NASA TECHNICAL MEMORANDUM

NASA TM-75679

PROPAGATION OF WAVES IN A MEDIUM WITH HIGH RADIATION PRESSURE

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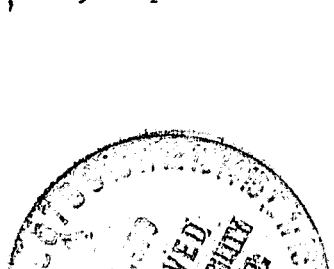
Translation of "Rasprostraneniye Voln v Srede s Vysokim Luchistym Davleniem", Academy of Sciences USSR, Institute of Space Research, Moscow, Report PR-421, 1978, pp 1-45



(NASA-TM-75679) PROPAGATION OF WAVES IN A MEDIUM WITH HIGH RADIATION PRESSURE (National Aeronautics and Space Administration) 40 p HC A03/MF A01 CSCL 03B

N80-10982

Unclas
G3/90 45889

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D. C. 20546 October, 1979

The propagation and mutual transformation of acoustic and thermal waves is investigated in media with a high radiative pressure. The investigation is carried out by means of the equations of hydrodynamics for matter and of the radiative transfer equations in a moving medium (in the first order with respect to v/c) in the Eddington approximation. A number of model problems which explain the physics of the phenomenon are solved in the first part of the paper: Waves in a homogeneous medium with an abrupt jump in opacity and in a medium of variable opacity. In the second part the outflow of waves is studied in the atmospheres of astrophysical objects; the accretion disks around black holes and very massive stars. The characteristics of the luminosity variability of Cygnus X-1 and of the nuclei of galaxies are examined in these models. The presence of convection and turbulence generates acoustic waves whose spectrum upon outflow into transparent layers is determined by the conditions of transmission and of damping. Variability of the radiation is connected with fluctuations of the photospheric temperature and that of the corona because of variable heating. The characteristic times of variability agree well with observations for all the objects, but it is difficult to obtain a sufficient amplitude for the brightness fluctuations for very massive stars.

I. THE BASIC EQUATIONS AND THE CASE OF A HOMOGENEOUS MEDIUM /5

INTRODUCTION

In astrophysical objects conditions are often encountered where the radiation pressure significantly exceeds the gas pressure, although the material mass is determined by the gas. A typical example are massive stars, which are well described by an Emden polytrope with n=3, and where the ratio β of the gas pressure to the radiative pressure equals (see [1])

$$\beta = P_g/P_r = \mu^{-1}(18 M_\odot/M)^{1/2}. \quad (1)$$

Here μ is the number of nucleons for one particle; for hydrogen $\mu=0.5$. At $M>72M_\odot$ the radiation pressure is already greater than the gas pressure, and for very massive stars with $M>10^4 M_\odot$, which are used as models for a quasar or for the nucleus of a galaxy (see [2]), $\beta<<1$. For these objects and also in the theoretical investigation of x-ray sources, a model of disk accretion onto a black hole is often considered (see [3] through [6]). For a fairly large luminosity L, $\beta<<1$ occurs for the central region of the disk.

Convective instability is a characteristic property of regions with a predominance of radiative pressure ($\beta<<1$). For very massive stars the presence of convection follows from the condition of radiative equilibrium (see [2] and also [7]), and convective instability of regions with $\beta<<1$ for disk accretion were shown in [8] (also see [9]).

Convection in the subphotospheric layers of a star or of a disk will unavoidably lead to the appearance of a mechanical, undulating energy flux which disappears at a small optical thickness $\tau<<1$, being transformed into heat. This leads to the formation of a hot gaseous /6
corona that is similar to the solar corona, whose temperature T_c is much greater than the photospheric temperature T_{ef}. One succeeds in explaining certain features of the Cygnus X-1 radiation source with the presence of a corona in an accretion disk around a black hole (see [8], [10] and [11]; moreover, the structure of the corona can be determined to a significant degree by the magnetic field (see [12]). It is also possible that the examination of the coronas around very massive stars will turn out to be necessary to explain the properties of certain quasars or active nuclei.

The flux of mechanical energy that is generated in the convective zone is transferred outward, by sound or Alfven and magnetosonic waves into layers with $\tau<<1$ through the outer radiative regions. If the radiation pressure is negligibly small in the convective zone and in the photosphere, then the mechanical energy flux (with no magnetic

$$F_m \simeq \tfrac{1}{2}\rho v^2 v_s,$$

field) is where v is the velocity of the material in the wave and v_s is the speed of sound that occurs practically without damping in a region where $\tau<1$. Transformation of the wave into a shock wave and dissipation of the magnetic field occur there, and the mechanical and magnetic energies go into heating the corona. If $\beta<<1$, then only a small fraction of the flux F_m that is generated in an optically thick region goes to heat the corona (see [8]). The great attentuation of the mechanical energy flux upon its outflow into a transparent region under the conditions $\beta<<1$ is connected with two circumstances. First, upon going over from a region where $\tau>>1$ to one where $\tau<<1$, the speed of sound which, inside this zone is related to the radiative pressure and outside of it, at $\tau<<1$, to the gas pressure, decreases: $v_s(\tau<<1)/v_s(\tau>>1) \approx \beta^{\frac{1}{2}}$. The second reason for the decrease of F_m is the strong damping of the sound waves that /7 is connected with radiative friction and with radiative heat conductivity.

A crude estimate for the total attenuation of the energy flux by β^{-1} times was used in [8]. The propagation of sound and thermal waves in a medium with $\beta<<1$ and the transfer of mechanical energy into the transparent layers above the photosphere are investigated in the present paper. The entire investigation is carried out on the basis of the equations of hydrodynamics for matter and of the equation of transfer for radiation, which is used in the Eddington approximation which allows one to describe the transparent and opaque regions in a single form. A flat geometry is considered and the effects of sphericity are neglected. Terms of the order of $\sim v/c$ are taken into account in the equation of transfer as is the case also in [13] and [14].

In the first part of the paper, a simple derivation of the equation of transfer in a moving medium is given in Paragraph 2 and the basic equations are shown there. In the following paragraphs the propagation of waves is investigated without a magnetic field under the conditions $\beta<<1$: of a wave in a homogeneous medium and in a medium with an abrupt jump of opacity. In the second part the

outflow of a wave from a plane, static atmosphere is invesgitated. Numerical estimates are made for the flux of mechanical energy in a model of disk accretion and in a very massive star. The estimate of a damping of β^{-1} times that has been used in [8] turned out to be fairly good. The condition for the outflow of a wave into a transparent region singles out a characteristic frequency which can be connected with the observed frequencies of the fluctuations and brightness variabilities in the x-ray sources Cygnus X-1 and Circinus X-1, and also in some nuclei of galaxies and in quasars. In conclusion, numerical estimates are carried out that are connected with the existence of the characteristic frequency and the observational consequences of the model are given.

I. Set-up of the problem and the basic equations. /8

a) The equation of transfer in a moving medium

Upon the outflow of a wave into a transparent region, it is necessary to use an equation of transfer with allowance for motion of the matter. In regions with $\tau \gg 1$ it is sufficient to limit oneself to the equations of radiative thermal conductivity; however, for $\tau \ll 1$, the flux and the density of the radiative energy are not connected unambiguously and it is necessary to consider at least the Eddington two-moment approximation. The equation of transfer in moving matter with allowance for $\sim v/c$ was derived by a fairly complex "geometric" method in [13] and [14]. Here we shall show a simpler method for deriving these equations that is mentioned in [14] but has not been carried out there. In an arbitrary inertial frame of reference and in the framework of the special theory of relativity, the equation of transfer has the form (see [15])

$$\frac{1}{c}\frac{\partial I_{\nu_0}}{\partial t_0} + \vec{n}_0 \cdot \nabla_0 I_{\nu_0} = j_0(\nu_0) - k_0(\nu_0) I_{\nu_0}, \quad I_{\nu_0} \equiv I(\vec{x}_0, \nu_0, \vec{n}_0, t_0). \tag{2}$$

Here $I\upsilon_0$ is the spectral intensity of the radiation, υ_0 is the frequency, j_0 and k_0 are the coefficients of emission and of absorption, which include scattering, and $\vec{n}_0$ is the direction of propagation of

the radiation. All the quantities with an index "o" are determined in the inertial frame of reference. The coefficients of emission and absorption are most simply expressed by the properties of the matter in the system with respect to which the matter is at rest. Let us call this system a Lagrangian system. Let us limit ourselves to a plane, one-dimensional case where all the quantities depend only on the one spatial variable z_o. Then

$$\vec{n}_o \cdot \nabla_o = \mu_o \frac{\partial}{\partial z_o}, \quad \mu_o = \cos(\hat{n_o v_o}). \tag{3}$$

If the matter is moving with a velocity v in the z-direction, then the frequency υ and the cosine μ of the angle in the Lagrangian system are connected with the quantities in the inertial system by relations (see [14]) that are written down with an accuracy $\sim v/c$: /9

$$\nu = \nu_o (1 - \mu_o v/c), \quad \nu_o = \nu (1 + \mu v/c), \tag{4}$$

$$\mu = \mu_o - v/c + \mu_o^2 v/c, \quad \mu_o = \mu + (1 - \mu^2) v/c.$$

The distribution function $n(\vec{p}, \vec{\alpha}, t)$ of the photons in phase space is related to the intensity I_υ by the relation (see [16])

$$n = \frac{c^2}{h^4} \frac{I_\nu}{\nu^3}. \tag{5}$$

From the invariance of the distribution function n with respect to Lorentz transformations, invariance follows for the quantity

$$I_{\nu_o}/\nu_o^3 = I_\nu/\nu^3. \tag{6}$$

Using Equation (6), one can write down the left-hand part of Equation (2) in an invariant form and then the invariance of the right-hand part of Equation (2) will be reduced to the relations

$$j_{\nu_o}/\nu_o^2 = j_\nu/\nu^2, \tag{7}$$

$$k_{\nu_o} \nu_o = k_\nu \nu.$$

The equation $dt_o/\upsilon_o = dt/\upsilon$ was taken into account here and Relations (6) and (7) were used in [13] and [14].

Let us write down Equation (2) in the (m,t) Lagrangian system where, with the necessary accuracy with respect to v/c

$$t = t_0, \quad m = \int_{z_0}^{z} \rho(z')\,dz' \tag{8}$$

/10

Taking the transformations and invariants (4), (6) and (7) into account, Equation (2) with allowance for Equation (3) to an accuracy of v/c, is written down in the form

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mu\frac{\partial I_\nu}{\partial z} + \frac{3\mu^2}{c}\frac{\partial z}{\partial t} I_\nu - \frac{\mu}{c}\frac{\partial z}{\partial t}\nu\frac{\partial I_\nu}{\partial \nu} - \tag{9}$$

$$- \frac{\mu(1-\mu^2)}{c}\frac{\partial v}{\partial z}\frac{\partial I_\nu}{\partial \mu} = j_\nu - k_\nu I_\nu.$$

One can write down the equation of continuity in Lagrangian coordinates in the form $dz/dm = 1/\rho$ or, after differentiation,

$$\partial v/\partial z = -(1/\rho)\partial\rho/\partial t. \tag{10}$$

After substituting Equation (10) into Equation (9), we shall finally obtain an equation which also follows from [14] for $r \to \infty$:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \mu\rho\frac{\partial I_\nu}{\partial m} - \frac{3\mu^2}{c\rho}\frac{\partial\rho}{\partial t} I_\nu + \frac{\mu^2\nu}{c\rho}\frac{\partial\rho}{\partial t}\frac{\partial I_\nu}{\partial\nu} + \tag{11}$$

$$+ \frac{\mu(1-\mu^2)}{c\rho}\frac{\partial\rho}{\partial t}\frac{\partial I_\nu}{\partial\mu} = j_\nu - k_\nu I_\nu.$$

If, in addition to pure absorption, scattering occurs, then one can, for a condition of local thermodynamic equilibriam (LTE), write down (see [17])

$$j_\nu = \alpha_\nu B_\nu(T) + \sigma_\nu \int I_\nu\, d\Omega/4\pi, \tag{12}$$

$$k_\nu = \alpha_\nu + \sigma_\nu, \quad \sigma_\nu = \sigma = \sigma_T n_e, \quad \sigma_T = 6.65\cdot 10^{-25}\ \text{cm}^2.$$

Here $\alpha\upsilon$ and σ are the coefficients of absorption and scattering for electrons, $B_\upsilon(T)$ is the Planck function, n_e is the concentration of electrons, and σ_T is the Thomson cross-section. The scattering is considered to be coherent and isotropic. These approximatons are

sufficiently good for the conditions being considered.

b) Equations for the moments in the Eddington Approximation /11

A system of angular moments that is obtained after multiplying Equation (11) by $\frac{1}{2}\mu^n$ and integration with respect to μ from -1 to +1 is often used for an approximate solution of the problem instead of Equation (11). The first two moment equations have the form

$$\frac{1}{c}\frac{\partial J_\nu}{\partial t} + \rho\frac{\partial H_\nu}{\partial m} - \frac{3}{c\rho}\frac{\partial \rho}{\partial t}K_\nu + \frac{\nu}{c\rho}\frac{\partial \rho}{\partial t}\frac{\partial K_\nu}{\partial \nu} + \frac{1}{\rho c}\frac{\partial \rho}{\partial t}(3K_\nu - J_\nu) = \alpha_\nu B_\nu(T) - \alpha_\nu J_\nu,$$

$$\frac{1}{c}\frac{\partial H_\nu}{\partial t} + \rho\frac{\partial K_\nu}{\partial m} + \frac{\nu}{c\rho}\frac{\partial \rho}{\partial t}\frac{\partial N_\nu}{\partial \nu} + \frac{1}{c\rho}\frac{\partial \rho}{\partial t}(N_\nu - H_\nu) = -(\alpha_\nu + \sigma)H_\nu. \qquad (13)$$

The following moments are introduced here:

$$J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu\, d\mu, \quad H_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu \mu\, d\mu,$$
$$K_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu \mu^2 d\mu, \quad N_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu \mu^3 d\mu. \qquad (14)$$

If one is not interested in effects connected with lines, then one can average Equation (13) with respect to frequency. We obtain

$$\frac{1}{c}\frac{\partial J}{\partial t} + \rho\frac{\partial H}{\partial m} - \frac{4}{c\rho}\frac{\partial \rho}{\partial t}K + \frac{1}{\rho c}\frac{\partial \rho}{\partial t}(3K - J) = \alpha_p B(T) - \int_0^\infty \alpha_\nu J_\nu\, d\nu, \qquad (15)$$

$$\frac{1}{c}\frac{\partial H}{\partial t} + \rho\frac{\partial K}{\partial m} - \frac{2}{c\rho}\frac{\partial \rho}{\partial t}H = -\int_0^\infty \alpha_\nu H_\nu\, d\nu - \sigma H. \qquad (16)$$

Here α_p is the absorption coefficient averaged with respect to the Planck function $B(T)=acT^4/4\pi$, $a=7.56\cdot10^{-15}$ in cgs units. The average absorption coefficients with the weights J_ν and H_ν are usually expressed by α_p and α_R, the Rosseland mean, respectively (see [14]) /12

$$\int_0^\infty \alpha_\nu J_\nu d\nu = \alpha_p J, \quad \int_0^\infty \alpha_\nu H_\nu d\nu = \alpha_p H,$$
$$J = \int_0^\infty J_\nu d\nu, \quad H = \int_0^\infty H_\nu d\nu, \quad K = \int_0^\infty K_\nu d\nu. \tag{17}$$

In later calculations we shall consider α_υ as independent of υ (the grey approximation) and $\alpha_p = \alpha_R = \alpha$. Let us notice that, upon averaging with respect to frequency, the third moment N_υ falls out of Equations (15) and (16) for the moments.

We shall solve the problem in the Eddington approximation, that is, let us set

$$K = \frac{1}{3} J. \tag{18}$$

This approximation is fulfilled with high accuracy for a large optical thickness τ, and for small τ values the error is small; we have $K = 0.41J$ in a plane, grey atmosphere at $\tau = 0$ (see [17] and [18]). Finally, in the Eddington approximation, the equations for the moments have the form:

$$\frac{1}{c}\frac{\partial J}{\partial t} + \rho \frac{\partial H}{\partial m} - \frac{4}{3c\rho}\frac{\partial \rho}{\partial t} J = \alpha (B - J), \tag{19}$$

$$\frac{1}{c}\frac{\partial H}{\partial t} + \frac{1}{3}\rho \frac{\partial J}{\partial m} - \frac{2}{c\rho}\frac{\partial \rho}{\partial t} H = -(\alpha + \sigma) H. \tag{20}$$

c) Hydrodynamic equations for matter

In a gravity field with a gravitational acceleration $g = \text{const} > 0$ the equation of motion has the form: /13

$$\frac{\partial^2 z}{\partial t^2} = -\frac{\partial p_0}{\partial m} - g + \frac{4\pi(\alpha + \sigma)}{\rho c} H, \quad v = \frac{\partial z}{\partial t}. \tag{21}$$

Here the gas pressure is $P_g = R\rho T/\mu$. $R = 8.3 \cdot 10^7$ in cgs units, and μ is the number of baryons for one particle. The last term in Equation (21) represents the force acting on the matter from the direction of the radiation flux. The equation which takes into account the exchange of energy between matter and radiation, has the form:

$$\frac{\partial E_g}{\partial t} + P_g \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \frac{4\pi\alpha}{\rho}(J - B), \qquad (22)$$

where $E_g = RT/\mu(\gamma-1)$ and γ is the ratio of specific heat for the gas. One must add the equation of continuity in Lagrangian coordinates Equation (10) to Equations (21) and (22). We shall show that the complete system of Equation (10) and Equations (19) through (22) leads to the correct form of the law of conservation of energy with an accuracy to terms $\sim(v/c)$. The density E_r, ergs/gm, pressure P_r, dynes/cm^2, and energy flux F_r, ergs/cm^2·sec, of the radiation are expressed by the moments J and H:

$$E_r = \frac{1}{\rho c}\int I_\nu \, d\Omega \, d\nu = \frac{2\pi}{\rho c}\int I_\nu \, d\mu \, d\nu = \frac{4\pi J}{\rho c}, \qquad (23)$$

$$P_r = \frac{1}{3}\rho E_r = \frac{4\pi}{3c} J,$$

$$F_r = \int I_\nu \mu \, d\Omega \, d\nu = 2\pi \int I_\nu \mu \, d\mu \, d\nu = 4\pi H.$$

Multiplying Equation (21) by v, Equation (19) by $4\pi/\rho$ and adding this equation to Equation (22), using $\partial v/\partial m$ for the expression $\partial\rho/\partial t$ in Equation (10), we obtain

$$\frac{\partial}{\partial t}\left(E_g + \frac{v^2}{2} + \frac{4\pi}{c}\frac{J}{\rho} + gz\right) =$$

$$= -\frac{\partial}{\partial m}(P_g v) - \frac{\partial}{\partial m}(4\pi H) - \frac{4\pi}{3c}\frac{\partial v}{\partial m} J + \frac{4\pi v}{\rho c}(\alpha+\sigma)H. \qquad (24)$$

Let us express the last term in Equation (24) by means of Equation (20). Neglecting terms of higher order than $(v/\rho c^2)\partial H/\partial t$ and $(vH/c^2)\partial v/\partial m$, we obtain, taking Equation (23) into account, an equation of the conservation of energy in the form: /14

$$\frac{\partial}{\partial t}\left(E_g + \frac{v^2}{2} + E_r + gz\right) = -\frac{\partial}{\partial m}(P_g v + P_r v) - \frac{\partial}{\partial m}(4\pi H). \qquad (25)$$

Here on the left stand the thermal, radiation, potential and kinetic energy for a unit of material mass under the derivation sign, the first term on the right expresses the work of the pressure forces, and the second term is the divergence of the radiation flux.

d) Selection of the absorption and scattering coefficients

It is more convenient to introduce the opacities

$$\varkappa_0 = (\alpha+\sigma)/\rho, \qquad \varkappa_1 = \alpha/\rho. \tag{26}$$

in Equations (19) through (22) instead of the coefficients α and σ.

Under the conditions considered below, the coefficient of braking absorption α is much less than the coefficient of scattering for electrons (see [18]):

$$\frac{\alpha}{\sigma} = \frac{0.1\, n^2 m_p T^{-7/2}}{6.6\cdot 10^{-25} n} \simeq \frac{n}{4T^{7/2}} < 10^{-3}. \tag{27}$$

On the other hand, at sufficiently high temperatures the energy of a photon changes due to Compton processes (for an invariant number of photons), which will lead to an exchange of energy between the gas and radiation and must be taken into account in Equations (19) and (22). In view of the great complexity of a rigorous allowance for non-coherent Compton scattering in a non-homogeneous medium, we shall limit ourselves to two limiting cases. In the first case we shall completely neglect absorption (pure scattering)

(I) $\varkappa_0 = \sigma/\rho = \varkappa_c \cdot 0.2\,(1+X_H)$ cm^2/gm, $\varkappa_1 = 0$, (28) /15

X_H is the concentration of hydrogen by weight.

In the second case let us consider Compton scattering which acts as a source of absorption:

(II) $\varkappa_0 = \varkappa_1 = \varkappa_c$. (29)

In the examples considered below the fraction of the mechanical energy flowing out from the photosphere is practically identical in Cases (I) and (II). Actually, the case of Compton scattering corresponds to an intermediate case $0 < \varkappa_1 < \varkappa_0$.

2. The linearized equations for small disturbances at $P_r >> F_g$

Let us introduce the Lagrangians of the small disturbances

$$\theta = \delta T, \quad y = \delta z, \quad j = \delta J, \quad h = \delta H, \quad \tilde{\rho} = \delta\rho \tag{30}$$

with respect to the equilibrium solution of Equations (10) and (19) through (22):

$$\rho(m), \quad T(m), \quad J(m) = B(m), \quad H = \text{const},$$
$$z(m), \quad B = acT^4/4\pi. \tag{31}$$

Let us also set all the quantities of Equations (30) $\alpha \exp(-i\omega t)$; then from Equations (10) and (19) through (22) we obtain:

$$\frac{dy}{dm} = -\frac{\tilde{\rho}}{\rho^2}, \tag{32}$$

$$\frac{i\omega}{c\rho} j - \frac{dh}{dm} - \frac{aT^4}{3\pi} i\omega \frac{\tilde{\rho}}{\rho^2} = \varkappa_1 \left(j - \frac{ac}{\pi} T^3 \theta\right), \tag{33}$$

$$\left(1 - \frac{i\omega}{c\varkappa_0\rho}\right) h + \frac{1}{3\varkappa_0} \frac{dj}{dm} + \frac{2i\omega}{\varkappa_0 c} H \frac{\tilde{\rho}}{\rho^2} = 0, \tag{34}$$

$$\omega^2 y = -\frac{d}{dm}\left(\frac{R\rho\theta}{\mu} + \frac{R\tilde{\rho}T}{\mu}\right) - \frac{4\pi\varkappa_0}{c} h, \tag{35}$$

$$\left[4\varkappa_1 acT^3 - i\omega \frac{R}{\mu(\gamma-1)}\right]\theta = 4\pi\varkappa_1 j - i\omega P_g \frac{\tilde{\rho}}{\rho^2}. \tag{36}$$ /16

Eliminating $\tilde{\rho}$ from Equations (33) through (36) by means of Equation (32), we shall obtain

$$\frac{i\omega}{c\rho} j - \frac{dh}{dm} + \frac{aT^4}{3\pi} i\omega \frac{dy}{dm} = \varkappa_1 \left(j - \frac{ac}{\pi} T^3 \theta\right), \tag{37}$$

$$\left(1 - \frac{i\omega}{c\varkappa_0\rho}\right) h + \frac{1}{3\varkappa_0} \frac{dj}{dm} - \frac{2i\omega}{\varkappa_0 c} H \frac{dy}{dm} = 0, \tag{38}$$

$$\omega^2 y = -\frac{d}{dm}\left(\frac{R\rho\theta}{\mu} - \rho P_g \frac{dy}{dm}\right) - \frac{4\pi\varkappa_0}{c} h, \tag{39}$$

$$\theta = \frac{4\pi\varkappa_1 j + i\omega P_g \, dy/dm}{4\varkappa_1 acT^3 - i\omega R/\mu(\gamma-1)}. \tag{40}$$

In Equation (28), Case (I), setting $\varkappa_s = 0$, we have from Equations (35) and (37) for Case (I):

$$\theta = -(\gamma-1)\rho T\, dy/dm, \tag{41a}$$

$$\frac{i\omega}{c\rho} j - \frac{dh}{dm} + \frac{aT^4}{3\pi} i\omega \frac{dy}{dm} = 0. \tag{42}$$

In Equation (29), Case (II), let us immediately allow for the smallness of the quantity $\beta = P_g/P_r \ll 1$, and also let us consider the case where the inequity

$$\rho\omega / \varkappa_s \rho c \ll 1. \tag{43}$$

is fulfilled. This inequality is fulfilled at oscillation frequencies that are not too high. Taking inequality (43) into account one can neglect the second term in the denominator of Equation (40) and obtain, for Case (II):

$$\theta = \frac{4\pi\varkappa_s j + i\omega P_g\, dy/dm}{4\varkappa_s acT^3} \tag{41b}$$

Taking Equation (41b) into account, the quantity in the right-hand part of Equation (37) equals /17

$$\varkappa_s (j - acT^3\theta/\pi) = -(i\omega P_g/4\pi)\, dy/dm. \tag{b}$$

By virtue of the inequality $\beta \ll 1$, this term is much smaller than the last term in the left-hand part of Equation (37) and one can neglect it. Thus, Equation (42) is valid both in Case (I) and also in Case (II). Let us now eliminate θ from Equation (39). Using Equation (41a), in Case (I) we obtain from Equation (39)

(I): $$\omega^2 y = -\frac{d}{dm}\left(\gamma\rho P_g \frac{dy}{dm}\right) - \frac{4\pi\varkappa_s}{c} h. \tag{44a}$$

In case (II), using Equation (41b), we obtain from Equation (39), with allowance for inequality (43):

(II): $$\omega^2 y = \frac{d}{dm}\left(\frac{\pi}{3}\frac{j}{c\rho} - \rho P_g \frac{dy}{dm}\right) - \frac{4\pi\varkappa_s}{c} h. \tag{44b}$$

Equations (38), (42) and (44) will be used in later calculations in regard to the variables y, h and j.

3. Waves in a homogeneous medium*

*This case was also considered in [29].

In this case H=0 all the equilibrium values in Equations (31) are constant, among them ρ=const, and instead of Inequality (43) one can use the stronger inequality

$$\omega/c\,\varkappa_0\rho \ll 1 \quad \text{or} \quad l=1/\varkappa_0\rho \ll c/\omega=L. \tag{45}$$

This inequality indicates the smallness of the free path of a photon $l=1/\varkappa_0\rho$ in comparison with the distance divided by 2π which a photon passes through freely in the period of the vibrations. One can look for the solution of Equations (38), (42) and (44) in a homogeneous medium in the form exp (ikz). Also taking Condition (45) into account to simplify Equation (38), we shall obtain /18

$$h_1 + \frac{ik}{3\varkappa_0\rho} j = 0, \tag{46}$$

$$-\frac{ik}{\rho} h_1 + \frac{j\omega}{c\rho} j - \frac{aT^4}{3\pi} \frac{k\omega}{\rho} y = 0,$$

(I) $$\frac{4\pi\varkappa_0}{c} h_1 + (\omega^2 - \gamma \frac{p_0}{\rho} k^2) y = 0,$$

(II) $$\frac{4\omega\varkappa_0}{c} h_1 - ik \frac{\pi}{3\rho} \frac{i}{c} \beta + (\omega^2 - \frac{p_0}{\rho} k^2) y = 0.$$

Equating the determinant of the system of Equations (46) to zero, introducing the notations

$$v_r^2 = \frac{4}{3} \frac{p_r}{\rho}, \quad v_g^2 = \gamma \frac{p_0}{\rho}, \quad l = \frac{1}{\varkappa_0\rho} \tag{47}$$

and allowing for the fact that $\beta \ll 1$, we obtain the dispersion equation in the form:

$$v_g^2 k^4 - k^2 \left(\omega^2 + 3i \frac{\omega}{cl} v_r^2\right) + \frac{3i}{cl} \omega^2 = 0. \tag{48}$$

For Case (I) the adiabatic speed of sound v_g from Equation (47) enters into Equation (48), and for Case (II) the isothermal speed of sound $v_T^2 = P_g/\rho$ enters instead of v_g, that is, Equations (47) and (48) are applicable for Case (II) if one sets $\gamma=1$.

In this paper only forced vibrations (according to the terminology of [19]) are considered, where ω is considered real and given, and k^2 is found from Equation (48). We obtain

$$k_2^2 = \left\{\omega^2 + 3i\frac{\omega}{c\ell}v_r^2 + \left[\left(\omega^2 + 3i\frac{\omega}{c\ell}v_r^2\right)^2 - \frac{12i\omega^2}{c\ell}v_g^2\right]^{1/2}\right\}. \quad (49)$$

$$k_1^2 = \frac{3i\omega^2}{c\ell v_r^2}\frac{1}{k_2^2}.$$

Taking the smallness of $v_g^2/v_r^2 \sim \beta << 1$ into account in Equation (49), we obtain /19

$$k_1^2 = \frac{\omega^2}{v_g^2} + i\frac{3\omega v_r^2}{c\ell v_g^2}, \quad (50)$$

$$k_2^2 = \frac{\omega^2}{v_r^2 - \frac{1}{3}i\omega c\ell} = \frac{\omega^2}{v_r^2\left[1 + \left(\frac{\omega c\ell}{3v_r^2}\right)^2\right]}\left(1 + i\frac{\omega c\ell}{3v_r^2}\right). \quad (51)$$

The solution of Equation (50) describes the propagation of waves that are associated with the gas pressure. If absorption is absent (Case I), then the waves are adiabatic, and in the case of strong coupling of gas and radiation (Case II) the speed of sound becomes isothermal. The solution of Equation (51) describes adiabatic waves that are associated with radiation pressure. The frequency

$$\omega_1 = 3v_r^2/c\ell \quad (52)$$

is critical for both types of waves. For $\omega << \omega_1$, we have

$$k_1 = \pm\frac{\sqrt{\omega\omega_1}}{\sqrt{2}v_g}(1+i)\left(1 - i\frac{\omega}{2\omega_1}\right), \quad (53a)$$

$$k_2 = \pm\frac{\omega}{v_r}\left(1 + i\frac{\omega}{2\omega_1}\right). \quad (53b)$$

For $\omega > \omega_1$, we obtain

$$k_1 = \pm\frac{\omega}{v_g}\left(1 + i\frac{\omega_1}{2\omega}\right), \quad (54a)$$

$$k_2 = \pm\frac{\sqrt{\omega\omega_1}}{\sqrt{2}v_r}(1+i)\left(1 - i\frac{\omega_1}{2\omega}\right). \quad (54b)$$

It follows from Equation (53) that the low-frequency disturbances of the radiation pressure P_r are propagated with a velocity v_r (Equation (53b)) almost without damping, and the propagation of low-frequency disturbances in the gas assumes the character of

"diffusion" waves (Equation (53a)) with a coefficient of diffusion $\chi_g = v_g^2/\omega \approx \beta c l/3$. These waves describe the leveling of the inhomogeneity in the gas ("entropy" disturbances). On the contrary, the high-frequency disturbances in the gas are propagated as sound waves /20 (Equation (54a)) and in the radiation they assume the character of thermal waves (Equation (54b)) with a coefficient of temperature conductivity $\chi_r = 1/3cl$ (compare with [20]). Let us remember that, for $\varkappa_0 = \varkappa_1$ the waves in a gas are isothermal.

4. A medium with an abrupt jump of opacity

Let us consider a medium in which ρ, T and P_g are homogeneous and the opacity is $\varkappa$. It undergoes an abrupt jump in the plane $z=0$. The quantities l and ω_1 undergo the same kind of abrupt jump in this plane. Let $\varkappa(z<0) \gg \varkappa(z>0)$. A wave which is propagated in the direction of increasing z can change the nature of its propagation at $z=0$ if $\omega_B \ll \omega \ll \omega_A$,

$$\omega_A = \frac{3v_r^2}{cl_A}, \quad \omega_B = \frac{3v_r^2}{cl_B}, \quad l_A = \frac{1}{\rho\varkappa(z<0)}, \quad l_B = \frac{1}{\rho\varkappa(z>0)}. \qquad (55)$$

Short-wavelength disturbances in a homogeneous medium $\omega \gg \omega_A$ rapidly damp out even in the case where their propagation has the nature of the wave of Equation (54a). The damping length equals $l_{d1} = 2v_g/\omega_2 = 2/3cv_g l/v_r^2$, which at small β can become on the order of the length of the free path l of the photons. On the other hand, the damping of long waves is very weak; from Equation (53b) we have $l_{d2} \approx 2v_1\omega_1/\omega^2 = 6v_r^3/cl\omega \to \infty$ for $\omega \to 0$. Thus, it makes sense to investigate the transition of only the fairly long waves through the abrupt jump at $\omega=0$. The case of Inequality (55) that is considered below is the most interesting one from the point of view of later applications.

The wave of Equation (53b), upon passing through an abrupt jump of opacity, is split into four waves: the two reflected waves of Equation (53) and the two transmission waves which, for $z>0$, $\omega>\omega_B$ already belong to waves of the Equation (54) type. It is obvious that, at the point of the abrupt jump of opacity, the y displacements, disturbances of the gas pressure $\tilde{P}_g = P_g(\tilde{\rho}/\rho + \theta/T)$ and of the first moment /21

j which characterizes the radiation pressure of Equation (23) remain continuous. The continuity of the second moment h also follows from the condition of the absence of an energy sink or source at the place of the abrupt jump. If A_0 is the amplitude of the incident wave, A_1 and A_2 are the amplitudes of the reflected waves of the Equation (53) type, and B_1 and B_2 are the amplitudes of the transmission waves of the Equation (54) type, then one can write down the continuity condition for the physical quantities at the abrupt jump at s=0 in the form

$$A_{0q} + A_{1q} + A_{2q} = B_{1q} + B_{2q}, \qquad q = y, h, j, \tilde{P}_g. \tag{56}$$

To obtain the amplitudes of A_1, A_2, B_1 and B_2 as functions of A_0, one must express the amplitudes for different q values by some defined q_0, using Equations (46), (32) and (41), and solve the four linear non-homogeneous equations which have been obtained. If one adopts $q_0=j$, then one can write down the remaining continuous quantities, using Conditions (45) and $\beta<<1$, in the form:

$$h = -i\frac{k}{3\varkappa_0\rho}j, \quad y = \frac{3\pi}{acT^4}\left(i - \frac{k^2c}{3\varkappa_0\rho\omega}\right)\frac{j}{k}, \tag{57}$$

$$\text{(I)}\ \frac{\tilde{P}_g}{P_g} = \frac{\pi\gamma}{acT^4}\left(3+i\frac{k^2c}{\varkappa_0\rho\omega}\right), \quad \text{(II)}\ \frac{\tilde{P}_g}{P_g} = \frac{\pi}{acT^4}\left(4+i\frac{k^2c}{\varkappa_0\rho\omega}\right)j.$$

The solution of the system of Equation (56), taking Equation (53), (54) and (57) into account, has the form for q=j.

$$A_{1j} = -2\frac{v_g}{v_r}\left[1 - (1-i)\sqrt{\frac{\omega}{2\omega_A}} - (1+i)\sqrt{\frac{\omega_B}{2\omega}}\right]A_{0j}, \qquad \text{/22}$$

$$A_{2j} = -\left[1 - 2\sqrt{\frac{\omega_B}{2\omega}}(1+i)\right]A_{0j},$$

$$B_{1j} = -2i\frac{v_g}{v_r}\frac{\omega_B}{\omega}\left[1 - (1-i)\sqrt{\frac{\omega}{2\omega_A}} - (1+i)\sqrt{\frac{\omega_B}{2\omega}}\right]A_{0j}, \tag{58}$$

$$B_{2j} = (1+i)\sqrt{\frac{2\omega_B}{\omega}}\left[1 - (1+i)\sqrt{\frac{\omega_B}{2\omega}}\right]A_{0j}.$$

For q=y we have

$$A_{1y} = -2\sqrt{\frac{\omega}{2\omega_A}}(1-i)\left[1-(1-i)\sqrt{\frac{\omega}{2\omega_A}} - (1+i)\sqrt{\frac{\omega_B}{2\omega}}\right]A_{0y},$$

$$A_{2y} = \left[1-2\sqrt{\frac{\omega_B}{2\omega}}(1+i)\right]A_{0y}, \qquad (59)$$

$$B_{1y} = 2\left[1-(1-i)\sqrt{\frac{\omega}{2\omega_A}} - (1+i)\sqrt{\frac{\omega_B}{2\omega}}\right]A_{0y},$$

$$B_{2y} = 2i\frac{\omega_B}{\omega}\left[1-(1+i)\sqrt{\frac{\omega_B}{2\omega}}\right]A_{0y}.$$

The relations (58) and (59) are valid for both Cases (I) and (II) if one assumes v_g to be equal to the adiabatic and the isothermal speeds of sound respectively. The terms $\sim\omega/\omega_A \sim \omega_B/\omega$, and v_g/v_r were neglected in comparison with unity in all quantities. It follows from Equations (58) and (59) that, upon passage through an abrupt jump of the type of Inequality (55), an almost complete reflection occurs for a radiation wave of the Equation (53b) type. The amplitudes of the remaining j-waves are significantly smaller than A_{2j}; the shift of the discontinuity point generates a transmission gas wave of almost the same amplitude

$$|B_{1y}| \sim |A_{0y}|. \qquad (60)$$

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Thus, upon passage through the abrupt jump of opacity, a wave transformation occurs such that the wave which possesses the minimum damping "survives". Relation (60) shows that y, the amplitude of the wave with minimum damping is almost continuous at the abrupt jump.

We also considered the question of a wave transformation for the case where the opacity (and the paths of the photons ℓ) change continuously. Here the system of Equations (38), (39) and (42) is reduced to one equation for the displacement of y: (a dashed line indicates a derivative with respect to s):

$$[l(y'' + \frac{\omega^2}{v_g^2} y)]'' + 3i\frac{\omega}{c}\frac{v_r^2}{v_g^2}(y'' + \frac{\omega^2}{v_r^2} y) = 0. \qquad (61$$

It was assumed that for $z \leq -\beta$ the path l is constant $l = l_A$, for $z > \beta$ $l = l_B$, and for $\beta \leq z \leq \beta$ the path increased monotonically from l_A to l_B, and, moreover, the relation $l(z)$ was selected in the form of a fifth degree polynomial in such a manner that

$$l' \propto (z^2 - \beta^2)^2. \qquad (62)$$

Then l, l', and l'' are continuous for all z. The parameter β, the half-width of the transition region, varied in different variants of the calculation from values $\beta \ll \lambda_g$ to $\beta \gg \lambda_g$, where

$$\lambda_g = 2\pi v_g/\omega \qquad (63)$$

is the wavelength in the gas (without allowance for damping).

Equation (61) was solved numerically by an implicit fifth order Adams type method (see [21]). In order to obtain a solution for Equation (61) which corresponds physically to a radiation wave going from the region $z \leq -\beta$, we started in the following manner. First, a "radiation" wave going in the direction of increasing z was given at $z = +\delta$. From this the initial conditions for y (that is, y, y', y'' and y''') were obtained for $z = +\beta$ and an integration "backward" of Equation (61) was carried out from $z = +\beta$ to $z = -\beta$. From the /24
y, y', y'' and y''' values at $z = -\beta$ the amplitudes of four waves (radiation and gas, incident and reflected) were found. Next the same procedure was repeated where the initial conditions for $z = +\beta$ corresponded to a purely "gas" wave. A linear combination of these two solutions in which the amplitude of the incident gas wave equals zero for $z = -\beta$ is also the solution sought (a radiation wave is incident; waves of both types are reflected and transmitted). To check the accuracy, a repeated calculation was carried out from $z = -\beta$ to β with the initial condition found for $z = -\beta$. The accuracy is usually better than 10^{-5} and only for $\beta \geq 10\lambda_g$ is an error of $\approx 10^{-3}$ accumulated.

The results of calculating amplitudes are shown in Table I for values of the parameters

$$\rho_B/\rho_A = 10^2, \quad v_g^2/v_r^2 = 10^{-2}, \quad 3\omega l_A/c = 10^{-3},$$
$$\omega/\omega_A = 10^{-1}, \quad \omega/\omega_B = 10,$$
$$l/\lambda_g = 0.01, \quad 0.1, \quad 1 \text{ и } 10. \tag{64}$$

It is evident from Table I that, for small values of l/λ_g, the results agree well with Equation (59) that have been obtained for an abrupt jump of opacity (upon comparison it is necessary to mention that the quantities discarded in Equation (59) amount to $\approx 10\%$ in this example). Even in the case of $l = \lambda_g$ one can use the relations for the modulus of the amplitude for an abrupt jump by an order of magnitude (this case is most important for us for later applications). And only in the case of an extended transition zone $l = 10\,\lambda_g$ are the amplitudes of the transmission gas waves smaller than would follow from the simple principle for the transmission of a wave with a minimum damping. Here the point is that the gas waves are formed in a region that is larger than their wave length, they are coherent and, by interfering, they extinguish each other (Ya. B. Zel'dovich turned our attention to this fact).

II. ASTROPHYSICAL APPLICATIONS /25

5. A plane equilibrium atmosphere

Let us now go over to considering the propagation of waves in astrophysical atmospheric objects with $\beta \ll 1$. We shall use the approximation of a plane atmosphere in a constant field of gravity which, in the majority of practical cases, allows one to obtain sufficient accuracy. Equations (10) and (19) through (22) for a static atmosphere assume the form

$$\frac{dz}{dm} = \frac{1}{\rho}, \quad \frac{dH}{dm} = \varkappa_*(B - J), \quad B = J,$$
$$\frac{1}{3}\frac{dJ}{dm} = -\varkappa_* H, \quad \frac{dP_g}{dm} = -g + \frac{4\pi\varkappa_*}{c} H. \tag{65}$$

The solution of the system of Equation (65) for a known steady flux has the form $J = B = \frac{acT^4}{4\pi}$, $T^4 = \frac{3}{4}\frac{\pi H}{ac}\left(1 + \frac{2}{3}\tau\right)$, $T_{ef} = T\left(\tau = \frac{2}{3}\right)$,

$$\rho = \frac{g\mu}{\varkappa_0 RT}\left(1 - \frac{H}{H_c}\right)\tau, \quad \frac{d\tau}{dz} = -\varkappa_0\rho, \quad H_c = \frac{g\mu}{4\pi\varkappa_0}; \quad \tau = \varkappa_0(M-m). \tag{66}$$

For $H \ll H_c$, the solution, Equation (66), is reduced to the well-known solution for a plane atmosphere (see [17]). For the solution, Equation (66), the characteristic frequency ω_1 is, from Equation (52), a function of the optical thickness τ:

$$\omega_1 = \frac{32\pi^2 H \varkappa_0}{3c^2}\left(1 + \frac{3}{2}\tau\right). \tag{67}$$

If a wave is generated at $\tau_0 \gg 1$ and its frequency $\omega < \omega_1(\tau_1)$, then, at $\omega > \omega_1(0)$, there exists such a τ_1 at which $\omega = \omega_1(\tau)$, and, at $\tau < \tau_1$, the nature of the wave's propagation changes. At $\tau \gg 1$ $z < 0$, the solution, Equation (66), has the form

$$J = B = \frac{acT^4}{4\pi}, \quad T^4 = \frac{12\pi H}{ac}\tau = \frac{3}{4}T_{ef}^4\tau = T_0^4\tau, \qquad \text{/26}$$

$$\rho = \left(\frac{4}{3}\right)^{1/4}\frac{g\mu}{\varkappa_0 RT_{ef}}\left(1 - \frac{H}{H_c}\right)\tau^{3/4} = \rho_0\tau^{3/4}, \tag{68}$$

$$z = \frac{1}{\varkappa_0\rho_0}\left[\left(\frac{2}{3}\right)^{1/4} - \tau^{1/4}\right] \quad \text{при} \quad z(2/3) = 0.$$

For $\tau \ll 1$, $z > 0$, from Equation (66) we have:

$$J = B = \frac{acT^4}{4\pi}, \; T^4 = \frac{1}{2}T_{ef}^4 = \frac{8\pi H}{ac}, \; \rho = \left(\frac{3}{2}\right)^{1/4}\rho_0\tau, \tag{69}$$

$$z = -z_0 \ln\frac{3}{2}\tau \text{ for } z\left(\frac{2}{3}\right) = 0, \; z_0 = \left(\frac{2}{3}\right)^{1/4} \cdot 1/\varkappa_0\rho_0.$$

With such a choice of the constants of integration the quantities T, ρ, and z are continuous for a transition from Equation (68) to Equation (69) at the point $\tau = 2/3$.

6. The propagation of waves in a plane atmosphere

The waves in a plane atmosphere are described by Equations (38), (42) and (44). Going over to the variable z instead of m, let us write them down in the form $\rho\left(1 - \frac{i\omega}{c\varkappa_0\rho}\right)h + \frac{1}{3\varkappa_0}\frac{dj}{dz} - \frac{2i\omega}{\varkappa_0 c}H\frac{dy}{dz} = 0,$

$$\frac{i\omega}{c}j - \frac{dh}{dz} + \frac{aT^4}{3\pi}i\omega\frac{dy}{dz} = 0, \tag{70}$$

(I) $$\omega^2 y = -\frac{1}{\rho}\frac{d}{dz}\left(\gamma P_y \frac{dy}{dz}\right) - \frac{4\pi\varkappa_0}{c} h,$$

(II) $$\omega^2 y = -\frac{1}{\rho}\frac{d}{dz}\left(P_y \frac{dy}{dz} - \frac{\pi}{3}\frac{j}{c\rho}\right) - \frac{4\pi\varkappa_0}{c} h.$$

a) The case of a large optical thickness

The equilibrium solution is defined by the system of Equation (68). One can obtain a solution of the system of Equation (70) in a quasi-classical approximation /27

$$y, h, j = \varphi(z)\exp\left(i\int k\,dz\right). \tag{71}$$

Leaving the main terms of the expansion, we obtain, by substituting Equation (71) in Equation (70),

$$\rho\left(1 - \frac{i\omega}{c\varkappa_0\rho}\right)h + \frac{ik}{3\varkappa_0} j + \frac{2k\omega}{\varkappa_0 c} Hy = 0, \tag{72}$$

$$\frac{i\omega}{c} j - ikh - \frac{aT^4}{3\pi} k\omega y = 0,$$

(I) $$\omega^2 y = k^2 \frac{\gamma P_y}{\rho} y - \frac{4\pi\varkappa_0}{c} h, \quad (II)\ \omega y = k^2 \frac{P_y}{\rho} y + \frac{ik\pi}{3}\frac{\rho}{\rho c} j - \frac{4\pi\varkappa_0}{c} h.$$

Let us use an inequality of the type of Inequalities (45)

$$\omega/c\,\varkappa_0\rho_0 \ll 1 \tag{73}$$

with ρ_0 from Equation (68). The system of Equation (72), which differs from the system of Equation (46) describing the propagation of waves in a homogeneous medium, only by the presence of the last term in the first equation. If one uses the inequalities

$$\beta \ll 1 \text{ and } \omega/kc\tau \ll 1, \tag{74}$$

which are assumed as being always fulfilled, then the dispersion equation is reduced to Equation (48) if the quantities v_g, v_r and $l = 1/\varkappa_0\rho$ are considered to be variables defined by the system of Equation (68).

As was noted in paragraph 5, the only type of waves that propagate with low damping in an optically thick medium are low-frequency ($\omega<<\omega_1$) waves that are defined by Equation (53b). The damping of this wave is given by its imaginary part which, in the case of Equation (71), has the form /28

$$\int \mathrm{Im}(k_2)dz = \frac{1}{2}\omega^2 \int \frac{dz}{\omega_1 v_r} = \frac{c\omega^2}{6\varkappa_0}\int \frac{dz}{\rho v_r^3} = \tag{75}$$

$$= -\frac{c^2\omega^2}{16\cdot 8\pi\varkappa_0^2 H}\left(\frac{3c}{\pi H\rho_0}\right)^{1/2}\int \frac{d\tau}{\tau^{15/8}} = \frac{c^2\omega^2}{112\pi\varkappa_0^2 H}\left(\frac{3c}{\pi H\rho_0}\right)^{1/2}\tau^{-7/8} = \mathcal{D}_1\tau^{-7/8}$$

The amplitude of the displacement A, which equals A_0 at $\tau=\tau_0$, has the form

$$A = A_0[f(\tau)/f(\tau_0)]\exp[-\mathcal{D}_1(\tau^{-7/8} - \tau_0^{-7/8})]. \tag{76}$$

To calculate the factor $f(\tau)$ in front of the exponential, one can use the following expansion terms after substituting Equation (71) into Equation (70). However, this requires fairly cumbersome calculations. Therefore, let us find $f(\tau)$ from simple physical considerations. Along with the wave damping, which is contained in the exponent's index, the amplitude of the wave must increase during its propagation outward through decreasing density. This intensification must also be exactly contained in the factor $f(\tau)$. If there were no damping, then the energy flux F transferred by the wave would be constant.

$$F = \rho f^2(\tau)\omega^2 d\omega/dk = \mathrm{const}. \tag{77}$$

Taking ρ from Equation (68) and k from Equation (53b) into acoount, we have

$$F \sim f^2(\tau)\tau^{7/8}, \quad f(\tau) \sim \tau^{-7/16}. \tag{78}$$

From Equations (78) and (76) we obtain the change of the wave's amplitude at $\omega<<\omega_1$ in the form

$$A = A(\tau_0)(\tau_0/\tau)^{7/16}\exp\left[\frac{c^2\omega^2}{112\pi\varkappa_0^2 H}\left(\frac{3c}{\pi H\rho_0}\right)^{1/2}(\tau_0^{-7/8} - \tau^{-7/8})\right], \tag{79}$$

$$\max(1,\tau_1) < \tau < \infty, \quad \omega(\tau_1) = \omega_1.$$

The value of τ_1 up to which Equation (79) is valid, equals, with allowance for Equations (67) and (68): /29

$$\tau_1 = \frac{\omega c^2}{16\pi H \varkappa_0} = \frac{\omega c}{a T_{ef}^4 \varkappa_0} = \frac{4}{9}\frac{c}{v_{rph}}\frac{\omega}{\varkappa_0 \rho_{ph} v_{rph}}, \qquad (80)$$

$$v_{rph}^2 = \frac{4}{9}\frac{a T_{ef}^4}{\rho}, \quad \rho_{ph} = \rho(\tau = 2/3) = (2/3)^{3/4}\rho_0.$$

In the majority of cases we shall consider waves for which $\tau_1 > 1$, then, from Equation (80) it follows that

$$\omega/\varkappa_0 \rho_0 v_{rph} > v_{rph}/c, \qquad (81)$$

which can be fulfilled simultaneously with Inequalities (73) and (74). If $\tau_1 > 1$ then, at $\tau < \tau_1$ the wave changes its character and begins to be determined by Relations (54). Although a gas wave (Equation (54a)) is dampled slightly after one vibration period, for a given length Δz it is, for $\tau = \tau_1$, damped much more strongly than a thermal wave (see Equation (54b)):

$$\mathrm{Im}[k_1(\tau_1)]/\mathrm{Im}[k_2(\tau_1)] = \tfrac{1}{2}(\omega_1/v_g)/(\omega_1/\sqrt{2}\,v_r) = v_r/\sqrt{2}\,v_g \gg 1 \qquad (82)$$

The amplitude of a thermal wave (Equation (54b)) in the quasi-classical approximation of Equation (71) has, using Condition (77) to determine $f(\tau)$ and the solution of Equation (8), the form:

$$F \sim f^2(\tau)\tau^{3/8}, \quad f(\tau) \sim \tau^{-3/16}, \qquad (83)$$

$$A = A(\tau_1)(\tau_1/\tau)^{3/16}\exp\left[\frac{4\sqrt{2}}{5}\left(\frac{\omega}{\varkappa_0\rho_0 c}\right)^{1/2}(\tau^{5/8} - \tau_1^{5/8})\right],$$

$$\max(1, \tau_2) < \tau < \tau_1.$$

The damping for gas and thermal waves in Equation (54) becomes identical for

$$\mathrm{Im}(k_1) = \mathrm{Im}(k_2), \quad \tau = \tau_2 = 2\tau_1 v_g^2/v_r^2 = \tfrac{1}{2}\gamma\beta\tau_1. \qquad (84)$$ /30

If $\tau_2 < 1$ in Equation (84), then the solution Equation (83) is extended to $\tau = 1$ after which one must use a solution for an optically thin region. Otherwise when,

$$\tau_2 > 1, \quad \frac{\omega}{\varkappa_0 \rho \cdot v_{rph}} > \frac{v_{rph}^2}{c v_g^2}, \tag{85}$$

there is a region $1<\tau<\tau_2$ where a gas wave (Equation (54a)) has less damping. Using Equations (54a), (71), (77) and (68), we obtain

$$F(\tau) \sim f^2(\tau)\tau^{3/8}, \quad f(\tau) \sim \tau^{-3/16},$$
$$A = A(\tau_1)(\tau_1/\tau)^{3/16} \exp\left[\frac{4}{3}\pi \frac{H}{\rho c^2}\left(\frac{4}{3}\right)^{1/8}\left(\frac{\mu}{\gamma R T_{ph}}\right)(\tau^{3/8} - \tau_1^{3/8})\right]. \tag{86}$$

b. The case of small optical thickness

In the limit of small optical thickness $\tau<<1$ the solution, Equation (69), of Equation (65) is valid and the system of Equation (70) is greatly simplified. At small τ, jump occurs, since $J=2H$ in Solution (69) and also $T=\text{const}$. One can estimate the terms with derivatives in Equation (70) by introducing the characteristic wavelength λ for a disturbance. By comparing the terms with h and dj/dz in the first of Equation (70) and with j and dh/dz in the second of Equation (70), it is easy to see that the terms with derivatives always predominate if

$$\rho \lambda \varkappa_0 << 1, \quad \omega\lambda/c << 1, \tag{87}$$

which is always valid for $\tau<<1$. In this case, from the first two of Equation (70) with allowance for Equation (69), it follows that: /31

$$j = \frac{\sigma i \omega}{c} H y, \quad h = \frac{i\omega}{3\pi} a T^4 y, \quad h = \frac{1}{3} j, \quad T^4 = \frac{1}{2} T_{ph}^4. \tag{88}$$

The constants of integration in Equation (88) which arise during the solution of Equation (70) have been set equal to zero. They are connected with boundary radiation sources which do not depend on the local y displacements. These sources are assumed to be absent. Substituting Equations (69) and (88) into the third of Equation (70) and taking Equation (69) into account, we shall obtain

$$v^2 y + v_g^2 \frac{d^2 y}{dz^2} - v_g^2 \varkappa_0 \rho_0 \left(\frac{3}{2}\right)^{1/4} \frac{dy}{dz} + i\omega \frac{\varkappa_0}{c} \frac{1}{3} a T^4 y = 0,$$
$$v_g^2 = \gamma R T/\mu. \tag{89}$$

Equation (89) is, with allowance for Inequalities (87), valid for Case (I) at the adiabatic velocity ($\gamma=5/3$) and for Case (II) for an isothermal v_g ($\gamma=1$). Equation (89) has an exact solution $y \alpha \exp(ikz)$. Substituting this form of solution into Equation (89) and introducing the characteristic length from Equation (69), we obtain

$$\omega^2 - k^2 v_g^2 - ikv_g^2/z_0 + i\omega\varkappa_0 \cdot \frac{4}{3} aT^4/c = 0. \tag{90}$$

From this, we have

$$k = -\frac{i}{2z_0} + \left[-\frac{1}{4z_0^2} + \frac{\omega^2}{v_g^2}\left(1 + i\frac{\varkappa_0}{c}\cdot\frac{4}{3}\frac{aT^4}{\omega}\right)\right]^{1/2}. \tag{91}$$

The "+" sign is chosen for waves propagating outwards. If

$$\omega > v_g/2z_0, \tag{92}$$

then the gas waves propagate in an isotropic atmosphere, otherwise the atmosphere vibrates as a whole (see [19]). Formally the propagation of some wave follows from Equation (86) at $\omega \ll v_g/2z_0$, but in this limiting case the second of Inequalities (87) is violated, and /32 therefore, Equation (91) is unsuitable for this case. If the radiative damping is sufficiently small

$$\frac{4}{3}\frac{\varkappa_0}{c}\frac{aT^4}{\omega} = \frac{4}{\gamma}\left(\frac{2}{3}\right)\frac{v_g}{\omega 2z_0}\frac{v_g}{c\beta_0} \ll 1,$$

$$\beta_0 = \frac{3\mathcal{R}}{\mu}\left(\frac{2}{3}\right)^{3/4}\frac{\rho_0}{aT^3} = \text{const} \text{ из (68)}, \tag{93}$$

then, from Equation (91) we have the expansion

$$k = -\frac{i}{2z_0} + \frac{\omega}{v_g}\left(1 + \frac{2}{3}i\frac{\varkappa_0}{c}\frac{aT^4}{\omega}\right). \tag{94}$$

The first term of Equation (94) determines the increase of amplitude in an exponential atmosphere for a constant energy flux. In fulfilling the condition $v_g \ll c\beta_0$ the radiative damping in Equation (94) is always weaker than the intensification, and the formation of a shock wave occurs. The wave amplitude in the case of Equation (94) changes, with allowance for Equation (69), according to the law:

$$A = A(\pi\cdot\tfrac{2}{3})\exp\left[\frac{z}{2z_0}\left(1 - \frac{8}{3\gamma}\frac{v_g}{c\beta_0}\right)\right] = A(\pi\cdot\tfrac{2}{3})\left(\tfrac{3}{2}\pi\right)^{\frac{1}{2}\cdot\frac{4}{3\gamma}\frac{v_g}{c\beta_0}}. \tag{95}$$

c) Calculation of the acoustic energy flux.

Let us find the dependence of the acoustic energy flux F on optical thickness, using relations that have been derived above. Let us introduce the dimensionless parameters

$$\alpha = \frac{\rho_0 c^2}{a T_0^4} = \frac{\rho_0 c^3}{12\pi H}, \quad \sigma = \frac{\omega}{\varkappa_0 \rho_0 c}. \tag{96}$$

We shall adopt from now on the value $\tau=2/3$ as the boundary between the transparent and opaque regions. Using Equations (77) and (79), we have for the region $\tau>\max(2/3, \tau_1)$

$$F(\tau) = F(\tau_0)\exp\left[\frac{9}{4}\sigma^2\alpha^{1/2}(\tau_0^{1/6} - \tau^{1/6})\right]. \tag{97}$$

For τ_1 2/3 in the region $\max(2/3,\tau_2)<\tau<\tau_1$, we have from Equation (83)

$$F(\tau) = F(\tau_1)\exp\left[\frac{8\sqrt{6}}{5}\sigma^{1/2}(\tau^{5/4} - \tau_1^{5/4})\right]. \tag{98}$$ /33

If $\tau_2>2/3$ also, then in the region $2/3<\tau<\tau_2$, using Equation (86), we have

$$F(\tau) = F(\tau_1)\exp\left[\frac{32\sqrt{3}}{27\sqrt{\gamma}}(\alpha\beta)^{-1/2}(\tau^{9/8} - \tau_2^{9/8})\right]. \tag{99}$$

In the transparent region $\tau<2/3$, we have from Equation (95)

$$F(\tau) = F(2/3)\left(\frac{3}{2}\tau\right)^{\frac{2}{3\gamma}}\frac{v_1}{c\beta_0}, \quad v_0 = v_0(\tau\ll 1). \tag{100}$$

As has been shown in Paragraph 5, upon the wave's going over into a region with different parameters and for a change of the type of wave, there is a characteristic quantity which "maintains itself", the amplitude of the wave with the smallest damping. In a plane stellar atmosphere the type of wave changes at the points $\tau=\tau_1$, τ_2 and 2/3. For finding the fraction of acoustic flux which arises at $\tau\gg\tau_1$ and which flows out into the region $\tau<2/3$, we shall use this condition of continuity. As follows from the previous examination, three cases are possible

I) $\tau_1<2/3$.

In this case the solution of Equation (97) immediately goes over into Equation (100), the wave velocity and the energy flux undergo an abrupt jump of $\sim\beta^{1/2}$, and we obtain /34

$$F(2/3) = \left(\frac{3}{4}\gamma\beta_0\right)^{1/2} F(\tau_0)\exp\left[+\frac{9}{4}\sigma^2\alpha^{1/2}\left(\tau_0^{-1/6} - \left(\frac{2}{3}\right)^{1/6}\right)\right]. \tag{101}$$

2) $\tau_1>2/3$, но $\tau_2<2/3$.

In this case, Solution (97) joins with Equation (98) and, moreover, in view of the preservation of the type of wave, the energy flux at $\tau=\tau_1$ is continuous, but at $\tau=2/3$, Equation (98) goes over into Equation (100) with an abrupt jump of the acoustic energy flux of $\sim\beta^{1/2}$. We have /34

$$F(2/3)=\left(\frac{3}{4}\gamma\beta_0\right)^{1/2}F(\tau_0)\exp\left[\frac{9}{7}\sigma^2\alpha^{2/3}\left(\tau_0^{-7/8}-\tau_1^{-7/8}\right)+\right.$$
$$\left.+\frac{8\sqrt{6}}{5}\sigma^{1/2}\left(\left(\frac{2}{3}\right)^{5/8}-\tau_1^{5/8}\right)\right]. \qquad (102)$$

3) $\tau_2>2/3$.

Here a continuous transition from Equation (97) to Equation (98) occurs, then a transition with an abrupt jump of $\sim\beta^{1/2}$ from Equation (98) to Equation (99) and, finally, a continuous transition from Equation (99) to Equation (100). We have

$$F(2/3)=\left(\frac{3\gamma}{4}\beta_0\right)^{1/2}F(\tau_0)\exp\left[\frac{9}{7}\sigma^2\alpha^{2/3}\left(\tau_0^{-7/8}-\tau_1^{-7/8}\right)+\right.$$
$$\left.+\frac{8\sqrt{6}}{5}\sigma^{1/2}\left(\left(\frac{2}{3}\right)^{5/8}-\tau_1^{5/8}\right)+\frac{32\sqrt{3}}{27\sqrt{\gamma}}(\alpha\beta)^{-1/2}\left(\left(\frac{2}{3}\right)^{9/8}-\tau_1^{9/8}\right)\right]. \qquad (103)$$

d) The outflow of waves from a plane atmosphere in the presence of a Corona

Condition (92) is, taking Expressions (68) and (69) for ρ_0 and z_0 into account, written down in the form

$$\omega>\left(\frac{\gamma\mu}{RT}\right)^{1/2}g\left(1-\frac{H}{H_c}\right)=\omega_c. \qquad (104)$$

Here T is the temperature of the gas in a transparent region. As was noted in [8], [10] and [11], for turbulent disks accreting onto a black hole, the presence of an acoustic energy flux and its transformation into heat, and also the heating of the gas by the disk's radiation, which creates a non-conservative radiative force, lead to

the formation of a corona in which the temperature T_c is considerably higher than the temperature $T=2^{-1/4}T_{ef}$ which is present in an equilibrium isothermal atmosphere. Coronas also apparently exist around very massive stars. The presence of a corona will lead to the weakening of Criterion (104) for wave transmission, which formally will be reduced to increases of T and v_g. This will lead to a broader band of outflowing waves and to a stronger acoustical heating. A seeming "shining through" of the star's atmosphere occurs upon the formation of a corona. The increase of the corona's temperature is limited by the fact that the intrinsic energy losses of the coronal gas, which are determined by Bremsstrahlung and inverse Compton radiation for a radiative flux of photons from the photosphere, become significant. A magnetic field can play an important role in the process of forming a corona (see [12]). /35

The transition zone from the photosphere from $\tau=2/3$ to the corona plays an important role in determining the fraction of the acoustic energy flux which flows outward.

If the thickness of the transition zone d_c is much less than the characteristic wavelength $\lambda_c=2\pi/k_c$, $k_c=$ /v_g with ω_c from Criterion (104) then one can evidently replace T by T_c in Criterion (104). In this case the band of frequencies increases for outflowing waves and the fraction of the outflowing acoustic flux increases. If $d_c >>$ $_c$, then the presence of a corona must not affect the frequency of the outflowing waves and one can use the equilibrium parameters of the gas* everywhere. In reality d can be $\sim\lambda_c$; therefore, we shall use the equations that have been obtained above with the equilibrium temperature T of the atmosphere for all estimates, but let us take into account the fact that the frequency of a transmitted wave can be of the same order or even somewhat less than $\omega_c(T)$ from Inequality (104).

* With a corona present, waves with $\omega_c(T_c)<\omega<\omega_c(T)$ flow out into the corona but their energy flux near the base of the corona is reduced by the factor $\exp(4\pi d_c/\lambda_c)$.

7. Numerical results

Let us calculate what fraction of the acoustic flux generated at large optical depths can flow out into the transparent atmosphere and heat it. The location of an acoustic wave's transition into a shock wave depends on the initial amplitude. We shall consider the value of the optical thickness for the transition as the free parameter τ_{out}. The calculation has been carried out for two different situations in which the role of radiative pressure is great: 1) an accretion disk, and 2) the atmosphere of a very massive star. /36

In the case of an accretion disk $R \sim 10 r_g$, where r_g is the Schwarzschild radius of a black hole, was considered as the region of maximum energy release. The mass of the black hole was taken as equal to $10 M_\odot$. The turbulence parameter $\alpha_t \approx 0.1$. Specifying the parameter $\dot{m}=\dot{M}/\dot{M}_{cr}$ determines all the properties of the disk if a model of the vertical structure has been chosen. We considered two variants of the vertical structure; the adiabatic structure that has been obtained in [8] with a polytropic index n=3 and the n=1 structure obtained in [9]. The results for different frequencies are presented in Table 2. In this table the limiting frequency ω_c from Inequality (104) is taken at the equilibrium atmospheric temperature without allowing for the existence of a corona. Therefore, in reality, waves with ω slightly lower than ω_c can also be transmitted.

The results of similar calculations for very massive stars (see [2]) that are considered as models for quasars are shown in Table 3.

From the results it is evident that the crude estimate of the fraction of the outflowing flux ($\sim\beta=P_g/P_r$) that has been used in [8] is actually valid in order of magnitude for frequencies that are typical for a disk (with $\lambda \sim z_o$). The high frequencies rapidly decay exponentially. This fact isolates the characteristic frequency (more precisely, the band of frequencies) among the waves which heat the

corona of the disk. Let us repeat the factors which lead to such an isolation: 1) an isolated region in the disk with a maximum temperature and maximum energy release; 2) low frequencies do not pass through into the corona (besides their generation is also impeded); and 3), the high frequencies decay rapidly.

8. Comparison with observational data /37

As follows from the previous examination, each object (a disk, or a very massive star) possesses a characteristic frequency that is determined in Inequality (104). Waves generated by convection with a frequency close to ω_c flow out into the atmosphere, disturb it, and can lead to the observed fluctuations of brightness. It is of interest to compare the characteristic frequencies resulting from theory with the observed times for the fluctuations of brightness for those objects in which one can suppose accretion disks or very massive stars.

a) The Cygnus X-1 X-ray source

This source, in which the presence of a black hole and accretion disk is supposed, is strongly fluctuating in the entire range of periods from 10^{-3} to 10^{-4} sec (see [22]). As follows from Table 2, the characteristic fluctuation period caused by the outflow of acoustic waves into the atmosphere amounts to from 5 to 10 millisecs. The detection of a quasi-period t≈10 millisec in the x-ray radiation of Cygnus X-1 (see [22]) may be associated with the outflow of waves into the atmosphere above a convective disk. In [22] the existence of a quasi-period of ∿10 millisec is associated with the revolution of hot spots around a black hole (see [23]). One can indicate several observational differences between these two mechanisms for the fluctuations. During the revolution of a hot spot which spirals in toward the black hole, the period of fluctuation in each series must decrease. At the same time the characteristic frequency which depends on the star's mass, does not change as a function of luminosity. If the same fluctuations are caused by convection, then the period in a

given series must be approximately constant, but upon an increase of luminosity, the characteristic period increases approximately proportionally to the luminosity, as follows from Table 2. Let us also notice the difference in the spectral variation of the two mechanisms: the spectrum in a pulse does not change during a revolution, but as a result of convection the hardest spectrum must be at /38
maximum brightness. However, an increase of the spectrum's hardness must be noticeable only on the shortest time scales (<10 millisecs). For longer intervals the connection of the spectrum with luminosity is considerably more complex, since different regions of the disk, the flares in which can be uncorrelated, give contributions to different parts of the spectrum. Therefore, the absence of a simple connection of the spectrum with luminosity that has been noted in [24] cannot serve as an argument against explaining the spectrum of Cygnus X-1 by a Compton process. Let us notice that the mechanism we have considered can give fluctuations that are weakly correlated in time and can imitate the white noise that has been obtained from an analysis of observations of the brightness fluctuations for Cygnus X-1 (see [24] and [25]).

b) The nuclei of active galaxies and quasars

The nature of quasars and of the compact nuclei of galaxies has been unclear up to now but the physical processes that have been considered in this paper occur in at least two existing models: the disks around very massive black holes (see [3]) and very large stars (see [2]). It is also of interest here to compare the observed properties of variability with the predictions of the model. Rapid fluctuations of luminosity with a quasi-period of $\sim$100 days are observed in the nucleus of the Seyfert galaxy NGC 4151 ($\sim$130 days see [26]) and in the object OJ 287 ($\sim$184 days), which is a BL Lacertae type object (see [27]). These quasi-periods agree well with the characteristic periods of the frequencies in a model for a very massive star of $\sim 10^8 M_\odot$ (see Table 3). According to the data of Table 3, one can estimate that, in a model of an accretion disk around a black hole, for a hole mass of $\sim 10^8 M_\odot$ these periods correspond to a luminosity

$L=0.1\ L_c$. As is especially noticed by observers (see, for example, [27]), "Phase change (which is evidently sharp) with preservation of the period is a feature of the periodicity of the rapid component in the nuclei of Seyfert galaxies". This property agrees well with /39 the convective-wave nature of rapid fluctuations.

A tendency for an increase of the quasi-period for the fluctuations with luminosity, an approximate constancy of the period in each given series of observations, and also an increase of the hardness of the spectrum at maximum brightness are the general properties of the convective-wave mechanism for brightness as fluctuations both in a model for a very large mass star and also in a model for a turbulent convective disk around a very massive black hole.

However, the role of radiation pressure is very great and the damping of acoustic waves is very strong in both these models. It is clear from Table 3 that in this case the outflowing acoustical flux amounts to no more than $\sim$1% of the flux generated at a large optical depth, which is significantly less than the observed amplitude of variability. It appears to us that the transfer of energy by other types of waves (for example, by magneto-acoustic waves) will also be strongly impeded in this case, since damping of them by radiative friction will occur effectively. This difficulty becomes especially important in a model of a very massive star, where there the mechanisms of variability that are specific for an accretion disk are not present (also see our paper [28]).

In conclusion, let us quote the main results of the paper.

1. Equations have been obtained which take into account v/c terms and which describe the propagation of plane linear waves in a medium with high radiative pressure, where the radiative transfer is described by the Eddington approximation.

2. The damping and transformation of waves have been investigated in model problems (an abrupt jump of opacity and a zone of variable opacity).

3. The outflow is considered of waves into the atmosphere and corona of an accretion disk and of a very massive star.

4. The characteristic times of variability have been obtained for the Cygnus X-1 source and for the active nuclei of galaxies and of quasars and the difficulties of the model for a very massive star in the latter case have been indicated. /40

The authors are grateful to Ya. B. Zel'dovich for important remarks. We thank Ch. Kunash and M. M. Basko for making available the material of [21].

TABLE 1 /41

The complex amplitudes of waves which were formed in the transition region with variable opacity. The amplitude of an incident radiation wave is $A_0=1$, the increase of a photon path $\ell_B/\ell_A = 10^2$, and the velocity ratio of the gas wave to the radiation wave is $v_g/v_r=0.1$. The frequency of the wave ω is selected such that $3\omega\ell_A/c = 10^{-3}$, $\omega/\omega_A=0.1$, and $\omega/\omega_B=10$. The parameter ξ is the half-width of the transition zone (see Equation (62)). A_1 and A_2 are the amplitudes of the reflected waves at the point $z=+\beta$, and B_1 and B_2 are those of the transmitted waves at the point $z=-b$. The case $b/\lambda_3 = 0$ is an analytical calculation of the abrupt jump (see Equation (59)), and the remaining variants are by numerical calculation.

b/λ_3	A_1	A_2	B_1	B_2
0	-0.247 + 0.247 i	0.553 - 0.447 i	1.105 + 0 i	0.045 + 0.155 i
0.01	-0.248 + 0.256 i	0.478 - 0.230 i	1.179 - 0.096 i	0.052 + 0.139 i
0.1	-0.250 + 0.229 i	0.478 - 0.234 i	1.169 + 0.091 i	0.051 + 0.139 i
1.0	-0.186 + 0.026 i	0.494 - 0.203 i	-0.027 + 1.022 i	0.040 + 0.138 i
10	-0.0052- 0.0012 i	0.316 + 0.171 i	-0.056 + 0.022 i	-0.049 + 0.083 i

TABLE 2

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The outflowing acoustical flux F in an accretion disk around a black hole of $M=10M_{\odot}$ for an outflow optical thickness τ_{out} (meaning: outflow optical thickness) for different values of the parameters $\dot{m}=\dot{M}/\dot{M}_{cr}$, $=P_g/P_r$, wavelengths λ relative to the half-thickness of the disk z_0 with corresponding values of the period t, and the oscillation frequencies ω in fractions of the limiting frequency ω_c obtained without allowance for the heating of the atmosphere.

$\dot{m}$	Model	β	λ/z_0	ω/ω_c	t, millisec	τ_1	τ_2	F/F$_0$		
								$\tau_{out}=0.5$	$\tau_{out}=0.3$	$\tau_{out}=0.1$
0.1	[9]	0.1	1/2	2.2	5.0	11	1.6	0.087	0.086	0.081
			1/4	4.4	2.5	22	3.3	0.019	0.019	0.018
	[8]	0.27	1/2	0.8	5.1	11	4.5	0.23	0.22	0.20
			1/4	2.4	1.7	33	13	0.045	0.044	0.043
0.3	[9]	0.01	1	4.2	10	3.7	0.06	0.036	0.029	0.017
			1/2	8.4	5	1.9	0.03	0.023	0.018	0.010
	[8]	0.032	1/2	2.7	5	3.8	0.18	0.040	0.037	0.030

TABLE 3

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The outflowing acoustical flux in a very massive star as a quasar model for different radii R (in units of the Schwarzschild radius r_g). The notations are the same as in Table 2.

$M/M_\odot$	β	R/r_g	γ	t sec	τ_1	τ_2	F/F_0		
							$\tau_{out}=0.3$	$\tau_{out}=0.1$	$\tau_{out}=0.01$
10^6	$8.5\cdot10^{-3}$	100	2	$1.0\cdot10^4$	4.7	0.06	$5.8\cdot10^{-2}$	$4.7\cdot10^{-2}$	$3.8\cdot10^{-2}$
			4	$5.0\cdot10^3$	9.4	0.12	$3.6\cdot10^{-2}$	$3.2\cdot10^{-2}$	$2.6\cdot10^{-2}$
		10	2	$1.9\cdot10^2$	2.7	0.034	$4.5\cdot10^{-2}$	$3.7\cdot10^{-2}$	$2.5\cdot10^{-2}$
			4	$9.7\cdot10^1$	5.4	0.068	$4.1\cdot10^{-2}$	$3.3\cdot10^{-2}$	$2.2\cdot10^{-2}$
10^8	$8.5\cdot10^{-4}$	100	2	$6.8\cdot10^6$	1.15	$1.5\cdot10^{-3}$	$1.2\cdot10^{-2}$	$6.0\cdot10^{-3}$	$1.5\cdot10^{-3}$
			4	$3.4\cdot10^6$	2.3	$2.9\cdot10^{-3}$	$1.2\cdot10^{-2}$	$5.7\cdot10^{-3}$	$1.4\cdot10^{-3}$
		10	2	$1.2\cdot10^5$	0.47	$0.6\cdot10^{-3}$	$8.7\cdot10^{-3}$	$2.7\cdot10^{-3}$	$3.0\cdot10^{-4}$
			4	$6.1\cdot10^4$	0.94	$1.2\cdot10^{-3}$	$8.7\cdot10^{-3}$	$2.7\cdot10^{-3}$	$3.0\cdot10^{-4}$

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