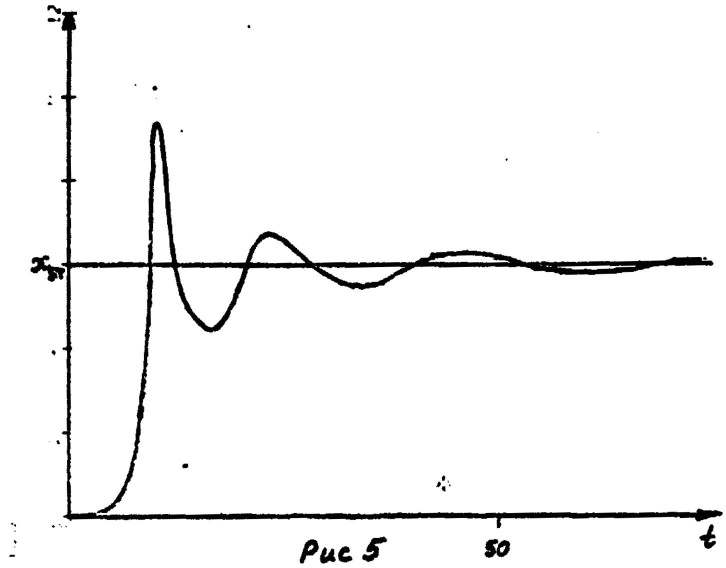


Figure 5. Appearance of the position of the center of the packet on a permanent establishment. /26 /20

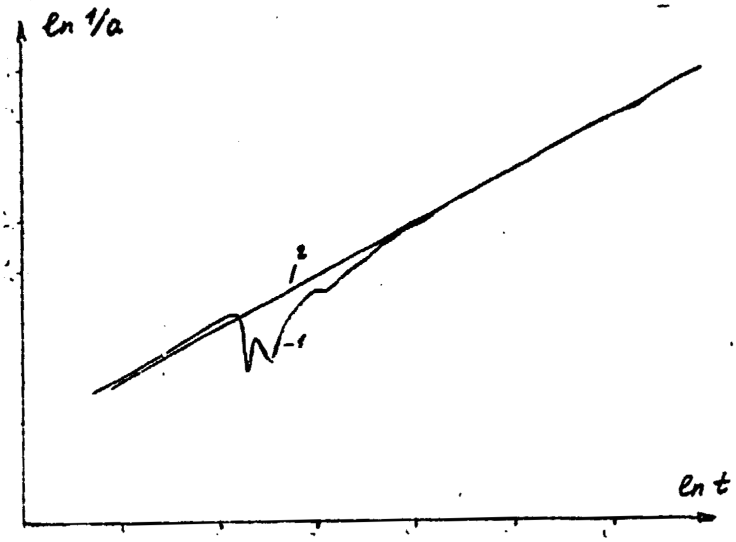
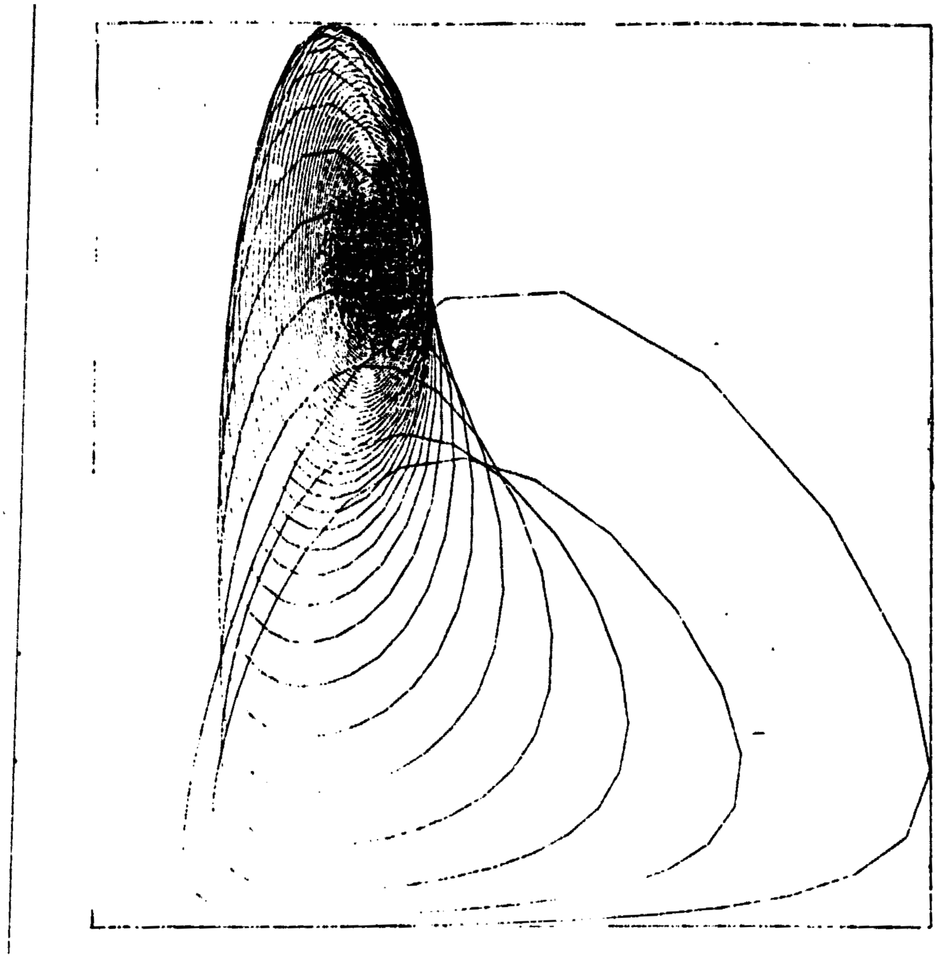


Figure 6. The relationship of the half-width of the peak to time in a logarithmic scale. (1 - calculated curve, 2 - a asymptotic.) /26 /20



/27

Figure 7. The typical phase curve of establishing a two-peak permanent establishment with abrupt inclusion of the increment. /20

4. Setting up a Multipeak Permanent Establishment. /14

We will study the character of establishment in this case where the distribution established consists of several peaks. If one assumes the existence of permanently established distribution, then one can describe nonstationary evolution of distribution for fairly large time periods. For this, we will assume that distribution of density can be prevented as a superposition equally depending on the time section $<n_k(t)>$ (which tends

toward stationary distribution) and the oscillation addition /15
$\tilde{n}_k(t)$. From formulas (2), (3) we find

$$\langle n_k(t)\rangle + \tilde{n}_k(t) = n_k(0)\, e^{\gamma_k t}\, e^{\int A_{kk'}\langle n_{k'}\rangle dk' dt}\, e^{\int_0^t A_{kk'}\tilde{n}_{k'} dk' dt} \quad (31)$$

The first three multipliers in the right part of expression (31) for large time intervals are a stationary distribution which, in accordance with what has been said above, consists of a series of δ-shaped peaks. Inasmuch as the fourth multiplier in (31) is limited in time (due to the oscillating character of $\tilde{n}_k$) we have when $t\to\infty$

$$n_k(t) \to n_{st_k} \exp\left(\int_0^t A_{kk'}\tilde{n}_{k'} dk' dt\right) \quad (32)$$

If a stationary solution exists, then even with the presence of an oscillation in the amplitudes of the peaks averaged for time, the value of n_k coincides with n_{st_k} which automatically is completed if

$$\left\langle \exp\left(\int_0^t A_{kk'}\tilde{n}_{k'} dk' dt\right)\right\rangle = 1 \quad (33)$$

For studying the general picture of establishing the spectrum for large time intervals, one can use the peak character $n_k(t)$. Because the position of the peaks is established most rapidly, we will assume that $n_k(t)$ consists of N oscillating peaks with amplitudes N_α, found at points corresponding to a stationary distribution. In this case, equation (1) is transformed to a system of nonlinear equations, the first of which is presented in reference [5]

$$\dot{N}_\alpha = N_\alpha \gamma_\alpha + \sum_{\beta=1}^{N} A_{\alpha\beta} N_\alpha N_\beta \quad (34)$$

Here N_α is the amplitude of a peak with the number α. In the case of a random number of peaks, the character of motion of the system close to the permanent establishment depends on the number of nondamped oscillations. To answer the question as to the number of nondamped oscillations in a general case, one mainly uses a linear theory. However, with a fairly large number of peaks, this is difficult to do in actual practice. An exception is the case of interaction of waves which describe an antisymmetrical matrix $\hat{A}$. From the fact that $Sp\hat{A}=0$ and $Re\lambda_1 \leq 0$ (a condition of stability of a stationary distribution, λ_1 -- the intrinsic number of $\hat{A}$) substantiation follows for all intrinsic frequencies of the system (absence of damping in all modes). These intrinsic nondamping modes can be excited in the system to a different level. For large numbers of such peaks this phenomenon is a singular secondary turbulence. It can be characterized by the functions of distribution according to the degree of excitation of these new modes. With a fairly high degree of excitation, the modes interact strongly and must be used by a system of nonlinear equations (31). The unusual character of this type of turbulence involves the fact that in a strongly perturbed distribution system, motion is realized with a finite number of degrees of freedom (on the order of the number of peaks). /16

The question as to excitation of secondary turbulence is very complex and requires a separate study. Here we will limit ourselves to separate qualitative expressions. Initial instability leading to primary turbulence (positive nature of γ at certain k) at the same time is included with a certain finite velocity. We will consider two separate cases: in the first of these the time of inclusion of the increment significantly increases the time for development of primary turbulence.

$$\frac{d}{dt}\frac{1}{\gamma} \ll 1 \tag{35}$$

In this case at any value of γ, the system succeeds in

becoming stationary and secondary turbulence does not develop. In the opposite extreme case of sudden inclusion of instab- /17
ility

$$\frac{d}{dt}\frac{1}{\gamma} \gg 1 \tag{36}$$

strongly developing secondary turbulence must occur.

We will study the simplest case where the matrix $A_{2\beta}$ in (34) is antisymmetrical and has a total of two peaks. Then we have

$$\begin{aligned} \dot{N}_1 &= -g N_1 (N_2 - N_{2s}) \\ \dot{N}_2 &= g N_2 (N_1 - N_{1s}) \end{aligned} \tag{37}$$

For this system

$$\frac{dN_1}{dN_2} = \frac{N_1 (N_{2s} - N_2)}{N_2 (N_1 - N_{1s})} \tag{38}$$

that is,

$$N_1 + N_2 - N_{1s} \ln N_1 - N_{2s} \ln N_2 = \mathrm{const} \tag{39}$$

The phase trajectories of the (37) system are closed lines with the center at a stationary position N_{1s}, N_{2s}. Thus, the system completes non-damping oscillation, not tending toward the permanent establishment. The amplitude of these oscillations depends on the initial conditions; then in the case of zero initial conditions, the span of oscillations is great. It is easy to prove that in this case the mean value of the exponent in (32) actually equals one. This condition is neutrally stable -- small excitation leads to a system with one cycle close to another. The introduction of noise into the right part of the model system of (37) results in the appearance of a focus instead of a cycle and to the establishment of equilibrium. The rate of approach to a permanent establishment is increased with an increase in noise.

A numerical study was made of the process of establishing a solution for equation (26) when

$$\gamma(x) = 1/(1+x^2) - 1/b \qquad (40)$$

which when b=1.5 corresponds to stationary distribution from /18
two peaks. Also the time of inclusion of the increment of instability was varied, producing

$$b(t) = 1 + 0{,}5\,(1 - \exp(-(t/t_B)^2)) \qquad (41)$$

With an abrupt inclusion of the increment (t_B=30) at first fairly rapidly (when $\gamma t \sim 60$) at points corresponding to stationary distribution, two fairly broad peaks are established whose area varies and whose width gradually contracts. It is convenient to draw the phase trajectories of such a process, having applied along the axis of the coordinates the area of the first and second peaks. A typical example of this phase trajectory is shown in Figure 7. The system is completed by slowly damping oscillations around a stationary position.

With slow inclusion of the increment (t_B=200 and especially t_B=400) the character of the process of establishment sharply changes and a quasi stationary condition for establishment is realized. Figure 8 shows phase trajectories of a system for different time inclusions (t_B=30, 200, 400, respectively). Numerical calculations show that in inequalities (35), (36) it is necessary to substitute one for the small parameter $\sim 10^{-2}$. This apparently involves the fact that for fairly narrow peaks n(x) differs noticeably from zero only at points where there is a small full increment Γ(x) (because

/28

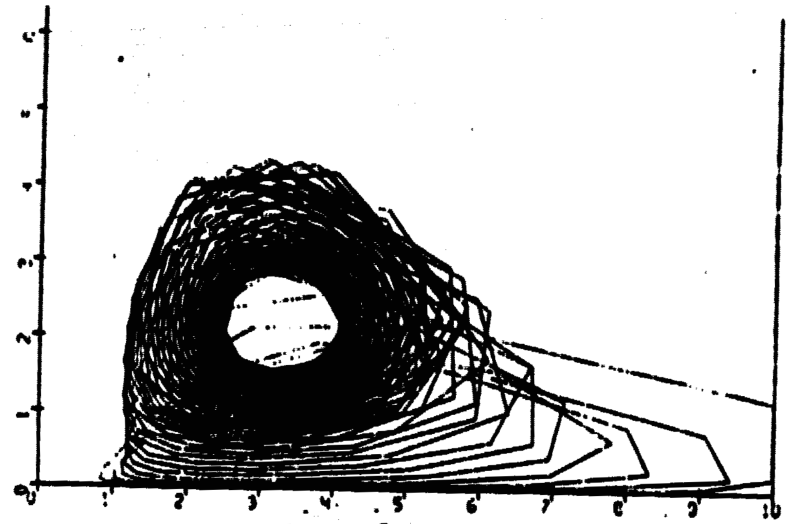


Fig. 8a

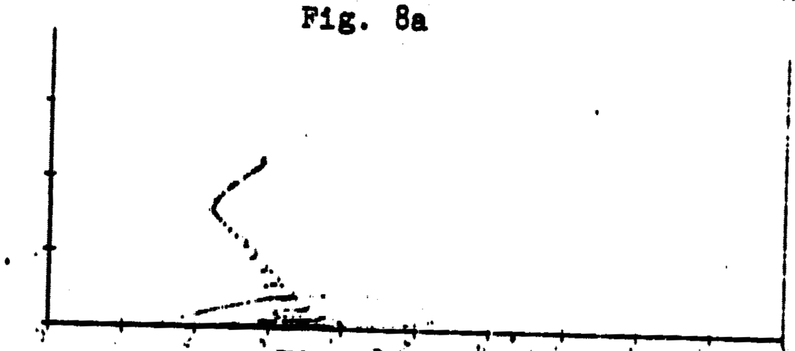


Fig. 8b

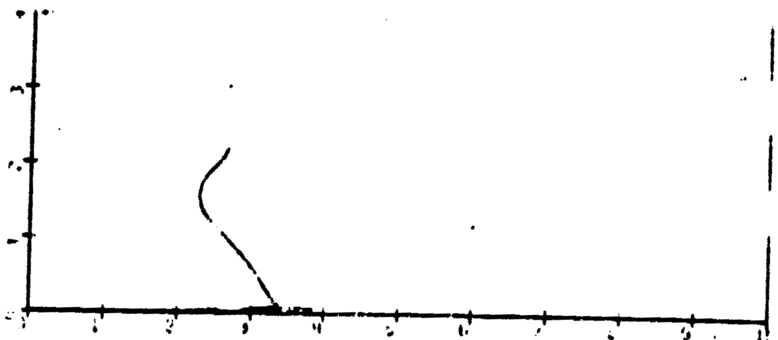


Fig. 8c

Figure 8. The relationship of the character of establishing a two-peak permanent establishment of the time of inclusion of the increment, time of inclusion: a) $t_B=30$, b) $t_B=200$, c) $t_B=400$ /20

in the permanent establishment $\Gamma(x)=0$).

It is natural that the rate of convergence toward a permanent establishment with an abrupt inclusion of the increment is increased with an increase in the noise level. Figure 9 shows phase trajectories of the system when $\epsilon_k=0.01$ and $\epsilon_k=0.001$ ($t_B=30$). Then, for a low noise level ($\epsilon_k=0.001$) the sections of the phase trajectory when $\gamma t>200$ are close to the maximum cycles of the model system (37) with the corresponding initial conditions.

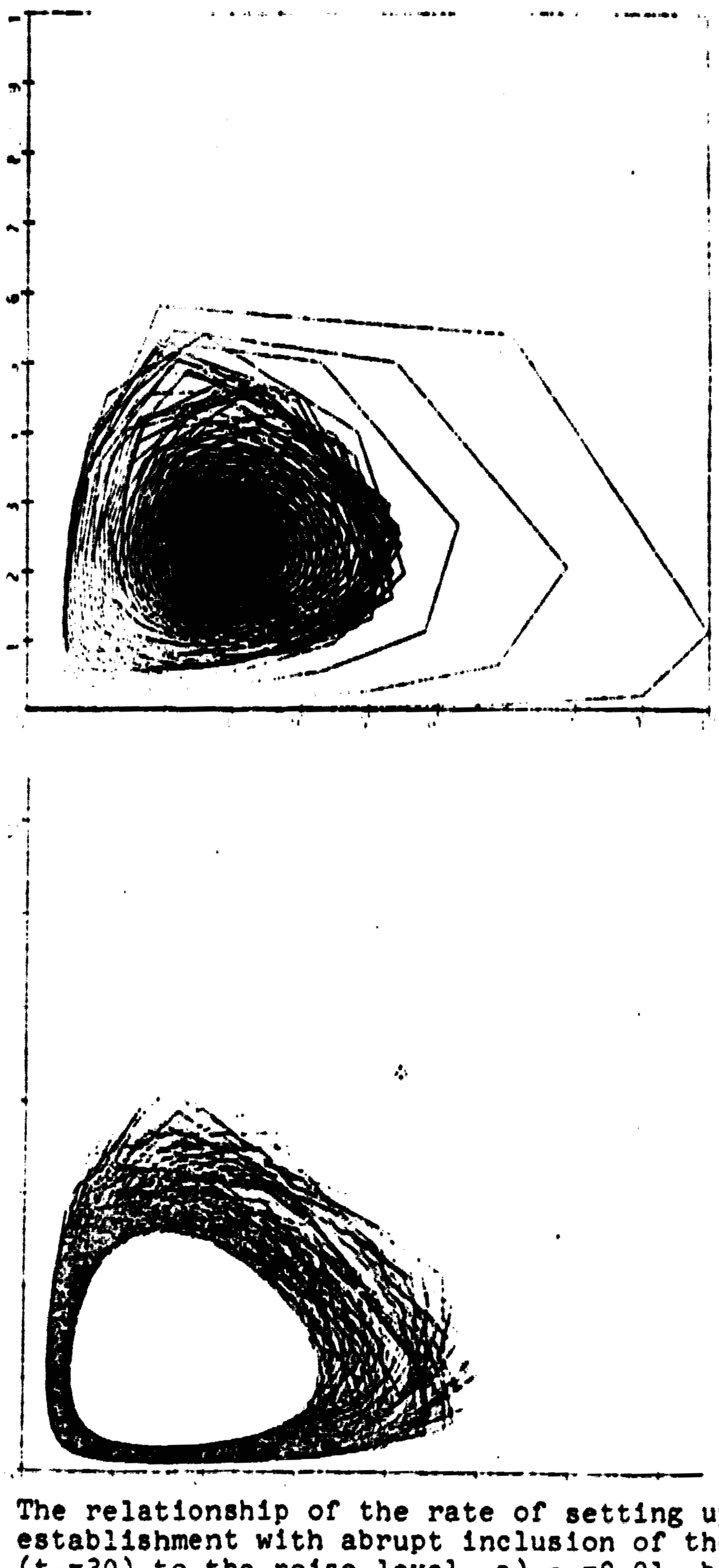
5. Conclusions /19

1. The character of the established spectrum of kinetic equation (1) is determined by the analytical properties of the nucleus $A_{kk'}$ and the linear increment γ_k. In a case of analytical $A_{kk'}$ and γ_k, the stationary spectrum, if it exists, is concentrated on varied dimensionality, the smallest dimensionality of the initial problem, at points on closed lines or surfaces.

2. As an example of establishing a single peak permanent establishment analytically and numerically it is pointed out that the height of the peak and the position of its center contain oscillation additives which decay with time. At first, the position of the peak is established significantly smaller than its area. Then distribution of the intensity within the peak is essentially not Gaussian. The time of establishing a permanent establishment is several hundreds of times larger than $1/\gamma_{max}$.

3. In a case when stationary distribution consists of many peaks, with the abrupt inclusion of the pumping increment into the system, secondary turbulence occurs. The position of the peaks is established and their amplitudes complete non-damped oscillations. In the distribution system considered,

/29



a

/30

b

Figure 9. The relationship of the rate of setting up a permanent establishment with abrupt inclusion of the increment (t_B=30) to the noise level. a) ε_k=0.01, b) ε_k=0.001.

a finite number of degrees of freedom is excited (on the order of the number of peaks).

4. As an example of establishing two-peak stationary distribution, the question of the character of secondary turbulence is studied in detail. It is pointed out that with abrupt inclusion of the increment, nonstability of the system completes non-damped (in the absence of noise) oscillations with large amplitude around a stationary position. During adiabatic inclusion of the increment (when the condition of adiabaticity is very rigid $\frac{d}{dt}\frac{1}{\gamma} \ll 10^{-2}$) a quasi stationary state of establishment is realized.