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ALGORITHMS FOR LOCATING CELESTIAL SOURCES  
OF  
X-RAYS AND GAMMA FLARES  
WITH THE AID OF SEVERAL SPACECRAFT

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An algorithm is suggested for defining the coordinates of X-ray and Gamma ray radiation sources by measuring the time lag of the transmission of radiation flares from various points of space in which spacecraft are located. Instances where the 2-x, 3-x and 4x spacecraft are used.

## I. The Problem of Localization

A sudden and intermittent increase in the intensity of x-ray or gamma radiation which is detected by spacecraft is commonly referred to as x-ray or gamma-ray flares [1]. Flares are observed comparatively seldom. Therefore, when registering them in  $n$  various points (which we will call reference points)  $M_i, i = \overline{0, n-1}$ , one may subsequently identify "temporal spectra" of  $S_i$  (i.e. the dependence of the radiations intensity on time), which pertain to one and the same flare. Using statistical methods considered in [2], the size of the time lag  $\tau_k, k = \overline{1, n-1}$  of the flare's transmission in points  $M_k$  (as compared to point  $M_0$ , which is taken for the original), may be determined by these spectra.

The rates of time lags are used for the localization of a flare's source, i.e. for determining three coordinates of the source  $\bar{r}_\gamma$  or only of the direction towards the source  $\bar{\ell}_\gamma$ , if it is presumed that the source may be considered infinitely remote. Together with  $\bar{r}_\gamma$  and  $\bar{\ell}_\gamma$  it is also necessary to determine the statistical characteristics of their errors  $\Delta \bar{r}_\gamma$  and  $\Delta \bar{\ell}_\gamma$ . The methods of locating sources are similar to the methods of hyperbolic navigation (Loran's system) [3]. In view of the fact that the solution to the problem of localization may not turn out to be simple, it is advisable to use additional information on the orientation of the axes of the detectors which register the flares. If the diagrams of the detectors' directionality are not spherical, then the information may be essential for uncovering the reason that the solution is not simple.

Thus it is supposed that when locating the sources the following initial data is used:

- a) the rate of time lags  $\tau_k, k = \overline{1, n-1}$  and the covariance matrix of their error  $K_\tau$  which is determined by the spectra  $S_i$  ( $i = \overline{0, n-1}$ ) [2], and also the moment of Moscow (or Greenwich) time  $t_0$  of the flare's trans-

emission in the point  $M_0$  (since the flare is not impulsive then  $t_0$  corresponds to some representative point of the spectrum  $S_0$ , its origin, for example);

- b) the radii-vectors  $\bar{r}_i$  of the points  $M_i$ , ( $i = 0, n-1$ ) and the covariance matrixes  $K_{r_i}$  of their errors  $\Delta \bar{r}_i$  at the moments  $t_i = t_0 + \tau_i$  ( $\tau_0$  will be made to equal zero) in some system of coordinants OXYZ;
- c) the single vectors of direction of the axes of the diagrams of the detectors' directionalities  $\bar{d}_i$  in points  $M_i$  at the moments  $t_i$  in the same system of coordinants OXYZ, and also the angles of semi-solution of the diagrams of directionalities  $\beta_i$ ,  $i = 0, n-1$ .

## 2. Basic Correlations

The quantities  $\tau_k$  will be considered, when it is convenient, as constitutes of the vector  $\bar{\tau}$ . All incidental vectors  $\Delta \bar{\tau}$  and  $\Delta \bar{r}_i$  will be considered uncorrelated between themselves, and their universal means as equal to zero:

$$\begin{aligned} E(\Delta \bar{\tau}) &= \bar{0}, & E(\Delta \bar{r}_i) &= \bar{0}, & i &= \overline{0, n-1} \\ E(\Delta \bar{\tau} \cdot \Delta \bar{r}_i^T) &= \bar{0}, & E(\Delta \bar{r}_i \cdot \Delta \bar{r}_j^T) &= \bar{0}, & i \neq j, & i, j = \overline{0, n-1} \end{aligned} \quad (1)$$

where "T" denotes transportation, and "E" denotes the operation of the universal mean.

We will transfer the origin of the system OXYZ to point  $M_0$  and designable the system  $o\xi\eta\zeta$ . The radii-vectors  $\bar{\rho}_k$  of point  $M_k$  in the system  $o\xi\eta\zeta$  will equal

$$\bar{\rho}_k = \bar{r}_k - \bar{r}_0, \quad k = \overline{1, n-1}, \quad (2)$$

and for the covariance matrixes  $K_{\rho_{kk}}$  of their errors  $\Delta \bar{\rho}_k$  in accordance with (1), one may write

$$K_{\rho_{kk}} = K_{r_k} + K_{r_0}, \quad k = \overline{1, n-1}. \quad (3)$$

It should be noted that the errors  $\Delta \bar{\rho}_k$ , as opposed to the errors  $\Delta \bar{r}_i$ , are correlated, since

$$K_{\rho_{kl}} = E(\Delta \bar{\rho}_k \cdot \Delta \bar{\rho}_l^T) = K_{r_0}. \quad (4)$$

The single vector  $\bar{\ell}_\gamma$  of direction towards an infinite remote source will be exactly the same in both systems, and the radius-vector of a source which is located in a finite distance will equal

$$\bar{\rho}_\gamma = \bar{r}_\gamma - \bar{r}_0.$$

We will suppose that the velocity of radiation propagation  $C$  is constant and the wave of radiation for infinitely remote sources is flat, and for a source located in a finite distance it is spherical.

Then, in the case of an infinitely remote source the quantities  $\bar{\tau}$ ,  $\bar{\rho}_k$  and  $\bar{\ell}_\gamma$  will be bound by the correlations

$$I \quad \begin{cases} R^T \bar{\ell}_\gamma + C \bar{\tau} = 0, & (5) \end{cases}$$

$$\begin{cases} |\bar{\ell}_\gamma| = 1, & (6) \end{cases}$$

$$\begin{cases} \bar{\ell}_\gamma^T \bar{d}_i \geq \cos \beta_i, \quad i = \overline{1, n-1}, & (7) \end{cases}$$

where  $R$  is the matrix whose columns are  $\bar{\rho}_k$ ,  $k = \overline{1, n-1}$ .

In the case of a source within a finite distance, the quantities  $\tau_k$ ,  $\bar{\rho}_k$  and  $\bar{\rho}_\gamma$  will be bound by the correlations

$$II \quad \begin{cases} |\bar{\rho}_k - \bar{\rho}_\gamma| - |\bar{\rho}_\gamma| = c \tau_k, \quad k = \overline{1, n-1}, & (8) \\ \bar{\rho}_\gamma^T \bar{d}_i \geq \rho_\gamma \cos \beta_i, \quad i = \overline{0, n-1}. & (9) \end{cases}$$

Algorithms of the definition  $\bar{\ell}_y$  and  $\bar{r}_y$ , with the help of systems I and II, respectively, will be considered for the cases of  $n = 2, 3, 4$ . In each of these cases we will suppose that

$$\text{rank}(D) = n - 1 \quad (10)$$

where

$$D = R^T R \quad (11)$$

If the conditions (10) is not fulfilled, then it is necessary to exclude one of the points  $M_k$  and consider the problem with smaller numbers of reference points.

It should be noted that when  $n > 2$  the solutions of systems I and II can depend on the choice of the original reference point

$M_0$  of  $M_i$ . We will consider, for example, the case where  $n = 3$ . Let  $\tau_{ij}$  be the rate of time lag of the transmission of a flare in point  $M_i$ , as compared to the point  $M_j$ , which is determined as a result of evaluating the spectra  $S_i$  and  $S_j$ . Depending on the choice of the initial point three sets of rates of time lags may be obtained:  $\tau_{10}$  and  $\tau_{20}$ ; and  $\tau_{01}, \tau_{02}$  and  $\tau_{12}$ . Since for any method of evaluating spectra the condition  $\tau_{ij} = -\tau_{ji}$ , (I2) should be fulfilled, only three quantities of  $\tau_{ij}$  can be independent. For example  $\tau_{10}, \tau_{02}$  and  $\tau_{21}$ , which for convenience we will indicate as  $\tau^{(1)}, \tau^{(2)}$  and  $\tau^{(3)}$ , respectively (if this is compared to quantities introduced earlier, then  $\tau_1 = \tau^{(1)}, \tau_2 = -\tau^{(2)}$ ) If the condition

$$\alpha \equiv \tau^{(1)} + \tau^{(2)} + \tau^{(3)} = 0, \quad (13)$$

is fulfilled, then, as is easily seen, the solutions of systems I and II do not depend on the choice of the original point. The condition (13) will be fulfilled, evidently, in the case that the spectra  $S_0, S_1$  and  $S_2$  are identical. Usually condition (13) is not fulfilled and then the difference of the quantity  $\alpha$  from zero will characterize the accuracy of the definition of  $\tau^{(k)}$ . Another way of using condition (13) is by bringing it into the interpretation of spectra with the definition of  $\tau^{(k)}$ . In this case all spectra

will be processed together. Specifically, if  $\tau^{(k)}$  is defined as the points of the extremum of several functions  $\varphi_k(\tau)$ , i.e. if

$$\frac{d\varphi_k(\tau)}{d\tau}|_{\tau \neq \tau_k} = 0, \quad b_k \equiv \frac{d^2\varphi_k(\tau)}{d\tau^2}|_{\tau = \tau_k} > 0 \text{ (or } < 0), \quad (14)$$

then the condition (13) may be considered by introducing amendments into the quantities  $\tau^{(k)}$ . It may be shown that the quantities  $\hat{\tau}^{(k)}$ , which guarantee the attainment of the conditional extremum of functions  $\Psi_k(\tau)$  (i.e. by calculating condition (13)), in a linear approach will equal

$$\hat{\tau}^{(k)} = \tau^{(k)} - \alpha / b_k (b_1^{-1} + b_2^{-1} + b_3^{-1}), \quad k = 1, 2, 3. \quad (15)$$

If all  $b_k$  are equal amongst themselves, then

$$\hat{\tau}^{(k)} = \tau^{(k)} - \alpha / 3, \quad k = 1, 2, 3. \quad (16)$$

These equations may be applied if  $b_k$  are not known.

### 3. The Case of Two.

The solution to system I will be the vector  $\bar{\ell}_y$ , which equals

$$\bar{\ell}_y = (\bar{\rho}_1 / \rho_1) \cos \alpha + (\bar{\kappa} / \kappa) \sin \alpha, \quad (17)$$

where

$$\cos \alpha = -c\tau_1 / \rho_1, \quad 0 \leq \alpha \leq \pi, \quad (18)$$

and  $\bar{\kappa}$  is the random vector, orthogonal  $\bar{\rho}_1$ :

$$\bar{\kappa}^T \bar{\rho}_1 = 0. \quad (19)$$

In the celestial sphere on an assemble of solutions of  $\bar{\ell}_y$  form the circumference of radius  $\alpha$  with the center at point  $\bar{\rho}_1 / \rho_1$ .



Errors of measurements  $\Delta \tau_1$  and  $\Delta \bar{\rho}_1$ , lead to the error  $\Delta \bar{\ell}$  of the definition of  $\bar{\ell}_y$ , which is directed along the normal to this circumference at point  $\bar{\ell}_y$ . The root-mean-square declination  $\sigma_\ell$  of the error  $\Delta \bar{\ell}$  depends upon the assumed quantity  $\bar{\ell}_y$  and equals [4]:

$$\sigma_\ell = \sigma_s / (\rho_1 \sin \alpha), \quad (20)$$

where

$$\sigma_s = (c^2 K_\tau + \bar{\ell}_y^T K_{\rho_{11}} \bar{\ell}_y)^{1/2}. \quad (21)$$

Since the dimension of the matrix  $K_\tau$  in this case equals I,  $K_\tau$  is simply the dispersion of error  $\Delta \tau_1$ .

The assemble of solutions of system II forms a surface which is the hyperboloid of the rotation whose focus lies in points  $M_0$  and  $M_1$ . The semi-major axis of the rotation equals  $|c\tau_1|/2$ . When  $\tau_1 > 0$  the solutions are the points of that cavity of the hyperboloid which encompasses the focus  $M_1$  and when  $\tau_1 < 0$  the solutions are the points of that cavity which encompass the focus  $M_0$ . If  $\tau_1 = 0$ , then the hyperboloid degenerates into a plane which is normal to  $\bar{\rho}_1$  and transient through its center. In consequence of the indeterminacy of the solution of system II, it is more expedient to solve the problem, which is reversed with respect to the problem of localization: the verification of the correspondence to system II of the coordinants of some cosmological objects which could be flare sources.

If  $\tilde{\rho}_y$  is the radius-vector of such a "suspicious" object, then a rate of time lag  $\tilde{\tau}_1$ , which corresponds to this object, can be determined:

$$\tilde{\tau}_k = (|\rho_k - \tilde{\rho}_y| - |\tilde{\rho}_y|) / c, k = 1. \quad (22)$$

The hypothesis that a region of space is a "suspected" flare source can be assumed by fulfilling the condition

$$|\tau_1 - \tilde{\tau}_1| / \sigma_\tau \leq K_\delta, \quad (23)$$

where  $K_\delta$  depends upon the assumed probability of  $\delta$  to repudiate the correct hypothesis (the level of significance). If the law of distribution of the error  $\Delta \tau_1$ , is normal, then the quantity  $K_\delta$  will be determined by the equation

$$\text{erf}(K_\delta/\sqrt{2}) = 1 - \delta. \quad (24)$$

#### 4. The Case of Three

In correspondence [ 4 ] with the solution of system I, vector  $\bar{\ell}_y$ , will equal

$$\bar{\ell}_y = ([\bar{n} \times \bar{\rho}_2] c\tau_1 + [\bar{\rho}_1 \times \bar{n}] c\tau_2 \pm \bar{n} \sin \varphi) / \kappa^2, \quad (25)$$

where

$$\bar{n} = \bar{\rho}_1 \times \bar{\rho}_2, \quad (26)$$

$$\cos \varphi = (c\bar{\tau}^T D^{-1} c\bar{\tau})^{1/2}, \quad 0 \leq \varphi \leq \pi/2. \quad (27)$$

The expression (27) defines the size of the angle  $\varphi$  between the directions towards the source and the reference plane which passes through points  $M_0$ ,  $M_1$  and  $M_2$ . The condition  $|\cos \varphi| \leq 1$  defines the region of possible values of  $\bar{\tau}$ . The region is limited by the ellipse

$$c\bar{\tau}^T D^{-1} c\bar{\tau} = 1, \quad (28)$$

which is written into the square  $|c\tau_1| \leq \rho_1$  and  $|c\tau_2| \leq \rho_2$ . On the border of the region (28) there exists a single solution  $\bar{\ell}_y$ , which lies in the reference plane. Within this area there exist two solutions of  $\bar{\ell}_y$ , which are symmetrical relative to the reference plane. For uncovering the non-simplicity it is necessary to appropriate, in this case, condition (7).

If  $\bar{\ell}_y$  does not lie in the reference plane, i.e.

$$\bar{\ell}_y^T \bar{x} \neq 0, \quad (29)$$

then the covariance matrix  $K_\ell$  of the error  $\Delta \bar{\ell}$  is determined in the following manner [4]:

$$K_\ell = S (c^2 K_\tau + K) S^T, \quad (30)$$

where  $S$  is the matrix of the dimension  $3 \times 2$ , whose columns are equal to:

$$\bar{S}_1 = [\bar{\ell}_y \times \bar{\rho}_2] / \bar{\ell}^T \bar{x}, \quad \bar{S}_2 = [\bar{\rho}_1 \times \bar{\ell}_y] / \bar{\ell}^T \bar{x}, \quad (31)$$

and  $K$  is the matrix of dimension  $2 \times 2$ , whose elements equal:

$$[K]_{kl} = \bar{\ell}_y^T K_{\rho_{kl}} \bar{\ell}_y, \quad k, l = 1, 2, \quad (32)$$

where the covariance matrixes  $K_{\rho_{kl}}$  are determined in accordance with (3) and (4).

At least one natural value of the matrix  $K_\ell$  is equal to zero. The corresponding natural vector is collinear to  $\bar{\ell}_y$ . Thus,  $K_\ell$  defines the dispersion of  $\Delta \bar{\ell}$  in the plane which is tangent to the celestial sphere at point  $\bar{\ell}_y$ . If  $\bar{\ell}_y$  lies in the reference plane, that is

$$\bar{\ell}_y^T \bar{x} = 0, \quad (33)$$

then the product of the error  $\Delta \bar{\ell}$ , which is orthogonal to the reference plane, turns out to be indeterminate. The root-mean-square declination of the projections of error  $\Delta \bar{\ell}$  on the reference plane can be determined in the same manner as in the case where  $n = 2$  (20).

For a source located within a finite distance, system II determines an assemble of possible solutions which form a line of

of intersection of two hyperboloids of the rotation. As in the case where  $n = 2$ , in this instance it is most expedient to solve the reversed problem, determining in accordance with (22) the quantities  $\bar{\tau}_1$  and  $\bar{\tau}_2$  for the coordinants of the supposed sources of flares and checking condition (23) for each quantity  $\bar{\tau}_k$ .

### 5. The Case of Four

When measurement errors are lacking the solution of system II would be the vector

$$\bar{S} = -R^{T^{-1}} c \bar{\tau} \quad (34)$$

The influence of the measurement errors leads to the failure of condition (6). Therefore, in the capacity of  $\bar{\ell}_y$  the normalized vector can be used:

$$\bar{\ell}_y = \bar{S} / S \quad (35)$$

However, it is more convenient to use the more general algorithm, which determines the solution of system II, and in a particular instance the solution of system I also. In accordance with the algorithm [4] the distance to a source  $\rho_y$  is determined from the quadratic equation

$$|\mu_y \bar{P} + \bar{S}| = 1, \quad (36)$$

$$\mu_y = 1/\rho_y, \quad (37)$$

$$\bar{P} = R^{T^{-1}} \bar{q}, \quad (38)$$

and the constituents  $q_k$  vector  $\bar{q}$  equal (39)

$$q_k = (\rho_k^2 - c^2 \tau_k^2) / 2, \quad k = 1, 2, 3.$$

Equation (36) has a single positive solution  $\mu_y > 0$  when  $S < I$ , i.e., if vector  $c\bar{\tau}$  lies within the ellipsoid

$$c\bar{\tau}^T D^{-1} c\bar{\tau} = 1. \quad (40)$$

When  $S=I$ , i.e. on the verification of the ellipsoid, there exists two solutions. One of them corresponds to an infinitely remote source ( $\mu_y = 0$ ), and the other corresponds to a source located at a distance

$$\rho_y = -P^2 / 2\bar{P}^T \bar{S}. \quad (41)$$

When  $S=I$  both solutions (36) may be positive. Each solution of  $\mu_y$  determines the direction towards a source at point  $M_0$  and equals

$$\bar{\ell}_0 = \mu_y \bar{P} + \bar{S}. \quad (42)$$

The accuracy of determining the distance to a source depends on the position of the source relative to the reference points. For the case where the distance to a source significantly exceeds the distance between reference points, a simple expression for the root-mean-square declination  $\sigma_p$  of the error of determining the distance can be obtained:

$$\begin{aligned} \sigma_p &\leq \sigma_s \rho_y^2 / \rho_{\min}^2, \\ \sigma_s &= 2 \left( \sigma_{\tau_{\max}}^2 + 2 \sigma_{r_{\max}}^2 \right)^{1/2}, \end{aligned} \quad (43)$$

where  $\rho_{\min}$  is the least of the distance  $\rho_1, \rho_2, \rho_3$ ;  $\sigma_{\tau_{\max}}^2$  is maximum natural value of the matrix  $K_{\tau}$ ; and  $\sigma_{r_{\max}}^2$  is the greatest of the maximum natural values of the matrixes  $K_{r_i}$ .

If it is demanded that  $\sigma_p$  be  $N$  times less than the determined distance  $\rho_y$ , then an expression for the greatest distance to sources which can be determined with the fulfillment of the following condition can be obtained:

$$\rho_{y \max} \begin{matrix} (\text{парсек}) \\ [\text{parsec}] \end{matrix} \leq \rho_{\min} (A.E) / N \cdot \Delta, \quad (44)$$

where  $\Delta$  is the relationship  $\sigma_s / \rho_{min}$ , expressed in angular seconds.

## 6. Conclusion

For the localization of infinitely remote sources it is sufficient to register a flare at three spacecrafts. The bivalency arising in this case can be eliminated with the use of directed detectors. However it is impossible to distinguish a flat-wave from a spherical. Therefore, the analysis of registrations of flares from nearby sources will determine the direction, which can differ considerably from the direction towards the true source. Thus, when using three  $sc$  it is necessary to verify (the reverse problem) the possibility of flare radiation from "suspicious" areas of space. The identification of wave form and the subsequent determination of the direction towards an infinitely remote source or coordinates of nearby sources are possible only with the use of four  $sc$ . In all cases of localization there occurs the degeneration of the measurement system when the source and any two reference points are located on one straight line. During the working of the problem it is necessary to exclude one of these points and conduct the localization using the remaining points.

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