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RECIPROCITY PRINCIPLE IN DUCT ACOUSTICS

Young-chung Cho
Lewis Research Center
Cleveland, Ohio

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RECIPROCITY PRINCIPLE IN DUCT ACOUSTICS

by Young-chung Cho
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

Various reciprocity relations in duct acoustics have been derived on the l of the spatial reciprocity principle implied in Green's functions for linear waves. The derivation includes the reciprocity relations between mode conversion coefficients for reflection and transmission in nonuniform ducts, and the relation between the radiation of a mode from an arbitrarily terminated duct and the absorption of an externally incident plane wave by the duct. Such relations are well defined as long as the systems remain linear, regardless of acoustic properties of duct nonuniformities which cause the mode conversions.

LIST OF SYMBOLS

A, B	uniform duct elements (Fig. 1)
$c(\vec{x})$	ambient sound speed
$f_j(\theta, \varphi)$	radiation directivity function of j duct mode (far-field measurement at spherical angle (θ, φ))
$h_\ell^{(1)}(kr)$	spherical Hankel function of the first kind
i	$\sqrt{-1}$
$j_\ell(kr)$	spherical Bessel function
k	$\lvert\vec{k}\rvert$
k_m, k_μ	axial wave numbers of m and μ modes
$\vec{k}$	wave (or propagation) vector in free space
$\hat{k}$	$\vec{k}/k$

$\hat{n}$	unit normal vector outward from a surface
$q_{j,\ell m}$	constant matrix element
R_{mn}	mode conversion coefficient (first subscript is mode number of incident wave; second subscript is mode number of reflected wave)
r	radial coordinate variable in a spherical coordinate system
S	surface of integration (Figs. 1 and 6)
S_A, S_B	cross-sectional surfaces of duct elements A and B
$S_{\ell m}(\alpha, \gamma)$	scattering matrix element for plane wave with incident angle (α, γ) and spherical wave (ℓm) scattered by the duct
S_W	surface made of duct wall
$T_{m\mu}$	mode conversion coefficient for transmission (first subscript is mode number of incident wave; second subscript is mode number of transmitted wave)
V	volume
x	duct axial coordinate variable
$\vec{x}$	spatial coordinate variables $((x_1, x_2, x_3)$ in Cartesian coordinate system; (r, θ, φ) in spherical coordinate system)
$\vec{x}_s, \vec{x}_s'$	spatial coordinate variables on a surface
$Y_{\ell m}$	spherical harmonics (asterisk superscript denotes complex conjugate)
α	polar angle of $-\vec{k}$
$\beta(\vec{x}_s, \vec{x}_s')$	complex function
$\beta_o(\vec{x}_s)$	admittance of locally reacting boundary

γ	azimuthal angle of $-\vec{k}$
$\delta(\vec{x}_s, \vec{x}_s')$	surface Dirac delta function
θ, θ'	polar angles in spherical coordinate system
$\rho(\vec{x}_s)$	mean density of air
$\sigma_n(\alpha, \gamma)$	amplitude of j duct mode induced by the external plane wave with incident angle (α, γ)
Φ_m, Φ_n	normalized eigenfunctions
φ, φ'	azimuthal angles in spherical coordinate system
Ψ	acoustical velocity potential function (subscript A or B is used to indicate duct element if necessary)
ω	angular frequency of wave

Subscripts:

A, B a, b	uniform duct elements A, B
j, ℓ, m, n, μ, ν, τ	mode numbers (each represents, collectively, a pair of integers for a mode of a three-dimensional duct)

Superscripts:

a, b	uniform duct elements A, B

INTRODUCTION

Modal analysis is a frequently used method of studying duct acoustics. The major merit of this method is that a mode remains undistorted in a uniform duct section. However, in the application of modal analysis to realistic systems, problems are often encountered. Some of the difficulties are due to duct nonuniformities such as duct termination, variation of duct cross section and wall impedance, alteration of duct axis (duct bending), axial gradients of mean fluid variables, etc. The complexities can be reduced by virtue of basic physics laws such as the reciprocity principle. Furthermore, such a principle helps one to gain physical insights into the problems, often without detailed analyses. This paper discusses in some detail the reciprocity principle in duct acoustics.

The reciprocity principle implies a symmetry property between a source and the excited field in systems of linear vibration.[1] The symmetry property is well elucidated in Green's functions for waves.[2] The reciprocity principle applies to a variety of systems of linear vibration, including those which are bounded, unbounded, or partially bounded. A ducted acoustics system may be considered as one which is partially bounded. Because a physics law should not depend on representations which are chosen for description of the system, the reciprocity principle, which is well established in spatial coordinates,[1-3] should be equally well describable in eigenfunction spaces. Note that eigenfunctions in a duct are in one-to-one correspondence with the duct modes.

Some limited discussions of the reciprocity principle with the duct mode representation are given in a number of previously reported works. Levine and Schwinger briefly discussed the reciprocity relation between the radiation directivity of the fundamental mode and the absorption of a plane wave by the duct.[4] The reciprocity relation between mode conversion coefficients for reflection of the propagating duct modes was used by Weinstein for discussion of the Wiener-Hopf solutions of radiation problems from hard-walled uniform circular and two-dimensional wave guides.[5] In his numerical studies of sound transmission through two-dimensional duct bends, Tam found that the mode conversion coefficients satisfy the reciprocity relation.[6] Eversman discussed the reciprocity principle for sound reflection and transmission in a two-dimensional hard-walled duct with partially variable cross section.[7] Koch found that the reciprocity relation is also satisfied by numerical results of the mode conversion coefficients for reflection and transmission in partially lined rectangular ducts of uniform cross section.[8] A more fundamental view of the reciprocity principle in duct acoustics was offered by Möhring.[9] His approach is not, however, applicable to a lossy system like a lined duct, because the scattering matrix fails to be unitary in such a system.

As noted in Ref. 1, Chapter V, the reciprocity principle applies to linear vibration systems involving dissipative force fields as well as conserved force fields if the dissipative forces are linear functions of the particle velocity. A wide range of dissipative forces belong to this type, including those pertinent to lossy boundaries involved in acoustic liners of ducts.[10] The reciprocity principle also applies to systems involving an inhomogeneous medium,[3] inclusive of gradients of mean density, pressure, and temperature, as long as the systems remain linear.

In this paper, the reciprocity relations are derived for mode conversions caused by the duct nonuniformities which have been mentioned thus far. The derivation is based on the reciprocity principle implied in Green's functions for waves, and includes the reciprocity relation between mode conversion coefficients for reflection and transmission in nonuniform ducts and for reflection due to an arbitrary duct termination. Also included is the reciprocity relation between the radiation directivity of a duct mode and the excitation of the mode from the absorption of an externally incident plane wave by the duct.

RECIPROCITY RELATIONS IN NONUNIFORM DUCTS

In this section, we consider sound reflection and transmission in a nonuniform duct as schematically illustrated in Fig. 1. The duct is composed of two uniform duct elements A and B, which are connected through a nonuniform transition section. The two uniform elements may differ from each other in cross-sectional shape and area, duct axis, wall impedance, and mean fluid variables. Also shown in the figure are the surfaces S_A, S_B, and S_W, which are, respectively, the cross sections of duct elements A and B, and the duct wall.

On the assumption of harmonic time dependence $e^{-i\omega t}$, the wave equation is, in the absence of mean flow, reduced to

$$\nabla^2\Psi(\vec{x}) + \left[\frac{\omega}{c(\vec{x})}\right]^2 \Psi(\vec{x}) = 0 \tag{1}$$

where the sound speed $c(\vec{x})$ may vary due to a spatial gradient of the mean fluid variables. The analysis includes boundary conditions on the duct walls which can be given by

$$\hat{n} \cdot \vec{\nabla}\Psi(\vec{x}_s) + \int_{S_W} \beta(\vec{x}_s, \vec{x}'_s)\Psi(\vec{x}'_s)d^2x'_s = 0 \tag{2}$$

where β is a complex function of $\vec{x}_s$ and $\vec{x}'_s$. This relation is a general form of homogeneous boundary condition. Note that boundary conditions for eigenvalue problems are always homogeneous.[2] This equation can be used to account for duct walls of extended reaction as well as for locally reacting walls. For the latter, β is given by

$$\beta(\vec{x}_s, \vec{x}'_s) = -i\omega\rho(\vec{x}'_s)\beta_o(\vec{x}'_s)\delta(\vec{x}_s, \vec{x}'_s) \tag{3}$$

where β_o is the wall admittance, and $\delta(\vec{x}_s, \vec{x}'_s)$ is the Dirac delta function.

Reciprocity for Transmission

We first derive the reciprocity relation for sound transmission in the duct. We use a method similar to one which is used for the proof of the spatial reciprocity principle in Ref. 3. Consider two different incident waves. One is composed of a single mode m generated in the duct element A (see Fig. 2), and the other of a single mode μ generated in the duct element B (see Fig. 3). Both incident waves propagate towards the nonuniform section.

The incident wave from A, upon arrival at the nonuniform section, is partly reflected back to A and partly transmitted to B, as schematically illustrated in Fig. 2. The wave will be attenuated unless the duct wall is hard. The resultant wave, denoted by Ψ_A, can be written in the uniform duct elements as follows:

At A,

$$\Psi_A = \underbrace{\Phi_m^a e^{ik_m^a x_a}}_{\text{Incident}} + \underbrace{\sum_n R_{mn}^a \Phi_n^a e^{-ik_n^a x_a}}_{\text{Reflected}} \tag{4a}$$

At B,

$$\Psi_A = \underbrace{\sum_\tau T_{m\tau}^a \Phi_\tau^b e^{ik_\tau^b x_b}}_{\substack{\text{Transmitted from} \\ \text{A to B}}} \tag{4b}$$

Similarly, the resultant wave Ψ_B due to the incident wave from B (see Fig. 3) can be written as follows:

At B,

$$\Psi_B = \underbrace{\Phi_\mu^b e^{-ik_\mu^b x_b}}_{\text{Incident}} + \underbrace{\sum_\nu R_{\mu\nu}^b \Phi_\nu^b e^{ik_\nu^b x_b}}_{\text{Reflected}} \tag{5a}$$

At A,

$$\Psi_B = \underbrace{\sum_\ell T_{\mu\ell}^b \Phi_\ell^b e^{-ik_\ell^a x_a}}_{\substack{\text{Transmitted from} \\ \text{B to A}}} \tag{5b}$$

Note that mode numbers m, n, and ℓ are used for duct element A; and μ, ν, and τ are used for element B. Each mode number represents, collectively, a pair of integers for a mode of a three-dimensional duct. The sub- or superscript a or b is used on Φ, k, x, and R to indicate the corresponding duct element, A or B. For the transmission coefficient T, the superscript refers to the duct element in which the incident wave is generated. It should also be noted that the two sets of eigenfunctions, $\left\{\Phi_m^a\right\}$ and $\left\{\Phi_\mu^b\right\}$, can differ from each other.

Let us consider the volume integral

$$I = \int_A \left(\Psi_A \nabla^2 \Psi_B - \Psi_B \nabla^2 \Psi_A\right) dx^3 \tag{6}$$

where V is the volume enclosed by the surfaces S_A, S_B, and S_W. It follows from Eq. (1) that the integrand is identically zero everywhere. Thus, the integral is equal to zero. By virtue of Green's theorem, the integration is transformed into

$$\int_{S_A, S_B, S_W} \left(\Psi_A \vec{\nabla} \Psi_B - \Psi_B \vec{\nabla} \Psi_A\right) \cdot \hat{n}\, dS = 0 \tag{7}$$

It is convenient to carry out the integration separately on the surfaces S_A, S_B, and S_W. It can be readily shown by using Eq. (2) that the integration over S_W vanishes. The integration on S_A is written as

$$I_A = -\int_{S_A} \left(\Psi_A \frac{\partial \Psi_B}{\partial x_a} - \Psi_B \frac{\partial \Psi_A}{\partial x_a}\right) dS \tag{8}$$

Substituting Eqs. (4) and (5) into Eq. (8) and utilizing the orthonormality of eigenfunctions yields

$$I_A = -2i T_{\mu m}^b k_m^a \tag{9}$$

Similarly, the integration on S_B results in

$$I_B = 2i T_{m\mu}^a k_\mu^b \tag{10}$$

It follows from Eqs. (9), (10), and (7) that

$$T_{m\mu}^a k_\mu^b = T_{\mu m}^b k_m^a \tag{11}$$

This equation specifies the reciprocity relation between the mode conversion coefficients for transmission, or simply, the transmission coefficients. Recall that the transmission coefficient $T^a_{m\mu}$ is the amplitude of mode μ of duct element B, which is induced by the incidence of mode m of duct element A. On the other hand, $T^b_{\mu m}$ is the amplitude of mode m of duct element A, induced by the incidence of mode μ of duct element B. Once the value of $T^a_{m\mu}$ is computed, then the value of $T^b_{\mu m}$ is determined (or vice versa) from Eq. (11).

Reciprocity for Reflection

For the reciprocity relation for reflection, we consider two waves, Ψ_A and Ψ'_A. Wave Ψ'_A is the resultant wave due to the incidence of a new single mode j ($\neq$m) of duct element A. It can be written in the uniform duct elements as follows:

At A,

$$\Psi'_A = \underbrace{\Phi^a_j \, e^{ik^a_j x_a}}_{\text{Incident}} + \underbrace{\sum_n R^a_{jn} \Phi^a_n \, e^{-ik^a_n x_a}}_{\text{Reflected}} \tag{12a}$$

At B,

$$\Psi'_A = \underbrace{\sum_\tau T^a_{j\tau} \Phi^b_\tau \, e^{ik^b_\tau x_b}}_{\substack{\text{Transmitted from} \\ \text{A to B}}} \tag{12b}$$

where $j \neq m$. We then have, in the place of Eq. (7),

$$\int_{S_A, S_B, S_W} \left(\Psi_A \vec{\nabla} \Psi'_A - \Psi'_A \vec{\nabla} \Psi_A \right) \cdot \hat{n} \, dS = 0 \tag{13}$$

The integral over S_W again vanishes. With the substitution of Eqs. (4) and (12) into Eq. (13), the integration on S_A and S_B is easily carried out. The contribution from the integration over S_B is also zero, and the integration over S_A gives

$$I_A = 2i\left(R^a_{mj}k^a_j - R^a_{jm}k^a_m\right) \tag{14}$$

It follows from Eqs. (13) and (14) that

$$R^a_{mj}k^a_j = R^a_{jm}k^a_m \tag{15}$$

This is the reciprocity relation for reflection. Note that this reciprocity relation is valid for reflection due to any type of duct nonuniformity if the linearity requirement is fulfilled, that is, if Eqs. (1) and (2) are valid, respectively, in the media and on the boundaries. This can be readily proven from the fact that in the derivation of Eq. (15), the integrations on S_B and S_W make no contributions to the integral in Eq. (13).

It should be mentioned that the reciprocity relations derived here are valid for the whole frequency range. In other words, Eqs. (11) and (15) are valid for real, imaginary, or complex values of axial wave number.

RECIPROCITY RELATION IN RADIATION PROBLEM

In this section, we derive the reciprocity relation between the radiation directivity of a duct mode and the excitation of the mode due to the absorption of an externally incident plane wave. The geometry of the problem is schematically illustrated in Figs. 4 and 5.

We consider again two incident waves. One is composed of the j mode of the uniform duct element A, which is propagating toward the duct termination. This wave is partly reflected back to duct element A, and is partly radiated. It will be also partly attenuated if the duct wall is soft. The resultant wave, denoted by Ψ_A, can be expressed as follows (see Fig. 4):

In duct element A,

$$\Psi_A = \underbrace{\Phi_j\, e^{ik_j x}}_{\substack{\text{Incident}\\ \text{in duct}}} + \underbrace{\sum_n R_{jn}\Phi_n\, e^{-ik_n x}}_{\text{Reflected in duct}} \tag{16a}$$

where the super- or subscript a has been dropped from the variables without the loss of clarity. The radiated wave can be written in terms of the spherical Hankel function of the first kind and the spherical harmonic, the product of which will be referred to as outgoing spherical wave functions:

$$\Psi_A = \underbrace{\sum_{\ell m} q_{j,\ell m} h_\ell^{(1)}(kr) Y_{\ell m}(\theta, \varphi)}_{\text{Radiated from duct}} \tag{16b}$$

Here, the radiation is into the three-dimensional space; ℓ and m are single integer numbers; and $q_{j,\ell m}$ is a constant matrix element.

In the far-field region, Eq. (16b) can be written as

$$\Psi_A \xrightarrow{r \to \infty} \frac{e^{ikr}}{r} f_j(\theta, \varphi) \tag{17}$$

where

$$f_j(\theta, \varphi) = \frac{-i}{k} \sum_{\ell m} (-i)^\ell q_{j,\ell m} Y_{\ell m}(\theta, \varphi) \tag{18}$$

It follows from Eq. (17) that $f_j(\theta, \varphi)$ completely determines the radiation directivity of the duct mode. It will be herein referred to as the radiation directivity function.

The other incident wave under consideration is an externally incident plane wave $e^{i\vec{k}\cdot\vec{x}}$. Since the present problem deals with a constant sound frequency, the magnitude of the wave (or propagation) vector is predetermined by $k = (\omega/c)$. Thus, this incident wave is specified primarily by the direction of its propagation.

$\hat{k}$. We will use the incident spherical angle (α, γ), which is in one-to-one correspondence with $\hat{k}$.

The externally incident wave is partly scattered by the duct and partly transmitted into the duct, as shown in Fig. 5. The scattered wave can be expressed in terms of the outgoing spherical wave functions. The resultant wave outside the duct is then written as

$$\Psi_B = \underbrace{\frac{1}{\sqrt{2\pi}} e^{i\vec{k}\cdot\vec{x}}}_{\text{Incident}} + \underbrace{\sum_{\ell m} S_{\ell m}(\alpha, \gamma) h_\ell^{(1)}(kr) Y_{\ell m}(\theta, \varphi)}_{\text{Scattered}} \tag{19a}$$

Here, the constant of the first term is the normalization factor of the plane wave in the sense of the Dirac delta, and $S_{\ell m}(\alpha, \gamma)$ is the scattering matrix element. The wave transmitted into the duct element A is written as

$$\Psi_B = \underbrace{\sum_n \sigma_n(\alpha, \gamma)\, \Phi_n\, e^{-ik_n x}}_{\substack{\text{Transmitted into} \\ \text{duct A}}} \tag{19b}$$

where $\sigma_n(\alpha, \gamma)$ is a constant matrix element.

We now consider the integral

$$\int_S \left(\Psi_A \vec{\nabla}\Psi_B - \Psi_B \vec{\nabla}\Psi_A\right) \cdot \hat{n}\, dS = 0 \tag{20}$$

where the surface S is shown in Fig. 6. This integral vanishes as long as Eq. (1) is valid throughout the medium enclosed by S. The contribution from the integration on the inner and outer walls of the duct is zero if the wall boundary condition can be given as in Eq. (2). Consequently, we have

$$I_A + I_B = 0 \tag{21}$$

where

$$I_A = -\int_{S_A} \left(\Psi_A \frac{\partial \Psi_B}{\partial x} - \Psi_B \frac{\partial \Psi_A}{\partial x} \right) dS \tag{22}$$

$$I_B = \lim_{r \to \infty} \int \left(\Psi_A \frac{\partial \Psi_B}{\partial r} - \Psi_B \frac{\partial \Psi_A}{\partial r} \right) r^2 \sin\theta \, d\theta \, d\varphi \tag{23}$$

Inserting Eqs. (16) and (19) into Eqs. (22) and (23) and utilizing Eqs. (18) and (21) yields

$$2\pi f_j(\alpha, \gamma) = ik_j \sigma_j(\alpha, \gamma) \tag{24}$$

Here, in evaluating the integral I_B, we have used the various function properties given in Appendix A.

Equation (24) specifies the reciprocal relation between the radiation directivity of a duct mode and the amplitude of that mode induced by external plane waves. Note that $\sigma_j(\alpha, \gamma)$ is the amplitude of the j mode of the duct, which is induced by an external plane wave incident to the duct termination with the incident angle (α, γ). On the other hand, $f_j(\alpha, \gamma)$ is the amplitude of the far-field measured at the spherical angle (α, γ) for the radiation of the j mode. If the radiation directivity of a duct mode is known, the amplitude of the mode induced by an external plane wave is computed by this equation. Or conversely, the radiation directivity of a duct mode can be indirectly determined by measuring the amplitude of the mode induced by external plane waves of various incident angles.

CONCLUDING REMARKS

Various reciprocity relations with modal analysis of duct acoustics have been derived. The existence of such relations are solely due to the linear characteristic of the vibration systems under consideration. The derivation is general and is valid regardless of the complexities of the duct nonuniformities involved. The reciprocity relations are useful in studies of sound propagation in nonuniform ducts and radiation from a duct. For instance, the numerical computation involved in such studies can be reduced by means of the reciprocity relation. For a radiation problem, the far-field directivity can be studied indirectly by measuring the transmission of external plane waves into the duct. Finally, it is always advantageous for studies of a system to have a rule which is not affected by complications of the system.

APPENDIX A

EQUATIONS PERTAINING TO SPHERICAL WAVES

Listed here are the various equations used in deriving Eq. (24).

The orthonormality of the spherical harmonics:

$$\int_0^{\pi}\int_0^{2\pi} Y_{\ell m}(\theta,\varphi)Y^*_{\ell' m'}(\theta,\varphi)\ \sin\theta\ d\theta\ d\varphi = \delta_{\ell\ell'}\,\delta_{mm'} \tag{A1}$$

The completeness of the spherical harmonics:

$$\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} Y_{\ell m}(\theta,\varphi)Y^*_{\ell m}(\theta',\varphi') = \delta(\cos\theta - \cos\theta')\delta(\varphi - \varphi') \tag{A2}$$

The relation between the plane wave and the spherical harmonics:

$$e^{i\vec{k}\cdot\vec{x}} = 4\pi\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} Y_{\ell m}(\alpha',\gamma')Y_{\ell m}(\theta,\varphi)(i)^{\ell} j_{\ell}(kr) \tag{A3}$$

where the vectors are given in terms of spherical coordinate variables as (see Fig. 5)

$$\vec{x} = (r,\theta,\varphi)$$

$$\vec{k} = (k,\alpha',\gamma')$$

and $j_\ell(kr)$ is the spherical Bessel function.

Also used is

$$Y_{\ell m}(\theta,\varphi) = (-1)^{\ell}Y_{\ell m}(\pi-\theta,\ \pi+\varphi) \tag{A4}$$

Note that the direction corresponding to the spherical angle $(\pi-\theta,\ \pi+\varphi)$ is the reverse of the direction corresponding to the spherical angle (θ,φ).

The asymptotic form of the spherical Hankel function:

$$h_\ell^{(1)}(kr) \xrightarrow{kr \to \infty} (-i)^{\ell+1} \frac{e^{ikr}}{kr} \tag{A5}$$

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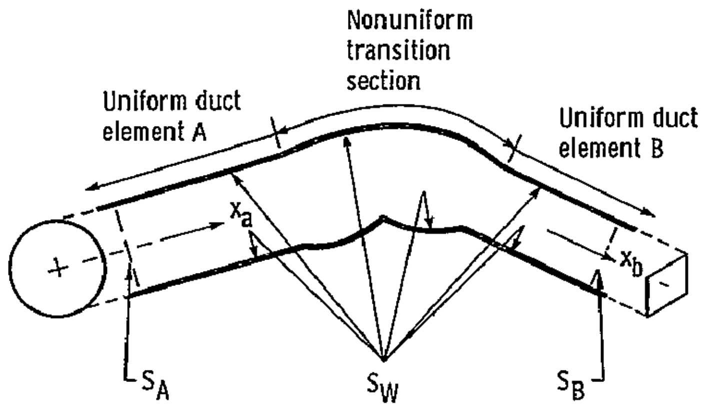

Figure 1. - Schematic of nonuniform duct.

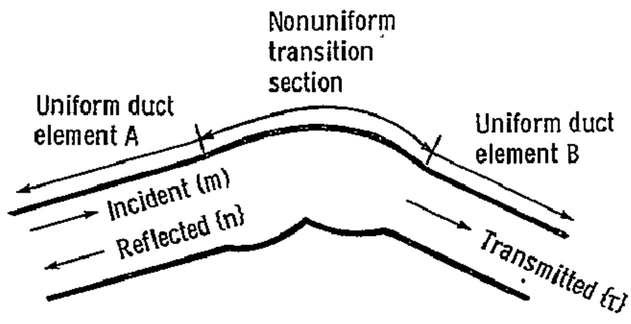

Figure 2. - Schematic representation of wave function Ψ_A given in Eqs. (4a) and (4b).

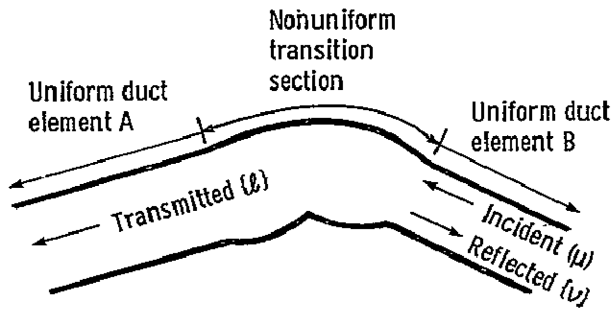

Figure 3. - Schematic representation of wave function ψ_B given in Eqs. (5a) and (5b).

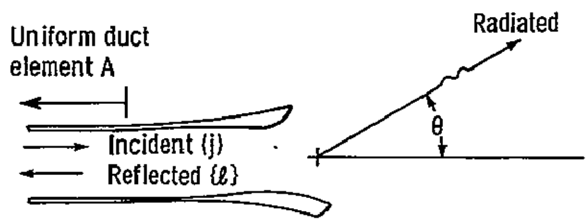

Figure 4. - Sound radiation from an arbitrarily terminated duct.

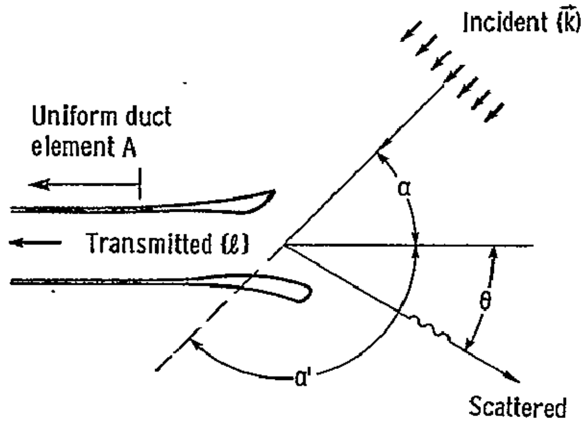

Figure 5. - Scattering and absorption of an external plane wave by a duct.

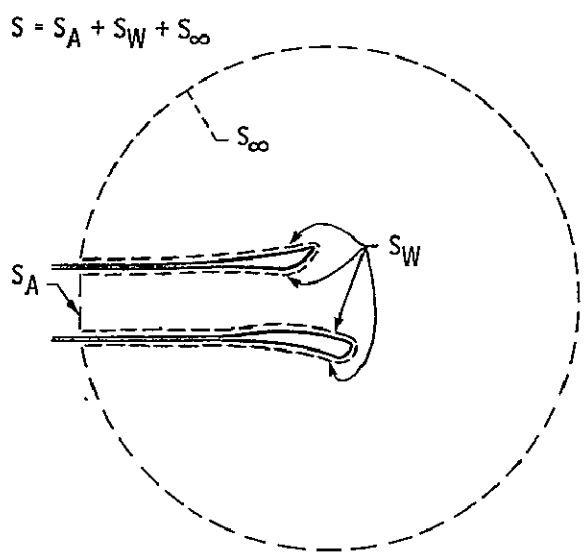

Figure 6. - Surface of integration, S, in Eq. (20).

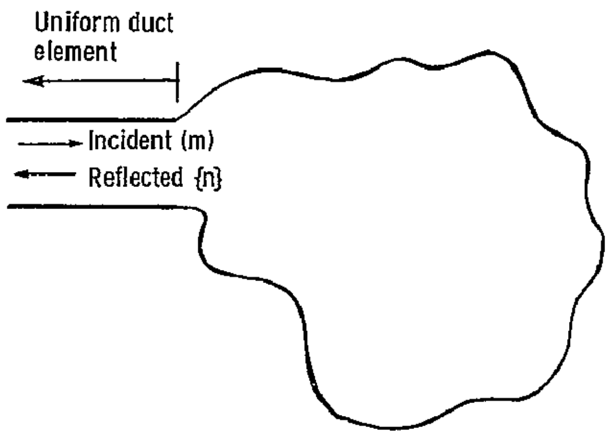

Figure 7. - Example case of sound reflection due to duct nonuniformity.

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